Applications and Interviews

A Structural Analysis of Two-Sided Simultaneous Search*

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Job Market Paper

November 17, 2009

Abstract

Most of the literature analyzing equilibrium effects of labor market policies assumes bilateral meetings between workers and firms. This ignores the frictions that arise when workers and firms meet in a multilateral way and cannot coordinate their application and hiring decisions. I analyze the magnitude of these frictions. For this purpose, I present an equilibrium search model of the labor market with an endogenous number of contacts between workers and firms. Firms post a wage and a recruitment technology that determines how many applicants they will interview. After observing these contracts, workers decide to which firms to apply. Sending more applications and interviewing more applicants are both costly activities but increase the probability to match. In equilibrium, contract dispersion arises endogenously and workers spread their applications over the different types of contracts. Estimation of the model on the Employment Opportunities Pilot Projects data set provides values for the cost of an application and the cost of an interview. These estimates are used to determine the output loss compared to a Walrasian world. Frictions on the worker and the firm side reduce output by approximately 1% each. There is a potential role for activating labor market policies, because I show that for the estimated parameter values welfare can be improved if unemployed workers increase their search intensity.

*This paper was originally titled *Simultaneous Search and Recruitment*. I am very grateful to Philipp Kircher for advice at various stages of this project. I thank Giuseppe Ferrero, Manolis Galenianos, Pieter Gautier, Veronica Guerrieri, Ali Hortacsu, Guido Menzio, Robert Shimer, and many others for their valuable comments. This paper has benefited from feedback during presentations at the University of Iowa, the 2009 Meeting of the Society of Economic Dynamics in Istanbul, the 2009 European Meeting of the Econometric Society in Barcelona, the Tinbergen Institute, the Search and Matching Workshop at the Philadelphia FED, the Fifth IZA Conference on Labor Market Policy Evaluation at Georgetown University, Chicago Booth, and the University of Chicago.

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1 Introduction

1.1 Motivation and Summary

In many Western economies, governments spend significant amounts of resources on labor market programs.[1] Many of these programs aim to shorten the unemployment spells of the targeted workers. Examples include counseling and monitoring of unemployed workers, training programs, wage subsidies, and programs that provide the workers with a bonus if they find a job quickly. Also unemployment insurance (UI) eligibility rules which require workers to exert a certain level of search effort every week or month aim to achieve the same goal by influencing search intensities. Empirical evaluations of these policies often show that they indeed reduce the duration of unemployment for the workers in question.

However, various authors have pointed out that such programs may generate equilibrium effects which should not be ignored. For example, if a certain fraction of the population starts to search harder as a result of the programs, the remaining workers in the economy suddenly face more competition. This may reduce their job finding rate. Moreover, there may be a change in the equilibrium wage levels or in the matching probabilities of the firms, which can create a feedback effect for the workers via the equilibrium number of job openings that is available. These general equilibrium effects typically offset the partial equilibrium effect to a considerable extent, as shown by for example Davidson & Woodbury (1993), Lise et al. (2005), and Cahuc et al. (2008).[2]

However, all these studies consider models in which workers contact at most one firm at a time. Either meetings are assumed to be bilateral, i.e. exactly one firm and one worker are involved in any given meeting, or workers are assumed to send one application per time period, such that firms can face multiple applicants but workers cannot face multiple firms. Although such assumptions are convenient from a technical point of view, it seems at odds with what is observed in actual labor markets. Typically, firms receive multiple applications for their vacancies and they have to decide which worker to hire. This recruitment decision may cause significant delays between the moment at which a worker applies to the job and the point at which the firm makes the hiring decision.[3] Such delays provide the workers with an incentive to send multiple applications simultaneously in order to reduce the risk of having to wait another period when failing to match in the current period.[4] Hence, multilateral meetings on both the worker and the firm side are an important feature of most labor markets.

When workers send multiple applications, the nature of competition between agents becomes

[1] In 2006, the United States spent approximately 52 billion dollars on labor market policies (OECD, 2009). The equivalent figure for the EU-15 was approximately 200 billion euros (Eurostat, 2009).

[2] In the latter two papers, the general equilibrium effects even reverse the conclusion about the desirability of the program. The importance of equilibrium effects has also been emphasized in different contexts, e.g. by Heckman et al. (1998) and Albrecht et al. (2009).

[3] For example, van Ours & Ridder (1992) find an average delay of about 2.5 months in Dutch data.

[4] See for example Morgan & Manning (1985).

more direct. In particular, firms must now take into account that if they make a job offer to a worker, the worker may reject it because he got a better offer from a different firm. This concern does not exist when workers meet at most one firm per time period. Clearly, the rejection of a job offer is more likely to happen when the worker in question has sent more applications. This effect has potentially important implications for the desirability of policies that aim to increase the search intensity of workers. Suppose that the rejection of a job offer is costly to the firm, either because if reduces the firm's hiring probability or because the firm incurs a cost for each worker it approaches. In that case, a worker who sends more applications to other firms generates a negative externality for the firm. Ignoring this externality may, depending on its size, lead to wrong policy recommendations. Nevertheless, the literature on this topic is very limited and, to the best of my knowledge, these effects have never been quantified.[5]

In this paper, I therefore analyze the magnitude of these effects. I present a model with multilateral meetings between workers and firms which incorporates all the externalities mentioned above and which can be used as a framework for policy analysis.[6] Firms post contracts in order to attract applicants. Unemployed workers observe these contracts and decide where to apply. An important novelty compared to existing models of multilateral interaction in the labor market is that both workers and firms can choose how many agents on the other side of the market they want to contact. Sending many applications reduces the worker's probability of remaining unemployed, but is costly. Therefore, a worker will typically decide to apply to multiple but not all firms. A similar structure holds for the firms. A firm contacts applicants by inviting them for an interview. Interviewing applicants is a costly but necessary activity before job offers can be made, hence it reduces the probability for a firm to remain unmatched. As a result, firms typically choose to interview multiple but not all applicants.

The limited number of contacts creates two types of frictions, which arise because workers and firms cannot coordinate their actions. The first is a standard search friction. Two workers may apply to the same firm, but only one of them can get the job. The other worker has wasted a costly application. A similar friction exists on the firm side, which I will call the recruitment friction. Two firms may interview the same candidate, but only one of them can hire him. The other firm has wasted a costly interview.[7] Together, these frictions are responsible for the externalities in the labor market. The two fundamental parameters of the model, i.e. the cost of an application and the cost of an interview, determine the magnitude of the frictions and the externalities. For example, if the

[5]Some anecdotal evidence exists which suggests that firms are concerned about too high levels of search intensity. O'Leary (2006) describes how many states in the US have moved away from specifying explicit criteria on the search intensity of unemployed workers in the law, because firms "do not want repetitive and burdensome employment applications that are filed merely to meet the UI work search requirements."

[6]Note that changes in search intensity may also have an effect on the level of sorting in a labor market, which depending on the production function may or may not be desirable. I abstract from such effects in this paper.

[7]Albrecht et al. (2004, 2006) discuss both frictions in a theoretical setting, but call them the 'urn-ball friction' and the 'multiple-application friction' respectively. See also Gautier & Moraga-González (2005).

cost of an application is really low, workers will decide to apply a lot and the search friction will be limited. However, firms now face a lot of competition for each worker, so the recruitment friction will be severe. On the other hand, if the cost of an interview is sufficiently close to zero, firms would be able to contact all applicants and the recruitment friction would be absent.

After specifying the model, I characterize the equilibrium. I show that different types of contracts are offered by the firm. Some firms offer low wages but contact many applicants, while other firms do the opposite. The number of contract types is equal to the maximum number of applications that workers send in any given period and firms are indifferent between all types. Workers face a trade-off between the wage and the job offer probability. Applications to low wage firms are more likely to turn into job offers than applications to high wage firms. I show that workers maximize the payoff from their application portfolio by spreading their applications over the different types of contracts.

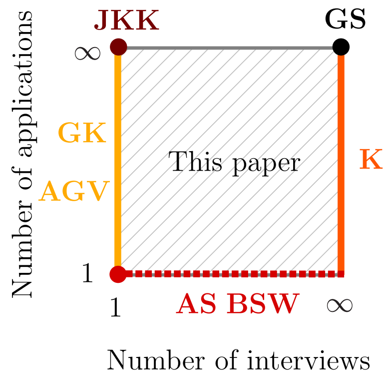
Estimation of the model on the Employment Opportunities Pilot Projects data set provides values for the cost of search and recruitment. An additional application is estimated to cost the worker 0.6% of one period of output, while firms incur a cost equal to 1.7% of periodical output for each interview. By simulating the equilibrium for different values of the two cost parameters, the market equilibrium can be compared with worlds in which one of the frictions or both of the frictions are absent. Output is 2.4% lower in the market equilibrium than in a Walrasian world, whereas unemployment is 4.1 percentage point higher. Search and recruitment frictions both contribute roughly equally much to the output loss.

Finally, I consider a social planner's problem to analyze the effect of UI eligibility rules that specify a minimum search intensity. The planner chooses a certain minimum level, after which workers decide whether they comply or not. I show that if unemployed workers increase their search intensity in reaction to these rules, firms indeed incur higher recruitment costs. Their matching probability goes down due to the increased competition that they face for any given candidate. However, for modest increases in the search intensity, steady state output goes up.

1.2 Related Literature

In addition to the literature on equilibrium effects of policy cited above, this paper adds to the existing literature in several ways. First of all, the paper extends the literature on micro-founded matching technologies by analyzing a model with an arbitrary number of contacts on both the supply and the demand side. Let A denote the number of applications per worker and let I be the number of interviews per firm.[8] Various authors have analyzed different combinations of A and I. For example, the seminal stable matching outcome of Gale & Shapley (1962) can be interpreted as a model with

[8] In the model described in this paper, both variables are endogenous and have a stochastic nature (see section 2). I abstract from that here in order to keep the comparison with literature as clear as possible.



Overview of related matching technologies. The vertical axis shows the number of applications A that workers send out. The horizontal axis displays the number of interviews I that a firm can conduct. GS = Gale & Shapley (1962), JKK = Julien et al. (2000), AS = Acemoglu & Shimer (2000), BSW = Burdett et al. (2001), AGV = Albrecht et al. (2006), GK = Galenianos & Kircher (2009), K = Kircher (2009). In Acemoglu & Shimer (2000) and Burdett et al. (2001), the first interview always leads to a match. The equilibrium outcome does therefore not depend on the number of interviews.

Figure 1: Related literature

$A \to \infty$ and $I \to \infty$.[9] This case, which has been applied extensively in the analysis of many different markets, would allow all workers in our model to apply to all firms and all firms to contact all workers.[10]

The other extreme is a situation with $A = 1$ and $I = 1$, i.e. all workers send one application and all firms can contact one worker. This is the matching technology explored by for example Acemoglu & Shimer (2000) and Burdett et al. (2001). Julien et al. (2000) study a model in which workers apply to all firms, and firms can bid for the services of one worker, i.e. $A \to \infty$ and $I = 1$. Albrecht et al. (2006) and Galenianos & Kircher (2009) also impose $I = 1$, but study the case in which workers send multiple applications simultaneously, i.e. $A > 1$.[11] Gautier & Moraga-González (2005), Kaas (2007) and Gautier et al. (2008) study a similar matching process in a random search setting. Finally, Kircher (2009) studies a model with $A > 1$ applications in which firms can contact all their applicants ($I \to \infty$). The model presented here includes all the matching technologies in these papers as special cases, as summarized in figure 1.

Second, the paper adds to the large literature on the estimation of equilibrium search models. Eckstein & van den Berg (2007) provide an extensive overview. A large part of this empirical

[9] The paper by Gale & Shapley (1962) includes a lot more heterogeneity on both sides of the market than most other models. I do not consider that aspect in this discussion.

[10] Parallel to the search literature, a large literature on matching has been developed with extensions, refinements and applications of the original model by Gale & Shapley (1962). See Roth (2008) for a recent overview.

[11] In a technical appendix to their paper, Albrecht et al. (2006) briefly discuss what happens if firms can contact a second applicant ($I = 2$).

literature is based on the random search model by Burdett & Mortensen (1998).[12] This model of on-the-job search has proven to be a very useful tool for estimation purposes, because of its clear insights and its analytical tractability even when allowing for e.g. worker and firm heterogeneity.[13] I use a directed search model instead. In these models, workers observe the wages that firms post before they apply.[14] The models have become very popular in the theoretical search literature and often provide strong empirical predictions, like e.g. a positive relationship between the wage that a firm posts and its hiring probability.[15] Nevertheless, the number of papers that confronts directed search models with the data is still very limited. Two exceptions are Menzio & Shi (2008), who do calibration for the US labor market, and Fu (2009), who estimates a model of college applications and admissions. To the best of my knowledge, this paper is the first to estimate a directed search model on microdata of the labor market.

Third, I contribute to the literature on recruitment decisions by firms. Barron et al. (1985), Barron et al. (1987) and Burdett & Cunningham (1998) study the determinants of employer search measures using the same data set as this paper. van Ours & Ridder (1992, 1993) and Abbring & van Ours (1994) analyze Dutch recruitment data and find evidence for simultaneous rather than sequential search by firms. All these papers have in common that they consider partial equilibrium models. Villena-Roldán (2008) develops an equilibrium model in which workers are heterogeneous and firms choose how many applicants to screen. However, all workers apply only once, which makes the model unsuitable for studying recruitment frictions.

Finally, this paper adds to the literature that develops measures for the magnitude of frictions and/or inefficiencies in the labor market. Ridder & van den Berg (2003) construct measures for search frictions that are invariant to the wage determination mechanism by using information on job durations. Gautier et al. (2008) solve a social planner's problem after estimating the distribution of workers' search cost with a random search model. Gautier & Teulings (2006, 2009) and Eeckhout & Kircher (2009) consider the efficiency loss due to mismatch. I use a slightly different approach. As discussed above, the model that I construct incorporates several other models as special cases, including models without search and/or recruitment frictions. A comparison of the equilibrium

[12]However, different models have been used as well. See Eckstein & Wolpin (1990) for an early example and Flinn (2006) for a more recent example.

[13]See for example van den Berg & Ridder (1998), Bontemps et al. (1999), and Bontemps et al. (2000).

[14]Early examples include Montgomery (1991), Peters (1991), Moen (1997), Acemoglu & Shimer (1999), and Burdett et al. (2001).

[15]Although nobody has formally tested directed versus random search, some empirical findings can be explained more easily with directed search models. For example, Hall & Krueger (2008) find in a recent survey of the US labor market that 84% of the white male non-college workers "knew exactly" or "had a pretty good idea" how much their current job would pay at the moment of the first interview. Further, Holzer et al. (1991) find, using the same data set as this paper, that firms in high-wage industries attract more applicants per vacancy than firms in low-wage industries. The theoretical analysis by Galenianos & Kircher (2009) suggests that directed search models may be better able to explain the downward sloping part in the wage distributions that is typically found in the data and cannot easily be fitted with random search models. As shown by Menzio & Shi (2008, 2009), directed search models also have the advantage that they are a lot more tractable when analyzing the dynamics out of steady state.

for different values of the search and recruitment cost parameters will therefore indicate how the frictions affect labor market outcomes. I study this issue in section 4.4.

The structure of the paper is as follows. Section 2 describes the model. The equilibrium is characterized in section 3. Section 4 presents the empirical part of this paper and section 5 concludes. Proofs are relegated to the appendix.

2 Model

Consider the steady state in a labor market in discrete time with a unit mass of workers and a positive measure of firms.[16] Both types of agents are infinitely-lived and risk-neutral. They maximize the sum of expected future periodical payoffs, discounted with the factor $\beta \in (0,1)$. All workers supply one indivisible unit of labor, while each firm has a position that can be filled by exactly one worker. A worker who is matched with a firm is called employed and creates 1 unit of output per period. Unemployed workers aim to match with firms with vacancies by sending applications. Throughout this paper, I assume that all firms are identical in their characteristics. In order to keep the exposition as simple as possible, I initially also assume that all worker are homogeneous, both in their productivity while employed and in the number of applications that they send while unemployed. For the moment, this number of applications is exogenously given. I relax these assumptions in the second part of this paper, after having characterized the equilibrium for the simpler setting.

Each period consists of several phases. First, there is a separation phase, in which a fraction $\delta \in (0,1)$ of the existing matches is destroyed by an exogenous job destruction shock. Workers hit by the shock become unemployed. I denote the measure of unemployed workers by u. In the second phase, vacancies are created. There is free entry of vacancies at a cost $k_V > 0$. In order to attract applications from unemployed workers, entering firms post a contract c . Each firm commits to its contract for the remainder of the period. A contract consists of two elements, which both are choice variables to the firm.

The first component of the contract is the wage $w \in \mathcal{W} \equiv [0,1]$ that the firm promises to pay to the worker it hires. The second component is a recruitment technology $r \in \mathcal{R} \equiv [1,\infty)$, which can be interpreted as a measure for the amount of time that the firm will spend on interviewing workers or as a measure for the number of recruiters that the firm has hired. As mentioned in the introduction, the frictions in the labor market are less severe if the firm spends a lot of time on interviewing. A higher value of r therefore increases the matching probability of both the firm and the workers

[16] It is straightforward to extend the model such that one can analyze the dynamics out of steady state, using a Block Recursive Equilibrium as in Menzio & Shi (2008, 2009). I do not pursue this approach because of data limitations. See section 4.1.

applying to the firm. However, choosing a higher value for r is also more costly for the firm. I assume that the cost of recruitment technology r equals $k_R(r-1) \geq 0$.[17] Summarizing, a contract c is pair (r, w). For future reference, let $\mathcal{C} = \mathcal{R} \times \mathcal{W}$ denote the set of possible contracts and $v(c)$ the mass of firms offering a contract c, with $v = \int_{\mathcal{C}} v(c)\, dc$.

The third phase is the search phase, during which unemployed workers can search for a job by sending applications. I assume that workers who lost their job during the separation phase cannot search in the current period. Workers observe all posted contracts and base their search decision on this information. Since there is a delay between sending the application letters and learning their result, workers typically have an incentive to send multiple applications simultaneously (see Morgan & Manning, 1985). Sending multiple applications is more costly than applying only once, since each application takes time and effort, but has two positive effects. First, it reduces the risk of not getting any job offer and remaining unemployed. Second, if there is wage dispersion in equilibrium, it increases the probability to get a really high wage offer. For the moment, I assume that all workers send A applications in each period, where A is a finite integer larger than 1. After characterizing the equilibrium for this case, I show that this assumption can easily be relaxed. In the empirical part of this paper, I allow workers to choose a level of search intensity which translates into a stochastic number of applications. In the current setup however, workers only have to decide to which contracts they want to send their A applications.

After the applications are sent, the recruitment phase starts. During the recruitment phase, each of the firms will interview a number of applicants. The number of interviews R that a firm can conduct (its 'interview capacity') is a random variable which follows a distribution around the firm's recruitment technology r. The intuition for this setup is as follows. In reality, a firm can roughly determine the number of applicants it is able to interview by hiring more or fewer recruiters or by allocating more or less time to recruitment. However, the exact capacity often also depends on factors that cannot be anticipated by the firm. For example, some recruiters may unexpectedly not be available at the moment of hiring, or the screening of a certain candidate takes longer or shorter than foreseen. Having a stochastic element in the recruitment capacity captures this. For computational reasons, it is convenient to have a distribution that generates tractable expressions for variables like the matching probability. As I will show in section 3.2, a geometric distribution does precisely this. Therefore, I assume that the actual interview capacity follows a geometric distribution with parameter $\frac{r-1}{r} \in [0, 1]$. Hence, the probability that the firm can interview $R \in \mathbb{N}_1$ applicants is given by $\left(\frac{r-1}{r}\right)^{R-1} \frac{1}{r}$, which implies that the expected capacity is exactly equal to the firm's investment r.[18]

[17] The first unit of capacity is free, since it assumed to be included in the entry cost. This assumption is not restrictive, but rules out a situation in which a large k_R dissuades firms from interviewing any candidates which eventually leads to the collapse of the market. As will become clear, the model now converges to the model in which all firms interview one applicant, as described in Galenianos & Kircher (2009), for $k_R \to \infty$.

[18] Compared to a situation in which firms can interview a deterministic number of applicants $\widetilde{R}$, the approach followed here has a second advantage. Estimating the continuous parameter r is computationally much easier than estimating the

The firm's interview capacity R is realized at the beginning of the recruitment phase and therefore not observed by the workers. Note that the actual number of interviews a firm conducts may be constrained by the number of applicants. If the firm receives a applications, the actual number of interviews will be $\min\{a,R\}$. If the number of applicants exceeds the interview capacity, candidates are chosen randomly since they all seem identical to the firm. Remaining applicants are rejected. During the interview the firm learns the outcome of a stochastic process that is iid over all firm-worker pairs. With probability $\phi \in (0,1]$, the firm and the worker form a good match and are indeed able to produce output. I call these candidates 'qualified'. With probability $1-\phi$ however, the match is a bad match which would be completely unproductive. The firm rejects these unqualified applicants.

In the next phase, matches are formed between workers and firms. For this, consider workers and firms as nodes in a bipartite network. The applications that are sent create links between the workers and the firms. The rejection of some applicants by the firms during the recruitment phase destroys some of these links. The matching is now assumed to be stable on the remaining network in the Gale & Shapley (1962) sense.[19] Hence, matches form such that no firm remains unmatched while one of its qualified candidates is hired by another firm at a lower wage or remains unemployed. If this were not true, both the firm and the worker could do better by deviating and forming a match together. Stability can be motivated by a process in which firms offer their job sequentially to the candidates, and workers are free to reconsider their options.[20] Ties are broken randomly.

In the last phase, production and consumption take place. Each match generates output, of which the worker consumes a fraction w. The firm obtains the remaining fraction $1-w$. Unemployed workers receive a payoff from unemployed benefits and/or household production, which together add up to $h \in (0,1)$.[21] After this, a new period starts.

In the next section, I derive the workers' and firms' optimal strategies. As standard in the literature, I impose symmetry, i.e. identical agents have identical strategies. Further, I require the strategies to be anonymous, in the sense that they cannot be conditioned on the identity of a specific worker or firm.

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discrete parameter $\widetilde{R}$.

[19] In the Gale & Shapley (1962) model, the network is complete. This paper allows for an arbitrary level of sparseness.

[20] In more detail: each firm randomly selects one of its qualified applicants, if any, and offers him the job. The applicant in question might get multiple offers. He compares them, tentatively accepts the best one and rejects all others. If a firm gets rejected, it can make new job offers if it still has other qualified applicants. After this, workers again compare all offers, potentially including the one tentatively accepted in the previous round, accept the best one for the moment and reject all other. The process continues until no firm can make an offer anymore. All offers that are tentatively accepted at the end of this process become matches between the worker and firm in question. This is known as the Deferred Acceptance Process, first described by Gale & Shapley (1962). For finite economies it converges in finite time. Since the labor market described here contains a continuum of agents, I impose stability by assumption, following Kircher (2009).

[21] The composition of h is irrelevant in the derivation of the market equilibrium. In section 4.4, I will make an assumption about the level of the unemployment benefits in order to calculate steady state output.

3 Equilibrium Analysis

3.1 Expected Payoffs

Consider an unemployed worker who has observed all posted contracts and has to decide where to apply. This decision is not trivial, since the worker does not only care about the wage that he can earn at a particular firm, but also about the probability that the firm will hire him. The worker realizes that relatively many workers may apply to firms which post high wages or large investments in recruitment, which implies that the matching probability at those firms will be lower. This provides the worker an incentive to consider firms with lower wages or recruitment investments as well. Although the worker can distinguish firms that post different contracts, the anonymity assumption implies that the worker applies to all firms that post the same contract with equal probability. The application strategy of the worker can therefore be written as

$$\mathbf{c} = (c_1, \ldots, c_A) = ((r_1, w_1), \ldots, (r_A, w_A)),$$

where c_i indicates to what contract the worker sends his i-th application. It will convenient to assume without loss of generality that $w_1 \leq \ldots \leq w_A$ and that the worker accepts the contract with the higher index in case of a tie. I denote the distribution of workers' strategies by G. In other words $G(\widetilde{\mathbf{c}})$ denotes the probability that an unemployed worker sends his first application to a contract (r, w) below $\widetilde{c}_1 = (\widetilde{r}_1, \widetilde{w}_1)$, his second application to a contract below $\widetilde{c}_2$, et cetera. The support of G is denoted by $\mathcal{G}$.

Let $\psi(c)$ denote the endogenous probability that an application to a firm posting $c = (r, w)$ results in a job offer. The result of each application is independent of the result of the workers' other applications. Therefore, the worker's job finding probability $\Psi_0(\mathbf{c})$, which equals the probability that the application strategy $\mathbf{c}$ results in at least one job offer, is given by

$$\Psi_0(\mathbf{c}) = 1 - \prod_{i=1}^{A} (1 - \psi(c_i)). \tag{1}$$

A worker accepts the job offer that gives him the highest wage. Hence, he ends up in a position paying w_i if he gets a job offer from that firm and if none of the applications to higher wages resulted in an offer. This happens with probability $\prod_{j=i+1}^{A} (1 - \psi(c_j)) \psi(c_i)$. The worker will then receive the value of employment at the wage w_i, which I denote by $V_E(w_i)$, but has to give up his outside option, i.e. the value of unemployment V_U. Summing over all possible values $i \in \{1, \ldots A\}$ gives the expected payoff of the strategy $\mathbf{c}$.

$$V_S(\mathbf{c}) = \sum_{i=1}^{A} \prod_{j=i+1}^{A} (1 - \psi(c_j)) \psi(c_i) (V_E(w_i) - V_U). \tag{2}$$

Workers choose the application strategy $\mathbf{c}$ that maximizes their expected payoff, i.e.

$$V_S^* = \max_{\mathbf{c}} V_S(\mathbf{c}) \tag{3}$$

A worker who is unemployed at the beginning of the production phase has an immediate payoff h. In the next period, the worker gets the value of unemployment V_U and the value of search, which equals $\max\{0, V_S^*\}$. Therefore, the worker's lifetime utility V_U satisfies

$$V_U = h + \beta\left(V_U + \max\{0, V_S^*\}\right). \tag{4}$$

On the other hand, a worker who is employed at the beginning of the production phase has an immediate payoff equal to his wage w. In the next period, the worker becomes unemployed with probability δ, otherwise he stays in his job. Hence, the lifetime utility $V_E(w)$ of a worker employed at a wage w satisfies

$$V_E(w) = w + \beta\left(\delta V_U + (1-\delta) V_E(w)\right). \tag{5}$$

Note that $V_E(w)$ is monotonically increasing in w, which implies that we can find an inverse function $V_E^{-1}(V)$ such that $V_E^{-1}(V_E(w)) = w$.

Next, consider a firm employing a worker and paying him a wage w. The match lasts until a job destruction shock takes place. The value function of the firm therefore equals

$$V_F(w) = 1 - w + \beta(1-\delta) V_F(w). \tag{6}$$

Firms with vacancies post a contract c in order to maximize their expected future payoff. Denote the distribution of contracts by $F(c)$, its density by $f(c)$ and its support by $\mathcal{F}$. Let $\eta(c)$ denote the hiring probability for a firm posting c, which indirectly depends on the number of firms $v(c)$ posting this specific contract. Then, firms solve

$$\max_{c} V_V(c),$$

where

$$V_V(c) = \eta(c) V_F(w) - k_V - k_R(r-1).$$

The free entry condition implies that firms will continue to enter and post a contract c until the expected payoff of this strategy becomes zero. Hence, in equilibrium we must have

$$V_V(c) \leq 0, \tag{7}$$

and $v(c) \geq 0$, with complementary slackness, for all $c \in \mathcal{C}$.

3.2 Queue Lengths

After specifying the value functions, I now turn to the matching probabilities for the workers and the firms. For this, consider a group of firms all posting same contract c and the group of unemployed workers each sending one or more applications to these firms. The anonymity assumption implies that the workers apply to each of the firms with equal probability. Hence, the number of applicants that a firm faces is a random variable. If the number of workers and firms were finite, it would follow a binomial distribution (see Albrecht et al., 2004). With a continuum of agents on both sides of the market, as I consider here, the number of applicants per firm converges to the Poisson distribution. This distribution can by characterized by one parameter, i.e. its mean, which equals the ratio of the total number of applications sent and the total number of firms at a given contract c. I denote by this mean by $\lambda(c)$. In the search literature, $\lambda(c)$ is called the queue length, since it describes how many applicants in expectation will be lining up for the job.

Before I consider the matching probabilities in the model presented here, it is instructive to briefly consider a few special cases, which already have been described in literature. For example, consider a world in which all workers send one application and all firms interview one applicant. Then it can be shown that the queue length $\lambda(c)$ determines the matching probabilities for the workers and the firms. After all, firms can be certain that their job offer will be accepted and hire thus as long as they have at least one qualified applicant, i.e. with probability $\phi\left(1-e^{-\lambda(c)}\right)$. Since there are $\lambda(c)$ as many workers as firms, an individual worker finds a job with probability $\phi\left(1-e^{-\lambda(c)}\right)/\lambda(c)$ (compare to e.g. Acemoglu & Shimer, 2000 and Burdett et al., 2001).

If workers start to send multiple applications, the recruitment technology of the firm becomes more important. If firms can still only interview one applicant, recruitment frictions arise because of congestion. A firm may offer the job to an applicant who rejects the offer because he got a better offer somewhere else. Hence, firms do not only care about the probability to have at least one applicant, but also about the probability that their job offer will get accepted. I denote this probability by $1-\Psi(c)$. Hence, firms now match with probability $\phi(1-\Psi(c))\left(1-e^{-\lambda(c)}\right)$, while the workers' job offer probability still equals $\phi\left(1-e^{-\lambda(c)}\right)/\lambda(c)$. This is the case analyzed by Galenianos & Kircher (2009), which corresponds with $k_R \to \infty$ (i.e. $r=1$) in the model presented in this paper.

On the other hand, if $k_R = 0$ (i.e. $r \to \infty$), firms can freely interview all their applicants. In this case, recruitment frictions do not play a role. The firm may still offer the job to a worker who rejects it because of a better offer somewhere else, but this worker does not cause congestion since the firm can now go back to any other applicants it had. Hence, the firm will match as long as it has at least one qualified applicant without better offers. Kircher (2009) shows that the expected number of such applicants per firm follows a Poisson distribution with mean

$$\mu(c) = \phi(1-\Psi(c))\lambda(c). \tag{8}$$

Clearly, $\mu(c) \leq \lambda(c)$. Both firms and workers only care about the queue length $\mu(c)$ now. The firm's hiring probability equals $1 - e^{-\mu(c)}$ and the workers' job offer probability $\phi\left(1 - e^{-\mu(c)}\right)/\mu(c)$.

For intermediate values of k_R, firms will typically interview neither one nor all applicants. The actual number of interviews that a firm conducts depends on its number of applicants and its interview capacity, both of which are endogenous random variables. However, the following lemma shows that a simple expression can be derived for the expected number of interviews.

Lemma 1. *A firm posting a contract $c = (r, w)$ and with queue length $\lambda(c)$ will in expectation interview $r\left(1 - e^{-\lambda(c)/r}\right)$ applicants.*

Note that the expected number of interviews equals $1 - e^{-\lambda(c)}$ for $r = 1$ and converges to $\lambda(c)$ for $r \to \infty$, exactly in line with the discussion above. If the firm interviews multiple candidates, it may be able to make a second offer if the first one gets rejected, provided it still has other qualified applicants. However, if the firm runs out of qualified applicants, it will remain unmatched for the remainder of the period, even if it still has applicants which it has not interviewed yet. Hence, congestion caused by recruitment frictions still arises, but to a lesser extent than for $k_R \to \infty$.

As I will show below, the firms' hiring probability and the workers' job offer probability are determined by a third queue length in this case. I denote this queue length by $\kappa(c)$. In order to distinguish between the three different queue lengths, I call $\lambda(c)$ the *gross queue length*, $\mu(c)$ the *net queue length,* and $\kappa(c)$ the *effective queue length*.[22] The effective queue length $\kappa(c)$ is defined as a weighted average of the gross and the net queue length, with $\frac{1}{r}$ and $\frac{r-1}{r}$ being the respective weights, i.e.

$$\kappa(c) = \frac{1}{r}\lambda(c) + \frac{r-1}{r}\mu(c). \tag{9}$$

Note that this definition implies that $\kappa(c) = \lambda(c)$ if $r = 1$ and $\kappa(c) \to \mu(c)$ if $r \to \infty$.

In order to derive expressions for the workers' job offer probability $\psi(c)$ and the firms' hiring probability $\eta(c)$, I consider a firm posting a contract c and I assume for the moment that the gross queue length $\lambda(c)$ and the net queue length $\mu(c)$ are exogenously given. This also pins down the queue length $\lambda(c) - \mu(c)$ of applicants that will never match with the firm. The actual number of applicants of each type follows a Poisson distribution. Given realizations for the number of qualified applicants without better offers and for the number of other applicants, it is straightforward to calculate the matching probabilities. Taking expectations then yields the following result.

Proposition 1. *A firm posting $c = (r, w)$ with gross queue length $\lambda(c)$, net queue length $\mu(c)$, and effective queue length $\kappa(c)$ hires with probability*

$$\eta(c) = \frac{\mu(c)}{\kappa(c)}\left(1 - e^{-\kappa(c)}\right). \tag{10}$$

[22] Kircher (2009) calls $\mu(c)$ the *effective queue length*, since it is the relevant notion in the world that he describes.

A worker that applies to the firm gets a job offer with probability

$$\psi(c) = \frac{\phi}{\kappa(c)}\left(1 - e^{-\kappa(c)}\right). \tag{11}$$

By convention, $\eta(c) = 0$ *and* $\psi(c) = \phi$ *if* $\kappa(c) = 0$.

The lemma shows that the job offer probability $\psi(c)$ indeed solely depends on $\kappa(c)$. The firm's hiring probability $\eta(c)$ depends on $\kappa(c)$ and, through $\mu(c)$, on the probability $1 - \Psi(c)$ that its job offer will get accepted. Hence, the matching probabilities have the same structure as in the special cases discussed above. They only depend on r and w through the queue lengths. This similarity in the structure is an important result. It implies that in the derivation of the equilibrium, we can build on the insights gained by Galenianos & Kircher (2009) and Kircher (2009), which means that the model despite its additional richness is tractable. Note further that equation (10) and (11) indeed collapse to the special cases for $r = 1$ (i.e. $k_R \to \infty$) and $r \to \infty$ (i.e. $k_R = 0$).

Note that the scope of proposition 1 and lemma 1 is not limited to directed search models, since they describes the matching technology. The applications that workers have sent during the application phase have created a network. The structure that is imposed implies that this network can be fully characterized by the queue lengths. The lemma and the proposition take these variables as given and therefore hold for other application processes, e.g. random search, as well. The application process however matters for the equilibrium expressions for the queue lengths. For example, under random search the gross queue length would be independent of the wage offered by a firm. This is not the case in the directed search setting of this paper. Firms that post higher wages or better recruitment technologies attract more applicants. Formally, the gross queue length is defined by the following integral equation

$$v\int_0^w \int_1^r \lambda(\widetilde{r},\widetilde{w})\, f(\widetilde{r},\widetilde{w})\, d\widetilde{r} d\widetilde{w} = u\sum_{i=1}^{A} G_i(c)\ \forall c \in \mathcal{C}, \tag{12}$$

where $G_i(c)$ denotes the marginal distribution of G with respect to c_i. The right hand side denotes the total mass of applications that are sent by the workers to contracts no higher than c. The left hand side represents the mass of applications received by firms posting a contract no higher than c. Both masses need to be the same for each possible c.

Equation (8) shows that once we know the gross queue length $\lambda(c)$, the acceptance probability is the only endogenous variable still required to calculate the net queue length $\mu(c)$. Consider a worker who gets a job offer with his i-th application. Let $\hat{G}_i(\mathbf{c}_{-i}; c)$ denote the conditional distribution of the remaining applications, given that the i-th application was sent to contract c. The worker will accept the offer w if and only if all applications sent to higher wages do not result in a job offer. This is the case with probability $\prod_{j=i+1}^{A}(1 - \psi(c_j))$. Finally, let $\hat{P}(i; w)$ denote the conditional probability that the worker who got the offer w applied there with his i-th application. Then the acceptance probability equals

$$1 - \Psi(c) = \sum_{i=1}^{A} \hat{P}(i;w) \int \prod_{j=i+1}^{A} \left(1 - \psi(w_j)\right) d\hat{G}_i\left(\mathbf{c}_{-i};c\right). \tag{13}$$

3.3 Equilibrium Definition and Summary

Before I define the equilibrium, consider the out-of-equilibrium beliefs. It is necessary to specify the firms' beliefs about the workers' response to a deviation in the contract posting. In particular, a deviating firm has to anticipate how many workers will apply, since that will affect the hiring probability. As standard in literature, I use the market utility condition. The market utility is defined as the highest level of utility from search that an unemployed worker can achieve by following the optimal application strategy. This is V_S, as defined in equation (3). The market utility condition postulates that workers will apply to the deviating firm in a way that provides them with exactly the same utility as they could have obtained by applying to non-deviating firms. The intuition is that if the deviant would provide the workers with a higher level of utility, all workers would have an incentive to apply to this firm, which would drive up the queue length and reduces the expected payoff. The reverse holds if the deviant offered a lower level of utility. Hence, $\kappa(c)$ has to be such that the market utility condition holds. I now define an equilibrium as follows:

Definition 1. An equilibrium is a tuple $\{v, F, G\}$ such that there exists $\kappa(\cdot)$ satisfying

1. $V_V(c) = V_V^* \equiv \max_{c'} V_V(c')$ for all $c \in \mathcal{F}$;
2. $V_V^* = 0$ if $v > 0$ and $V_V^* \leq 0$ if $v = 0$;
3. $V_S(\mathbf{c}) = V_S^* \equiv \max_{\mathbf{c}'} V_S(\mathbf{c}')$ for all $c \in \mathcal{G}$.
4. $\kappa(\cdot)$ is consistent with equations (8) to (13) and fulfills the market utility condition.

The first and third condition guarantee optimal behavior by firms and workers respectively. Firms with vacancies choose the contract c that maximizes their discounted future payoff. Workers choose the application portfolio $\mathbf{c}$ that maximizes their value of search. The second condition represents free entry. Firms continue to post a certain contract until its value becomes zero. If no firm posts a specific contract, it must be the case that the associated value is non-positive. The last condition ensures that the effective queue length satisfies its requirements.

In the next subsections, I analyze the existence of an equilibrium. I start by analyzing the worker's optimal strategy, after which I turn to the firm's decision problem. I derive the following result.

Summary 1. An equilibrium exists and has the following properties: in total, A different types of contracts are posted; firms that post a higher wage invest less in recruitment; each worker applies exactly once to each type of contract.

3.4 Workers' Application Behavior

An unemployed worker observes all contracts posted by the firms and has to decide to which firms he wishes to apply. When considering a firm, the worker is concerned about two things: the wage w that he can earn in the firm and the probability $\psi(c)$ that the firm will hire him. Hence, while observing the contracts, the wage is of direct interest to him, whereas he only cares about the recruitment technology through the job offer probability. This probability is a function of the effective queue length $\kappa(c)$, as shown in equation (11). Hence, everything else being equal, the worker prefers firms with short effective queue lengths. Whether the short queue lengths are the result of only few other workers applying to the same firm, i.e. a low $\lambda(c)$, or of a good recruitment technology, i.e. a high r, is irrelevant to the worker.[23]

Since workers only care about the wage and the job offer probabilities, an equilibrium relation exists between these two variables. This relationship has to be negative, i.e. applications to firms that offer higher wages are less likely to result in a job offer. After all, a worker would not apply to a certain firm if he can get a higher wage with a larger probability somewhere else. So, after observing the contracts, the worker knows in equilibrium what job offer probability is associated with each one of them. He then chooses the application portfolio that maximizes his expected lifetime utility. Hence, his A applications must solve

$$\max_{(c_1,\dots,c_A)\in\mathcal{F}^A} \sum_{i=1}^{A} \prod_{j=i+1}^{A} \left(1-\psi(c_j)\right)\psi(c_i)\left(V_E(w_i)-V_U\right)+V_U. \tag{14}$$

Note that this optimization problem has a recursive structure. The application to a firm offering w_i is only relevant if all applications to higher wage firms, i.e. the applications to wages $w_{i+1},\dots,w_A$, fail to result in a job offer. This simplifies the derivation of the workers' optimal strategy considerably. Define V_a as the maximum value for the worker provided by the first a applications and let $V_0 = V_U$. Then the worker's optimal choice for the i-th application ($i \in \{1,\dots,A\}$) solves

$$\max_{c\in\mathcal{F}} \psi(c)V_E(w)+(1-\psi(c))V_{i-1}. \tag{15}$$

This recursive structure provides several important insights. First, the worker's decision where to apply does not depend on the exact number of applications that he sends. This simplifies the implementation of heterogeneity in the number of applications. I discuss this in more detail in section 4. Second, note that this recursive equation describes the equilibrium relationship between the job offer probability $\psi(c)$ and the wage w. Since $\psi(c)$ only depends on $\kappa(c)$, it also defines a relationship between $\kappa(c)$ and w. This confirms that in equilibrium $\kappa(c)$ and $\psi(c)$ only depend

[23] Since r does not enter the worker's decision problem directly, the derivation of the worker's optimal search strategy is analogous to the analysis in Galenianos & Kircher (2009) and Kircher (2009). I therefore keep the exposition brief and refer to their papers for additional details. See also Chade & Smith (2006).

on r through w. The intuition for this result is as follows: if two firms offer the same wage but a different recruitment technology, then the one with the better technology is more attractive and will receive more applications. Hence, the gross queue length $\lambda(c)$ will go up until workers are again indifferent between both firms. This will be the case if the effective queue length $\kappa(c)$ is the same.

Now, let $\bar{w}_i$ denote the highest wage that yields utility V_i if the outside option equals V_{i-1}, i.e.

$$\bar{w}_i = \sup\{w \in \mathcal{W} | \psi(c) V_E(w) + (1-\psi(c)) V_{i-1} = V_i\} \ \forall i \in \{1,\ldots,A\}$$

Additionally, define $\bar{w}_0$ as the lowest wage that would receive applications. The following lemma states that the values $\bar{w}_i$ bound the interval to which application i can be sent in equilibrium.

Lemma 2. *The optimal application strategy for any worker is to send application $i \in \{1,\ldots,A\}$ to a wage in the interval $\mathcal{W}_i \equiv [\bar{w}_{i-1}, \bar{w}_i]$.*

This lemma confirms that workers do not necessarily want to send all their applications to the firm offering the highest wage. Specifically, workers send their first application to a low wage, although higher wages may be offered as well. The reason is that workers also care about the probability to be hired and a high wage implies a low job offer probability. The lemma also reveals a second characteristic of the equilibrium. Note that the first application provides the worker with a value V_1. The second application is sent to a wage w in the interval $\mathcal{W}_2$. By the definition of $\bar{w}_1$, this application by itself provides a value that is at most V_1 (if $w = \bar{w}_1$) and typically strictly less (if $w < \bar{w}_1$). The reason why workers nevertheless want to send their second application to this interval rather than to the interval $\mathcal{W}_1$ is the fact that the outside option is different. The outside option to the first application is the value of unemployment V_U, whereas failure of the second application still yields $V_1 > V_U$ to the worker. Because of this better safety net, the worker is willing to take more risk with his second application, i.e. apply to a firm that offers a higher wage but a lower job offer probability.

Since lemma 2 restricts the workers' application behavior, it has implications for the job offer probability $\psi(c)$ and the effective queue length $\kappa(c)$. For example, no worker will apply to wages below $\bar{w}_0$. Therefore, the queue length at firms offering such wages equals 0 and the corresponding job offer probability equals ϕ.[24] Firms posting wages in $[\bar{w}_0, \bar{w}_1]$ receive the first application of each worker. This application must yield a utility V_1 to the worker, hence the job offer probability is such that the condition $V_1(x) = \psi(c) V_E(w) + (1-\psi(c)) V_0$ holds. In a similar manner conditions for the remaining intervals can be derived. This is summarized in the following lemma.

[24] A deviant applying to a firm posting a wage below $\bar{w}_0$ faces no competition from other workers and will get an offer if he is qualified for the job.

Lemma 3. *In any equilibrium, the job offer probability* $\psi(c)$ *satisfies the following conditions*

$$\begin{aligned} \psi(c) &= \phi \quad \forall w \in [0, \bar{w}_0], r \in \mathcal{R} && (16) \\ \psi(c) V_E(w) + (1-\psi(c)) V_{i-1} &= V_i \quad \forall w \in [\bar{w}_{i-1}, \bar{w}_i], r \in \mathcal{R}, i \in \{1, \ldots, A\}, && (17) \end{aligned}$$

for some tuple $(V_1, \ldots, V_A)$*,* $V_0 = V_U$*,* $\bar{w}_0 = V_E^{-1}(V_1)$*, and* $\bar{w}_i = V_E^{-1}\left(\frac{V_i^2 - V_{i-1}V_{i+1}}{2V_i - V_{i-1} - V_{i+1}}\right)$*. By equation* (11)*, the conditions* (16) *and* (17) *also determine the effective queue length* $\kappa(c)$*.*

Given a vector of values $(V_0, \ldots, V_A)$, this lemma describes the workers' application behavior at all possible contracts, including the ones not offered in equilibrium. This will prove to be convenient in the derivation of the firms' optimal strategies in the next subsection.

3.5 Firms' Contract Posting

I now analyze the strategy of a firm. Consider a tuple $(V_0, \ldots, V_A)$ and the associated cut-off values $(\bar{w}_0, \ldots, \bar{w}_{A-1})$, which by lemma 3 describe the application behavior of workers. Firms have to decide what contract $c = (r, w)$ to post, taking into account this application behavior as well as the strategies followed by the other firms. The free entry condition implies that firms will continue to choose c until the expected payoff of this choice becomes zero. Hence, in equilibrium we must have

$$\eta(c) V_F(w) - k_V - k_R(r-1) \leq 0,$$

and $v(c) \geq 0$, with complementary slackness, for all $c \in \mathcal{C}$.

Lemma 3 implies that we can distinguish between A different types of firms. Specifically, I define a firm to be of type $i \in \{1, \ldots, A\}$ if it posts a wage in the interval $\mathcal{W}_i = [\bar{w}_{i-1}, \bar{w}_i]$ and receives the i-th application of a worker.[25] In choosing its optimal strategy, a firm of type i must take into account that a worker may reject its wage offer because he got a better offer at a different firm. Clearly, the probability $\Psi(c)$ that the worker rejects the offer is lower for higher type firms and it is zero for the firms offering the highest wages. Moreover, by lemma 3, we know that this probability is constant within the interval $\mathcal{W}_i$, since workers send one application to a given interval only. Individual firms take $\Psi(c)$ as given, since it only depends on the behavior of workers and higher type firms. Denote the rejection probability in interval $\mathcal{W}_i$ by Ψ_i. Substituting this variable and the equations (8) and (9) into (10) shows that for all wages in the interval $\mathcal{W}_i$ the hiring probability $\eta(c)$ can be written as

$$\eta(c) = \frac{r\phi(1-\Psi_i)}{1+(r-1)\phi(1-\Psi_i)}\left(1 - e^{-\kappa(c)}\right). \tag{18}$$

Note that the first factor on the right hand side is independent of the exact value of $w \in \mathcal{W}_i$, but only

[25] Posting a wage in the interval $[0, \bar{w}_0]$ does not lead to any applications and can therefore not be part of an equilibrium strategy.

depends on the recruitment technology r. On the other hand, the second factor only depends on the contract through the effective queue length $\kappa(c)$.

By substituting (18) into the firm's objective function, we can write the optimization problem of a firm of type i as

$$\begin{aligned} &\max_{w\in\mathcal{W}_i, r\in\mathcal{R}} \frac{r\phi(1-\Psi_i)\left(1-e^{-\kappa(c)}\right)V_F(w)}{1+(r-1)\phi(1-\Psi_i)} - k_V - k_R(r-1) \\ &\text{s.t.} \quad \psi(c)V_E(w) + (1-\psi(c))V_{i-1} = V_i, \end{aligned}$$

where the constraint specifies the level of utility that the firm has to provide to the workers in order to attract applicants. This optimization problem can be solved by exploiting the fact that there exists a one-to-one relationship between the wage and the effective queue length. Each wage w provides the firm with a certain queue length κ. Rather than by choosing a wage, the firm can therefore also solve this optimization problem by choosing an effective queue length. First, using equation (5) and (11), I rewrite the constraint as

$$w(\kappa) \equiv (1-\beta(1-\delta))\left(V_{i-1} + \kappa\frac{V_i - V_{i-1}}{\phi(1-e^{-\kappa})}\right) - \beta\delta V_U,$$

which specifies the wage that the firm needs to post in order to attract a certain queue κ. Evaluating the firm's value function (6) in this expression yields

$$V_F(w(\kappa)) = V_E(1) - V_{i-1} - \kappa\frac{V_i - V_{i-1}}{\phi(1-e^{-\kappa})}.$$

Substituting this into the objective function eliminates the wage. Instead, we get an expression that depends on the effective queue length κ and the recruitment technology r only. To be precise, the firm solves

$$\max_{\kappa\in\mathcal{K}_i, r\in\mathcal{R}} \frac{r\phi(1-\Psi_i)\left((1-e^{-\kappa})(V_E(1)-V_{i-1}) - \frac{\kappa}{\phi}(V_i - V_{i-1})\right)}{1+(r-1)\phi(1-\Psi_i)} - k_V - k_R(r-1) \qquad (19)$$

where $\mathcal{K}_i \equiv [\bar{\kappa}_{i-1}, \bar{\kappa}_i]$ and $\bar{\kappa}_i$ is the effective queue length that corresponds to $\bar{w}_i$.

We can now show that in equilibrium firms of different types will never offer the same wage. As a result, wage dispersion is a fundamental characteristic of any equilibrium.

Lemma 4. *In any equilibrium, for all $i, j \in \{1,\ldots,A\}$, firms of type i and $j \neq i$ do not post the same contract. In particular, they do not post the same wage.*

The fact that in equilibrium κ does not directly depend on r makes it possible to maximize the objective function over both variables independently. One can show that the objective function is strictly concave in both $\kappa \in \mathcal{K}_i$ and $r \in \mathcal{R}$. This implies that all firms of type i choose the same optimal queue length κ_i^* and recruitment technology r_i^*, which are determined by either the solution to the first order conditions or the boundary values. I summarize this in the following lemma.

Lemma 5. *Consider a firm of type $i \in \{1,\ldots,A\}$. In any equilibrium, the firm's optimal effective queue length κ_i^* equals*

$$\kappa_i^* = \max\left\{\log\frac{\phi\left(V_E\left(1\right)-V_{i-1}\right)}{V_i - V_{i-1}}, \bar{\kappa}_{i-1}\right\} \tag{20}$$

and the firm's optimal recruitment technology r_a^* is given by

$$r_i^* = \max\left\{\sqrt{\frac{1-\phi\left(1-\Psi_i\right)}{\phi\left(1-\Psi_i\right)}\frac{k_V - k_R}{k_R}}, 1\right\} \tag{21}$$

This lemma provides several important insights. First of all, note that in equilibrium exactly A different contracts will be offered. Hence, $\mathcal{F} = \{c_1^*,\ldots,c_A^*\}$, where w_i^* is the wage that corresponds to κ_i^*. Workers will send one application to each of the A types of contracts. As the lemma shows, firms that post low wages and are therefore likely to be rejected by applicants (i.e. Ψ_i is large), will buy more interview capacity than firms that offer higher wages. Hence, firms face a trade-off between paying a high wage and investing a lot in recruitment. Further, equation (21) reveals that firms also invest more in recruitment if only few candidates are qualified (i.e. ϕ is small), or if the cost per interview k_R is small compared to the fixed cost of opening a vacancy k_V, which is in line with intuition.

Note that $\bar{\kappa}_0 = 0$, which implies that the optimal queue length κ_1^* for the lowest wage firms is given by the interior solution $\log\frac{\phi(V_E(1)-V_0)}{V_1-V_0}$. In general, it is complicated to derive whether the interior or the boundary solution prevails for κ_i^*, $i \in \{2,\ldots,A\}$. Only in special cases such a result can be obtained. Galenianos & Kircher (2009) show that in a model with one interview per firm ($k_R \to \infty$), the optimal queue length equals the boundary value $\bar{\kappa}_{i-1}$ for $i \in \{2,\ldots,A\}$. In a similar model but with an unlimited number of interviews ($k_R = 0$), the optimal queue length for a type i firm is defined by the solution to the first order conditions, as shown by Kircher (2009). Such a general result cannot be derived for the model presented here. Numerical simulations however indicate that for a large range of parameter values for k_R, the boundary solutions arise. Only for k_R very close to zero, the interior solution may occur.

Each optimal effective queue length κ_i^* is associated with a gross queue length λ_i^* and a net queue length μ_i^*. By combining equations (8) and (21), we can express the optimal recruitment technology as a function of these queue lengths:

$$r_i^* = \max\left\{\sqrt{\frac{\lambda_i^* - \mu_i^*}{\mu_i^*}\frac{k_V - k_R}{k_R}}, 1\right\}.$$

3.6 Equilibrium Outcome

The previous subsections have derived some important characteristics of the equilibrium. In particular, the optimal recruitment technology for a firm posting the i-th contract has been determined. After substituting this expression, the model is isomorphic to the one described in section 6 of Galenianos & Kircher (2005). Hence, an equilibrium can be shown to exist.

Proposition 2. *An equilibrium exists. It satisfies the properties derived in lemma 2 to 5.*

Multiplicity of the equilibrium cannot be ruled out analytically, but simulations seem to support the conjecture by Galenianos & Kircher (2005) that the equilibrium is in fact unique for a wide range of parameter values.[26]

The efficiency properties of the equilibrium also follow directly from the literature. Galenianos & Kircher (2009) prove that when firms can contact only one applicant, the decentralized market equilibrium is not constrained efficient. A social planner could increase output by changing the search and entry behavior of respectively the workers and the firms, even when he is subject to the same frictions as the market. Kircher (2009) shows that constrained efficiency is only obtained if firms can contact all applicants. The intuition for this result is as follows. Recall that workers only care about the effective queue length $\kappa(c)$ at a given firm. Hence, if a firm changes the wage it posts, workers adjust their application behavior until the new effective queue length at the deviant is such that the market utility condition holds. However, firms do not only care about $\kappa(c)$, but also about the probability that the worker accepts the job offer, which depends on $\mu(c)$. Hence, in general firms cannot price the queue of applicants they are interested in. Only if $\kappa(c)$ and $\mu(c)$ coincide, such pricing is possible. This is the case if firms can contact all applicants, which in terms of the model described here implies that the equilibrium is constrained efficient if and only if $k_R = 0$.

Kircher (2009) also shows that if $k_R = 0$ and the number of applications tends to infinity, i.e. $A \to \infty$, the Walrasian outcome is obtained. This suggest a natural procedure to investigate the magnitude of the search frictions in the model. Given parameter values, the steady state output in the market equilibrium can be calculated. This can be compared to the output in a world in which $k_R = 0$. The difference is a measure for the magnitude of the recruitment friction. Finally, output can be calculated for the Walrasian outcome, which then sheds light on the size of the search friction. I follow this approach in section 4.4. First, I extend the model by endogenizing search intensity.

3.7 Endogenous Search Intensity

In the previous subsections, the number of applications sent by the workers was exogenously given. I will now argue that this assumption can easily be relaxed. The reason is that the nature of the equilibrium does not change, as can be seen from the recursive expression for the worker's decision

[26] This may change when the search intensity is endogenized, see section 3.7.

problem. Given a value of unemployment, wages and job offer probabilities, the optimization problem that the worker solves for his i-th application does not depend on the total number of times $a \geq i$ that he applies. This makes it straightforward to let workers choose the number of applications that they send at a weakly increasing cost, as shown by Kircher (2009).

However, optimization over a discrete variable is cumbersome, so I use a slightly different approach and let search intensity be a continuous variable.[27] I choose a specification that is similar to the one for the firms' recruitment technology. Workers choose their search intensity $\alpha \in \mathbb{R}_+$, which is a measure for the time they allocate to searching a job. The cost of a choice α is $k_A \alpha \geq 0$. The search intensity determines the expected number of applications that a worker can send. However, as with the interview capacity, there is a stochastic element as well, e.g. because a particular application takes more or less time than initially expected. Hence, the actual number of applications follows a certain distribution that depends on α. I denote the fraction of workers with search intensity α that can send $a \in \mathbb{N}_0$ applications by $p(a|\alpha)$ and the cumulative distribution function by $P(a|\alpha) = \sum_{i=0}^{a} p(i|\alpha)$. The model imposes some restrictions on this application distribution. For example, interpreting α as search intensity requires first order stochastic dominance in α. Studying simultaneous search implies $P(1|\alpha) < 1$ for all $\alpha > 0$. In order to avoid an infinite number of contracts, I assume that the support of the distribution is bounded. Workers send at most A applications per period, where A is a potentially large but finite integer.

Having heterogeneity in the number of applications introduces small changes in some of the equations. Appendix B presents the updated versions. However, most equilibrium properties remain the same. Still A different contracts are offered. Workers applying A times send one application to each type of contract. Unemployed who send $a < A$ applications, apply to the a types of contracts offering the lowest wages. In order to complete the description of the equilibrium with an endogenous number of applications, consider the worker's choice of search intensity. Suppose that a worker has already decided on the optimal application vector $\mathbf{c}$ but still has to choose α. He takes the behavior of the firms and the other workers as given and solves $\max_\alpha V_S(\alpha) - k_A \alpha$, where

$$V_S(\alpha) = \sum_{i=0}^{A} p(i|\alpha) V_i. \tag{22}$$

The solution to this optimization problem is unique if $V_S(\alpha)$ is strictly concave in α. Note that $V_S(\alpha)$ can be rewritten as

$$V_S(\alpha) = V_0 + \sum_{i=1}^{A} (1 - P(i-1|\alpha))(V_i - V_{i-1}).$$

[27] See Kaas, 2007 for a similar approach.

Hence, strict concavity requires

$$\frac{d^2 V_S(\alpha)}{d\alpha^2} = -\sum_{i=1}^{A} \frac{d^2 P(i-1|\alpha)}{d\alpha^2} (V_i - V_{i-1}) < 0.$$

In that case, all workers choose the same level of search intensity, as determined by the first order condition of the optimization problem. Because of the boundedness of $V_S(\alpha)$ concavity holds for sufficiently large α. However, in general it is not obvious that the second derivative is strictly negative for all α. In particular, it may be positive for low values of α, as pointed out by Kaas (2007) in a similar setting. In that case, the equilibrium may require that the workers mix between $\alpha = 0$ and the α implied by the first order condition. Since the sign of the second derivative depends on the endogenous objects V_i, it is not clear which distribution functions give strict concavity. In the empirical part, I will therefore choose a particular distribution, assume that the condition is satisfied in the equilibrium calculation, and verify this assumption ex post.

Endogenous search intensity may be a source of multiplicity of the equilibrium. For example, if the cost of search k_A is low, workers will typically apply a lot, This forces firms to offer high wages, which may provide workers with an incentive to indeed send many applications. The reverse holds if he cost of an application is high. Such multiplicity is a potential problem in the estimation. I solve this by estimating the search intensity α rather than the search cost k_A, as I will explain in section 4.2.

4 Estimation

4.1 Data

The model discussed above has implications for four key variables at the firm level. For given parameter values, it determines 1) the number of applicants that a firm has, 2) the number of interviews that the firm conducts, 3) the number of job offers that the firm makes, and 4) the wage that the firm pays. Unfortunately, the availability of microdata containing these four variables is very limited. One of the very few data sets with this information is the Employment Opportunities Pilot Projects (EOPP) data set, which I use in this paper. It contains the information from a two-wave longitudinal survey that was designed to evaluate the impact of US labor market programs developed by the Office of the Assistant Secretary for Policy, Evaluation and Research, and funded by the Department of State's Employment and Training Administration.[28]

The Employment Opportunities Pilot Project was introduced in the summer of 1979. It consisted of an intensive job search program combined with a work and training program, organized at 10

[28]Several other authors have used the EOPP data set, e.g. Barron et al. (1985), Barron et al. (1987), and Burdett & Cunningham (1998). See these papers for additional information about the survey. The papers are different in the sense that they do not use a two-sided equilibrium search model to analyze the data.

pilots sites throughout the country. The program was aimed at unemployed workers with a low family income, and tried to place eligible workers in private-market jobs at one of the pilot sites during a job search assistance program. If these attempts failed, the worker was offered a federally-assisted work or training position. The program was in full operation by the summer of 1980, but was phased out during 1981 by the new Administration.

In order to evaluate the program, a survey was sent to firms at the ten pilot sites and twenty control sites which where selected on the basis of their similarity to the pilot sites. The first wave of the survey took place between March and June 1980. The second wave was conducted between February and July 1982 and aimed to re-interview all respondents to the first survey. The response rate was about 70%.

The data set is not representative for the US labor market as a whole. Due to the nature of the labor market program, low wage workers are overrepresented.[29] Further, the data set does not include workers in agriculture, government and non-profit organizations. Moreover, the pilot sites are disproportionally concentrated in Gulf Coast cities and underrepresent cities in the Northeast of the US. The probability for a firm in one of the sites to be included in the survey depended on its size and location and varied between 0.006 for the smallest establishments to close to 1 for establishments with more than 200 employees (see Barron et al., 1985). The data set contains sample weights to account for the heterogeneity in the sampling probability.

The survey sent to the firms included various questions on the recruitment process for the last hired worker. The second survey was a lot more comprehensive than the first one. In particular, the first survey did not include a question on the number of applicants per vacancy. For this reason, I only utilize the 1982 data in this paper. This data set contains information on all four variables listed above for hires between January 1980 and September 1981. Further, the data set contains a number of firm and worker characteristics, like gender, age, experience, education level, sector, and location.[30] I use these variables to control for productivity differences, as I explain in more detail in the next subsection.

The data set contains information on both the last subsidized and the last non-subsidized hire. I restrict the sample to the latter group. I select workers who are 18 years or older and I omit observations with missing or unreliable values.[31] The data set includes two wage variables. The first one reports the wage of the individual at the survey date, but is by definition only available if the workers is still employed at the firm. The second wage variable, which reports the wage that

[29]Note that this is not a concern if we want to study the effect of recruitment frictions on the desirability of active labor market programs. These programs are often targeted at unemployed workers, among which low wage workers are typically overrepresented as well. Nevertheless, it would certainly be interesting to estimate the model on a more representative sample of workers or on data from specific, well-defined submarkets, like e.g. the job market for economists. However, the required data is currently not available. For future research in this area, surveys like the one conducted Hall & Krueger (2008) could be an important source of information.

[30]The sector is coded according to the SIC 1972 (4 digits). I use the first digit.

[31]For example, a small fraction of the reported wages is below the minimum wage. Re-estimating the model with those observations included does not lead to results that qualitatively differ from the ones present in section 4.3.

the firm would offer to a new hire in the same position at the moment of the survey. Conditional on observing both, the correlation between the two variables is more than 90%. I choose the second variable since it is observed more often and because it excludes the influence of wage increases since the moment of hiring. Table 1 displays some descriptive statistics for the final sample.[32]

The table shows that there is considerable variation in the number of applicants and the number of interviews per firm. The variation in the number of job offers is smaller: most firms only need to make one job offer to hire the worker.

4.2 Estimation Strategy and Identification

In this subsection, I discuss how the model presented above can be estimated on the EOPP data. I start with extending the model by allowing for heterogeneity in worker productivity. Despite its richness along several dimensions, the EOPP data does not contain sufficient information for non-parametric identification of all elements of the model. Therefore, I describe some parametric assumptions that I need to make. Finally, I briefly discuss the identification of the remaining parameters and I derive the log-likelihood.

The model described so far assumes that all qualified workers are equally productive, i.e. each of them generates one unit of output per period when matched with a firm. A world in which firms face applicants with different productivity levels would be much harder to analyze, because it makes the firm's decision problem more complex. For example, always offering the job to the best applicant is generally not the optimal strategy, since that implies that the competition for these workers will be much higher than for slightly less productive workers. Instead, firms will randomize over their best candidates, while rejecting applicants with much lower productivity. This implies that the firm's strategy is a function of its total pool of applicants, which can be composed in infinitely many ways. Hence, the assumption of homogeneous workers keeps the model tractable.

However, one of the variables that will be included in the estimation is the wage that the firm pays to the worker. For this, homogeneity of workers is a problematic assumption, since differences in productivity are a key determinant of wage differences. Hence, not controlling for heterogeneity in productivity would attribute too much wage dispersion to the frictions. I therefore choose the following approach. I allow workers to differ in their productivity, but I assume that the productivity is public information and that a separate submarket exists for each productivity type.[33] Hence, workers with different skill sets do not compete with each other. This structure can be generated endogenously by assuming that firms that create a vacancy need to choose and post a production technology that allows them to match with one specific type of workers only. Let y be the produc-

[32] Section 4.3 presents some histograms of the data, while discussing the goodness of fit of the model.

[33] See van den Berg & Ridder (1998) for a similar approach.

	Mean	Std.dev.	Min	Max
Dependent variables				
Number of applicants	16.30	29.55	1	600
Number of interviews	5.26	6.96	1	50
Number of offers	1.23	1.02	1	35
Wage	4.20	1.42	3.09	14.69
Independent variables				
Male	0.45	0.50	0	1
Age	27.27	9.29	18	70
Experience	4.15	5.38	0	40
Some high school	0.07	0.25	0	1
High school	0.56	0.50	0	1
Some college	0.21	0.40	0	1
College	0.04	0.21	0	1
Other education	0.11	0.31	0	1
Extraction	0.02	0.14	0	1
Construction	0.08	0.26	0	1
Manufacturing	0.08	0.26	0	1
Transport	0.07	0.26	0	1
Wholesale	0.17	0.37	0	1
Retail	0.24	0.42	0	1
Financial	0.07	0.25	0	1

Descriptive statistics for the sample used in the estimation. Wages are measured in 1982 dollars. 1 dollar in 1982 has the same buying power as $2.23 in 2008 (Bureau of Labor Statistics, 2009). Age and experience are measured in years. The gender, education and sector variables are dummies. The reference categories for the estimation (female, elementary school, service sector) are omitted.

Table 1: Descriptive statistics

tivity of a worker. The associated density is denoted by $f_y(y)$ and has support $\mathcal{Y}$ on (a subset of) $\mathbb{R}_+$. There is a continuum of workers at each productivity level y, such that the expression for the matching probabilities given in proposition 1 hold in each submarket.

In order to avoid that the potentially infinite number of worker types creates a dimensionality problem in the number of parameters that needs to be estimated, I assume that the equilibrium is identical in each of the submarkets. This is the case if all relevant periodical costs and payoffs are scaled linearly by the productivity of the worker. Hence, creating a job for a highly productive worker is more costly than creating a job for a low-skilled worker. The effect is linear, such that a firm opening a vacancy in submarket y incurs an entry cost $k_V y$. Likewise, the recruitment cost equals $k_R(r-1)y$. Unemployed workers receive a payoff from household production equal to hy and incur a search cost $k_A \alpha y$. An employed worker earns wy, while the firm keeps $(1-w)y$. This guarantees that in each submarket the exact same contracts are posted.[34]

In the model, there is no uncertainty about a worker's productivity: workers always have full information, and a firm learns the productivity of an applicant before making him a job offer. However, the econometrician lacks such detailed information. Instead, the econometrician only observes a number of characteristics for each worker. I therefore assume that a worker's productivity is a function of these characteristics and an unobserved error term in the following way:

$$\log y = x'\gamma + \varepsilon,$$

where x denotes the vector of observed characteristics, γ is a vector of coefficients and $\varepsilon \sim N\left(0, \sigma^2\right)$ is the unobserved component. In the estimation, x will include the following variables: the worker's gender, age (and its square), amount of relevant work experience (and its square), education, as well as indicators for the sector and the location of the job.[35] Both γ and σ then follow from the maximum likelihood estimation, as I discuss below.

Unfortunately, the data is not rich enough to identify all model elements non-parametrically. I will therefore choose some parameter values and functional forms exogenously. With respect to the number of applications, I assume a truncated Poisson distribution with the search intensity α as its

[34]The assumed cost and payoff structure is more reasonable than it may seem. Recall that the search intensity and the recruitment technology reflect units of time. Consequently, the multiplicativity of the associated costs in y arises naturally. Further, the assumption that the wage equals wy is without loss of generality, since firms choose and post both w and y. The assumption of linearity of household production in y is arguably the most debatable one, but is made in several other recent papers, see for example Burdett et al. (2008).

[35]Although the last two variables are often labeled as job characteristics, I choose to interpret them as worker characteristics. Clearly, workers have a choice in the sector and the region in which they work, hence their choice most likely contains some information about their productivity.

parameter. Hence, the fraction of workers $p(a)$ that sends a applications satisfies:

$$p(a|\alpha) = \begin{cases} e^{-\alpha}\frac{\alpha^a}{a!} & \text{for } a \in \{0,\dots,A-1\} \\ 1-\sum_{i=1}^{A-1} e^{-\alpha}\frac{\alpha^i}{i!} & \text{for } a = A, \end{cases}$$

where A is a finite integer and $\alpha > 0$.[36] Note that if $\alpha \to 0$, all workers send zero applications. On the other hand, if $\alpha \to \infty$, all workers apply A times. The distribution converges to a regular Poisson distribution for $A \to \infty$. The theoretical analysis in the previous sections shows that calculation of the equilibrium requires solving for A different wage levels. In order to avoid dimensionality problems, I will set A equal to 10 in the estimation. A Poisson distribution is a natural choice, since it is the discrete time equivalent of the Poisson process typically used in continuous time models. Further, it has the attractive feature that it has discrete support while depending on a continuous parameter. Estimation of such a continuous parameter is significantly less cumbersome than estimating a discrete number of applications. As mentioned before, endogenous search intensity may be a source of multiplicity of the equilibrium. In order to avoid problems with the estimation, I therefore do not estimate the search cost k_A directly, but I treat the search intensity α as a parameter and estimate that instead. The search cost can then be calculated ex post by equating it to the marginal benefit of an extra unit of search intensity.

A second assumption that needs to be made concerns the length of a period. In theory, the data contains some information about this length. To see this, suppose that the number of applications that workers send is deterministic. Then, compare a period of one week in which workers send one application to a period of two weeks in which workers send two applications. In both cases, the gross queue length λ is the same given a fixed number of firms, but other equilibrium outcomes are very different. In the first case, we are in a world similar to the one described in Burdett et al. (2001). There no congestion occurs, so firms have less incentive to interview many candidates. Moreover, no wage dispersion will be present in the data after controlling for observed and unobserved productivity differences. This is all not true in the second case. However, estimating the length of a period in this way would lead to very indirect identification. Therefore, I choose to fix the length exogenously by setting it equal to one month. Since the matching process may consist of multiple rounds of job offers, a much shorter length seems implausible. The EOPP data contains some information on vacancy durations. Firms report the number of days between the start of recruitment and the moment at which the employee started to work. I do not utilize this variable in the empirical analysis, since it is quite noisy due to rounding (see Burdett & Cunningham, 1998). However, the mean duration in the sample is 16 days and the 90-th percentile equals 30 days, which suggests that a period length of a month seems very reasonable. In line with the chosen period length, I set

[36] I have also experimented with a specification that allows for extra mass at $a = 0$. However, the estimation results in that case converge to this more parsimonious specification.

$\beta = 0.9957$, which corresponds to an annual discount factor of 0.95.

The data also lacks detailed information about the number of qualified applicants, the payoff of unemployed workers, the entry of firms, and the destruction of matches, which therefore have to be determined in an indirect way. Consider the job destruction rate first. I determine this variable by matching the unemployment rate u as implied by the model to the true value. Data from the US Bureau of Labor Statistics (2009) shows that the average unemployment rate during the sample period was equal to 6.79% for workers older than 18. The steady state assumption implies that inflow into unemployment equals outflow, i.e. $(1-u)\,\delta = u\Psi_0$, which pins down δ as function of u and the workers' job finding probability Ψ_0. Likewise, I restrict k_V by fixing the number of vacancies v. Unfortunately, there are no reliable sources that provide direct information on the number of vacancies in the US around 1980. Therefore, I follow the approach of Davis et al. (2009). They detrend the Conference Board's Help-Wanted Index using an HP filter with smoothing parameter of 10^5, and rescale the deviations such that they match the mean of the more direct measure of the vacancy rate in the Job Openings and Labor Turnover Survey (JOLTS) in the overlapping period (2001 - 2007).[37] This gives an average value $v = 0.0273$ for the sample period.

Since the data does not report the number of qualified applicants, there is no direct information that can be used to identify the probability ϕ that an applicant is qualified for the job. Moreover, the effect of ϕ on the firm's matching probability cannot be exploited to estimate its value, since the data only includes firms that managed to hire a worker. I therefore choose to fix ϕ exogenously. For various reasons, relatively high values seem plausible. First of all, the sample concerns mainly low-skilled workers for which screening should be relatively less important. Second, the relatively small number of interviews per firm implies that meeting unqualified applicants is not a great concern to the firms. For example, if only one in every ten workers would be qualified, it is hard to explain why firms on average interview only 6 out of 16 applicants (see table 1). Therefore, I will estimate the model using $\phi = 0.9$ and do some robustness checks in section 4.6.

Finally, consider the level of household production. In the recent literature there has been some debate about the right value of this variable. Shimer (2005) shows that for $h = 0.4$ the standard Mortensen & Pissarides (1994) model has difficulties explaining the cyclical behavior of unemployment and vacancies. Hagedorn & Manovskii (2008) use a different calibration strategy in a similar model and find a much higher value, 0.955. Hall & Milgrom (2008) obtain a value of 0.71. I follow Hagedorn & Manovskii (2008) for two reasons. First, a small difference between household and market productivity seems likely, given the fact that we consider a sample of low-skilled workers. Second, although household production is not directly related to one of the observed vari-

[37] The Help-Wanted Index (HWI) is a monthly measure of help-wanted print advertising in a sample of 51 U.S. newspapers, constructed by the Conference Board. It is the only vacancy-related measure for the U.S. economy that provides a long, high-frequency time series. The Job Openings and Labor Turnover Survey (JOLTS), conducted by the Bureau of Labor Statistics of the U.S. Department of Labor, was only introduced in December 2000. For more information about the data or the detrending and rescaling procedure, see Davis et al. (2009).

ables, the model imposes enough structure on the data to let the maximum likelihood estimation also determine h. If I do this, I find values for h close to the estimate by Hagedorn & Manovskii (2008).

After making the above assumptions, several parameters remain: the search intensity α, the interview cost k_R, and the productivity parameters γ and σ. These parameters are all closely related to the information that we observe in the data. I will now discuss how I use the data to obtain estimates for them and derive the log-likelihood.

Each observation in the data is a tuple (z_A, z_I, z_O, z_W, x), respectively denoting 1) the number of applicants at a given firm, 2) the number of interviews the firm conducts, 3) the number of job offers it makes, 4) the wage it pays to the worker it hires, and 5) the characteristics of the worker. The analysis in section 3 shows that in equilibrium we can distinguish between A different firms types in each submarket, each associated with a specific wage w_i and recruitment technology r_i, $i \in \{1, \ldots, A\}$. The data does not tell us the type of a certain firm. The likelihood of an observation is therefore calculated by initially conditioning on each possible type and subsequently taking the expectation with respect to the type. Note further that a firm is only included in the data set if it manages to match with a worker. This creates a selection issue: firms with many applicants or firms with a high acceptance probability are more likely to be observed. Hence, the likelihood needs to be conditioned on the hiring probability η_i of a firm.

Consider each element of the observation separately, starting with the number of applicants. A firm of type i faces a number of applicants hat is the realization of a Poisson distribution. The mean is equal to the firm's gross queue length λ_i, which is determined in equilibrium by its choice of (r_i, w_i). The firm therefore has exactly z_A applicants with probability $e^{-\lambda_i} \frac{\lambda_i^{z_A}}{z_A!}$. For the labor market as a whole, the number of applications received by the firms has to equal the number of applications that are sent by the workers. The latter figure depends on the number of unemployed, which I fixed, and the distribution of applications $P(a|\alpha)$. Hence, the data on z_A identifies α.

The number of interviews that a firm conducts depends on its number of applicants and its interview capacity. If we observe that the number of interviews z_I is strictly smaller than the number of applicants z_A, it follows that the number of interviews was restricted by the capacity. The capacity follows a geometric distribution with mean r_i, hence the probability that the firm has a capacity of z_I equals $\left(\frac{r_i-1}{r_i}\right)^{z_I-1}\left(\frac{1}{r_i}\right)$. In case we observe that the firm interviews all applicants (i.e. $z_I = z_A$), the capacity must have been at least z_A. The probability of this event is given by $\left(\frac{r_i-1}{r_i}\right)^{z_I-1}$. A high value for z_I is more likely when r_i is high. Hence, z_I contains information on r_i. Note that r_i depends on k_R, k_V, ϕ and Ψ_i. Since ϕ is fixed, k_V is determined by the number of vacancies, and Ψ_i is determined by the structure of the equilibrium, z_I identifies k_R. Specifically, a higher value of k_R leads to fewer interviews.

The number of job offers that a firm needs to make in order to attract a worker follows a geometric distribution as well. The probability of success (i.e. the worker accepts the offer) is $1-\Psi_i$. However, acceptance needs to occur before the firm runs out of qualified applicants. The conditional probability that the firm hires with the z_O-th offer and had $z_O \leq j \leq z_I$ qualified candidates is given by $\Psi_i^{z_O-1}(1-\Psi_i)\sum_{j=z_O}^{z_I}\binom{z_I}{j}\phi^j(1-\phi)^{z_I-j}$.

Finally, the productivity parameters γ and σ follow from the wage information. A worker with productivity y who is employed at a type i firm gets a wage equal to $w_i y$. Hence, if a wage z_W is observed, the following equation must hold

$$\begin{aligned} \log z_W &= \log w_i + \log y \\ &= \log w_i + x'\gamma + \varepsilon. \end{aligned} \tag{23}$$

Hence, the unobserved productivity component of the worker must equal $\varepsilon_i \equiv \log z_W - \log w_i - x'\gamma$. The likelihood of this equals $f_\varepsilon(\varepsilon_i)$. Equation (23) is a standard Mincer equation, augmented with the term $\log w_i$, which captures the worker's success in the matching process. Given parameter values for α, k_V, k_R, ϕ and δ, the equilibrium wage rates $(w_1^*,\ldots,w_A^*)$ are fixed. We do not observe which of these wage rates the worker earns, but after conditioning on each possible value and averaging, the identification of γ and σ is standard.

Summarizing, the likelihood of an observation is given by

$$L = \frac{\sum_{i=1}^{A} v_i e^{-\lambda_i}\frac{\lambda_i^{z_A}}{z_A!}\left(\frac{r_i-1}{r_i}\right)^{z_I-1}\left(\frac{1}{r_i}\right)^{\mathbb{I}\{z_I<z_A\}}\Psi_i^{z_O-1}(1-\Psi_i)\sum_{j=z_O}^{z_I}\binom{z_I}{j}\phi^j(1-\phi)^{z_I-j}f_\varepsilon(\varepsilon_i)}{\sum_{i=1}^{A} v_i\eta_i},$$

where v_i denotes the measure of firms of type i and $\mathbb{I}\{z_I < z_A\}$ is an indicator function which equals 1 if the condition $z_I < z_A$ is satisfied and 0 otherwise. I maximize the log of this likelihood, summed over all observations. The standard errors are calculated with the delta method from the inverse of the Hessian.

4.3 Estimation Results

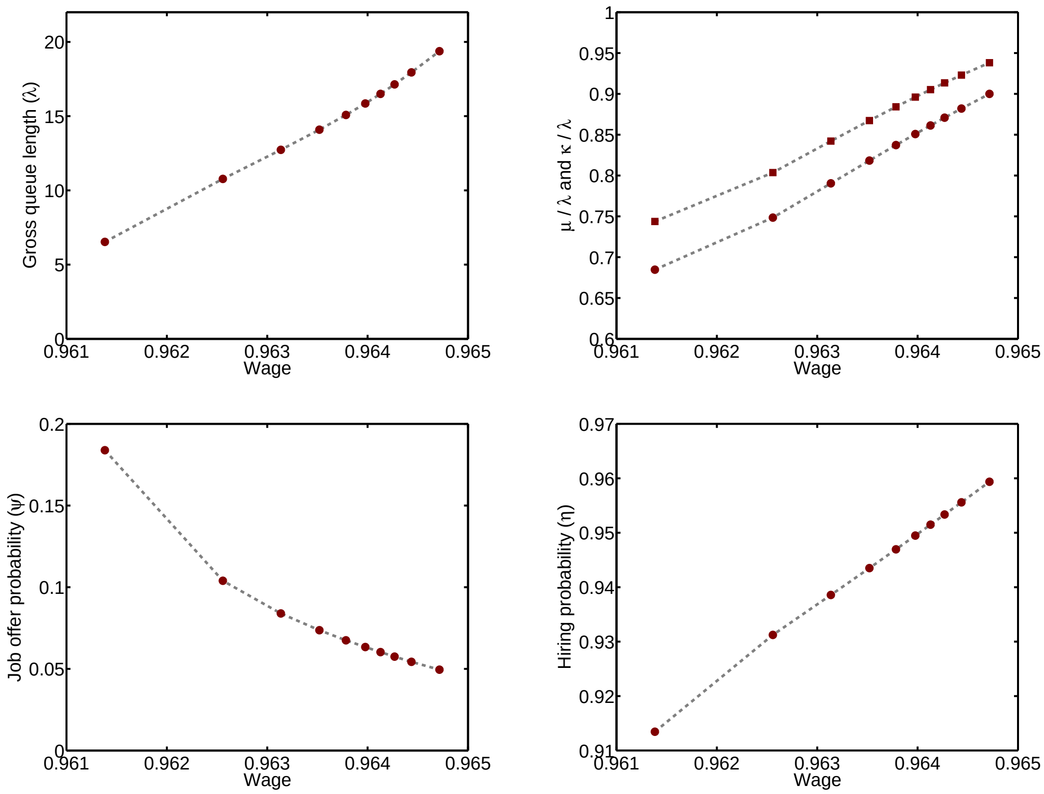
The results of the estimation are presented in table 2. The table lists the parameters α, k_A, k_V, k_R, and δ, which specify the equilibrium within a specific submarket y, as well as the coefficients γ and σ, which determine the distribution of submarkets. As derived in section 3, exactly A different contracts are offered for each productivity level y. These contracts are shown in table 3. Figure 2 displays the queue lengths and the matching probabilities at each of the posted contracts.

Note first of all that the estimation results confirm the theoretical results derived in the previous section. For example, firms that post higher wages invest less in recruitment. Nevertheless, they attract more applicants, leading to longer gross, net, and effective queue lengths. As a result, high wage firms are more likely to match. Reversely, applications sent to high wage firms are less likely

	Estimate	Std.err.
Model parameters		
α	4.259	0.134
k_A	0.006	-
k_V	1.052	-
k_R	0.017	0.001
δ	0.027	-
Productivity coefficients		
Male	0.174	0.023
Age	0.264	0.070
Age-square	-0.038	0.010
Experience	0.160	0.053
Experience-square	-0.041	0.022
Some high school	-0.032	0.082
High school	0.077	0.074
Some college	0.135	0.075
College	0.261	0.083
Other education	0.159	0.076
Extraction	0.268	0.068
Construction	0.112	0.042
Manufacturing	0.127	0.041
Transport	0.043	0.039
Wholesale	0.049	0.029
Retail	-0.106	0.028
Financial	0.017	0.041
Pilot site dummies	included	
σ	0.221	0.028
Statistics		
Observations	594	
Avg log-likelihood	-7.342	

Estimation results obtained with maximum likelihood. The model parameters α, k_A, k_V, k_R, and δ characterize the equilibrium for a given productivity level y. The productivity coefficients γ and the standard deviation σ of the unobserved productivity component jointly determine a worker's output y when matched with a firm. Age and experience are measured in decades. The gender, education and sector variables are dummies. The reference category is female, elementary school, service sector.

Table 2: Estimation results



Queue lengths and matching probabilities in the estimated equilibrium. The top left panel show the gross queue length λ as a function of the wage. The top right panel displays the net queue length μ (circles) and the effective queue length κ (squares) as a fraction of λ. The bottom left panel contains the worker's job offer probability at each of the posted wages, while the bottom right panel shows the hiring probability for the firm. The lines connecting the dots serve illustrative purposes only.

Figure 2: Queue lengths and matching probabilities

Contract	w	r
1	0.9614	5.341
2	0.9626	4.563
3	0.9631	4.052
4	0.9635	3.709
5	0.9638	3.470
6	0.9640	3.296
7	0.9641	3.158
8	0.9643	3.031
9	0.9644	2.879
10	0.9647	2.624

Contracts that are posted in equilibrium. Each contract specifies a wage w and a recruitment technology r.

Table 3: Posted contracts

to generate job offers. The average firm fills its vacancy within a month with probability 0.930. This value is slightly higher than typically found in the literature[38], but is well in line with what is observed in the data. According to the reported vacancy durations, a fraction 0.937 of the firms matches within 30 days after posting the vacancy. The workers' job finding probability equals 0.374, which is close to the value of 0.414 found by Shimer (2005) for the time period January 1980 - September 1981. The estimate of 0.027 for the separation rate δ is a bit lower than the 0.042 obtained by Shimer.[39]

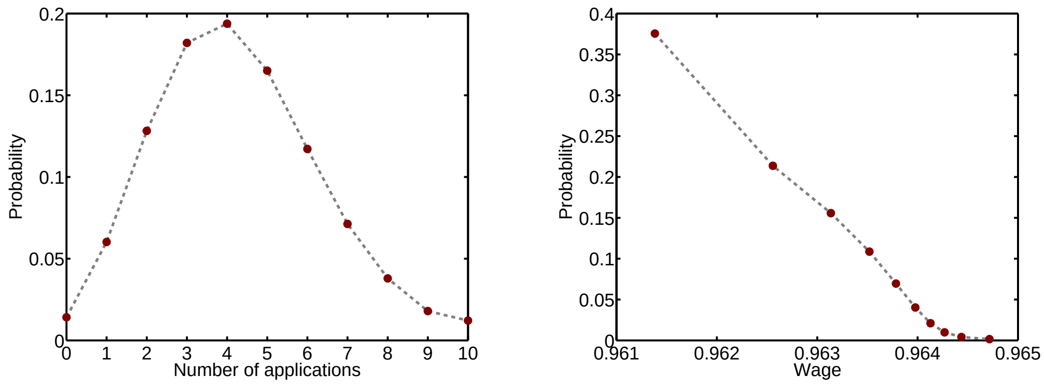
The firm's entry cost is estimated to be approximately 1.1 months of production. This estimate may seem slightly high at first sight, but can be explained by that fact that the number of firms present in the market is relatively low, despite a high matching probability and a low job destruction probability. The only way to reconcile these facts is by having a high entry costs. Moreover, note in that in reality the entry costs for a firm likely consist of two components: a job creation cost and the cost of posting a vacancy. The former component is only incurred once, but is typically much higher than the latter. Here, I capture both components with the same variable k_V, which implies that this variable implicitly includes the job creation cost, but spread over all periods in which the firm has a vacancy. Since firms hire with a probability close to 1, the resulting value is large.[40]

Next, consider the recruitment costs of the firm. The estimation results indicate that an extra

[38] For example, den Haan et al. (2000) find a value of 0.71 per month.

[39] One can think of various explanations for this difference. First, the actual length of a period may differ from the month chosen here. Second, the steady state assumption, although standard, may be quite strong. According to the NBER, a short recession occurred between January 1980 and July 1980, followed by a short period of growth until July 1981 and a stronger recession thereafter. Unfortunately, the data is not rich enough to analyze the dynamics out of steady state.

[40] Gautier et al. (2008) find even larger estimates of the entry cost in the estimation of a random search model on Dutch data.



Probability distributions of the number of applications sent by the workers and the wages posted by the firms. The dark-red dots indicate the mass points.

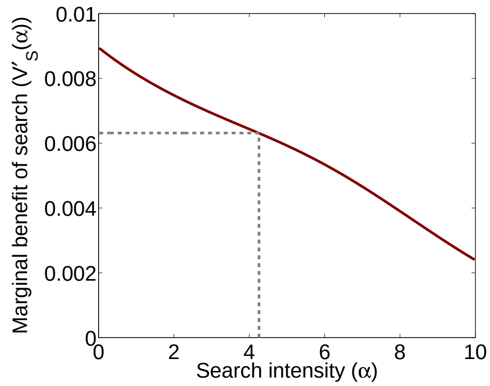
Figure 3: Application and wage distributions

unit of interview capacity requires an additional investment of $k_R = 0.017$. Intuition for this number can be obtained by converting it into a time cost. If a firm and a worker produce $1/30 = 0.033$ units of output per day, then the estimate for k_R corresponds to 0.50 days of output. The average firm chooses $r = 4.50$, which implies that the total cost of recruitment equals 0.058, or 1.75 days of production. Barron et al. (1997) find 11.24 hours with a different methodology on the same data.

The search intensity α of the workers is estimated to equal 4.26. The left panel of figure 3 shows the distribution of the number of applications that workers send.[41] In order to determine the cost of an application, we first need to verify that the gains from search as defined in equation (22) are strictly concave in α. Figure 4 shows that this is indeed the case: the marginal benefits $V_S'(\alpha)$ are downward sloping. This implies that all workers choose their search intensity by equating marginal benefit to marginal cost. Hence, the marginal cost k_A of an extra unit of search intensity equals 0.006. In order to interpret this value, suppose that one day of household production generates a payoff equal to $0.955/30 = 0.032$. Then, the cost of an application corresponds to 0.20 days of household production. The total time spent on search by an unemployed worker amounts to 0.84 days per month. Using data from the American Time Use Surveys, Krueger & Mueller (2008) find a value of 16 hours per month.

The estimates for γ do not show surprising results. The wage is increasing in the education level and is higher for men than for women. Age and experience affect productivity in a concave way. The relatively high coefficient for the extraction industry is present in the raw data as well and is most likely the result of the specific sample of workers being selected. The estimate for σ falls within the range of values found by other studies.

[41] Note that this estimate for α implies that $p(A) = 0.012$, which means that limiting A to 10 is hardly restrictive.



The dark-red line represents the marginal benefits of search for an individual worker, given equilibrium behavior by the firms and the other workers. The worker equates marginal benefit to marginal cost (horizontal light-gray line) in order to determine his search intensity (vertical light-gray line).

Figure 4: Costs and benefits of search

The right panel of figure 3 shows the probability distribution of the posted wages. It is worth noting that this distribution is downward sloping, in contrast with models based on the Burdett & Mortensen (1998) model. The distribution of the wages earned by employed workers is not shown, since it is virtually identical to the distribution of the posted wages. The reason is that all firms match with probability close to 1. Note further that the estimated amount of frictional wage dispersion is low. Multiple causes underlie this result. First, limited wage dispersion is a standard finding in well calibrated search models of the labor market, as shown by Hornstein et al. (2009). Second, the high value of home production restricts the amount of wage dispersion that can arise. Third, I limit the amount of dispersion even more by allowing for unobserved heterogeneity in productivity in a very flexible way. The model attributes a large part of the remaining variation in the wages after controlling for worker characteristics to the unobserved productivity component ε rather than to frictions.[42]

In order to consider the fit of the model, I use the estimated parameter values to simulate data for each of the dependent variables, i.e. the number of applicants per firm, the number of interviews per firm, the number of job offers per firm and the wages. Figure 5 displays the distributions of the simulated data along with those of the actual data. It shows that in general the model matches the

[42]The exact magnitude of frictional wage dispersion is still an open question in the literature, which I do not aim to answer here. Different assumptions about productivity and wages can of course be made. For example, adding on-the-job search to the model would probably help in generating more wage dispersion, but is left for future research because of the associated computational complexities. However, it is important to stress that changing the model along such dimensions has a very limited effect on the estimates for its key parameters since wage data is not the main source of their identification.

data quite well. The fit of the number of job offers is particularly good. For the number of applicants and interviews, the model has some difficulties fitting the large amount of dispersion present in the data. It predicts too few very low and very high values. However, the fit seems good around the median in both cases. For the unobserved wage component $\log(w) + \varepsilon$, the model slightly underestimates the height of the mode. Note that the graph confirms that once the unobserved productivity component ε is taken into account the amount of wage dispersion implied by the model is not smaller than in the data.

4.4 Magnitude of Frictions

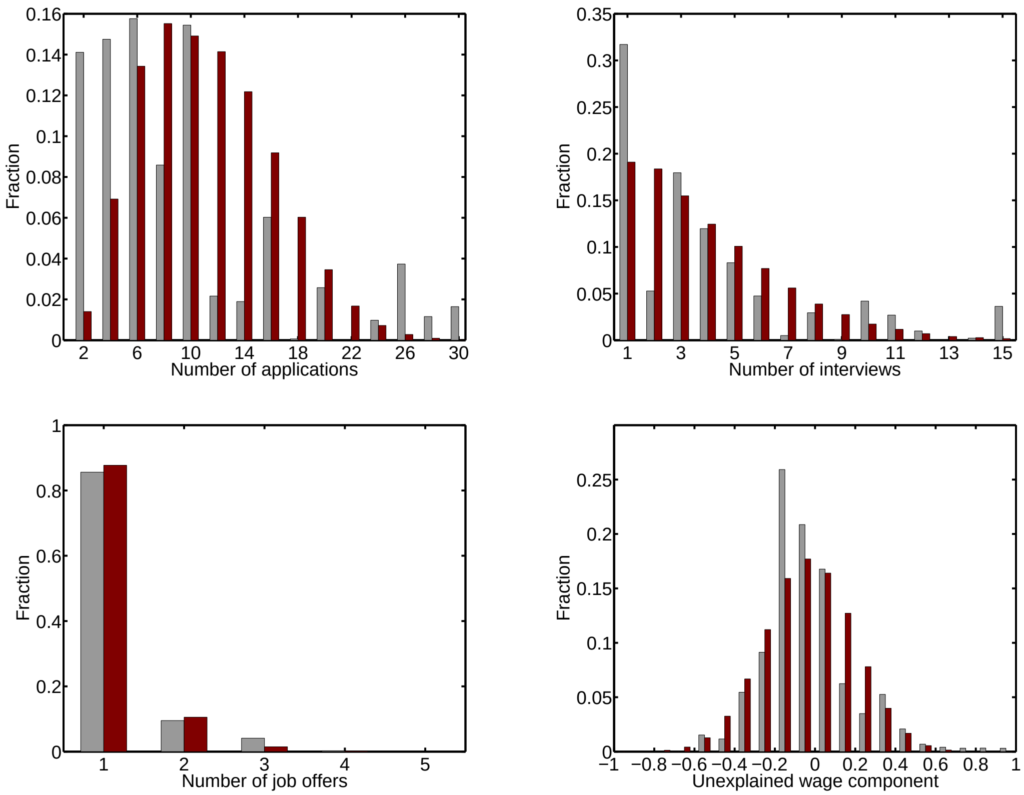
The estimation results indicate that both sending applications and conducting job interviews are costly activities. This means that both search and recruitment frictions arise, causing the externalities described before. I analyze the magnitude of these frictions by considering the effect of an exogenous change in the search cost k_A and/or recruitment cost k_R on steady state output.[43] Because of the efficiency unit assumption, a change in one of the parameters will change the equilibrium in any submarket in the same way. As a result, we do not need to aggregate across the submarkets in order to determine welfare effects, but we can focus on one particular submarket instead. The main component of steady state output in such a submarket is the production by matched firm-worker pairs. In equilibrium, there is a mass $1-u$ of such pairs in each submarket, each producing 1 unit. Firms with a vacancy pay the entry cost k_V and the recruitment cost $(r_i - 1)\,k_R$. Finally, unemployed workers incur a search cost αk_A, but produce in the household.

With respect to household production, one more assumption needs to be made. So far, it was irrelevant what components are included in h. However, when considering welfare, it is not the worker's private value of non-market time but the social value that is the relevant measure. In particular, if h includes unemployment benefits, then those do not add to total output, because they are simply a transfer. However, the fraction of h that consists of unemployment benefits cannot be identified with the model and data used in this paper. Therefore, I assume that unemployment benefits equal $b = 0.4$, roughly in line with the highest replacement rates in the US. Household production and the worker's value of leisure then add up to $h - b$. Hence, (net) steady state output Y, measured in efficiency units, equals

$$Y = (1-u) + u\,(h - b - \alpha k_A) - \sum_{i=1}^{A} v_i\,((r_i - 1)\,k_R + k_V)\,. \tag{24}$$

This expression equals 1 if all workers are employed all the time. However, such an equilibrium is clearly unattainable. The maximum level that can be realized is the level of output in a Walrasian

[43] To keep the exposition simple, I abstract from discounting and the transition path towards the steady state. This has very limited effects on the outcomes, since the periodical discount factor is close to 1.



Fit of the model for the distribution of the number of applicants per firm, the number of interviews per firm, the number of job offers per firm, and the posted wages. Light-gray bars represent the data and dark-red bars the model. All graphs are conditional on the firm hiring a worker. The bottom right panel shows the distribution of $\log(w) + \varepsilon$, i.e. the unexplained component in the wages after controlling for the effect of the observable characteristics x. The bin width is 2 for the applications, 1 for the interviews and offers, and 0.1 for the wages. The graphs do not display the entire domain of the distributions; outliers are omitted.

Figure 5: Fit of the model

	α	u	v	Output
$k_A = 0.006$ and $k_R = 0.017$ (Estimated model)	4.259	0.068	0.027	0.938
$k_A = 0.006$ and $k_R \to \infty$ (One interview)	3.860	0.089	0.031	0.926
$k_A = 0.006$ and $k_R = 0$ (Unrestricted interviewing)	3.883	0.049	0.026	0.949
$k_A = 0$ and $k_R = 0$ (Walrasian outcome)	∞	0.027	0.027	0.960

Equilibrium outcomes for several values of the search cost k_A and the recruitment cost k_R. If $k_R = 0$, firms can freely interview all applicants, while $k_R \to \infty$ implies that firms will interview at most one worker. The number of applications tends to infinity if $k_A = 0$.

Table 4: Comparative statics

world.[44] This outcome is obtained by setting the recruitment cost and the search cost equal to zero and by removing the upper bound on the application distribution, i.e. $k_R = 0$, $k_A = 0$ and $A \to \infty$.[45] Frictions are completely eliminated in this case and the matching process is fully efficient. Both firms with vacancies and unemployed workers match with probability 1, such that $v = u = \frac{\delta}{1+\delta}$. Hence, output Y^* in that case is given by

$$Y^* = \frac{1 + \delta\,(h - b - k_V)}{1 + \delta}.$$

This expression equals 0.960 for the estimated values of the parameters, which is 2.42% higher than in the estimated equilibrium. Besides the estimated equilibrium and the Walrasian outcome, I consider two other scenarios. I calculate steady state output for two different values of the recruitment cost, while keeping all other parameters constant at their estimated values. The outcomes are reported in table 4.

The first scenario is the one in which the recruitment cost goes to infinity. This implies that all firms will interview one candidate only ('one interview'), as in Galenianos & Kircher (2009). Fewer matches are formed for any given level of search intensity, which means that the job offer and hiring probability go down. In general, the effect of such a reduction in the matching probability on the level of search intensity that workers choose is not obvious. Workers may start to apply more in order to compensate for the fact that it is harder to match, but they may also start to apply

[44] Unemployment continues to exist in a Walrasian world since in each period a fraction δ of the employees is hit by the job destruction shock and cannot search for a job in that period.

[45] Kircher (2009) provides a formal proof of convergence to the Walrasian outcome.

less because they get discouraged.[46] The table shows that the latter effect dominates here. The unemployment rate goes up by about two percentage points and the vacancy rate increases slightly. Steady state output is reduced by approximately 1.27% compared to the estimated equilibrium.

In the second scenario, I set the recruitment costs equal to zero. This leads to an equilibrium in which firms contact all applicants ('unrestricted interviewing'), as described in Kircher (2009). Everything else being equal, this increased number of contacts increases the matching probability for both workers and firms. Search intensity is reduced, implying that now the other of the two above described effects dominates. The unemployment rate goes down and output goes up. Again, the effects are considerable in magnitude. Unemployment is about two percentage points lower and output is 1.25% higher than in the estimated equilibrium. Hence, in terms of output and unemployment the estimated equilibrium is approximately halfway between the extremes obtained when $k_R = 0$ and $k_R \to \infty$.

Figure 6 shows how these two scenarios influence the equilibrium probability distributions for the number of applicants per firm, the number of interviews per firm, the number of job offers per firm, and the posted wages. The effects are large. For example, eliminating the recruitment cost reduces wages and the number of applications that firms receive, but increases as expected the number of interviews and job offers. On the other hand, if the recruitment cost tends to infinity, the distribution of interviews and job offers becomes degenerate at 1. The distribution of the number of applications is bimodal in that case, while wages are similar to the ones in the estimated equilibrium.

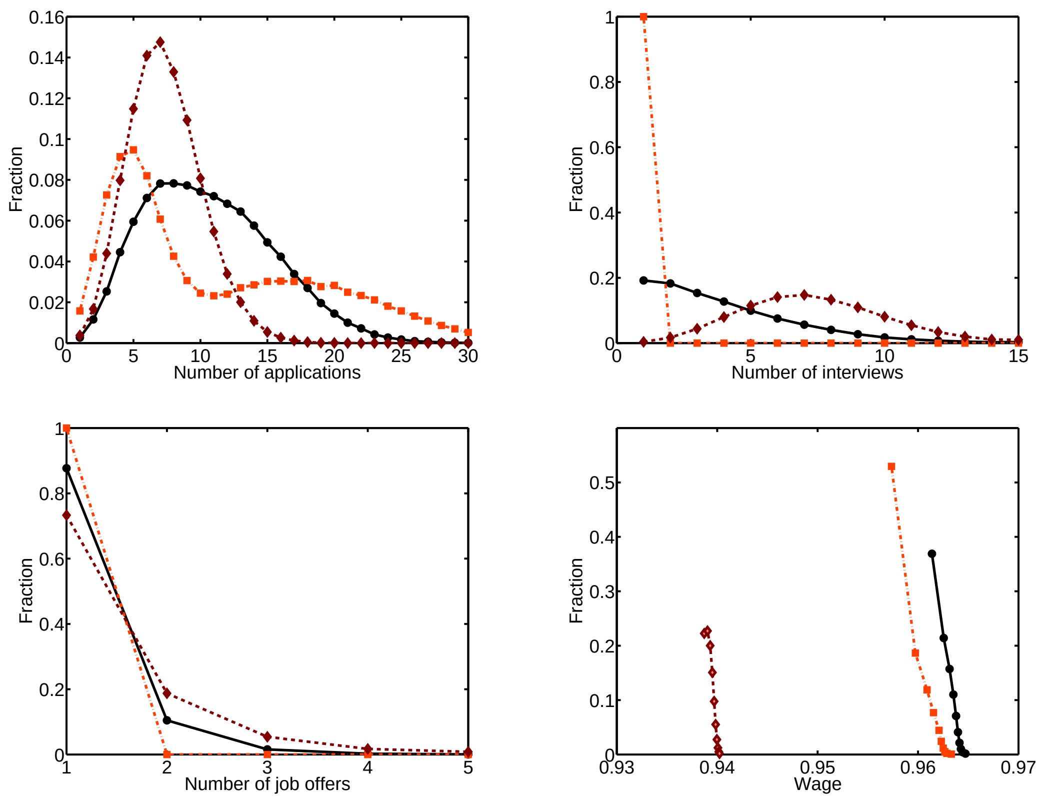
The results in table 4 can now be used to assess the relative importance of the search and the recruitment frictions. Starting from the estimated equilibrium, elimination of the recruitment frictions increases steady state output by 1.25%. If we then eliminate search frictions as well, we reach an output that is 2.42% higher than in the original situation.[47] Hence, recruitment costs account for 1.25% / 2.42% = 52% of the output loss compared to the Walrasian outcome, whereas search costs account for the remaining 48%. In other words, at the estimated parameter values, recruitment frictions seem roughly equally important as search frictions in this respect. This suggests that studies which ignore recruitment frictions potentially draw wrong inference about e.g. the desirability of certain labor market policies.

4.5 Social Planner

In order to illustrate how this model can be used for policy analysis, I consider a social planner's problem. I aim to capture the effects of the unemployment insurance (UI) eligibility criteria dis-

[46] See Shimer (2004) for a discussion of this issue.

[47] In both cases, the increase consists of two effects: 1) the direct effect of the elimination of the cost (while holding the equilibrium constant) and 2) the indirect effect via a change in the equilibrium outcomes. The indirect effect greatly dominates both times, as can be shown by calculating output while only taking into account the direct effect. Starting from the estimated equilibrium, elimination of the recruitment cost would then yield an output equal to 0.939. Likewise, starting from the 'unrestricted interviewing' case, elimination of the search cost would give an output equal to 0.951.



Probability distributions of the number of applicants per firm, the number of interviews per firm, the number of job offers per firm, and the posted wages. All graphs are conditional on the firm hiring a worker. The black circles represent the estimated equilibrium ($k_R = 0.017$). The orange squares correspond to the scenario in which all firms interview one candidate ($k_R \rightarrow \infty$). The dark-red diamonds display the case in which firms interview all applicants ($k_R = 0$).

Figure 6: Comparative statics

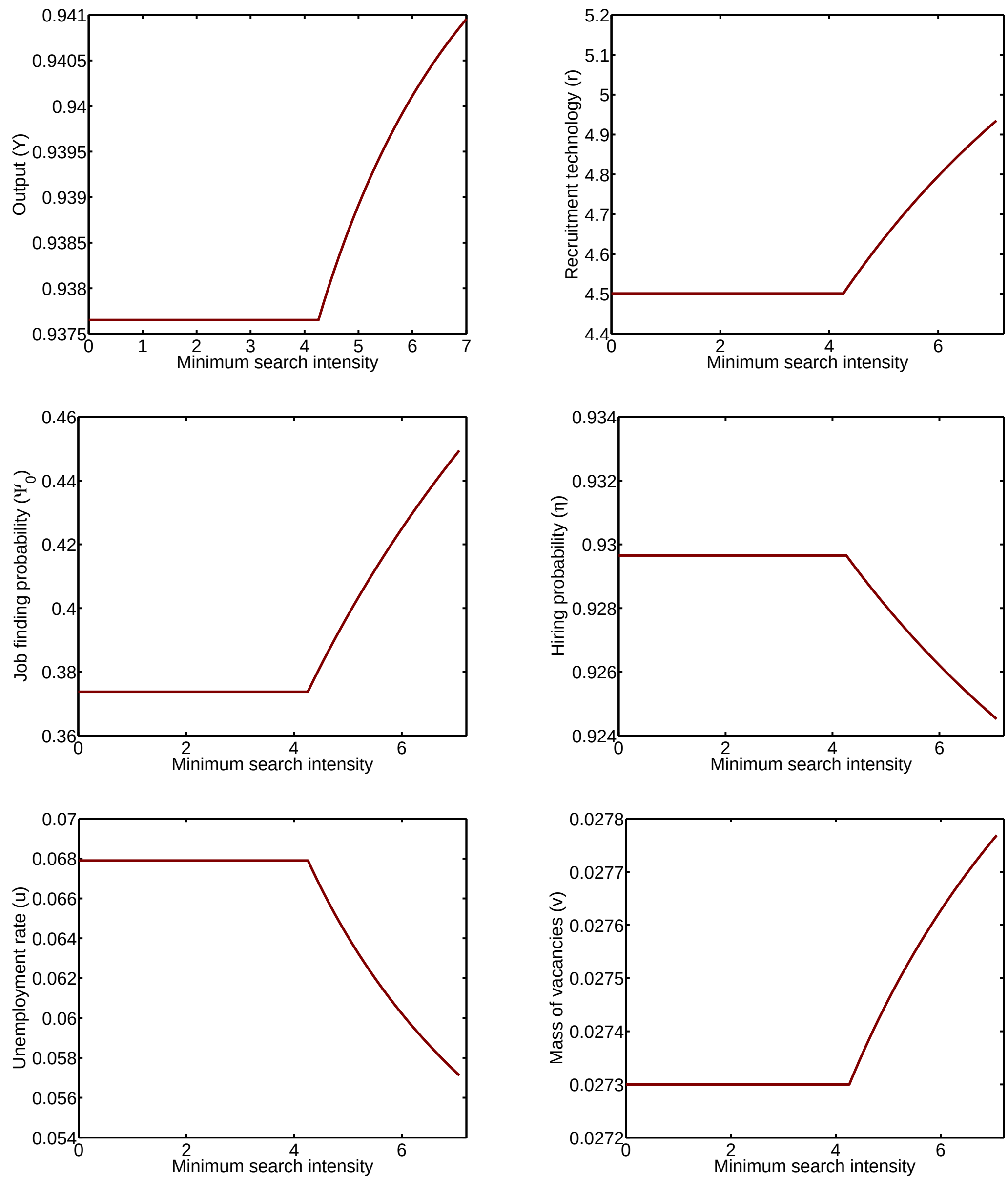
cussed in the introduction with the following setup. The planner sets the minimum search intensity $\underline{\alpha}$ that is required to be eligible for UI benefits, but everything else is determined by the decentralized market.[48] In particular, workers choose whether they want to comply with the rules, or whether they rather prefer to forgo the benefits. I determine the equilibrium for minimum search intensities between 0 and 7. The results are displayed in figure 7.

Two results are immediate. First, if the minimum search intensity is not binding because it is set below the estimated market level of 4.26, the equilibrium does not change. Therefore, I focus on higher values of $\underline{\alpha}$ in the discussion below. Second, for any reasonable value of $\underline{\alpha}$, workers always decide to comply, since the gain from getting unemployment benefits ($b = 0.4$) is much larger than the cost of a few more applications ($k_A = 0.006$). Other results are less straightforward because of the large number of externalities included in the model. The top right panel of figure 7 shows that firms respond to the increase in search intensity by investing more in recruitment. For example, setting $\underline{\alpha}$ equal to 7, causes firms to choose a 10% higher value of r. This is consistent with the fact that any job offer is now more likely to be rejected. The increase in r is however not sufficiently large to completely offset the decrease in the hiring probability η caused by this increase in competition, as shown in the fourth panel of the figure.

Hence, the firms are now worse off than before along two dimensions. However, they benefit from the decrease in wages that is the result of the more intensive search by the workers (not shown in the figure). In fact, this effect dominates, which translates into additional entry of vacancies (bottom right panel). Workers benefit from the increase in the number of vacancies since it increases their job finding probability Ψ_0 (third panel). As a result, unemployment falls. This change is quite large, e.g. requiring workers to write two additional letters every month leads to a reduction in the unemployment rate of 0.8 percentage point. An unemployed worker is nevertheless worse off than before, in the sense that the value of unemployment V_U decreases. The lower wages and the higher search costs dominate the positive effect of the better matching possibilities. However, the key question in analyzing the policy is what happens to net output Y, which includes all costs and benefits. As the top left panel shows, this welfare measure goes up. Although the program aggravates the negative externalities and leads to higher recruitment costs for the firms, the positive externalities that result from the increase in the workers' search intensity dominate.

Note that output does not reach a maximum for $\underline{\alpha}$ in the interval (4.26, 7). In principle, one can extend the analysis to find the optimal level for the minimum level of search. This would yield $\underline{\alpha} = 16.2$ as the optimum. However, this result has to be interpreted with a lot of caution. First, the potential existence of multiple equilibria raises the question how the market will respond to the policy. For a small change, convergence to an equilibrium 'nearby' seem a reasonable assumption. For larger changes, such an assumption is much more questionable. Second, for large values of $\underline{\alpha}$,

[48] Note that the actual rules are in terms of the number of applications instead of the search intensity. I abstract from that here.



Equilibrium outcomes in the social planner's problem. The planner specifies the minimum search intensity $\underline{\alpha}$ that is required in order to receive UI benefits. Workers choose whether to comply or not.

Figure 7: Social planner's problem

	$\phi = 0.8$	$\phi = 0.9$	$\phi = 1$
Search intensity (α)	4.173	4.259	4.193
Search cost (k_A)	0.008	0.006	0.006
Entry cost (k_V)	0.988	1.052	1.111
Recruitment cost (k_R)	0.026	0.017	0.008
Job destruction probability (δ)	0.026	0.027	0.028
Output gain from elimination recruitment costs	1.47%	1.25%	1.13%
Output difference with a Walrasian world	2.66%	2.42%	2.28%
Relative importance search frictions	45%	52%	48%
Relative importance recruitment frictions	55%	48%	52%
Welfare gain from $\underline{a} = 7$	0.36%	0.35%	0.39%

Table 5: Results of the robustness checks

the assumption that $A = 10$ becomes restrictive. This limits the negative impact of the recruitment friction, which leads to an overestimate of output for such high values of $\underline{\alpha}$. Third, the optimal level of $\underline{\alpha}$ strongly depends on the level of unemployment benefits b, which is a variable that was not estimated.[49] The main conclusion from the exercise here should therefore be as follows: given the estimated parameter values, a small increase in the required search intensity is likely to increase steady state output.

4.6 Robustness

In the estimation and the welfare analysis, I fixed the probability ϕ that an interviewed applicant is qualified for the job at 0.9. In this subsection I check how sensitive the estimation results are to the value for ϕ by repeating the analysis for $\phi = 0.8$ and $\phi = 1$. If $\phi = 1$, interviews are clearly not necessary for screening but only because of the assumption that an interview is required before a job offer can be made. It is nevertheless useful to consider this value as a limit case. The results of the robustness exercise are listed in table 5.

The values in the table indicate that the results are fairly robust to changes in ϕ. Not surprisingly, the estimate for the cost of recruitment is slightly higher for lower values of ϕ in order to explain why firms interview relatively few applicants. To compensate for this, the estimate of the entry cost decreases when ϕ decreases.[50] The potential gains in output from an elimination of the recruitment frictions are larger when k_R is higher. The output loss compared to a Walrasian world is also decreasing in ϕ. All other estimates hardly change. In particular, the welfare effects of imposing a minimum search intensity $\underline{\alpha} = 7$ are remarkably constant.

[49] For example, if $b = 0$ and $h = 0.955$, the pattern is reversed. In that case, an increase in search intensity decreases output. The intuition for this result is clear: if workers are almost equally productive at home as in a job, the social gains from search are very small.

[50] Recall that I fix the measure of vacancies in the estimation procedure.

Hence, the results in this paper do not seem to be driven by the particular choice of ϕ. Of course, an approach in which also this parameter would be estimated from microdata is preferred. However, this requires information on the number of interviewed applicants that each firm considers to be qualified for its job.[51] Unfortunately, such detailed data does not exist at this moment. I leave this issue therefore for further research.

5 Conclusion

This paper analyzes the magnitude of labor market frictions that arise when workers and firms cannot coordinate their search and recruitment decisions. For this purpose, a directed search model is presented in which both workers and firms decide how many agents on the other side of the market they want to contact. Firms post a wage and recruitment technology that determines how many applicants they will interview. After observing these contracts, workers decide how many times and to which firms to apply. Since workers typically send multiple applications, any firm faces the risk that its job offer gets rejected. In that case, the firm can make a job offer to a different applicant, but only if it has interviewed him. Hence, interviewing more applicants is more costly but increases the probability to match. The same holds for sending more applications.

In equilibrium, contract dispersion arises. Some firms offer low wages but make large investments in recruitment, while other firms do the opposite. The number of contract types is equal to the maximum number of applications that workers may send in any given period and firms are indifferent between all types. Workers face a trade-off between the wage and the probability to get a job offer. Applications to low wage firms are more likely to turn into job offers than applications to high wage firms. It is shown that workers maximize the payoff from their application portfolio by spreading their applications over the different types of contracts.

Estimation of the model provides values for the cost of search and recruitment. An additional application is estimated to cost the worker 0.6% of one period of production, while firms incur a cost equal to 1.7% of periodical output for each interview. Given a period length of a month, these figures correspond to respectively 0.20 and 0.50 days. By simulating the equilibrium for different values of the two cost parameters, the market equilibrium is compared to the Walrasian outcome. Output is 2.4% lower in the market equilibrium than in the Walrasian world, whereas unemployment is 4.1 percentage point higher. Search and recruitment frictions both contribute roughly equally much to the output loss. Solving a social planner's problem shows that there is a potential role for active labor market programs. In particular, programs that marginally increase the level of search intensity of unemployed workers negatively affect the firms' hiring probability, but increase steady state output.

An interesting avenue for future work would be to allow for more productivity differences

[51] Ideally, the data would be even more detailed, for example by including the characteristics and the firm's evaluation of each applicant.

among workers and firms. This would make the model suitable to study settings in which heterogeneity plays a larger role than in the sample that I use here. Examples include college admissions and the academic job market for economists. A second area for future research concerns the natural rate of unemployment. There is a widely held belief that the rise of the Internet has reduced search costs and perhaps also recruitment costs. Given an estimate of the change in those costs, the model presented in this paper could be used to analyze how much of the sharp decline in the unemployment rate that many Western economies experienced in the nineties can be attributed to a reduction in the frictions in the labor market.

A Proofs

A.1 Proof of Lemma 1

Proof. Consider a firm with queue length λ and recruitment technology r. The number of applicants i follows a Poisson distribution with mean λ, while the interview capacity j follows geometric distribution with parameter $\rho \equiv \frac{r-1}{r}$. The actual number of interviews is equal to $\min\{i,j\}$. Hence, the expected number of interviews equals

$$\begin{aligned}
\sum_{i=0}^{\infty}\sum_{j=1}^{\infty}\min\{i,j\}\, e^{-\lambda}\frac{\lambda^i}{j!}\left(\frac{r-1}{r}\right)^{j-1}\frac{1}{r} &= \sum_{i=0}^{\infty} e^{-\lambda}\frac{\lambda^i}{i!}\sum_{j=1}^{i-1} j\rho^{j-1}\frac{1}{r}+\sum_{i=0}^{\infty} ie^{-\lambda}\frac{\lambda^i}{i!}\sum_{j=i}^{\infty}\rho^{j-1}\frac{1}{r} \\
&= r\sum_{i=0}^{\infty} e^{-\lambda}\frac{\lambda^i}{i!}\left(1-\rho^{i-1}\left(\rho+\frac{i}{r}\right)\right)+\sum_{i=0}^{\infty} ie^{-\lambda}\frac{\lambda^i}{i!}\rho^{i-1} \\
&= r\left(1-e^{-\lambda/r}-\frac{\lambda}{r}e^{-\lambda/r}\right)+\lambda e^{-\lambda/r} \\
&= r\left(1-e^{-\lambda/r}\right).
\end{aligned}$$

□

A.2 Proof of Lemma 1

Proof. Consider the job offer probability ψ first.[52] Let $\rho = \frac{r-1}{r}$. When a worker applies to a firm, he will in general face a number of competitors, i.e. other applicants at the same firm. Let m denote the number of competitors who are qualified and will accept the job if offered to them. On the other hand, let n denote the number of competitors who are either unqualified or will reject the job in favor of a better offer. In order to simplify terminology, I refer to the first type of competitors as 'direct competitors'. The second type is called 'indirect competitors', since they only reduce the worker's matching probability by causing congestion.

A number of applicants will be selected for an interview. Each applicant is equally likely to be selected and to be qualified. Therefore, I assume without loss of generality that if the firm has multiple qualified candidates, it makes offers among them in the same order as in which it conducted the interviews. Hence, one can interpret the recruitment as a process with multiple rounds, which ends when the firm selects a qualified applicant who accepts the job, or when the maximum possible number of interviews has been held.

For the worker it is not relevant in which round he gets the job offer. Getting a job offer in round i and getting one in round j are mutually exclusive events, since the firm will offer the job to each worker at most once. Hence, the probability that the worker gets a job offer equals the probability

[52] In this proof, I omit the dependence of the job offer probability, the matching probability and the queue lengths on the contract c in order to keep notation as simple as possible.

that he gets an offer in round i, summed over all possible i. In order for the worker to get a job offer in round i, several things must happen: (i) the firm must be able to interview at least i applicants, (ii) only indirect competitors must have been selected in round $1,\ldots,i-1$, (iii) the worker gets selected in round i itself and (iv) he is qualified. The probability that this occurs equals

$$\phi\rho^{i-1}\frac{n}{m+n+1}\cdot\frac{n-1}{m+n}\cdot\ldots\cdot\frac{n+2-i}{m+n+3-i}\cdot\frac{1}{m+n+2-i}=\phi\rho^{i-1}\frac{n!}{(n+1-i)!}\frac{(m+n+1-i)!}{(m+n+1)!},$$

as long as $i\leq n+1$ and zero otherwise. Hence, the total probability that the worker gets a job offer equals

$$\hat{\psi}(m,n|r)=\phi\sum_{i=1}^{n+1}\rho^{i-1}\frac{n!}{(n+1-i)!}\frac{(m+n+1-i)!}{(m+n+1)!},$$

which goes to $\frac{1}{m+1}$ for $r\to\infty$.

Actually, the number of competitors that a worker faces is a random variable. Both m and n follow Poisson distributions with respective parameters μ and $\nu=\lambda-\mu$. Hence, the probability to compete with exactly m direct and n indirect competitors equals

$$f_{m,n}(m,n|\phi\mu,\nu)=e^{-\mu}\frac{\mu^m}{m!}e^{-\nu}\frac{\nu^n}{n!}.$$

The ex ante probability to get a job offer therefore equals

$$\psi=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\hat{\psi}(n,m|r)f_{m,n}(m,n|\mu,\lambda-\mu).$$

Substituting the expression for $\hat{\psi}(m,n|r)$ and changing the order of summation yields

$$\psi=\phi\sum_{i=1}^{\infty}\rho^{i-1}\sum_{m=0}^{\infty}\sum_{n=i-1}^{\infty}\frac{n!}{(n+1-i)!}\frac{(m+n+1-i)!}{(m+n+1)!}f_{m,n}(m,n|\mu,\lambda-\mu).$$

The last part of the right hand side can be rewritten as follows

$$\phi\sum_{m=0}^{\infty}\sum_{n=i-1}^{\infty}\frac{n!}{(n+1-i)!}\frac{(m+n+1-i)!}{(m+n+1)!}f_{m,n}(m,n|\mu,\lambda-\mu)=\frac{\phi(\lambda-\mu)^{i-1}}{\lambda^i}\left(1-e^{-\lambda}\sum_{j=0}^{i-1}\frac{\lambda^j}{j!}\right).$$

Substituting this into the expression for ψ and applying a change in the order of summation gives

$$\psi=\phi\sum_{i=1}^{\infty}\frac{\rho^{i-1}(\lambda-\mu)^{i-1}}{\lambda^i}-\phi e^{-\lambda}\sum_{j=0}^{\infty}\frac{\lambda^j}{j!}\sum_{i=j+1}^{\infty}\frac{\rho^{i-1}(\lambda-\mu)^{i-1}}{\lambda^i}.$$

Since $\sum_{i=j+1}^{\infty} \frac{\rho^{i-1}(\lambda-\mu)^{i-1}}{\lambda^i} = \frac{1}{\kappa}\left(\frac{\rho(\lambda-\mu)}{\lambda}\right)^j$ with $\kappa = \frac{1}{r}\lambda + \frac{r-1}{r}\mu$, this implies

$$\begin{aligned}\psi &= \frac{\phi}{\kappa}\left(1 - e^{-\lambda}\sum_{j=0}^{\infty}\frac{\rho^j(\lambda-\mu)^j}{j!}\right) \\ &= \frac{\phi}{\kappa}\left(1-e^{-\kappa}\right).\end{aligned}$$

An expression for the firm's matching probability η can be derived in a similar way. A firm with m qualified applicants who would accept the job and n other applicants, hires in round i with probability

$$\rho^{i-1}\frac{n}{m+n}\cdot\frac{n-1}{m+n-1}\cdot\ldots\cdot\frac{n+2-i}{m+n+2-i}\cdot\frac{m}{m+n+1-i} = \rho^{i-1}\frac{m\cdot n!}{(n+1-i)!}\frac{(m+n-i)!}{(m+n)!},$$

as long as $i \leq n+1$ and zero otherwise. Hence, given m and n the total hiring probability equals

$$\hat{\eta}(m,n|r) = \sum_{i=1}^{n+1}\rho^{i-1}\frac{m\cdot n!}{(n+1-i)!}\frac{(m+n-i)!}{(m+n)!},$$

which goes to 1 for $r\to\infty$.

Both m and n follow Poisson distributions with respective parameters μ and $\nu = \lambda-\mu$. Hence, the ex ante matching probability is equal to

$$\begin{aligned}\eta &= \sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\hat{\eta}(n,m|r)\, f_{m,n}(m,n|\mu,\lambda-\mu) \\ &= \sum_{i=1}^{\infty}\rho^{i-1}\sum_{m=1}^{\infty}\sum_{n=i-1}^{\infty}\frac{m\cdot n!}{(n+1-i)!}\frac{(m+n-i)!}{(m+n)!} f_{m,n}(m,n|\mu,\lambda-\mu).\end{aligned}$$

The last part of the right hand side can be rewritten as follows

$$\sum_{m=1}^{\infty}\sum_{n=i-1}^{\infty}\frac{m\cdot n!}{(n+1-i)!}\frac{(m+n-i)!}{(m+n)!} f_{m,n}(m,n|\mu,\lambda-\mu) = \frac{\mu(\lambda-\mu)^{i-1}}{\lambda^i}\left(1-e^{-\lambda}\sum_{j=0}^{i-1}\frac{\lambda^j}{j!}\right),$$

which implies

$$\begin{aligned}\eta &= \mu\psi \\ &= \frac{\mu}{\kappa}\left(1-e^{-\kappa}\right).\end{aligned}$$

□

A.3 Proof of Lemma 2

Proof. The proof is a generalization of the proof provided by Galenianos & Kircher (2009). Consider application $i > 1$ and suppose it is not sent to a contract offering a wage in the interval $[\bar{w}_{i-1}, \bar{w}_i]$. Then there are two possibilities: 1) the application is sent to a contract offering a wage $w > \bar{w}_i$ or 2) the application is sent to a contract offering a wage $w < \bar{w}_{i-1}$. By the definition of $\bar{w}_i$, the first case would give the worker a payoff that is strictly lower than V_i, which cannot be optimal. In the second case, the worker could do better by deviating and sending the application to $\bar{w}_{i-1}$. Let $\bar{c}_i$ denote the contract $(r, \bar{w}_i)$, then the utility gain from deviating equals

$$\left(\psi(\bar{c}_{i-1}) V_E(\bar{w}_{i-1}) + (1-\psi(\bar{c}_{i-1})) V_{i-1}\right) - \left(\psi(c) V_E(w) + (1-\psi(c)) V_{i-1}\right) =$$
$$\left(\psi(\bar{c}_{i-1}) (V_E(\bar{w}_{i-1}) - V_{i-2}) - \psi(c)(V_E(w) - V_{i-2})\right) + (\psi(c) - \psi(\bar{c}_{i-1}))(V_{i-1} - V_{i-2}) > 0.$$

The first term is non-negative, since $\bar{w}_{i-1}$ maximizes $\psi(\bar{c}_{i-1})(V_E(\bar{w}_{i-1}) - V_{i-2})$ by definition. The second term is positive, since $\bar{w}_{i-1} > w$ implies that $\psi(\bar{c}_{i-1}) < \psi(c)$. □

A.4 Proof of Lemma 3

Proof. The proof is a generalization of the proof provided by Kircher (2009). First, note that for wages w below $\bar{w}_0 = V_E^{-1}(V_1)$, the market utility cannot be obtained. As a result, $\kappa(c) = 0$ and $\psi(c) = \phi$. At $w = V_E^{-1}(V_1)$, the market utility can only be obtained if $\kappa(c) = 0$ and $\psi(c) = \phi$. Lemma 2 establishes that the first application is sent to a wage in the interval $\left(V_E^{-1}(V_1), \bar{w}_1\right]$. As a result, $\psi(c) V_E(w) + (1-\psi(c)) V_0 = V_1$ needs to hold for all wages in this interval in order to satisfy the market utility condition. A similar argument applies to the other interval. Application $i > 1$ is sent to the interval $[\bar{w}_{i-1}, \bar{w}_i]$, which implies that the effective queue length $\kappa(c)$ and the job offer probability $\psi(c)$ are governed by equation (17). The effective queue length has to be continuous in the wage. This implies that $\bar{w}_i$ is determined as the wage at which equation (17) holds for both i and $i+1$. Solving the two equations yields

$$V_E(\bar{w}_i) = \frac{V_i^2 - V_{i-1} V_{i+1}}{2V_i - V_{i-1} - V_{i+1}}.$$

□

A.5 Proof of Lemma 4

Proof. Suppose that the lemma does not hold and that firms of type i offer the same contract (r, w) as firms of type $i+1$. By lemma 3, we must then have that $w = \bar{w}_i$. Let ψ_i denote the associated job offer probability. If the worker gets a job offer from both firms, but no higher one, if will accept the offer from firm $i+1$ by construction. As a result, the acceptance probabilities for both firms are related as follows

$$1-\Psi_i = (1-\psi_i)(1-\Psi_{i+1}) < 1-\Psi_{i+1}.$$

Consider first $\bar{w}_i \in [0,1)$. If the firms of type i offer a marginally higher wage $w' \downarrow \bar{w}_i$, their acceptance probability jumps up to $1-\Psi_{i+1}$. Clearly, such a deviation is profitable. Next, consider $\bar{w}_i = 1$. In that case, the firms make zero profits. At wages slightly below one, the queue length is lower but still positive. This deviation therefore yields higher profits to the firm. $\square$

A.6 Proof of Lemma 5

Proof. Firms solve

$$\max_{\kappa\in\mathcal{K}_i, r\in\mathcal{R}} \frac{r\phi\,(1-\Psi_i)\left(\left(1-e^{-\kappa}\right)(V_E(1)-V_{i-1})-\frac{\kappa}{\phi}(V_i-V_{i-1})\right)}{1+(r-1)\,\phi\,(1-\Psi_i)} - k_V - k_R\,(r-1)\,.$$

Start with the optimization over the effective queue length. The second derivative of the objective function with respect to κ equals

$$-\frac{r\phi\,(1-\Psi_i)\,e^{-\kappa}\,(V_E(1)-V_{i-1})}{1+(r-1)\,\phi\,(1-\Psi_i)},$$

which is strictly negative for all $\kappa\in\mathcal{K}_i$. Consequently, the objective function is strictly concave in the effective queue length.

The first order condition is given by

$$\frac{r\phi\,(1-\Psi_i)\left(e^{-\hat{\kappa}_i}(V_E(1)-V_{i-1})-\frac{1}{\phi}(V_i-V_{i-1})\right)}{1+(r-1)\,\phi\,(1-\Psi_i)} = 0.$$

Solving for $\hat{\kappa}_i$ yields

$$\hat{\kappa}_i = \log\frac{\phi\,(V_E(1)-V_{i-1})}{V_i-V_{i-1}}$$

If $\hat{\kappa}_i\in\mathcal{K}_i$, the optimal effective queue length is defined by this expression, i.e. $\kappa_i^*=\hat{\kappa}_i$. On the other hand, if $\hat{\kappa}_i<\bar{\kappa}_{i-1}$, then $\kappa_i^*=\bar{\kappa}_{i-1}$.

Next, consider the optimization over the recruitment technology r. In order to simplify notation, define

$$\widetilde{V}_{F,i} \equiv \left(1-e^{-\kappa_i^*}\right)(V_E(1)-V_{i-1}) - \frac{\kappa_i^*}{\phi}(V_i-V_{i-1}) = \max_{\kappa\in\mathcal{K}_i}\left(1-e^{-\kappa}\right)V_F(w(\kappa))\,,$$

The optimization problem can then be written as

$$\max_{r\in\mathcal{R}} \frac{r\phi\left(1-\Psi_i\right)\widetilde{V}_{F,i}}{1+\left(r-1\right)\phi\left(1-\Psi_i\right)} - k_V - k_R\left(r-1\right).$$

The first derivative of the objective function with respect to r equals

$$\frac{\phi\left(1-\Psi_i\right)\left(1-\phi\left(1-\Psi_i\right)\right)}{\left(1+\left(r-1\right)\phi\left(1-\Psi_i\right)\right)^2}\widetilde{V}_{F,i} - k_R,$$

whereas the second derivative is equal to

$$-2\frac{\phi^2\left(1-\Psi_i\right)^2\left(1-\phi\left(1-\Psi_i\right)\right)}{\left(1+\left(r-1\right)\phi\left(1-\Psi_i\right)\right)^3}\widetilde{V}_{F,i}.$$

The latter is strictly negative for all $r \in \mathcal{R}$, which implies that the objective function is strictly concave. Solving the first order condition yields

$$\widehat{r}_i = \sqrt{\frac{1-\phi\left(1-\Psi_i\right)}{\phi\left(1-\Psi_i\right)}}\sqrt{\frac{\widetilde{V}_{F,i}}{k_R}} - \frac{1-\phi\left(1-\Psi_i\right)}{\phi\left(1-\Psi_i\right)}.$$

Substituting this into the objective function gives after some manipulation the following relationship

$$\sqrt{\frac{k_V-k_R}{k_R}} = \sqrt{\frac{\widetilde{V}_{F,i}}{k_R}} - \sqrt{\frac{1-\phi\left(1-\Psi_i\right)}{\phi\left(1-\Psi_i\right)}},$$

which implies that

$$\widehat{r}_i = \sqrt{\frac{1-\phi\left(1-\Psi_i\right)}{\phi\left(1-\Psi_i\right)}\frac{k_V-k_R}{k_R}}.$$

Hence, the optimal recruitment technology r_i^* equals $\widehat{r}_i$ if $\widehat{r}_i > 1$, and 1 otherwise. □

A.7 Proof of Proposition 2

Proof. The proof closely follows Galenianos & Kircher (2005). □

B Search Heterogeneity

When workers face uncertainty about the number of applications a that they will send, all relevant expressions can be calculated by conditioning on a first and subsequently taking the expectation over a. For example, a worker sending $a \in \{1,\ldots,A\}$ applications fails to get a job offer with probability $\prod_{i=1}^{a}\left(1-\psi\left(c_i\right)\right)$. The ex ante matching probability therefore equals

$$\Psi_0(\mathbf{c}) = 1 - \sum_{a=1}^{A} p(a|\alpha) \prod_{i=1}^{a} (1 - \psi(c_i)). \tag{1'}$$

A similar logic applies to the value of search, which now is a function of both α and $\mathbf{c}$.

$$V_S(\alpha, \mathbf{c}) = \sum_{a=1}^{A} p(a|\alpha) \sum_{i=1}^{a} \prod_{j=i+1}^{a} (1 - \psi(c_j)) \psi(c_i) (V_E(w_i) - V_U) \tag{2'}$$

Workers choose the application strategy $\{\alpha, \mathbf{c}\}$ that maximizes their expected payoff, i.e.

$$V_S^* = \max_{\alpha, \mathbf{c}} V_S(\alpha, \mathbf{c}) - k_A \alpha. \tag{3'}$$

Workers' application behavior determines the gross queue length $\lambda(c)$ at each of the vacancies. Formally, we have

$$v \int_0^w \int_1^r \lambda(\widetilde{r}, \widetilde{w}) f(\widetilde{r}, \widetilde{w}) d\widetilde{r} d\widetilde{w} = u \sum_{i=1}^{A} (1 - P(i-1|\alpha)) G_i(c) \ \forall c \in \mathcal{C}, \tag{12'}$$

where $G_i(c)$ denotes the marginal distribution with respect to c_i. The right hand side again denotes the total mass of applications that are sent by the workers to contracts no higher than c, now taking into account the heterogeneity. The left hand side still represents the mass of applications received by firms posting a contract no higher than c.

Finally, the acceptance probability is now defined by

$$1 - \Psi(c) = \sum_{i=1}^{A} \sum_{a=i}^{A} \mathcal{P}(a, i; w) \int \prod_{j=i+1}^{a} (1 - \psi(w_j)) d\hat{G}_i(\mathbf{c}_{-i}; c), \tag{13'}$$

where $\mathcal{P}(a, i; w)$ denotes the conditional probability that the worker who got the offer w sent there his i-th out of a applications.

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