Taxation in Matching Markets*

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Abstract

We analyze the effects of transfer taxation on stable outcomes in matching markets. Taxes can make inefficient outcomes stable by causing workers to prefer firms from which they receive high idiosyncratic match utility, but to which they are not well-suited. We show that under proportional taxation, efficiency can be non-monotonic in the tax rate. However, in wage markets – where agents on one side of the market (workers) refuse to match without a positive transfer (wage) – efficiency is strictly decreasing in the tax rate. We find analogous results for lump sum taxation. In addition to highlighting a cost of taxation that does not appear to have been examined previously, our model provides a continuous link between the canonical models of matching with and without transfers.

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1 Introduction

We study the distortionary effects of income taxation on the stable outcomes in markets with individual-specific match utilities. Taxation causes workers to prefer firms from which they receive high idiosyncratic match utility, but to which they are not well-suited. This can reduce market efficiency by changing the set of matchings that are stable in the sense that no agent wants to deviate. The *matching distortion* we identify differs from the well known effects of taxation on intensive (e.g., Blundell et al. (1998); Saez (2004)) and extensive (e.g., Meyer (2002); Saez (2002)) labor supply; it affects the allocation of workers to firms without necessarily changing provision of labor. This matching distortion arises even in markets with frictionless matching, and thus differs from the well known effects of search costs on matching efficiency (e.g., Mortensen and Pissarides (2001); Boone and Bovenberg (2002)) and of taxation on search behavior (e.g., Gentry and Hubbard (2004); Holzner and Launov (2012)).

We analyze a model of matching with transfers. Agents have heterogeneous rankings of potential match partners. Transfers may be "taxed," causing some of each transfer payment to be taken from the agents.[1] For example, in the case of a proportional tax τ, an agent receives fraction $(1 - \tau)$ of the amount her partner gives up (see Section 3). Taxation lowers the value of transfers, causing agents to prefer match partners that provide higher individual-specific match utility over those offering higher transfers; this distorts away from efficient matching.

As in standard taxation models, full efficiency is achieved as the level of taxation approaches 0. Moreover, unlike in most other models of taxation, for any given matching market there is some strictly positive tax threshold below which full efficiency is always obtained. Unfortunately, however, in general markets lowering the tax does not always increase efficiency: A fully efficient match may be stable in the presence of a high tax, but unstable in the presence of a lower tax, and stable again in the presence of an even lower tax.

While the impact of taxation on matching efficiency can be non-monotonic in general, monotonicity does obtain in a class of markets particularly important for economic analysis: markets in which agents on one side of the market ("workers") refuse to match without positive transfers ("wages") from their match partners ("managers"). In these *wage markets*, reducing taxation always improves the efficiency of stable matchings.

Although our results are presented in the language of labor markets, they also have implications for understanding marriage markets. Taxation can be reinterpreted as representing frictions in the (often non-monetary) transfers that occur between partners. For example, it may be the case that the utility a woman gives up by washing the dishes is greater than the utility her partner receives from her doing so.[2] Our non-monotonicity results indicate that it is hard to predict the effect of a reduction in intrahousehold transfer frictions on matching efficiency.

Of the vast literature on taxation, our work is most closely related to the research on the effect of taxation on workers' occupational choices (e.g., Parker (2003); Sheshinski (2003);

[1] We do not explicitly model the central authority that collects the tax. Our welfare analysis focuses on total match utility, implicitly assuming that the social value of tax revenue equals the private value.

[2] A similar idea is modeled by Arcidiacono et al. (2011), who treats sexual intercourse as an imperfect transfer from women to men in the context adolescent relationships.

Powell and Shan (2012); Lockwood et al. (2012)); however, this prior work only reflects part of the matching distortion because it does not model the two-sidedness of the market. If workers and firms *both* have heterogeneous preferences over match partners, then matching distortions can reduce productivity even without any aggregate shift in workers from one firm (or industry) to another.

Our approach is also related to the literature on taxation in Roy models. The utility that a manager or firm in our model gets from a worker could be thought of the productivity of the worker in that firm or sector. However, Roy models generally assume that an agent's pre-tax wage in a sector is unchanged when the tax changes, while we allow the surplus sharing between firms and individuals to adjust when the tax changes (Rothschild and Scheuer (2012); Boadway et al. (1991)). While some Roy models allow the productivity of individuals to be affected by other workers in the industry, it is also reasonable to think that when taxes cause an industry's labor supply to increase, firms may offer workers a smaller share of their outputs as wages.

Our work also introduces a new link between the literatures on matching with and without transfers: Absent taxation, our framework is equivalent to matching with perfect transfers (e.g., Koopmans and Beckmann (1957); Shapley and Shubik (1971); Becker (1974)); under maximal taxation, it corresponds to the standard model of matching without transfers (e.g., Gale and Shapley (1962); Roth (1982)). Thus, the intermediate tax levels we consider introduce a continuum of models between the two existing extremes. While prior work has analyzed frameworks that can embed our intermediate transfer models (Crawford and Knoer (1981); Kelso and Crawford (1982); Quinzii (1984)), it has focused on the structure of the set of stable outcomes *within* fixed transfer models. To the best of our knowledge, our work is the first to show how the efficiency of stable outcomes changes *across* intermediate transfer models.

The remainder of the paper is organized as follows: Section 2 introduces our general model. Sections 3 and 4 analyze the cases of proportional and lump sum taxation, respectively. Section 5 discusses further connections to the literature and concludes.

2 General Model

We study a two-sided, many-to-one matching market with fully heterogeneous preferences. We refer to agents on one side of the market as *managers*, denoted $m \in M$; we call agents on the other side *workers*, denoted $w \in W$. Our notation and language are also consistent with modeling marriage markets, in which case M can be taken to be the set of men and W can be taken to be the set of women. Consequently, and also to simplify our exposition, we use male-gendered pronouns when referring to managers $m \in M$ and female-gendered pronouns when referring to workers $w \in W$ (without intending any statement concerning gender roles in reality).

Before introducing our models of taxation, we describe our underlying matching framework.

Each agent $i \in M \cup W$ derives utility from being matched to agents on the other side of the market. We denote these *match utilities* by α^m_Y and γ^w_m, with α^m_Y denoting the utility $m \in M$ obtains from matching with the set of workers $Y \subseteq W$ and γ^w_m denoting the utility

$w \in W$ obtains from matching with $m \in M$. We normalize the utility of being unmatched ("the outside option") to 0, setting $\alpha_m^m = \gamma_w^w = 0$ for all $m \in M$ and $w \in W$. In the labor market context, α_Y^m may be the productivity of the set of workers Y when employed by manager m and γ_m^w may be the utility or disutility worker w gets from working for m.[3]

Note that it is possible for workers to disagree about the relative desirabilities of potential managers and for managers to disagree about the relative values of potential workers. We impose no structure on worker match utilities and only enough structure on manager preferences to ensure the existence of equilibria (as we discuss later in this section). Specifically, we assume that managers' preferences satisfy the standard Kelso and Crawford (1982)/Hatfield and Milgrom (2005) *substitutability* condition: the availability of new workers cannot make a manager want to hire a worker he would otherwise reject.[4]

A *matching* μ is an assignment of agents such that each manager is either matched to himself (*unmatched*) or matched to a set of workers who are also matched to him. Denoting the power set of W by $\wp(W)$, a matching is then a mapping μ such that

$$\begin{aligned}\mu(m) &\in (\wp(W) \cup \{m\}) \quad \forall m \in M,\\ \mu(w) &\in (M \cup \{w\}) \quad \forall w \in W,\end{aligned}$$

with $w \in \mu(m)$ if and only if $\mu(w) = m$. The *total match utility* of μ is

$$\mathfrak{M}(\mu) \equiv \sum_{m \in M} \alpha_{\mu(m)}^m + \sum_{w \in W} \gamma_{\mu(w)}^w.$$

We say that a matching $\hat{\mu}$ is *efficient* if it maximizes total match utility (among all possible matchings), i.e. if $\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\mu)$ for all matchings μ.

We allow for the possibility of (at least partial) transfers between matched agents. We denote the transfer from m to w by $t^{m \to w}$; if m receives a (positive) transfer from w, then $t^{m \to w} < 0$. A transfer vector t identifies (prospective) transfers between all manager–worker pairs, not just between those pairs that are matched. We also include in the vector t "transfers" $t^{i \to i}$ for all agents $i \in M \cup W$, with the understanding that $t^{i \to i} = 0$. For notational convenience, we denote by $t^{m \to Y}$ the total transfer from manager m to workers in Y:

$$t^{m \to Y} \equiv \sum_{w \in Y} t^{m \to w}.$$

With these conventions, we have the following lemma.

Lemma 1. *For a given matching μ, the sum of transfers managers pay to their match partners equals the sum of the transfers paid by workers' match partners,*

$$\sum_{m \in M} t^{m \to \mu(m)} = \sum_{m \in M} \sum_{w \in \mu(m)} t^{m \to w} = \sum_{w \in W} t^{\mu(w) \to w}. \tag{1}$$

[3]Also, although it may seem that α_Y^m should be positive and γ_m^w should be negative, for our general analysis we do not make sign assumptions. That is, we allow for the possibility of highly demanded internships and for counterproductive employees.

[4]Substitutability plays no role in our analysis other than ensuring, through appeal to previous work (Kelso and Crawford (1982)), that the objects of study exist. Thus, we leave the formal discussion of the substitutability condition to the Appendix.

In the presence of taxation, a worker might not receive an amount equal to that which her match partner gives up; in general, a (weakly increasing) *transfer function* $\xi(\cdot)$ converts managers' transfer payments into the amounts that workers receive, post-tax. For all our transfer functions, we use the convention that $\xi(t^{w\to w}) = 0$ for all $w \in W$. Note that if $\xi(t^{m\to w}) \neq t^{m\to w}$, then (1) will generally *not* be equal to the sum of the amounts that the workers receive.

An *arrangement* $[\mu; t]$ consists of a matching and transfer vector.[5] We assume that agent utility is quasi-linear in transfers and that there are no externalities. With these assumptions, the utilities of arrangement $[\mu; t]$ for manager $m \in M$ and worker $w \in W$ are

$$u^m([\mu; t]) = \alpha^m_{\mu(m)} - t^{m\to\mu(m)},$$
$$u^w([\mu; t]) = \gamma^w_{\mu(w)} + \xi(t^{\mu(w)\to w}).$$

Note that the utility of a worker $w \in W$ depends on the transfer function $\xi(\cdot)$.

Our analysis focuses on the arrangements that are stable, in the sense that no agent wants to deviate.

Definition. An arrangement $[\mu; t]$ is *stable given transfer function* $\xi(\cdot)$ if the following conditions hold:

1. Each agent (weakly) prefers his or her assigned match partner(s) (with the corresponding transfer(s)) to being unmatched, that is,

$$u^i([\mu; t]) \geq 0 \qquad\qquad \forall i \in M \cup W.$$

2. Each manager (weakly) prefers his assigned match partners (with the corresponding transfers) to any alternative set of workers (with the corresponding transfers), that is,

$$u^m([\mu; t]) = \alpha^m_{\mu(m)} - t^{m\to\mu(m)} \geq \alpha^m_Y - t^{m\to Y}, \qquad\qquad \forall m \in M, Y \subseteq W;$$

and each worker (weakly) prefers her assigned match partner (with the corresponding transfer) to any alternative manager (with the corresponding transfer), that is,

$$u^w([\mu; t]) = \gamma^w_{\mu(w)} + \xi(t^{\mu(w)\to w}) \geq \gamma^w_m + \xi(t^{m\to w}) \qquad\qquad \forall m \in M, w \in W.$$

A matching μ is *stable given transfer function* $\xi(\cdot)$ if there is some transfer vector t such that the arrangement $[\mu; t]$ is stable given $\xi(\cdot)$; in this case t is said to *support* μ (given $\xi(\cdot)$).

Arguments of Kelso and Crawford (1982) show that the stability concept we use is equivalent to the other standard stability concept of matching theory, which rules out the possibility of "blocks" in which groups of agents jointly deviate from the stable outcome (potentially adjusting transfers).[6] The assumption of substitutable preferences ensures that at least one stable arrangement always exists.[7]

[5]Here were use the term "arrangement" instead of "outcome" for consistency with the matching literature (e.g., Hatfield et al. (2012)), which uses the latter term when the transfer vector only includes transfers between agents who are matched to each other.

[6]Our stability concept is defined in terms of arrangements; the block-based definition is defined only in terms of a matching and the transfers *between matched partners*. Kelso and Crawford (1982) used the term *competitive equilibrium* for the former concept and used *the core* to refer to the latter.

[7]Results of Kelso and Crawford (1982) guarantee the existence of a stable arrangement in our framework. Details are provided in the Appendix.

Some of our analysis focuses on markets in which workers have nonpositive valuations for matching, so that they will only match if paid positive "wage" transfers. Formally, we say that a market is a *wage market* if

$$\gamma_m^w \leq 0 \tag{2}$$

for all $w \in W$ and $m \in M$; it is a *strictly positive wage market* if the inequality in (2) is *strict* for all $w \in W$ and $m \in M$.[8]

For simplicity, our examples are set in *one-to-one matching markets*, in which each manager matches to at most one worker. For such markets, we abuse notation slightly by only specifying match utilities for manager–worker pairs and writing w in place of the set $\{w\}$ (e.g., $\alpha^m_{\{w\}}$ is denoted α^m_w).

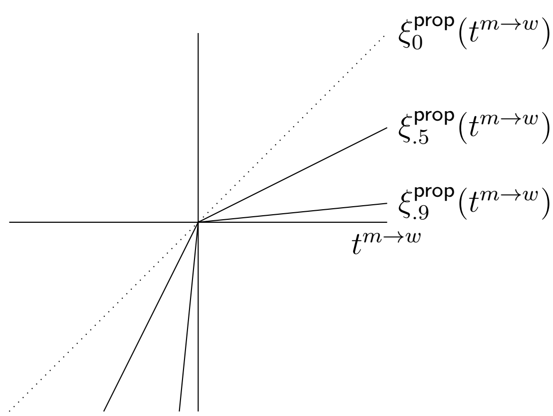
3 Proportional Taxation

First we analyze proportional (linear) taxation systems, of the type used in some US states and dozens of countries around the world. These taxes take the form of a fixed percentage deduction of each agent's income. Formally, under proportional tax τ, if an agent pays p, then his or her partner receives $(1-\tau)p$. The associated transfer function $\xi_\tau^{\mathsf{prop}}(\cdot)$ is

$$\xi_\tau^{\mathsf{prop}}(t^{m\to w}) \equiv \begin{cases} (1-\tau)t^{m\to w} & t^{m\to w} \geq 0 \\ \frac{1}{(1-\tau)}t^{m\to w} & t^{m\to w} < 0. \end{cases}$$

Figure 1 illustrates the transfer function $\xi_\tau^{\mathsf{prop}}(\cdot)$ for different levels of τ.

Figure 1: Transfer Function $\xi_\tau^{\mathsf{prop}}(\cdot)$.



If an arrangement $[\mu; t]$ or matching μ is stable given $\xi_\tau^{\mathsf{prop}}(\cdot)$, then we say it is *stable under tax* τ. We analyze how the set of stable matchings changes as τ decreases from 1 to 0.

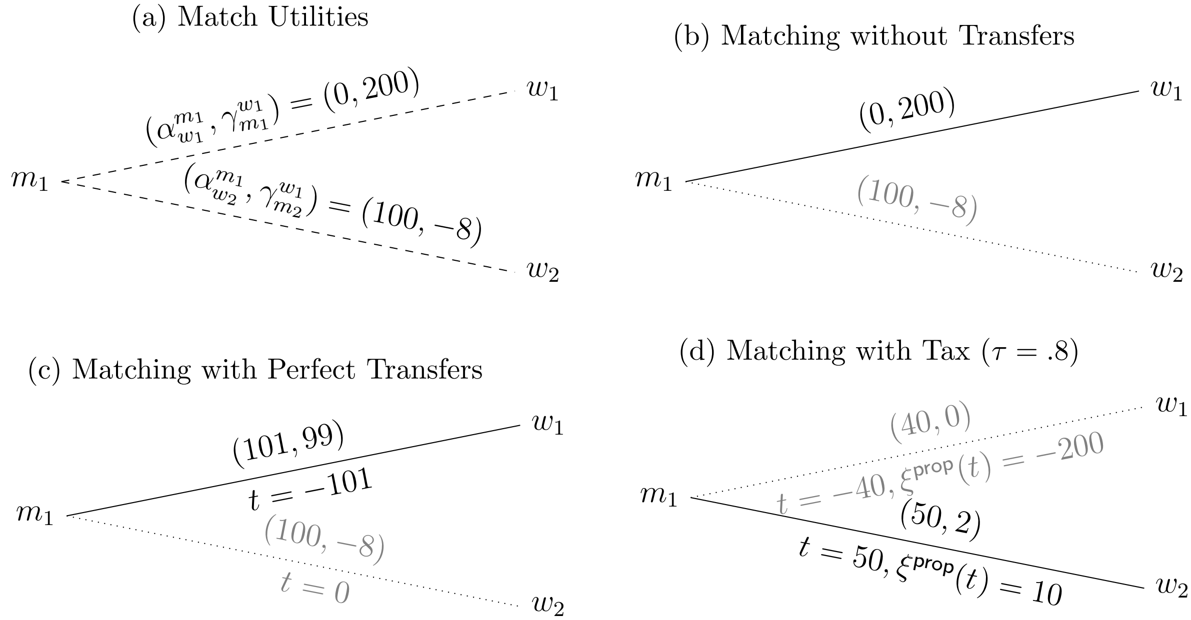
[8]Formally, the restriction to wage markets is stronger than is required for our results. Our results rely on the fact that all transfers flow from managers to workers in equilibrium; requiring nonpositive worker match utilities is sufficient for this conclusion, but not necessary.

The case $\tau = 1$ corresponds to the standard Gale and Shapley (1962) setting in which transfers are not allowed,[9] so that inefficient matchings can be stable. When $\tau = 0$, by contrast, only efficient matchings are stable (see, e.g., Shapley and Shubik (1971); Hatfield et al. (2012)). Given this, one might expect that as the tax level τ decreases, the match utilities of stable matchings should always (weakly) increase. Unfortunately, a simple example shows that this is not true in general.

3.1 Possible Inefficiencies of Tax-Reduction in General Markets

Figure 2: Example with a Proportional Tax on Transfers

Utilities, net of transfers, are above the lines (manager's, worker's).
Possible supporting transfers (when applicable) are below the lines.
Solid lines indicate the stable matching.



Example 1. Consider a one-to-one market with one manager, $M = \{m_1\}$, two workers, $W = \{w_1, w_2\}$, and match utilities as pictured in Figure 2a. Worker w_1 receives high utility from matching with m_1. Manager m_1 is indifferent towards worker w_1 and receives moderate utility from matching with w_2. Worker w_2 has a mild preference for being unmatched, rather than matching with m_1.

As illustrated in Figure 2b, when $\tau = 1$ (or when transfers are not allowed), the only stable matching $\hat{\mu}$ has $\hat{\mu}(m_1) = w_1$. This happens to be the efficient matching; therefore, it

[9]When $\tau = 1$, the set of stable matchings is the same as in the case that transfers are not allowed. The associated arrangements are not exactly the same, however, because the supporting transfer vectors are not equal to 0. However, if μ is stable when $\tau = 1$, then there is a transfer vector t supporting μ such that $t^{m \to w} = 0$ for all $m \in M$ and $w \in \mu(m)$; the arrangement $[\mu; t]$ therefore replicates the utilities that arise under μ when transfers are not allowed.

is also stable when $\tau = 0$, as shown in Figure 2c. This matching yields total match utility $\mathfrak{M}(\hat{\mu}) = 200$.

Figure 2d shows that for $\tau = .8$, an inefficient matching $\tilde{\mu}$, for which $\tilde{\mu}(m_1) = w_2$, is stable. This matching generates a total match utility $\mathfrak{M}(\tilde{\mu}) = 92$. Even if w_1 transfers 200—her maximal utility of matching—to m_1, there is a transfer m_1 can offer to w_2 that is sufficient to attract w_2, while still providing m_1 more utility than he would obtain from matching with w_1 (and receiving $(1 - .8)(200) = 40$).

Not only is an inefficient matching stable under tax $\tau = .8$, but the efficient matching $\hat{\mu}$ is *not* stable at this tax level. Indeed, the efficient matching $\hat{\mu}$ is unstable under any tax $\tau \in (.6, .9)$. For that range, $(100 - 200(1 - \tau))(1 - \tau) - 8 > 0$, so that the maximum m_1 can transfer to w_2 while still preferring her to w_1 is sufficient to outweigh the disutility w_2 gets from matching to m_1.

Although Example 1 shows that total match utility of stable matchings may decrease when the tax rate falls, an arrangement stable under a higher tax rate can never Pareto dominate an arrangement that is stable under a lower tax rate.

Proposition 1. *Suppose that $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$, and that $[\tilde{\mu}; \tilde{t}]$ is stable under tax $\tilde{\tau}$, with $\tilde{\tau} > \hat{\tau}$. Then, $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau}$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$).*

To see the intuition behind Proposition 1, we consider the case in which $\tilde{\tau} = 1$: If $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau} = 1$) Pareto dominates $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$), then every manager $m \in M$ (weakly) prefers $\tilde{\mu}(m)$ to $\hat{\mu}(m)$ with the transfer $\hat{t}^{m \to \hat{\mu}(m)}$.[10] But then, because $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$, we see that m must be paying a weakly positive transfer to $\tilde{\mu}(m)$ under $\hat{t}$ (i.e. $\hat{t}^{m \to \tilde{\mu}(m)} \geq 0$). An anlogous argument shows that each worker $w \in W$ must be paying a weakly positive transfer to $\tilde{\mu}(w)$ under $\hat{t}$ (i.e. $\xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\tilde{\mu}(w) \to w}) \leq 0$). Moreover, Pareto dominance implies that at least one manager or worker must be paying a *strictly* positive transfer. But then, that agent must pay a strictly positive transfer and receive a weakly positive transfer—impossible.

3.2 Efficiency of Tax-Reduction in Wage Markets

Example 1 shows that, in general markets, decreasing the tax rate on transfers may *decrease* the total match utility of stable matchings. Our next result shows, however, that this cannot happen in wage markets. Thus, in labor markets where firms always pay workers, decreasing the tax rate always makes (weakly) more efficient matchings stable.

In wage markets, payments flow from managers to workers; hence, any stable matching can be supported by a non-negative transfer vector.[11] Thus, the transfer function $\xi^{\mathsf{prop}}_{\tau}(\cdot)$ takes the simpler form

$$\xi^{\mathsf{prop}}_{\tau}(t^{m \to w}) = (1 - \tau) t^{m \to w} \geq 0.$$

[10] To see this, we first note that under tax $\tilde{\tau} = 1$, an arrangement with transfers of 0 among match partners Pareto dominates any other arrangement associated to the same matching. Thus, the transfers between match partners under $[\tilde{\mu}; \tilde{t}]$ can be asumed to be 0. Then, the comparison between $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau} = 1$) to $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$) amounts to a comparison of agents' match utilities under $\tilde{\mu}$ to their total utilities under $[\hat{\mu}; \hat{t}]$.

[11] There may be a supporting transfer vector where some off-path transfers are negative, but in that case there is always another supporting transfer vector that replaces those negative transfers with 0s. Our results only require the existence of a non-negative supporting transfer vector.

As all positive transfers are paid from managers to workers, there cannot be a scenario in which, when the tax is reduced, a manager m_1 can transfer enough to get a worker w_2 he prefers, but when the tax falls more, a different worker w_1 can "buy m_1 back." Thus, situations as in Example 1 cannot arise. Our next result shows that this intuition extends to wage markets more generally.

Theorem 1. *In a wage market with proportional taxation, a decrease in taxation (weakly) increases the total match utility of stable matchings. That is, if in a wage market, matching $\tilde{\mu}$ is stable under tax $\tilde{\tau}$, matching $\hat{\mu}$ is stable under tax $\hat{\tau}$, and $\hat{\tau} < \tilde{\tau}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\tilde{\mu}).$$

To prove Theorem 1, we let $\hat{t} \geq 0$ and $\tilde{t} \geq 0$ be transfer vectors supporting $\hat{\mu}$ and $\tilde{\mu}$ respectively. The stability of $[\hat{\mu}; \hat{t}]$ implies that

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)}, \tag{3}$$

$$\gamma^w_{\hat{\mu}(w)} + (1 - \hat{\tau})\hat{t}^{\hat{\mu}(w) \to w} \geq \gamma^w_{\tilde{\mu}(w)} + (1 - \hat{\tau})\hat{t}^{\tilde{\mu}(w) \to w}. \tag{4}$$

Summing (3) and (4) across agents and regrouping terms, we find:

$$\begin{aligned}\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) &= \sum_{m \in M} (\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}) + \sum_{w \in W} (\gamma^w_{\hat{\mu}(w)} - \gamma^w_{\tilde{\mu}(w)}) \\ &\geq \hat{\tau} \sum_{m \in M} \left(\hat{t}^{m \to \hat{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)}\right). \end{aligned} \tag{5}$$

Intuitively, if we had $\sum_{m \in M} \left(\hat{t}^{m \to \hat{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)}\right) < 0$, then lowering the tax from $\tilde{\tau}$ to $\hat{\tau}$ would increase workers' relative preference for $\tilde{\mu}$ over $\hat{\mu}$, as the tax change has a larger effect on large transfers. Since $\hat{\mu}$ is stable under the lower tax $\hat{\tau}$, the difference in (5) must thus be positive; this implies Theorem 1.

Although total match utility in wage markets increases as the tax is reduced, *individual* utility may be non-monotonic. For example, pursuant to a tax decrease, a manager may be made worse off because his match partner is now able to receive more from an alternative manger and switches to that manager. Even if a given manager's match is unchanged, his total utility may decrease because he is forced to increase his transfer to compensate for an increased offer from a competitor.

Individual managers' match utilities *may* decrease with a decrease in the tax rate, but the sum of workers' match utilities *must* decrease.

Proposition 2. *In a wage market with proportional taxation, if an $\tilde{\mu}$ is stable under tax $\tilde{\tau}$, and $\hat{\mu}$ is stable under tax $\hat{\tau} < \tilde{\tau}$, then workers' aggregate match utility must be (weakly) higher under $\tilde{\mu}$ than under $\hat{\mu}$. That is,*

$$\sum_{w \in W} \gamma^w_{\tilde{\mu}(w)} \geq \sum_{w \in W} \gamma^w_{\hat{\mu}(w)}.$$

Note that Proposition 2 does not imply that lower taxes necessarily make workers worse off: workers might receive higher transfers to compensate for their lower match utilities.

Finally, we show that if two distinct matchings $\hat{\mu}$ and $\tilde{\mu}$ are both stable under tax τ, then either managers and workers must disagree as to which matching is preferred, or both groups must be indifferent between the two matchings. This is a consequence of the following more general result.

Proposition 3. *In a wage market with proportional taxation, if two distinct matchings $\tilde{\mu}$ and $\hat{\mu}$ are both stable under tax τ, then*

$$\sum_{w\in W}\left(\gamma^{w}_{\tilde{\mu}(w)}-\gamma^{w}_{\hat{\mu}(w)}\right)=(1-\tau)\sum_{w\in W}\left(\alpha^{m}_{\hat{\mu}(m)}-\alpha^{m}_{\tilde{\mu}(m)}\right). \tag{6}$$

Thus, if the managers are not indifferent between $\tilde{\mu}$ and $\hat{\mu}$, then the only tax rate τ under which both $\tilde{\mu}$ and $\hat{\mu}$ can be stable is

$$\tau=1+\frac{\sum_{w\in W}\left(\gamma^{w}_{\tilde{\mu}(w)}-\gamma^{w}_{\hat{\mu}(w)}\right)}{\sum_{w\in W}\left(\alpha^{m}_{\tilde{\mu}(m)}-\alpha^{m}_{\hat{\mu}(m)}\right)}. \tag{7}$$

For τ as defined in (7) to be less than 1, the fraction in (7) must be negative, so that that managers and workers in aggregate disagree about which matching they prefer.

In order for there to be multiple values of τ at which a given two matchings are both stable, it must be that both managers and (following (6)) workers are indifferent between those two matchings.

Corollary 1. *In a wage market with proportional taxation, if there is more than one tax under which two distinct matchings $\tilde{\mu}$ and $\hat{\mu}$ both are stable, then $\mathfrak{M}(\tilde{\mu})=\mathfrak{M}(\hat{\mu})$.*

Corollary 1 implies that for generic match utilities, there is at most one value of τ at which two matchings $\tilde{\mu}$ and $\hat{\mu}$ are both stable; in this case, since there are finitely many matchings, there is a unique stable matching under almost every tax τ.

4 Lump Sum Taxation

While not typically phrased in the exact language of taxation, lump sum taxes are present throughout labor markets. They might take the form of costs for hiring (e.g., employee health care costs) or for entering employment (e.g., licensing requirements). In the marriage market context, lump sum taxes arise in settings such as online matching services that charge a fee to put participants in contact with each other.

4.1 Lump Sum Taxation of Transfers

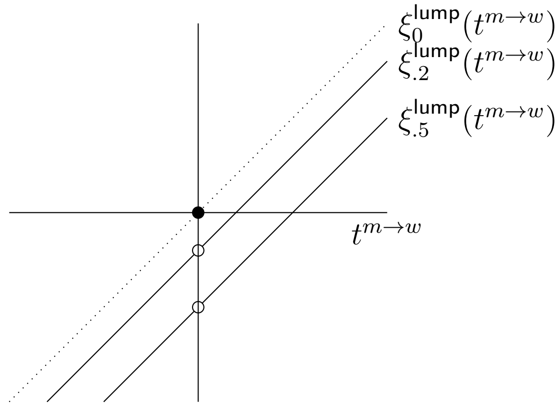
We first consider a lump sum tax that is levied only on (nonzero) transfers between match partners.[12] Such a *lump sum tax on transfers*, f, corresponds to the transfer function

$$\xi^{\mathsf{lump}}_{f}(t^{m\to w})\equiv\begin{cases}t^{m\to w}-f & t^{m\to w}\neq 0\\ t^{m\to w} & t^{m\to w}=0.\end{cases}$$

[12] An alternative approach to lump sum taxation, which we discuss in the next section, imposes a flat fee on all matches.

Figure 3 shows this transfer function. Under this tax structure, the case $f = 0$ corresponds to the standard (Shapley and Shubik (1971)) model of matching with transfers and the case $f = \infty$ corresponds to (Gale and Shapley (1962)) matching without transfers. We say that

Figure 3: Transfer Function $\xi_f^{\mathsf{lump}}(\cdot)$.



an arrangement or matching is *stable under lump sum tax* f if it is stable given transfer function $\xi_f^{\mathsf{lump}}(\cdot)$.

A lump sum tax on transfers has an extensive margin effect that makes being unmatched more attractive relative to matching with a transfer. In non-wage markets,[13] a lump sum tax on transfers can also encourage matchings in which transfers are unnecessary.[14] As our next example illustrates, this second distortion can cause the total match utility of stable matchings to be non-monotonic in the size of the lump sum tax.

Example 2. Consider a one-to-one market with two managers—$M = \{m_1, m_2\}$—three workers—$W = \{w_1, w_2, w_3\}$—and match utilities as pictured in Figure 4a. Worker w_1 likes m_1—who has a slight preference for w_2—but w_2 strongly prefers m_2. Manager m_2, meanwhile, has a strong preference for w_3, who slightly dislikes him. When transfers are not allowed (or when there is a high lump sum tax on transfers, $f \geq 195$), the only stable matching is the matching μ_1 in which $\mu_1(m_1) = w_1$ and $\mu_1(m_2) = w_2$, as shown in Figure 4b. This matching yields total match utility of $\mathfrak{M}(\mu_1) = 360$.

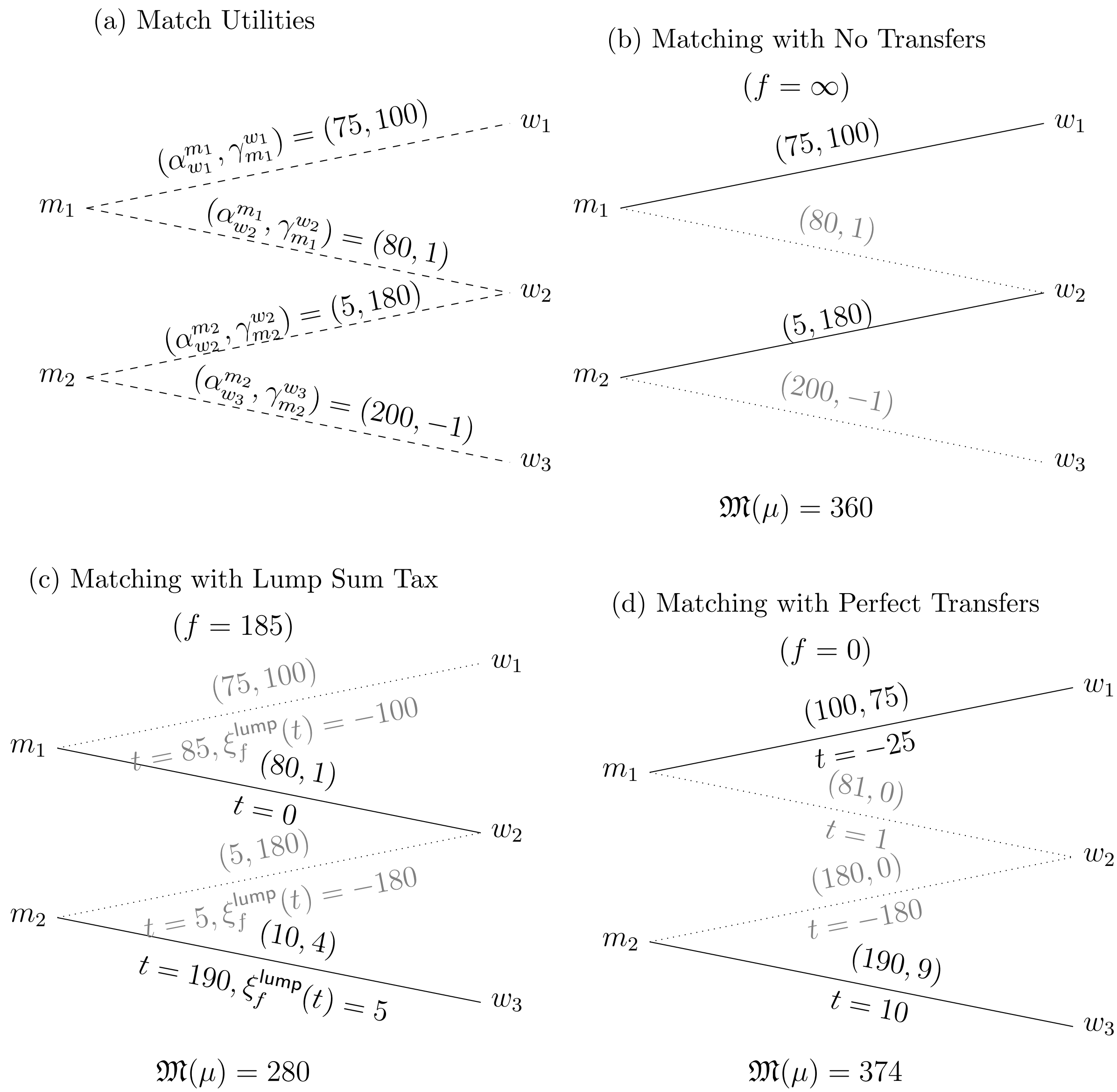
When the the lump sum tax is lowered to $f = 185$, only the matching μ_2 in which $\mu_2(m_1) = w_2$ and $\mu_2(m_2) = w_3$ is stable; this matching gives a total match utility $\mathfrak{M}(\mu_2) = 281$, as shown in Figure 4c. When $f = 185$, the tax is low enough that m_2 can convince w_3 to match with him, but not low enough for w_1 to convince to m_1 to stay with her when he has the option of matching with w_2 (which he has once m_2 does not want to match with her). The first change relative to μ_1 (m_2 switching to w_3) progresses towards the efficient matching, shown in Figure 4d, but the second change relative to μ_1 (m_1 switching to w_2)

[13]Since it is difficult to observe transfers in non-wage markets, such as marriage markets, it is somewhat hard to imagine taxing them. Nevertheless, taxing transfers could correspond to a lump sum tax on gifts between spouses and taxing matches could correspond to marriage license fees.

[14]To see this, consider the case of one-to-one matching markets. In such markets, lump sum taxes on transfers promotes pairings–(m, w)–where the match utility $\alpha_w^m + \gamma_m^w$ is evenly distributed between the two partners ($\alpha_w^m \approx \gamma_m^w$), so that transfers are unnecessary.

Figure 4: Example with Lump Sum Tax on Transfers

Utilities, net of transfers, are above the lines (manager's, worker's).
Possible supporting transfers (when applicable) are below the lines.
Solid lines indicate the stable math.



does not. Lowering the lump sum tax from 200 to 185 decreases total match utility of the stable matching.

In strictly positive wage markets, all matchings require a transfer, so a lump sum tax on transfers does not distort agents' preferences among match partners—for a given transfer vector, if a worker prefers manager m_1 to m_2 without a tax, she also prefers m_1 to m_2 under a lump sum tax. Thus, in strictly positive wage markets, the matching distortion of the lump sum tax is only on the extensive margin—the decision of *whether* to match—under a higher lump sum tax, fewer agents find matching desirable. This intuition is captured in the following lemma.

Lemma 2. *In strictly positive wage markets, reduction in a lump sum tax on transfers (weakly) increases the number of workers matched in stable matchings. That is, if matching $\tilde{\mu}$ is stable under lump sum tax $\tilde{f}$, matching $\hat{\mu}$ is stable under lump sum tax $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\#(\hat{\mu}) \geq \#(\tilde{\mu}),$$

where $\#(\mu)$ denotes the number of workers matched in matching μ.

In non-wage markets, the conclusion of Lemma 2 is not true, in general, because distortion among match partners can dominate the extensive margin effect.[15]

Since lump sum taxes do not distort among match partners in strictly positive wage markets, in such markets they can only reduce the efficiency of stable matchings by reducing the number of workers matched. This observation, when combined with Lemma 2, gives the following result.

Theorem 2. *In strictly positive wage markets, a reduction in a lump sum tax on transfers (weakly) increases the total match utility of stable matchings. That is, if $\tilde{\mu}$ is stable under lump sum tax $\tilde{f}$, $\hat{\mu}$ is stable under lump sum tax $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\tilde{\mu}).$$

Theorem 2 indicates that in strictly positive wage markets, match utility increases monotonically as lump sum taxation decreases. We can also bound the total match utility loss from a given lump sum tax.

Proposition 4. *In a strictly positive wage market, let $\hat{\mu}$ be an efficient matching, and let $\tilde{\mu}$ be stable under lump sum tax on transfers $\tilde{f}$. Then,*

$$0 \leq \mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) \leq \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})).$$

Finally, we can show that, for a *fixed* limit on the number of workers matched in the presence of a lump sum tax, stable matchings in strictly positive wage markets must generate the maximal match utility possible.

[15] To see this in the context of Example 2, consider adding a manager m_3 who receives match utility ε when matched with worker w_3, and gets negative match utility from all other partners. If w_1, w_2, and w_3 respectively receive 0, 0, and ε from matching with m_3 and the lump sum tax is $f \geq 195$, then w_3 will match with m_3—so that all three workers are matched. When $f = 185$, meanwhile, the matching is the same as without m_3—so worker w_1 ends up unmatched.

Proposition 5. *In a strictly positive wage market, a matching $\tilde{\mu}$ can be stable under a lump sum tax on transfers only if*

$$\tilde{\mu} \in \underset{\{\mu : \#(\mu) \leq \#(\tilde{\mu})\}}{\arg\max} \{\mathfrak{M}(\mu)\}.$$

Proposition 5 shows that a lump sum tax is an efficient way for a market designer to limit the number of matches (in strictly positive wage markets), since matchings stable under lump sum taxation have maximal utility, given the tax's implied limit on the number of agents matched. Analogously, if a market designer wants to encourage matches, a flat subsidy will maximize total match utility, given the tax-induced lower bound on the number of agents matched. This suggests that if a government wants to use tuition subsidies to encourage people to go to school, then uniform tuition subsidies are more efficient than subsidies proportional to the cost of tuition.

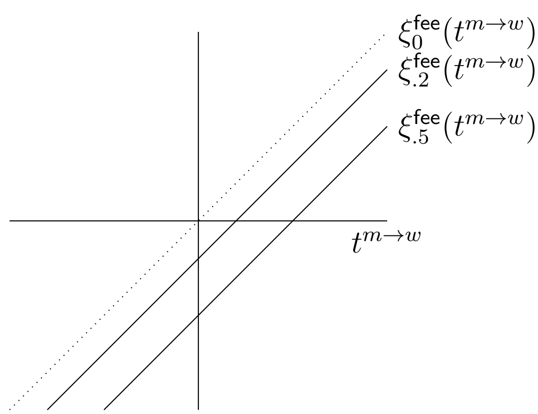
4.2 Lump Sum Taxation of Matches

Some fee structures tax *all* pairings, rather than just those that include nonzero transfers. Such *flat fees for matching* can also be interpreted in the language of taxation: they correspond to the transfer function

$$\xi_f^{\mathsf{fee}}(t^{m \to w}) \equiv t^{m \to w} - f.$$

Figure 5 shows this transfer function for different levels of f.

Figure 5: Transfer Function $\xi_f^{\mathsf{fee}}(\cdot)$.



Unlike lump sum taxes on transfers, flat fees for matching never distort among match partners—even in non-wage markets. They only have extensive margin effects, and thus markets with flat fees for matching behave similarly to strictly positive wage markets with lump sum taxes on transfers.[16] As we show in the Appendix, the conclusions of Lemma 2, Theorem 2, and Propositions 4 and 5 *always* hold in markets with flat fees for matching.

[16]Indeed, in strictly positive wage markets, lump sum taxation of transfers is equivalent to lump sum taxation of matchings because workers never match without receiving a strictly positive transfer.

5 Discussion

Before concluding, we briefly remark on some structure that is common to both models of taxation.

The Effect of Very Small Taxes

Unlike in non-matching models of taxation, in our setting there is always a non-zero tax that does not generate matching distortions. To see this in the proportional tax setting, let $\hat{\mu}$ be an efficient matching. Our results show that if $\tilde{\mu}$ is stable under $\tilde{\tau}$ then[17]

$$\tilde{\tau} \geq \frac{\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu})}{\sum_{m \in M}(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)})}. \tag{8}$$

For any inefficient matching $\tilde{\mu}$, there is a strictly positive minimum tax $\underline{\tau}(\tilde{\mu})$ at which $\tilde{\mu}$ could possibly be stable. Since there are a finite number of possible matchings, we can just take the minimum of this threshold across inefficient matchings,

$$\underline{\tau}^* = \min_{\{\mu : \mathfrak{M}(\mu) < \mathfrak{M}(\hat{\mu})\}} \underline{\tau}(\mu).$$

For $\tau < \underline{\tau}^*$ only an efficient matching can be stable.[18] The argument for the case of lump sum taxation is similar.

Structure of the Set of Stable Arrangements

Results of Kelso and Crawford (1982) and Hatfield and Milgrom (2005) imply that for any fixed τ, or f, if there are multiple stable arrangements, then workers' and managers' interests are opposed. If all managers prefer $[\mu; t]$ to $[\hat{\mu}; \hat{t}]$, then all workers prefer $[\hat{\mu}; \hat{t}]$ to $[\mu; t]$. Moreover, there exists a manager optimal (worker pessimal) stable arrangement which the managers weakly prefer to all other stable arrangements and a worker optimal (manager pessimal) stable arrangement which the managers weakly prefer. In wage markets with proportional taxation, where there is generically one stable matching, this implies that generically the opposition of interests carries over to the set of supporting transfer vectors.

Conclusion

We analyze the matching distorition that arises taxes on transfers affect the matching of workers to managers. This matching distortion is not necessarily monotonic in the tax level. In wage markets, however, matching distortions always decrease as taxes are reduced. The matching distortion we identify affects the "allocative margin" and is distinct from distortions on the intensive or extensive margins.

A natural extension would examine the interaction of allocative and intensive margin effects in a model that allows for non-binary labor supply decisions. It would also be valuable

[17] See Equation (38) of the Appendix.

[18] One caveat is that if there are multiple efficient matchings (all of which are stable when $\tau = 0$), some of them may *not* be stable in the limit as $\tau \to 0$ or $f \to 0$.

to analyze how the magnitude of the matching distortion depends on the variance and heterogeneity of agents' preferences. Such work might inform the estimation of the losses due to matching distortions in real-world labor markets.

Appendix

Existence of Stable Arrangements

In this section, we use results from the literature on matching with contracts to show the existence of stable arrangements in our framework. For a given transfer vector t, the *demand of manager $m \in M$*, denoted $D^m(t)$, is

$$D^m(t) \equiv \arg\max_{Y \subseteq W} \left\{ \alpha_Y^m - t^{m \to Y} \right\}.$$

Definition (Kelso and Crawford (1982))**.** The preferences of manager $m \in M$ are *substitutable* if for any transfer vectors t and $\tilde{t}$ with $\tilde{t} \geq t$, there exists, for each $Y \in D^m(t)$, some $\tilde{Y} \in D^m(\tilde{t})$ such that

$$\tilde{Y} \supseteq \{w \in Y : t^{m \to w} = \tilde{t}^{m \to w}\}.$$

That is, the preferences of $m \in M$ are substitutable if an increase in the "prices" of some workers cannot decrease demand for the workers whose prices remain unchanged.[19]

Theorem 2 of Kelso and Crawford (1982) shows that under the assumption that all managers' preferences are substitutable, there is an arrangement $[\mu; t]$ that is *strict core*, in the sense that:[20]

- Each agent (weakly) prefers his or her assigned match partner(s) (with the corresponding transfer(s)) to being unmatched, that is,

$$u^i([\mu; t]) \geq 0 \qquad\qquad \forall i \in M \cup W.$$

- There does not exist a manager $m \in M$, a set of workers $Y \subseteq W$, and a transfer vector $\tilde{t}$ such that

$$\begin{aligned} \alpha_Y^m - \tilde{t}^{m \to Y} &\geq \alpha_{\mu(m)}^m - t^{m \to \mu(m)}, && \text{and} \\ \gamma_m^w + \xi(\tilde{t}^{m \to w}) &\geq \gamma_{\mu(w)}^w + \xi(t^{\mu(w) \to w}) && \forall w \in Y, \end{aligned}$$

  with strict inequality for at least one $i \in (\{m\} \cup Y)$.

The Kelso and Crawford (1982) (p. 1487) construction of competitive equilibria from strict core allocations then implies that there is some transfer vector $\hat{t}$, having $\hat{t}^{\mu(w) \to w} = t^{\mu(w) \to w}$ (for each $w \in W$), such that $[\mu; \hat{t}]$ is stable in our sense.

[19] Theorem A.1 of Hatfield et al. (2012) shows that in our setting the Kelso and Crawford (1982) substitutability condition is equivalent to the choice-based substitutability condition of Hatfield and Milgrom (2005), that we describe in the main text: *the availability of new workers cannot make a manager want to hire a worker he would otherwise reject.*

[20] Strictly speaking, Kelso and Crawford (1982) have one technical assumption not present in our framework: they assume that $\alpha_w^m + \gamma_m^w \geq 0$, in order to ensure that all workers are matched. However, examining the Kelso and Crawford (1982) arguments reveals that this extra assumption is not necessary to ensure that a strict core arrangement exists—the Kelso and Crawford (1982) salary adjustment processes can be started at some arbitrarily low (negative) salary offer and all of the steps and results of Kelso and Crawford (1982) remain valid, with the caveat that some workers may not be matched at core outcomes.

Proof of Lemma 1

We let $\mathfrak{B}$ be the set of managers who are matched at μ and let $\mathscr{B}$ be the set of workers who are matched at μ. This means that

$$\begin{aligned} \mu(m) &\subseteq \mathscr{B} && \forall m \in \mathfrak{B}, \\ \mu(w) &\in \mathfrak{B} && \forall w \in \mathscr{B}. \end{aligned}$$

These observations, combined with the fact that $t^{m\to m} = t^{w\to w} = 0$, enable us to show that

$$\begin{aligned} \sum_{m\in M} t^{m\to\mu(m)} &= \sum_{m\in\mathfrak{B}} t^{m\to\mu(m)} + \sum_{m\in M\setminus\mathfrak{B}} t^{m\to\mu(m)}, \\ &= \sum_{m\in\mathfrak{B}} t^{m\to\mu(m)}, \\ &= \sum_{m\in\mathfrak{B}} \sum_{w\in\mu(m)} t^{m\to w}, \\ &= \sum_{w\in\mathscr{B}} t^{\mu(w)\to w}, \\ &= \sum_{w\in\mathscr{B}} t^{\mu(w)\to w} + \sum_{w\in W\setminus\mathscr{B}} t^{\mu(w)\to w}, \\ &= \sum_{w\in W} t^{\mu(w)\to w}. \end{aligned}$$

Proof of Proposition 1

First, we show that the outcomes stable under full taxation ($\tilde{\tau} = 1$) cannot Pareto dominate those stable under tax $\hat{\tau} < 1$.

Claim. *Suppose that $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau} < 1$, and that $[\tilde{\mu}; \tilde{t}]$ is stable under tax $\tilde{\tau} = 1$. Then, $[\tilde{\mu}; \tilde{t}]$ (under tax $\tilde{\tau} = 1$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau} < 1$).*

Proof. As no transfers get through under full taxation, an arrangement stable under full taxation is most likely to Pareto dominate some other arrangement when all transfers between match partners are 0. Thus, we assume that $\tilde{t}^{\tilde{\mu}(w)\to w} = 0$ for each $w \in W$, and suppose that $[\tilde{\mu}; \tilde{t}]$ (under full taxation) Pareto dominates $[\hat{\mu}; \hat{t}]$ (under tax $\hat{\tau}$). This would imply that

$$\alpha^m_{\tilde{\mu}(m)} = \alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m\to\mu(m)} \geq \alpha^m_{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)}, \tag{9}$$
$$\gamma^w_{\tilde{\mu}(w)} = \gamma^w_{\tilde{\mu}(w)} + \xi^{\mathsf{prop}}_{\tilde{\tau}}(\tilde{t}^{\tilde{\mu}(w)\to w}) \geq \gamma^w_{\hat{\mu}(w)} + \xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\hat{\mu}(w)\to w}), \tag{10}$$

with strict inequality for some m or w. However, stability of $[\hat{\mu}; \hat{t}]$ under tax $\hat{\tau}$ implies that

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}, \tag{11}$$
$$\gamma^w_{\hat{\mu}(w)} + \xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\hat{\mu}(w)\to w}) \geq \gamma^w_{\tilde{\mu}(w)} + \xi^{\mathsf{prop}}_{\hat{\tau}}(\hat{t}^{\tilde{\mu}(w)\to w}). \tag{12}$$

Combining (9) and (11) gives

$$0 \geq -\hat{t}^{m\to\tilde{\mu}(m)}, \tag{13}$$

for each $m \in M$, while combining (10) and (12) gives

$$0 \geq \xi_{\hat{\tau}}^{\mathsf{prop}}(\hat{t}^{\tilde{\mu}(w) \to w}), \tag{14}$$

for each $w \in W$. Strict inequality must hold in (13) or (14) for some m or w.

In the first of these cases, we have

$$\hat{t}^{m' \to \tilde{\mu}(m')} > 0$$

for some $m' \in M$; hence, there exists at least one $w \in \hat{\mu}(m')$ for whom

$$\hat{t}^{\tilde{\mu}(w) \to w} > 0. \tag{15}$$

But (15) contradicts (14).

In the second case, we have

$$0 > \xi_{\hat{\tau}}^{\mathsf{prop}}(\hat{t}^{\tilde{\mu}(w') \to w'}), \tag{16}$$

for some $w' \in W$. If we take $m = \tilde{\mu}(w')$, then (16) and (14) together imply that

$$0 > \sum_{w \in \tilde{\mu}(m)} \hat{t}^{\tilde{\mu}(w) \to w} = \hat{t}^{m \to \tilde{\mu}(m)},$$

contradicting (13). □

For $\tilde{\tau} < 1$, $\xi_{\tilde{\tau}}^{\mathsf{prop}}(\cdot)$ is strictly increasing and the conclusion of the proposition follows from the following more general result.

Proposition 1'. *Suppose that $\tilde{\xi}(\cdot)$ is strictly increasing, that $[\hat{\mu}; \hat{t}]$ is stable under $\hat{\xi}(\cdot)$, and that $[\tilde{\mu}; \tilde{t}]$ is stable under $\tilde{\xi}(\cdot)$, with $\tilde{\xi}(\cdot) \leq \hat{\xi}(\cdot)$. Then, $[\tilde{\mu}; \tilde{t}]$ (under $\tilde{\xi}(\cdot)$) cannot Pareto dominate $[\hat{\mu}; \hat{t}]$ (under $\hat{\xi}(\cdot)$).*

Proof. The case for $\tilde{\tau}$ Pareto dominance of $[\tilde{\mu}; \tilde{t}]$ (under $\tilde{\xi}(\cdot)$) over $[\hat{\mu}; \hat{t}]$ (under $\hat{\xi}(\cdot)$) would imply that

$$\alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)}, \tag{17}$$

$$\gamma^w_{\tilde{\mu}(w)} + \tilde{\xi}(\tilde{t}^{\tilde{\mu}(w) \to w}) \geq \gamma^w_{\hat{\mu}(w)} + \hat{\xi}(\hat{t}^{\hat{\mu}(w) \to w}), \tag{18}$$

with strict inequality for some m or w. However, stability of $[\hat{\mu}; \hat{t}]$ under $\hat{\xi}(\cdot)$ implies that

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)}, \tag{19}$$

$$\gamma^w_{\hat{\mu}(w)} + \hat{\xi}(\hat{t}^{\hat{\mu}(w) \to w}) \geq \gamma^w_{\tilde{\mu}(w)} + \hat{\xi}(\hat{t}^{\tilde{\mu}(w) \to w}) \geq \gamma^w_{\tilde{\mu}(w)} + \tilde{\xi}(\hat{t}^{\tilde{\mu}(w) \to w}), \tag{20}$$

where the second inequality in (20) follows from the fact that $\hat{\xi}(\cdot) \geq \tilde{\xi}(\cdot)$.

Combining (17) and (19) gives

$$\hat{t}^{m \to \tilde{\mu}(m)} \geq \tilde{t}^{m \to \tilde{\mu}(m)}, \tag{21}$$

for each $m \in M$, while combining (18) and (20) gives

$$\begin{aligned} \tilde{\xi}(\tilde{t}^{\tilde{\mu}(w)\to w}) &\geq \tilde{\xi}(\hat{t}^{\tilde{\mu}(w)\to w}) \\ \tilde{t}^{\tilde{\mu}(w)\to w} &\geq \hat{t}^{\tilde{\mu}(w)\to w} \end{aligned} \tag{22}$$

for each $w \in W$, where the second line of (22) follows from the fact that $\tilde{\xi}(\cdot)$ is strictly increasing. Strict inequality must hold in (21) or (22) for some m or w.

In the first of these cases, we have

$$\hat{t}^{m'\to\tilde{\mu}(m')} > \tilde{t}^{m'\to\tilde{\mu}(m')}$$

for some $m' \in M$; hence, there exists at least one $w \in \hat{\mu}(m')$ for whom

$$\hat{t}^{\tilde{\mu}(w)\to w} > \tilde{t}^{\tilde{\mu}(w)\to w}. \tag{23}$$

But (23) contradicts (22).

In the second case, we have

$$\tilde{t}^{\tilde{\mu}(w')\to w'} > \hat{t}^{\tilde{\mu}(w')\to w'} \tag{24}$$

for some $w' \in W$. If we take $m = \tilde{\mu}(w')$, then (24) and (22) together imply that

$$\sum_{w\in\tilde{\mu}(m)} \tilde{t}^{\tilde{\mu}(w)\to w} > \sum_{w\in\tilde{\mu}(m)} \hat{t}^{\tilde{\mu}(w)\to w};$$

hence, we find that

$$\tilde{t}^{m\to\tilde{\mu}(m)} > \hat{t}^{m\to\tilde{\mu}(m)},$$

contradicting (21). □

Proof of Theorem 1

If $\hat{\mu} = \tilde{\mu}$, then the theorem is trivially true. Thus, we consider a wage market in which $[\tilde{\mu}; \tilde{t}]$ is stable under tax $\tilde{\tau}$, $[\hat{\mu}; \hat{t}]$ is stable under tax $\hat{\tau}$, $\tilde{\tau} > \hat{\tau}$, and $\tilde{\mu} \neq \hat{\mu}$.

The stability conditions for the managers imply that

$$\alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \tilde{t}^{m\to\hat{\mu}(m)}, \tag{25}$$

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}; \tag{26}$$

these inequalities together imply that

$$\sum_{m\in M} \left(\tilde{t}^{m\to\hat{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)}\right) \geq \sum_{m\in M} \left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right). \tag{27}$$

As the market is a wage market, we have

$$\xi^{\mathsf{prop}}_{\tau}(\tilde{t}^{\tilde{\mu}(w)\to w}) = (1-\tilde{\tau})\tilde{t}^{\tilde{\mu}(w)\to w} \quad \text{and} \quad \xi^{\mathsf{prop}}_{\tau}(\hat{t}^{\hat{\mu}(w)\to w}) = (1-\hat{\tau})\hat{t}^{\hat{\mu}(w)\to w};$$

hence, the stability conditions for the workers imply that

$$\gamma^w_{\tilde{\mu}(w)} + (1-\tilde{\tau})\tilde{t}^{\tilde{\mu}(w)\to w} \geq \gamma^w_{\hat{\mu}(w)} + (1-\tilde{\tau})\tilde{t}^{\hat{\mu}(w)\to w}, \tag{28}$$

$$\gamma^w_{\hat{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\hat{\mu}(w)\to w} \geq \gamma^w_{\tilde{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\tilde{\mu}(w)\to w}. \tag{29}$$

Summing these inequalities and applying Lemma 1, we obtain

$$(1-\hat{\tau})\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(w)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq (1-\tilde{\tau})\sum_{m\in M}\left(\tilde{t}^{m\to\hat{\mu}(m)} - \tilde{t}^{m\to\tilde{\mu}(m)}\right). \tag{30}$$

Combining (27) and (30), we find that

$$(1-\hat{\tau})\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(w)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq (1-\tilde{\tau})\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(w)} - \hat{t}^{m\to\tilde{\mu}(m)}\right). \tag{31}$$

Since $\hat{\tau} < \tilde{\tau}$, (31) implies that

$$\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq 0. \tag{32}$$

Next, using (26) and (29), we find that

$$\begin{aligned}
\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) &= \sum_{m\in M}(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}) + \sum_{w\in W}(\gamma^w_{\hat{\mu}(w)} - \gamma^w_{\tilde{\mu}(w)}) \\
&\geq \sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) - (1-\hat{\tau})\sum_{w\in W}\left(\hat{t}^{\hat{\mu}(w)\to w} - \hat{t}^{\tilde{\mu}(w)\to w}\right), \\
&= \hat{\tau}\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \geq 0,
\end{aligned}$$

where the final inequality follows from (32).

Proof of Proposition 2 and Derivation of Equation (8)

Summing (28) across women, we find that

$$\begin{aligned}
\sum_{w\in W}\left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)}\right) &\geq (1-\tilde{\tau})\sum_{w\in W}\left(\tilde{t}^{\hat{\mu}(w)\to w} - \tilde{t}^{\tilde{\mu}(w)\to w}\right) && (33)\\
&\geq (1-\tilde{\tau})\sum_{w\in W}\left(\hat{t}^{\hat{\mu}(w)\to w} - \hat{t}^{\tilde{\mu}(w)\to w}\right) && (34)\\
&\geq 0, && (35)
\end{aligned}$$

where the inequality (34) follows from (27), and the inequality (35) follows from (32). Thus, we see Proposition 2—the workers receive higher match utility under $\tilde{\mu}$ than under $\hat{\mu}$.

Furthermore, this implies that

$$\sum_{m\in M}\left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)}\right) \geq 0, \tag{36}$$

so that we may calculate the lowest tax under which a given inefficient match $\tilde{\mu}$ can be stable. Combining (25) and (33), we find that

$$\sum_{w\in W}\left(\gamma^{w}_{\hat{\mu}(w)}-\gamma^{w}_{\tilde{\mu}(w)}\right)\geq(1-\tilde{\tau})\sum_{m\in M}\left(\alpha^{m}_{\hat{\mu}(m)}-\alpha^{m}_{\tilde{\mu}(m)}\right). \tag{37}$$

The inequality in (36) allows us to rearrange (37) to obtain

$$\frac{\sum_{w\in W}\left(\gamma^{w}_{\hat{\mu}(w)}-\gamma^{w}_{\tilde{\mu}(w)}\right)}{\sum_{m\in M}\left(\alpha^{m}_{\hat{\mu}(m)}-\alpha^{m}_{\tilde{\mu}(m)}\right)}\geq(1-\tilde{\tau}),$$

so that we find

$$\begin{aligned}\tilde{\tau}&\geq\frac{\sum_{m\in M}\left(\alpha^{m}_{\hat{\mu}(m)}-\alpha^{m}_{\tilde{\mu}(m)}\right)}{\sum_{m\in M}\left(\alpha^{m}_{\hat{\mu}(m)}-\alpha^{m}_{\tilde{\mu}(m)}\right)}+\frac{\sum_{w\in W}\left(\gamma^{w}_{\hat{\mu}(w)}-\gamma^{w}_{\tilde{\mu}(w)}\right)}{\sum_{m\in M}\left(\alpha^{m}_{\hat{\mu}(m)}-\alpha^{m}_{\tilde{\mu}(m)}\right)}\\&=\frac{\mathfrak{M}(\hat{\mu})-\mathfrak{M}(\tilde{\mu})}{\sum_{m\in M}\left(\alpha^{m}_{\hat{\mu}(m)}-\alpha^{m}_{\tilde{\mu}(m)}\right)}.\end{aligned} \tag{38}$$

Proofs of Proposition 3 and Corollary 1

Suppose that in a wage market, both $[\tilde{\mu};\tilde{t}]$ and $[\hat{\mu};\hat{t}]$ are stable under tax τ. The stability conditions for the managers imply that

$$\alpha^{m}_{\tilde{\mu}(m)}-\tilde{t}^{m\to\tilde{\mu}(m)}\geq\alpha^{m}_{\hat{\mu}(m)}-\tilde{t}^{m\to\hat{\mu}(m)}, \tag{39}$$

$$\alpha^{m}_{\tilde{\mu}(m)}-\hat{t}^{m\to\tilde{\mu}(m)}\leq\alpha^{m}_{\hat{\mu}(m)}-\hat{t}^{m\to\hat{\mu}(m)}, \tag{40}$$

so that

$$\tilde{t}^{m\to\hat{\mu}(m)}-\tilde{t}^{m\to\tilde{\mu}(m)}\geq\hat{t}^{m\to\hat{\mu}(m)}-\hat{t}^{m\to\tilde{\mu}(m)}. \tag{41}$$

Meanwhile, the stability conditions for the workers imply that

$$\gamma^{w}_{\tilde{\mu}(w)}+(1-\tau)\tilde{t}^{\tilde{\mu}(w)\to w}\geq\gamma^{w}_{\hat{\mu}(w)}+(1-\tau)\tilde{t}^{\hat{\mu}(w)\to w}, \tag{42}$$

$$\gamma^{w}_{\tilde{\mu}(w)}+(1-\tau)\hat{t}^{\tilde{\mu}(w)\to w}\leq\gamma^{m}_{\hat{\mu}(w)}+(1-\tau)\hat{t}^{\hat{\mu}(w)\to w}, \tag{43}$$

so that

$$(1-\tau)(\tilde{t}^{\hat{\mu}(w)\to w}-\tilde{t}^{\tilde{\mu}(w)\to w})\leq(1-\tau)(\hat{t}^{\hat{\mu}(m)\to w}-\hat{t}^{\tilde{\mu}(m)\to w}). \tag{44}$$

Summing (41) and (44) across agents and using Lemma 1, we find that

$$\sum_{m\in M}\left(\tilde{t}^{m\to\hat{\mu}(m)}-\tilde{t}^{m\to\tilde{\mu}(m)}\right)=\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)}-\hat{t}^{m\to\tilde{\mu}(m)}\right).$$

For this to hold, we must have equality in (41) for each $m\in M$. But this implies equality in (39) and (40), for each $m\in M$. Similarly, it requires that (44) hold with equality for each $w \in W$, which implies equality in (42) and (43), for each $w \in W$. Combining these equalities, and summing across workers $w \in W$, shows that

$$\begin{aligned} \sum_{w \in W} \left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)} \right) &= (1-\tau) \sum_{m \in M} \left(\tilde{t}^{m \to \hat{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \right), \\ &= (1-\tau) \sum_{m \in M} \left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)} \right). \end{aligned} \tag{45}$$

If the managers are not indifferent in aggregate between $\tilde{\mu}$ and $\hat{\mu}$, so that

$$\sum_{m \in M} \left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)} \right) \neq 0, \tag{46}$$

we have,

$$\tau = 1 + \frac{\sum_{w \in W} \left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)} \right)}{\sum_{w \in M} \left(\alpha^m_{\tilde{\mu}(m)} - \alpha^m_{\hat{\mu}(m)} \right)}. \tag{47}$$

This shows Proposition 3.

To see Corollary 1, it suffices to observe that (47) pins down a unique tax rate in the case that (46) holds. Thus, if there are two tax rates under which matchings $\tilde{\mu}$ and $\hat{\mu}$ are both stable, then we must have

$$\sum_{m \in M} \left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)} \right) = 0. \tag{48}$$

But then, we also have

$$\sum_{w \in W} \left(\gamma^w_{\tilde{\mu}(w)} - \gamma^w_{\hat{\mu}(w)} \right) = 0, \tag{49}$$

by (45). Combining (48) and (49), we find that

$$\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) = \sum_{m \in M} \left(\alpha^m_{\hat{\mu}(m)} - \alpha^m_{\tilde{\mu}(m)} \right) + \sum_{w \in W} \left(\gamma^w_{\hat{\mu}(w)} - \gamma^w_{\tilde{\mu}(w)} \right) = 0,$$

as desired.

Proof of Lemma 2

In a strictly positive wage market, all matches are accompanied by a strictly positive transfer; hence, a lump sum tax on transfers is equivalent to a flat fee for matching. Thus, Lemma 2 follows from the following slightly more general result.

Here and hereafter, we say that an arrangement or matching is *stable under flat fee f* if it is stable given transfer function $\xi^{\mathsf{fee}}_f(\cdot)$.

Lemma 2'. *Reduction of a flat fee for matching (weakly) increases the number of workers matched in stable matchings. That is, if matching $\tilde{\mu}$ is stable under flat fee $\tilde{f}$, matching $\hat{\mu}$ is stable under flat fee $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\#(\hat{\mu}) \geq \#(\tilde{\mu}),$$

where $\#(\mu)$ denotes the number of workers matched in matching μ.

Proof. As $[\tilde{\mu}; \tilde{t}]$ is stable under flat fee $\tilde{f}$, we have

$$\alpha^m_{\tilde{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \geq \alpha^m_{\hat{\mu}(m)} - \tilde{t}^{m \to \hat{\mu}(m)}$$
$$\gamma^w_{\tilde{\mu}(w)} + \tilde{t}^{\tilde{\mu}(w) \to w} - \tilde{f} \cdot \{1_{\tilde{\mu}(w) \neq w}\} \geq \gamma^w_{\hat{\mu}(w)} + \tilde{t}^{\hat{\mu}(w) \to w} - \tilde{f} \cdot \{1_{\hat{\mu}(w) \neq w}\}$$

where $\{1_{\mu(w) \neq w}\}$ is an indicator function that equals 1 if w is matched in matching μ and 0 if w is unmatched in matching μ. Summing these inequalities across agents, and using Lemma 1, we find that

$$\sum_{m \in M} \left(\alpha^i_{\tilde{\mu}(i)} - \alpha^i_{\hat{\mu}(i)}\right) + \sum_{w \in W} \left(\gamma^i_{\tilde{\mu}(i)} - \gamma^i_{\hat{\mu}(i)}\right) + \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0. \tag{50}$$

Similarly, as $[\hat{\mu}; \hat{t}]$ is stable under flat fee $\hat{f}$,

$$\alpha^m_{\hat{\mu}(m)} - \hat{t}^{m \to \hat{\mu}(m)} \geq \alpha^m_{\tilde{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)}$$
$$\gamma^w_{\hat{\mu}(w)} + \hat{t}^{\hat{\mu}(w) \to w} - \hat{f} \cdot \{1_{\hat{\mu}(w) \neq w}\} \geq \gamma^w_{\tilde{\mu}(w)} + \hat{t}^{\tilde{\mu}(w) \to w} - \hat{f} \cdot \{1_{\tilde{\mu}(w) \neq w}\};$$

these inequalities yield

$$\sum_{m \in M} \left(\alpha^i_{\hat{\mu}(i)} - \alpha^i_{\tilde{\mu}(i)}\right) + \sum_{w \in W} \left(\gamma^i_{\hat{\mu}(i)} - \gamma^i_{\tilde{\mu}(i)}\right) + \hat{f} \cdot (\#(\tilde{\mu}) - \#(\hat{\mu})) \geq 0. \tag{51}$$

upon summation.

Adding (50) and (51) shows that

$$(\tilde{f} - \hat{f})(\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0.$$

Thus, if $\tilde{f} > \hat{f}$, we have $\#(\hat{\mu}) \geq \#(\tilde{\mu})$; this proves the result. □

Proof of Theorem 2

As in the proof of Lemma 2, Theorem 2 follows from the following slightly more general result.

Theorem 2'. *A reduction in a flat fee for matching (weakly) increases the total match utility of stable matchings. That is, if $\tilde{\mu}$ is stable under flat fee $\tilde{f}$, $\hat{\mu}$ is stable under flat fee $\hat{f}$, and $\hat{f} < \tilde{f}$, then*

$$\mathfrak{M}(\hat{\mu}) \geq \mathfrak{M}(\tilde{\mu}).$$

Proof. Using (51) and Lemma 5, we find that

$$\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) = \sum_{m \in M} \left(\alpha^i_{\hat{\mu}(i)} - \alpha^i_{\tilde{\mu}(i)}\right) + \sum_{w \in W} \left(\gamma^i_{\hat{\mu}(i)} - \gamma^i_{\tilde{\mu}(i)}\right) \geq \hat{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0; \tag{52}$$

this proves Theorem 5. □

Proof of Proposition 4

As in the proof of Lemma 2, Proposition 4 follows from the following slightly more general result.

Proposition 4'. *Let $\hat{\mu}$ be an efficient matching, and let $\tilde{\mu}$ be stable under flat fee $\tilde{f}$. Then,*

$$0 \leq \mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) \leq \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})).$$

Proof. This is immediate from (50). □

Proof of Proposition 5

As in the proof of Lemma 2, Proposition 5 follows from the following slightly more general result.

Proposition 5'. *A matching $\tilde{\mu}$ can be stable under a flat fee only if*

$$\tilde{\mu} \in \underset{\{\mu \,:\, \#(\mu) \leq \#(\tilde{\mu})\}}{\arg\max} \left\{ \mathfrak{M}(\mu) \right\}.$$

Proof. From (50), we see that if $[\tilde{\mu}; \tilde{t}]$ is stable under lump sum tax $\tilde{f}$, then for any matching $\hat{\mu} \neq \tilde{\mu}$,

$$\mathfrak{M}(\tilde{\mu}) - \mathfrak{M}(\hat{\mu}) + \tilde{f} \cdot (\#(\hat{\mu}) - \#(\tilde{\mu})) \geq 0. \tag{53}$$

If fewer workers are matched in $\hat{\mu}$ than in $\tilde{\mu}$ (i.e. $\#(\tilde{\mu}) \geq \#(\hat{\mu})$), (53) implies that

$$\mathfrak{M}(\tilde{\mu}) - \mathfrak{M}(\hat{\mu}) \geq \hat{f} \cdot (\#(\tilde{\mu}) - \#(\hat{\mu})) \geq 0,$$

so that $\tilde{\mu}$ must have higher total match utility than $\hat{\mu}$. □

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