Bidding for Teams*

Benoit Julien†, John Kennes‡, and Moritz Ritter§

May 9, 2013

This is a preliminary draft. Suggestions are welcomed!

Abstract

This paper contributes to both the theory of team formation and the theory of competing auctions. Optimal team membership is determined by economic factors such as complementarities and returns to scale. Teams compete for new and existing members by advertising membership auctions with reserve prices. In equilibrium, an organization with complementarities between several types of team members and/or increasing returns to scale typically advertises a negative reserve price, which is equivalent to advertising an amenity (for example, free health club membership). However, in the case of an organization exhibiting constant, or decreasing returns to scale, the equilibrium membership selling mechanism is a simple auction without a reserve price.

1 Introduction

> "Everyday experience reveal that there exists some most preferred or "optimal" membership for almost any activity in which we engage, and that this membership varies in some relation to economic factors. European hotels have more communally shared bathrooms than their American counterparts. Middle and low income communities organize swimming-bathing facilities; high income communities are observed to enjoy privately owned swimming pools." James Buchanan (1965), An Economic Theory of Clubs.

In this paper we develop a theory of teams/clubs along the lines first envisioned by Buchanan (1965). The essential goal is to explain the distribution of memberships across clubs, teams, and other cooperative units using a simple economic model. The main idea is that teams can be formed either for the purposes of work or for the purposes of shared consumption. A team might be required to be of sufficient size as when a group of hunters attempt to corner a stag,

*The authors wish to thank Ian King, Ben Lester, Guido Menzio, Ken Burdett, Jim Albrecht, Espen Moen, Dale Mortensen and Robert Shimer for helpful comments.

†UNSW; Email: benoit.julien@unsw.edu.au

‡Department of Economics and Business, Aarhus University, Universitetsparken, Building 1322, DK-8000 Aarhus C, Denmark; Email: jkennes@econ.au.dk

§Temple University; Email: moritz@temple.edu

or be of sufficient specialization as when a group of musicians play in a symphony orchestra. More prosaic examples of a team are a group of employees working together inside a firm, or the members of a social networking site sharing stories and making joint plans of action.

The important friction that both limits and defines the creation of teams in our model is the problem of coordination. We study how teams emerge in competitive equilibrium when teams compete by auctions with reserve prices (McAfee 1993, Wolinsky 1988, Peters and Severinov 1997). This provides an extension of this theory to the problem of individual platforms hiring/selling multiple units. Here, we find that negative reserve prices are an essential tool concerning the creation of teams. This means that many activities such as the offering of heath care services by a firm serve an economic purpose of increasing coordination towards larger more productive organizations in the economy as a whole.

Our model also forges a link between standard games of coordination and anti-coordination. In small markets, the coordination frictions related to team formation are closely related to problems of coordination for which there typically exists multiple equilibria. However, following our analysis, such environments can typically be expressed as an anti-coordination games in larger markets. In this case, we can analyze these environments using analytical tools related to the study of directed search, which is now a mainstream method in the analysis of labor markets.

We present a very simple model of team formation in a large market with complementarities. We follow the analogy of Buchanan and consider a world of 'hotels' that attempt to attract males and females. We explain that this world will be subject to too many (low quality) hotels in equilibrium if prices are determined by local Bertrand competition. This environment can be made efficient if hotels pay a tax to limit the number of hotels and thus reduce problems of coordination. We also show that hotels can compete in quality and achieve a similar effect. This competing auction equilibrium is equivalent to one having auctions with negative reserve prices.

We then extend the problem of team formation to an environment where agents are homogenous but each platform has initial increasing returns to scale. We show that the social planner may wish to have some agents quit if there are too many and that each platform will have different queue lengths depending on existing membership. We also show that platforms will advertise negative reserve prices as a function of their existing membership where the magnitude of the negative reserve price falls as a function of the number of incumbent members on the platform. If the platform has constant or decreasing returns to scale, the platform advertises a simple auction without a reserve price.

The last section discusses some extensions of the analysis. We explain how allocations might also be directed by price posting commitments without a need for amenities to coordinate trade. We also consider the problem of creating larger islands as in Mortensen (2008) and explain that the directed search equilibrium of that environment is the Walrasian outcome. Finally, we explain how we must extend the notion of our bidding procedures if we wish to extend the theory to more general problems involving asymmetric numbers of complementary inputs..

The paper is organized as follows. In section 2, we present the problem of island assignment and discuss how this environment can be characterized as either a game of coordination or anti-coordination. In section 3, we present the basic 'hotels' model. In section 4, we consider the problem of competing auctions when organizations have existing members and the optimal size of the organization is arbitrarily large. Section 5 then considers some other extension of our analysis. The final section offers concluding remarks.

2 Preliminaries

This section introduces two canonical matching problems that are relevant to the analysis of team formation in directed search equilibrium. The concept of a directed search equilibrium is widely understood to mean the following.

Definition. In a directed search equilibrium, the set of identical seller types advertise identical contracts, and the set of identical buyer types choose identical location strategies.

The key idea is that this equilibrium captures important elements of anonimity and coordination in competitve decentralized environments.

2.1 Anti-coordination game

The standard model of directed search can be cast as a problem of coordinating the matching of agents to locations . A simple example is when there are two spatially separated males (sellers) and two spatially separate females (buyers), where the females must simultaneously choose one of these male location in order to create a match. This can be characterized as an anti-coordination game where a female gets a payoff of zero if the other female chooses the same location (Competition with the other female bids away the rents of the match with the male), and a payoff of y_i if she chooses male $i \in \{1, 2\}$ and the other female chooses the other male.[1]

The normal form of the game involving the location decision of the two females is given by

F_1/F_2	1	2
1	$0, 0$	y_1, y_2
2	y_2, y_1	$0, 0$

In the mixed strategy equilibrium of this assignment game, we find that probability π_1 that a female visits male 1 is given by

$$\pi_1 = \frac{y_1}{y_1 + y_2}$$

This assignment game has the 'nice' property that a more attractive male has a higher probability of trade than the less attractive male. In this sense, the directed assignment equilibrium is better than purely random matching. This assignment is also considered to be the reasonable outcome of this game because it is the only equilibrium of the game where the identical females are assumed to adopt identical mixed strategies.

[1](Refer to Julien, Kennes and King 2000)

2.2 Coordination game

Another variation of the location game is to assume that the problem of coordination involves two islands, a man, and a woman. In this game, if both the man and the woman choose the same island, they can both enjoy $y/2$ units of output and if they choose different islands then they receive nothing. The question then is how do they choose islands if they are uncoordinated over which island to visit.

An obvious point is that this is a simple battle of the sexes game with two pure strategy equilibria and mixed strategy equilibrium. The normal form game is simply

M/F	1	2
1	$\frac{y}{2},\frac{y}{2}$	$0,0$
2	$0,0$	$\frac{y}{2},\frac{y}{2}$

In this case, the mixed strategy is the one where the men and women randomize over the two islands. To see how this game differs from the standard directed search game, consider the case where one island is better than another island so that the return on island 1 is greater than on island 2 - thus $y_1 > y_2$. This changes the normal form of the coordination game as follows.

M/F	1	2
1	$\frac{y_1}{2},\frac{y_1}{2}$	$0,0$
2	$0,0$	$\frac{y_2}{2},\frac{y_2}{2}$

Proposition 1. *There are two directed search equilibrium of the coordination game.*

The logic of the directed search equilibrium is to assume that agents choose to play the mixed strategy equilibrium. Therefore, the expected payoffs of visiting the two islands are equated. This means that probability that a male or female visits an island is given by

$$\hat{\pi}^1 = \frac{y_2}{y_1 + y_2}$$

In this version of a directed search equilibrium, a reduction in the quality of an island has the perverse effect of increasing the probability that men and women locate on this island. If agents follow this strategy, then we have the perverse effect that an increase in the quality of one island will lower the overall value of trade (Note that we can easily interpret high y_1 as a high wage offer to a couple.)

The mixed strategy equilibrium of the coordination game can be considered unrealistic, because there exists another equilibrium that improves the welfare of both buyers, and satisfies the usual assumption made concening directed search equilibrium. In particular, suppose that all agents play a strategy of choosing the best island. This means that

$$\tilde{\pi}^2 = \begin{cases} 0 & if \quad y_1 > y_2 \\ 1/2 & if \quad y_1 = y_2 \\ 1 & if \quad y_2 > y_1 \end{cases}$$

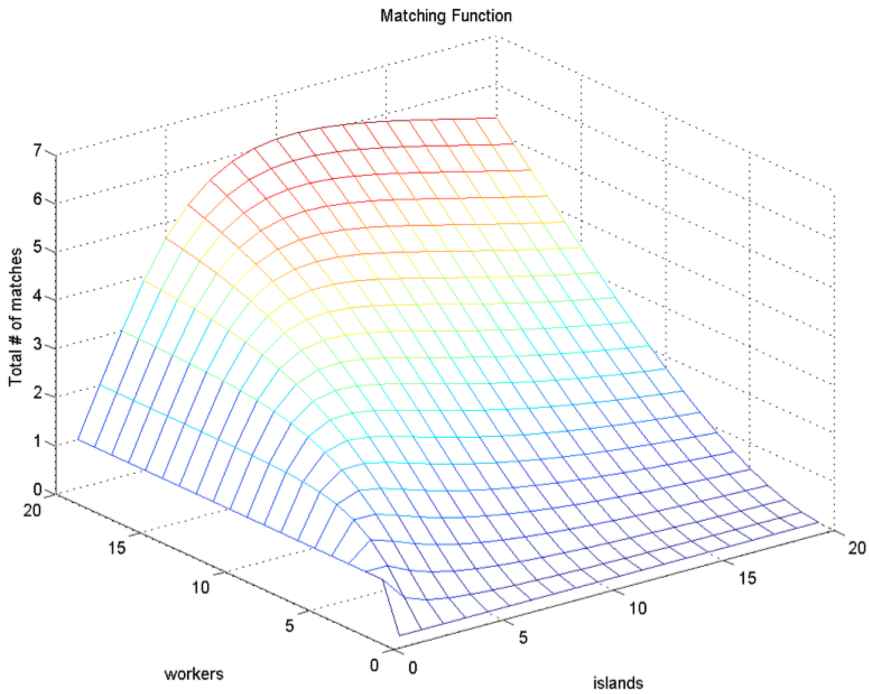
In this case, the equilibrium is to have both agents visit the high quality island. The idea suggested by this equilibrium is that the islands can pursue a higher quality (or lower prices) as a means to facilitate coordination. This includes any credible commitments/announcements by any two agents.

2.3 Matching technology

We can also relate the problem of coordination and team formation by mapping out the matching technology for an economy where a firm must hire at least two agents to produce and the agents are randomizing over island locations. A match occurs then if two or more agents choose the same location. The matching technology for this environment is

$$X(M,N) = N\left(1 - (1-1/N)^M - M(1-1/N)^{M-1}\right)$$

where N is the number of firms, $(1-1/N)^M$ is the probability no worker visits the firm, and $M(1-1/N)^{M-1}$ is the probability exactly one worker visits. The matching technology for this problem of critical mass teams formation is illustrated below.



The figure illustrates that number of matches first increases then decreases as the number of firms decreases. This example serves to illustrate that the problem of team coordination is different than partnership coordination. The fundamental issue concerns the value of critical mass in team formation, which may differ fundamentally from that in partnership formation economies. Nevertheless, the analysis in the subsequent sections will demonstrate that directed search equilibrium are generally found in the region where the matching function is well-behaved.

Therefore, such models are anti-coordination games even when teams require critical mass or complementary inputs

3 The basic "hotel" model

Assume that there is a fixed and equal number of men and women, which we can normalize to one, and that islands can enter this environment if they pay a cost c. Production requires exactly one man, one woman and one slot (i.e. room) on an island. If an island enters the market, it has a single room that it can sell. If n islands enter this economy, the average queue length of people of each sex at the islands is $q = 1/n$. If the men and women treat all islands equally and pursue identical mixed strategies, the aggregate matching function is given by

$$X(q) = n\left(1 - e^{-q}\right)^2 \tag{1}$$

where $q = 1/n$ is the expected queue length of men and women at each island, and the rate at which an island is filled is then given by $x(q) = (1 - e^{-q})^2$.

3.1 The social planning problem

The social planner's problem is to maximize the number of expected matches times their output subject to the costs of entry for islands. Therefore, the social planner chooses the number of islands $n = 1/q$ to maximize:

$$S_P^* = \max_{\{q\}} \frac{(1 - e^{-q})^2}{q} y - \frac{c}{q}. \tag{2}$$

where the first term to the right of the max operator is the number of matches times the output of a match, and the second term is the total cost of the $n = 1/q$ islands. This is a well defined maximization problem with a solution

$$yx\left(q\right)\eta_{X,q} = c \tag{3}$$

where

$$\eta_{X,q} = \frac{\partial X\left(q\right)q}{\partial q X\left(q\right)}$$

which states that the social planner equates the social cost of each island with the output of a match weighted by the hiring probability and the elasticity of the matching function.

3.2 Simple auctions

Suppose that islands earn the surplus of the island whenever there are more couples than slots. In this case, the couples extract all the surplus whenever there is one couple on the island. The free entry condition for islands in this equilibrium is given by

$$\pi_1^1 y = c \tag{4}$$

where $\pi_1^1 = (1 - e^{-q} - qe^{-q})^2$ is the probability of more couples than slots (i.e. multiple couples) - here the subscript and superscript refers to the valuation of the slot by the first and second highest valuation couple, respectively. The seller earns zero profits when there is no bidding couple, which occurs with probability $\pi_0^0 = (e^{-q})^2$ and when there is only one couple, which occurs with probability $\pi_1^0 = (1 - e^{-q} - qe^{-q})^2 - (e^{-q})^2$. If we use this condition to determine the number of matches, then there might be more islands than would maximize the number of matches.

3.3 The Mortensen tax

Consider the case where island entry costs are sufficiently high to keep the equilibrium to the left of the match maximizing number of islands. We can take equation () and rewrite it as follows.

$$\left[\eta_{X,q} x(q) + \left(qe^{-q}\right)^2\right] y = c \tag{5}$$

Therefore, comparing this to the social planning problem, we observe too many islands in the simple auction equilibrium. One solution to this problem is to tax islands by an amount

$$t = \left(qe^{-q}\right)^2 y,$$

which is the same rule proposed by Mortensen (2009). The Mortensen tax in the unit capacity island environment is equivalent to the probability that there is one couple at the island times the output of the island.

3.4 Competing auctions

In this section we assume that the islands may offer amenities to the agents who locate on their islands. The idea is that the island could offer an additional 'free service' that is of value to a couple if they decide to locate on the island. An example might be a firm that offers access to a recreational centre in addition to the tasks needed to complete the job. Under an auction, a team will collect this benefit only if there are no other competing teams on the island. The timing of this posted subsidy game is as follows: Stage 1. Simultaneous entry of islands; Stage 2. Simultaneous posting of the extra benefits d at each island; Stage 3. Men and women the posted benefits at each island and make simultaneous choices over which island to visit; Stage 4. Couples bid for the slots on the islands

Proposition 2. *There exists a unique directed search equilibrium to the competing auction game, which implements the social planners solution*

Proof. Suppose that all island offer amenities d. If there are $n = 1/q$ islands in the economy, the expected queue length at each is q. The payoff to an island offering amenities d and enjoying expected queue length q is given by

$$\pi(q,d) = \left(1 - e^{-q} - qe^{-q}\right)^2 y - d\left(2qe^{-q}\left(1 - e^{-q}\right) - \left(qe^{-q}\right)^2\right) \tag{6}$$

and the expected return to a female (or male) locating on this island is given by

$$F(q,d) = \left(1 - e^{-q} - qe^{-q}\right) e^{-q}(y+d) + qe^{-q}e^{-q}\left(\frac{y+d}{2}\right) \tag{7}$$

Now consider the return to an island that deviates and offers amenities d'. The expected returns on both islands must be the same.

$$F(q,d) = F(q',d') \tag{8}$$

Therefore, the dividend of the deviating firm and the associated queue are then related as follows

$$d' = d(q',q,d) = \frac{F(q,d)}{\left(1 - e^{-q'} - \frac{q'e^{-q'}}{2}\right) e^{-q'}} - y \tag{9}$$

We can use the implicit function theorem to verify that $\partial q'/\partial d' > 0$. We can also cast the deviating island's problem as a choice of q' to maximize

$$q^* = \arg\max_{q'} \pi\left(q', d'(q',q,d)\right) \tag{10}$$

The first order condition of this seller's problem is given by

$$\left(1 - e^{-q*}\right) e^{-q*} y - \left(1 - e^{-q} - \frac{qe^{-q}}{2}\right) e^{-q}\left(y + d(q^*,q,d)\right) = 0 \tag{11}$$

In equilibrium, the expected queue length of the deviating island is equal to the expected queue length of the non-deviating islands. Thus

$$q^* = q$$

Therefore, we can use the first order condition of the seller's maximization problem to solve for the equilibrium amenity, d^*. We find that

$$d^* = \frac{\left(\frac{q^* e^{-q^*}}{2}\right)}{\left(1 - e^{-q^*} - \frac{q^* e^{-q^*}}{2}\right)} y$$

This gives revenue to the competing auction of

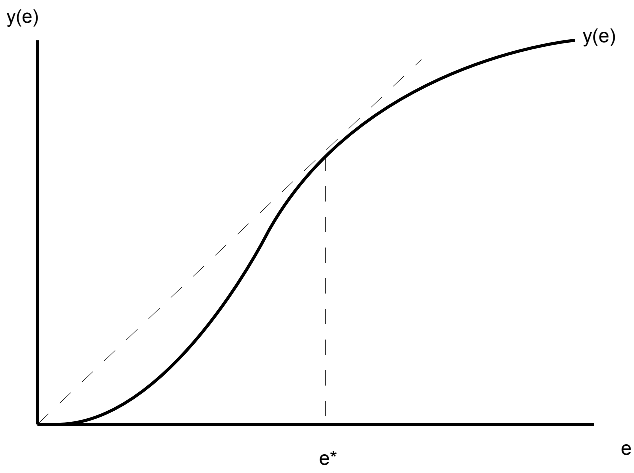
$$\pi(q^*,d^*) = yx(q^*)\left(1 - \eta_{x,q}\right)$$

which is equivalent to the returns required by the social planner. □

So the decentralized advertisement of amenities and the selling of island slots by an auction can function in the same way as the Mortensen tax.

4 Returns to scale

Consider the matching problem with homogenous workers and existing employees. There is a fixed number of n islands. We assume that some workers are initially distributed across islands and let $n(e)$ denote the number of islands with e employees at the start of the period. Any worker who enters this economy must be paid an expected wage of W. People also can quit a firm at the start of the period and also earn an expected wage of W. An island with e workers produce $y(e)$ units of output. We allow for the possibility that the production function has initially increasing returns to scale. Let e^* denote the maximum output per worker for the firm. The following figure describes the technology of each firm.



Matching is a three stage game. In the first stage, firms choose a selling mechanism (We start by restricting this to a simple auction without a reserve price). In the second stage, employed workers can choose to quit the firm and become an unemployed worker, the unemployed can choose to be a job searcher, and job searchers are assigned to an employer subject to coordination frictions. In the third stage, the workers bid competitively for jobs. The model is solved by backwards induction.

4.0.1 Bidding game

Consider how wages are determined as a function of the number of bidders at the island (firm).

Proposition 3. *The bidding function given e bidders at a firm is as follows*

$$w(e) = \begin{cases} \triangle_e & if \quad e \geq e^* \\ \frac{y(e)}{e} & if \quad e < e^* \end{cases} \tag{12}$$

where $\Delta_e = y(e) - y(e-1)$ *denotes the marginal productivity of the eth worker on an island.*

4.0.2 Expected payoffs of workers

Let $q(e)$ denote the queue length in a submarket for firms with e incumbent employees. Let $\hat{W}(e)$ be the expected utility of an incumbent employee at a firm with e incumbent workers. If the queue length for any particular firm type is $q(e)$, the expected payoff of an incumbent worker in this submarket is given by

$$\hat{W}(e) = \sum w(e+z) \, Pr(z \mid q(e)) \tag{13}$$

where

$$Pr(z \mid q(e)) = (q(e))^z \, e^{-q(e)}/z!$$

is the usual formula for number of arriving bidders as a function of queue length. Similarly, the expected payoff of a job searcher choosing such a firm is given by

$$W(e) = \sum w(e+z+1) \, Pr(z \mid q(e)) \tag{14}$$

Note that

$$W(e), \hat{W}(e) \leq y(e^*)/e^*$$

given that wages are determined by equation (1) and $y(e^*)/e^*$ is the maximum wage.

4.0.3 The queueing decision of a job searcher

Let W denote the common expected utility of a job searcher. We have

$$W = W(e)$$

for all employers that have positive queues, $q(e) > 0$. Note that we can substitute $W = W(e)$ into equation (3). This gives:

$$W = w(e+1)e^{-q} + w(e+2)qe^{-q} + w(e+3)\frac{q^2}{2}e^{-q} + w(e+4)\frac{q^3}{3!}e^{-q} + ...$$

Using this expression, we have a formula for the queue length as a function of W. Let

$$q(W,e) = \Gamma^{-1}(W,e) \tag{15}$$

where $W(e) \equiv \Gamma(q(e))$. Note that the function $w(e+z))$ may be first increasing and then decreasing in z depending on returns to scale. Therefore, in the case of IRS ($e < e*$), there are two different queue lengths consistent with W, because wages are initially increasing with respect to the number of job applicants. We are generally interested in the larger of these queue lengths, because in the case of the smaller queue lengths incumbent workers have smaller expected incomes than job searchers. This cannot be an equilibrium.

4.0.4 The quitting decision of incumbent workers

Equation (2) and (4) can be used to solve for $\hat{W}(e)$ as a function of W.

$$\hat{W}(e, W) = \sum w(e+z) \, Pr(z \mid q(W, e))$$

Given this common value of W, we have $\hat{W}(e) > \hat{W}(\underline{e}), \hat{W}(\bar{e})$ for $\underline{e} < e < \bar{e}$. A worker at a type e firm can choose to stay $\sigma(e) = 0$, or search $\sigma(e) = 1$. This decision is the solution to the following problem. Each worker maximizes

$$\sigma^*(e \mid W) = \arg\max_{\sigma(e)} \left\{ (1 - \sigma(e)) \hat{W}(e, W) + \sigma(e) W \right\} \tag{16}$$

If any worker chooses to remain at the firm, we must have $\hat{W}(e) \geq W$ since worker can always quit.

Assume that the quitting decisions are made sequentially. So that firms which are too big will shrink until they are of a sustainable size.

4.0.5 Social Planning problem

Can solve the social planning problem. Consider the value of a firm that has e employees. Suppose that we can bring in workers at some costs W. Suppose also that each of the current employees also have alternative uses of value also equal to W. What is (i) the optimal queue length at each firm type, and (ii) the size of the firm. Consider the following program.

$$Z(e) = \max_{q,x} \left\{ \sum y(e - x + z) \, Pr(z \mid q) - Wq + Wx, 0 \right\}$$

where x is the number of quits and q is the queue length. To solve the social planner problem, write

$$Z(e) = \max_{q,x} \left\{ \left[y(e-x) e^{-q} + y(e-x+1) q e^{-q} + y(e-x+2) \frac{q^2}{2!} e^{-q} + ... \right] - Wq + Wx, 0 \right\}$$

First order condition for the social planner $q^*(e)$ is given by

$$\sum_{i=0}^{\infty} \left[y(e-x+i+1) - y(e-x+i) \right] \frac{(q^*(e))^i}{i!} e^{-q^*(e)} = W$$

- If DRS everywhere, then social planning solution equivalent to decentralized equilibrium.
- If IRS, then social planner will generally prefer to have longer queues of workers than the decentralized equilibrium, because $\triangle_e > \frac{y(e)}{e}$
- If IRS, then social planner will generally prefer to keep active smaller (larger?) firm sizes than the decentralized equilibrium.

4.0.6 The Mortensen tax

Formula for Mortensen tax for a firm with e incumbent worker is given by

$$t(e)=\sum_{i=0}^{\infty}\left[y(e-x+i+1)-y(e-x+i)\right]\frac{(q^{*}(e))^{i}}{i!}e^{-(q^{*}(e))}-\sum w(e+z+1)\,Pr(z\mid q^{*})$$

$$t(e)=\sum_{i=0}^{e^{*}-e}\left[\triangle_{e}-\frac{y(e+i)}{e+i}\right]\frac{(q^{*}(e))^{i}}{i!}e^{-(q^{*}(e))}$$

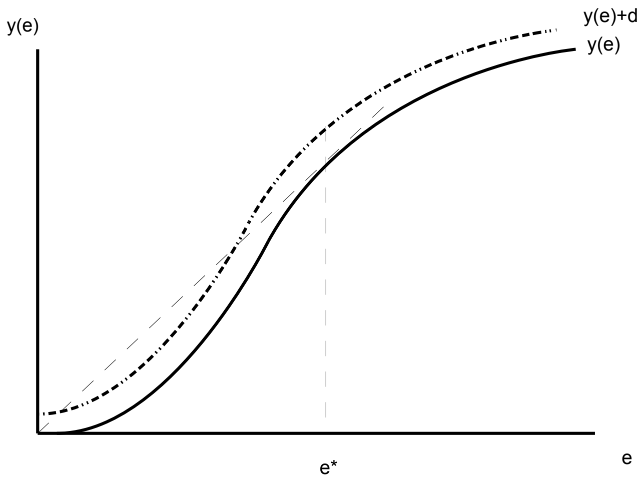
- If everywhere DRS, the social planner is equivalent to decentralized economy and so no Mortensen tax.
- The Mortensen tax is larger for firms with fewer incumbent workers.

4.0.7 Competing auctions

Consider the payoff of a firm that advertises a subsidy d. This changes the schedule of match productivities to be

$$y(e)+d$$

This gives a new schedule of output as a function of the number of workers at the firm.



So the payoff of a worker is given by

$$w(e,d)=\begin{cases}\triangle_{e}=w(e) & if \quad e\geq e^{*}\\ \frac{y(e)+d}{e} & if \quad e<e^{*}\end{cases} \tag{17}$$

where $\Delta_e = y(e) - y(e-1)$ denotes the marginal productivity of the eth worker on an island. Note that

$$\frac{\partial w(e,d)}{\partial d}=\begin{cases}0 & if \quad e\geq e^{*}\\ \frac{1}{e} & if \quad e<e^{*}\end{cases} \tag{18}$$

This formula tell us that a higher dividend will cause higher wages only if there is IRS. Otherwise, the workers will bid the entire gains of dividend back to the firm. How do queue lengths change as we increase wages by increasing dividends? The formula for queue length as a function of d must satisfy the free entry condition below.

$$W = W(e,d) = w(e+1,d)e^{-q(e,d)} + w(e+2,d)q\left(e,d\right)e^{-q(e,d)}$$

$$+w(e+3,d)\frac{q\left(e,d\right)^{2}}{2}e^{-q(e,d)} + w(e+4,d)\frac{q\left(e,d\right)^{3}}{3!}e^{-q(e,d)} + ...$$

What is $q'(d,e) \equiv \frac{\partial q'(d,e)}{\partial d}$? Totally differentiate equation () above to get an expression

$$0 = \frac{1}{e+1}e^{-q(e,d)} - w(e+1,d)q'\left(e,d\right)e^{-q(e,d)}$$

$$+\frac{1}{e+2}q\left(e,d\right)e^{-q(e,d)} - w(e+2,d)\left[q'\left(e,d\right)e^{-q(e,d)} + q\left(e,d\right)q'\left(e,d\right)e^{-q(e,d)}\right]$$

$$+\frac{1}{e+3}\frac{q\left(e,d\right)^{2}}{2}e^{-q(e,d)} - w(e+3,d)\left[q'\left(e,d\right)q\left(e,d\right)e^{-q(e,d)} + \frac{q\left(e,d\right)^{2}}{2}q'\left(e,d\right)e^{-q(e,d)}\right]$$

We can verify that this $q'\left(e,d\right)$ is well behaved when there is IRS. This means that higher d implies higher q. This follows from the fact that wages are increasing provided that there are IRS.

Proposition 4. *The social planning problem of the variable returns to scale model is implement by competing auctions*

Proof. The sellers problem is

$$S(e) = \max_{d}\left\{\sum\left(y\left(e+z\right)\right)Pr(z \mid q\left(d,e\right)) - Wq\left(d,e\right)\right\}$$

The first order condition of the seller's problem is the following.

$$\left[\sum_{i=0}^{\infty}\left[y\left(e+i+1\right) - y\left(e+i\right)\right]\frac{\left(q^{**}\left(e\right)\right)^{i}}{i!}e^{-q^{**}(e)}\right]q'(d,e) = Wq'(d,e)$$

Thus

$$\sum_{i=0}^{\infty}\left[y\left(e-x+i+1\right) - y\left(e-x+i\right)\right]\frac{\left(q^{*}\left(e\right)\right)^{i}}{i!}e^{-q^{*}(e)} = W$$

which is the same condition as the social planner. □

5 Other extensions

This section draws out some further implications of theory.

5.1 Posted prices

An alternative pricing mechanism to an auction is to assume that the island commits to a price for each slot. In this case, the male and female of a couple each gets a payoff of (1-p)/2 if they are assigned to a room and zero otherwise. The owner of the island gets a payoff of p if the room is sold and zero otherwise. The timing of the price posting game is as follows. Stage 1. Simultaneous entry of islands; Stage 2. Simultaneous posting of hotel prices p at each island; Stage 3. Simultaneous choice of islands by men and women; Stage 4. One couple is selected at random.

Proposition 5. *The posted price equilibrium of the hotel matching game implements the social planners solution without amenities*

Proof. We solve the pricing game of islands after they have paid the sunk cost of entry. If all islands advertise a price p and buyers randomize over these islands, the payoff to an island advertising price p and expecting a queue length of q is given by

$$\pi^P(p,q) = p\left[\left(1-e^{-q}\right)\left(1-e^{-q}\right)\right]$$

The expected payoff of a woman (or man) of choosing an island with posted price p and expected queue length q is given by

$$F^P(q,p) = \left(\frac{1-e^{-q}}{q}\right)\left(1-e^{-q}\right)\left(\frac{1-p}{2}\right)$$

Now consider the possibility that a seller deviates and advertises a price p' other than p. In the mixed strategy equilibrium, the men and women are indifferent to visiting either type of seller. Thus

$$F^P(q,p) = F^P\left(q',p'\right)$$

Therefore, we can solve for the following 'reaction function'

$$p'\left(q',p,q\right) = 1 - \frac{\left(1-e^{-q}\right)^2}{\left(1-e^{-q'}\right)^2}\frac{q'}{q}(1-p)$$

This reaction function of buyers can be substituted into the island's pricing problem. The seller can then be seen to choosing an optimal q^* to maximize

$$q^* = \arg\max_{q'} \pi^P\left(p\left(q',p,q\right),q'\right)$$

The first order condition of the pricing problem is

$$2\left(1-e^{-q*}\right)e^{-q*} - \left(1-e^{-q}\right)^2\frac{(1-p)}{q} = 0$$

In equilibrium, all sellers are identical and have the same q, thus

$$q* = q$$

Therefore, using the seller's first order condition

$$2\left(1-e^{-q}\right)e^{-q}-\left(1-e^{-q}\right)^2\frac{(1-p)}{q}=0$$

or

$$p^* = 1-2\frac{qe^{-q}}{1-e^{-q}}$$

We can then substitute this price and the associated queue length into the island's payoff function to get

$$\pi^P(p^*,q^*)=p\left[\left(1-e^{-q}\right)\left(1-e^{-q}\right)\right]$$

Thus the equilibrium posted price gives each island seller identical revenues as the receive when they advertise auctions with benefits, and as the social planner would give them to encourage efficient entry. □

5.2 Bidding procedures

It is straightforward to redo the above analysis when males and females have different opportunity costs such that there are different numbers of males and females searching for islands. In this case, it is straightforward to show that efficient entry of islands is implemented if islands pay the following Mortensen tax.

$$t = yq_fe^{-q_f}q_me^{-q_m}$$

Now suppose that there is a posted amenity d such that the couples will get this additional surplus if there not more couples than slots. Again the total payout of this subsidy must be equivalent to the magnitude of the Mortensen tax. The issue is to figure out if an efficient lottery exists to determine the probability that females bid for males.

Proposition 6. *If males and females have different opportunity costs of entry in the hotel matching game, the bidding procedure when there is exactly one male and one female features the following lottery to decide who makes an offer*

$$\hat{\tau}=\frac{\frac{q_f}{q_m}\frac{\partial x}{\partial q_f}}{\frac{q_m}{q_f}\frac{\partial x}{\partial q_m}+\frac{q_f}{q_m}\frac{\partial x}{\partial q_f}}$$

Proof. The expected returns at the islands for males and females must equal the expected returns given by the planner. Thus

$$(y+d)\left(e^{-q_m}\left(1-e^{-q_f}\right)-(1-\hat{\tau})q_fe^{-q_f}e^{-q_m}\right)=ye^{-q_m}\left(1-e^{-q_f}\right)$$

$$(y+d)\left(e^{-q_f}\left(1-e^{-q_m}\right)-\hat{\tau}q_m e^{-q_f}e^{-q_m}\right)=ye^{-q_f}\left(1-e^{-q_m}\right)$$

We can eliminate d and solve for the optimal lottery. □

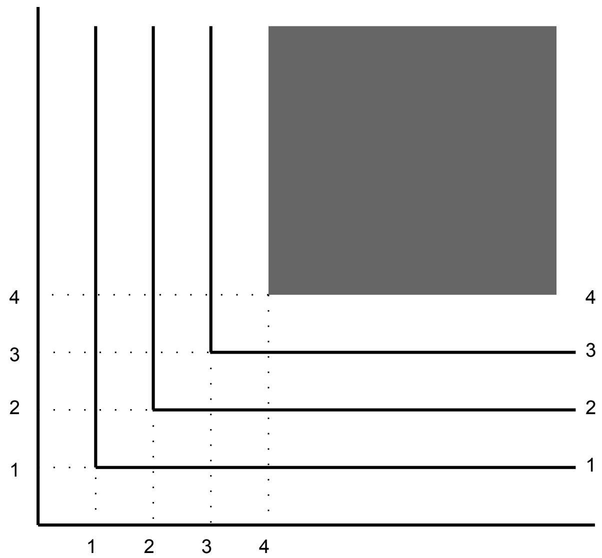
So if each island posts an amenity d it must also give a lottery $\hat{\tau}$ to determine who bids for who on the island when there is exactly one male and one female on the island.

5.3 Island matching

It is easy to relax the assumption that each island can accommodate exactly one pairwise match. Consider the case where the island can accommodate a maximum of k couples.

$$y(m,f)=\begin{cases}\min\{m,f\} & if \quad \min\{m,f\}\leq k\\ k & if \quad \min\{m,f\}>k\end{cases} \tag{19}$$

For example, if $k=4$, this means that the production function has the following isoquants regarding output and inputs.



Let $x_k(q)$ denote the number of trades that are expected at each island if the islands capacity k and the expected queue length at the island is q. The matching technology for the whole economy as function of the number islands is simply

$$X_K(n)=nx_K(1/n)$$

The function for $x_k(q)$ is given by a simple recursive formula

$$\begin{aligned}\triangle x_K(q) &= x_{K+1}(q)-x_K(q)\\ &= \left(1-e^{-q}-qe^{-q}-\frac{q^2}{2}e^{-q}-...-\frac{q^K}{K!}e^{-q}\right)^2\end{aligned}$$

The idea is that the extra capacity of moving from one slot to two slots raises the number of trades by the probability that there are two or more couples, and so on. because given the unit quantities of men and women the queue length at each island is a function of the number of islands, that is $q = 1/n$. Finally, we can define the marginal contribution of additional islands for islands of a given capacity by

$$\frac{\partial x_{K+1}}{\partial q} = \frac{\partial x_K}{\partial q} + 2\left(1 - e^{-q} - qe^{-q} - \frac{q^2}{2}e^{-q} - ... - \frac{q^K}{K!}e^{-q}\right)\frac{q^K}{K!}e^{-q}$$

In all cases, extra capacity for a given queue length at islands increases the number of islands. However, for larger capacity islands, the optimal queue length is longer (queue length is inversely related to number of islands). Therefore, optimality will generally have few islands as island capacity increases.

The social planner chooses the number of islands to maximize the number of matches subject to the cost of extra islands. That is

$$S_K^* = \max_{\{q\}} \frac{x_K(q)}{q} y - \frac{c_K}{q}.$$

and the first order condition is given by

$$y\left(x_K - q\frac{\partial x_K}{\partial q}\right) = c_K$$

The simple auction equilibrium is given by

$$y \triangle_K = c_K$$

The mortensen tax for islands of capacity K is given by transfer from islands that gives a private benefit from island entry equal to the social benefit. Thus

$$\begin{aligned} t^K &= y\left(x_K - q\frac{\partial x_K}{\partial q} - \triangle_K\right) \\ &= y\sum_{i=1}^{K}\left[i\left(\frac{q^i}{i!}e^{-q}\right)^2\right]y \end{aligned}$$

where q is determined by equation (). Clearly that the social planning solution and the auction with amenities equilibrium coincide for all levels of capacity. It also follows that if the social planner chooses a particular island capacity, then this will also be the decentralized equilibrium. In particular

$$S^* = max\{S_1^*, ..., S_Z^*\}$$

where z is the largest possible island. For a given cost, this means always selecting the largest island. An interesting case is the limit where we move to large capacity islands such that Walrasian markets are possible.

Proposition 7. *If the cost of creating a large island falls to zero, then there exists one island in equilibrium and the Mortensen tax is zero.*

Proof. Suppose $c_\infty = 0$, then there is one large capacity island and the equilibrium amenity as a fraction of output is zero. The formula for the Mortensen tax in this case is as follows.

$$t_\infty = -\sum_{i=1}^{i=\infty} i \left(\frac{q^i e^{-q}}{i!} \right)^2$$

when $q = 1/n$ goes to infinity (the number of islands gets small).

$$\frac{q^i}{e^q} \frac{\infty}{\infty} \to \frac{1}{e^q} \to 0$$

□

This means that the limiting behavior of this market is Walrasian even if pricing on each island is determined by auctions. This is somewhat paradoxical because large islands can reap large rewards in the non-limiting case. However, in the limit as costs of large island creation get small, all agents coordinate on a single large island, and a second island does not enter even though the cost of entering are becoming infinitely small.

6 Conclusions

This paper has developed a theory of teams that can explain the distribution of team memberships and the mechanics behind their creation. A key finding of our analysis is the importance of negative reserve prices, which are equivalent to advertised amenities, in the competing auction equilibrium. The implication of this theory is that team formation may require large up front expenditures as a means of coordination. It will be interesting to explore whether these expenditures are possible in richer environments that are subject to borrowing constraints and other types of economic frictions. It will also be interesting to take this sort of model to the data to determine whether the mechanisms identified in this theory of organizations can better explain various observations such as the distribution of firm sizes and the fact that smaller firms pay lower wages than larger firms.

References

[1] Burdett, K., S. Shi and R. Wright (2001): "Pricing and Matching with Frictions", *Journal of Political Economy*, Vol. 109, 1060-1085

[2] Buchanan, James, M. (1965) An Economic Theory of Clubs, *Economica*, Vol 32, No 125. pp 1-14.

[3] Coase, R. (1937), "The Nature of the Firm", *Economica* 4 (16): 386–405,

[4] Julien, B., Kennes, J. & I. King, 2000. "Bidding for Labor," *Review of Economic Dynamics*, vol. 3(4), pages 619-649, October.

[5] Hawkins, W. (2011) "Competitive Search, Efficiency, and Multi-Workers Firms, mimeo, University of Rochester.

[6] Lester, B. , 2010. "Directed search with multi-vacancy firms," *Journal of Economic Theory*, vol. 145(6), pages 2108-2132, November.

[7] McAfee (1993) Mechanism Design by Competing Sellers, *Econometrica*, Vol 61, pages 1281-1312.

[8] Mortensen, D. (2009) "Island Matching," *Journal of Economic Theory*, Vol 144, pages 2336-2353.

[9] Peters, M. and S. Severinov (1997), "Competition among sellers who offer auctions instead of prices." *Journal of Economic Theory,* 75, 141–179.

[10] Shimer, R. 2007. "Mismatch," *American Economic Review*, vol. 97(4), pages 1074-1101, September.

[11] Wolinsky, A. 1988 "Dynamic Markets with Competitive Bidding", *Review of Economic Studies*, Vol. 55, No. 181, January 1988, pp. 71-84