Security Design in a Production Economy with Flexible Information Acquisition*

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Abstract

We synthesize debt and non-debt securities in a production economy under a security design problem. We highlight the investor's ability to acquire costly information (not known to the entrepreneur) on the entrepreneur's project and then to screen it through financing decisions. This leads to a new informational friction: real production depends on information acquisition, but they are performed separately by the entrepreneur and the investor. Debt is optimal when the dependence is weak (i.e., the friction is not severe), and convertible preferred stock (a combination of debt and equity) is optimal when the dependence is strong (i.e., the friction is severe). The optimality of the two securities in different circumstances constructs new pecking orders for financing private businesses, which are consistent with empirical facts. Flexible information acquisition characterizes securities' information sensitivity and the investor's attention allocation in screening in a state-contingent way. It helps capture the nature of screening and synthesize the two securities. It also enables us to work with arbitrary feasible securities over continuous states without distributional assumptions.

Keywords: security design, production economy, flexible information acquisition.

JEL: D82, D86, G24, G32, L26

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1 Introduction

Both debt and non-debt securities are commonly viewed as optimal financing approaches in different real-world corporate finance contexts, but it is usually hard to synthesize their optimality in different circumstances under a simple unified framework. We achieve this goal. The landscape of security design research is that a penniless entrepreneur with a potential project proposes specific contracts to an investor to get finance. The entrepreneur is often modeled as an expert who is more informed about the project. Yet, this prevailing approach misses a crucial aspect. In financing entrepreneurial production, some investors are more capable of assessing projects' uncertain market prospects drawing upon their industry experience, and thus screening projects through information acquisition and the ensuing financing decisions. For instance, start-ups look for venture capital for finance, and most venture capitalists are former founders of successful start-ups, so they can better evaluate whether new technologies match the market.[1] As suggested by Tirole (2006), a deficiency of the classical corporate finance literature is that such an information advantage of investors is overlooked.[2] Our paper fills the gap by uncovering the interaction between investors' security design and entrepreneurs' endogenous information acquisition and screening.[3] It enables us to synthesize debt and non-debt securities and to construct new pecking orders[4] well suited for financing private businesses, which are consistent with empirical evidence.

The investor's endogenous information advantage over the entrepreneur[5] leads to a new informational friction in a production setting. Specifically, in our model, the investor can acquire costly but flexible information about the project's uncertain cash flow before making a financing decision. Only when the investor believes the project is good enough, can it be financed. Hence, the entrepreneur's real production depends on the investor's information acquisition, but they are separately performed, which constructs the friction at the heart of our model.

[1]A famous example features Peter Thiel, the first outside investor of Facebook. Thiel himself was the former founder of Paypal. In August 2004, Thiel made a $500,000 investment in Facebook in the form of convertible notes.

[2]Some exceptions are surveyed by Bond, Edmans and Goldstein (2012). However, most of those papers focus on the role of competitive financial markets in soliciting or aggregating the information of investors or speculators (for instance, Boot and Thakor, 1993, Fulghieri and Lukin, 2001, Axelson, 2007, Garmaise, 2007, Hennessy, 2013, on security design) rather than the role of screening by individual investors. In reality, most firms are private and do not have easy access to a competitive financial market. A burgeoning security design literature highlights individual investors' endogenous information advantage directly (Dang, Gorton and Holmstrom, 2011, Yang, 2013), but these papers are pertaining to the asset-backed securities market as an exchange economy and not fit for the corporate finance setting with production.

[3]In our model, the terms information acquisition and screening mean the same thing. Henceforth, we use them interchangeably in verbal discussion to ease understanding. As our model does not feature entrepreneurs' private information, our notion of screening is however different from the notion of separating (different types of entrepreneurs) commonly used in the literature.

[4]Our notion of pecking order speaks to the entrepreneur's preference of one optimal security over the other in different economic circumstances. It implies orders of optimal securities over the dimensions of certain parameters. It is more general than the classical concept in Myers and Majluf (1984), featuring the dimension of financing cost.

[5]We do not attempt to deny that entrepreneurs in reality may have private information about their technologies, which has been discussed extensively in past literature. Rather, we highlight the overlooked fact that investors may acquire information and be more informed about the potential match between new technologies and the market. Consequently, this also implies that our model does not feature any signaling mechanism.

Facing this friction, the entrepreneur designs a security that incentivizes the investor to acquire information in favor of the entrepreneur. Two conflicting forces arise from the friction. On the one hand, the entrepreneur wants to compensate the investor more, because doing this induces the investor to acquire information and to screen the project more effectively, leading to a higher social surplus. This first force comes from the dependence of real production and information acquisition. The second force, on the other hand, is from their separation: as the entrepreneur shares the social surplus with the investor, she also wants to retain more. The optimal security reduces the friction by reconciling the two forces.

We predict and synthesize two optimal securities: standard debt and convertible preferred stock. The optimality of them in different circumstances constructs new pecking orders for financing entrepreneurial production. Our predictions help bridge the security design literature and the classical pecking order theory, because our pecking orders come from an optimization over a very general feasible security space, as opposed to monotone securities or even a given set of securities like debt and equity. Moreover, in solving for optimal securities, we do not have restrictions on the priors or information structures. The cash flow is built over continuous states and it admits arbitrary distributions, as opposed to finite states or continuous states with given distributional assumptions often seen in previous literature. Hence, our predictions are sharp and robust.

When the dependence of real production on information acquisition is weak, namely, the friction is not severe, the optimal security is debt that does not induce information acquisition. This case corresponds to scenarios where the project's ex-ante market prospects are already good enough or the screening cost is high. This prediction is consistent with the evidence that conventional start-ups and mature private businesses rely heavily on plain-vanilla debt finance from investors who are not good at screening like families, friends, and banks (see Berger and Udell, 1998, Kerr and Nanda, 2009, Robb and Robinson, 2014). The intuition is clear: since the benefit of screening does not justify its cost, the entrepreneur finds it optimal to deter information acquisition by issuing debt, the least information-sensitive security. The investor thus makes the investment decision based on the prior. Interestingly, our intuition for the optimality of debt is different from the conventional wisdom, as our mechanism does not feature adverse selection.[6]

In contrast, when the dependence of real production on information acquisition is strong, namely, the friction is severe, the optimal security is participating convertible preferred stock (could be viewed as a combination of debt and equity) that induces the investor to acquire information. This case corresponds to scenarios where the project's ex-ante market prospects are not good enough or the cost of screening is low. This prediction is new to the security

[6]Notable results regarding debt as the least information-sensitive security to mitigate adverse selection include Myers and Majluf (1984), Gorton and Pennacchi (1990), DeMarzo and Duffie (1999), Dang, Gorton and Holmstrom (2011), Yang (2013). None of those papers considers production.

design literature, but also fits empirical facts closely (Sahlman, 1990, Gompers, 1999). Especially, convertible preferred stocks have been used in almost all the contracts between entrepreneurs and venture investors, and nearly half of them are participating, as documented in Kaplan and Stromberg (2003). Participating convertible preferred stock is in particular popular for earlier rounds of investment (Kaplan and Stromberg, 2003), when the friction is more severe.

The optimality of convertible preferred stock is more subtle. Intuitively, when screening takes place, the investor offers information and money, both of which need to be compensated properly. First, screening makes sense to the entrepreneur only if the investor screens in a potentially good project or screens out a potentially bad project.[7] That is, any project with a higher ex-post cash flow should have a better chance to be financed ex-ante, due to screening. Hence, the entrepreneur is willing to compensate the investor to produce such information, when its benefit exceeds its cost. Specifically, the entrepreneur designs a security with generous and increasing payments over states, encouraging the investor to acquire adequate and proper information to differentiate any different states. Second, the residual of the security is also (weakly) increasing, for the entrepreneur to retain as much as possible. These two points above justify the equity part, in exchange for the investor's information. Moreover, the investor's information after screening is still imperfect, albeit with a perhaps better posterior. Thus, the face value of the convertible preferred stock needs to be higher than the investment requirement, ensuring that the investor would not reject the project. This finally justifies the debt part, in exchange for the investor's money. These intuitions further suggest that straight or leveraged equity alone is not optimal for financing new projects, consistent with the reality (Kaplan and Stromberg, 2003, Lerner, Leamon and Hardymon, 2012).

A new concept, flexible information acquisition, characterizes the above nature of screening and helps establish the predictions on the payment structure, visually, the "shape," of securities. Securities with different shapes incentivize the investor to screen the project with different intensity, and also to allocate attention to different states of the cash flow. For instance, debt is less likely to induce screening due to its flatter shape than that of equity. Moreover, in screening, a debt holder only allocates her attention to states with low cash flows, as the payments are constant over states with high cash flows so that it does not help to differentiate these states. In contrast, a levered equity holder pays attention to states with high cash flows as she benefits from the upside payments. An arbitrary security determines the investor's incentives of screening and attention allocation through this state-contingent way accordingly, and they in turn affect the entrepreneur's incentives to design the security. In characterizing these incentives, the traditional approach of exogenous information asymmetry is inadequate. Recent models of endogenous information acquisition also fail to capture such flexibility of incentives, since they only consider the amount

[7] Our model features continuous state, but we use the notions of good and bad at times to help develop intuitions.

or precision of information (see Veldkamp, 2011, for a review).[8] Our approach following Yang (2012, 2013) takes rational inattention (Sims, 2003, Woodford, 2008) as a foundation, but has a different focus.[9] It captures not only how much, but also what kind of information the investor acquires through state-contingent attention allocation. In our setting, when screening is desirable, the optimal security encourages the investor to allocate adequate and proper attention to all states so as to effectively differentiate any potentially good project from a bad one, and thus delivers the highest possible ex-ante profit to the entrepreneur. This mechanism generates the exact shape of participating convertible preferred stock that cannot be achieved in previous literature.

We map the above friction-based pecking order of debt and convertible preferred stock to three empirical dimensions: the ex-ante profitability (NPV) of the project, the uncertainty of the project, and the cost of screening. We perform comparative statics of the optimal securities over the three dimensions. These comparative statics further suggest that different projects are endogenously financed by different securities, and potentially different types of investors. The role of screening varies as well, but is still unified under the friction: the different extent of dependence of real production on information acquisition.

The generality of our model in synthesizing debt and non-debt securities stems from identifying the costs and benefits of screening in different economies. In our production economy (a primary financial market), the aggregate cash flow depends on the financing decision. Compared to our work, Yang (2013) considers a model where a seller has an asset in-place and proposes an asset-backed security to a more patient buyer to raise liquidity. The buyer can also flexibly acquire information before purchase. That model features an exchange economy (a secondary financial market) as the aggregate cash flow is fixed. In such an exchange economy, social surplus depends negatively on information acquisition. Debt is uniquely optimal there because it mitigates the buyer's adverse selection to the greatest extent. On the contrary, in our model, social surplus may depend positively on information acquisition. Adverse selection is no longer the focus, and debt may no longer be optimal when information acquisition is desirable. This contrast is reminiscent of Hirshleifer (1971) that distinguishes between information value in an exchange economy and that in a production economy. Earlier mechanism design literature on information gathering also hints this difference, and suggests that the contribution of information provision on liquidity would differ accordingly (Cremer and Khalil, 1992, Cremer, Khalil and Rochet, 1998a,b).[10]

Along with the sharp predictions, our parsimonious framework admits a variety of theoretical

[8]We call those information acquisition technologies rigid, because they impose parametric restrictions on the signals, while flexible information acquisition allows for any conditional distribution over the fundamental.

[9]The original rational inattention concept mainly captures the bounded rationality of agents: they have limited capacity of attention and cannot pay full attention to all payoff-related states or variables. Flexible information acquisition highlights the opposite: agents have the capacity to allocate attention to different states of the economy in a flexible way. This renders flexible information acquisition a match to strategic interactions like security design, while the original rational inattention works for decision problems to generate rigidity-related phenomena.

[10]But they do not focus on security design and do not predict the forms of optimal contracts in different markets.

corporate finance contexts and real-world scenarios of financing entrepreneurial production. On the one hand, we highlight the investor as an screening expert. The acknowledgement of investors' screening dates back to Knight (1921) and Schumpeter (1942). Besides numerous anecdotal evidence (see Kaplan and Lerner, 2010, Da Rin, Hellmann and Puri, 2011, for reviews), recent empirical literature (Chemmanur, Krishnan and Nandy, 2012, Kerr, Lerner and Schoar, 2014) has also identified the direct screening by various investors. As the cost of screening pertains to both the project's nature and the investor's information expertise, it also allows us to cover various investors, including families and friends, banks, and venture capitalists. On the other hand, we highlight two aspects of the entrepreneur, capturing private businesses that account for most firms. First, the entrepreneur is financially constrained. Second, the entrepreneur's human capital is inalienable, which means the investor can not take over the project, and the entrepreneur has bargaining power in designing the security; however, even relaxing these assumptions does not affect our results.[11] These settings fit the notion of entrepreneur-led financing proposed by Admati and Pfleiderer (1994), as well as the argument in Rajan (2012) that entrepreneurs' human capital is important in the early stage of firms' life cycles. This paper, to the best of our knowledge, is the first to investigate the interplay between security design and screening in a production economy, and deliver predictions that are both sharp and consistent with empirical evidence on the contracts between real-world entrepreneurs and investors.

Related Literature. In addition to the security design literature pointing out debt as most information-insensitive (as mentioned above), this paper is related to a series of theoretical papers that predict non-debt optimal securities and potentially invalidates the classical pecking order in various circumstances with hidden information (see Brennan and Kraus, 1987, Constantinides and Grundy, 1989, Nachman and Noe, 1994, Chemmanur and Fulghieri, 1997, Inderst and Mueller, 2006, Chakraborty and Yilmaz, 2011, Chakraborty, Gervais and Yilmaz, 2011, Fulghieri, Garcia and Hackbarth, 2013). Besides those papers, closer are Boot and Thakor (1993), Fulghieri and Lukin (2001), Axelson (2007), Garmaise (2007), and Hennessy (2013), all of which highlight the competitive financial markets' role in soliciting or aggregating investors' private information. These papers, however, do not consider individual investors' screening directly. Compared to these papers employing exogenous information asymmetry or rigid information acquisition, our model delivers a clear interaction between security design and screening. This allows us to synthesize debt and non-debt securities, and to identify their optimality in different circumstances. Also, previous models can only admit discrete states, or continuous states with distributional assumptions,[12] or restricted sets of feasible securities.[13] With flexible information acquisition, we can model

[11] The results of optimal securities continue to hold, either if the project is transferrable or if the entrepreneur does not have full bargaining power in designing the security. See subsection 3.3 and subsection 6.2.

[12] For example, the monotone likelihood ratio property (MLRP), or various forms of stochastic dominance.

[13] Most of the security design literature only considers monotone securities, or even requires monotone residuals. We endogenize these properties.

arbitrary securities on continuous states with arbitrary distributions and information structures, and thus characterize the optimal securities in a more rigorous and robust way. Finally, our framework also allows for a new moral hazard interpretation of screening, suggesting a bridge between hidden information and hidden action which are often addressed as separate frictions in the classical contract design literature.

A new strand of literature on the real effects of rating agencies (see Kashyap and Kovrijnykh, 2013, Opp, Opp and Harris, 2013) is also related. On behalf of investors, the rating agency screens the firm, who does not know its own type. Information acquisition may improve social surplus through ratings and the resulting investment decisions. Different from this literature, we study how different shapes of securities interact with the incentives to allocate attention in acquiring information and as a result the equilibrium financing choice. Flexible information acquisition also allows us to combine the two roles of rating agencies and investors, and thus to flesh out the impact of endogenous screening on security design.

Our model also contributes to the venture contract design literature by highlighting screening. Security design is one focus of modern research in entrepreneurial finance and innovation, but existing literature mostly pays attention to control rights (Berglof, 1994, Hellmann, 1998, Kirilenko, 2001), monitoring (Ravid and Spiegel, 1997, Schmidt, 2003, Casamatta, 2003, Hellmann, 2006), and refinancing and staging of finance (Admati and Pfleiderer, 1994, Bergemann and Hege, 1998, Cornelli and Yosha, 2003, Repullo and Suarez, 2004), leaving screening untouched. Also, a majority of these models achieve only one class of optimal security, working with restrictive sets of feasible securities, discrete states, or restrictive distributional assumptions. In contrast, our model unifies debt and (participating) convertible preferred stock in a more general framework and provides a consistent mapping of their optimality to different real-world circumstances.

Finally, a growing behavioral literature highlights attention allocation in the interaction of firms and consumers. Close to our setting is Bordalo, Gennaioli and Shleifer (2013) arguing that consumers may pay different attention to different product attributes through distorted payoff perceptions, and such attention allocation interacts with product design and competition.[14] Complementing this literature, our work focuses on attention allocation across states through information acquisition, and delivers insights within the rational Bayesian updating paradigm.

The rest of the paper is organized as follows. Section 2 specifies the the economy and elaborates the concept of flexible information acquisition. The optimal securities are characterized and discussed in Section 3. Section 4 characterizes our new pecking orders. Section 5 performs comparative statics on the optimal securities. Section 6 demonstrates the robustness of our results by discussing and extending our model. All proofs are attached in Appendix A.2.

[14] An implication of this mechanism in financial markets is that financial innovations may draw investors' attention to returns instead of risks, which leads to neglected risks highlighted by Gennaioli, Shleifer and Vishny (2012).

2 Model

We present our stylized model of a production economy, focusing on the interplay between security design and flexible screening. We make assumptions to highlight the key friction: the dependence and separation of real production and information acquisition.

2.1 Financing Entrepreneurial Production

Consider an economy with two dates, $t = 0, 1$, and a single consumption good. There are two agents: a penniless entrepreneur and a deep-pocket investor, both risk neutral. Their utility function is the sum of consumptions over the two dates: $u = c_0 + c_1$, where c_t denotes an agent's consumption at date t. In what follows we use subscripts E and I to indicate the entrepreneur and the investor, respectively.

We consider the finance of the entrepreneur's risky project. To initiate the project at date 0, the underlying technology requires an investment $k > 0$. If financed, the project generates a non-negative verifiable random cash flow θ at date 1. The project cannot be initiated partially. Hence, the entrepreneur has to raise k, by selling a security to the investor at date 0. The payment of a security at date 1 is a mapping $s : \mathbb{R}_+ \to \mathbb{R}_+$ such that $s(\theta) \in [0, \theta]$ for any θ. We only focus on the cash flow aspect of projects and securities.

We specify the processes of security design and information acquisition, both at date 0. The agents have a common prior Π on the potential project's future cash flow θ, and neither party has any private information ex-ante.[15] The entrepreneur designs the security, and then proposes a take-it-or-leave-it offer to the investor at price k. Facing the offer, the investor acquires information about θ in the manner of rational inattention (Sims, 2003, Woodford, 2008, Yang, 2012, 2013), updates her belief on θ, and then decides whether to accept the offer. We model this process through flexible information acquisition, where the information acquired is measured by reduction of entropy. The information cost per unit reduction of entropy is μ, which is interpreted as the cost of screening. We will elaborate flexible information acquisition in subsection 2.2.

The implicit assumptions in the settings reflect the key features in financing entrepreneurial production, highlighting the role of screening. First, the entrepreneur can only undertake the project by external finance. This is consistent with evidence that entrepreneurs and private firms are often financially constrained (Evans and Jovanovic, 1989, Holtz-Eakin, Joulfaian and Rosen, 1994). Even in mature firms, managers seek for outside finance since the internal capital market does not function well for new risky projects (Stein, 1997, Scharfstein and Stein, 2000). Second,

[15]We can interpret this setting as that the entrepreneur may still have some private information about the future cash flow, but she does not have any effective ways to signal that to the investor, absent the investor's potential screening behavior. The signaling channel has been extensively discussed in the literature and already well understood, so we choose to leave it aside for highlighting our focus on screening.

the investor is able to acquire information about the cash flow and thus screen the project through her financing decision. This point not only accounts for empirical evidence, but also differs our work from most previous security design literature that features the entrepreneur's exogenous information advantage. These two points together lead to the dependence and separation of real production and information acquisition, which is the key friction in our model.

It is worth noting what aspects of finance in the production economy are abstracted away, and to what extent they affect our work. First, to focus on screening, we put classical moral hazard aside as it does not help deliver insights. To ignore moral hazard is common in the security design literature, especially when hidden information is highlighted (see DeMarzo and Duffie, 1999, for a justification). Interestingly, screening in our context has a natural but new moral hazard interpretation, and we discuss it in subsection 6.1. Second, the bargaining process and the allocation of control rights are not our focus. We assume that the entrepreneur's human capital is inalienable, so that she has the bargaining power to design the security and a direct project transfer is impossible. This notion of entrepreneur-led financing is also common in literature (Brennan and Kraus, 1987, Constantinides and Grundy, 1989, Admati and Pfleiderer, 1994). Together with the differentiation argument in Rajan (2012), this assumption also broadly corresponds to the earlier incomplete contract literature suggesting the ownership go to the entrepreneur when firms are young (Aghion and Tirole, 1994). In subsection 3.3 we will formally show that even if the project is transferrable, to transfer the project at any fixed price is not optimal. Moreover, in subsection 6.2 we discuss a general allocation of bargaining powers between the two agents, and we show that our main results are unaffected unless the investor's bargaining power is too strong.[16] Third, we do not model the staging of finance, and thus we interpret the cash flow θ in our model as already taking the consequence of investors' exiting into account. Hence, each round of investment may be mapped to our model separately with a different prior. Fourth, we do not model competition among investors. The third and fourth points pertain to the micro-structure of financial markets, which is tangential to the focus of our friction. Last, risk neutrality enables us to focus on screening as opposed to risk sharing, which is of less interest for our purpose.

2.2 Flexible Information Acquisition

We model the investor's screening by flexible information acquisition (Yang, 2013).[17] It better captures the nature of screening, and allows us to work with arbitrary securities over continuous states without distributional assumptions and to deliver sharp predictions. Fundamentally, the entrepreneur is able to design the security's payoff structure in an arbitrary way, which may result in arbitrary allocation of attention for the investor when she screens the project. It thus calls for

[16]The results are different if the investor's bargaining power is strong, but these results are still intuitive based on the friction point of view. See subsection 6.2 for details.

[17]For more detailed expositions of flexible information acquisition, see Woodford (2008) and Yang (2012, 2013).

an equally flexible account of screening to capture the interaction between the shape of securities and the incentives to allocate attention. This goal is not achievable through classical information acquisition technologies.

The key of flexible information acquisition is that it captures not only how much but also what aspects of information that an agent acquires. Consider an agent who chooses a binary action $a \in \{0, 1\}$ and receives a payoff $u(a, \theta)$, where $\theta \in \mathbb{R}_+$ is the fundamental, distributed according to a continuous probability measure Π over $\mathbb{R}_+$. Before making a decision, the agent may acquire information through a set of binary-signal information structures, each signal corresponding to one optimal action.[18] Specifically, she may choose a measurable function $m : \mathbb{R}_+ \to [0, 1]$, the probability of observing signal 1 if the true state is θ, and acquire binary signals $x \in \{0, 1\}$ parameterized by $m(\theta)$. $m(\theta)$ is chosen to ensure that the agent's optimal action is 1 (or 0) when observing 1 (or 0). By choosing different functional forms of $m(\theta)$, the agent can make her signal correlated with the fundamental in any arbitrary way.[19] Intuitively, for instance, if the agent's payoff is sensitive to fluctuations of the state within some range $A \subset \mathbb{R}_+$, she would pay more attention to this range by making $m(\theta)$ covarying more with θ in A. This gives us a desirable account to model an agent's incentive to acquire different aspects of information.

The conditional probability $m(\cdot)$ has a natural interpretation of screening. In our setting of financing entrepreneurial production, conditional on a cash flow θ, $m(\theta)$ is the probability of the project being screened in to get financed. It is state-contingent, capturing the investor's incentive to allocate attention in screening a project. In particular, the absolute value of the first order derivative $|dm(\theta)/d\theta|$ represents the screening intensity: when it is larger, the investor differentiates the states around θ better. Thus, we call $m(\cdot)$ a screening rule in what follows.

We then characterize the cost of information acquisition. As in Woodford (2008) and Yang (2013), the amount of information conveyed by a screening rule $m(\cdot)$ is defined as the expected reduction of uncertainty through observing the signal generated, where the uncertainty associated with a distribution is measured by Shannon (1948)'s entropy. Formally, we use the concept of mutual information, which is defined as the difference between agents' prior entropy and expected posterior entropy:

$$\begin{aligned} I(m) &= H(prior) - H(posterior) \\ &= -g(\mathbb{E}[m(\theta)]) - (-\mathbb{E}[g(m(\theta))]) , \end{aligned}$$

where $g(x) = x \cdot \ln x + (1-x) \cdot \ln(1-x)$, and the expectation operator $\mathbb{E}(\cdot)$ is with respect to

[18] In general, an agent can choose any information structure. But an agent always prefers binary-signal information structures in binary decision problems. See Woodford (2008) and Yang (2012) for formal discussions.

[19] Technically, this allows agents to choose signals drawn from any conditional distribution of the fundamental, as opposed to classical information acquisition technologies that often involve restrictions on the signals to be acquired.

θ under the probability measure Π. Denote by $M = \{m \in L(\mathbb{R}_+, \Pi) : \theta \in \mathbb{R}_+, m(\theta) \in [0,1]\}$ the set of binary-signal information structures, and $c : M \to \mathbb{R}_+$ the cost of information. The cost is assumed to be proportional to the associated mutual information:

$$c(m) = \mu \cdot I(m) ,$$

where $\mu > 0$ is the marginal cost of information acquisition per reduction of entropy.[20,21]

Built upon flexible information acquisition, the agent's problem is to choose a functional form of $m(\theta)$ to maximize her expected payoff minus the information cost. We characterize the optimal screening rule $m(\theta)$ in the following proposition. We denote $\Delta u(\theta) = u(1,\theta) - u(0,\theta)$, which is the the payoff gain of taking action 1 over action 0. We also assume that $\Pr[\Delta u(\theta) \neq 0] > 0$ to exclude the trivial case where the agent is always indifferent between the two actions. The proof is in Yang (2013) (see also Woodford, 2008, for an earlier treatment).

Proposition 1. *Given u, Π, and $\mu > 0$, let $m^*(\theta) \in M$ be an optimal screening rule and*

$$\bar{\pi}^* = \mathbb{E}[m^*(\theta)]$$

be the corresponding unconditional probability of taking action 1. Then,

i) the optimal screening rule is unique;

ii) there are three cases for the optimal screening rule:

a) $\bar{\pi}^* = 1$, i.e., $Prob[m^*(\theta) = 1] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \Delta u(\theta)\right)\right] \leqslant 1\,; \tag{2.1}$$

b) $\bar{\pi}^* = 0$, i.e., $Prob[m^*(\theta) = 0] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(\mu^{-1} \cdot \Delta u(\theta)\right)\right] \leqslant 1\,;$$

c) $0 < \bar{\pi}^* < 1$ and $Prob[0 < m^*(\theta) < 1] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(\mu^{-1} \cdot \Delta u(\theta)\right)\right] > 1 \text{ and } \mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \Delta u(\theta)\right)\right] > 1\ ; \tag{2.2}$$

[20] Although the cost $c(m)$ is linear in mutual information $I(m)$, it does not mean it is linear in information acquisition. Essentially, mutual information $I(m)$ is a non-linear functional of the screening rule $m(\cdot)$ and the prior Π, micro-founded by the information theory.

[21] The cost function following rational inattention also implies that all states are homogenous in terms of the cost of information acquisition. That is, it is equally costly to differentiate any states. See Woodford (2012) and Yang (2013) for extensive discussions on this point.

in this case, the optimal screening rule $m^*(\theta)$ is determined by the equation

$$\Delta u(\theta) = \mu \cdot \left(g'(m^*(\theta)) - g'(\bar{\pi}^*)\right) \tag{2.3}$$

for all $\theta \in \mathbb{R}_+$, where

$$g'(x) = \ln\left(\frac{x}{1-x}\right) .$$

Proposition 1 fully characterizes the agent's possible optimal decisions of information acquisition. Case a) and Case b) correspond to the scenarios where there exists an ex-ante optimal action 1 or 0. These two cases do not involve information acquisition. They correspond to the scenarios where the prior is extreme or the cost of information acquisition is sufficiently high. In contrast, Case c), the more interesting one, involves information acquisition. Especially, the optimal screening rule $m^*(\theta)$ is not constant in this case, and neither action 1 nor 0 is ex-ante optimal. This case corresponds to the scenario where the prior is not extreme, or the cost of information acquisition is sufficiently low. In Case c) where information acquisition is involved, the agent equates the marginal benefit of information to the marginal cost of information. By doing so, the agent chooses the shape of $m^*(\theta)$ according to the shape of payoff gain $\Delta u(\theta)$ and her prior Π.[22] In the next section we will see that the shape of $m^*(\theta)$ plays a critical role in characterizing how the investor screens a project.

3 Security Design

We consider the entrepreneur's security design problem. Denote the optimal security of the entrepreneur by $s^*(\theta)$. The game between the entrepreneur and the investor is a dynamic Bayesian game. Concretely, the entrepreneur designs the security, and then the investor screens the project given the security. Hence, we apply Proposition 1 to the investor's problem, given the entrepreneur's security, and then solve for the entrepreneur's optimal security backwards. To distinguish from the decision problem above, we denote the investor's optimal screening rule as $m_s(\theta)$, given the security $s(\theta)$. The investor's optimal screening rule given the entrepreneur's optimal security $s^*(\theta)$ will be denoted by $m_s^*(\theta)$.

We formally define the equilibrium as follows.

Definition 1. *Given u, Π, k and $\mu > 0$, the sequential equilibrium is defined as a collection of the entrepreneur's optimal security $s^*(\theta)$ and the investor's optimal screening rule $m_s(\theta)$ for any generic security $s(\theta)$, such that*

i) the investor optimally acquires information at any generic information set induced by $s(\theta)$:

[22] See Woodford (2008), Yang (2012, 2013) for more examples on this decision problem.

$m_s(\theta)$ is prescribed by Proposition 1,[23] *and*

ii) the entrepreneur designs the optimal security:

$$s^*(\theta) \in \arg \max_{0 \leqslant s(\theta) \leqslant \theta} \mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \, .$$

According to Proposition 1, there are three cases pertaining to the investor's behavior, given the entrepreneur's optimal security. First, the investor may optimally choose not to acquire information and accept the entrepreneur's optimal security directly. This implies that the project would be financed for sure. Second, the investor may optimally acquire some information, induced by the entrepreneur's optimal security, and then accept the entrepreneur's optimal security with a positive (but less than one) probability. In this case, the project would be financed with a positive (but less than one) probability from an ex-ante perspective. Third, the investor may directly reject the entrepreneur's optimal security without acquiring information, which implies that the project would not be financed. All the three cases are accommodated by the equilibrium definition. This last case, however, represents the outside option of the entrepreneur, who can always propose nothing to the investor and drop the project. Thus, we will focus on the first two types of equilibrium. The following lemma helps distinguish the first two cases from the last case.

LEMMA **1.** *The project can be financed with a positive probability in equilibrium if and only if*

$$\mathbb{E}\left[\exp(\mu^{-1} \cdot (\theta - k))\right] > 1 \, . \tag{3.1}$$

Lemma 1 is an intuitive investment criterion. It implies that the security is more likely to be accepted by the investor, if the prior of the cash flow is better, if the initial investment k is smaller, or if the cost of screening μ is lower. When condition (3.1) is violated, the investor would reject the security, whatever it is.

Condition (3.1) appears different from the ex-ante NPV criterion suggesting that a project should be financed for sure when $\mathbb{E}[\theta] - k > 0$. In our model with screening, by Jensen's inequality, condition (3.1) suggests that any project with positive ex-ante NPV would be financed with a positive probability. Moreover, some projects with negative ex-ante NPV may be financed with a positive probability as well. This is consistent with our idea that real production depends on information acquisition. Thanks to screening, the ex-ante NPV criterion based on a fixed prior is generalized to a new information-adjusted one to admit the potential of belief updating.

The following Corollary 1 implies that the entrepreneur will never propose all the cash flow to the investor if the project would be financed. This corollary is straightforward, but we highlight it as it helps illustrate our key friction by showing that the interests of the entrepreneur and the

[23] The specification of belief for the investor at any generic information set is also implicitly given by Proposition 1, provided the definition of $m_s(\theta)$.

investor are not perfectly aligned. It also helps establish some important results later. Intuitively, to retain a little bit more would still result in a finance with a positive probability and give the entrepreneur a positive expected payoff.

Corollary **1.** *When the project can be financed with a positive probability, $s^*(\theta) = \theta$ is not an optimal security.*

In what follows, we assume that condition (3.1) is satisfied, and characterize the entrepreneur's optimal security, focusing on the first two types of equilibrium with a positive screening cost $\mu > 0$. As we will see, the entrepreneur's optimal securities in these two cases are different, which implies that the investor screens the project in different manners. We further show that to transfer the project at a given price is always not optimal, which also justifies the security design approach. We finally consider two limiting case with infinite and zero screening cost to offer more intuition.

3.1 Optimal Security without Inducing Information Acquisition

In this subsection, we consider the case in which the entrepreneur's optimal security is directly accepted by the investor without information acquisition. In other words, the entrepreneur finds screening not worthwhile in this case and wants to design a security to deter it. Concretely, this means $Pr\left[m_s(\theta) = 1\right] = 1$. We first consider the investor's problem of screening, given the entrepreneur's security, then characterize the optimal security.

Given a security $s(\theta)$, the investor's payoff gain by accepting the security over rejecting it is

$$\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = s\left(\theta\right) - k\,. \tag{3.2}$$

According to Proposition 1 and conditions (2.1) and (3.2), any security $s(\theta)$ that is accepted by the investor without information acquisition must satisfy

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\cdot\left(s\left(\theta\right) - k\right)\right)\right] \leqslant 1\,. \tag{3.3}$$

If the left hand side of the inequality (3.3) is strictly less than one, the entrepreneur could lower $s(\theta)$ to some extent to increase her expected payoff gain, without affecting the investor's incentives. Hence, condition (3.3) always holds as an equality in equilibrium.

By backward induction, the entrepreneur's problem is to choose a security $s(\theta)$ to maximize her expected payoff

$$u_E(s(\cdot)) = \mathbb{E}\left[\theta - s(\theta)\right]$$

subject to the investor's information acquisition constraint

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\cdot\left(s\left(\theta\right) - k\right)\right)\right] = 1\,,$$

and the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$.[24]

As we would see, the entrepreneur's optimal security in this case follows a debt. We characterize this optimal security by the following proposition, along with its graphical illustration in Figure 1. It is easy to see that the face value of the debt is unique in this case.

PROPOSITION **2.** *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to accept the security without acquiring information in equilibrium, then it takes the form of a debt:*

$$s^*(\theta) = \min(\theta, D^*)$$

where the unique face value D^* is determined by

$$D^* = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu) > k\,,$$

in which λ is a positive constant determined in equilibrium.[25] *Also, the expected payment of the optimal debt satisfies*

$$\mathbb{E}[s^*(\theta)] > k\,.$$

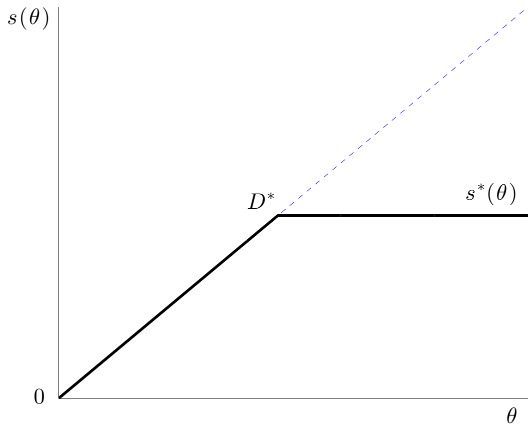

Figure 1: The Unique Optimal Security without Information Acquisition

It is intuitive to have debt as the optimal security when the entrepreneur finds it optimal not to induce information acquisition. Since screening is not worthwhile and thus the entrepreneur wants to design a security to deter it, debt is the least information-sensitive one to provide the desired

[24]With this feasibility condition, the entrepreneur's individual rationality constraint $\mathbb{E}[\theta - s(\theta)] \geqslant 0$ is automatically satisfied, which is also true for the later case with information acquisition. This comes from the fact that the entrepreneur has no money to start. It also implies that the entrepreneur always prefers to undertake the project, which is consistent with real-world practices. However, it is not correct to interpret this as that the entrepreneur would like to contract with any investor, as we do not model the competition among different investors.

[25]Our focus is the qualitative nature of the optimal security, instead of quantities. Thus, we do not solve for the face value in closed form, which is less tractable and does not help deliver insights. This also applies to the other optimal security discussed in subsection 3.2.

expected payoff to the entrepreneur. From another perspective, the optimal security renders the investor to break even between acquiring and not acquiring information. Hence, thanks to flexible information acquisition, any mean-preserving spread of the optimal security, which gives the entrepreneur the same expected payoff, would induce the investor to acquire unnecessary information. This implies that the optimal security should be as flat as possible when the limited liability constraint is not binding, which leads to debt.

Interestingly, although the investor only provides physical investment in this case, the expected payment $\mathbb{E}[s^*(\theta)]$ exceeds the investment requirement k. The extra payment exceeding k works as a premium to make the investor comfortable to accept the offer for sure, as the investor may worry about financing a potentially bad project without screening.

Debt accounts for the real-world scenarios in which new projects are financed by fixed-income securities. On the one hand, when a project's market prospects are good and thus not much extra information is needed, it is optimal to deter or mitigate investor's costly information acquisition by issuing debt, which is the least information-sensitive. Interestingly, the rationale for debt in our model does not feature adverse selection, but rather a cost-benefit trade-off of screening. On the other hand, empirical evidence suggests that many conventional businesses and less revolutionary start-ups rely heavily on plain vanilla debt finance from investors who are not good at screening like families, friends, and tranditional banks (for example, Berger and Udell, 1998, Kerr and Nanda, 2009, Robb and Robinson, 2014), as opposed to more sophisticated financial contracts with venture capital organizations or buyout funds.

3.2 Optimal Security Inducing Information Acquisition

In this subsection, we characterize the entrepreneur's optimal security if it induces the investor to acquire information and to accept the security with a positive probability (but less than one). In other words, the entrepreneur finds screening favorable in this case and wants to design a security to incentivize it. According to Proposition 1, this means $Prob\,[0 < m_s(\theta) < 1] = 1$.

Again, according to Proposition 1 and conditions (2.2) and (3.2), any generic security $s(\theta)$ that induces the investor to acquire information must satisfy

$$\mathbb{E}\left[\exp\left(\mu^{-1}\left(s\left(\theta\right)-k\right)\right)\right] > 1 \tag{3.4}$$

and

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\left(s\left(\theta\right)-k\right)\right)\right] > 1\ , \tag{3.5}$$

Given such a security $s(\theta)$, Proposition 1 and condition (2.3) also prescribe that the investor's optimal screening rule $m_s(\theta)$ is uniquely characterized by

$$s(\theta) - k = \mu \cdot \left(g'(m_s(\theta)) - g'(\overline{\pi}_s)\right) \ , \tag{3.6}$$

where

$$\overline{\pi}_s = \mathbb{E}[m_s(\theta)]$$

is the investor's unconditional probability of accepting the security and it does not depend on θ. In what follows, we denote the unconditional probability induced by the entrepreneur's optimal security $s^*(\theta)$ by $\overline{\pi}_s^*$.

We derive the entrepreneur's optimal security backwards. Taking into account of investor's response $m_s(\theta)$, the entrepreneur chooses a security $s(\theta)$ to maximize

$$u_E(s(\cdot)) = \mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \tag{3.7}$$

subject to (3.4), (3.5),[26] (3.6), and the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$.[27]

To fix idea, we first offer an intuitive roadmap to investigate the optimal security and the associated optimal screening rule, highlighting their key properties. Then we give a formal proposition to characterize the optimal security and discuss its implications. The detailed derivation of the optimal security is presented in Appendix A.1.[28]

First, the investor's optimal screening rule $m_s^*(\theta)$, induced by the optimal security $s^*(\theta)$, must increase in θ. When the entrepreneur finds it optimal to induce information acquisition, she benefits from screening of the investor. Effective screening makes sense only if the investor screens in a potentially good project and screens out a bad one; otherwise it incurs a lower social surplus. Under flexible information acquisition, this implies that $m_s^*(\theta)$ should be more likely to generate a good signal and to result in a successful finance for a higher cash flow θ, while to generate a bad signal and to result in a rejection for a lower θ. Therefore, $m_s^*(\theta)$ should be increasing in θ. As we will see, the monotonicity of $m_s^*(\theta)$ and the shape of $s^*(\theta)$ are highly related to each other.

To induce an increasing optimal screening rule $m_s^*(\theta)$, the optimal security $s^*(\theta)$, proposed by the entrepreneur, must be increasing in θ as well, according to the first order condition of information acquisition (3.6). Intuitively, this monotonicity reflects the dependence of real production on information acquisition: the entrepreneur is willing to compensate the investor more in an event of higher cash flow to encourage effective screening. As opposed to the classical security design literature that often restricts the feasible set to non-decreasing securities (for

[26]According to Proposition 1, both conditions (3.4) and (3.5) should not be binding for the optimal security; otherwise the investor would not acquire information.

[27]Again, the entrepreneur's individual rationality constraint $\mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \geqslant 0$ is automatically satisfied.

[28]To facilitate understanding, the intuitive investigation of the optimal security is not organized in the same order as the derivation goes in the Appendix, but all the claims in the main text are guaranteed by the formal proofs.

example, Innes, 1990, DeMarzo and Duffie, 1999, DeMarzo, 2005, among others), our prediction of an increasing optimal security without such constraints is significant.

We also argue that the non-negative constraint $s(\theta) \geqslant 0$ is not binding for the optimal security $s^*(\theta)$ for any $\theta > 0$. Suppose $s^*(\widetilde{\theta}) = 0$ for some $\widetilde{\theta} > 0$. Since $s^*(\theta)$ is increasing in θ, for all $0 \leqslant \theta \leqslant \widetilde{\theta}$ we must have $s^*(\theta) = 0$. This violates the argument above that $s^*(\theta)$ must be increasing in θ. Intuitively, zero payments in states with low cash flows give the investor too little incentive to acquire information, which is not optimal for the entrepreneur. The security with zero payments in states with low cash flows looks closest to levered common stock, which is the least used security between entrepreneurs and investors (Kaplan and Stromberg, 2003, Kaplan and Lerner, 2010, Lerner, Leamon and Hardymon, 2012).

To shot a closer examination of the optimal security, it is instructive to follow a perturbation argument on the entrepreneur's security design problem, which gives the entrepreneur's first order condition. Specifically, denote by $r^*(\theta)$ the marginal contribution to the entrepreneur's expected payoff $u_E(s(\cdot))$ by any feasible perturbation to the optimal security $s^*(\theta)$.[29] As $s^*(\theta) > 0$ for any $\theta > 0$, it is intuitive to show that for any $\theta > 0$:

$$r^*(\theta) \begin{cases} = 0 & \text{if} \quad 0 < s^*(\theta) < \theta \\ \geqslant 0 & \text{if} \quad s^*(\theta) = \theta \end{cases},$$

which is further shown to be equivalent to

$$(1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} = \mu & \text{if} \quad 0 < s^*(\theta) < \theta \\ \geqslant \mu & \text{if} \quad s^*(\theta) = \theta \end{cases}, \tag{3.8}$$

where w^* is a constant determined in equilibrium.

We argue that the optimal security $s^*(\theta)$ follows the 45° line in states with low cash flows and then increases in θ with some smaller slope in states with high cash flows. That is, the residual of the optimal security, $\theta - s^*(\theta)$, increases in θ as well in states with high cash flows. According to the entrepreneur's first order condition (3.8) and the monotonicity of $m_s^*(\theta)$, if $s^*(\widehat{\theta}) = \widehat{\theta}$ for some $\widehat{\theta} > 0$, it must be $s^*(\theta) = \theta$ for any $0 < \theta < \widehat{\theta}$. Similarly, if $s^*(\widehat{\theta}) < \widehat{\theta}$ for some $\widehat{\theta} > 0$, it must be $s^*(\theta) < \theta$ for any $\theta > \widehat{\theta}$. In addition, Corollary 1 rules out $s^*(\theta) = \theta$ for all $\theta > 0$ as an optimal security. Thus, since $s^*(\theta)$ is increasing in θ, the limited liability constraint can only be binding in states with low cash flows.[30] Importantly, according to condition (3.8) and again the monotonicity of $m_s^*(\theta)$, when the limited liability constraint is not binding in states with high

[29] Formally, $r^*(\theta)$ is the Frechet derivative, the functional derivative used in the calculus of variations, of $u_E(s(\cdot))$ at $s^*(\theta)$. It is analogical to the commonly used derivative of a real-valued function of a single real variable but generalized to accommodate functions on Banach spaces.

[30] In the formal proofs we further show that the limited liability constraint must be binding for some states $(0, \widehat{\theta})$ with $\widehat{\theta} > 0$.

cash flows, not only $s^*(\theta)$ but also $\theta - s^*(\theta)$ are increasing in θ. In other words, $s^*(\theta)$ is dual monotone when it deviates from the the 45° line in states with high cash flows.

The shape of the optimal security $s^*(\theta)$ reflects the friction of the economy. Recall that the monotonicity of $s^*(\theta)$ reflects the dependence of information and real production. The monotonicity of $\theta - s^*(\theta)$, however, reflects their separation: the entrepreneur wants to retain as much as possible even if she wants to incentivize the investor to screen the project. Specifically, the area between $s^*(\theta)$ and the 45° line not only captures the entrepreneur's retained benefit, but also reflects the degree to which the allocation of resources is inefficient when screening is desirable. This is intuitive: the dependence renders the investor to get all the resources, but the separation prevents the entrepreneur from proposing such a deal, as shown in Corollary 1. The competition of the two forces is alleviated in a most efficient way: to reward the investor more but also retain more in better states. In this sense, our prediction of the dual monotonicity comes endogenously from the friction of the economy, rather than by assumption like existing literature (for example, Nachman and Noe, 1994, Biais and Mariotti, 2005, Garmaise, 2007, among many others).

Formally, the following proposition characterizes the optimal security $s^*(\theta)$ that induces the investor to acquire information. We interpret it as a participating convertible preferred stock.

PROPOSITION **3.** *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to acquire information in equilibrium, then it takes the following form of a participating convertible preferred stock with a face value $\widehat{\theta} > 0$:*

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad 0 \leqslant \theta \leqslant \widehat{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta > \widehat{\theta} \end{cases},$$

where $\widehat{\theta}$ is determined in equilibrium and the unconstrained part $\widehat{s}(\theta)$ satisfies:

i) $\widehat{\theta} < \widehat{s}(\theta) < \theta$;

ii) $0 < d\widehat{s}(\theta)/d\theta < 1$.

Finally, the corresponding optimal screening rule satisfies $dm_s^*(\theta)/d\theta > 0$.

Proposition 3 offers a clear prediction on the entrepreneur's optimal security when screening is favorable. It is closest to participating convertible preferred stock, $d\widehat{s}(\theta)/d\theta$ as the converting rate,[31] which grants the holder a right to receive both the face value and their equity participation as if it was converted, in the real-world event of a public offering or sale. The payoff

[31] In Appendix A.1, we provide the implicit function that determines $d\widehat{s}(\theta)/d\theta$. We have further shown that $d^2\widehat{s}(\theta)/d\theta^2 < 0$, which implies that the unconstrained part is concave, as illustrated in Figure 2. We interpret this as a state-contingent converting rate, with which the entrepreneur retains more shares in better states. It is consistent with the use of contingent contracts in venture finance and private equity buyouts when screening is relevant, also documented in Kaplan and Stromberg (2003).

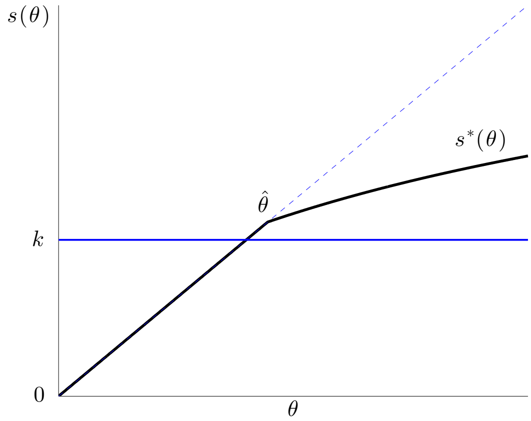

Figure 2: The Unique Optimal Security with Information Acquisition

structure shown in Figure 2 may be also interpreted as debt plus equity (common stock),[32] or participating convertible debt.[33] This prediction is consistent with the empirical evidence of venture contracts documented in Kaplan and Stromberg (2003), in which convertible preferred stocks account for 94.4% of total contracts used,[34] among which 40.8% are participating, and the participating feature is in particular popular in earlier rounds of investment. As suggested by Kaplan and Stromberg (2003), participating convertible preferred stock is even preferable to straight convertible preferred stock for screening purpose. Our model implies the same: straight convertible preferred stock is better than debt but still not optimal, because its flat payoffs in intermediate states do not provide enough incentive for the investor to differentiate these states. Our prediction also fits in line with earlier evidence (Sahlman, 1990, Bergemann and Hege, 1998, Gompers, 1999) on the popularity of participating convertible preferred stock and the combination of debt and equity in financing young firms and new projects. For brevity, in what follows we refer to the optimal security in this case as convertible preferred stock.

Flexible information acquisition plays an important role in predicting the shape of convertible

[32]In terms of cash flow rights, the package of redeemable preferred stock and common stock is used as equivalent to participating convertible preferred stock in practice. But the package is less popular in practice, since it is harder to assign reasonable value to each component of the package, for the following reasons not fully modeled in our work. On the one hand, an expensive common stock component leaves too much downside risk to the investor and perhaps too little tax deferral benefit as well. On the other hand, with a cheap common stock component and an expensive redeemable preferred stock component, the expected large payout to the preferred stock holder at the public offering or sale may cause significant adverse effect on the project's public market value. In contrast, the mandatory conversion feature of participating convertible preferred stock helps overcome the dilemma in reality. Although the entrepreneur still pays back the face value to the investor, it is included in the value of the converted common stocks, and thus the investor do not take it out of the market value. See Lerner, Leamon and Hardymon (2012) for more detailed discussion on this point. Also see Cornelli and Yosha (2003) for a theoretical exposition on the advantage of convertible securities over a combination of debt and equity when characteristics other than the cash flow rights are taken into account.

[33]Compared to equity (common stock), debt and preferred stock are identical in our model as it only features two tranches and no dividends.

[34]If we include convertible debt and the combination of debt and equity, this number increases to 98.1%.

preferred stock. When screening is desirable, on the one hand, a globally increasing security incentivizes the investor to allocate adequate attention to all states so as to differentiate any potentially good project from a bad one. On the other hand, a higher converting rate $d\widehat{s}(\theta)/d\theta$ induces the investor to screen the underlying project more intensively at a higher cost of the entrepreneur.[35] Hence, the entrepreneur weighs the benefit of screening again its cost by choosing the optimal converting rates state-contingently to ensure the highest possible ex-ante profit. Thanks to this mechanism, we are also able to work with arbitrary securities on continuous states with arbitrary distributions, which cannot be achieved in previous models.

Our prediction of the multiple of convertible preferred stock, defined as the ratio of the face value $\widehat{\theta}$ to the investor's initial investment k, is also consistent with the empirical evidence (Kaplan and Stromberg, 2003, Lerner, Leamon and Hardymon, 2012). The multiple is a key characteristic of convertible preferred stock, sometimes viewed analogous to the role of returns.

Corollary **2.** *The multiple of convertible preferred stock, as the optimal security $s^*(\theta)$ that induces information acquisition in equilibrium, is greater than one. In other words, $\widehat{\theta} > k$.*

Like the debt case, this property again comes from the fact that the entrepreneur should offer the investor a premium for accepting the offer at a positive probability. Even after screening, the investor's information is still imperfect. Thus, this premium makes the investor comfortable not to reject the offer, because even with information acquisition the investor may still end up financing a potentially bad project.

Finally, a comparison between the production economy and an exchange economy helps highlight why our model is able to predict both debt and non-debt securities. In a production economy (a primary financial market), costly information contributes to the output, while it only helps reallocate existing resources in an exchange economy (a secondary financial market). Yang (2013) considers security design and information acquisition in a comparable exchange economy. In that model, a seller has an asset in-place and proposes a security to a less patient buyer to raise liquidity. The buyer can acquire information about the asset's cash flow before her purchase. Debt is shown to be uniquely optimal because it mitigates the buyer's adverse selection to the greatest extent. In that exchange economy, information is socially wasteful for the following reasons. First, information acquired by the buyer makes herself better off at the expense of the seller through endogenous adverse selection and it results in illiquidity. Second, information is costly per se. As a result, to discourage information acquisition is desirable. On the contrary, in this paper, the entrepreneur and the investor are jointly exposed to the cash flow of the project if the investor accepts the security, and not so if the security is rejected. Thus, our model features a production economy in which the social surplus may depend positively on costly information. In this case,

[35] In the Appendix A.1, we formally show that the screening intensity $|d\widehat{m}(\theta)/d\theta|$ in the converting region is increasing in $d\widehat{s}(\theta)/d\theta$.

adverse selection is no longer the focus. Instead, the entrepreneur may want to design a security that encourages the investor to acquire information in favor of herself. Therefore, debt may no longer be optimal when information acquisition is desirable.

3.3 Project Transfer

This subsection considers the potential for transferring the project at a fixed non-negative price and shows that it is not optimal even though agents are risk neutral in our model. Equivalently, it also represents a scenario where the entrepreneur works for the investor as a worker and gets a fixed wage payment, as there is no moral hazard at the entrepreneur's side. According to Rajan (2012), entrepreneurs' human capital is often inalienable because of the high differentiation of young firms, which justifies our security design approach. Here, we further argue that, even if the project is transferrable, the entrepreneur still finds such a transfer not optimal.

The key to understand the idea is to view project transfer as a feasible security, as in Figure 3, and show that this security is not optimal. When the entrepreneur proposes a project transfer to the investor at a fixed price $p \geqslant 0$, it is equivalent for her to propose a security $s(\theta) = \theta - p$ without the non-negative constraint $s(\theta) \geqslant 0$. To see why, if the investor accept the offer of transfer and undertake the project, she gets the entire cash flow θ and pay p as an upfront cost. This interpretation allows us to analyze project transfer in our security design framework.

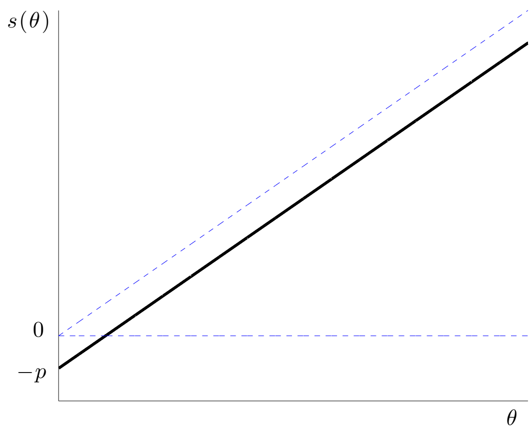

Figure 3: Project Transfer as A Security

To see why the equivalent security $s(\theta) = \theta - p$ is feasible but not optimal, it is important to observe that the non-negative constraint $s(\theta) \geqslant 0$ is binding in neither case of security design. Hence, it is equivalent to consider a larger set of feasible securities, which is still restricted by the limited liability constraint $s(\theta) \leqslant \theta$ but allows negative payoffs to the investor. As debt and convertible preferred stock are still the only two optimal securities in this generalized problem, and $s(\theta) = \theta - p$ representing project transfer is feasible, we conclude that project transfer is not optimal to the entrepreneur for any transfer price p. Intuitively, project transfer is not optimal because it does not follow the least costly way to compensate the investor, no matter whether information acquisition is induced.

Proposition **4.** *When the project can be financed with a positive probability, to transfer the project at a fixed price $p \geqslant 0$ is not optimal for the entrepreneur.*

The timing of the economy are important for Proposition 4. Consider an alternative timing in which the investor can only acquire information after the transfer. The friction of the economy is no longer present, because real production and information acquisition are both performed by the investor after the transfer. Hence, under this timing, project transfer is optimal to the entrepreneur through setting a price that is equivalent to the expected profit of the investor. In this case, the entrepreneur's bargaining power is too strong in the sense that she can prevent the investor from acquiring information when proposing the transfer deal, which essentially removes the friction of the economy. In practice, however, it is common and reasonable that investors have the option to acquire information about the pending project before a potential transfer. This justifies our timing and suggests that a transfer is not optimal when screening is important.

3.4 Optimal Security When Cost of Screening is Infinity or Zero

This subsection looks at the optimal securities in two limiting cases when the cost of screening approaches infinity or zero, offering another perspective to understand the interaction of security design and screening.[36] When the cost of screening μ is infinity, the investor is not able to acquire information. Thus, both the entrepreneur and the investor make decisions based on the common prior. The investor accepts the entrepreneur's offer if and only if the expected payment $\mathbb{E}[s(\theta)]$ exceeds the investment requirement k, regardless of the shape of the security. On the contrary, when μ is zero, the investor always acquires information. In particular, she gets known of the true state of the cash flow θ. Thus, when θ is lower than k, the investor rejects the offer regardless of its payments. Instead, when θ exceeds k, the investor accepts the offer as long as the entrepreneur promises a payment no less than k. Taking the entrepreneur's optimization into account, we immediately have the following results.

Proposition **5.** *For any project,*

i) if $\mu = \infty$ and $\mathbb{E}[\theta] \geqslant k$, the optimal security $s^*(\theta)$ satisfies $\mathbb{E}[s^*(\theta)] = k$ and $0 \leqslant s^*(\theta) \leqslant \theta$ for any $\theta \in \Theta$;

ii) if $\mu = 0$, the optimal security $s^*(\theta)$ satisfies

[36] In the two limiting cases, Proposition 1 and Definition 1 are not applicable. However, as the two cases are both straightforward, we omit a formal equilibrium definition for brevity.

$$\begin{cases} 0 \leqslant s^*(\theta) \leqslant \theta & \textit{if} \quad 0 \leqslant \theta < k \\ s^*(\theta) = k & \textit{if} \quad \theta \geqslant k \end{cases} .$$

It is helpful to attach the two limiting cases to the two types of generic equilibrium discussed in subsection 3.1 and subsection 3.2 according to the respective information acquisition status, to deliver clearer intuition. In particular, we focus on the optimal debt $s^*_\infty(\theta)$ in the first case and the optimal (degenerate) convertible preferred stock $s^*_0(\theta)$ in the second case, among the infinitely many optimal securities in these limiting cases.

Corollary **3.** *For any project,*

i) if $\mu = \infty$ and $\mathbb{E}[\theta] \geqslant k$, the optimal debt $s^*_\infty(\theta)$ satisfies

$$s^*_\infty(\theta) = \min(\theta, D^*_\infty),$$

where $D^*_\infty > k$ is chosen to satisfy $\mathbb{E}[s^*_\infty(\theta)] = k$;

ii) if $\mu = 0$, the optimal (degenerate) convertible preferred stock $s^*_0(\theta)$ satisfies

$$s^*_0(\theta) = \begin{cases} \theta & \textit{if} \quad 0 \leqslant \theta \leqslant k \\ \widehat{\theta}_0 = k & \textit{if} \quad \theta > k \end{cases} .$$

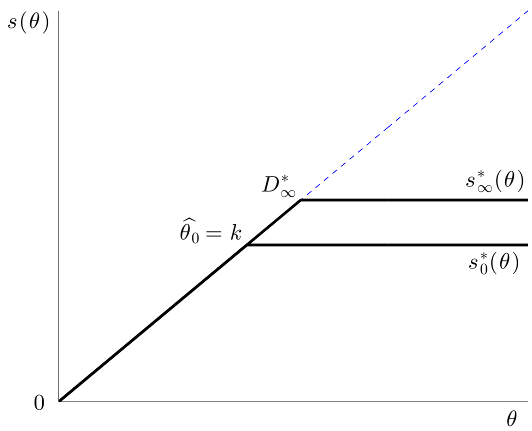

Figure 4: Optimal Securities When the Cost of Screening is Infinity or Zero

Two observations uncover the interaction of security design and screening in the limiting cases, providing an alternative to better understand the role of screening. First, compared to the optimal debt that does not induce information acquisition in the cases with a positive screening cost μ, the optimal debt $s^*_\infty(\theta)$ has the lowest face value D^*_∞, ceteris paribus. This is clearly seen from Proposition 2, which suggests that the face value D^* in those generic cases satisfies $\mathbb{E}[s^*(\theta)] > k$, the extra payment exceeding k as a premium to ensure the investor not to acquire information, while D^*_∞ is chosen to satisfy $\mathbb{E}[s^*_\infty(\theta)] = k$. The intuition is as follows. When $\mu = \infty$, because the investor simply cannot acquire information, there is no need for the entrepreneur to offer the premium. In addition, any security satisfying $\mathbb{E}[s^*(\theta)] = k$ is equivalent to the least expensive debt $s^*_\infty(\theta)$, since the shape of securities is no longer relevant when screening is physically absent.

Similarly, compared to the optimal convertible preferred stock that induces information acquisition in the cases with a positive screening cost μ, the optimal (degenerate) convertible preferred stock $s^*_0(\theta)$ has both the lowest face value $\widehat{\theta}_0 = k$ and the lowest (degenerate) converting rate zero, ceteris paribus. This is seen from Corollary 2 and Proposition 3, which indicate a multiple larger than one and a positive converting rate. The intuition for the lowest face value is however subtly different from the debt case. Since the investor knows the true state and can make investment decisions in a state-contingent way, she would not worry about a potentially bad investment. Thus, there is no need for the entrepreneur to offer the premium exceeding the investment k as suggested by Corollary 2. The intuition for the lowest converting rate is also different. When $\mu = 0$, although the investor acquires information to the greatest extent, it is completely costless. So again there is no need for the entrepreneur to compensate for that. In all, the debt-like convertible preferred stock $s^*_0(\theta)$ finds the least expensive way for the entrepreneur to encourage the investor to acquire information.[37]

4 Pecking Orders of the Optimal Security

A natural question is: in our production economy, when debt is optimal, and when instead convertible preferred stock is optimal? Having characterized the optimal securities with and without inducing screening, we take them together and determine the optimal security given characteristics of the production economy. It also helps generalize the pecking order theory (Myers and Majluf, 1984): the entrepreneur chooses different optimal securities and thus different capital structures in different circumstances. The new pecking orders help synthesize debt and convertible preferred stock, which are often viewed as two distinct securities in many aspects. Our approach also bridges the security design literature and the classical pecking order theory, since our new pecking orders come from security design over a general space of feasible securities, rather than a set of commonly observed ones like debt and equity.

Our new pecking orders are presented against two dimensions: the ex-ante NPV dimension and an efficiency dimension. The first dimension offers an intuitive perspective to investigate the

[37] Although the optimal security $s^*_0(\theta)$ looks like debt, there are two reasons why we prefer to interpret is as convertible preferred stock. First, in our theoretical framework, debt does not induce information acquisition. Second, the face value of debt should be larger than k to ensure an expected payment no less than k. In an equilibrium with information acquisition, as μ approaches zero, we conjecture that the optimal convertible preferred stock converges to $s^*_0(\theta)$ pointwisely. To formally show this is beyond the interest and scope of this paper, but we provide numerical examples in Section 5 to support it.

optimality of the two optimal securities in different circumstances, and highlights two different roles of screening: screening in and out. The second allows us to reveal the relationship between the friction of the production economy and the optimal security in a more fundamental manner. Whether convertible preferred stock or debt is optimal depends on whether or not the entrepreneur wants to encourage screening, which further depends on the different extent of dependence of real production on information acquisition. If the dependence is strong, the friction of the economy is severe. Screening is thus more valuable in this case, so that the entrepreneur finds it more worthwhile to induce screening and proposes convertible preferred stock. Otherwise, the friction is not severe and inducing costly screening is not worthwhile, so that debt is optimal.

4.1 NPV Dimension for the Pecking Order

We first benchmark our new pecking order to the ex-ante NPV dimension.

PROPOSITION **6.** *When the project is financed with a positive probability:*

i). If $\mathbb{E}[\theta] \leqslant k$, the optimal security $s^*(\theta)$ is convertible preferred stock; or

ii). If $\mathbb{E}[\theta] > k$, $s^*(\theta)$ may be convertible preferred stock or debt.[38]

Intuitively, a negative NPV project may only be financed by convertible preferred stock: only through screening could it be potentially found good and worth financing. This is also consistent with the conventional wisdom that a negative NPV project can never be financed by debt with a given and fixed belief.

Interestingly, convertible preferred stock may be optimal for financing both negative and positive NPV projects, but the underlying mechanisms of screening are subtly different. In both cases, the dependence of real production on information acquisition is strong.

When the project has a zero or negative NPV, convertible preferred stock is used to encourage the investor to screen in a potentially good project. In this case, the investor will never finance the project if she is unable to screen it, because it only incurs an expected loss even if the entrepreneur promises all the cash flow. Thus, if it would be financed, the only way is to use convertible preferred stock to encourage screening. This implies that the dependence of real production on information acquisition is strong due to the relatively poor prior, and thus the friction is severe. When the investor acquires information, she may expect either a good signal that leads to a successful deal or a bad signal that results in a rejection, but the ex-ante probability of finance becomes positive since a potentially good project can be screened in. Hence, the entrepreneur is better off by proposing convertible preferred stock.

[38]We do not give explicit conditions to distinguish between the two optimal securities when the ex-ante NPV is positive. Technically, doing this requires restrictions on the prior, which does not help deliver insights in general. However, we fully separate the two optimal securities over the efficiency dimension.

In contrast, when the project has a positive NPV, convertible preferred stock may still be used, but to encourage the investor to screen out a potentially bad project. Here, the dependence of real production on information acquisition is still strong due to a relatively mediocre prior. In the status quo where the investor is unable to screen the project, the entrepreneur can finance the positive NPV project for sure by proposing debt with a high face value. However, when the investor can acquire information, the entrepreneur might find such a sure finance too expensive or even impossible, because by doing so she retains too little for herself. Instead, the entrepreneur could retain more by offering a less generous convertible preferred stock and invite the investor to screen the project. Although this results in a probability of finance less than one, the entrepreneur's total expected profit is higher since a potentially bad project may be screened out, which justifies convertible preferred stock as optimal.

Finally, debt may be optimal for some circumstances with a positive NPV project. When the prior is sufficiently good, the dependence of real production on information acquisition is weak, and thus the benefit from screening does not justify the cost. Hence, it is optimal for the entrepreneur to propose debt to deter costly screening while still retain enough for herself.

4.2 Efficiency Dimension for the Pecking Order

We then benchmark our pecking order to a more fundamental efficiency dimension. To understand how the optimal security evolves with the severity of the friction, we consider a frictionless centralized economy in which real production and information acquisition are aligned. We define a new efficiency dimension with help of this centralized economy. We show that, if and only if the friction in the decentralized economy is not severe in the sense that an optimal security can achieve efficiency, the optimal security is debt and screening is not induced in equilibrium. If and only if the friction is severe in the sense that even an optimal security cannot achieve efficiency, the optimal security is convertible preferred stock and screening is induced. This dichotomy again highlights the close connection among the shape of the optimal security, the role of screening, and the extent of the friction in the production economy.

We start by defining the expected social surplus and the efficiency dimension. The expected surplus of our decentralized economy is the difference between the expected profit of the project and the cost of screening, both of which are functions of the screening rule. Thus, an optimal security in the decentralized economy achieves efficiency if the associated optimal screening rule maximizes expected social surplus in equilibrium.

DEFINITION **2.** *An optimal security in the decentralized economy achieves efficiency if and only* *if the associated optimal screening rule* $m_s^*(\theta)$ *satisfies:*

$$m_s^*(\theta) \in \arg \max_{0 \leqslant m(\theta) \leqslant 1} \mathbb{E}[m(\theta) \cdot (\theta - k)] - \mu \cdot I(m(\theta))\,. \tag{4.9}$$

To facilitate discussion, we characterize a frictionless centralized economy that helps benchmark the friction in the corresponding decentralized economy. In the centralized economy, u, Θ, Π, k and μ are given as the same. However, we suppose the entrepreneur has enough initial wealth and can also screen the project. Thus, real production still depends on information acquisition, but they are aligned. In this economy, security design is irrelevant. The entrepreneur's problem is to decide whether to undertake the project directly, to screen it, or to skip it directly. The entrepreneur's payoff gain by undertaking the project over skipping it is

$$\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = \theta - k\,.$$

We denote the screening rule in the centralized economy by $m_c(\theta)$ and the optimal one by $m_c^*(\theta)$. Thus, the entrepreneur's problem in the centralized economy is

$$\max_{0 \leqslant m_c(\theta) \leqslant 1} \mathbb{E}[m_c(\theta) \cdot (\theta - k)] - \mu \cdot I(m_c(\theta))\,. \tag{4.10}$$

By construction, the entrepreneur's objective (4.10) in the centralized economy is just the expected social surplus in the decentralized economy. It is convenient to work with the centralized economy to analyze the efficiency of equilibria in the corresponding decentralized economy.

Since the optimal screening rules are unique for both the centralized and decentralized economies, efficiency is achieved if and only if information is acquired in the same manner in both the decentralized economy and the centralized economy.

Lemma **2.** *An optimal security in the decentralized economy achieves efficiency if and only if the associated optimal screening rule* $m_s^*(\theta)$ *satisfies*

$$Prob[m_s^*(\theta) = m_c^*(\theta)] = 1\,,$$

where $m_c^*(\theta)$ *is the optimal screening rule in the corresponding centralized economy.*

The efficiency concept in Lemma 2 highlights the role of screening in the production economy and provides a natural measure of the severity of friction in the decentralized economy. Fundamentally, we may view the expected social surplus (4.10) or (4.9) in our production economy as a production function, information characterized by the screening rule $m_c(\theta)$ or $m(\theta)$ as the sole input. This again fits in line with our idea that real production depends on information acquisition. Consequently, efficiency is achieved if and only if the optimal security in the decentralized economy delivers the same equilibrium allocation of input, information acquisition, as what the centralized economy does. If the optimal security achieves this job, the friction in the decentralized economy is not severe as it can be effectively removed by optimal security design. Otherwise, the friction is severe since it cannot be completely removed even if an optimal security is used.

Thanks to the efficiency concept in Definition 2 and Lemma 2, we are able to characterize our pecking order of optimal securities over the efficiency dimension, as follows.

PROPOSITION **7.** *In the decentralized production economy, when the project is financed with a positive probability:*

i) the optimal security $s^*(\theta)$ is debt if and only if the friction of the decentralized economy is not severe in the sense that an optimal security achieves efficiency; and

ii) $s^*(\theta)$ is convertible preferred stock if and only if the friction is severe in the sense that even an optimal security cannot achieve efficiency.

This pecking order against the efficiency dimension is important not only because it distinguishes between the two optimal securities given the characteristics of the production economy, but also because it wraps up the roles of optimal securities and screening in reducing the friction of our economy. In the decentralized economy, real production is performed by the entrepreneur while information acquisition by the investor. The separation is always present and unchanged in spite of different exogenous characteristics of the economy. Hence, the severity of friction is reflected by the extent to which real production depends on information acquisition. If the friction is severe, the dependence is strong, which makes screening worthwhile and thus renders convertible preferred stock as optimal. Similarly, if the friction is not severe, the dependence is weak. As a consequence, the benefit of screening does not justify its cost and thus debt is optimal.

Our pecking orders shed new lights on unifying empirical evidence. They are particularly well suited for private businesses. In practice, debt financing is popular for conventional projects and for investors who have less expertise in screening, that is, when the informational friction is not severe. Instead, financing with convertible preferred stock (or a combination of debt and equity) is common for innovative projects in particular in earlier rounds that need more screening and for investors who are more capable of doing so, that is, when the friction is severe.

5 Comparative Statics of the Optimal Security

To build more intuitions, we numerically perform comparative statics of the shape of optimal securities with respect to three empirical dimensions: the profitability of the project, the uncertainty of the project, and the cost of screening.[39] When the environment varies, the role of screening

[39]We are not aware of any analytical comparative statics pertaining to functionals. An analytical comparative statics requires a total order, which is not applicable for our security space. Even for some ordered characteristics

changes and the way through which the entrepreneur incentivizes screening changes accordingly, which leads to different shapes of optimal securities.

5.1 Profitability of the Project

First, we consider the effects of varying the profitability of the project on the shape of the optimal security $s^*(\theta)$. We fix the project's market prospects, the prior distribution of the cash flow θ, and the cost of screening, μ. Thus, a decrease in the investment requirement k implies that the project is more profitable ex-ante.

We show the results in Figure 5. The investment k takes three increasing values: 0.4, 0.475, and 0.525. The optimal security is debt when $k = 0.4$, and is convertible preferred stock for another two projects when k takes larger values, one with positive ex-ante NPV and the other negative. In particular, the face value $\widehat{\theta}$ and the converting rates $d\widehat{s}(\theta)/d\theta$ of the convertible preferred stock are both increasing in k. For the prior of the cash flow θ, we take a normal distribution with mean 0.5 and standard deviation 0.125, and then truncate and normalize this distribution to the interval $[0, 1]$. The screening cost μ is fixed at 0.2.

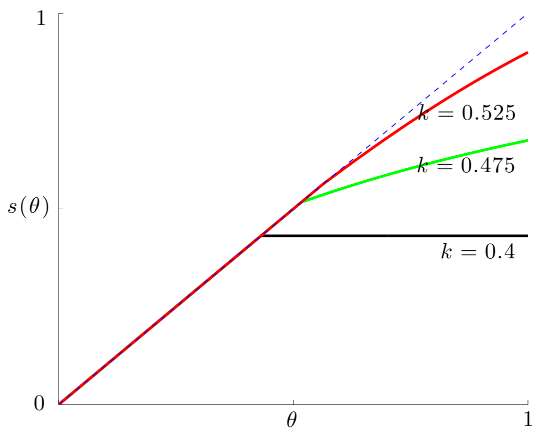

Figure 5: Change of Investment with $\mathbb{E}[\theta] = 0.5, \mu = 0.2$

The comparative statics with respect to the profitability of the project serves as a detailed illustration of Proposition 6. When the project is sufficiently profitable ex-ante, the friction is not severe, and thus it is financed by debt without inducing screening. When the project looks mediocre in terms of its profitability but still has a positive ex-ante NPV, the friction becomes severe, and information acquisition becomes worthwhile to screen out a potentially bad project. When the project is not profitable in the sense that its NPV is negative, the friction is more

of the optimal security, for instance, the face value, analytical comparative statics are not achievable. Thus, we rely on numerical results to deliver intuitions and leave analytical work to future research. Numerical analysis in our framework is tractable but intensively technical, and the algorithm and codes are available upon request.

severe. The only way to finance it is to propose convertible preferred stock and to expect a potentially good project being screened in. Especially, for the last project, screening is more valuable, and thus the entrepreneur is willing to compensate the investor more generously to induce more effective screening.

5.2 Uncertainty of the Project

We consider the effects of varying the project's uncertainty on the optimal security $s^*(\theta)$. Concretely, we consider varying prior distributions of the cash flow θ with the same mean, ranked by second order stochastic dominance.[40] We also fix the investment requirement k as well as the cost of screening μ. Note that, the effect of varying uncertainty cannot be accounted by any argument involving risks, because both the entrepreneur and the investor are risk neutral. Instead, we still focus on the friction and the role of screening to explain these effects.

Interestingly, the comparative statics with respect to the project's uncertainty depend on the sign of its ex-ante NPV. As highlighted in Proposition 6, the role of screening differs when the projects have different signs of NPV. This further leads to different patterns of comparative statics when the uncertainty varies.

First, we consider projects with a positive ex-ante NPV and increasing uncertainty. The results are in Figure 6, the left panel illustrating the priors of the cash flow θ and the right panel the evolution of the optimal security. As shown, the optimal security is debt when the project is the least uncertain. Convertible preferred stock becomes optimal for financing more uncertain projects, the face value $\widehat{\theta}$ and the converting rates $d\widehat{s}(\theta)/d\theta$ both increasing in uncertainty. For the priors, we take normal distributions with mean 0.5 and standard deviations 0.125 and 0.25, and then truncate and normalize them to the interval $[0, 1]$. We also construct a third distribution, in which the project is so uncertain that the cash flow has more chances to take extreme values in $[0, 1]$. The investment is $k = 0.4$, and the cost of screening is $\mu = 0.2$.

The comparative statics in this case demonstrate how varying uncertainty affects the role of screening out a potentially bad project, given a positive ex-ante NPV. When the the project is the least uncertain, it is least likely to be bad, which implies that screening-out is least relevant and thus debt financing is optimal. When the uncertainty becomes greater, the project is more likely to be bad, and screening-out becomes more valuable. Hence, the entrepreneur finds it optimal to propose a more generous convertible preferred stock to induce screening-out.

Then we consider projects with a negative ex-ante NPV. We focus on the projects that may be financed with a positive probability, thanks to screening-in through convertible preferred stock. The results are in Figure 7: both the face value $\widehat{\theta}$ and the converting rate $d\widehat{s}(\theta)/d\theta$ of convertible

[40] There are other ways to measure the project's uncertainty. For comparative statics, our idea is to find a partial order of uncertainty over the space of distributions, while to keep the project's ex-ante NPV constant. Thus, second order stochastic dominance is a natural choice.

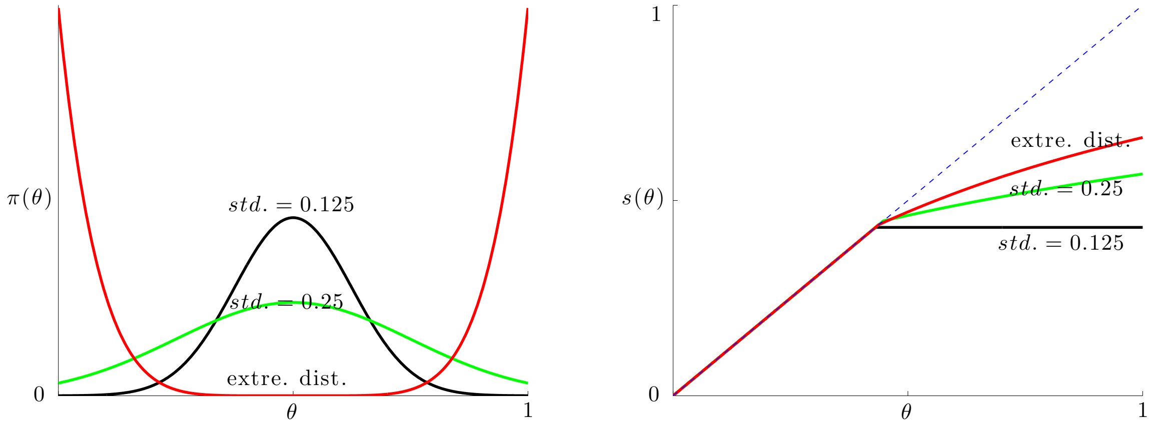

Figure 6: Change of Uncertainty: $k = 0.4 < \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

preferred stock are decreasing in uncertainty. The priors are generated as we did in Figure 6. The investment is $k = 0.525$ and the cost of screening is $\mu = 0.2$.

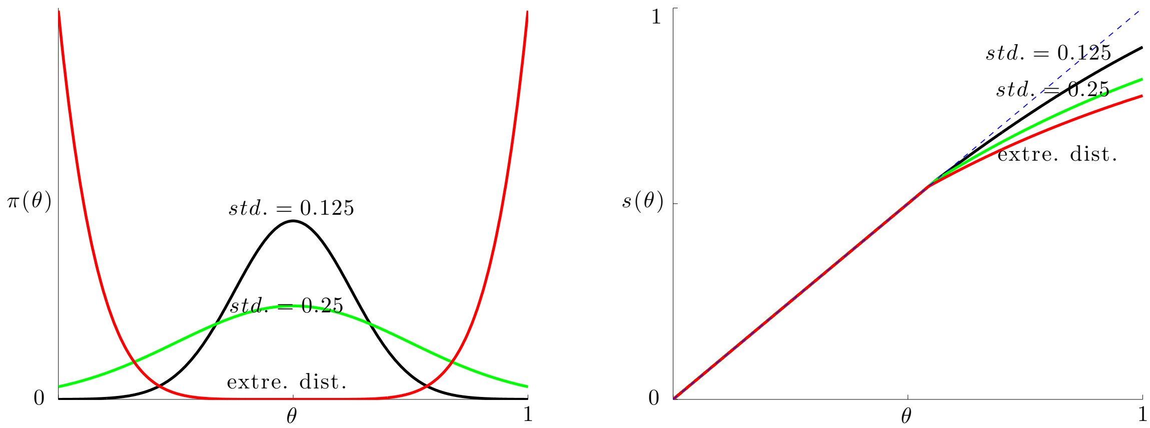

Figure 7: Change of Uncertainty: $k = 0.525 > \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

The comparative statics in this case are also intuitive due to the role of screening-in. Given a negative ex-ante NPV, the investor screens in a potentially good project. In contrast to the case with a positive NPV, when the project is more uncertain, the negative NPV project is instead more likely to be good. Thus, to acquire costly information to screen in a potentially good project becomes less necessary. Therefore, the entrepreneur wants to propose a less generous convertible preferred stock for less costly screening. Not surprisingly, the resulting convertible preferred stock moves away from the 45° line when the project is more uncertain.

5.3 Cost of Screening

We consider the effects of changing the screening cost μ on the optimal security $s^*(\theta)$, with the prior of the cash flow θ and the investment requirement k fixed. The comparative statics again depend on the sign of the project's ex-ante NPV, and fundamentally, the different roles of screening-in and screening-out.

First, we consider a positive ex-ante NPV project with increasing cost of screening. The results are shown in Figure 8. The cost of screening μ takes five increasing values: 0, 0.2, 0.4, 1, and infinity.[41] When μ is low, convertible preferred stock is optimal because the friction is severe and screening-out is called for,[42] while debt is optimal for high values of μ when the friction is not severe. The investment is $k = 0.4$. The prior distribution of the cash flow θ is also fixed: we take a normal distribution with mean 0.5 and standard deviation 2, and then truncate and normalize it to the interval $[0, 1]$.

Interestingly, both the face value $\widehat{\theta}$ or D^* and the converting rates $d\widehat{s}(\theta)/d\theta$ exhibit non-monotonic patterns not present in other comparative statics. On the one hand, $\widehat{\theta}$ first increases in the screening cost μ when the optimal security is convertible preferred stock. but then D^* decreases in μ when the optimal security turns debt with high values of μ. On the other hand, similarly, $d\widehat{s}(\theta)/d\theta$ first increases in μ from zero (degenerate convertible preferred stock) and then decreases to zero (debt) again, pointwisely.

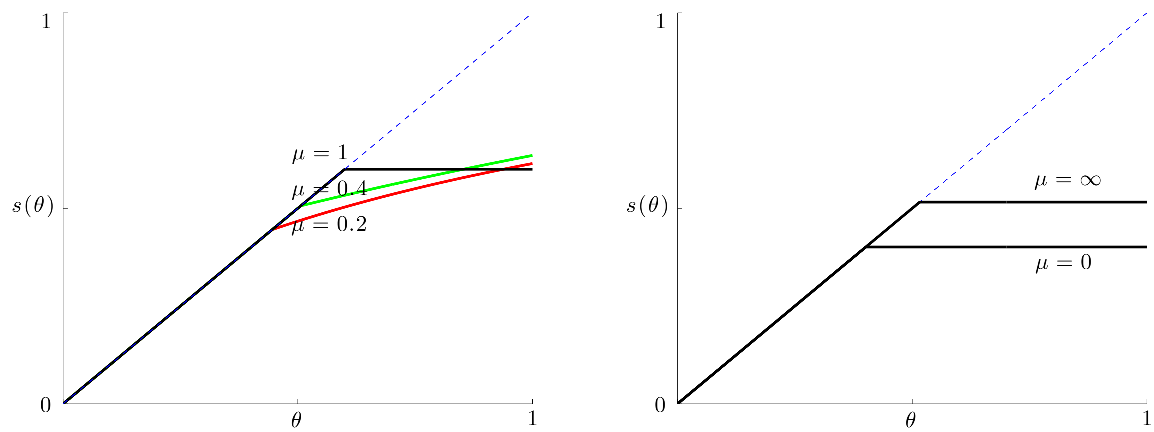

Figure 8: Change of Screening Cost: $k = 0.4 < \mathbb{E}[\theta] = 0.5$

The intuition for the non-monotonic patterns of the face value and the converting rate is more nuanced. We first consider the face value. When the screening cost μ takes high values and the optimal security is debt, the investor does not acquire information and enjoys a premium for accepting the offer for sure. In this case, increasing μ makes the investor less able to acquire

[41] The limiting cases have been characterized in subsection 3.4. We plot them in a separate panel for clarity.

[42] When $\mu = 0$, the optimal convertible preferred stock is degenerate like a debt.

information if she would do so. Thus, the entrepreneur is able to offer the investor a lower premium to ensure her not to acquire information as μ increases. In contrast, when μ takes low values and the optimal security is convertible preferred stock, the investor still enjoys a premium for financing a potentially bad project by chance. In this case, increasing μ makes it harder for the investor to screening out a potentially bad project, so that she would ask for a higher premium for financing a bad project at a higher chance and still accepting the offer at a positive probability.

Regarding the converting rates $d\widehat{s}(\theta)/d\theta$ of convertible preferred stock, its non-monotonicity again stems from the interaction of security design and the role of screening-out. When the screening cost μ takes low values, as μ increases, it becomes necessary for the entrepreneur to compensate the investor more for the higher screening cost. Thus, the entrepreneur proposes higher converting rates. However, when μ takes higher values, the entrepreneur may find incentivizing screening-out too costly. As μ increases, she instead proposes lower converting rates along with a much higher face value (a higher premium), to make the investor screen less intensively but still comfortable to accept the offer. When μ is too high, the investor just gives up and uses debt with a sufficiently high face value to achieve a sure finance without inducing any screening-out.

Finally, we consider a negative NPV project with increasing cost of screening, in which case only convertible preferred stock is optimal and screening-in is called for. The results are in Figure 9. The screening cost μ takes there increasing values: 0.075, 0.125, and 0.225. As shown, both the face value $\widehat{\theta}$ and the converting rate $d\widehat{s}(\theta)/d\theta$ of convertible preferred stock are increasing in μ. The investment is $k = 0.6$. The prior is the same as the last case: a normal distribution with mean 0.5 and standard deviation 2, and then truncated and normalized to $[0, 1]$.

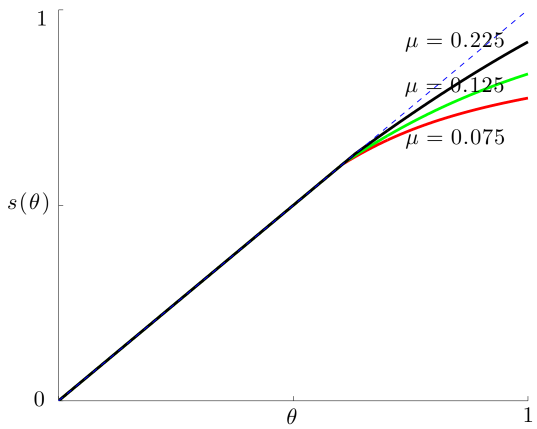

Figure 9: Change of Screening Cost: $k = 0.6 > \mathbb{E}[\theta] = 0.5$

The intuition is again straightforward under the unified account of screening. When financing the project with a negative NPV, the investor acquires information to screen in a potentially good project. As screening-in through convertible preferred stock is the only way to finance a project with a negative ex-ante NPV, the entrepreneur has no other choices but to compensate the investor more when the cost of screening is higher. This results in a more generous convertible preferred stock.

6 Discussion and Extension

In this section, we demonstrate the generality and robustness of our framework by reinterpreting and extending our baseline model.

6.1 Moral Hazard Interpretation of Screening

This subsection reinterprets the investor's screening as a hidden effort, by mapping our setting to the standard principal-agent moral hazard setting. By doing this, we show that our framework admits a new bridge between hidden information and hidden effort, which are often addressed as separate frictions in the contract design literature. This reinterpretation also provides an easy way to understand the pecking order over the efficiency dimension as in Proposition 7.

First, it is instructive to interpret information acquisition, captured by the screening rule $m(\theta)$, as the sole effort in the centralized production economy discussed in subsection 4.2. We could see this from the production function in the entrepreneur's problem (4.10):

$$\mathbb{E}[m(\theta) \cdot (\theta - k)] - \mu \cdot I(m(\theta)) \,,$$

where $m(\theta)$ is the only choice variable (functional). The entrepreneur chooses the optimal screening rule $m_c^*(\theta)$ and achieves the first-best allocation. It corresponds to the first-best allocation in typical moral hazard models if the effort is observable.

In contrast, in our baseline model as a decentralized production economy, we may interpret the investor as a worker, whose hidden effort is to acquire information. This effort is unobservable and non-verifiable, but if determines production, which is observable and verifiable. Parallel to the standard hidden effort models, the entrepreneur (principal) designs an optimal security (contract) $s^*(\theta)$ to induce a screening rule (hidden effort) $m^*(\theta)$ by the investor (agent). This reinterpretation is consistent with our Definition 1 of the equilibrium. It is also consistent with the welfare implications of standard hidden effort models, where the first-best can only be achieved if the optimal effort level is zero. As Proposition 7 indicates, first-best is achieved if and only if the optimal screening rule satisfies $Prob[m^*(\theta) = 1] = 1$, that is, the investor does not acquire information (zero effort) in equilibrium. In this case, the optimal security is debt. On the contrary, when the optimal screening rule satisfies $0 < Prob[m^*(\theta) = 1] < 1$, suggesting information acquisition (positive effort) in equilibrium, the economy only achieves second-best. In this case, the optimal security is convertible preferred stock.

6.2 General Allocation of Bargaining Powers

This subsection extends our baseline model to a more general setting that allows for arbitrary allocation of bargaining powers between the entrepreneur and the investor. It demonstrates that our framework and qualitative results are robust to the allocation of bargain powers.

Without loss of generality, let the entrepreneur's bargaining power in security design be $1-\alpha$ and the investor's α. Suppose a third party in the economy knows α, designs the security and proposes it to the investor. The investor acquire information according to the security and decides whether or not to accept this offer. The third party's objective function is a weighted average of the entrepreneur's and the investor's utilities. The weights represent the bargaining powers of the two, respectively. When $\alpha = 0$, this reduces to our baseline model. The derivations for the results in the baseline model can be recycled here so we do not state them twice.

In this setting, the third-party's objective function, namely, the payoff gain is

$$u_T(s(\theta)) = \alpha \cdot (\mathbb{E}[(s(\theta)-k)\cdot m(\theta)] - \mu \cdot I(m)) + (1-\alpha)\cdot \mathbb{E}[(\theta - s(\theta))\cdot m(\theta)]\,.$$

We can show that, with information acquisition, the equation that governs information acquisition is still as same as condition (3.6):

$$s(\theta) - k = \mu \cdot \left(g'(m_s(\theta)) - g'(\overline{\pi}_s)\right)\ ,$$

while the other that characterizes the optimality of the unconstrained optimal security is now

$$r(\theta) = (2\alpha - 1)\cdot m(\theta) + (1-\alpha)\cdot \mu^{-1}\cdot m(\theta)\cdot(1-m(\theta))\cdot(\theta - s(\theta) + w)\,.$$

The following two propositions characterize the optimal security in the general setting.[43]

Proposition 8. *When $0 \leqslant \alpha < 1/2$ and information acquisition happens in equilibrium, the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding screening rule $\widehat{m}_s(\theta)$ satisfy*

$$\frac{d\widehat{s}(\theta)}{d\theta} = \frac{1-\widehat{m}_s(\theta)}{1-\frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} \in (0,1)$$

and

$$\frac{d\widehat{m}_s(\theta)}{d\theta} = \frac{\mu^{-1}\cdot\widehat{m}_s(\theta)\cdot(1-\widehat{m}_s(\theta))^2}{1-\frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} > 0\ .$$

[43] The proofs for the benchmark model can be recycled for the extended model here, so we do not repeat them in the appendix.

Also, all the results from Lemma 5 to Lemma 8 and from Proposition 1 to Proposition 7 still hold.

Proposition **9.** *When $1/2 \leqslant \alpha \leqslant 1$, the optimal security features $s^*(\theta) = \theta$.*

Our generalized results show that, when the investor has some but not too strong bargaining power in security design, all the qualitative results remain unchanged. Nevertheless, if the investor has strong bargaining power, the optimal security would favor her to the greatest extent: a complete takeover. This is intuitive given our friction point of view. If the entrepreneur dominates, she still plays a considerable role in real production, which depends on the investor's information acquisition. Thus, the presence of friction still calls for a meaningful security design that follows our interaction between the shape of securities and the incentive of screening. On the contrary, if the investor dominates, she may take over the project and effectively remove the friction of the economy. In this case, real production and information acquisition is allied, and security design is of less interest. This corresponds to the fact that buyouts and takeovers are popular for mature and old companies, in which the role of entrepreneurs or founders is no longer inalienable, a point also highlighted in Rajan (2012) and Lerner, Leamon and Hardymon (2012).

7 Conclusion

Highlighting a new informational friction, this paper have investigated security design in a production economy. Real production depends on information acquisition while these two are separately performed by an entrepreneur and an investor. New pecking orders of optimal securities has been predicted: debt, which does not induce screening, is optimal when the dependence is weak and thus the friction is not severe, and convertible preferred stock, which induces screening, is optimal when the dependence is strong and thus the friction is severe. Both the optimal securities and the pecking orders are supported by empirical evidence.

This paper contributes to several fronts of security design and perhaps the broader corporate finance and contract design literature. Information insensitive and sensitive securities have been unified to a new but single origin: in financing different projects with different extents of friction, the investor's expertise of screening is called for in different manners, which further shapes the optimal securities. The comparison between the production economy and an exchange economy further highlights this origin. Thanks to the new concept of flexible information acquisition, we have been able to identify the nature of screening, and work with arbitrary securities on continuous states without any distributional assumptions.

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A Appendix

A.1 Derivation of Convertible Preferred Stock as the Optimal Security

This appendix derives the optimal security $s^*(\theta)$ when it induces information acquisition. To make the intuition clearer, we proceed by two steps.

First, we solve for an "unconstrained" optimal security without the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$.[44] We denote the solution by $\widehat{s}(\theta)$. We also denote the corresponding screening rule by $\widehat{m}_s(\theta)$. The unconstrained optimal security actually recovers the unconstrained part $\widehat{s}(\theta)$ of the eventual optimal security in Proposition 3. After that, we resume the feasibility condition and characterize the optimal security $s^*(\theta)$. This two-step approach streamlines our presentation.

Lemma **3.** *In an equilibrium with information acquisition, the unconstrained optimal security $\widehat{s}(\theta)$ and its corresponding screening rule $\widehat{m}_s(\theta)$ are determined by*

$$\widehat{s}(\theta) - k = \mu \cdot \left(g'\left(\widehat{m}_s(\theta)\right) - g'\left(\overline{\pi}_s^*\right)\right) \ , \tag{A.1}$$

where

$$\overline{\pi}_s^* = \mathbb{E}\left[m_s^*(\theta)\right] \ ,$$

and

$$(1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) = \mu \, , \tag{A.2}$$

where

$$w^* = \mathbb{E}\left[(\theta - s^*(\theta)) \frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right] \left(1 - \mathbb{E}\left[\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\right)^{-1} \ ,$$

in which $\overline{p}_s^*$ and w^* are two constants determined in equilibrium, and $s^*(\theta)$ and $m_s^*(\theta)$ are the solutions of the original constrained problem.

Lemma 3 exhibits the relationship between the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding screening rule $\widehat{m}_s(\theta)$. Condition (A.1) specifies how the investor responses to the unconstrained optimal security by adjusting her screening rule. On the other hand, condition (A.2) is derived from the entrepreneur's optimization problem. It indicates the entrepreneur's optimal choices of payments across states, given the investor's screening rule. In equilibrium, $\widehat{s}(\theta)$ and $\widehat{m}_s(\theta)$ are jointly determined.

Although it is mathematically difficult to solve the system of equations (A.1) and (A.2), we are able to deliver important analytical characteristics of the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding screening rule $\widehat{m}_s(\theta)$.

[44] More precisely, $\widehat{m}_s(\theta)$ is not the solution to the entrepreneur's unconstrained problem (without the feasibility condition), but is a translation of that solution. This will be seen clearer in the statement of Lemma 3.

Lemma 4. *In an equilibrium with information acquisition, the unconstrained optimal security* $\widehat{s}(\theta)$ *and the corresponding screening rule* $\widehat{m}_s(\theta)$ *satisfy*

$$\frac{\partial \widehat{m}_s(\theta)}{\partial \theta} = \mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2 > 0 \ , \tag{A.3}$$

and

$$\frac{\partial \widehat{s}(\theta)}{\partial \theta} = 1 - \widehat{m}_s(\theta) \in (0, 1) \, . \tag{A.4}$$

We have several interesting observations from Lemma 4. First, condition (A.3) implies that the unconstrained optimal screening rule $\widehat{m}_s(\theta)$ is strictly increasing. Second, condition (A.4) implies that the unconstrained optimal security $\widehat{s}(\theta)$ is also strictly increasing. These are because, according to Proposition 1, we have $Prob[0 < \widehat{m}_s(\theta) < 1] = 1$ in this case, and thus the right hand sides of (A.3) and (A.4) are positive. It follows immediately that the residual of the unconstrained optimal security, $\theta - \widehat{s}(\theta)$, is also strictly increasing. Last, the unconstrained optimal security $\widehat{s}(\theta)$ is strictly concave. This is because conditions (A.3) and (A.4) imply

$$\frac{\partial^2 \widehat{s}(\theta)}{\partial \theta^2} = -\mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2 < 0 \ .$$

Therefore, the unconstrained optimal security $\widehat{s}(\theta)$ is an increasing concave function of θ.

The monotonicity of the unconstrained screening rule $\widehat{m}_s(\theta)$ as shown in (A.3) is intuitive. Screening makes sense only when the investor screens in a potentially good project and screens out a potentially bad project. In other words, a better cash flow would be more likely to generate a good signal and result in a successful finance, while a worse cash flow would to more likely to generate a bad signal and result in a rejection. This implies an increasing $\widehat{m}_s(\theta)$.

Moreover, the monotonicity of $\widehat{m}_s(\theta)$ has two important implications on the shape of the unconstrained optimal security $\widehat{s}(\theta)$. On the one hand, according to condition (A.1), $\widehat{s}(\theta)$ is increasing because $\widehat{m}_s(\theta)$ is increasing. This reflects the dependence of real production on information acquisition: the entrepreneur is willing to compensate the investor more in an event of higher cash flow to encourage screening. On the other hand, according to condition (A.2), $\theta - \widehat{s}(\theta)$ is also increasing because $\widehat{m}_s(\theta)$ is increasing. This however reflects the separation of real production and information acquisition: the entrepreneur wants to retain as much as possible. Again as $\widehat{m}_s(\theta)$ is increasing, the competition of the two effects above implies that the least costly way for the entrepreneur to encourage the investor to acquire information is to reward the investor more but also retain more in better states.

Given Lemma 4, it is instructive to have the following lemma to illustrate the possible relative positions between the unconstrained optimal security and the feasibility constraints.

Lemma 5. *Three possible relative positions between the unconstrained optimal security* $\widehat{s}(\theta)$ *and* *the feasibility constraints* $0 \leqslant s(\theta) \leqslant \theta$ *may occur in equilibrium, in the* $\theta \sim s$ *space:*

i) $\widehat{s}(\theta)$ *intersects with the* $45°$ *line* $s = \theta$ *at* $(\widehat{\theta}, \widehat{\theta})$, $\widehat{\theta} > 0$, *and does not intersect with the horizontal axis* $s = 0$*;*

ii) $\widehat{s}(\theta)$ *goes through the origin* $(0, 0)$, *and does not intersect with either the* $45°$ *line* $s = \theta$ *or the horizontal axis* $s = 0$ *for any* $\theta \neq 0$*;*

iii) $\widehat{s}(\theta)$ *intersects with the horizontal axis* $s = 0$ *at* $(\widetilde{\theta}, 0)$, $\widetilde{\theta} > 0$, *and does not intersect with the* $45°$ *line* $s = \theta$.

In the three different cases, it is easy to imagine that the actual optimal security $s^*(\theta)$ will be constrained by the feasibility condition in different ways. For example, $s^*(\theta)$ will be constrained by the $45°$ line $s = \theta$ in Case i) while by the horizontal axis $s = 0$ in Case iii). By imposing the feasibility conditions, we have the following characterization for $s^*(\theta)$:

Lemma **6.** *In an equilibrium with information acquisition, the corresponding optimal security* $s^*(\theta)$ *satisfies*

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad \widehat{s}(\theta) > \theta \\ \widehat{s}(\theta) & \text{if} \quad 0 \leqslant \widehat{s}(\theta) \leqslant \theta \\ 0 & \text{if} \quad \widehat{s}(\theta) < 0 \end{cases},$$

where $\widehat{s}(\theta)$ *is the corresponding unconstrained optimal security.*

Lemma 6 is helpful because it tells us how to construct an optimal security $s^*(\theta)$ from its corresponding unconstrained optimal security $\widehat{s}(\theta)$. Concretely, $s^*(\theta)$ will follow $\widehat{s}(\theta)$ when the latter is within the feasible region $0 \leqslant s \leqslant \theta$. When $\widehat{s}(\theta)$ goes out of the feasible region, the resulting optimal security will follow one of the feasibility constraints that is binding.

We apply Lemma 6 to the three cases of the unconstrained optimal security $\widehat{s}(\theta)$ described in Lemma 5. This gives the three potential cases of the optimal security $s^*(\theta)$, respectively.

Lemma **7.** *In an equilibrium with information acquisition, the optimal security* $s^*(\theta)$ *may take one of the following three forms:*

i) When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ *intersects with the* $45°$ *line* $s = \theta$ *at* $(\widehat{\theta}, \widehat{\theta})$, $\widehat{\theta} > 0$, *we have*

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad 0 \leqslant \theta < \widehat{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta \geqslant \widehat{\theta} \end{cases};$$

ii) When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ *goes through the origin* $(0, 0)$, *we have* $s^*(\theta) = \widehat{s}(\theta)$ *for* $\theta \in \mathbb{R}_+$*;*

iii) When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ *intersects with the horizontal* *axis* $s = 0$ *at* $(\widetilde{\theta}, 0)$, $\widetilde{\theta} > 0$, *we have*

$$s^*(\theta) = \begin{cases} 0 & \textit{if} \quad 0 \leqslant \theta < \widetilde{\theta} \\ \widehat{s}(\theta) & \textit{if} \quad \theta \geqslant \widetilde{\theta} \end{cases} .$$

The optimal security $s^*(\theta)$ takes different shapes in the three potential cases. In Case i), $s^*(\theta)$ follows a debt in states with low cash flows but increases in states with high cash flows. In Case iii), $s^*(\theta)$ has zero payment in states with low cash flows, while is an increasing function in states with high cash flows. Case ii) lies in between as a cut-off case, in which $s^*(\theta)$ is an increasing function.

We proceed by determining whether these three potential cases are valid solutions to the entrepreneur's problem in an equilibrium with information acquisition. Interestingly, not all the three cases can occur in equilibrium.

Lemma **8.** *If the entrepreneur's optimal security* $s^*(\theta)$ *induces the investor to acquire information in equilibrium, then it must follow Case i) in Lemma 7, which corresponds to a participating convertible preferred stock with a face value* $\widehat{\theta} > 0$.

Together with the lemmas already established, Lemma 8 immediately leads to Proposition 3. Intuitively, Case ii) and Case iii) in Lemma 7 cannot sustain an equilibrium with information acquisition because the investor is underpaid. Recall that the investor provides two types of inputs. The first is the investment required to initiate the project, and the second is the costly information to screen the project. As a result, the entrepreneur wants to make sure that the investor is sufficiently compensated for both inputs to be willing to accept the security. This argument is further strengthened by Corollary 2, which suggested that $\widehat{\theta}$ should be larger than the investment requirement k.

A.2 Proofs

This appendix provides all proofs omitted above.

Proof of Lemma 1. We first prove the "only if" part. Suppose that

$$\mathbb{E}\left[\exp(\mu^{-1}(\theta - k))\right] \leqslant 1 .$$

According to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the investor would reject the offer without acquiring information. Since $s(\theta) \leqslant \theta$, the project cannot be initiated in this case.

Then we prove the "if" part. Let $t \in (0,1)$. Since $\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right]$ is continuous in t, there exists $t < 1$ such that

$$\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right] > 1\,.$$

Hence, according to Proposition 1, the security $s_t(\theta) = t \cdot \theta$ would be accepted by the investor with a positive probability. Moreover, let $m_t(\theta)$ be the corresponding screening rule. As $s_t(\theta)$ would be accepted with a positive probability, $m_t(\theta)$ cannot be always zero. Hence, the entrepreneur's expected payoff is $\mathbb{E}[(1-t) \cdot \theta \cdot m_t(\theta)]$, which is strictly positive.

Note that the security $s_t(\theta)$ is a feasible security. Hence, the optimal security $s^*(\theta)$ would also be accepted with a positive probability and delivers positive expected payoff to the entrepreneur. This concludes the proof. □

Proof of Corollary 1. The proof is straightforward following the above proof of Lemma 1. Proposing $s^*(\theta) = \theta$ gives the entrepreneur a zero payoff, while proposing $s_t(\theta) = t \cdot \theta$ constructed in the proof of Lemma 1 gives her a strictly positive expected payoff. This suggests $s^*(\theta) = \theta$ is not optimal. □

Proof of Proposition 2. The Lagrangian of the entrepreneur's problem is

$$\mathscr{L} = \mathbb{E}\left[\theta - s(\theta) + \lambda \cdot \left(1 - \exp\left(\mu^{-1} \cdot (k - s(\theta))\right)\right) + \eta_1(\theta) \cdot s(\theta) + \eta_2(\theta) \cdot (\theta - s(\theta))\right]\,,$$

where λ, $\eta_1(\theta)$ and $\eta_2(\theta)$ are multipliers.

The first order condition is

$$\frac{d\mathscr{L}}{ds(\theta)} = -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (q - s(\theta))\right) + \eta_1(\theta) - \eta_2(\theta) = 0\,. \tag{A.5}$$

We first consider a special case that is helpful for us to solve the optimal security. If $0 < s(\theta) < \theta$, the two feasibility conditions are not binding. Thus $\eta_1(\theta) = \eta_2(\theta) = 0$, and the first order condition is simplified as

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - s(\theta))\right) = 0\,.$$

By rearrangement, we get

$$s(\theta) = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu)\,. \tag{A.6}$$

We denote the right hand side of (A.6), which is irrelevant of θ, as D^*. By definition, we have $D^* > 0$. Also, it is straightforward to have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) = 0\,. \tag{A.7}$$

In what follows, we characterize the optimal solution $s^*(\theta)$ for different regions of θ.

First, we consider the region of $\theta > D^*$. We show that $0 < s^*(\theta) < \theta$ in this region by contradiction.

If $s^*(\theta) = \theta > D^*$, we have $\eta_1(\theta) = 0$ and $\eta_2(\theta) \geqslant 0$. From the first order condition (A.5) we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - \theta)\right) = \eta_2(\theta) \geqslant 0\,. \tag{A.8}$$

On the other hand, as $\theta > D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - \theta)\right)\,. \tag{A.9}$$

Conditions (A.7), (A.8), and (A.9) construct a contradiction. So we must have $s^*(\theta) < \theta$ if $\theta > D^*$.

Similarly, if $s^*(\theta) = 0$, we have $\eta_1(\theta) \geqslant 0$ and $\eta_2(\theta) = 0$. Again from the first order condition (A.5) we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot k\right) = -\eta_1(\theta) \leqslant 0\,. \tag{A.10}$$

On the other hand, as $D^* > 0$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) < -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot k\right)\,. \tag{A.11}$$

Conditions (A.7), (A.10), and (A.11) construct another contradiction. So we must have $s^*(\theta) > 0$ if $\theta > D^*$.

Therefore, we have shown that $0 < s^*(\theta) < \theta$ for $\theta > D^*$. From the discussion above for this specific case, we conclude that $s^*(\theta) = D^*$ for $\theta > D^*$.

We then consider the region of $\theta < D^*$. We show that $s^*(\theta) = \theta$ in this region.

Since $s^*(\theta) \leqslant \theta < D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - s^*(\theta))\right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right)\,. \tag{A.12}$$

From condition (A.7), the right hand side of this inequality (A.12) is zero. Together with the first order condition (A.5), the inequality (A.12) implies that $\eta_1(\theta) = 0$ and $\eta_2(\theta) > 0$. Therefore, we have $s^*(\theta) = \theta$ in this region.

Also, from the first order condition (A.5) and the condition (A.7), it is obvious that $s^*(D^*) = D^*$.

To sum up, the entrepreneur's optimal security without inducing the investor to acquire information features a debt with face value D^* determined by condition (A.6).

We need to check that there exists $D^* > 0$ and the corresponding multiplier $\lambda > 0$ such that

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\cdot\left(\min(\theta,D^*)-k\right)\right)\right]=1\,, \tag{A.13}$$

where D^* is determined by condition (A.6).

Consider the left hand side of condition (A.13). Clearly, it is continuous and monotonically decreasing in D^*. When D^* is sufficiently large, the left hand side of (A.13) approaches $\mathbb{E}\left[\exp\left(-\mu^{-1}\cdot(\theta-k)\right)\right]$, a number less than one, which is guaranteed by the condition (3.3) as well as the feasibility condition $s(\theta)\leqslant\theta$. On the other hand, when $D^*=0$, it approaches $\exp\left(\mu^{-1}\cdot k\right)$, which is strictly greater than one. Hence, there exists $D^*>0$ such that the condition (A.13) holds.

Moreover, from the condition (A.6), we also know that D^* is continuous and monotonically increasing in λ. When λ approaches zero, D^* approaches negative infinity, while when λ approaches positive infinity, D^* approaches positive infinity as well. Hence, for any $D^*>0$ there exists a corresponding multiplier $\lambda>0$.

Last, suppose $D^*\leqslant k$. It is easy to see that this debt would be rejected by the investor due to Proposition 1, a contradiction.

Finally, by condition (3.3) again, since the optimal security $s^*(\theta)$ satisfies

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\cdot\left(s^*(\theta)-k\right)\right)\right]=1\,,$$

Jensen's inequality implies that $\mathbb{E}[s^*(\theta)]>k$ given $\mu>0$. This concludes the proof. □

Proof of Lemma 3. We derive the entrepreneur's optimal security $s^*(\theta)$ and the corresponding unconstrained optimal security $\widehat{s}(\theta)$ through variational methods. Specifically, we characterize how the entrepreneur's expected payoff responds to the perturbation of her optimal security.

Let $s(\theta)=s^*(\theta)+\alpha\cdot\varepsilon(\theta)$ be an arbitrary perturbation of the optimal security $s^*(\theta)$. Note that the investor's optimal screening rule $m_s(\theta)$ appears in the entrepreneur's expected payoff $u_E(s(\cdot))$, according to condition (3.7), and it is implicitly determined by the proposed security $s(\theta)$ through the functional equation (3.6). Hence, we need to first characterize how $m_s(\theta)$ varies with respect to the perturbation of $s^*(\theta)$. Taking derivative with respect to α at $\alpha=0$ for both sides of (3.6) leads to

$$\begin{aligned}\mu^{-1}\varepsilon(\theta) &= g''\left(m_s^*(\theta)\right)\cdot\left.\frac{\partial m_s(\theta)}{\partial\alpha}\right|_{\alpha=0}\\ &\quad -g''\left(\overline{\pi}_s^*\right)\cdot\mathbb{E}\left.\frac{\partial m_s(\theta)}{\partial\alpha}\right|_{\alpha=0}.\end{aligned}$$

Take expectation of both sides and we get

$$\begin{aligned}
&\mathbb{E}\left[\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0}\right] \\
= \ &\mu^{-1}\cdot\left(1-\mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\right]\cdot g''\left(\overline{\pi}_s^*\right)\right)^{-1}\cdot\mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\varepsilon(\theta)\right] .
\end{aligned}$$

Combining the above two equations, for any perturbation $s(\theta)=s^*(\theta)+\alpha\cdot\varepsilon(\theta)$, the investor's screening rule $m_s(\cdot)$ is characterized by

$$\begin{aligned}
\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0} = \ &\mu^{-1}\cdot\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\varepsilon(\theta) \\
&+\frac{\mu^{-1}\cdot\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\cdot\mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\varepsilon(\theta)\right]}{\left(g''\left(\overline{\pi}_s^*\right)\right)^{-1}-\mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\right]} .
\end{aligned} \tag{A.14}$$

We interpret condition (A.14). The first term of the right hand side of (A.14) is the investor's local response to $\varepsilon(\theta)$. It is of the same sign as the perturbation $\varepsilon(\theta)$. When the payment of the security increases at state θ, the investor is more likely to accept the security at this state. The second term measures the investor's average response to perturbation $\varepsilon(\theta)$ over all states. It is straightforward to verify that the denominator of the second term is positive due to Jensen's inequality. As a result, if the perturbation increases the investor's payment on average over all states, she is more likely to accept the security as well.

Now we can calculate the variation of the entrepreneur's expected payoff $u_E(s(\cdot))$, according to condition (3.7). Taking derivative of $u_E(s(\cdot))$ with respect to α at $\alpha=0$ leads to

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0}=\mathbb{E}\left[\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0}(\theta-s(\theta))\right]-\mathbb{E}\left[m_s^*(\theta)\cdot\varepsilon(\theta)\right] . \tag{A.15}$$

Substitute (A.14) into (A.15) and we get

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0}=\mathbb{E}\left[r(\theta)\cdot\varepsilon(\theta)\right] , \tag{A.16}$$

where

$$r(\theta)=-m_s^*(\theta)+\mu^{-1}\cdot\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\cdot\left(\theta-s^*(\theta)+w^*\right) \tag{A.17}$$

and

$$w^*=\mathbb{E}\left[\left(\theta-s^*(\theta)\right)\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\left(1-\mathbb{E}\left[\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\right)^{-1} .$$

Note that w^* is a constant that does not depend on θ and will be endogenously determined in the equilibrium. Besides, $r(\theta)$ is the Frechet derivative of the entrepreneur's expected payoff $u_E(s(\cdot))$ at $s^*(\theta)$, which measures the marginal contribution of any perturbation to the entrepreneur's expected payoff when the security is optimal. Specifically, the first term of (A.17) is the direct contribution of perturbing $s^*(\theta)$ disregarding the variation of $m_s^*(\theta)$, and the second term measures the indirect contribution through the variation of $m_s^*(\theta)$. This Frechet derivative $r(\theta)$ plays an important role in shaping the entrepreneur's optimal security.

To further characterize the optimal security, we discuss the Frechet derivative $r(\theta)$ in detail. Recall that the optimal security would be restricted by the feasibility condition $0 \leqslant s^*(\theta) \leqslant \theta$. Let

$$A_0 = \{\theta \in \Theta : \theta \neq 0, s^*(\theta) = 0\} \ ,$$

$$A_1 = \{\theta \in \Theta : \theta \neq 0, 0 < s^*(\theta) < \theta\} \ ,$$

and

$$A_2 = \{\theta \in \Theta : \theta \neq 0, s^*(\theta) = \theta\} \ .$$

Clearly, $\{A_0, A_1, A_2\}$ is a partition of $\Theta \backslash \{0\}$. Since $s^*(\theta)$ is the optimal security, we have

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0} \leqslant 0$$

for any feasible perturbation $\varepsilon(\theta)$.[45] Hence, condition (A.16) implies

$$r(\theta) \begin{cases} \leqslant 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geqslant 0 & \text{if } \theta \in A_2 \end{cases} . \tag{A.18}$$

According to Proposition 1, when the optimal security $s^*(\theta)$ induces the investor to acquire information, we have $0 < m_s^*(\theta) < 1$ for all $\theta \in \Theta$. Hence, condition (A.18) can be rearranged as

$$\frac{r(\theta)}{m_s^*(\theta)} = -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} \leqslant 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geqslant 0 & \text{if } \theta \in A_2 \end{cases} . \tag{A.19}$$

Recall condition (3.6), given the optimal security $s^*(\theta)$, the investor's optimal screening rule $m_s^*(\theta)$ is characterized by

$$s^*(\theta) - k = \mu \cdot \left(g'(m_s^*(\theta)) - g'(\overline{\pi}_s^*)\right) \ , \tag{A.20}$$

where

$$\overline{\pi}_s^* = \mathbb{E}[m_s^*(\theta)]$$

[45] A perturbation $\varepsilon(\theta)$ is feasible with respect to $s^*(\theta)$ if there exists $\alpha > 0$ such that for any $\theta \in \Theta$, $s^*(\theta) + \alpha \cdot \varepsilon(\theta) \in [0, \theta]$.

is the investor's unconditional probability of accepting the optimal security $s^*(\theta)$. Conditions (A.19) and (A.20) as a system of functional equations jointly determine the optimal security $s^*(\theta)$ when it induces the investor's information acquisition.

Finally, when we focus on the unconstrained optimal security $\widehat{s}(\theta)$, note that is would not be restricted by the feasibility condition. Hence, the corresponding Frechet derivative $r(\theta)$ would be always zero at the optimum. On the other hand, the investor's optimal screening rule would not be affected. As a result, the conditions (A.20) and (A.19) become

$$\widehat{s}(\theta) - k = \mu \cdot \left(g'(\widehat{m}_s(\theta)) - g'(\overline{\pi}_s^*)\right) ,$$

where

$$\overline{p}_s^* = \mathbb{E}\left[m_s^*(\theta)\right] ,$$

and

$$(1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) = \mu ,$$

where

$$w^* = \mathbb{E}\left[(\theta - s^*(\theta)) \frac{g''(\overline{\pi}_s^*)}{g''(m_s^*(\theta))}\right]\left(1 - \mathbb{E}\left[\frac{g''(\overline{\pi}_s^*)}{g''(m_s^*(\theta))}\right]\right)^{-1} ,$$

in which $\overline{p}_s^*$ and w^* are two constants that do not depend on θ. This concludes the proof. □

Proof of Lemma 4. From Lemma 3, $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfies the two equations (A.1) and (A.2). By condition (A.2), we get

$$\widehat{m}_s(\theta) = 1 - \frac{\mu}{\theta - \widehat{s}(\theta) + w^*} . \tag{A.21}$$

Substituting (A.21) into (A.1) leads to

$$\mu^{-1}(\widehat{s}(\theta) - k) = g'\left(\frac{\mu}{\theta - \widehat{s}(\theta) + w^*}\right) - g'(\overline{\pi}_s^*) .$$

Taking derivatives of both sides of the above equation with respect to θ leads to

$$\begin{aligned}
\mu^{-1} \cdot \frac{d\widehat{s}(\theta)}{d\theta} &= g''(\widehat{m}_s(\theta)) \cdot \frac{d\widehat{m}_s(\theta)}{d\theta} \\
&= g''(\widehat{m}_s(\theta)) \cdot \frac{\mu \cdot \left(1 - \frac{d\widehat{s}(\theta)}{d\theta}\right)}{(\theta - \widehat{s}(\theta) + w^*)^2} \\
&= \frac{1 - \frac{d\widehat{s}(\theta)}{d\theta}}{\theta - \widehat{s}(\theta) + w^* - \mu} ,
\end{aligned}$$

where we use

$$g''(x) = \frac{1}{x(1-x)}$$

while deriving the third equality. Rearrange the above equation, and we get

$$\begin{aligned}\frac{d\widehat{s}\,(\theta)}{d\theta} &= \frac{\mu}{\theta-\widehat{s}\,(\theta)+w^*} \\ &= 1-\widehat{m}_s\,(\theta)\ \ ,\end{aligned}$$

where the last equality follows (A.21).

Again, taking derivatives of both sides of the above equation with respect to θ leads to

$$\begin{aligned}\mu^{-1}\cdot\frac{d\widehat{s}\,(\theta)}{d\theta} &= g''\left(\widehat{m}_s\,(\theta)\right)\cdot\frac{d\widehat{m}_s\,(\theta)}{d\theta} \\ &= \frac{1}{\widehat{m}_s\,(\theta)\,(1-\widehat{m}_s\,(\theta))}\cdot\frac{d\widehat{m}_s\,(\theta)}{d\theta}\ .\end{aligned}$$

Hence

$$\begin{aligned}\frac{d\widehat{m}_s\,(\theta)}{d\theta} &= \mu^{-1}\cdot\widehat{m}_s\,(\theta)\cdot(1-\widehat{m}_s\,(\theta))\cdot\frac{d\widehat{s}\,(\theta)}{d\theta} \\ &= \mu^{-1}\cdot\widehat{m}_s\,(\theta)\cdot(1-\widehat{m}_s\,(\theta))^2\,.\end{aligned}$$

This completes the proof. □

Proof of Lemma 5. From Lemma 4, it is easy to see that the slope of $\widehat{s}(\theta)$ is always less than one. Hence, Lemma 5 is straightforward. □

Proof of Lemma 6. We proceed by discussing three cases.

Case 1: We show that $\widehat{s}(\theta)>\theta$ would imply $s^*(\theta)=\theta$.

Suppose $s^*(\theta)<\theta$. Then we have $s^*(\theta)<\widehat{s}(\theta)$. Since both $(s^*(\theta),m_s^*(\theta))$ and $(\widehat{s}(\theta),\widehat{m}_s(\theta))$ satisfy condition (3.6), we must have $m_s^*(\theta)<\widehat{m}_s(\theta)$. Therefore,

$$\begin{aligned}\frac{r(\theta)}{m_s^*(\theta)} &= -1+\mu^{-1}\cdot(1-m_s^*(\theta))\cdot(\theta-s^*(\theta)+w^*) \\ &> -1+\mu^{-1}\cdot(1-\widehat{m}_s(\theta))\cdot(\theta-\widehat{s}(\theta)+w^*) \\ &= 0\ ,\end{aligned}$$

which implies $s^*(\theta)=\theta$, a contradiction.

Note that, the logic for the inequality above is as follows. Since $(\widehat{s}(\theta),\widehat{m}_s(\theta))$ satisfies condition (A.2), we must have $\theta-\widehat{\theta}+w^*>0$. Hence, $\widehat{s}(\theta)>s^*(\theta)$ implies that

$$\theta-s^*(\theta)+w^*>\theta-\widehat{s}(\theta)+w^*>0\,.$$

Also by noting that

$$1-m_s^*(\theta)>1-\widehat{m}_s(\theta)>0\,,$$

we get the inequality above.

Hence, we have $s^*(\theta) = \theta$ in this case.

Case 2: We show that $\widehat{s}(\theta) < 0$ would imply $s^*(\theta) = 0$.

Suppose $s^*(\theta) > 0$. Then we have $s^*(\theta) > \widehat{s}(\theta)$. By similar argument we know that $m_s^*(\theta) > \widehat{m}_s(\theta)$. Therefore,

$$
\begin{aligned}
\frac{r(\theta)}{m_s^*(\theta)} &= -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&< -1 + \mu^{-1} \cdot (1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) \\
&= 0 \, ,
\end{aligned}
$$

which implies $s^*(\theta) = 0$. This is a contradiction. Hence, we have $s^*(\theta) = 0$ in this case.

Case 3: We show that $0 \leqslant \widehat{s}(\theta) \leqslant \theta$ would imply $s^*(\theta) = \widehat{s}(\theta)$.

Suppose $\widehat{s}(\theta) < s^*(\theta)$. Then similar argument suggests $r(\theta)/m_s^*(\theta) < 0$, which implies $s^*(\theta) = 0 < \widehat{s}(\theta)$. This is a contradiction.

Similarly, suppose $s^*(\theta) < \widehat{s}(\theta)$. Similar argument suggests that $r(\theta)/m_s^*(\theta) > 0$, which implies $s^*(\theta) = \theta > \widehat{s}(\theta)$. This is, again, a contradiction. Hence, we have $s^*(\theta) = \widehat{s}(\theta)$ in this case.

This concludes the proof. □

Proof of Lemma 7. Apply Lemma 5 to Lemma 6, then Lemma 7 is straightforward. □

Proof of Lemma 8. We prove by contradiction. Suppose that the last two cases in Lemma 7 can occur in equilibrium. Hence, there exists a $\widetilde{\theta} \geqslant 0$, such that $s^*(\theta) = 0$ when $0 \leqslant \theta \leqslant \widetilde{\theta}$ and $s^*(\theta) = \widehat{s}(\theta)$ when $\theta > \widetilde{\theta}$.

Note that, $s^*(\theta)$ is strictly increasing when $\theta > \widetilde{\theta}$. Also, since we focus on the equilibrium with information acquisition, there must exist a θ'' such that $s^*(\theta'') > k$; otherwise the optimal security would be rejected without information acquisition. Therefore, there exists a $\theta' > \widetilde{\theta}$ such that $s^*(\theta') = \widehat{s}(\theta') = k$. Recall condition (A.1), we have

$$
m_s^*(\theta') = \bar{\pi}_s^* \, .
$$

Moreover, notice that we have $s^*(\theta') \in (0, \theta')$, we have

$$
\begin{aligned}
0 = r(\theta') &= -m_s^*(\theta') + \mu^{-1} \cdot m_s^*(\theta') \cdot (1 - m_s^*(\theta')) \cdot (\theta' - s^*(\theta') + w^*) \\
&= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k + w^*) \\
&= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) + \mathbb{E}[r(\theta)] \, ,
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{E}[r(\theta)] &= -\bar{\pi}_s^* + \mu^{-1}\left(\mathbb{E}\left[\frac{(\theta - s(\theta)) \cdot g''(\bar{\pi}_s^*))}{g''(m(\theta))}\right] / g''(\bar{\pi}_s^*) + w^* \mathbb{E}\left[\frac{1}{g''(m(\theta))}\right]\right) \\
&= -\bar{\pi}_s^* + \mu^{-1}\left(w^* \cdot \left(1 - \mathbb{E}\left[\frac{g''(\bar{\pi}_s^*)}{g''(m(\theta))}\right]\right) / g''(\bar{\pi}_s^*) + w^* \mathbb{E}\left[\frac{1}{g''(m(\theta))}\right]\right) \\
&= -\bar{\pi}_s^* + \frac{\mu^{-1} w^*}{g''(\bar{\pi}_s^*)} \\
&= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot w^* \,.
\end{aligned}
$$

We can express the expectation term $\mathbb{E}[r(\theta)]$ in another way. Note that, for any $\theta \in [0, \widetilde{\theta}]$, by definition we have

$$
\begin{aligned}
r(\theta) &= -m_s^*(\theta) + \mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&= -\widehat{m}_s(\widetilde{\theta}) + \mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \cdot (\theta - 0 - \theta^* + \theta^* + w^*) \\
&= r(\widetilde{\theta}) - \mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \cdot (\widetilde{\theta} - \theta) \\
&= -\mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \cdot (\widetilde{\theta} - \theta) \,.
\end{aligned}
$$

Also, as $s^*(\theta) = \widehat{s}(\theta)$ for any $\theta > \widetilde{\theta}$, we have $r(\theta) = 0$ for all $\theta > \widetilde{\theta}$. Hence,

$$
\mathbb{E}[r(\theta)] = -\mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \,.
$$

Therefore, we have

$$
\begin{aligned}
\mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) &= -\mathbb{E}[r(\theta)] && \text{(A.22)} \\
&= \mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \,. && \text{(A.23)}
\end{aligned}
$$

Now we take the tangent line of $s^*(\theta)$ at $\theta = \widetilde{\theta}$. The tangent line intersects $s = k$ at $\widetilde{\theta}'$, which is given by

$$
\frac{k}{\widetilde{\theta}' - \theta'} = \left.\frac{ds^*(\theta)}{d\theta}\right|_{\theta'} = 1 - \widehat{m}_s(\widetilde{\theta}) \,.
$$

Hence, we have

$$
\widetilde{\theta}' = \widetilde{\theta} + \frac{k}{1 - \widehat{m}_s(\widetilde{\theta})} \,.
$$

Also, note that we have shown that for any $\theta \geqslant \widetilde{\theta}$, we have

$$
\frac{ds^*(\theta)}{d\theta} = \frac{d\widehat{s}(\theta)}{d\theta} = 1 - \widehat{m}_s(\theta) = 1 - m_s^*(\theta) \,.
$$

Hence,

$$\frac{d^2 s^*(\theta)}{d\theta^2} = -\mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta))^2 < 0\,.$$

Therefore, $s^*(\theta)$ is strictly concave for $\theta \geqslant \widetilde{\theta}$, and consequently, we also have $\widetilde{\theta}' < \theta'$. As a result, from conditions (A.22) and (A.23), we have

$$\begin{aligned} \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot (\widetilde{\theta}' - k) &< \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \\ &= \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1-\widehat{m}_s(\widetilde{\theta})} \cdot k \right)\,. \end{aligned}$$

By Jensen's inequality, we get

$$\bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) > \mathbb{E}\left[m_s^*(\theta) \cdot (1 - m^*(\theta)) \right]\,.$$

Therefore, we have

$$\begin{aligned} \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) &> \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1-\widehat{m}_s(\widetilde{\theta})} \cdot k \right) \\ &> \mathbb{E}\left[m_s^*(\theta) \cdot (1 - m^*(\theta)) \right] \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1-\widehat{m}_s(\widetilde{\theta})} \cdot k \right)\,. \end{aligned}$$

Expand the expectation term above and rearrange, we get

$$\begin{aligned} & \widehat{m}_s(\widetilde{\theta})^2 \cdot k \cdot Prob[\theta \leqslant \widetilde{\theta}] + \int_{\widetilde{\theta}}^{+\infty} m_s^*(\theta) \cdot (1 - m_s^*(\theta)) d\Pi(\theta) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1-\widehat{m}_s(\widetilde{\theta})} \cdot k \right) \\ < \;& \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (-\theta) d\Pi(\theta) \\ \leqslant \;& 0\,. \end{aligned}$$

Nevertheless, the left hand side of the above inequality should be positive, which is a contradiction. This concludes the proof. □

Proof of Proposition 4. We first consider the case with a positive transfer price $p > 0$. Suppose the corresponding security $s(\theta) = \theta - p$ is optimal in a generalized security design problem without the non-negative constraint. However, this security can be accommodated by neither Proposition 2 or Proposition 3, which two exclusively characterize the optimal security in the generalized security design problem, a contradiction.

By Corollary 1, we know that the security $s(\theta) = \theta$ that represents transfer with a zero price is not optimal. This concludes the proof. □

Proof of Corollary 2. First, note that $s^*(\theta)$ is strictly increasing and continuous. Also, note that there exists a θ'' such that $s^*(\theta'') > k$; otherwise, the offer will be rejected without information acquisition.

Therefore, there exists an unique θ' such that $s^*(\theta') = k$, which ensures that $m_s^*(\theta') = \bar{\pi}_s^*$, and

$$\begin{aligned} r(\theta') &= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta') + w^*) \\ &= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta')) + \mathbb{E}[r(\theta)] \ . \end{aligned}$$

Note that $\mathbb{E}[r(\theta)] > 0$ and $\theta' - s^*(\theta') \geqslant 0$, we have $\theta' < \widehat{\theta}$. As $\theta' = s^*(\theta') = k$, it follows that $\widehat{\theta} > \theta' = k$. This concludes the proof. □

Proof of Proposition 6. When we have $\mathbb{E}[\theta] \leqslant k$ and

$$\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right] > 1 \,,$$

according to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the security would induce the investor to acquire information and accept it with positive (but less than one) probability. The only optimal security for this case is convertible preferred stock. This concludes the proof. □

Proof of Lemma 2. The "if" part is straightforward, following the definition of efficiency. The "only if" part is ensured by the fact that the optimal screening rule is always unique given a security, established in Proposition 1. □

Proof of Proposition 7. We state a useful lemma to begin. It allow us to focus on the first two types of equilibrium for welfare analysis.

Lemma **9.** *A project is initiated with a positive probability in the decentralized economy if and only if it is initiated with a positive probability in the corresponding centralized economy.*

Proof of Lemma 9. With the objective function (4.10) in the centralized economy, the entrepreneur's optimal screening rule $m_c^*(\theta)$ is characterized by Proposition 1. Specifically, the investor will initiate the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 1] = 1$ if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] \leqslant 1 \,,$$

will skip the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 0] = 1$ if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] \leqslant 1 \,,$$

and will initiate the project with probability $0 < \bar{\pi}_c^* < 1$, $\bar{\pi}_c^* = \mathbb{E}[m_c^*(\theta)]$, if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] > 1 \text{ and } \mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1\,,$$

in which $m_c^*(\theta)$ is determined by

$$\theta - k = \mu \cdot (g'(m_c^*(\theta)) - g'(\bar{\pi}_c^*))\,.$$

It is straightforward to observe that, the project is initiated with a positive probability in the frictionless centralized economy if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1\,. \tag{A.24}$$

Note that, condition (A.24) is the same as condition (3.1) in Lemma 1 that gives the investment criterion in a corresponding decentralized economy. This concludes the proof. $\square$

Thanks to Lemma 9, once we prove the "only if" parts of both cases of debt and convertible preferred stock, the "if" parts would follow.

First, consider the case when $s^*(\theta)$ is debt. In this case, we have $Prob[m_s^*(\theta) = 1] = 1$, and

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (s^*(\theta) - k)\right)] \leqslant 1\,,$$

both from Proposition 1. Since $s^*(\theta) < \theta$ when $\theta > D^*$, it follows that

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] \leqslant 1\,,$$

which implies that $Prob[m_c^*(\theta) = 1] = 1$, also by Proposition 1. Hence, we know that

$$Prob[m_s^*(\theta) = m_c^*(\theta)] = 1\,,$$

which suggests that $s^*(\theta)$, as debt, achieves efficiency, according to Lemma 2.

Second, consider the case when $s^*(\theta)$ is convertible preferred stock that induces information acquisition. In this case, we have $Prob[0 < m_s^*(\theta) < 1] = 1$, and

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (s^*(\theta) - k)\right)] > 1\,,$$

again both from Proposition 1. Since $s^*(\theta) < \theta$ when $\theta > \widehat{\theta}$, the relationship between

$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)]$ and 1 is ambiguous. If

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] \leqslant 1\,,$$

we have $Prob[m_c^*(\theta) = 1] = 1$, and information acquisition is not induced in the centralized economy. It follows that

$$Prob[m_s^*(\theta) = m_c^*(\theta)] \neq 1\,.$$

Otherwise, if

$$\mathbb{E}[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)] > 1\,,$$

suppose we also have $Prob[m_s^*(\theta) = m_c^*(\theta)] = 1$, then according to condition (A.20), we have

$$Prob[s^*(\theta) = \theta] = 1\,,$$

which violates Corollary 1. A contradiction. As a result, from Lemma 2, we know that $s^*(\theta)$, as convertible preferred stock, cannot achieve efficiency. This concludes the proof. □