Attention, Coordination, and Bounded Recall*

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Preliminary

Abstract

I consider a flexible model of coordination under dispersed information where, prior to committing their actions, agents choose the attention to allocate to an arbitrarily large number of information sources. The analysis identifies channels that are responsible for inefficiency in the equilibrium allocation of attention. Finally, the results for the case of perfect recall (where the agents remember the effect of each source on posterior beliefs) are compared to the case of bounded recall (where posterior beliefs are consistent with the allocation of attention, but where agents are unable to recognize the influence of individual sources).

Keywords: endogenous information, strategic complementarity/substitutability, externalities, efficiency, welfare

JEL classification: C72, D62, D83, E50

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1 Introduction

Many strategic interactions occur under incomplete information about relevant fundamentals affecting preferences and technology: For example, firms make real and nominal decisions under limited information about the demand for their products an the cost for their inputs; consumers choose consumption bundles with limited information about their own needs and other consumers' behavior (think of a buyer choosing which platform to join in a multi-sided market); traders choose portfolios with limited information about the profitability of stocks and the riskiness of bonds; voters choose candidates with limited information about their valiance.

Such limited information may either reflect limits on what is known to society as a whole (the long-run profitability of stocks, for example, is unknown to anyone), or individual constraints on the amount of information that each single decision maker can process. Time and cognitive capacity is limited, implying that the information that individuals use for most of their decisions is significantly less precise than what is in the public domain. Furthermore, in many situations of interest, individuals experience difficulty decomposing their own beliefs. While they may remember the sources of information they paid attention to, they may be unable to recall the effect of each single source. Such inability need not be relevant for a single decision maker (as long as posterior beliefs are a sufficient statistics for the information contained in the different sources). However, such inability plays a major role in a strategic setting, for it impacts the decision makers' ability to coordinate with one another.

This paper investigates how attention is allocated to a large number of information sources in the presence of strategic effects. It relates possible inefficiencies in the equilibrium allocation of attention to primitive conditions and shows how the allocation of attention is affected by the inability to perfectly recall the inferences that come from the specific sources of information.

The analysis is conducted within the family of Gaussian-quadratic economies extensively used in the literature (see, e.g., Angeletos and Pavan, 2007, and the references therein). The information structure, instead, is taken from a recent paper by Myatt and Wallace (2012). Agents have access to an arbitrarily large set of information sources. Each source is defined by its "*accuracy*" (equivalently, by the "sender noise", defined to be the precision of the content of the source) and by its "*transparency*" (equivalently, by the extent to which additional attention to the source leads to a marginal reduction in the idiosyncratic interpretation of its content).

Combining the flexibility of the payoff structure of Angeletos and Pavan (2007) with the flexibility of the information structure of Myatt and Wallace (2012) provides an ideal framework to identify possible inefficiencies in the equilibrium allocation of attention. It is also ideal to investigate how the allocation of attention is affected by bounded recall, that is, by the agents' inability to decompose their posterior beliefs into the inferences based on the individual sources of information. Formally, this impossibility implies a measurability constraint on the agents' strategies. Agents allocate attention to various sources of information but their actions (investment and consumption decisions) must be measurable in their posterior beliefs (as opposed to the individual signals that generate them). Such a restriction appears relevant in complex environments, with a large number of sources, where it is unlikely that agents be able to recall the influence of each single source and respond separately to changes in their informational content (see, e.g., Kahneman (1973, 2011), and Kahneman, Slovic and Tversky (1982) for studies documenting such a difficulty).

In the first part of the paper, I consider the case of perfect recall, where agents are able to identify the influence of each source on their posterior beliefs. I show that there exists a unique symmetric equilibrium and is such that any source that receives positive attention is characterized by a ratio between its transparency and its marginal cost of attention exceeding a critical threshold. In the special case where the marginal cost of attention is the same across all sources (as is necessarily the case when the attention cost depends only on the total amount of time spent listening to the sources), the result thus extends the finding in Myatt and Wallace (2012) that only the most transparent sources receive attention to the more flexible payoff specification considered here.

I then compare the equilibrium allocation of attention to the efficient allocation of attention (defined to be the one that maximizes the ex-ante utility of a representative agent). I show that in economies that are efficient in their use of information, possible inefficiencies in the allocation of attention originate in the dispersion of individual actions around the mean action. In particular, the attention allocated to any given source is inefficiently low in economies where agents suffer from the dispersion of individual actions, whereas it is inefficiently high in economies where they benefit from such a dispersion. Likewise, the attention allocated to any given source is inefficiently low in economies where the sensitivity of individual actions to fundamentals under complete information falls short of the first-best level and is inefficiently high in economies where such a sensitivity exceeds the first-best level.

The above results extend findings in Colombo, Femminis and Pavan (2013) to the more general information structure considered in the present paper. The most interesting result, however, pertains economies in which inefficiencies in the allocation of attention originate in the equilibrium use of information. I show that, when agents are excessively concerned about aligning their actions with the actions of others, they allocate too much attention to sources that are highly transparent and too little attention to sources whose transparency is low but whose accuracy is high.

The above results can be interpreted by understanding that what creates a discrepancy between the equilibrium and the efficient allocation of attention is the interaction of two forces: (i) the value that each agent assigns to reducing the dispersion of her actions around the mean action relative to the value that the planner assigns to the same reduction (as shown in Colombo, Femminis and Pavan (2013), the planner takes into account externalities that the individual fails to internalize); and (ii) the reduction in the dispersion of individual actions that obtains when agents respond to the increase in attention by following the equilibrium strategy, relative to the reduction that obtains when they respond by following the efficient strategy (as shown in Angeletos and Pavan (2007), inefficiencies in the equilibrium use of information in turn originate in a discrepancy between the equilibrium and the efficient value of coordination). The above results build on these insights and adapt them to the general information structure considered in the paper.

In the second part of the paper, which contains the core results, I then show how the above conclusions are affected by the inability to decompose posterior beliefs. The first insight is that, with bounded recall, the benefit that each individual assigns to an increase in the attention allocated to any given source combines the familiar reduction in the dispersion of her action around the mean action (Colombo, Femminis and Pavan (2013)), with the change in the distribution of the individual's own average action around its complete-information counterpart. This second effect is absent under perfect recall and has important implications. Compared to the benchmark with full recall, agents reduce the attention allocated to sources of low and high publicity (these are sources or relatively low and high transparency) and increase the attention they allocate to sources of intermediate publicity.

To understand the result, observe that sources of low publicity are sources whose ratio between transparency and accuracy is low. They may serve well in predicting the fundamentals but are poor coordination devices because of their high sensitivity to idiosyncratic interpretations. In a world of perfect recall, paying a lot of attention to such sources is justified by the possibility to identify their effect on posterior beliefs thus limiting the impact of such idiosyncratic interpretations on the dispersion of individual actions around the mean action. Such a possibility is absent with bounded recall.

Sources of high publicity, instead, are sources whose ratio between transparency and accuracy is high. They may not serve well in predicting the underlying fundamentals but are powerful coordination devices. With bounded recall, however, paying a lot of attention to such sources may lead to a high volatility of an agent's average action around its complete-information counterpart. Because such a volatility contributes negatively to payoffs, agents marginally cut the attention allocated to such sources to redirect it towards sources of intermediate publicity.

I conclude by investigating how bounded recall affects the (in)efficiency of the equilibrium allocation of attention. Inefficiencies now originate not only in the discrepancy between the private and the social value of reducing the dispersion of individual actions in the cross section of the population but also in the discrepancy between the private and the social value in reducing the dispersion of average actions around their complete-information counterparts. Despite these novel effects, economies in which agents value coordination more than the planner continue to feature an excessively high allocation of attention to sources whose publicity is higher than average and an excessively low attention to sources whose publicity is lower than average. The opposite conclusion holds in economies where agents value coordination less than the planner.

The rest of the paper is organized as follows. I briefly review the pertinent literature below. Section 2 contains all results for the case with full recall, while Section 3 contains the results for the case of bounded recall. Section 4 concludes. All proofs are in the Appendix at the end of the document.

1.1 Related literature

The paper belongs to the literature on information acquisition in coordination games. The closest works are Myatt and Wallace (2012) and Colombo, Femminis and Pavan (2013). The first paper shares with the present one the specification of the information structure but considers a more restrictive payoff specification which is meant to capture strategic interactions resembling Keynes' beauty contests. Allowing for a more flexible payoff specification is essential to the questions addressed in this paper. In fact, the beauty-contest specification of Myatt and Wallace (2012) makes the game a potential game where the potential function is social welfare. This specification is appropriate for the analysis in that paper (which is primarily positive), but does not permit one to identify sources of inefficiency in the equilibrium allocation of attention.

The payoff specification in the present paper is the same as in Colombo, Femminis and Pavan (2013)—which in turn is the same as in Angeletos and Pavan (2007, 2009). The information structure is, however, more general. That paper identifies sources of inefficiency in the acquisition of private information in an environment with a single perfectly private and a single perfectly public signal. The precision of the public signal is exogenous and processing public information is costless. In contrast, in the present paper, agents have access to an arbitrary large number of information sources; processing the information of each source is costly (albeit the cost may vary with the source); each source is defined by its accuracy and its transparency; and the publicity of a source is determined endogenously by the attention allocated to the source by all decision makers. This richer information structure permits one to identify which dimension (accuracy versus transparency) is favored in equilibrium, how this selection depends on the payoff structure, and of whether efficiency requires more or less weight to each of these dimensions. It is also instrumental to the analysis of the effects of bounded recall on the allocation of attention.

Related are also Hellwig and Veldkamp (2009), Chahrour (2012), and Llosa and Venkateswaran (2013). Hellwig and Veldkamp are the first to examine how complementarities in actions lead to complementarities in information acquisition. The information structure in that paper is different from the one in the present paper in that it assumes that the publicity of each source (the extent to which its noise is correlated across agents) is exogenous and that the attention allocated to each source is a binary choice. This last property is responsible for equilibrium multiplicity. In contrast, the (symmetric) equilibrium is unique in this paper, as well as in the other papers cited above.

Chahrour (2012) studies optimal central bank disclosures in an economy where processing information is costly and where agents may miscoordinate on which sources they pay attention to. The framework in that paper and the questions addressed are quite different from the ones in the present paper.

Llosa and Venkateswaran (2012) compare the equilibrium acquisition of private information to the efficient acquisition of private information in three business-cycles models. As in the present paper, they find that efficiency in the use of information is no guarantee of efficiency in the acquisition. That paper, however, does not seek to identify general sources of inefficiency in the acquisition process and instead focuses on the comparison of different business cycle specifications.

All works mentioned above consider games with a continuum of actions and continuous payoffs. Information acquisition in games of regime change (where payoffs are discontinuous and players have binary actions) is examined in Szkup and Trevino (2013) and in Yang (2013). The first paper considers a canonical information structure with one perfectly private and one perfectly public signal and where the latter is costless to process. The latter considers a very flexible information structure and shows how the possibility to learn asymmetrically across states (which is particularly appealing in discontinuous games) leads to multiple equilibria.

The paper is also related to the literature on rational inattention as pioneered by Sims (see, e.g., Sims (2003, 2011) for an overview and Maćkowiak and Wiederholt (2009) for an influential business cycle application). The closest paper is Maćkowiak and Wiederholt (2012). That paper compares the equilibrium allocation of attention to the efficient allocation of attention assuming that decision makers can absorb any information as long as the reduction in entropy is below a given capacity threshold. In contrast, in the present paper, I consider a smooth cost function. The information structure is also different and permits me to investigate which dimension (transparency versus accuracy) receives more weight in equilibrium and whether such weight is socially efficient.

Importantly, none of the above papers considers the effects of bounded recall on the allocation of attention, which is the focus of this paper and its distinctive contribution.

Finally, the paper is related to the literature that investigates the effects of limited memory on individual decision making (see, e.g., Mullainathan (2002), Wilson (2004), and Kocer (2010)). This literature, however, does not investigate how bounded memory influences the allocation of attention in a strategic setting, nor the discrepancy between the equilibrium and the efficient allocation of attention. The effects of bounded recall in strategic interactions are examined in the literature on dynamic (and repeated) games with imperfect information (see, e.g., Mailath and Samuelson (2006) and the references therein). The formalization of bounded recall as well as the questions addressed in that literature are however very different from what I do in the present paper.

2 Perfect Recall

2.1 Environment

Agents, Attention, and Information. The economy is populated by a continuum of agents of measure one, indexed by i and uniformly distributed over $[0,1]$. Each agent i has access to $N \in \mathbb{N}$ sources of information about an underlying fundamental variable θ which is drawn from a Normal distribution with mean zero and precision $\pi_\theta \equiv \sigma_\theta^{-2}$ (σ_θ^2 is thus the variance of the distribution).[1]

[1] That the prior mean is zero simplifies the formulas, without any important effect on the results.

The information contained in each source $n = 1, ..., N$ is given by

$$y_n = \theta + \varepsilon_n,$$

where ε_n is normally distributed, independent of θ and of any ε_s, $s \neq n$, with mean zero and precision η_n. The parameter η_n thus controls for the quality of the n-th source of information. By paying attention $z^i \equiv (z_n^i)_{n=1}^N \in \mathbb{R}_+^N$ to the various sources, each agent $i \in [0, 1]$ then receives $x^i \equiv (x_n^i)_{n=1}^N \in \mathbb{R}^N$ private signals about θ with each signal $n = 1, ..., N$ given by

$$x_n^i = y_n + \xi_n^i,$$

where ξ_n^i is idiosyncratic noise, normally distributed, with mean zero and precision $t_n z_n^i$, independent of θ, $\varepsilon \equiv (\varepsilon_n)_{n=1}^N$, and of ξ_s^j, with $s = 1, ..., N$ for $j \neq i$, and with $s = 1, ..., n-1, n+1, ..., N$ for $j = i$. The parameter $\eta_i \in \mathbb{R}_+$ thus proxies the *accuracy* of the source (the "sender's noise" in the language of Myatt and Wallace (2012)), whereas the parameter t_n proxies the *transparency* of the source (the extent to which a marginal increase in the attention allocated to the source reduces its idiosyncratic interpretation by the agents).

Actions and Payoffs. Let $k^i \in \mathbb{R}$ denote agent i's action, $K \equiv \int_j k^j dj$ the mean action, and $\sigma_k^2 \equiv \int_j \left[k^j - K\right]^2 dj$ the dispersion of individual actions in the cross-section of the population. Each agent's payoff is given by the (expectation of) the Bernoulli utility function

$$u\left(k^i, K, \sigma_k, \theta\right) - C(z^i)$$

where $C(z^i)$ denotes the attention cost incurred by the agent. I assume that C is increasing, convex and continuously differentiable.[2]

As is standard in the literature, I assume that U is approximated by a second-order polynomial and that dispersion σ_k has only a second-order non-strategic external effect, so that $u_{k\sigma} = u_{K\sigma} = u_{\theta\sigma} = 0$ and that $u_\sigma(k, K, 0, \theta) = 0$, for all (k, K, θ).[3] The quadratic specification of the utility function ensures the linearity of the agents' best responses and simplifies the analysis.

In addition to the above conditions, I assume that partial derivatives satisfy the following conditions: (i) $u_{kk} < 0$, (ii) $\alpha \equiv -u_{kK}/u_{kk} < 1$, (iii) $u_{kk} + 2u_{kK} + u_{KK} < 0$, (iv) $u_{kk} + u_{\sigma\sigma} < 0$,

[2] As explained in Myatt and Wallace (2012), the assumption that C is convex need not be compatible with an entropy-based cost function (that is, a cost function increasing in the coefficient of mutual information between y and x^i, as assumed in certain models of rational inattention). With that type of cost-function, equilibrium uniqueness cannot be guaranteed for sufficiently high degrees of coordination. However, even in that case, social welfare continues to be concave in the allocation of attention, meaning that the efficient allocation of attention remains unique. Besides, all our key results pertaining (a) the comparison between the equilibrium allocation of attention and the efficient allocation of attention and (b) the comparison of the equilibrium with full recall and the equilibrium with bounded recall are established by looking at the *gross* private benefit of increasing the attention allocated to any given source. As such, all these results extend to a situation where the attention cost is concave.

[3] The notation u_k denotes the partial derivative of u with respect to k, whereas the notation u_{kK} denotes the cross derivative with respect to k and K. Similar notation applies to the other arguments of the utility function.

and (v) $u_{k\theta} \neq 0$. Condition (i) imposes concavity at the individual level, so that best responses are well defined. Condition (ii) implies that the slope of best responses is less than one, which in turn guarantees uniqueness of the equilibrium actions, for any given allocation of attention. Conditions (iii) and (iv) guarantee that the first-best allocation is unique and bounded. Finally, Condition (v) ensures that the fundamental θ affects equilibrium behavior, thus making the analysis non-trivial.

Timing. Agents simultaneously choose the attention they allocate to the various sources of information. They then receive private information. Finally, they simultaneously commit their actions.

2.2 The equilibrium allocation of attention

To solve for the equilibrium allocation of attention, I start by describing how an agent's action is affected by the quality of her information, which in turn is affected by the attention allocated to the different sources of information. These first steps parallel the analysis in Angeletos and Pavan (2009, Proposition 3).

First note that, under complete information about θ, the unique equilibrium features each agent taking an action $k^i = \kappa$ where $\kappa \equiv \kappa_0 + \kappa_1\theta$ with $\kappa_0 \equiv \frac{-u_k(0,0,0,0)}{u_{kk}+u_{kK}}$ and $\kappa_1 \equiv \frac{-u_{k\theta}}{u_{kk}+u_{kK}}$. Now consider the problem of an agent $j \in [0,1]$ with information x^j who allocated attention z^j to the various sources of information. Optimality requires that the agent's action k^j satisfies

$$k^j = \mathbb{E}[(1-\alpha)\kappa + \alpha K \mid z^j, x^j], \tag{1}$$

where $\alpha \equiv \frac{u_{kK}}{|u_{kk}|}$ measures the slope of individual best responses to aggregate activity.

Now suppose that all agents allocate attention z to the various sources of information. The *precision* of each source $s = 1, ..., N$ is then given by

$$\pi_s \equiv \frac{\eta_s z_s t_s}{z_s t_s + \eta_s}$$

which is increasing in the accuracy η_s of the source, in its transparency t_s, and in the attention z_s allocated to it.

Next, let

$$\varphi_s^j \equiv \varepsilon_s + \xi_s^i$$

denote the total noise in the information that agent j receives from source s, and denote by

$$\rho_s \equiv corr(\varphi_s^j, \varphi_s^i) = \frac{z_s t_s}{z_s t_s + \eta_s}$$

the correlation in the noise among any two different agents $i, j \in [0,1]$, $i \neq j$. Using the terminology in Myatt and Wallace (2012), hereafter I will refer to ρ_s as to the source's *endogenous publicity*. Finally, let

$$C'_n(z) \equiv \frac{\partial C(z)}{\partial z_n}$$

denote the marginal cost of increasing the attention allocated to the n-th source of information. The following result is then true:

Proposition 1 *There exists a unique symmetric equilibrium. In this equilibrium, the attention $\hat{z}$ that each agent $i \in [0,1]$ allocates to the various sources of information is such that, for any source $n = 1, ..., N$ that receives strictly positive attention*[4]

$$C_n'(\hat{z}) = \frac{|u_{kk}|}{2} \frac{(\kappa_1 \gamma_n(\hat{z}))^2}{(\hat{z}_n)^2 t_n} \tag{2}$$

where

$$\gamma_n(z) \equiv \frac{\frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}}{\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha)\pi_s(z)}{1-\alpha\rho_s(z)}} \text{ with } \pi_s(z) = \frac{\eta_s z_s t_s}{z_s t_s + \eta_s} \text{ and } \rho_s(z) = \frac{\pi_s(z)}{\eta_s},\ s = 1, ..., N. \tag{3}$$

Given the equilibrium allocation of attention $\hat{z}$, the equilibrium actions are given by

$$k^i = k(x^i; \hat{z}) = \kappa_0 + \kappa_1 \left(\sum_{n=1}^N \gamma_n(\hat{z}) x_n^i \right) \text{ all } i \in [0,1], \text{ all } x^i \in \mathbb{R}^N. \tag{4}$$

To understand the result, note that, when all agents follow the strategy in (4), in equilibrium, the dispersion of individual actions in the cross section of the population is given by

$$Var[k - K \mid z,\ k(\cdot; z)] = \kappa_1^2 \sum_{s=1}^N \frac{\gamma_s^2(z)}{z_s t_s}.$$

Differentiating $Var[k - K \mid z, k(\cdot; z)]$ with respect to z_n while keeping fixed the strategy $k(\cdot; z)$ as defined in (4) (for all agents, including agent i), then reveals that the private benefit of increasing the attention allocated to each source of information is given by

$$\frac{|u_{kk}|}{2} \left| \frac{\partial}{\partial z_n} Var[k - K \mid z,\ k(\cdot; z)] \right| = \frac{|u_{kk}|}{2} \frac{(\kappa_1 \gamma_n(z))^2}{(z_n)^2 t_n}. \tag{5}$$

In other words, in equilibrium, the marginal benefit that each agent assigns to paying more attention to any given source of information coincides with the marginal reduction in the dispersion of the individual's action around the mean action, weighted by the importance $|u_{kk}|/2$ that the individual assigns to such a reduction. Importantly, the reduction in dispersion is computed by holding fixed the equilibrium strategy $k(\cdot; z)$. This is intuitive, given that, from the usual envelope arguments, the individual expects her information to be used optimally once collected. As I show below, this interpretation helps understanding the sources of inefficiency in the equilibrium allocation of attention.

[4] For any source that receives no attention, the following condition must hold:

$$C_n'(\hat{z}) \geq \frac{|u_{kk}|}{2} \frac{(\kappa_1)^2 (1-\alpha)^2 t_n}{\left[\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha)\eta_s \hat{z}_s t_s}{(1-\alpha)\hat{z}_s t_s + \eta_s}\right]^2}.$$

Also note that, fixing the equilibrium allocation of attention $\hat{z}$, the influence $\kappa_1\gamma_n$ that each source exerts on the equilibrium actions always increases with the source's precision (given by π_n) and it increases with the source's endogenous publicity (given by ρ_n) when agents value positively aligning their actions with the actions of others (i.e., when $\alpha > 0$), whereas it decreases when they value it negatively (i.e., when $\alpha < 0$). In turn, both the precision π_n and the publicity ρ_n of any given source increase with the source's accuracy η_n and with its transparency t_n. Finally, note that when $\alpha \to 0$ the sensitivity of the equilibrium actions to each source of information converges to $\kappa_1\delta_n$ with

$$\delta_n \equiv \frac{\pi_n}{\pi_\theta + \sum_{s=1}^N \pi_s}.$$

This makes sense, for this limit corresponds to a single decision maker's problem, in which case the relative influence of any two sources of information converges to their relative informativeness as captured by the ratio between the two sources' precisions. In contrast, when $\alpha \to 1$, $\gamma_n \to 0$ for all $n = 1, ..., N$. This also makes sense: as the agents' concern about aligning their actions with the actions of others grows large, they ignore all sources of information that contain idiosyncratic noise and simply base their actions on the common prior.

I now turn to the equilibrium allocation of attention. To facilitate the intuition, consider the case where $\pi_\theta = 0$ (this corresponds to an improper prior over the entire real line) and where the attention cost depends on z only through the total attention assigned to the various sources of information. That is, assume that there exists a strictly increasing, differentiable, convex function $c : \mathbb{R}_+ \to \mathbb{R}_+$ such that, for any $z \in \mathbb{R}_+^N$, $C(z) = c\left(\sum_{s=1}^N z_s\right)$. The relative attention allocated to any two sources of information $n, n' \in \{1, ..., N\}$ that receive strictly positive attention in equilibrium is then given by

$$\frac{\hat{z}_n}{\hat{z}_{n'}} = \frac{\gamma_n}{\gamma_{n'}}\sqrt{\frac{t_{n'}}{t_n}}$$

Substituting for γ_n and $\gamma_{n'}$, I then have that

$$\hat{z}_n = \frac{\eta_n}{t_n}\left(\frac{1}{\eta_{n'}}\sqrt{t_{n'}t_n}\hat{z}_{n'} + \frac{\sqrt{t_n} - \sqrt{t_{n'}}}{(1-\alpha)\sqrt{t_{n'}}}\right) \tag{6}$$

Any two sources with the same transparency thus receive attention proportional to their accuracy. More generally, (6) suggests that the attention that a source receives in equilibrium is increasing in its accuracy, but nonmonotone in its transparency. The intuition is the following. When transparency is low, paying a lot of attention to a source is not worth the cost, given that the reduction in the idiosyncratic interpretation of its content is small. Likewise, when transparency is high, reducing the attention allocated to the source leads only to a small increase in its idiosyncratic interpretation. As a result, attention is maximal for intermediate degrees of transparency.

This intuition is confirmed in the following proposition which extends results in Myatt and Wallace (2012) to the more flexible payoff specification considered here (note that the result does not require any assumption on the cost function, in addition to those assumed in the model setup):

Corollary 1 *There exists a threshold $R > 0$ such that, in the unique symmetric equilibrium, for any source that receives positive attention*

$$\frac{t_n}{C'_n(\hat{z})} > R,$$

whereas for any source that receives no attention $t_n/C'_n(\hat{z}) \leq R$.

In the special case where the cost is linear and all sources receive positive attention in equilibrium, I can arrive to a close-form characterization for the attention allocated to the various sources of information.

Example 1 *Suppose that $C(z) = \bar{c} \cdot \sum_{s=1}^{N} z_s$ for some $\bar{c} \in \mathbb{R}_{++}$ and assume that $\bar{c}$ is sufficiently small that all sources receive positive attention in equilibrium. The attention that each source receives is then given by*

$$\hat{z}_n = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)} \left[\frac{(1-\alpha)\kappa_1 \sqrt{\frac{|u_{kk}|}{2\bar{c}}} + \sum_{s=1}^{N} \frac{\eta_s}{\sqrt{t_s}}}{\pi_\theta + \sum_{s=1}^{N} \eta_s} - \frac{1}{\sqrt{t_n}} \right]. \tag{7}$$

The example illustrates the general properties discussed above that attention is increasing in accuracy but nonmonotone in transparency. It also shows that, under the assumed cost function, as the value of coordination α increases, the attention allocated to sources of low transparency decreases, whereas the attention allocated to sources of high transparency increases.[5] Finally, it shows that the total amount of attention decreases with the coordination motive, α.[6]

2.3 The efficient allocation of attention

I now turn to the allocation of attention that maximizes the ex-ante utility of a representative agent, hereafter referred to as the efficient allocation of attention.

First, observe that, for any allocation of attention z, the efficient use of information consists in all agents following the unique strategy $k^*(\cdot; z)$ that solves the functional equation[7]

$$k\,(x; z) = \mathbb{E}\left[(1-\alpha^*)\,\kappa^* + \alpha^* K \;\mid z, x\right] \text{ for all } (z, x), \tag{8}$$

[5] Formally,

$$\frac{\partial \hat{z}_n}{\partial \alpha} < 0 \text{ if } \sqrt{t_n} \leq \left(\frac{\pi_\theta + \sum_{s=1}^{N} \eta_s}{\sum_{s=1}^{N} \frac{\eta_s}{\sqrt{t_s}}} \right) \text{ and } \frac{\partial \hat{z}_n}{\partial \alpha} > 0 \text{ if the previous inequality is reversed.}$$

[6] This is not immediate to see, but can be verified by differentiating $\hat{Z} = \sum_n \hat{z}_n$ with respect to α and using the property that

$$\left(\sum_{j=1}^{N} \frac{\eta_j}{\sqrt{t_j}} \right)^2 \leq \sum_{j=1}^{N} \frac{\eta_j}{t_j} \sum_{s=1}^{N} \eta_s.$$

[7] The result about the efficient use of information follow from Angeletos and Pavan (2009).

where $\kappa^* = \kappa_0^* + \kappa_1^* \theta$ is the first-best allocation[8], $K = \mathbb{E}\left[k\left(x; z\right) \mid z, \theta, \varepsilon\right]$ is the average action, and

$$\alpha^* \equiv \frac{u_{\sigma\sigma} - 2u_{kK} - u_{KK}}{u_{kk} + u_{\sigma\sigma}} \tag{9}$$

is the socially optimal degree of coordination (that is, the level of complementarity, or substitutability, that the planner would like the agents to perceive in order for the equilibrium of the economy to coincide with the efficient allocation.) Because (8) differs from the equilibrium optimality condition (1) only by the fact that α is replaced by α^* and κ by κ^*, it is then immediate that the efficient strategy takes the linear form

$$k^*(x; z) = \kappa_0^* + \kappa_1^* \left(\textstyle\sum_{n=1}^{N} \gamma_n^*(z) x_n \right), \tag{10}$$

where $\gamma_n^*(z)$ is defined as $\gamma_n(z)$ but with α^* replacing α.

Next note that, for any given attention z, welfare under the efficient use of information $k^*(\cdot; z)$ can be expressed as

$$w^*(z) \equiv \mathbb{E}[u(\kappa^*, \kappa^*, 0, \theta)] - \mathcal{L}^*(z) - C(z),$$

where $\mathbb{E}[u(\kappa^*, \kappa^*, 0, \theta)]$ is expected welfare under the first-best allocation and where

$$\mathcal{L}^*(z) \equiv \frac{|u_{kk} + 2u_{kK} + u_{KK}|}{2} Var[K - \kappa^* \mid z, k^*(\cdot; z)] + \frac{|u_{kk} + u_{\sigma\sigma}|}{2} Var[k - K \mid z, k^*(\cdot; z)]$$

combines the welfare losses that derive from the volatility of the average action K around its first-best counterpart with the losses that derive from the dispersion of individual actions in the cross section of the population.

I now turn to the efficient allocation of attention. This is instrumental to the understanding of what inefficiency, if any, arises in the way attention is allocated in equilibrium, and on how such inefficiency relates to the way information is then used in equilibrium.

Using the envelope theorem and observing that, holding constant the strategy $k^*(\cdot; z)$, the volatility of the aggregate action around its complete-information counterpart, $Var[K - \kappa^* \mid z, k^*(\cdot; z)]$, is independent of the attention allocated to the various sources of information, I then have that the (gross) social benefit of increasing the attention allocated to any source n is given by[9]

$$\frac{|u_{kk} + u_{\sigma\sigma}|}{2} \left| \frac{\partial}{\partial z_n} Var[k - K \mid z, k^*(\cdot; z)] \right| = \frac{|u_{kk} + u_{\sigma\sigma}|}{2} \frac{\left(\kappa_1^* \gamma_n^*(z)\right)^2}{\left(\hat{z}_n\right)^2 t_n} \tag{11}$$

[8] The scalars κ_0^* and κ_1^* are given by $\kappa_0^* = \frac{u_k(0,0,0) + u_K(0,0,0)}{-(u_{kk} + 2u_{kK} + u_{KK})}$ and $\kappa_1^* = \frac{u_{k\theta} + u_{K\theta}}{-(u_{kk} + 2u_{kK} + u_{KK})}$, respectively.

[9] As in the equilibrium case, the expression in (11) applies to sources that receive strictly positive attention (that is, for which $z_n > 0$). The marginal benefit of increasing the attention allocated to a source that receives zero attention is simply the limit of the right-hande side of (11) as $z_n \to 0$ which is equal to

$$\frac{|u_{kk} + u_{\sigma\sigma}|}{2} \frac{(\kappa_1^*)^2 (1 - \alpha^*)^2 t_n}{\left[\pi_\theta + \sum_{s=1}^{N} \frac{(1-\alpha^*) \eta_s z_s t_s}{(1-\alpha^*) z_s t_s + \eta_s} \right]^2}.$$

where $\partial Var[k-K \mid z, k^*(\cdot;z)]/\partial z_n$ is computed holding fixed the efficient strategy $k^*(\cdot;z)$ that maps information into individual actions. In other words, the social benefit of allocating more attention to any given source is given by the reduction in the dispersion of individual actions around the mean action that obtains when agents allocate more attention to that source, weighted by the social aversion to dispersion $|u_{kk}+u_{\sigma\sigma}|/2$. The following result then follows from the arguments above:

Proposition 2 *Suppose that the planner can control the agents' actions. There exists a unique allocation of attention z^* that maximizes welfare and is such that, for any source n that receives positive attention,*[10]

$$C_n'(z^*) = \frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{\left(\kappa_1^*\gamma_n^*(z^*)\right)^2}{\left(z_n^*\right)^2 t_n}$$

where $\kappa_1^*\gamma_n(z^*)$ represents the influence of the source under the efficient use of information, with

$$\gamma_n^*(z) \equiv \frac{\frac{(1-\alpha^*)\pi_n(z)}{1-\alpha^*\rho_n(z)}}{\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha^*)\pi_s(z)}{1-\alpha^*\rho_s(z)}} = \frac{\frac{(1-\alpha^*)\eta_n z_n t_n}{(1-\alpha^*)z_n t_n+\eta_n}}{\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha^*)\eta_s z_s t_s}{(1-\alpha^*)z_s t_s+\eta_s}}.$$

Comparing the private benefit (5) to the social benefit (11) of allocating more attention to any given source of information then permits me to establish the following conclusion:

Corollary 2 *Let $\hat{z}$ denote the equilibrium allocation of attention. Suppose that the planner can control the agents' actions. Then, starting from $\hat{z}$, forcing the agents to pay more attention to a source n that receives positive attention in equilibrium (i.e., for which $\hat{z}_n > 0$) increases welfare if*

$$|u_{kk}|(\kappa_1\gamma_n(\hat{z}))^2 < |u_{kk}+u_{\sigma\sigma}|(\kappa_1^*\gamma_n^*(\hat{z}))^2 \tag{12}$$

and decreases it if the inequality in (12) is reversed, where $\kappa_1\gamma_n(\hat{z})$ and $\kappa_1^*\gamma_n^*(\hat{z})$ denote, respectively, the sensitivity of the equilibrium and of the efficient strategy to the n-th source of information, when the attention allocated to the various sources is $\hat{z}$. Likewise, forcing the agents to pay attention to a source n that receives no attention in equilibrium (i.e., for which $\hat{z}_n = 0$) increases welfare if

$$\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1^*)^2(1-\alpha^*)^2 t_n}{\left[\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha^*)\eta_s \hat{z}_s t_s}{(1-\alpha^*)\hat{z}_s t_s+\eta_s}\right]^2} > C_n'(\hat{z})$$

and decreases it if the inequality is reversed.

[10] As in the equilibrium case, for any source that receives no attention, the following condition must hold:

$$C_n'(z^*) \geq \frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1^*)^2(1-\alpha^*)^2 t_n}{\left[\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha^*)\eta_s z_s t_s}{(1-\alpha^*)z_s t_s+\eta_s}\right]^2}.$$

To understand the result, recall from the analysis above that both the private and the social (gross) marginal benefit of allocating more attention to any given source of information come from the marginal reduction in the dispersion of individual actions around the mean action.[11] The magnitude of this reduction depends on the sensitivity of individual actions to the source of information, which is given by $\kappa_1 \gamma_n$ under the equilibrium strategy and by $\kappa_1^* \gamma_n^*$ under the efficient strategy. The weight that the planner assigns to reducing the cross-sectional dispersion of individual actions is $|u_{kk} + u_{\sigma\sigma}|$, while the weight that the individual agent assigns to reducing the dispersion of her action around the mean action is $|u_{kk}|$.

Increasing the attention allocated to a source that receives positive attention in equilibrium then increases welfare if and only if the marginal reduction in the dispersion of actions under the equilibrium strategy, weighted by the importance that each agent assigns to dispersion, falls short of the marginal reduction in dispersion under the efficient allocation, weighted by the importance that the planner assigns to dispersion.

Likewise, forcing the agents to pay attention to a source that receives no attention in equilibrium increases welfare if and only if the marginal cost exceeds the private benefit of reducing dispersion, but falls short of the social benefit.

Put it differently, efficiency in the allocation of attention requires both (i) efficiency in the use of information and (ii) alignment between the private and the social value of reducing the dispersion of individual actions, which obtains when and only when $u_{\sigma\sigma} = 0$.

The following result generalizes and extends a result in Colombo, Femminis and Pavan (2013) to the more flexible information structure considered in this paper. Let $\hat{z}$ denote the equilibrium allocation of attention and z^* the allocation of attention that maximizes welfare when the planner can control the use of information. Furthermore, let $\#\hat{N}$ denote the cardinality of the set of sources that receive positive attention in equilibrium, and $\#N^*$ the cardinality of the set of sources that receive positive attention when the planner can control both the allocation of attention and the use of information.

Proposition 3 *Let $\hat{z}$ denote the equilibrium allocation of attention and z^* the allocation of attention that maximizes welfare when the planner can control the agents' actions.*

(i) Consider economies that are efficient in their use of information ($\kappa = \kappa^*$ and $\alpha = \alpha^*$). The attention allocated to each source is inefficiently low if $u_{\sigma\sigma} < 0$ and inefficiently high if $u_{\sigma\sigma} > 0$ (meaning that, for any n, $z_n^* \geq \hat{z}_n$ if $u_{\sigma\sigma} < 0$ and $z_n^* \leq \hat{z}_n$ if $u_{\sigma\sigma} > 0$, with the inequalities strict if $\hat{z}_n > 0$).

(ii) Consider economies that are efficient under complete information and where inefficiencies in the allocation of attention originate in the way information is used in equilibrium ($\kappa = \kappa^*$, $u_{\sigma\sigma} = 0$ but $\alpha \neq \alpha^*$). When $\alpha > \alpha^*$, agents pay too much attention to sources that are transparent

[11] Both marginal reductions are computed holding constant, respectively, the equilibrium and the efficient use of information, that is, the mappings $k(\cdot; z)$ and $k^*(\cdot; z)$, by usual envelope arguments.

and too little attention to sources that are opaque (Formally, there exists $R^* > 0$ such that, if $t_n/C'_n(\hat{z}) > R^*$, then $z^*_n \leq \hat{z}_n$, whereas, if $t_n/C'_n(\hat{z}) < R^*$, then $\hat{z}_n \leq z^*_n$, with the inequalities strict if $\hat{z}_n > 0$). The opposite conclusions hold for $\alpha < \alpha^*$. Furthermore, when $C(z) = c\left(\sum_{s=1}^N z_s\right)$ with $c(\cdot)$ increasing, convex, and differentiable, the equilibrium total attention is inefficiently low and too few sources receive positive attention meaning that $\sum_{s=1}^N \hat{z}_s \leq \sum_{s=1}^N z^*_s$ and $\#\hat{N} \leq \#N^*$) if $\alpha > \alpha^*$, whereas the opposite is true if $\alpha < \alpha^*$.

(iii) Consider economies in which the inefficiency in the allocation of attention originates in the inefficiency of the complete-information actions ($u_{\sigma\sigma} = 0$, $\alpha = \alpha^*$, but $\kappa \neq \kappa^*$). The attention allocated to each source is inefficiently low if $\kappa^*_1 > \kappa_1$ and inefficiently high if $\kappa^*_1 < \kappa_1$ (meaning that, for any n, $z^*_n \geq \hat{z}_n$ if $\kappa^*_1 > \kappa_1$ and $z^*_n \leq \hat{z}_n$ if $\kappa^*_1 < \kappa_1$, with the inequalities strict if $\hat{z}_n > 0$).

Let's start with part (i). Because in these economies the equilibrium use of information is efficient, the marginal reduction in the dispersion of individual actions under the equilibrium strategy coincides with the marginal reduction under the efficient strategy. That the equilibrium use of information is efficient, however, does not guarantee that the private and the social marginal benefit of increasing the attention allocated to a source of information are the same. The reason is that the private benefit fails to take into account the direct, non-strategic, effect that the dispersion of individual actions has on payoffs, as captured by $u_{\sigma\sigma}$. Because this externality has no strategic effects, it is not internalized and is thus a source of inefficiency in the allocation of attention. In particular, the attention given in equilibrium to each source falls short of the efficient level (weakly) in the presence of a negative externality from dispersion, $u_{\sigma\sigma} < 0$, while it exceeds the efficient level (weakly) in the presence of a positive externality, $u_{\sigma\sigma} > 0$.

Next, consider part (ii) and take an economy for which $\alpha > \alpha^*$. Because there are no direct externalities from dispersion (i.e., $u_{\sigma\sigma} = 0$), the weight that each agent assigns to a reduction in the dispersion of her action around the mean action coincides with the weight assigned by the planner. The discrepancy between the private and the social value of increasing the attention allocated to any given source then comes from the inefficiency of the equilibrium use of information; in particular, from the fact, in these economies, equilibrium actions are too sensitive to sources that are relatively public, (i.e., for which ρ_n is high) and too little sensitive to sources that are relatively private (i.e., for which ρ_n is low). The publicity of a source is, however, endogenous, for it depends on the attention allocated to it. Taking into account how attention depends on the primitive properties of the sources, the proposition then shows that agents pay too much attention to those sources that are highly transparent relative to their cost and too little attention to those sources that are either opaque or for which the attention cost is high.

To see this last result more explicitly, consider an economy satisfying the conditions in Example 1 above, where all sources receive strictly positive attention in equilibrium. It is easy to see that, as long as the difference $\alpha - \alpha^*$ is not too high so that the planner also wants the agent to assign positive attention to all sources, then the efficient allocation of attention satisfies the analog of the conditions in (7) with α^* replacing α. It is then also easy to see that $z_n^* > \hat{z}_n$ for those sources for which

$$\sqrt{t_n} < \frac{\pi_\theta + \sum_{s=1}^N \eta_s}{\sum_{s=1}^N \left(\eta_s/\sqrt{t_s}\right)}$$

whereas $z_n^* < \hat{z}_n$ for those sources for which the above inequality is reversed.

The proposition also shows that, when the cost depends only on the total attention, (a) too few sources receive some attention in equilibrium and (b) the total amount of attention that each agent devotes to the various sources is inefficiently low. The opposite conclusions hold for economies where the value that the agents assign to coordinating their actions falls short of the efficient one, i.e., for which $\alpha < \alpha^*$.

Lastly, consider part (iii). When $\kappa_1 < \kappa_1^*$, the complete-information equilibrium responds too little to changes in the fundamentals relative to the first-best allocation, κ^*. As a result, the agents' incentives to learn about θ are inefficiently low, and hence in equilibrium all sources of information receive too little attention relative to what is efficient.

I conclude this section by looking at how welfare changes with the attention allocated to the various sources of information (around the equilibrium level) when the agents' actions are determined by the equilibrium rule $k(\cdot;\hat{z})$ as opposed to the efficient rule $k^*(\cdot;\hat{z})$.

In the Appendix, I show that, in this case, the gross marginal benefit of inducing the agents to increase the attention allocated to any given source is

$$\begin{aligned} &\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1\gamma_n(\hat{z}))^2}{(\hat{z}_n)^2 t_n} + |u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)\left\{\sum_{s=1}^N\left(\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_s t_s}\right)\frac{\partial\gamma_s(\hat{z})}{\partial z_n}\right\} \\ &+\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\left\{\sum_{s=1}^N\frac{\partial\gamma_s(\hat{z})}{\partial z_n}\right\}. \end{aligned} \tag{13}$$

The first term in (13) is the direct marginal effect of a reduction in the cross-sectional dispersion of individual actions that obtains as a result of an increase in z_n, holding fixed the equilibrium use of information $k(\cdot;\hat{z})$. The second term combines the marginal effects of changing the equilibrium rule $k(\cdot;\hat{z})$ on (a) the volatility of the aggregate action K around its complete-information counterpart κ and (b) the dispersion of individual actions. Finally, the last term, which is relevant only in economies that are inefficient under complete information, captures the effect of changing the rule $k(\cdot;\hat{z})$ on the way the "error" due to incomplete information $K-\kappa$ covaries with the inefficiency of the complete-information allocation. Clearly, by usual envelope arguments, these last two terms are absent in economies where the equilibrium use of information is efficient (that is, in economies where $k(\cdot;z) = k^*(\cdot;z)$, which is the case if and only if $\alpha = \alpha^*$ and $\kappa = \kappa^*$) or, alternatively, when the planner can dictate to the agents how to use their information.

Comparing (13) to the private value (5) of increasing the attention allocated to any given source (evaluated at the equilibrium level), then permits me to arrive to the following result.

Proposition 4 *Suppose that the planner cannot change the way the agents use their information in equilibrium.*

(a) Consider economies that are either efficient in their use of information ($\kappa = \kappa^*$ and $\alpha = \alpha^*$) or in which the inefficiency in the allocation of attention originates in the inefficiency of the complete-information actions ($u_{\sigma\sigma} = 0$, $\alpha = \alpha^*$, but $\kappa \neq \kappa^*$). The same conclusions hold true as for the case where the planner can dictate to the agents how to use their information (as in, respectively, parts (i) and (iii) of Proposition 3).

(b) Consider economies that are efficient under complete information and in which inefficiencies in the allocation of attention originate in the way information is used in equilibrium ($\kappa = \kappa^*$, $u_{\sigma\sigma} = 0$ but $\alpha \neq \alpha^*$). There exists a threshold $M > 0$ such that, starting from the equilibrium allocation of attention $\hat{z}$, inducing the agents to increase the attention allocated to any source for which $\hat{z}_n > 0$ increases welfare if

$$sign\left\{\alpha - \alpha^*\right\} = sign\left\{\frac{C_n'(\hat{z})}{t_n} - M\right\}$$

and decreases it otherwise.

Consider first part (b) and take an economy where agents are over-concerned about aligning their actions ($\alpha > \alpha^*$). In these economies, equilibrium actions are too sensitive to sources that are highly transparent and too little sensitive to sources that are relative opaque. Now use Proposition 1 to observe that the sensitivity $\kappa_1\gamma_n$ of the equilibrium actions to each source is increasing in the attention allocated to that source and decreasing in the attention allocated to any other source. By inducing the agents to reallocate their attention from sources of high transparency to sources of low transparency, the planner then also induces the agents to use their information in a more efficient manner. This effect thus reinforces the conclusions in Proposition 3 for the case where the planner can control the way agents map their information into their actions.

Now, consider part (a) and focus on economies in which the inefficiency in the allocation of attention originates in the inefficiency of the complete-information allocation. Specifically, suppose that the complete-information equilibrium actions respond too little to variations in the fundamentals (i.e., $\kappa_1 < \kappa_1^*$). Relative to the case where the planner can control the agents' actions, the extra benefit

$$\frac{|u_{kk} + 2u_{kK} + u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^* - \kappa_1}{\kappa_1}\right)\left\{\sum_{s=1}^N \frac{\partial \gamma_s(z)}{\partial z_n}\right\}$$

of inducing the agents to pay more attention to any given source comes from the fact that the equilibrium actions then become more responsive to that source and less to any other source. The net effect of this adjustment is to partially correct the inefficiency of the complete-information allocation by bringing the aggregate action K,on average, closer to its first-best counterpart κ^*. This novel effect then adds to the benefit of reducing the dispersion of individual actions in the cross-section of the population thus reinforcing the conclusions of Proposition 3.

3 Bounded Recall

3.1 Environment

Suppose now that agents are unable to keep track of the influence of individual sources on posterior beliefs. I model this situation as follows. Agents understand how their beliefs are affected by the attention they allocate to the various sources. However, when it comes to committing their actions, they respond only to variations in their posterior beliefs (as opposed to responding separately to variations in the individual sources). Apart from this modification, the environment is the same as in the previous section.

Given the attention z^j allocated to the different sources and the information $x^j \equiv (x_1^j, ..., x_N^j)$ received from them, agent j's posterior beliefs about θ continue to be Normal with mean

$$\bar{x}^j(x^j; z^j) = \sum\nolimits_{n=1}^N \delta_n(z^j) x_n^i, \text{ with } \delta_n(z^j) \equiv \frac{\pi_n(z^j)}{\pi_\theta + \sum_{s=1}^N \pi_s(z^j)} \text{ and } \pi_s(z^j) \equiv \frac{\eta_s z_s^j t_s}{z_s^j t_s + \eta_s}$$

and precision $\pi_\theta + \sum_{s=1}^N \pi_s(z^j)$. Each signal $x_n^j = \theta + \varepsilon_n + \xi_n^j$ has the same statistical properties as in the previous section. The only difference is that agent j is now unable to decompose $\bar{x}^j$ into the various signals x^j. This is equivalent to assuming that, given the attention z^j, agent j receives a single signal

$$X^j(x^j; z^j) \equiv \frac{\bar{x}^j(x^j; z^j)}{\sum_{s=1}^N \delta_s(z^j)} = \left(\frac{\pi_\theta + \pi_X(z^\#)}{\pi_X(z^\#)} \right) \bar{x}^j(x^j; z^j) = \theta + \sum\nolimits_{n=1}^N \frac{\pi_n(z^j)}{\pi_X(z^j)} (\varepsilon_n + \xi_n^j)$$

with precision

$$\pi_X(z^j) \equiv \sum\nolimits_{s=1}^N \pi_s(z^j).$$

Bounded recall then amounts to imposing that agent j's actions be measurable in the sigma algebra generated by the random variable $X^j(z^j)$. Importantly, note that this restriction matters only because of strategic effects. In fact, because $\bar{x}^j$ is a sufficient statistics for $(\bar{x}^j, x^j)$ with respect to θ, in the absence of strategic effects, the agent's actions and payoff would be the same as in the case of perfect recall.

3.2 Equilibrium allocation of attention

Following steps similar to those in Section 2.2, I can establish the following result. Let

$$\rho_X(z) \equiv \sum\nolimits_{s=1}^N \frac{\pi_s(z)}{\pi_X(z)} \rho_s(z)$$

denote the weighted average of the endogenous publicity of the various sources of information, where the weights are the relative precisions.

Proposition 5 *There is a unique symmetric equilibrium. In this equilibrium, given the attention $z^{\#}$ allocated to the various sources, individual actions are given by*

$$k^i = k(\bar{x}^i; z^{\#}) = \kappa_0 + \kappa_1 \gamma(z^{\#}) \cdot \bar{x}^j \tag{14}$$

all $i \in [0,1]$, where

$$\gamma(z) \equiv \left(\frac{\frac{(1-\alpha)\pi_X(z)}{1-\alpha\rho_X(z)}}{\pi_\theta + \frac{(1-\alpha)\pi_X(z)}{1-\alpha\rho_X(z)}} \right) \left(\frac{\pi_\theta + \pi_X(z^{\#})}{\pi_X(z^{\#})} \right).$$

Furthermore, for any $i \in [0,1]$ and any source $n = 1, ..., N$ that receives strictly positive attention in equilibrium,

$$C_n'(z^{\#}) = -\frac{|u_{kk}|}{2} \frac{\partial}{\partial z_n} Var\left[k - K; z^{\#}, k(\cdot; z^{\#})\right] - \frac{|u_{kk}|}{2}(1-\alpha) \frac{\partial}{\partial z_n} Var\left[K - \kappa; z^{\#}, k(\cdot; z^{\#})\right] \tag{15}$$

where the derivatives are computed holding fixed the mapping $k(\cdot; z^{\#})$ given by (14).

There are important differences relative to the case of perfect recall. First, the marginal benefit of increasing the attention allocated to each source now has two components. The first one is the marginal reduction in the dispersion of individual actions around the mean action. This component is similar to the one in the model with perfect recall, and is computed holding fixed all agents' strategies by the usual envelope reasoning. Importantly, in a symmetric equilibrium, the reduction of dispersion of individual actions around the mean action is the same irrespective of whether one changes only the individual's allocation of attention or all agents' allocation of attention (this observation, which is formally proved in the appendix, will turn out to be important when comparing the equilibrium with the efficient allocation of attention).[12]

The second component reflects the fact that, with bounded recall, not only the second moment but also the first moment of the distribution of each agent's own action is affected by the allocation of attention (this even if one holds fixed the mapping $k(\cdot; z^{\#})$ by usual envelope arguments). The reason is that a change in the allocation of attention changes the weights δ_n that the posterior mean $\bar{x}^j$ assigns to the different sources, and hence impacts the first moment of the distribution of $\bar{x}^j$. The second component in the right-hand side of (15) thus represents the marginal benefit of bringing an agent's own average action, which in a symmetric equilibrium coincides with the average action in the cross-section of the population, closer to the complete-information equilibrium action. Importantly, note that, while the weight the individual assigns to reducing the dispersion of his own action around the mean action continues to be given by the curvature of individual payoffs u_{kk}, the weight the individual assigns to reducing the volatility of his average action around the

[12] This property was also true in the model with bounded recall. There the result was obvious given that the distribution of the average action K was independent of the allocation of attention. In contrast, with bounded recall, the distribution of the average action depends on the allocation of attention, even holding fixed the agents' strategies. The reason is that the allocation of attention impacts the weights assigned by the posterior means to the various sources of information and hence the mean of the distribution of the agents' posteriors.

complete-information equilibrium is given by $|u_{kk}|(1-\alpha) = -(u_{kk}+u_{kK})$, which takes into account also the response of the agent's action to variations in the average action.

The next result specializes the analysis to the case of an improper prior. The role of an improper prior is to make the sensitivity of the equilibrium actions to the posterior means invariant to the quality of information (formally, when $\pi_\theta = 0$, $\gamma = 1$, irrespective of the allocation of attention). Because, in practice, the role of a proper prior can be recovered by assuming there exists a source that is infinitely transparent and hence almost public, the results below do not appear particularly sensitive to the simplification assumed.

Corollary 3 *Assume $\pi_\theta = 0$. In the unique symmetric equilibrium, for any source of information that receives strictly positive attention*

$$C_n'(z^{\#}) = \frac{|u_{kk}|\,(\kappa_1)^2}{\left(\pi_X(z^{\#})\right)^2}\frac{\eta_n^2 t_n}{\left(z_n^{\#}t_n+\eta_n\right)^2}\left\{\frac{1}{2}+\alpha\left[\rho_n(z^{\#})-\rho_X(z^{\#})\right]\right\}. \tag{16}$$

As with perfect recall, an increase in the coordination motive α thus raises the marginal benefit of increasing the attention allocated to sources of information that are more public than average (i.e., for which $\rho_n > \rho_X$) and decreases the benefit of allocating attention to sources that are less public than average (i.e., for which $\rho_n < \rho_X$). As a result, an increase in the coordination motive induces the agents to concentrate their attention to a smaller number of information sources. Furthermore, as one can easily see from (16), any source that receives positive attention is characterized by a sufficiently high degree of endogenous publicity, relative to the mean level. Namely, for any source that receives positive attention

$$\rho_n > \rho_X - \frac{1}{2\alpha}.$$

The next result, which is one the key predictions of the paper, shows how bounded recall changes the allocation of attention relative to the benchmark with perfect recall.

Proposition 6 *Suppose $\pi_\theta = 0$. Let $\hat{z}$ be the allocation of attention in the unique symmetric equilibrium of the game with perfect recall. There exists ρ', ρ'' with $0 \leq \rho' < \rho_X(\hat{z}) < \rho'' \leq 1$ ($\rho'' < 1$ for α large enough) such that, starting from $\hat{z}$, any agent with bounded recall is better off by (a) locally increasing the attention to any source for which $\rho_n(\hat{z}) \in [\rho', \rho'']$ and (b) locally decreasing the attention to any source for which $\rho_n(\hat{z}) \notin [\rho', \rho'']$.*

Recall that the endogenous publicity of a source is given by

$$\rho_n(\hat{z}) = \frac{\pi_n(\hat{z})}{\eta_n} = \frac{\hat{z}_n t_n}{\hat{z}_n t_n + \eta_n}.$$

Sources of low publicity are thus sources whose endogenous precision $\pi_n(\hat{z})$ is small relative to the source's exogenous accuracy. A low publicity in turn may reflect either a low transparency or little attention allocated by the agent. The information contained in such sources is thus subject to significant idiosyncratic noise. In a world with perfect recall, paying attention to such sources may however be justified by the source's high accuracy, which permits the decision maker to forecast well the underlying fundamentals. With bounded recall, the predictive power of such sources is, however, diminished by the impossibility to respond separately to variations in the sources' content.

Sources of high publicity are, instead, sources of potentially low accuracy but which receive significant attention under full recall because of their transparency. These sources thus serve primarily as coordination devices. With bounded recall, however, the coordination value of such sources is again diminished by the impossibility to respond separately to variations in the source's content. As a result, bounded recall induces the decision maker to reallocate attention from sources of extreme publicity to sources of intermediate publicity.

Using the relationship between the equilibrium publicity of a source and its exogenous transparency, the following result translates the conclusions in the previous proposition in terms of transparency.

Corollary 4 *Suppose $\pi_\theta = 0$. Let $\hat{z}$ be the allocation of attention in the unique symmetric equilibrium of the game with perfect recall. There exists $t', t'' \in \mathbb{R}_+$ such that, starting from $\hat{z}$, any agent with bounded recall is better off by (a) locally increasing the attention to any source for which $t_n \in [t', t'']$ and (b) locally decreasing the attention to any source for which $t_n \notin [t', t'']$.*

3.3 Efficient allocation of attention

Now consider the efficient allocation of attention in the presence of bounded recall. First note that, because the planner's problem is concave, it is never optimal to induce different agents to allocate different attention to the various sources of information. This in turn means that, for any symmetric allocation of attention z, efficiency in the agents' actions requires that, for any agent $i \in [0,1]$, almost any $\bar{x}^i$

$$k^i = k^{**}(\bar{x}^i; z) = \kappa_0^* + \kappa_1^* \gamma^{**}(z) \bar{x}^i \tag{17}$$

with

$$\gamma^{**}(z) = \left(\frac{\frac{(1-\alpha^*)\pi_X(z)}{1-\alpha^*\rho_X(z)}}{\pi_\theta + \frac{(1-\alpha^*)\pi_X(z)}{1-\alpha^*\rho_X(z)}} \right) \left(\frac{\pi_\theta + \pi_X(z^\#)}{\pi_X(z^\#)} \right)$$

where π_X and ρ_X are as defined above. This implies that, for any z, the maximum welfare that can be achieved by having the agents follow the rule $k^{**}(\cdot; z)$ defined by (17 (17) is given by

$$w^*(z) \equiv \mathbb{E}[u(\kappa^*, \kappa^*, 0, \theta)] - \mathcal{L}^*(z) - C(z), \tag{18}$$

where $u(\kappa^*, \kappa^*, 0, \theta)$ continues to denote welfare under the first-best allocation and where

$$\mathcal{L}^*(z) \equiv \frac{|u_{kk} + u_{\sigma\sigma}|}{2} Var[k - K \mid z, k^{**}(\cdot; z)] + \frac{|u_{kk} + 2u_{kK} + u_{KK}|}{2} Var[K - \kappa^* \mid z, k^{**}(\cdot; z)]$$

continues to denote the welfare losses due to incomplete information. Using the fact that $|u_{kk} + 2u_{kK} + u_{KK}| = (1 - \alpha^*) |u_{kk} + u_{\sigma\sigma}|$, I then have the following result.

Proposition 7 *Suppose that the planner can dictate to the agents how to respond to their posterior beliefs. Let z^{**} denote the allocation of attention that maximizes welfare in the presence of bounded recall. Then, for any source that receives strictly positive attention,*

$$
\begin{aligned}
C_n'(z^{**}) = &-\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{\partial}{\partial z_n}Var[k-K \mid z^{**}, k^{**}(\cdot; z^{**})] \\
&-(1-\alpha^*)\,\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{\partial}{\partial z_n}Var[K-\kappa^* \mid z^{**}, k^{**}(\cdot; z^{**})]
\end{aligned}
$$

where all derivatives are computed holding fixed the mapping from the agents' posteriors into their actions, as given by (17).

To appreciate how bounded recall affects the inefficiency in the allocation of attention (as identified in Propositions 2 and 3 in the previous section), it is useful to consider a diffused prior, in which case the results are particularly sharp.

Proposition 8 *Suppose that the prior is diffused (i.e., $\pi_\theta = 0$). Let $z^{\#}$ denote the allocation of attention in the unique symmetric equilibrium and z^{**} the allocation of attention that maximizes welfare when the planner can dictate to the agents how to respond to their posterior beliefs. (a) The same conclusions as in parts (i) and (iii) of Proposition 3 hold for the comparison between $z^{\#}$ and z^{**}.*

(b) Consider economies in which the complete-information actions are first-best efficient (i.e., $\kappa = \kappa^*$) and in which there are no externalities from the dispersion of individual actions (i.e., $u_{\sigma\sigma} = 0$). Suppose $\alpha > \alpha^*$. Then, starting from $z^{\#}$, the planner would like the agents to reduce the attention allocated to sources whose publicity is higher than average (i.e., for which $\rho_n(z^{\#}) > \rho_X(z^{\#})$) and increase the attention allocated to sources whose publicity is lower than average (i.e., for which $\rho_n(z^{\#}) < \rho_X(z^{\#})$) and for which $z_n^{\#} > 0$. The opposite conclusion holds for economies in which $\alpha < \alpha^*$.

The result in part (a) follows from arguments similar to those for perfect recall. Thus consider part (b). Note that, under bounded recall, in such economies, the individual actions are always efficient (meaning that $k^{**}(\cdot; z) = k(\cdot; z)$, for any z), despite possible discrepancies between the equilibrium degree of coordination (α) and the efficient degree of coordination (α^*).[13] Furthermore, because there are no externalities from dispersion ($u_{\sigma\sigma} = 0$), the importance that each agent assigns to reducing the dispersion of her action around the mean action coincides with the importance assigned by the planner. Possible discrepancies between the private and the social value of increasing the attention allocated to any given source now originate in the discrepancy between the importance that the individual and the planner assign to reducing the volatility of the *average action* around the complete-information counterpart (which coincides with the first-best allocation by assumption). As I show in the Appendix, increasing the attention allocated to any given source increases such a

[13] To see this, note that, when $\pi_\theta = 0$, under bounded recall, $\gamma = 1 = \gamma^*$ for all z, independently of α.

volatility if the source's publicity is higher than average and decrease it if it is lower. Now economies where agents coordinate too much ($\alpha > \alpha^*$) are also economies in which agents care less than the planner about reducing such a volatility. As a result, in these economies, the planner would like the agents to reduce the attention allocated to sources whose publicity is higher than average and increase the attention to sources whose publicity is lower than average.

4 Conclusions

I compared the equilibrium allocation of attention to the efficient allocation of attention in a flexible, yet tractable, model featuring a rich set of payoff interdependencies and an arbitrary number of information sources differing in their accuracy and transparency. I then examined the effects of bounded recall, defined to be the inability to keep track of the effects of individual sources on posterior beliefs.

In future work, it would be interesting to endogenize the agents' ability to recall and examine how the latter is influenced by the accuracy and transparency of the different sources. It would also be interesting to extend the analysis to a dynamic setting where agents choose whether or not to pay additional attention to the various sources as a function of their current beliefs. Both extensions are challenging but worth examining.

A last word concerns the welfare effects of bounded recall. As it is the case with other distortions too, equilibrium welfare can be higher with bounded recall because it may induce the agents to allocate their attention more efficiently. In future work it would be interesting to characterize the conditions under which this happens.

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Appendix

Proof of Proposition 1. When all agents allocate attention z to the various sources of information, the continuation game that starts after the agents receive their signals x^j and must choose their actions has a unique continuation equilibrium where all agents follow the linear strategy (4). This step follows from arguments similar to those that lead to Proposition 3 in Angeletos and Pavan (2009) and hence the proof is omitted.

Next, let

$$U^j(z^j;\hat{z}) = \mathbb{E}[u(k^j, K, \sigma_k, \theta)|z^j] - C(z^j)$$

denote agent j's expected payoff when all agents $i \neq j$ pay attention $\hat{z}$ to the different sources of information and then choose their actions according to (4), whereas agent j allocates attention z^j to the various sources and then chooses his actions optimally. It is easy to show that $U^j(z^j;\hat{z})$ is continuously right-differentiable in z_n^j, any n, any $(z^j;\hat{z})$, and that, for any $z_n^j > 0$ the derivative $\partial U^j(z^j;\hat{z})/\partial z_n^j$ coincides with the partial derivative of the agent's expected payoff under the optimal (linear) strategy. If follows that, in any symmetric equilibrium, for any source n that receives positive attention,

$$\frac{\partial U^j(\hat{z};\hat{z})}{\partial z_n^j} = \frac{u_{kk}}{2}\frac{\partial}{\partial z_n^j} Var[k^j - K \mid \hat{z}, k(\cdot;\hat{z})] - C_n'(\hat{z}) \tag{19}$$

where the derivative in the right hand side of (19) is computed holding fixed the mapping $k(\cdot;\hat{z})$ from each agent's information to his actions, as given by (4). To see this, note that, by usual envelope arguments, the left hand side of (19) coincides with the partial derivative of the agent's payoff, holding fixed the mapping from the agent's information to his actions and letting this mapping be given by (4) by optimality. Next observe that, when all agents (including agent j) follow (4), then

$$U^j(z^j;\hat{z}) = \mathbb{E}[u(K, K, \sigma_k, \theta) \mid z^j, k(\cdot;\hat{z})] + \frac{u_{kk}}{2} Var[k^j - K \mid z^j, k(\cdot;\hat{z})] - C(z^j) \tag{20}$$

where the first term in the right-hand side of (20) is the expected payoff of an agent whose action coincides with the average action in the population in every state, while the second term is the ex-ante dispersion of an agent's action around the mean action. Note that, when all agents follow the linear strategy in (4) — more generally, when their actions are determined by any linear mapping of their signals — then the distribution of K is independent of the allocation of attention. Furthermore, when all agents follow the mapping in (4),

$$Var[k^j - K \mid z^j, k(\cdot;\hat{z})] = \kappa_1^2 \sum\nolimits_{n=1}^{N} \frac{(\gamma_n(\hat{z}))^2}{z_n^j t_n}.$$

I conclude that, in any symmetric equilibrium, for any source of information that receives strictly positive attention, the following optimality condition must hold:

$$C_n'(\hat{z}) = \frac{|u_{kk}|}{2} \frac{(\kappa_1 \gamma_n(\hat{z}))^2}{\left(\hat{z}_n^j\right)^2 t_n}.$$

By continuity of the right-hand derivative $\partial U_+^j(z^j;\hat{z})/\partial z_n^j$, I also have that, for any source that receives no attention, the following optimality condition must hold

$$C_n'(\hat{z}) \geq \frac{|u_{kk}|\kappa_1^2(1-\alpha)^2 t_n}{2\left[\pi_\theta + \sum_{s=1}^N \frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})}\right]^2},$$

which is equivalent to the condition that $\partial U_+^j(\hat{z};\hat{z})/\partial z_n^j \leq 0$ at $\hat{z}_n = 0$.

Lastly, to see that the symmetric equilibrium is unique, let $\mathcal{U}$ denote the family of quadratic payoff functions satisfying all the conditions in the model setup. From arguments similar to those that lead to Proposition 2 in Angeletos and Pavan (2009), one can show that, given any $u \in \mathcal{U}$, there exists a unique $u' \in \mathcal{U}$ such that any symmetric equilibrium of the game where payoffs are given by u coincides with an efficient allocation for the economy with payoffs given by u'. That the efficient allocation for the economy with payoffs given by $u^{'}$ is unique follows from the fact that the planner's problem that consists in choosing a vector $z \in \mathbb{R}_+^N$ along with a function $k : \mathbb{R}^N \to \mathbb{R}$ so as to maximize the ex-ante expectation of u' is strictly concave. This in turn implies that the symmetric equilibrium for the economy with payoffs given by u is also unique, which establishes the result. Q.E.D.

Proof of Corollary 1. From Proposition 1, any source that receives strictly positive attention in equilibrium must satisfy (2). Substituting for

$$\gamma_n(\hat{z}) = \frac{\frac{(1-\alpha)\hat{z}_n t_n \eta_n}{(1-\alpha)\hat{z}_n t_n + \eta_n}}{\pi_\theta + \sum_{l=1}^N \frac{(1-\alpha)\hat{z}_l t_l \eta_l}{(1-\alpha)\hat{z}_l t_l + \eta_l}}$$

condition (2) can be rewritten as

$$\hat{z}_n = \frac{\eta_n}{\sqrt{t_n}} \left\{ \frac{1}{\sqrt{C_n'(\hat{z})}} \sqrt{\frac{|u_{kk}|}{2}} \frac{\kappa_1}{M_1(\hat{z})} - \frac{1}{(1-\alpha)\sqrt{t_n}} \right\} \tag{21}$$

where

$$M_1(z) \equiv \pi_\theta + \sum_{l=1}^N \frac{(1-\alpha)\eta_l z_l t_l}{(1-\alpha) z_l t_l + \eta_l} > 0. \tag{22}$$

For the right-hand-side in (21) to be positive, it must be that

$$\frac{t_n}{C_n'(\hat{z})} > R \equiv \frac{2\left(M_1(\hat{z})\right)^2}{(1-\alpha)^2 \kappa_1^2 |u_{kk}|}, \tag{23}$$

which establishes the first claim in the Corollary.

Next, I prove that, for any source that receives no attention in equilibrium, condition (23) must be violated. To see this, suppose that, by contradiction, there exists a source n for which (23) holds and such that $\hat{z}_n = 0$. Suppose that the individual were to increase locally the attention allocated to this source. The continuity of the right-hand derivative of the agent's payoff $\partial U_+^j(\hat{z};\hat{z})/\partial z_n^j$ implies that the net effect on the agent's expected payoff is

$$\frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(\hat{z}))^2}{(\hat{z}_n)^2\,t_n}-C_n'(\hat{z})=\frac{|u_{kk}|\kappa_1^2}{2}\frac{(1-\alpha)^2t_n}{(M_1(\hat{z}))^2}-C_n'(\hat{z})>0,$$

contradicting the optimality of the equilibrium allocation of attention. Q.E.D.

Proof of Example 1. Suppose that all sources receive strictly positive attention in equilibrium. The amount of attention allocated to each source n is then equal to

$$\hat{z}_n=\sqrt{\frac{|u_{kk}|\kappa_1^2}{2\bar{c}}}\frac{\gamma_n(\hat{z})}{\sqrt{t_n}}. \tag{24}$$

It follows that the influence of each source n is given by

$$\gamma_n(\hat{z})=\sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}\sqrt{t_n}\hat{z}_n. \tag{25}$$

Combining the above with the fact that

$$\gamma_n(\hat{z})=\frac{\frac{(1-\alpha)\pi_n(\hat{z})}{1-\alpha\rho_n(\hat{z})}}{\pi_\theta+\sum_{s=1}^N\frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})}} \tag{26}$$

I then have that

$$\sum_{n=1}^N\gamma_n(\hat{z})=\frac{\sum_{s=1}^N\frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})}}{\pi_\theta+\sum_{s=1}^N\frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})}}=\sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}\sum_{n=1}^N\sqrt{t_n}\hat{z}_n.$$

This implies that

$$\pi_\theta+\sum_{s=1}^N\frac{(1-\alpha)\pi_s(\hat{z})}{1-\alpha\rho_s(\hat{z})}=\frac{\pi_\theta}{1-\sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}\sum_{s=1}^N\sqrt{t_s}\hat{z}_s}.$$

Replacing the latter expression into the definition of $\gamma_n(\hat{z})$ in (26) and using the fact that

$$\frac{(1-\alpha)\pi_n(\hat{z})}{1-\alpha\rho_n(\hat{z})}=\frac{(1-\alpha)\eta_n\hat{z}_nt_n}{\hat{z}_nt_n(1-\alpha)+\eta_n}$$

I then have that

$$\gamma_n(\hat{z})=\frac{\frac{(1-\alpha)\eta_n\hat{z}_nt_n}{\hat{z}_nt_n(1-\alpha)+\eta_n}}{\frac{\pi_\theta}{1-\sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}\sum_{s=1}^N\sqrt{t_s}\hat{z}_s}}.$$

Combining this expression with (24) I then have that

$$\hat{z}_n=\left[\frac{1}{\pi_\theta\sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}}-\frac{1}{\pi_\theta}\sum_{s=1}^N\sqrt{t_s}\hat{z}_s\right]\frac{1}{\sqrt{t_n}}\eta_n-\frac{\eta_n}{(1-\alpha)t_n}. \tag{27}$$

Multiplying both sides of (27) by $\sqrt{t_n}$, summing over n, and rearranging, I then obtain that

$$\frac{1}{\pi_\theta}\sum_{s=1}^{N}\sqrt{t_s}\hat{z}_s = \frac{\frac{\sum_{s=1}^{N}\eta_s}{\pi_\theta\sqrt{\frac{2\bar{c}}{|u_{kk}|\kappa_1^2}}} - \frac{1}{(1-\alpha)}\sum_{s=1}^{N}\frac{\eta_s}{\sqrt{t_s}}}{\pi_\theta + \sum_{s=1}^{N}\eta_s}. \tag{28}$$

Replacing (28) into (27), I conclude that

$$\hat{z}_n = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)}\left[\frac{(1-\alpha)\sqrt{\frac{|u_{kk}|\kappa_1^2}{2\bar{c}}} + \sum_{s=1}^{N}\frac{\eta_s}{\sqrt{t_s}}}{\pi_\theta + \sum_{s=1}^{N}\eta_s} - \frac{1}{\sqrt{t_n}}\right]$$

as claimed. Q.E.D.

Proof of Proposition (3). The proof for parts (i) and (iii) follows directly from the results in Corollary 2. Thus consider part (ii). Note that, in these economies, for any z, and any n, the discrepancy between the social and the private benefit of increasing the attention allocated to source n is proportional to the difference[14]

$$\frac{(\gamma_n^*(z))^2}{(z_n)^2 t_n} - \frac{(\gamma_n(z))^2}{(z_n)^2 t_n}. \tag{29}$$

Using the expressions for γ and γ^*, the difference in (29) is equal to

$$m^*(z)\frac{t_n\eta_n^2}{[(1-\alpha^*)z_nt_n+\eta_n]^2} - m(z)\frac{t_n\eta_n^2}{[(1-\alpha)z_nt_n+\eta_n]^2}$$

where

$$m(z) \equiv \frac{(1-\alpha)^2}{\left[\pi_\theta + \sum_{l=1}^{N}\frac{(1-\alpha)z_lt_l\eta_l}{(1-\alpha)z_lt_l+\eta_l}\right]^2} \text{ and } m^*(z) \equiv \frac{(1-\alpha^*)^2}{\left[\pi_\theta + \sum_{l=1}^{N}\frac{(1-\alpha^*)z_lt_l\eta_l}{(1-\alpha^*)z_lt_l+\eta_l}\right]^2}.$$

It follows that, starting from the equilibrium allocation of attention $\hat{z}$, the social benefit of increasing the attention allocated to source n exceeds the private benefit if and only if

$$\frac{(1-\alpha)\hat{z}_nt_n+\eta_n}{(1-\alpha^*)\hat{z}_nt_n+\eta_n} \geq \sqrt{\frac{m(\hat{z})}{m^*(\hat{z})}}. \tag{30}$$

Now note that, when $\alpha > \alpha^*$, $m(\hat{z}) < m^*(\hat{z})$. This means that the social benefit exceeds the private benefit for any source that receives no attention in equilibrium. The opposite conclusion holds when $\alpha < \alpha^*$. Thus consider sources that receive strictly positive attention in equilibrium. Using (21),

$$\hat{z}_nt_n = \frac{\eta_n\sqrt{t_n}}{\sqrt{C_n'(\hat{z})}}Q(\hat{z}) - \frac{\eta_n}{(1-\alpha)} \tag{31}$$

where

$$Q(z) \equiv \sqrt{\frac{|u_{kk}|}{2}}\frac{\kappa_1}{M_1(z)}$$

[14] Note that the comparison here applies also to sources that receive no attention, i.e., for which $z_n = 0$.

with the function $M_1(\cdot)$ as defined in (22). Using (31), the left-hand-side of (30) becomes

$$\frac{(1-\alpha)Q(\hat{z})\sqrt{\frac{t_n}{C_n'(\hat{z})}}}{(1-\alpha^*)Q(\hat{z})\sqrt{\frac{t_n}{C_n'(\hat{z})}+\frac{\alpha^*-\alpha}{1-\alpha}}}$$

which is decreasing in $t_n/C_n'(\hat{z})$ for $\alpha > \alpha^*$ and increasing in $\frac{t_n}{C_n'(\hat{z})}$ for $\alpha < \alpha^*$.

I conclude that, when $\alpha > \alpha^*$, there exists a critical value $R^* > 0$ such that, starting from the equilibrium allocation of attention $\hat{z}$, the planner would like the agents to locally increase the attention allocated to any source of information that receives positive attention in equilibrium and such that $t_n/C_n'(\hat{z}) < R^*$ and decrease the attention allocated to any source that receives positive attention and for which $t_n/C_n'(\hat{z}) > R^*$. The opposite conclusions hold for $\alpha < \alpha^*$.

Lastly, consider the case where $C(z) = c(\hat{Z})$ with $\hat{Z} \equiv \sum_{s=1}^{N} z_s$ and with $c(\cdot)$ strictly increasing, convex, and continuously differentiable. To prove the two claims in the proposition, note that, in these economies, the efficient allocation $(z^*, k(\cdot; z^*))$ coincides with the equilibrium allocation of another economy that differs from the original one only in the degree of coordination. It thus suffices to show that the equilibrium total attention $\hat{Z}$ (as well as the number of sources $\#\hat{N}$ that receive strictly positive attention in equilibrium) decrease with α.

Let $\hat{N}(\alpha)$ denote the subset of sources that receive strictly positive attention when the equilibrium degree of coordination is α. Now use the results in the proof of Corollary 1 to see that the attention allocated in equilibrium to each source n is given by

$$\hat{z}_n(\alpha) = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)} \max\left\{T(\alpha) - \frac{1}{\sqrt{t_n}}; 0\right\} \tag{32}$$

where

$$T(\alpha) = \frac{1-\alpha}{\sqrt{c'(\hat{Z}(\alpha))}} \sqrt{\frac{|u_{kk}|}{2} \frac{\kappa_1}{M_1(\hat{z}(\alpha))}} \tag{33}$$

with

$$\hat{Z}(\alpha) = \sum_{l=1}^{N} \hat{z}_l(\alpha) \tag{34}$$

and

$$M_1(\hat{z}(\alpha)) = \pi_\theta + \sum_{l=1}^{N} \frac{(1-\alpha)\eta_l \hat{z}_l(\alpha) t_l}{(1-\alpha)\hat{z}_l(\alpha) t_l + \eta_l}. \tag{35}$$

Combining (32)-(35), we obtain that, for any α, $T(\alpha)$ is the unique solution to the equation

$$\frac{T}{1-\alpha}\sqrt{c'\left(\sum_{l=1}^{N} \frac{\eta_l}{\sqrt{t_l}(1-\alpha)} \max\left\{T - \frac{1}{\sqrt{t_l}}; 0\right\}\right)} \left(\pi_\theta + \sum_{l=1}^{N} \frac{\eta_l \sqrt{t_l} \max\left\{T - \frac{1}{\sqrt{t_l}}; 0\right\}}{\sqrt{t_l} \max\left\{T - \frac{1}{\sqrt{t_l}}; 0\right\} + 1}\right) = \kappa_1 \sqrt{\frac{|u_{kk}|}{2}}. \tag{36}$$

Because the left-hand-side of (36) is increasing in both α and T, we then have that $T(\alpha)$ is decreasing in α. This means that the critical level of transparency required for each source to receive positive attention in equilibrium increases with α. In turn, this implies that that $\hat{N}(\alpha') \subset \hat{N}(\alpha)$ for any $\alpha' > \alpha$, which in turn implies that $\#\hat{N}(\alpha)$ decreases with α, as claimed.

Next, to see that the total attention $\hat{Z}(\alpha)$ also decreases with α, follow steps similar to those in the proof of Example 1 to see that, for each source $n \in \hat{N}(\alpha)$ that receives strictly positive attention,

$$\hat{z}_n(\alpha) = \frac{\eta_n}{\sqrt{t_n}(1-\alpha)} \left[\frac{(1-\alpha)\sqrt{\frac{|u_{kk}|\kappa_1^2}{2c'\left(\sum_{s\in\hat{N}(\alpha)} \hat{z}_s(\alpha)\right)}} + \sum_{s\in\hat{N}(\alpha)} \frac{\eta_s}{\sqrt{t_s}}}{\pi_\theta + \sum_{s\in\hat{N}(\alpha)} \eta_s} - \frac{1}{\sqrt{t_n}} \right].$$

Holding fixed $\hat{N}(\alpha)$, it follows that

$$\frac{\partial \hat{z}_n(\alpha)}{\partial \alpha} = -\frac{\eta_n}{\sqrt{t_n}} \frac{\sqrt{|u_{kk}|\kappa_1^2} c'' \left(\sum_{s\in\hat{N}(\alpha)} \hat{z}_s(\alpha)\right)\left(\sum_{s\in\hat{N}(\alpha)} \frac{\partial \hat{z}_s(\alpha)}{\partial\alpha}\right)}{\left[2c'\left(\sum_{s\in\hat{N}(\alpha)} \hat{z}_s(\alpha)\right)\right]^{3/2}\left(\pi_\theta + \sum_{s\in\hat{N}(\alpha)} \eta_s\right)} + \frac{\eta_n \sum_{s\in\hat{N}(\alpha)} \frac{\eta_s}{\sqrt{t_s}}}{\sqrt{t_n}\left(\pi_\theta + \sum_{s\in\hat{N}(\alpha)} \eta_s\right)(1-\alpha)^2}$$
$$- \frac{\eta_n}{t_n\,(1-\alpha)^2}.$$

Summing over all sources in $\hat{N}(\alpha)$ we then have that

$$\sum_{s\in\hat{N}(\alpha)} \frac{\partial \hat{z}_s(\alpha)}{\partial\alpha} = -\left(\frac{\sqrt{|u_{kk}|\kappa_1^2} c'' \left(\sum_{s\in\hat{N}(\alpha)} \hat{z}_s(\alpha)\right)\left(\sum_{s\in\hat{N}(\alpha)} \frac{\partial \hat{z}_s(\alpha)}{\partial\alpha}\right)}{\left[2c'\left(\sum_{s\in\hat{N}(\alpha)} \hat{z}_s(\alpha)\right)\right]^{3/2}\left(\pi_\theta + \sum_{s\in\hat{N}(\alpha)} \eta_s\right)} \right)\left(\sum_{s\in\hat{N}(\alpha)} \frac{\eta_n}{\sqrt{t_n}}\right)$$
$$+ \frac{\left(\sum_{s\in\hat{N}(\alpha)} \frac{\eta_s}{\sqrt{t_s}}\right)^2}{\left(\pi_\theta + \sum_{s\in\hat{N}(\alpha)} \eta_s\right)(1-\alpha)^2} - \sum_{s\in\hat{N}(\alpha)} \frac{\eta_n}{t_n\,(1-\alpha)^2}.$$

Rearranging,

$$\frac{\partial\left(\sum_{s\in\hat{N}(\alpha)} \hat{z}_s(\alpha)\right)}{\partial\alpha} \left\{ 1 + \frac{\sqrt{|u_{kk}|\kappa_1^2} c'' \left(\sum_{s\in\hat{N}(\alpha)} \hat{z}_s(\alpha)\right)\left(\sum_{s\in\hat{N}(\alpha)} \frac{\eta_n}{\sqrt{t_n}}\right)}{\left[2c'\left(\sum_{s\in\hat{N}(\alpha)} \hat{z}_s(\alpha)\right)\right]^{3/2}\left(\pi_\theta + \sum_{s\in\hat{N}(\alpha)} \eta_s\right)} \right\} \tag{37}$$
$$= \frac{\left(\sum_{s\in\hat{N}(\alpha)} \frac{\eta_s}{\sqrt{t_s}}\right)^2}{\left(\pi_\theta + \sum_{s\in\hat{N}(\alpha)} \eta_s\right)(1-\alpha)^2} - \sum_{s\in\hat{N}(\alpha)} \frac{\eta_n}{t_n\,(1-\alpha)^2}.$$

Because the term in curly brackets in the left-hand-side of (37) is positive, the sign of $\frac{\partial}{\partial\alpha}\left(\sum_{s\in\hat{N}(\alpha)} \hat{z}_s(\alpha)\right)$ is determined by the sign of the right-hand-side of (37) which in turn coincides with the sign of the following expression

$$\frac{\left(\sum_{s\in\hat{N}(\alpha)} \frac{\eta_s}{\sqrt{t_s}}\right)^2}{\pi_\theta + \sum_{s\in\hat{N}(\alpha)} \eta_s} - \sum_{s\in\hat{N}(\alpha)} \frac{\eta_n}{t_n}. \tag{38}$$

Hereafter, we show that

$$\left(\sum_{j\in\hat{N}(\alpha)} \frac{\eta_j}{\sqrt{t_j}}\right)^2 - \left(\sum_{j\in\hat{N}(\alpha)} \frac{\eta_j}{t_j}\right)\left(\sum_{j\in\hat{N}(\alpha)} \eta_j\right) \leq 0$$

which implies that the sign of (38) is always negative.

To see this, it suffices to note that

$$\left(\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s}{\sqrt{t_s}}\right)^2-\left(\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s}{t_s}\right)\left(\sum_{s\in\hat{N}(\alpha)}\eta_s\right)=\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s^2}{t_s}+\sum_{s\in\hat{N}(\alpha)}\sum_{k\in\hat{N}(\alpha),k\neq s}\frac{\eta_s\eta_k}{\sqrt{t_s}\sqrt{t_k}}$$
$$-\sum_{s\in\hat{N}(\alpha)}\frac{\eta_s^2}{t_s}-\sum_{s\in\hat{N}(\alpha)}\sum_{k\in\hat{N}(\alpha),k\neq s}\frac{\eta_s\eta_k}{t_s}$$
$$=\sum_{s,k\in\hat{N}(\alpha),k\neq s}\left[\eta_s\eta_k\left(\frac{2}{\sqrt{t_st_k}}-\frac{1}{t_s}-\frac{1}{t_k}\right)\right]<0.$$

Along with the property established above that $\hat{N}(\alpha')\subset\hat{N}(\alpha)$ for any $\alpha'>\alpha$, this result implies that $\hat{Z}(\alpha)$ decreases with α. Q.E.D.

Derivation of Condition (13). I start by noting that, for any given z, welfare under the equilibrium strategy $k(\cdot;z)$ is given by (see Angeletos and Pavan (2007)):

$$w(z)\equiv\mathbb{E}[u(k,K,\sigma_k,\theta)\mid z,k(\cdot;z)]-C(z)=\mathbb{E}[W(\kappa,0,\theta)]-\mathcal{L}(z)-C(z),\tag{39}$$

where $W(K,0,\theta)\equiv u(K,K,0,\theta)$ is the payoff that each agent obtains when all agents take the same action ($W(\kappa,0,\theta)$ is thus welfare under the complete-information equilibrium allocation $\kappa=\kappa_0+\kappa_1\theta$), whereas

$$\mathcal{L}(z)\equiv\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\cdot Var[k-K\mid z,k(\cdot;z)]+\frac{|u_{kk}+2u_{kK}+u_{KK}|}{2}\cdot Var[K-\kappa\mid z,k(\cdot;z)]$$
$$-\,Cov\left[K-\kappa,W_K(\kappa,0,\theta)\mid z,k(\cdot;z)\right]$$

are the welfare losses due to incomplete information. The first two terms in $\mathcal{L}$ measure the welfare losses due to, respectively, the dispersion of individual actions around the aggregate action and the volatility of the aggregate action around its complete-information counterpart. The last term captures losses (or gains) due to the correlation between the 'aggregate error' due to incomplete information, $K-\kappa$, and W_K, the social return to aggregate activity. Following steps similar to those in Angeletos and Pavan (2007) one can show that:

$$Cov\left[K-\kappa,W_K(\kappa,0,\theta)\mid z,k(\cdot;z)\right]=|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\frac{\left[\sum_n\gamma_n(z)-1\right]}{\pi_\theta},$$

$$Var[K-\kappa\mid z,k(\cdot;z)]=\kappa_1^2\frac{\left[\sum_{s=1}^N\gamma_s(z)-1\right]^2}{\pi_\theta}+\sum_{s=1}^N\frac{(\kappa_1\gamma_s(z))^2}{\eta_s},$$

and

$$Var[k-K\mid z,k(\cdot;z)]=\sum_{s=1}^N\frac{(\kappa_1\gamma_s(z))^2}{z_st_s}.$$

Welfare under the equilibrium strategy $k(\cdot;z)$ can thus be expressed as

$$\begin{aligned} w(z) = & \mathbb{E}[W(\kappa,0,\theta)] \\ & -\frac{|u_{kk}+u_{\sigma\sigma}|\kappa_1^2}{2}\cdot\left\{\sum_{s=1}^{N}\frac{(\gamma_s(z))^2}{z_s t_s}\right\} \\ & -\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{2}\cdot\left\{\frac{\left[\sum_{s=1}^{N}\gamma_s(z)-1\right]^2}{\pi_\theta}+\sum_{s=1}^{N}\frac{(\gamma_s(z))^2}{\eta_s}\right\} \\ & +|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2\cdot\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\cdot\frac{\sum_{s=1}^{N}\gamma_s(z)-1}{\pi_\theta}-C(z). \end{aligned}$$

The (gross) marginal effect on welfare of an increase in the attention allocated to the n-th source is thus equal to

$$\begin{aligned} & \frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1\gamma_n(z))^2}{(z_n)^2 t_n}-|u_{kk}+u_{\sigma\sigma}|\kappa_1^2\cdot\left\{\sum_{s=1}^{N}\frac{\gamma_s(z)}{z_s t_s}\frac{\partial\gamma_s(z)}{\partial z_n}\right\} \\ & -|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2\left\{\frac{\left[\sum_{s=1}^{N}\gamma_s(z)-1\right]\left(\sum_{s=1}^{N}\frac{\partial\gamma_s(z)}{\partial z_n}\right)}{\pi_\theta}+\sum_{s=1}^{N}\frac{\gamma_s(z)}{\eta_s}\frac{\partial\gamma_s(z)}{\partial z_n}\right\} \\ & +\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\left\{\sum_{s=1}^{N}\frac{\partial\gamma_s(z)}{\partial z_n}\right\}. \end{aligned} \tag{40}$$

Substituting $|u_{kk}+2u_{kK}+u_{KK}|=(1-\alpha^*)|u_{kk}+u_{\sigma\sigma}|$, I can rewrite the sum of the second and third addendum in (40) as

$$\begin{aligned} & -|u_{kk}+u_{\sigma\sigma}|\kappa_1^2\left\{\frac{(1-\alpha^*)\left[\sum_{s=1}^{N}\gamma_s(z)-1\right]\left(\sum_{s=1}^{N}\frac{\partial\gamma_s(z)}{\partial z_n}\right)}{\pi_\theta}+\sum_{s=1}^{N}\left[\frac{1-\alpha^*}{\eta_s}+\frac{1}{z_s t_s}\right]\gamma_s\frac{\partial\gamma_s(z)}{\partial z_n}\right\} \\ & =-|u_{kk}+u_{\sigma\sigma}|\kappa_1^2\left\{\sum_{s=1}^{N}\left(\frac{\left[\frac{1-\alpha^*}{\eta_s}+\frac{1}{z_s t_s}\right]\frac{(1-\alpha)\pi_s(z)}{1-\alpha\rho_s(z)}-(1-\alpha^*)}{\pi_\theta+\sum_{n=1}^{N}\frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}}\right)\frac{\partial\gamma_s(z)}{\partial z_n}\right\}. \end{aligned}$$

Using

$$\pi_s(z)\equiv\frac{\eta_s z_s t_s}{z_s t_s+\eta_s}\text{ and }\rho_s(z)=\frac{z_s t_s}{z_s t_s+\eta_s}$$

I then have that

$$\begin{aligned} & \left[\frac{1-\alpha^*}{\eta_s}+\frac{1}{z_s t_s}\right]\frac{(1-\alpha)\pi_s(z)}{1-\alpha\rho_s(z)}-(1-\alpha^*) \\ & =\frac{[(1-\alpha^*)z_s t_s+\eta_s](1-\alpha)}{[(1-\alpha)z_s t_s+\eta_s]}-\frac{[(1-\alpha)z_s t_s+\eta_s](1-\alpha^*)}{[(1-\alpha)z_s t_s+\eta_s]} \\ & =\frac{[(1-\alpha^*)z_s t_s+\eta_s](1-\alpha)-[(1-\alpha)z_s t_s+\eta_s](1-\alpha^*)}{[(1-\alpha)z_s t_s+\eta_s]} \\ & =-\frac{\eta_s(\alpha-\alpha^*)}{[(1-\alpha)z_s t_s+\eta_s]}. \end{aligned}$$

The sum of the second and third addendum in (40) can thus be rewritten as

$$|u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)\left\{\sum_{s=1}^N\left(\frac{\eta_s}{[(1-\alpha)z_st_s+\eta_s]\left[\pi_\theta+\sum_{n=1}^N\frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}\right]}\right)\frac{\partial\gamma_s(z)}{\partial z_n}\right\}.$$

Next, note that

$$\frac{\eta_s}{[(1-\alpha)z_st_s+\eta_s]\left[\pi_\theta+\sum_{n=1}^N\frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}\right]}=\frac{\gamma_s(z)}{(1-\alpha)z_st_s}.$$

I conclude that the gross marginal benefit of increasing the attention allocated to the n-th source is given by

$$\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1\gamma_n(z))^2}{(z_n)^2t_n}+|u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)\left\{\sum_{s=1}^N\left(\frac{\gamma_s(z)}{(1-\alpha)z_st_s}\right)\frac{\partial\gamma_s(z)}{\partial z_n}\right\}$$
$$+\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\left\{\sum_{s=1}^N\frac{\partial\gamma_s(z)}{\partial z_n}\right\}.$$

Q.E.D.

Proof of Proposition 4. Consider first part (b). Using (13), note that, in these economies, the net benefit of inducing the agents to pay more attention to source n is given by

$$\frac{\partial w(\hat{z})}{\partial z_n}=\frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(\hat{z}))^2}{(\hat{z}_n)^2t_n}+|u_{kk}|\kappa_1^2(\alpha-\alpha^*)\left\{\sum_{s=1}^N\left(\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_st_s}\right)\frac{\partial\gamma_s(\hat{z})}{\partial z_n}\right\}-C_n'(\hat{z}).$$

Using the fact that the private net marginal benefit is equal to

$$\frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(\hat{z}))^2}{(\hat{z}_n)^2t_n}-C_n'(\hat{z})$$

I then have that the social benefit exceeds the private benefit if and only if

$$sign\left\{\alpha-\alpha^*\right\}=sign\left\{\sum_{s=1}^N\left(\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_st_s}\right)\frac{\partial\gamma_s(\hat{z})}{\partial z_n}\right\}.$$

Next, observe that

$$\sum_{s=1}^N\left(\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_st_s}\right)\frac{\partial\gamma_s(\hat{z})}{\partial z_n}$$
$$=-\sum_{s=1}^N\left\{\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_st_s}\frac{\frac{(1-\alpha)\eta_s\hat{z}_st_s}{(1-\alpha)\hat{z}_st_s+\eta_s}}{\left(\pi_\theta+\sum_{l=1}^N\frac{(1-\alpha)\eta_l\hat{z}_lt_l}{(1-\alpha)\hat{z}_lt_l+\eta_l}\right)^2}\frac{\partial}{\partial z_n}\left(\frac{(1-\alpha)\eta_n\hat{z}_nt_n}{(1-\alpha)\hat{z}_nt_n+\eta_n}\right)\right\}$$
$$+\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_nt_n}\frac{\frac{\partial}{\partial z_n}\left(\frac{(1-\alpha)\eta_n\hat{z}_nt_n}{(1-\alpha)\hat{z}_nt_n+\eta_n}\right)}{\pi_\theta+\sum_{l=1}^N\frac{(1-\alpha)\eta_l\hat{z}_lt_l}{(1-\alpha)\hat{z}_lt_l+\eta_l}}$$
$$=\frac{\frac{\partial}{\partial z_n}\left(\frac{(1-\alpha)\eta_n\hat{z}_nt_n}{(1-\alpha)\hat{z}_nt_n+\eta_n}\right)}{\pi_\theta+\sum_{l=1}^N\frac{(1-\alpha)\eta_l\hat{z}_lt_l}{(1-\alpha)\hat{z}_lt_l+\eta_l}}\left\{\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_nt_n}-\sum_{s=1}^N\left(\frac{(\gamma_s(\hat{z}))^2}{(1-\alpha)\hat{z}_st_s}\right)\right\}.$$

Clearly,

$$\frac{\frac{\partial}{\partial z_n}\left(\frac{(1-\alpha)\eta_n\hat{z}_n t_n}{(1-\alpha)\hat{z}_n t_n+\eta_n}\right)}{\pi_\theta+\sum_{l=1}^N\frac{(1-\alpha)\eta_l\hat{z}_l t_l}{(1-\alpha)\hat{z}_l t_l+\eta_l}}>0.$$

Hence,

$$sign\left\{\sum_{s=1}^N\left(\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_s t_s}\right)\frac{\partial\gamma_s(\hat{z})}{\partial z_n}\right\}=sign\left\{\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n}-\sum_{s=1}^N\left(\frac{(\gamma_s(\hat{z}))^2}{(1-\alpha)\hat{z}_s t_s}\right)\right\}.$$

This means that the social benefit exceeds the private benefit if and only if

$$sign\{\alpha-\alpha^*\}=sign\left\{\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n}-M_0(\hat{z})\right\},$$

where $M_0(z)\equiv\sum_{s=1}^N\left(\frac{(\gamma_s(z))^2}{(1-\alpha)z_s t_s}\right)>0$ does not depend on the source of information. Now observe that

$$\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n}=\frac{\eta_n}{(1-\alpha)\hat{z}_n t_n+\eta_n}\frac{1}{M_1(\hat{z})}$$

where $M_1(\cdot)$ is the function defined in (22). Lastly use (2) to note that, for any source that receives positive attention in equilibrium,

$$(1-\alpha)\hat{z}_n t_n+\eta_n=M_2(\hat{z})\sqrt{\frac{t_n\eta_n^2}{C_n'(\hat{z})}}$$

where $M_2(z)\equiv\sqrt{\frac{|u_{kk}|\kappa_1^2(M_1(z))^2(1-\alpha)^2}{2}}$. I conclude that there exists a constant

$$M(\hat{z})\equiv[M_0(\hat{z})M_1(\hat{z})M_2(\hat{z})]^2>0$$

such that

$$sign\left\{\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n}-M_0(\hat{z})\right\}=sign\left\{\frac{C_n'(\hat{z})}{t_n}-M(\hat{z})\right\}.$$

The result in the proposition then follows.

Next, consider part (a). The result for the economies that are efficient in their use of information follows directly from Proposition 3, given that, in these economies, the impossibility to dictate to the agents how to use their information is inconsequential. The result for the economies where the inefficiency in the allocation of attention originates in the inefficiency of the complete-information actions follows from (13) along with the fact that, in these economies,[15]

$$\frac{\partial w(\hat{z})}{\partial z_n}=\frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(\hat{z}))^2}{(\hat{z}_n)^2 t_n}+\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\left\{\sum_{s=1}^N\frac{\partial\gamma_s(z)}{\partial z_n}\right\}$$
$$-C_n'(\hat{z})$$

and the fact that

$$\sum_{s=1}^N\gamma_s(z)=\frac{1}{\frac{\pi_\theta}{\sum_{s=1}^N\frac{(1-\alpha)\eta_s z_s t_s}{(1-\alpha)z_s t_s+\eta_s}}+1}$$

[15] If $\hat{z}_n=0$, then interpret the derivative as the right-hand derivative.

is increasing in z_n. Q.E.D.

Proof of Proposition 5. First I prove that, when all agents assign attention z to the various sources of information, the continuation game that starts when the agents, after observing their posterior beliefs, must choose their actions has a unique continuation equilibrium where all agents follow the linear strategy

$$k^i = k(\bar{x}^i; z) = \kappa_0 + \kappa_1 \gamma(z)\bar{x}^i. \tag{41}$$

To see this, recall that observing the posterior mean $\bar{x}^i$ is informationally equivalent to observing the signal

$$\frac{\bar{x}^i}{\sum_{n=1}^N \delta_n(z)} = \left(\frac{\pi_X(z) + \pi_\theta}{\pi_X(z)}\right)\bar{x}^i \equiv \theta + \sum\nolimits_{n=1}^N \frac{\pi_n(z)}{\pi_X(z)}(\varepsilon_n + \xi_n^i)$$

with precision $\pi_X(z) \equiv \sum_{s=1}^N \pi_s(z)$ and with an error whose correlation across any pair of agents $i, j \in [0,1]$, $j \neq i$, is given by

$$\begin{aligned}\rho_X(z) &\equiv Corr\left(\sum\nolimits_{n=1}^N \frac{\pi_n(z)}{\pi_X(z)}(\varepsilon_n + \xi_n^j); \sum\nolimits_{n=1}^N \frac{\pi_n(z)}{\pi_X(z)}(\varepsilon_n + \xi_n^i)\right) \\ &= \sum\nolimits_{s=1}^N \left(\frac{\pi_s(z)}{\pi_X(z)}\right)\rho_s(z).\end{aligned}$$

This game is isomorphic to the one in Section 2, with the only difference that each agent receives a single signal. From Proposition 1 I then have that, in the unique continuation equilibrium, individual actions are given by (41).

Next, I characterize the allocation of attention in any symmetric equilibrium. To this purpose, suppose that all agents $i \neq j$ assign attention $z^i = z$ to the different sources of information and then use (41) to determine their actions. Let $U^j(z^j; z)$ denote the payoff of agent j when he assigns attention z^j to the different sources and then chooses optimally the mapping from his posterior into his actions. Using the envelope theorem, in any symmetric equilibrium, for any source for which $z_n^\# > 0$, $\partial U^j(z^\#; z^\#)/\partial z_n^j$ must coincide with the partial derivative of the agent's expected payoff with respect to z_n^j, holding fixed the mapping $k(\cdot; z^\#)$ from the agent's posterior means to his actions and letting this mapping be the one in (41).[16]

Next observe that, when all agents (including agent j) follow (41), then

$$\begin{aligned}U^j(z^j; z) &= \mathbb{E}[u(K, K, \sigma_k, \theta) \mid z^j, z] + \mathbb{E}[u_k(K, K, \sigma_k, \theta)(k^j - K) \mid z^j, z] \\ &\quad + \frac{u_{kk}}{2}\mathbb{E}[\left(k^j - K\right)^2 \mid z^j, z] - C(z^j)\end{aligned}$$

where the first term in the right-hand side of (20) is the expected payoff of an agent whose action coincides with the average action in the population in every state. Importantly, note that, because the mapping $k(\cdot; z)$ is kept fixed, $\mathbb{E}[u(K, K, \sigma_k, \theta) \mid z^j, z]$ is independent of the agent's own

[16] Furthermore, for any source for which $z_n^\# = 0$, the right-hand derivative $\partial U_+^j(z^\#; z^\#)/\partial z_n^j$ must coincide with the limit for $z_n \to 0^+$ of the derivative $\partial U^j((z_n, z_{-n}^\#); (z_n, z_{-n}^\#))/\partial z_n^j$ by continuity of the right-hand derivative.

information and that all expectations are computed assuming all agents' actions are determined by following the linear strategy in (41).

Next observe that

$$\mathbb{E}[\left(k^j - K\right)^2 \mid z^j, z] = \mathbb{E}[\left(k^j - K^j\right)^2 + (K^j - K)^2 + 2\left(k^j - K^j\right)\left(K^j - K\right) \mid z^j, z]$$

where $K^j \equiv \mathbb{E}[k^j \mid (\theta, \varepsilon), z^j]$ denotes the agent's own average action given (θ, ε), when his attention is z^j. Using the fact that, for any z and z^j, $k^j - K^j = \kappa_1 \gamma(z) \left\{\sum_n \delta_n(z^j) \xi_n^j\right\}$ is orthogonal to $K^j - K = \kappa_1 \gamma(z) \left\{\sum_n (\delta_n(z^j) - \delta_n(z))(\theta + \varepsilon_n)\right\}$, I then have that

$$\begin{aligned}\frac{\partial}{\partial z_n^j} \mathbb{E}[\left(k^j - K\right)^2 \mid z, z] &= \frac{\partial}{\partial z_n^j} \mathbb{E}[\left(k^j - K^j\right)^2 \mid z, z] + \frac{\partial}{\partial z_n^j} \mathbb{E}[\left(K^j - K\right)^2 \mid z, z] \\ &= \frac{\partial}{\partial z_n^j} \mathbb{E}[\left(k^j - K^j\right)^2 \mid z, z] = \frac{\partial}{\partial z_n} Var\left[k - K \mid z, k(\cdot; z)\right]\end{aligned}$$

where all derivatives are computed holding fixed the agents' strategies, as given by (41). Note that the second equality follows from the fact that, at a symmetric equilibrium (i.e., for $z^j = z$),

$$\frac{\partial}{\partial z_n^j} \mathbb{E}[\left(K^j - K\right)^2 \mid z, z] = 0$$

whereas the third equality uses the fact that, in a symmetric equilibrium, the dispersion of each agent's action around his own average action coincides with the dispersion of each agent's action around the mean action in the cross-section of the population (in the notation for such dispersion, I explicitly write the strategy $k(\cdot; z)$ to make clear that the distribution of individual and aggregate actions is obtained by letting the agents follow the mapping in (41)). Importantly, note that the derivative

$$\frac{\partial}{\partial z_n} Var\left[k - K \mid z, k(\cdot; z)\right]$$

is again computed holding fixed the agents' strategies and takes into account the fact that, an increase in z_n affects the dispersion of individual actions both directly by changing the distribution of x_j and indirectly by changing the weights $\delta_s(z)$ in the agents' posterior means.

Finally, consider the term $\mathbb{E}[u_k(K, K, \sigma_k, \theta)(k^j - K) \mid z^j, z]$. Using the fact that

$$u_k(K, K, \sigma_k, \theta) = u_k(\kappa, \kappa, 0, \theta) + \left(u_{kk} + u_{kK}\right)(K - \kappa),$$

along with the fact that $u_k(\kappa, \kappa, 0, \theta) = 0$ by definition of the complete-information equilibrium, I then have that

$$\begin{aligned}\mathbb{E}[u_k(K, K, \sigma_k, \theta)(k^j - K) \mid z^j, z] &= \left(u_{kk} + u_{kK}\right) \cdot \mathbb{E}[(K - \kappa)(k^j - K) \mid z^j, z] \\ &= \left(u_{kk} + u_{kK}\right) \cdot \mathbb{E}[(K - \kappa)(K^j - K) \mid z^j, z]\end{aligned}$$

where the second equality uses the fact that $k^j - K^j$ is orthogonal to $K - \kappa$. Now observe that

$$\frac{\partial}{\partial z_n^j}\mathbb{E}[(K-\kappa)(K^j-K) \mid z, z] = \mathbb{E}\left[(K-\kappa)\frac{\partial(K^j-K)}{\partial z_n^j} \mid z, z\right]$$
$$= \kappa_1\gamma(z)\mathbb{E}\left[(K-\kappa)\left(\sum_{s=1}^N \frac{\partial\delta_s(z)}{\partial z_n}(\theta+\varepsilon_s)\right) \mid z, z\right]$$
$$= \kappa_1^2\gamma(z)\cdot Cov\left[\left(\gamma(z)\sum_{s=1}^N \delta_s(z)(\theta+\varepsilon_s)-\theta\right); \left(\sum_{s=1}^N \frac{\partial\delta_s(z)}{\partial z_n}(\theta+\varepsilon_s)\right) \mid z, z\right]$$
$$= \kappa_1^2\gamma(z)\cdot\left\{\left(\gamma(z)\sum_{s=1}^N \delta_s(z)-1\right)\left(\sum_{s=1}^N \frac{\partial\delta_s(z)}{\partial z_n}\right)\frac{1}{\pi_\theta}+\gamma(z)\sum_{s=1}^N\left(\delta_s(z)\frac{\partial\delta_s(z)}{\partial z_n}\right)\frac{1}{\eta_s}\right\}.$$

Let $\frac{\partial}{\partial z_n}Var\left[K-\kappa \mid z, k(\cdot;z)\right]$ denote the marginal change in the dispersion of K around κ that obtains when one changes the attention allocated to the n-th source, holding fixed the strategy in (41). Then observe that

$$\frac{1}{2}\frac{\partial}{\partial z_n}Var\left[K-\kappa \mid z, k(\cdot;z)\right] \tag{42}$$
$$= \frac{1}{2}\frac{\partial}{\partial z_n}Var\left[\kappa_1\left(\gamma(z)\sum_{s=1}^N \delta_s(z)(\theta+\varepsilon_s)-\theta\right) \mid z, k(\cdot;z)\right]$$
$$= \frac{\kappa_1^2}{2}\frac{\partial}{\partial z_n}Var\left[\left(\gamma(z)\sum_{s=1}^N \delta_s(z)-1\right)\theta+\gamma(z)\sum_{s=1}^N \delta_s(z)\varepsilon_s\right]$$
$$= \frac{\kappa_1^2}{2}\frac{\partial}{\partial z_n}\left[\left(\gamma(z)\sum_{s=1}^N \delta_s(z)-1\right)^2\frac{1}{\pi_\theta}+(\gamma(z))^2\sum_{s=1}^N \delta_s(z)^2\frac{1}{\eta_s}\right]$$
$$= \kappa_1^2\gamma(z)\left\{\left(\gamma(z)\sum_{s=1}^N \delta_s(z)-1\right)\left(\sum_{s=1}^N \frac{\partial\delta_s(z)}{\partial z_n}\right)\frac{1}{\pi_\theta}+\gamma(z)\sum_{s=1}^N\left(\delta_s(z)\frac{\partial\delta_s(z)}{\partial z_n}\right)\frac{1}{\eta_s}\right\}$$
$$= \frac{\partial}{\partial z_n^j}\mathbb{E}[(K-\kappa)(K^j-K) \mid z, z].$$

Combining the different pieces and using the fact that $|u_{kk}|(1-\alpha) = -(u_{kk}+u_{kK})$, I conclude that

$$\frac{\partial U^j(z;z)}{\partial z_n^j} = -\frac{|u_{kk}|}{2}\frac{\partial}{\partial z_n}Var\left[k-K \mid z, k(\cdot;z)\right] \tag{43}$$
$$-\frac{|u_{kk}|}{2}(1-\alpha)\frac{\partial}{\partial z_n}Var\left[K-\kappa \mid z, k(\cdot;z)\right]-C_n'(z).$$

Clearly, in any symmetric equilibrium, for any source of information $n = 1, ..., N$ that receives strictly positive attention, it must be that the above derivative vanishes, which yields (15) in the main text.

Finally, the uniqueness of the symmetric equilibrium follows from arguments similar to those that establish uniqueness in the model with perfect recall; the proof is thus omitted for brevity. Q.E.D.

Proof of Corollary 3. The proof consists in expressing the various terms in (15) as a function of the parameters of the information structure. To simplify the exposition, hereafter I drop z from the arguments of the various functions when there is no risk of confusion.

First observe that

$$\frac{\partial}{\partial z_n} Var\left[k - K \mid z, k(\cdot; z)\right] = (\kappa_1 \gamma)^2 \frac{\partial}{\partial z_n} Var\left(\sum_{s=1}^{N} \delta_s \xi_s\right).$$

Next observe that

$$\frac{\partial}{\partial z_n} Var\left(\sum_{s=1}^{N} \delta_s \xi_s\right) = \frac{\partial}{\partial z_n}\left[\sum_{s=1}^{N} \frac{\delta_s^2}{t_s z_s}\right] = \sum_{s=1}^{N} \frac{2\delta_s}{t_s z_s}\frac{\partial \delta_s}{\partial z_n} - \frac{\delta_n^2}{t_n z_n^2} = 2\sum_{s \neq n} \frac{\delta_s}{t_s z_s}\frac{\partial \delta_s}{\partial z_n} + 2\frac{\delta_n}{t_n z_n}\frac{\partial \delta_n}{\partial z_n} - \frac{\delta_n^2}{t_n z_n^2}$$

$$= -2\sum_{s \neq n} \frac{\delta_s}{t_s z_s}\frac{\pi_s \frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2} + 2\frac{\delta_n}{t_n z_n}\left(-\frac{\pi_n \frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2} + \frac{\frac{\partial \pi_n}{\partial z_n}}{\pi_\theta + \pi_X}\right) - \frac{\delta_n^2}{t_n z_n^2}$$

$$= -2\frac{\frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2}\sum_{s=1}^{N} \frac{\delta_s \pi_s}{t_s z_s} + 2\frac{\delta_n}{t_n z_n}\frac{\frac{\partial \pi_n}{\partial z_n}}{\pi_\theta + \pi_X} - \frac{\delta_n^2}{t_n z_n^2} = -2\frac{\frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2}\left(\sum_{s=1}^{N} \frac{\delta_s \pi_s}{t_s z_s} - \frac{\delta_n}{t_n z_n}(\pi_\theta + \pi_X)\right) - \frac{\delta_n^2}{t_n z_n^2}$$

$$= -2\frac{\frac{\partial \pi_n}{\partial z_n}}{(\pi_\theta + \pi_X)^2}\left(\frac{\pi_X}{\pi_\theta + \pi_X}\sum_{s=1}^{N} \frac{\pi_s}{\pi_X}\frac{\pi_s}{t_s z_s} - \frac{\pi_n}{t_n z_n}\right) - \frac{\pi_n}{(\pi_\theta + \pi_X)^2}\frac{\pi_n}{t_n z_n}\frac{1}{z_n}.$$

Now recall that $\pi_s = \frac{\eta_s z_s t_s}{z_s t_s + \eta_s}$, which means that $\frac{\pi_s}{t_s z_s} = \frac{\eta_s}{z_s t_s + \eta_s}$ and that $\frac{\partial \pi_n}{\partial z_n} = \frac{\pi_n}{z_n}\frac{\pi_n}{t_n z_n}$. I thus have that

$$\frac{\partial}{\partial z_n} Var\left(\sum_{s=1}^{N} \delta_s \xi_s\right) = \frac{-2}{(\pi_\theta + \pi_X)^2}\frac{\pi_n}{t_n z_n}\frac{\pi_n}{z_n}\left[\frac{\pi_X}{\pi_\theta + \pi_X}\sum_{s=1}^{N}\left(\frac{\pi_s}{\pi_X}\right)\frac{\pi_s}{t_s z_s} - \frac{\pi_n}{t_n z_n}\right] - \frac{1}{(\pi_\theta + \pi_X)^2}\frac{\pi_n}{t_n z_n}\frac{\pi_n}{z_n}$$

$$= -\frac{1}{(\pi_\theta + \pi_X)^2}\frac{\eta_n^2 t_n}{(z_n t_n + \eta_n)^2}\left\{1 + 2\left[\frac{\pi_X}{\pi_\theta + \pi_X}\sum_{s=1}^{N}\left(\frac{\pi_s}{\pi_X}\right)\frac{\eta_s}{z_s t_s + \eta_s} - \frac{\eta_n}{z_n t_n + \eta_n}\right]\right\}.$$

This means that

$$-\frac{|u_{kk}|}{2}\frac{\partial}{\partial z_n} Var\left[k - K \mid z, k(\cdot; z)\right]$$

$$= \frac{|u_{kk}|(\kappa_1 \gamma)^2}{2(\pi_\theta + \pi_X)^2}\frac{\eta_n^2 t_n}{(z_n t_n + \eta_n)^2}\left\{1 + 2\left[\frac{\pi_X}{\pi_\theta + \pi_X}\sum_{s=1}^{N}\left(\frac{\pi_s}{\pi_X}\right)\frac{\eta_s}{z_s t_s + \eta_s} - \frac{\eta_n}{z_n t_n + \eta_n}\right]\right\}.$$

Next, use (42) to observe that

$$-\frac{|u_{kk}|(1-\alpha)}{2}\frac{\partial}{\partial z_n} Var\left[K - \kappa \mid z, k(\cdot; z)\right]$$

$$= -|u_{kk}|(1-\alpha)\kappa_1^2 \gamma\left\{\left(\gamma\sum_{s=1}^{N}\delta_s - 1\right)\left(\sum_{s=1}^{N}\frac{\partial \delta_s}{\partial z_n}\right)\frac{1}{\pi_\theta} + \gamma\sum_{s=1}^{N}\left(\delta_s\frac{\partial \delta_s}{\partial z_n}\right)\frac{1}{\eta_s}\right\}.$$

Note that

$$\sum_{s=1}^{N}\frac{\partial \delta_s}{\partial z_n} = \frac{\frac{\partial \pi_n}{\partial z_n}\pi_\theta}{(\pi_X + \pi_\theta)^2} \quad \text{and that} \quad \gamma\sum_{s=1}^{N}\delta_s - 1 = \frac{\gamma \pi_X}{\pi_X + \pi_\theta} - 1 = -\frac{(1-\gamma)\pi_X + \pi_\theta}{\pi_X + \pi_\theta}.$$

Hence

$$\left(\gamma\sum_{s=1}^{N}\delta_s-1\right)\left(\sum_{s=1}^{N}\frac{\partial\delta_s}{\partial z_n}\right)\frac{1}{\pi_\theta}=-\frac{(1-\gamma)\pi_X+\pi_\theta}{(\pi_X+\pi_\theta)^3}\frac{\partial\pi_n}{\partial z_n}.$$

Also note that

$$\gamma\sum_{s=1}^{N}\left(\delta_s\frac{\partial\delta_s}{\partial z_n}\right)\frac{1}{\eta_s}=\gamma\sum_{s=1}^{N}\left(\frac{\partial\delta_s}{\partial z_n}\right)\frac{\rho_s}{\pi_\theta+\pi_X}=-\gamma\frac{\partial\pi_n}{\partial z_n}\sum_{s=1}^{N}\left(\frac{\pi_s\rho_s}{(\pi_\theta+\pi_X)^3}\right)+\gamma\frac{\partial\pi_n}{\partial z_n}\frac{\rho_n}{(\pi_\theta+\pi_X)^2}.$$

It follows that

$$\begin{aligned}
&-\frac{|u_{kk}|(1-\alpha)}{2}\frac{\partial}{\partial z_n}var\left[K-\kappa\mid z,k(\cdot;z)\right]\\
&=-|u_{kk}|(1-\alpha)\kappa_1^2\gamma\left\{-\frac{(1-\gamma)\pi_X+\pi_\theta}{(\pi_X+\pi_\theta)^3}\frac{\partial\pi_n}{\partial z_n}-\gamma\frac{\partial\pi_n}{\partial z_n}\sum_{s=1}^{N}\left(\frac{\pi_s\rho_s}{(\pi_\theta+\pi_X)^3}\right)+\gamma\frac{\partial\pi_n}{\partial z_n}\frac{\rho_n}{(\pi_\theta+\pi_X)^2}\right\}\\
&=|u_{kk}|(1-\alpha)\kappa_1^2\gamma^2\frac{\frac{\partial\pi_n}{\partial z_n}}{(\pi_X+\pi_\theta)^2}\left\{\frac{(1-\gamma)\pi_X+\pi_\theta}{(\pi_X+\pi_\theta)\gamma}-\left(\rho_n-\frac{\pi_X}{\pi_\theta+\pi_X}\rho_X\right)\right\}\\
&=\frac{|u_{kk}|(1-\alpha)\kappa_1^2\gamma^2}{(\pi_X+\pi_\theta)^2}\frac{\eta_n^2t_n}{(z_nt_n+\eta_n)^2}\left\{\frac{(1-\gamma)\pi_X+\pi_\theta}{(\pi_X+\pi_\theta)\gamma}-\left(\rho_n-\frac{\pi_X}{\pi_\theta+\pi_X}\rho_X\right)\right\}.
\end{aligned}$$

I conclude that, in any symmetric equilibrium, for any source that receives strictly positive attention

$$\begin{aligned}
C_n'(z)=&\frac{|u_{kk}|\left(\kappa_1\gamma\right)^2}{(\pi_\theta+\pi_X)^2}\frac{\eta_n^2t_n}{(z_nt_n+\eta_n)^2}\left\{\frac{1}{2}+\frac{\pi_X}{\pi_\theta+\pi_X}\sum_{s=1}^{N}\left(\frac{\pi_s}{\pi_X}\right)\frac{\eta_s}{z_st_s+\eta_s}-\frac{\eta_n}{z_nt_n+\eta_n}\right\}\\
&+\frac{|u_{kk}|(1-\alpha)\left(\kappa_1\gamma\right)^2}{(\pi_X+\pi_\theta)^2}\frac{\eta_n^2t_n}{(z_nt_n+\eta_n)^2}\left\{\frac{(1-\gamma)\pi_X+\pi_\theta}{(\pi_X+\pi_\theta)\gamma}+\frac{\pi_X}{\pi_\theta+\pi_X}\rho_X-\rho_n\right\}
\end{aligned}$$

or, equivalently,

$$\begin{aligned}
C_n'(z)=&\frac{|u_{kk}|\left(\kappa_1\gamma\right)^2}{(\pi_\theta+\pi_X)^2}\frac{\eta_n^2t_n}{(z_nt_n+\eta_n)^2}\left\{-\frac{1}{2}+\frac{\pi_X}{\pi_\theta+\pi_X}+\frac{(1-\gamma)\pi_X+\pi_\theta}{(\pi_X+\pi_\theta)\gamma}\right\}\\
&-\alpha\frac{|u_{kk}|\left(\kappa_1\gamma\right)^2}{(\pi_\theta+\pi_X)^2}\frac{\eta_n^2t_n}{(z_nt_n+\eta_n)^2}\left\{\frac{(1-\gamma)\pi_X+\pi_\theta}{(\pi_X+\pi_\theta)\gamma}+\frac{\pi_X}{\pi_\theta+\pi_X}\rho_X-\rho_n\right\}
\end{aligned}$$

When $\pi_\theta=0$, $\gamma=1$ and the above reduces to

$$C_n'(z)=\frac{|u_{kk}|\left(\kappa_1\right)^2}{(\pi_X)^2}\frac{\eta_n^2t_n}{(z_nt_n+\eta_n)^2}\left\{\frac{1}{2}+\alpha\left(\rho_n-\rho_X\right)\right\}.$$

Q.E.D.

Proof of Proposition 6. From Proposition 1, one can see that, with perfect recall, the gross benefit of increasing (locally) the attention to any source of information, around the equilibrium level $\hat{z}_n$, is given by (again, I drop the dependence of the various functions on z when there is no risk of confusion):

$$\frac{|u_{kk}|(\kappa_1)^2}{2}\frac{1}{(\hat{z}_n)^2 t_n}\left(\frac{\frac{\hat{\pi}_n}{1-\alpha\hat{\rho}_n}}{\sum_{s=1}^N \frac{\hat{\pi}_s}{1-\alpha\hat{\rho}_s}}\right)^2 \tag{44}$$

$$= \frac{|u_{kk}|(\kappa_1)^2}{2(\hat{\pi}_X)^2}\frac{(\eta_n\hat{z}_n t_n)^2}{(\hat{z}_n t_n+\eta_n)^2(\hat{z}_n)^2 t_n}\frac{1}{(1-\alpha\hat{\rho}_n)^2\left(\sum_{s=1}^N \frac{\hat{\pi}_s}{\hat{\pi}_X}\frac{1}{1-\alpha\hat{\rho}_s}\right)^2}$$

$$= \frac{|u_{kk}|(\kappa_1)^2}{2(\hat{\pi}_X)^2}\frac{\eta_n^2 t_n}{(\hat{z}_n t_n+\eta_n)^2}\frac{1}{(1-\alpha\hat{\rho}_n)^2\left(\sum_{s=1}^N \frac{\hat{\pi}_s}{\hat{\pi}_X}\frac{1}{1-\alpha\hat{\rho}_s}\right)^2}$$

where

$$\hat{\pi}_X \equiv \sum\nolimits_{s=1}^N \hat{\pi}_s,\ \ \hat{\pi}_s \equiv \frac{\eta_s\hat{z}_s t_s}{\hat{z}_s t_s+\eta_s} \text{ and } \hat{\rho}_s \equiv \frac{\hat{\pi}_s}{\eta_s}.$$

From (16), one can also see that, starting from $\hat{z}$, the gross benefit of increasing (locally) the attention to the n-th source for an agent with bounded recall is given by

$$\frac{|u_{kk}|(\kappa_1)^2}{2(\hat{\pi}_X)^2}\frac{\eta_n^2 t_n}{(\hat{z}_n t_n+\eta_n)^2}\left\{1+2\alpha(\hat{\rho}_n-\hat{\rho}_X)\right\} \tag{45}$$

where $\hat{\rho}_X \equiv \sum_{s=1}^N \frac{\hat{\pi}_s}{\hat{\pi}_X}\hat{\rho}_s$. Comparing (44) with (45), it is then easy to see that the gross benefit is larger in the presence of bounded recall if

$$1+2\alpha(\hat{\rho}_n-\hat{\rho}_X) > \frac{1}{(1-\alpha\hat{\rho}_n)^2\left(\sum_{s=1}^N \frac{\hat{\pi}_s}{\hat{\pi}_X}\frac{1}{1-\alpha\hat{\rho}_s}\right)^2} \tag{46}$$

and lower if the inequality is reversed. Now, let $f(\rho)$ the function defined by

$$f(\rho) \equiv \frac{1}{1-\alpha\rho}$$

and note that this function is convex. By Jensen inequality, it then follows that

$$\sum_{s=1}^N \frac{\hat{\pi}_s}{\hat{\pi}_X}\frac{1}{1-\alpha\hat{\rho}_s} > \frac{1}{1-\alpha\hat{\rho}_X}.$$

Furthermore, fixing the term

$$\sum_{s=1}^N \frac{\hat{\pi}_s}{\hat{\pi}_X}\frac{1}{1-\alpha\hat{\rho}_s},$$

the function

$$\frac{1}{(1-\alpha\hat{\rho}_n)^2\left(\sum_{s=1}^N \frac{\hat{\pi}_s}{\hat{\pi}_X}\frac{1}{1-\alpha\hat{\rho}_s}\right)^2}$$

in the right-hand-side of (46) is convex in $\hat{\rho}_n$.

The properties above imply that, for $\hat{\rho}_n = \rho_X$, the inequality in (46) holds. Along with the fact that the left-hand-side of (46) is linear in $\hat{\rho}_n$ and the right-hand-side is convex in $\hat{\rho}_n$, they also imply that there exist $\rho', \rho'' \in [0,1]$ with $0 \leq \rho' < \hat{\rho}_X < \rho'' \leq 1$ such that the inequality in (46) holds if and only if $\hat{\rho}_n \in [\rho', \rho'']$. Lastly note that, when $\alpha = 1$, the inequality in (46) is reversed for ρ close to 1. By continuity, I then have that $\rho'' < 1$ for α large enough. Likewise, one can verify that, for α large enough, the inequality in (46) can be reversed when evaluated at ρ_n close to zero. This means that there can be situations in which $\rho' > 0$ for α large enough. Q.E.D.

Proof of Corollary 4. The result follows from Proposition 6 along with the fact that, when the attention cost depends only on total attention, there is a monotone relationship between the exogenous transparency of the sources and their endogenous publicity. Namely, if $t_n > t_{n'}$, then $\delta_n(\hat{z}) \geq \delta_{n'}(\hat{z})$. The proof follows from arguments similar to those in Proposition 5 in Myatt and Wallace and is thus omitted. Q.E.D.

Proof of Proposition 7. The result follows directly from applying the envelope theorem to the welfare function under the efficient actions, as given in (18). Q.E.D.

Proof of Proposition 8. Using the derivations in the proof of Corollary 3, the private benefit of increasing the attention allocated to any source of information is given by

$$\frac{|u_{kk}|\,(\kappa_1)^2}{(\pi_X)^2} \frac{\eta_n^2 t_n}{(z_n t_n + \eta_n)^2} \left\{ \frac{1}{2} + \alpha\,(\rho_n - \rho_X) \right\} \tag{47}$$

where I drop the dependence of π_X, ρ_n and ρ_X on z to ease the exposition. Because $\gamma(z) = \gamma^{**}(z) = 1$ for all z, when $\pi_\theta = 1$, following steps similar to those in the proof of proof of Corollary 3, it is then easy to see that the social benefit of increasing the attention allocated to any source is given by

$$\frac{|u_{kk} + u_{\sigma\sigma}|\,(\kappa_1^*)^2}{(\pi_X)^2} \frac{\eta_n^2 t_n}{(z_n t_n + \eta_n)^2} \left\{ \frac{1}{2} + \alpha^*\,(\rho_n - \rho_X) \right\}. \tag{48}$$

The results in the proposition then follows by comparing (47) with (48). Q.E.D.