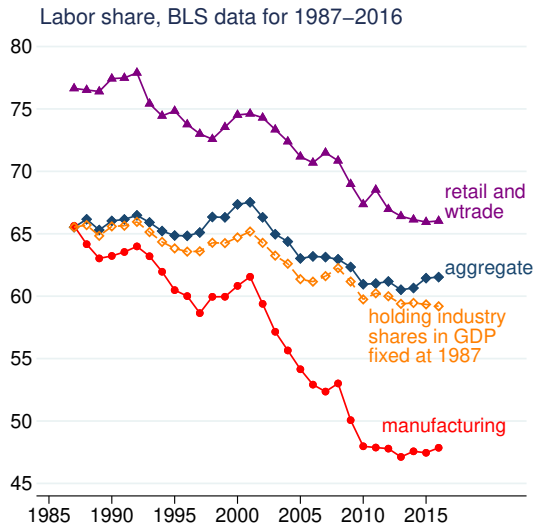


Not a Typical Firm: The Joint Dynamics of Firms, Labor Shares, and Capital-Labor Substitution

Joachim Hubmer Pascual Restrepo

University of Toronto
December 17th, 2020

The Decline in the US Labor Share



Two broad explanations:

- **Technology:** substitution of capital for labor in widening range of tasks as automation advances and capital becomes cheaper.

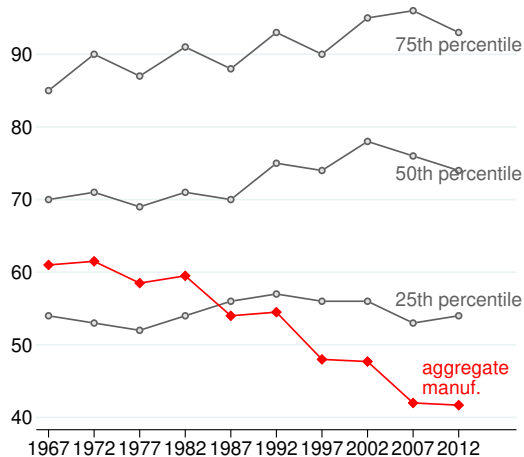
(Karabarbounis– Neiman 2014; Edden–Gagl 2018; Hubmer 2020; Acemoglu–Restrepo, 2018)

- **Concentration:** rising competition reallocates economic activity to high markup firms

(Barkai 2020; De Loecker–Eeckhout–Unger 2020; Autor–Dorn–Katz–Patterson–V.Reenen 2020)

The Role of Firms in the Decline of the Labor Share

Labor share decomposition in manufacturing
Kehrig & Vincent (2020)



- Decline not uniform across firms, and not visible for median firm (Autor et al. 2020; Kehrig-Vincent, 2020)
- Challenges **technology view**:
 - rules out simple story where all firms face same factor prices and have access to same technologies
 - suggests key role for reallocation rather than capital-labor substitution

This Paper

1. Develop firm dynamics model with **costly capital–labor substitution within tasks**: matches firm-level labor and market share dynamics as capital prices falling
 - typical firm: no change in task allocation $\Rightarrow K$ and L complements \Rightarrow LS rises
 - top firms: more tasks automated $\Rightarrow K$ and L substitutes \Rightarrow LS declines
2. Extend to non-CES demand system to account for **rising competition** and reallocation to more productive firms: K–L substitution remains important
 - 90% of the decline in labor share in manufacturing
 - 40% of the decline in labor share in retail; rising competition more important
3. “Model-free” accounting of contribution of markups/reallocation to LS decline
 - important to allow for **differences in technology across firms** (cf. DeLoecker, Eeckhout, Unger 2020)

Outline

1. Model with CES demand to show implications of capital–labor substitution across firms
2. Model with non-CES demand to compare effects of competition and capital–labor substitution
3. Model-free bounds on effect of reallocation and changes in markups on labor share

Model: Overview

- Standard **firm dynamics model** with **task level substitution and costly automation**.
 - Firms differ in productivity z (exog) and capital share parameter α (endog).
 - To raise α , firms have to pay a (constant) fixed cost for each automated task.
 - Matches studies on adoption of new capital-intensive techs (Lashkari et al. 2019; Acemoglu et al. 2020; US Census 2020) [details](#)
 - New technologies diffuse in the economy through imitation by entrants.

Model: Production Function

- Production requires a **continuum of tasks**

$$y = z \cdot \left(\int_0^1 \mathcal{Y}(x)^{\frac{\eta-1}{\eta}} dx \right)^{\frac{\eta}{\eta-1}}$$

- Tasks $\in [0, \alpha]$ are automated** and can be produced by capital:

$$\mathcal{Y}(x) = \begin{cases} \psi_k(x)k(x) + \psi_\ell(x)\ell(x) & \text{if } x \leq \alpha \\ \psi_\ell(x)\ell(x) & \text{if } x > \alpha \end{cases} \quad \frac{\psi_\ell(x)}{\psi_k(x)} \text{ increasing in } x$$

- Unit cost of production (if all tasks in $[0, \alpha]$ produced by capital)

$$c(z, \alpha) = \frac{1}{z} \left[\Psi_k(\alpha) \left(\frac{R}{q} \right)^{1-\eta} + \Psi_\ell(\alpha) W^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

share parameters: $\Psi_k(\alpha) = \int_0^\alpha \psi_k(x)^{\eta-1} dx$ and $\Psi_\ell(\alpha) = \int_\alpha^1 \psi_\ell(x)^{\eta-1} dx$

Model: Value Functions for Entrants and Incumbents

- Value function of incumbent with technology (z, α) :

$$V(z, \alpha) = \pi(z, \alpha) + \int \max \left\{ 0, -c_f + \max_{\alpha' \in [\alpha, 1]} \left\{ -c_a \cdot (\alpha' - \alpha) + \beta \mathbb{E}[V(z', \alpha') | z] \right\} \right\} dG(c_f)$$

- Value function of (potential) entrants with productivity signal z :

$$V_e(z, \alpha_0) = \int \max \left\{ 0, -c_f + \max_{\alpha' \in [\alpha_0, 1]} \left\{ -c_a \cdot (\alpha' - \alpha_0) + \beta \mathbb{E}[V(z', \alpha') | z] \right\} \right\} dG(c_f)$$

- Diffusion of automation through imitation $\alpha_0 = \bar{\alpha}$
- Exogenous process for productivity $\log z' = \rho_z \log z + \epsilon'$, with $\epsilon' \sim N(\mu_z, \sigma_z)$

Model: Equilibrium

- Final goods and the supply of capital and labor
 - final good combines output of all firms in CES aggregator with $\sigma > 1$
 - capital produced from final good at constant rate q
 - labor fixed at L and mobile across firms
- We start economy in steady state and consider perfect foresight transitions
- Transitional dynamics following uniform increase in q (investment-specific technical change)

Equilibrium: Optimal Automation

- Let α_t^* denote the α that minimizes production costs:

$$\frac{W_t}{\psi_\ell(\alpha_t^*)} = \frac{R}{q_t \cdot \psi_k(\alpha_t^*)}$$

- Optimal automation choice is an increasing function of z : $\alpha_{t+1} = \max\{\alpha_t, \hat{\alpha}_t(z_t)\}$.
 - For low enough z , no automation: $\hat{\alpha}_t(z_t) = 0$.
 - In the limit, firms that grow enough automate fully: $\lim_{z_t \rightarrow \infty} \hat{\alpha}_t(z_t) = \alpha_{t+1}^*$.
- Automation episodes as $\hat{\alpha}_t(z_t)$ crosses current α_t for growing and large firms.

Equilibrium: Two Key Elasticities

Labor share of cost is given by:

$$\varepsilon_\ell(\alpha) = \frac{\Psi_\ell(\alpha) W^{1-\eta}}{\Psi_k(\alpha) \left(\frac{R}{q}\right)^{1-\eta} + \Psi_\ell(\alpha) W^{1-\eta}}$$

- **Short-run $K - L$ elasticity** (fixed α) equals the task substitution elasticity η .
 \Rightarrow maps to firm-level short-run $K - L$ elasticity

- **Long-run $K - L$ elasticity** (adjusting α to α^*) equals

$$\eta_{LR} = \eta + \frac{\partial \ln \Psi_k(\alpha) / \Psi_\ell(\alpha)}{\partial \ln \alpha} \bigg/ \frac{\partial \ln \psi_\ell(\alpha) / \psi_k(\alpha)}{\partial \ln \alpha} > \eta$$

- As firms grow and $\hat{\alpha}_{t+1}(z) \rightarrow \alpha_{t+1}^*$, elasticity goes from η to η_{LR}

Model: Calibration

- Standard firm dynamics parametrization to match entry, exit, firm size, markup details
- Set technology parameters to match key facts on manufacturing labor share:
 - task-substitution elasticity $\eta = 0.4$: match short-run K-L elasticity (Oberfield-Raval 2020).
 - specify $\psi_\ell(x) = \left(x^{\frac{1-\eta-\gamma}{\gamma}} - 1\right)^{\frac{1}{1-\eta-\gamma}}$ and $\psi_k(x) = 1$. $\eta + \gamma$ gives long-run aggregate $K - L$ elasticity: target 1.35 (Hubmer, 2020; Karabarbounis & Neiman, 2014).
 - $d \ln q = 1.39$ to match observed decline in aggregate mf. labor share 1982–2012 (−17.8pp)
 - choose automation cost c_a to target change in median labor share (+3pp).

automation cost

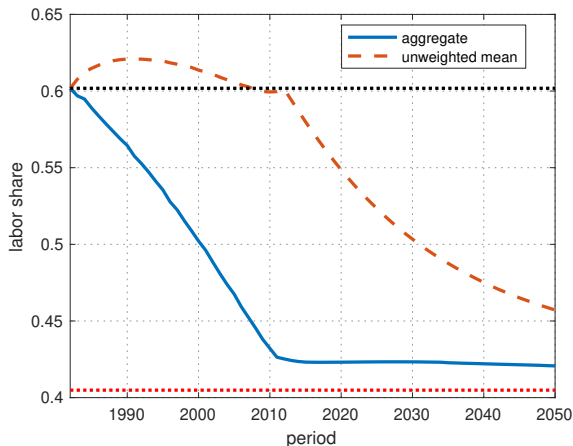
Model: Aggregate Labor Share Dynamics

- Economy starts in steady state in 1982
→ all firms have same labor share.
- In terms of **aggregate factor shares**,
fast transition.
- In terms of **average firm**, **slow transition**.

shock

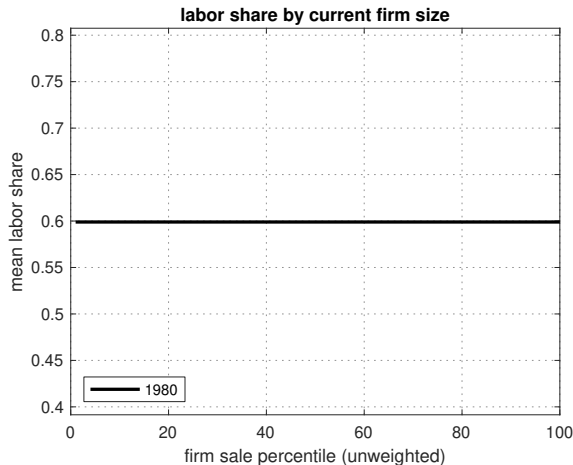
other aggregates

example firm history



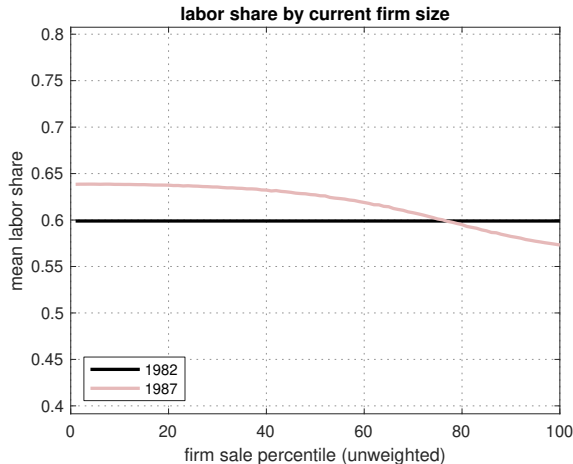
Model: Firm-level Labor Share Dynamics

- In 1982 **st. state**, **uniform technology**.
 - Note: Labor share is proportional to elasticity $\varepsilon_{\ell}(\alpha)$.



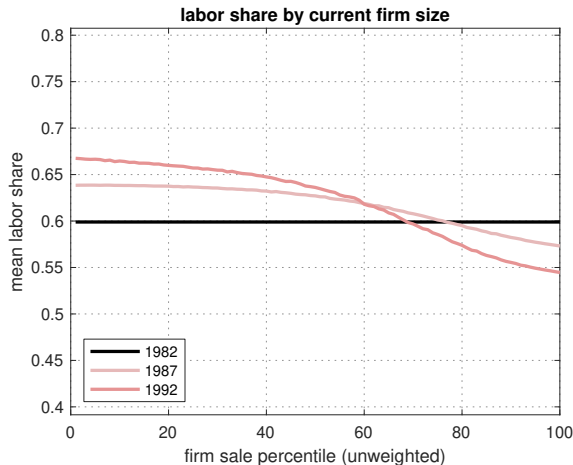
Model: Firm-level Labor Share Dynamics

- In 1982 **st. state**, **uniform technology**.
 - Note: Labor share is proportional to elasticity $\varepsilon_\ell(\alpha)$.
- As capital becomes cheaper and wages rise, **labor shares of small non-automating firms increase**.



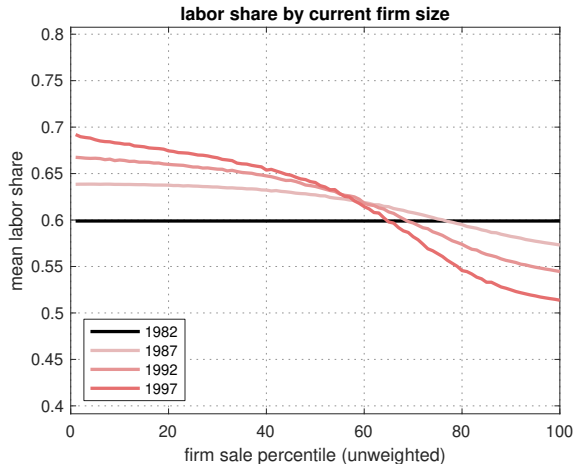
Model: Firm-level Labor Share Dynamics

- In 1982 **st. state**, **uniform technology**.
 - Note: Labor share is proportional to elasticity $\varepsilon_\ell(\alpha)$.
- As capital becomes cheaper and wages rise, **labor shares of small non-automating firms increase**.
- **Firms that become large** reaching top sales percentiles are the ones **reducing their labor shares**.



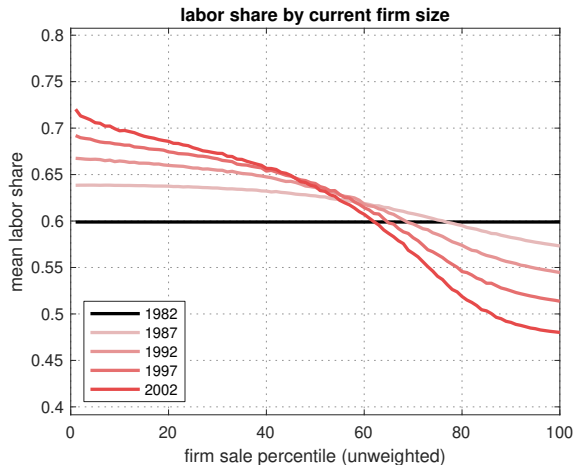
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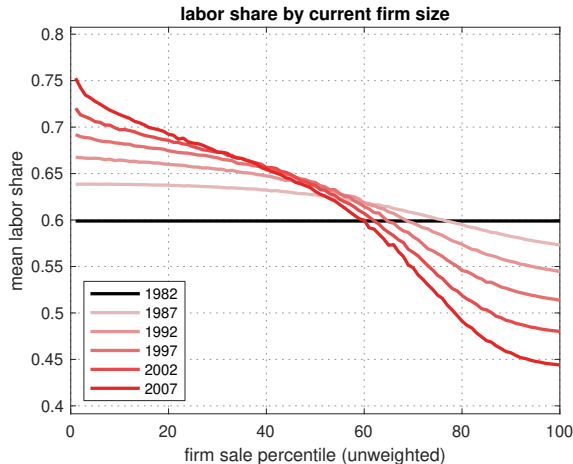
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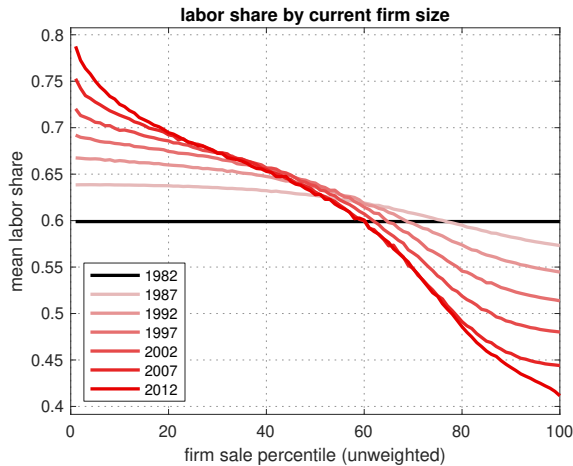
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- **Firms that become large** reaching top sales percentiles are the ones **reducing their labor shares**.



Decomposition of Labor Share in Manufacturing 1982–2012

- The labor share in an industry is

$$\lambda := \sum_i s_i^y \times \lambda_i$$

λ_i = labor share firm i ,

s_i^y = share firm i in value added

- Melitz–Polanec decomposition in Autor et al. (2020)

$$\begin{aligned} \Delta \lambda &= \Delta \bar{\lambda} && (\text{unw. mean}) \\ &+ \Delta \text{cov}(\lambda_i, s_i^y) && (\text{covariance}) \\ &+ \text{entry} + \text{exit} \end{aligned}$$

	Data	CES Model
Δ Unweighted survivors' mean	-0.2	3.0
Δ Covariance	-18.7	-20.9
Entry	5.9	0.6
Exit	-5.5	-0.3
Δ Aggregate	-18.5	-17.6

Data: Autor et al (2020), manufacturing, compensation share of value added. In p.p.

Unpacking the Changing Covariance Between Size and Labor Share

- Covariance can be further decomposed

$$\begin{aligned}\Delta \text{cov}(\lambda_i, s_i^y) \\ &= \text{cov}(\lambda_i, \Delta s_i^y) \quad (\text{market share dynamics}) \\ &+ \text{cov}(\Delta \lambda_i, s_i^y) \quad (\text{labor share dynamics}) \\ &+ \text{cov}(\Delta \lambda_i, \Delta s_i^y) \quad (\text{cross dynamics}).\end{aligned}$$

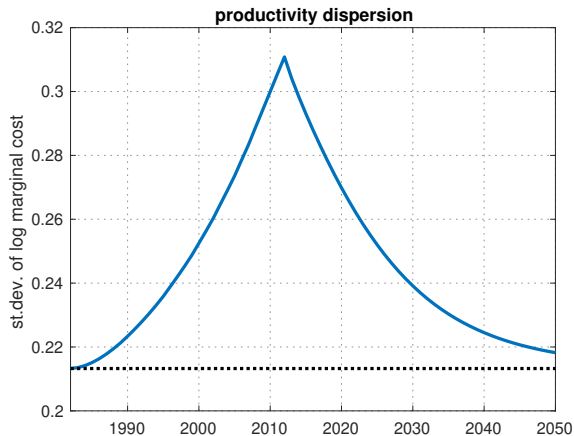
- Uncertainty about exact contribution of these terms in data...
- Kehrig and Vincent (2020): cross dynamics important in manufacturing

	Data (in p.p.)	CES Model (in p.p.)
Market share dynamics	4.7	0.0
Labor share dynamics*	-4.3	-4.1
Cross dynamics	-23.1	-13.7

Note: Kehrig and Vincent (2020) data from balanced panel of manufacturing establishments. (*) The labor share dynamics term includes the unweighted mean for survivors.

Productivity dispersion

- endogenous automation choice → **endogenous (temporary) increase in productivity dispersion**
- broadly **in line with data**: Decker, Haltiwanger, Jarmin, Miranda (2020) find 5 log points increase 1980s to 2000s (TFP, U.S. manufacturing)

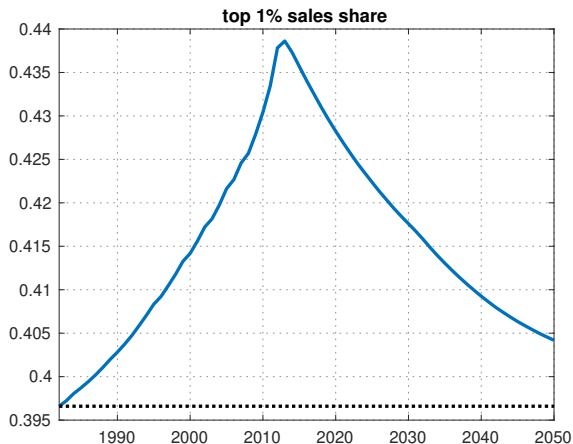


Sales concentration

- endogenous (temporary) increase in productivity dispersion → **endogenous (temporary) increase in sales concentration**
- broadly **in line with data:**

	Data	CES Model
Δ CR4	0.060	0.038
Δ CR20	0.052	0.063

Autor et al (2020) data, manufacturing, 1982–2012. In p.p.



Summarizing Findings from CES Model

Simple firm dynamics model with endogenous automation:

1. matches aggregate and firm labor share dynamics
2. generates increasing concentration of sales and dispersion of productivity

Now: incorporate differences in markups to separate role of reallocation to large firms due to tighter competition from $K - L$ substitution.

Outline

1. Model with CES demand to show implications of capital-labor substitution across firms
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Size-dependent Markups and Rising Competition

- Non-CES demand system: Kimball aggregator $H(\cdot)$ implicitly defines aggregate output Y ; λ is (exogenous) proxy for “market size”

$$\int_{\theta} \lambda \cdot H\left(\frac{y(\theta)}{\lambda \cdot Y}\right) m(\theta) d\theta = 1, \quad \theta = (z, \alpha)$$

- Normalizing price of final good to 1 yields demand function

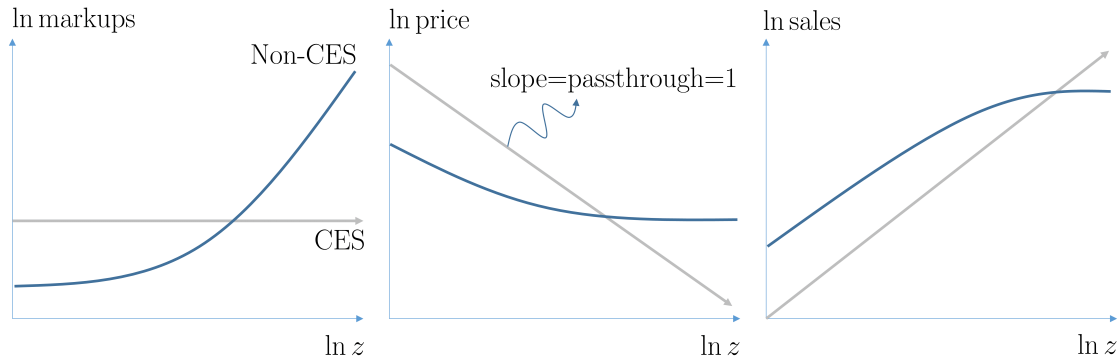
$$y(\theta) = Y \cdot \lambda \cdot D\left(\frac{p(\theta)}{\rho}\right), \quad \rho = \text{comp. price index} \neq 1, \quad (H' = D^{-1})$$

- Key assumptions:** Marshall's second laws:

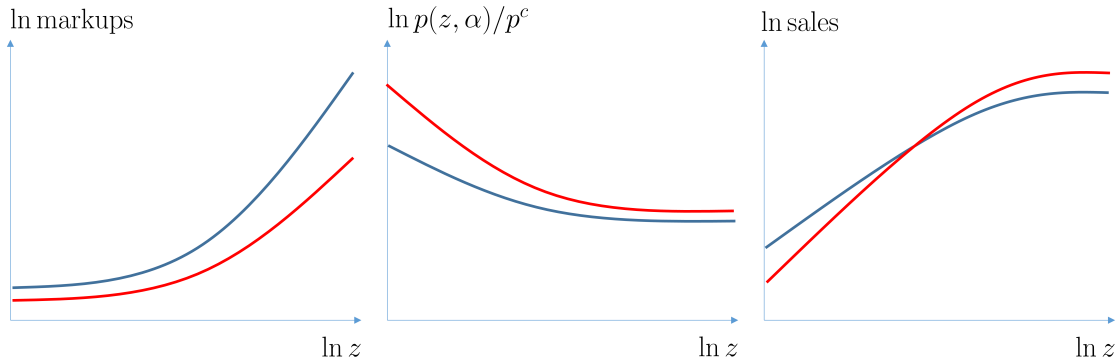
$$-\frac{D'(x)}{D(x)}x \text{ greater than 1 and increasing in } x; \quad x + \frac{D(x)}{D'(x)} \text{ positive and log-concave}$$

(markups higher for prod firms) (passthroughs lower for prod firms)

Steady State Markups, Pricing and Sales



Steady State Effects of Rising Competition, $\lambda \uparrow$



- $\lambda \uparrow$ increases market size, pushing firms towards elastic section of their demand curve
- reallocates economic activity towards top firms; but also lowers markups for all firms
- **net increase in the aggregate markup if z has a log-convex distribution**

Calibrating the Non-CES Demand system: Manufacturing 1982–2012

- $H(\cdot)$: Klenow-Willis aggregator \Rightarrow demand elasticity $:= \sigma \cdot \text{rel. quantity}^{-\frac{\nu}{\sigma}}$
 - $\sigma = 6.1$: matches aggregate markup of 15%
 - $\nu/\sigma = 0.22$: matches difference between median and aggregate labor share in 1982
- Productivity process given by

$$z = \exp \left(F_{\text{Weibull}(\zeta, k)}^{-1}(\Phi(\tilde{z})) \right), \quad \tilde{z}' = \rho_z \tilde{z} + \epsilon'_z$$

which ensures that

$$\ln z \sim \text{Weibull}(\zeta, n) \Rightarrow \mathbb{P}(\ln z \geq x) = \exp \left(- \left(\frac{x}{\zeta} \right)^n \right)$$

- $n = 0.78, \zeta = 0.086$ to match top sales shares (CR4 and CR20)
- more log convex than Pareto ($n = 1$); but not too much! [details](#)

Transitional Dynamics: Manufacturing 1982–2012

Automation and rise in comp. calibrated to match

- decline in labor share
- observed rise in concentration

Inferred shocks

- $d \ln q = 1.40$
- $d \ln \lambda = 0.04$

		MODEL			
		DATA	FULL	q -SHOCK	λ -SHOCK
	Δ aggregate LS ^a	−17.8	−17.3	−16.3	−0.2
	Δ median LS ^a	3.0	2.6	1.4	0.2
	Δ CR4 ^b	6.0	6.0	3.8	1.9
	$\Delta \ln$ agg. markup	.	1.1	1.0	0.1

In percentage points. [a] Kehrig–Vincent (2020). [b] Autor et al (2020): Average manuf. industry sales concentration.

Calibrating the Non-CES Demand system: Retail 1982–2012

- $H(x)$: Klenow-Willis aggregator \Rightarrow demand elasticity $:= \sigma \cdot \text{rel. quantity}^{-\frac{\nu}{\sigma}}$
 - $\sigma = 6.1 \rightarrow \sigma = 9.0$
 - $\nu/\sigma = 0.22 \rightarrow \nu/\sigma = 0.20$
- Productivity process given by

$$z = \exp \left(F_{\text{Weibull}(\zeta, n)}^{-1}(\Phi(\tilde{z})) \right), \quad \tilde{z}' = \rho_z \tilde{z} + \epsilon'_z$$

which ensures that

$$\ln z \sim \text{Weibull}(\zeta, n) \Rightarrow \mathbb{P}(\ln z \geq x) = \exp \left(- \left(\frac{x}{\zeta} \right)^n \right)$$

- $n = 0.78, \zeta = 0.086 \rightarrow n = 0.54, \zeta = 0.023$
- more log convex than in manufacturing [details](#)

Transitional Dynamics: Retail 1982–2012

Automation and rise in comp. calibrated to match

- decline in labor share
- observed rise in concentration

Inferred shocks

- $d \ln q = 0.75$ (1.40 in manuf)
- $d \ln \lambda = 0.41$ (0.04 in manuf)

		MODEL			
		DATA	FULL	q -SHOCK	λ -SHOCK
	Δ aggregate LS ^a	−10.2	−10.3	−4.2	−2.0
	Δ uw. mean LS ^b	4.4	4.2	−2.3	1.4
	Δ CR4 ^b	14.0	11.5	0.1	10.8
	Δ CR20 ^b	16.3	20.4	0.3	18.8
	$\Delta \ln$ agg. markup	.	3.3	0.1	2.9

In percentage points. [a] BEA Multifactor Productivity Tables. [b] Autor et al (2020).

Model-based decomposition of sectoral labor share changes

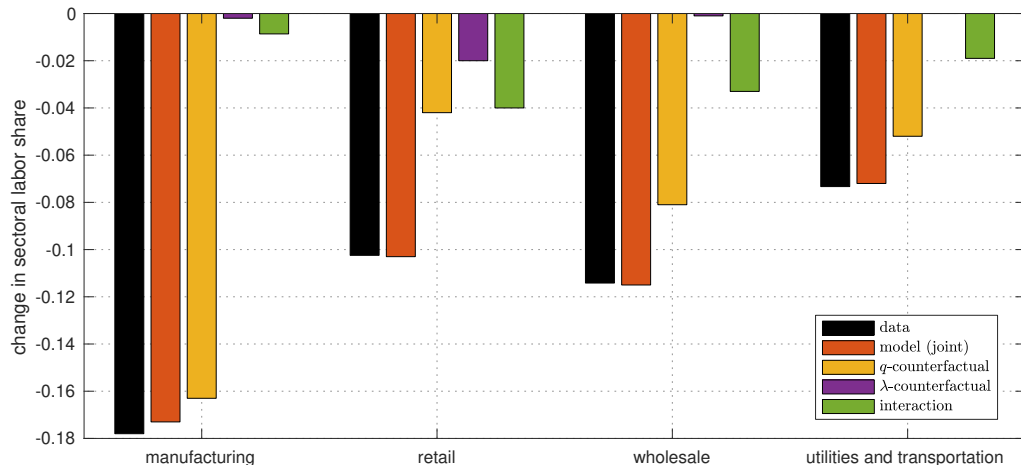


Figure: Data: Kehrig & Vincent for manufacturing, BLS MFP Tables for all other sectors.

Outline

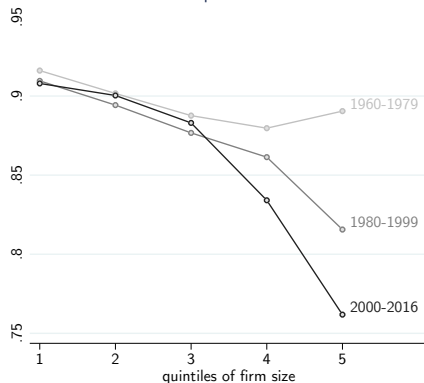
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Model-free Bounds on Markup Changes and Reallocation

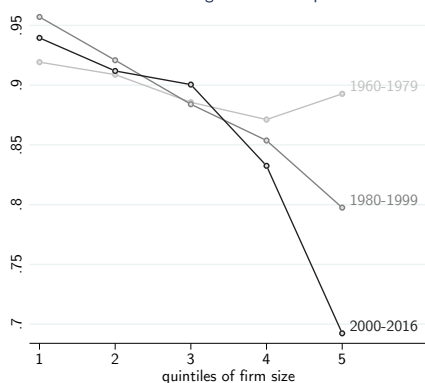
- So far, only accounted for **size-dependent component of markups**.
 - In the data, much more markup variation.
- Now allow for more general markup variation and reallocation (Baqaee–Farhi, 2020).
 - Greater generality comes at cost of requiring **firm-level markup estimates**.
 - Strong assumptions are inevitable (Compustat; prices). [details](#)
- We follow the literature (DeLoecker–Eeckhout–Unger, 2020) in using the **production function approach** to recover output elasticities $\varepsilon_{v,f,t} \Rightarrow \text{markup } \mu_{f,t} = \frac{\varepsilon_{v,f,t}}{s_{v,f,t}}$.
 - We allow **technology** ($\varepsilon_{v,f,t}$) **to vary by** time period, industry, **and firm size**.

Finding: Clockwise Rotation in Elasticities

Output elasticity wrt variable inputs,
estimated for firms in Compustat



Output elasticity wrt variable inputs,
estimated for manufacturing firms in Compustat

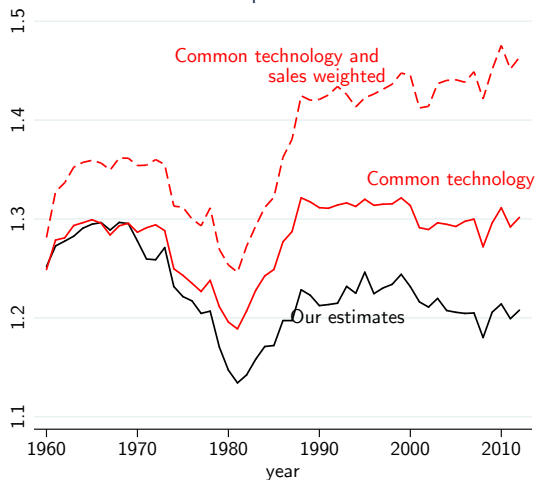


As in model, clockwise rotation in $\varepsilon_{v,f,t} \Rightarrow$ top firms switching to cap-intensive techs.

Finding: Aggregate Markup

- Common technology, sales weighting (DLEU headline): replicate strong increase in aggregate markup
- Common technology, cost-weighting (Edmond, Midrigan, Xu, 2018; BF): mild increase
- Our estimate with heterogenous technology and cost-weighting: no trend

Markups,
estimated for firms in Compustat



Contribution to Labor Share Decline

- We can write the labor share of an industry i as

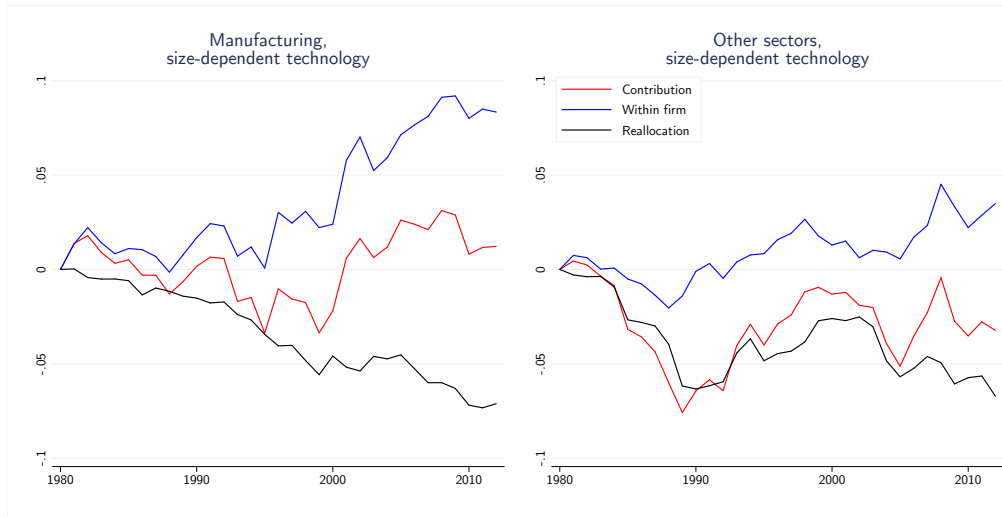
$$s_{\ell t} := \frac{\varepsilon_{\ell t}}{\mu_t},$$

- Holding fixed common technology $\varepsilon_{\ell,t}$ (model and data pre-1980), compute **counterfactual labor share that reflects only changes in markups**:

$$d \ln s_{\ell t}^{cf} := -d \ln \mu_t \quad \text{where} \quad \frac{1}{\mu_t} := \sum_f \omega_{f,t} \frac{1}{\mu_{f,t}}.$$

- We further want to distinguish changes in this counterfactual labor share due to **within firm** and **reallocation component** of markups.
 - Expect that at firm level, markups fall \rightarrow **positive within component**.
 - ... and reallocation to high-markup firms \rightarrow **negative reallocation component**.

Finding: Contribution to Labor Share Decline



Reallocation explains one fifth to one half of LS decline (within component weakens effect).

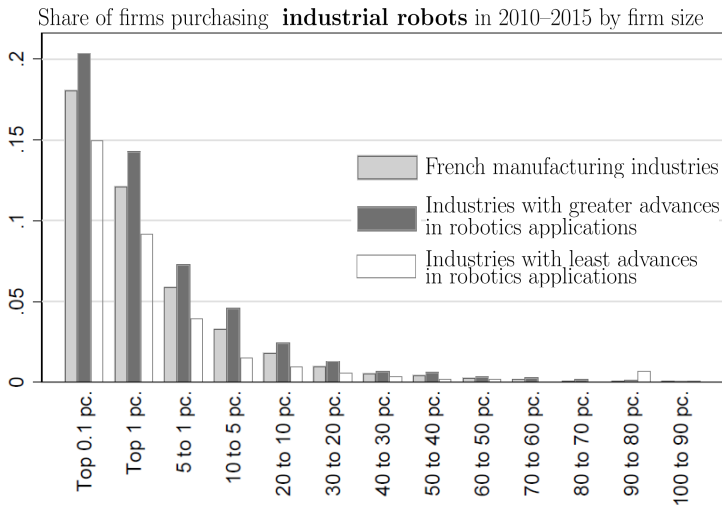
Concluding Remarks

- Model of **K–L substitution with a fixed cost per task matches firm-level facts** on the decline of labor share
- When **accounting for rising competition** and reallocation to more productive firms, **K–L substitution retains an important role**
 - explains 90% of the decline in labor share in manufacturing
 - explains 50% of the decline in labor share in retail
- Model-free bounds on role of competition and concentration support these findings
 - highlights importance of allowing for **differences in technology across firms**

Appendix Slides

Skewed Adoption of Capital Intensive Technologies

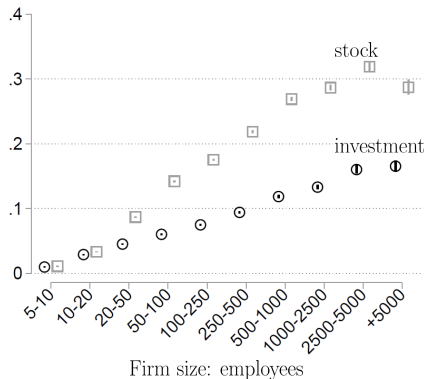
Acemoglu–Lelarge–Restrepo 2020: Industrial robots in France [return](#)



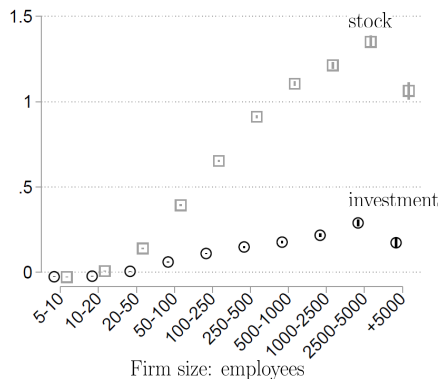
Skewed Adoption of Capital Intensive Technologies

Lashkari–Bauer–Boussard 2019: IT in France [return](#)

Investment and stock of **software** per worker
(in thousand euros per worker)



Investment and stock of **IT-hardware** per worker
(in thousand euros per worker)



Skewed Adoption of Capital Intensive Technologies

US Census 2020: Advanced Business Techs in the US [return](#)

Adoption of **advanced business technologies*** by US firms in 2017
share of firms reporting use by size and age bins

Age \ Size	1 to 9 Employees	10 to 49 Employees	50 to 249 Employees	250 or more Employees
0 to 5 Years	0.08	0.14	0.15	0.16
6 to 10 Years	0.07	0.13	0.16	0.16
11 to 20 Years	0.06	0.13	0.16	0.19
21 or more Years	0.06	0.11	0.19	0.25

Within manufacturing

Age \ Size	1 to 9	10 to 49	50 to 249	250 or more
0 to 5	0.13	0.20	0.31	0.28
6 to 10	0.10	0.18	0.30	0.35
11 to 20	0.10	0.20	0.31	0.43
21 or more	0.08	0.19	0.31	0.37

* includes: augmented reality, automated vehicles, machine learning, machine vision, RFID, robotics, touchscreens, voice recognition.

Model: Incumbents

Incumbents begin period t with productivity z and automation level α .

1. Collect static profits (assume simple CES demand w/ elast. $\sigma > 1 \Rightarrow$ constant markup):

$$\pi(z, \alpha) = \sigma^{-\sigma} (\sigma - 1)^{\sigma-1} \cdot Y \cdot c(z, \alpha)^{1-\sigma}.$$

2. Draw stochastic fixed operating cost c_f and decide whether to continue.
3. If continue, **decide whether and how much ($\alpha' \geq \alpha$) to automate at cost $c_a \cdot (\alpha' - \alpha)$.**
4. Draw next period's productivity level:

$$\log(z') = \rho_z \log(z) + \sigma_z \epsilon'$$

Model: Entrants

Every period t , mass M_e of potential entrants decides whether to enter:

1. A potential entrants receives a productivity signal z_e from some distribution.
2. Potential entrants start with $\alpha = \alpha_0$, which we set to equal the mean α of incumbents.
3. Decide whether to enter at stochastic fixed entry cost c_f .
4. If enter, decide whether and how much ($\alpha' > \alpha_0$) to automate at cost $c_a \cdot (\alpha' - \alpha_0)$.
5. Draw next period's productivity level (production starts in $t + 1$):

$$\log(z') = \rho_z \log(z_e) + \sigma_z \epsilon'$$

Table: Calibration of the CES-demand model for manufacturing

	PARAMETER		MOMENT	DATA	MODEL
<i>I. Parameters governing steady state in 1982</i>					
$\ln q_0$	Inverse capital price	-5.35	Manufacturing labor share from Kehrig and Vincent (2020)	60.1%	60.2%
σ	Demand elasticity	11.0	Aggregate markup from Barkai (2020)	1.10	1.10
σ_z	Std. dev. of $\ln z$ innovations	0.095	Top 4 firms' sales share in 1982 from Autor et al. (2020)	40.0%	40.0%
<i>II. Parameters governing firm dynamics</i>					
\underline{c}_f	Minimum fixed cost	$6.0 \cdot 10^{-6}$	Entry (=exit) rate from Lee and Mukoyama (2015)	0.062	0.063
ξ	Dispersion fixed cost	0.330	Size of exiters from Lee and Mukoyama (2015)	0.490	0.485
μ_e	Entrant productivity	0.935	Size of entrants from Lee and Mukoyama (2015)	0.600	0.598
<i>III. Parameters related to the elasticity of substitution</i>					
η	Task substitution elasticity	0.4	Short-run K-L elasticity from Oberfield and Raval (2014)	0.40	0.40
γ	Comparative advantage	0.95	Long-run K-L elasticity from Hubmer (2020)	1.35	1.35

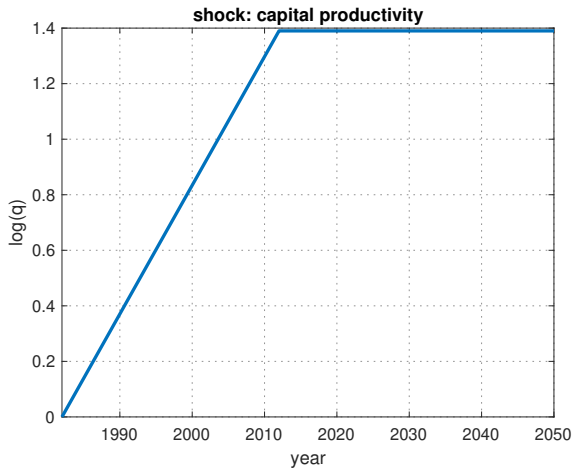
The Automation Cost c_a

On average, in the relevant range, how much does it cost to increase α by an amount that decreases the labor share by one percentage point?

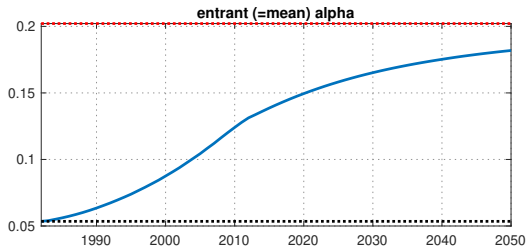
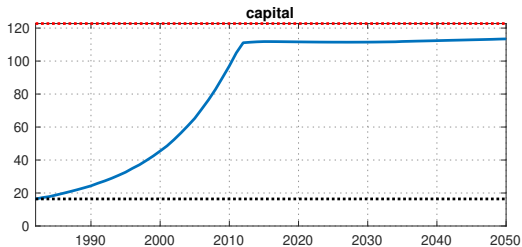
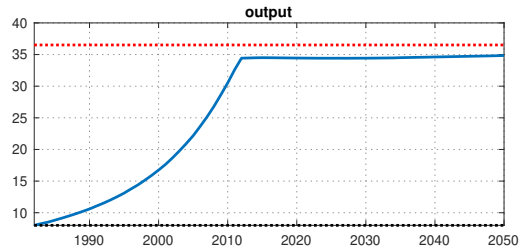
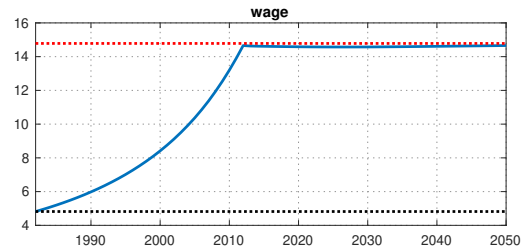
⇒ The equivalent of just 0.6%–1.2% of average annual firm sales (across all firms, not just automating ones)

return

Model: Time series of shock (investment-specific technical change q)



Model: Aggregate Variables



Model: History of One Particular Firm

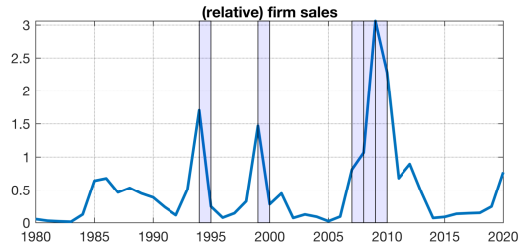
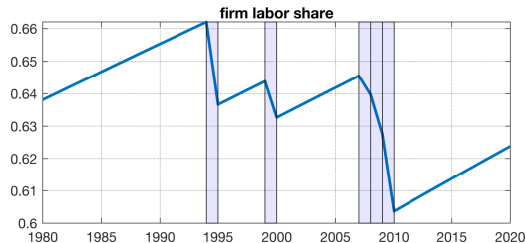
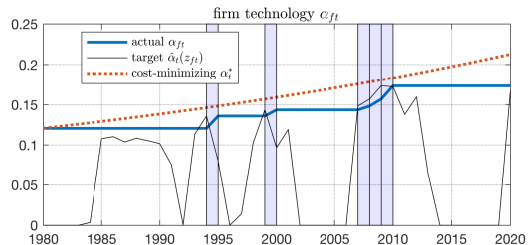


Table: Steady state calibration of the non-CES demand model: Manufacturing

	PARAMETER		MOMENT	DATA	MODEL
<i>I. Parameters governing steady state in 1982</i>					
$\ln q_0$	Inverse capital price	-6.55	Aggregate labor share	60.1%	60.2%
σ	Demand elasticity	6.10	Aggregate markup	1.150	1.150
ν/σ	Demand supra-elasticity	0.22	Median labor share ratio	1.169	1.101
ζ	Weibull scale	0.086	Top 20 firms' sales share	69.7%	69.7%
n	Weibull shape	0.78	Top 4 firms' sales share	40.0%	40.0%
<i>II. Parameters governing firm dynamics</i>					
\underline{c}_f	Minimum fixed cost	$4.6 \cdot 10^{-6}$	Entry (=exit) rate	0.062	0.063
ξ	Dispersion fixed cost	0.310	Size of exiters	0.490	0.488
μ_e	Entrant productivity	0.876	Size of entrants	0.600	0.601

Notes: Aggregate and median LS correspond time averages in manufacturing sector 1967–1982 (Kehrig and Vincent, 2020); median displayed as ratio over aggregate. Aggregate markup from Barkai (2020). Concentration measures are from Autor et al. (2020): manufacturing sector in 1982. Model equivalents refer to top 1.1% and top 5.5% of firms ranked by sales (on average 364 firms per 4-digit manufacturing industry). Data moments in Panel II follow the model with CES demand. All eight parameters jointly calibrated to match the eight corresponding moments.

Table: Steady state calibration of the non-CES demand model: Retail

	PARAMETER		MOMENT	DATA	MODEL
<i>I. Parameters governing steady state in 1982</i>					
$\ln q_0$	Inverse capital price	-7.35	Aggregate labor share	70.4%	70.5%
σ	Demand elasticity	9.0	Aggregate markup	1.150	1.150
ν/σ	Demand supra-elasticity	0.20	Median labor share ratio	1.169	1.106
ζ	Weibull scale	0.023	Top 20 firms' sales share	29.9%	29.9%
n	Weibull shape	0.54	Top 4 firms' sales share	15.1%	15.1%
<i>II. Parameters governing firm dynamics</i>					
c_f	Minimum fixed cost	$5.2 \cdot 10^{-7}$	Entry (=exit) rate	0.062	0.062
ξ	Dispersion fixed cost	0.250	Size of exiters	0.490	0.488
μ_e	Entrant productivity	0.868	Size of entrants	0.600	0.599

Notes: Aggregate LS corresponds to the BLS MFP estimate for the retail sector. The ratio median-to-aggregate is from Kehrig and Vincent (2020); refers to manufacturing, since in retail the data does not allow to compute the labor share of value added. Aggregate markup from Barkai (2020). Two concentration measures are from Autor et al. (2020), correspond to retail in 1982. Model equivalents refer to top 0.023% and 0.116% of firms ranked by sales (on average 17,259 firms per 4-digit retail industry). All eight parameters jointly calibrated to match eight corresponding moments.

Markup estimation: Assumptions return

A1 differences in the price of variable inputs reflect quality,

A2 revenue is given by a revenue production function of the form

$$\ln y_{ft} = z_{ft} + \varepsilon_{vc(f)t}^R \cdot \ln v_{ft} + \varepsilon_{kc(f)t}^R \cdot \ln k_{ft} + \epsilon_{ft},$$

where $c(f)$ denotes groups of firms with a common technology and same process for their revenue productivity, and ϵ_{ft} is an i.i.d ex-post shock orthogonal to k_{ft} and v_{ft}

A3 unobserved productivity z_{ft} evolves according to a Markov process of the form

$$z_{ft}, t = g(z_{ft-1}) + \zeta_{ft},$$

where ζ_{ft} is orthogonal to k_{ft} and v_{ft-1} , and

A4 the gross output PF exhibits CRTS in capital and variable input \Rightarrow quantity elasticities

$$\varepsilon_{vft} = \varepsilon_{vc(f)t}^R / \left(\varepsilon_{vc(f)t}^R + \varepsilon_{kc(f)t}^R \right)$$

Markup estimation: Method

1. First-stage regression to purge measurement error:

$$\ln \tilde{y}_{ft} = \mathbb{E}[\ln y_{ft} | \ln x_{ft}, \ln k_{ft}, \ln v_{ft}, t, c(f)] = h(\ln x_{ft}, \ln k_{ft}, \ln v_{ft}; \theta_{c(f)t}).$$

2. Second stage: Given any pair of revenue elasticities $\varepsilon_{vc(f)t}^R$ and $\varepsilon_{kc(f)t}^R$ compute

$$\tilde{z}_{ft} = \ln \tilde{y}_{ft} - \varepsilon_{vc(f)t}^R \cdot \ln v_{ft} - \varepsilon_{kc(f)t}^R \cdot \ln k_{ft},$$

estimate the flexible model

$$\tilde{z}_{ft} = g(\tilde{z}_{ft-1}; \theta_{c(f)t}) + \tilde{\zeta}_{ft},$$

and form the following moment conditions that identify the revenue elasticities:

$$\mathbb{E} [\tilde{\zeta}_{ft} \otimes (\ln k_{ft}, \ln v_{ft-1})] = 0.$$