“We’re Number 1:” Price Wars for Market Share Leadership

Luís Cabral
New York University and *CEPR*

Preliminary and incomplete: February 2013

Abstract. I examine the dynamics of oligopolies when firms derive subjective value from being the market leader. In equilibrium, prices alternate between high levels and occasional price wars, which take place when market shares are similar and market leadership is at stake. The stationary distribution of market shares is bimodal, that is, most of the time there is a stable market leader. Finally, even though shareholders do not value market leadership per se, a corporate culture that values market leadership may increase shareholder value.

W R Berkley Term Professor of Economics, Stern School of Business, New York University; IME and PSPS Research Fellow, IESE; and Research Fellow, CEPR (London); luis.cabral@nyu.edu.

1. Introduction

In many industries, the prevailing managerial attitude places a disproportionate weight on being Number 1 — the market share leader. For example, in May 2012 Airbus "accused Boeing of trying to start a price war after the U.S. company pledged to work aggressively to regain a 50% share of the market." A February 2011 headline announced that "IBM reclaims server market share revenue crown in Q4," adding that "IBM and HP will continue to duke it out." HP, in turn, seems to care enough about the issue to post a message on its website proclaiming "The Real Story about Server Market Share: HP is a leader in the server market, the latest market share results from IDC confirm it." And during a 2007 interview with a group of bloggers, SAP CEO Henning Kagermann stated, "We are not arrogant, we are the market leader."

In this paper, I examine the implications of the Number 1 bias for market competition. I do not develop a theory to explain why managers derive utility from being market leaders, though I discuss some rational and behavioral reasons for this pattern. I consider a model with two sellers and multiple buyers, all of whom live forever. Buyers reassess their choice of seller at random points in time. Buyers have preference for sellers and for money.

I show that a simple assumption as the Number 1 effect leads to a rich theory of price wars, mobility barriers, and the evolution of market shares. I also show that a corporate culture that emphasizes the importance of market leadership may increase shareholder value even if shareholders do not care about market share per se.

Specifically, I show that a firm's utility from begin market leader implies a price drop when market shares are close to 50%, and thus a lot is at stake. Moreover, I provide conditions such that, fearful of entering into a price war, competition is softened at states close to the price war region, to the extent that shareholder value increases with respect to the no Number 1 effects regime. The softening of price competition also implies that the stationary distribution of market shares is bimodal, that is, most of the time one firm is larger than the other one — and occasionally price wars for market share take place.

In the previous IO literature, price wars are seen as a "necessary evil" for price collusion. In my model, it's the firm's desire to be a market leader that leads to aggressive pricing. This may be an "evil" to the extent that firm value is lower in price war states. However, an equilibrium with Number 1 effects — and the resulting cyclical price wars — may increase firm value at asymmetric states. The reason is that the threat of a price war creates a mobility barrier that protects large firms from small firms. If industry profits are higher at asymmetric states, then Number 1 effects may also increase shareholder value in the steady state. In this case, the drop in value at symmetric states is more than compensated by the shift in probability mass towards asymmetric states.

■ **Related literature and contribution.** The paper makes several contributions to the industrial organization and strategy literatures. First, it studies the implications of a fairly pervasive phenomenon, namely firms' desire to be market share leaders. Baumol (1962) and others have developed models where firms follow objectives other than profit maximization. However, to the best of my knowledge this is the first paper in the industrial organization literature to explicitly consider pricing dynamics when Number 1 effects are in place.

Second, I develop a realistic theory of price wars. For all of the richness of industrial organization theory, the core theory of price wars is still connected almost exclusively to collusion models. In Green and Porter (1982), price wars result from the breakdown of collusive equilibria during periods of (unobservable) low demand. Rotemberg and Saloner (1985) suggest that price wars correspond to firms refraining from collusion during periods of observable high demand. By contrast, I assume that firms do not collude (they play Markov strategies). Instead of a repeated game, I assume firms play a dynamic game where the state is defined by each firm's market share. In this context, price wars emerge in periods where a firms' value function is particularly steep, that is, during periods when a firm's gain from increasing market share is particularly high.

Third, I propose a new meaning of the concept of mobility barriers. In a seminar paper, Caves and Porter (1977) proposed an extension of the theory of entry barriers, one that goes beyond the movement of a firm from zero output to some positive level: for example, in some cases established firms enter a new segment of a given industry. Exogenous or endogenous impediments to such segment entry are denoted mobility barriers. My theory of dynamic price competition suggests an additional instance of intra-industry mobility: a firm that is a market share follower becoming a market share leader. To the extent that the stationary distribution of market shares is bimodal (as I will show is frequently the case), this shift in relative positions is sufficiently "discontinuous" that the analogy of mobility barriers is meaningful. The barrier I will consider is endogenous and results from the leader's aggressive price behavior when the laggard's market share becomes threateningly close to the leader's.

Fourth, I provide an instance where corporate culture has a clear influence on the way firms compete. Specifically, I provide conditions such that a deviation from profit maximization may in effect lead to higher firm value. Vickers (1985) and Fershtman and Judd (1987) have shown that profit seeking shareholders may have an interest in delegating decisions to managers based on incentive mechanisms that differ from profit maximization.[1] Specifically, if the firms' strategic variables are strategic complements (as is the case in my model) then equilibrium delegation contracts ask managers to pay less importance on profits than shareholders: such contracts "soften" price competition and lead to overall higher profits than in the "normal" price competition game. My approach is very different, and so are the results, essentially because my approach is dynamic whereas Vickers' (1985) and Fershtman and Judd (1987) is static. Number 1 effects ask firms to place more weight on market shares than shareholders would. This makes firms more, not less, aggressive. From a static point of view, this effect is bad news for shareholders, for excessively aggressive pricing means lower equilibrium profits. However, the price wars that follow from Number 1 effects are rare; and the negative effect of overly aggressive pricing is more than compensated by the deterrence effect that the threat of a price war has when firms are in an asymmetric state. In this sense, my results relate to the so-called topsy-turvy principle in collusion through repeated interaction: the greater the credible punishment that firms can find, the greater the equilibrium profit they can sustain under collusion (see Shapiro, 1989).

My paper is also related to three other strands of the economics literature. First, the

1. The idea goes back to (at least) Shelling's (1960) observation that

> The use of thugs or sadists for the collection of extortion or the guarding of prisoners, or the conspicuous delegation of authority to a military commander of known motivation, exemplifies a common means of making credible a response pattern that the original source of decision might have been thought to shrink from or to find profitless, once the threat had failed.

literature on dynamic oligopoly competition. In this context, continuation value functions are typically increasing in current market shares and there is a trade off between current profit and future market share (sometimes referred to as market share "harvesting" and market share "investing"). Examples of this pattern include switching costs (Klemperer, 1987), learning curves (Cabral and Riordan, 1994), and network effects (Cabral, 2010). Number 1 effects provide an additional reason why firms care about market shares. One important difference of my approach is that a firm's value function may not be monotonic with respect to market shares: even though each period's payoff is increasing in market shares, a small firm's prospect of entering into a price war with a large firm may imply that it's continuation be decreasing in market share.

Second, my work relates to the literature on tournaments. Nearly all of the economics applications of this literature, beginning with Lazear and Rosen (1981), has been limited to issues of personnel economics. By contrast, I consider the case when market competition is a sort of tournament where ordinal relative positioning matters (in addition to profits).

Finally, my work also relates to the recent literature on behavior industrial organization. Most of this literature deals with cases when consumer behavior departs from full information and full rationality (see Ellison, 2005, for a survey). Some papers deal with the case when competing firms behave behaviorally. For example, Al-Najjar, Baliga and Besanko (2008) consider the case when firms cannot distinguish between different types of cost (fixed, sunk, variable), which leads to distorted pricing decisions. To the best of my knowledge, my work is the first attempt at measuring the effects of a market leadership bias.[2]

■ **Roadmap.** The rest of the paper is structured as follows. In Section 2 I present the basic model. Section 3 presents the core results regarding price wars, mobility barriers and firm value. Section 4 develops a series of extensions of the basic model. Section 5 considers the meta game where shareholders choose their company's "culture," in particular the weight that managers should place on being market share leaders. Section 6 concludes the paper.

2. Model

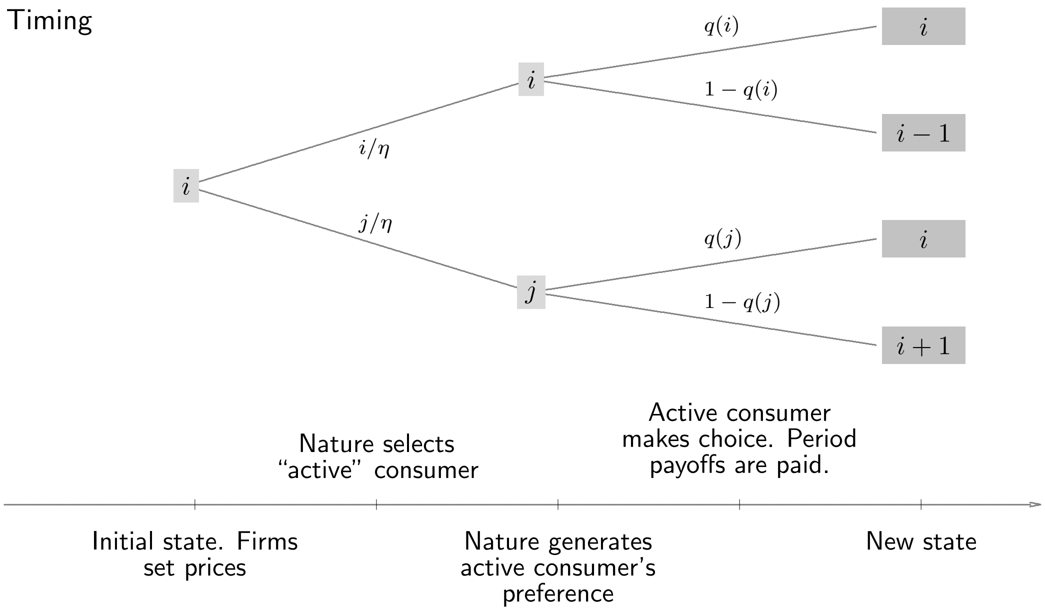
Consider a duopoly with two firms, A and B. I will use i and j to designate a firm generically, that is, $i = A, B$. Time is discrete and runs indefinitely: $t = 1, 2, ...$ The total number of consumers is given by η.

The model dynamics are given by the assumption that agents make "durable" decisions infrequently. Specifically, at random moments in time a consumer is called to re-assess its decision regarding the firm it buys from.[3] The timing of this process is described in Figure 1. Each period starts with each firm having a certain number of consumers, i and

2. In a duopoly context, my paper assumes that there is a discontinuity at 50% in the relation between market shares and payoffs. In this sense, there is an interesting relation to the recent literature on fairness where 50% plays a special role. Andreoni and Bernheim (2009) observe a high frequency of 50–50 splits in dictator and related games and suggest that individuals like to be perceived as fair. By contrast, I assume that in a market competition context managers like to be perceived as superior.

3. Similarly to Cabral (2011), the assumption of discrete time with exactly one consumer being "active" in each period may be interpreted as the reduced form of a continuous time model where each consumer becomes "active" with a constant hazard rate ν. The relevant discount factor is then computed as $\delta \equiv \eta\,\nu/(r + \eta\,\nu)$, where r is the continuous time interest rate.

Figure 1
Timing



j, attached to it (where $i + j = \eta$). Firm set prices $p(i)$ and $p(j)$. I constraint prices to be a function of the state (i, j), that is, I restrict firms to play Markov strategies. Since the total number of consumers is constant, the state space is one-dimensional and can be summarized by i.

After firms set prices, Nature chooses a particular agent, whom I will call the "active" agent. Each agent becomes active with equal probability. Then Nature generates active agent's preferences: values ζ_A and ζ_B, corresponding to consumer specific preference for each firm's product. I assume these values are drawn from a cdf $\Omega(\zeta)$ and that $\xi \equiv \zeta_A - \zeta_B$ is distributed according to cdf $\Phi(\cdot)$. The active consumer then chooses one of the firms and period payoffs are paid: the sale price to the firm that makes a sale and utility minus price to the consumer who makes a purchase. In addition to these underlying utilities, I will also consider the extra utility terms from being number one: specifically, the market leader gets an extra utility θ from being a market leader. More generally, I will assume that firms get an extra benefit $\Theta(i)$ when at state i.

There are two sources of randomness in the model. One is that each period one consumer is selected by Nature to be an active consumer. Second, Nature generates utility shocks for the active agent such that the difference $\xi_i \equiv \zeta_i - \zeta_j$ is distributed according to cdf $\Phi(\xi)$. Many of the results below require relatively mild assumptions regarding Φ:

Assumption 1. *(i) $\Phi(\xi)$ is continuously differentiable; (ii) $\phi(\xi) = \phi(-\xi)$; (iii) $\phi(\xi) > 0$, $\forall \xi$; (iv) $\Phi(\xi)/\phi(\xi)$ is strictly increasing.*

I will focus on symmetric Markov equilibria, which are characterized by a pricing strategy $p(i)$, where i is the number of living consumers who have purchased from firm i. In the remainder of the section, I first derive the determinants of consumer demand. Next, I derive the firm value functions and the resulting pricing strategy. Putting together demand and pricing, I derive a master equation that determines the evolution of market shares. The section concludes with two preliminary results: one regarding equilibrium existence and uniqueness; and another one regarding the stationary distribution of market shares.

■ Consumer demand. At state i, an active consumer chooses firm i if and only if,

$$\zeta_i - p(i) > \zeta_j - p(j) \tag{1}$$

or simply

$$\xi_i \equiv \zeta_i - \zeta_j > x(i)$$

where

$$x(i) \equiv p(i) - p(j) \tag{2}$$

Firm i's demand function is simply given by

$$q(i) = 1 - \Phi\big(x(i)\big) \tag{3}$$

Notice that

$$\frac{\partial q(i)}{\partial p(i)} = -\phi\big(x(i)\big) \tag{4}$$

■ Price. Suppose that firms' costs are zero and that the only revenue source is sales to "active" consumers. (Later I will consider the possibility of revenues from non-active consumers as well.) Firm i's value function is then given by

$$\begin{aligned} v(i) = {} & q(i)\, p(i) \\ & + \frac{i}{\eta}\,\Big(q(i)\Big(\Theta(i) + \delta\, v(i)\Big) + \big(1 - q(i)\big)\Big(\Theta(i-1) + \delta\, v(i-1)\Big)\Big) \\ & + \frac{j}{\eta}\,\Big(q(i)\Big(\Theta(i+1) + \delta\, v(i+1)\Big) + \big(1 - q(i)\big)\Big(\Theta(i) + \delta\, v(i)\Big)\Big) \end{aligned} \tag{5}$$

where $q(i)$ is firm i's demand. where $i = 0, \ldots, \eta - 1$ and $j = \eta - 1 - i$.[4] With probability $q(i)$, firm i attracts the new consumer and receives $p(i)$. With probability j/η the active agent was a firm j consumer, in which case the state switches to $i+1$; whereas with probability i/η the active agent was a firm i consumer, in which case the state remains at i. With probability $q(j) = 1 - q(i)$, the rival firm makes the current sale and firm i gets no revenues. With probability j/η the active agent was a firm j consumer, in which case the state remains at i; whereas with probability i/η the active agent was a firm i consumer, in which case the state drops to $i-1$.

Define

$$w(i) \equiv \Theta(i+1) + \delta\, v(i+1) - \Theta(i) - \delta\, v(i) \tag{6}$$

In words, this denotes firm i's value from poaching a customer from firm j. This is divided into two different components: the immediate value in terms of market leadership, $\Theta(i+1)$ if you make the same minus $\Theta(i)$ if you do not; and the discounted future value, $\delta\, v(i+1)$ if you make the sale minus $\delta\, v(i)$ if you do not.

4. Notice that, for the extreme case $i = 0$, (5) calls for values of $v(\cdot)$ which are not defined. However, these values are multiplied by zero.

The first order condition for maximizing the right-hand side with respect to $p(i)$ is given by

$$q(i) + \frac{\partial q(i)}{\partial p(i)}\, p(i) + \frac{i}{\eta}\, \frac{\partial q(i)}{\partial p(i)}\, w(i-1) + \frac{j}{\eta}\, \frac{\partial q(i)}{\partial p(i)}\, w(i) = 0$$

or simply

$$p(i) = \frac{1 - \Phi\big(x(i)\big)}{\phi\big(x(i)\big)} - \frac{i}{\eta}\, w(i-1) - \frac{j}{\eta}\, w(i) \tag{7}$$

where I substitute (3) for $q(i)$ and (4) for $\partial q(i) / \partial p(i)$.

If there were no Number 1 effects, then $w(i-1) = w(i) = 0$ and we have a standard static product differentiation model and only the first term on the right-hand side would exist, where $x(i) = p(i) - p(j)$. We thus conclude that, in the presence of Number 1 effects, firms will lower their price to the extent of what they have to gain from making the next sale, which is given by $i/\eta\, w(i-1) + j/\eta\, w(i)$: From firm i's perspective, with probability i/η, the next sale is a battle for keeping one of its customers, that is, it's the difference between the continuation value of state i and the continuation value of state $i-1$. With probability j/η, the next sale is a battle for attracting a rival customer, that is, it's the difference between the continuation value of state $i+1$ and the continuation value of state i.

Plugging this back into the value function (5) yields

$$v(i) = \frac{\Big(1 - \Phi\big(x(i)\big)\Big)^2}{\phi\big(x(i)\big)} + \frac{i}{\eta}\, \Big(\Theta(i-1) + \delta\, v(i-1)\Big) + \frac{j}{\eta}\, \Big(\Theta(i) + \delta\, v(i)\Big) \tag{8}$$

Under static oligopolistic we would only have the first term on the right-hand side. The additional terms suggest that a firm's value corresponds to the value in case it loses the challenge for the next consumer: with probability i/η, the battle for keeping one of its consumers; and with probability j/η, the battle for capturing on of the rival's consumers. This is the intuition underlying the Bertrand trap (Cabral and Villas-Boas, 2005): to the extent that firms lower their price by the value of winning a sale, their expected value is the value corresponding to losing the same (zero in the standard symmetric Bertrand model, the first term on the right-hand side if there is differentiation).

This system (8) can be solved sequentially:

$$v(i) = \left(1 - \frac{j}{\eta}\, \delta\right)^{-1} \left(\frac{\Big(1 - \Phi\big(x(i)\big)\Big)^2}{\phi\big(x(i)\big)} + \frac{i}{\eta}\, \Big(\Theta(i-1) + \delta\, v(i-1)\Big) + \frac{j}{\eta}\, \Theta(i) \right) \tag{9}$$

Finally, I will also be interested in distinguishing firm value (the function that firm decision makers maximize) from shareholder value (the firm's financial gain). The latter is given by

$$\begin{aligned} s(i) = {} & q(i)\, p(i) \\ & + \frac{i}{\eta}\, \Big(q(i)\, \delta\, s(i) + \big(1 - q(i)\big)\, \delta\, s(i-1) \Big) \\ & + \frac{j}{\eta}\, \Big(q(i)\, \delta\, s(i+1) + \big(1 - q(i)\big)\, \delta\, s(i) \Big) \end{aligned} \tag{10}$$

■ **Market shares.** Recalling that $x(i) = p(i) - p(j)$ and subtracting (7) from the corresponding $p(j)$ equation, we get

$$\begin{aligned} p(i) - p(j) = \; & \frac{1 - \Phi\big(x(i)\big)}{\phi\big(x(i)\big)} - \frac{i}{\eta}\, w(i-1) - \frac{j}{\eta}\, w(i) \\ & - \frac{1 - \Phi\big(x(j)\big)}{\phi\big(x(j)\big)} + \frac{j}{\eta}\, w(j-1) + \frac{i}{\eta}\, w(j) \end{aligned}$$

or simply

$$x(i) = \frac{1 - 2\,\Phi\big(x(i)\big)}{\phi\big(x(i)\big)} - \frac{i}{\eta}\Big(w(i-1) - w(j)\Big) - \frac{j}{\eta}\Big(w(i) - w(j-1)\Big) \tag{11}$$

where I use the fact that $1 - \Phi\big(x(j)\big) = \Phi\big(x(i)\big)$. We may re-write (11) as

$$x(i) = \frac{1 - 2\,\Phi\big(x(i)\big)}{\phi\big(x(i)\big)} + \Gamma(i) \tag{12}$$

where

$$\Gamma(i) \equiv -\frac{i}{\eta}\Big(w(i-1) - w(j)\Big) - \frac{j}{\eta}\Big(w(i) - w(j-1)\Big) \tag{13}$$

For given $\Theta(i)$ and $v(i)$ functions, all summarized in $\Gamma(i)$, equation (12) is the "master equation" determining the evolution of market shares (in expected value). Recall that $q(i) = 1 - \Phi\big(x(i)\big)$, so a higher $x(i)$ implies a lower probability that firm i makes the next sale. If $\Theta(i) = v(i) = 0$, then we have a standard static product differentiation model: all terms on the right-hand side except the first one are zero and as a result $x(i) = 0$ too: each firm makes a sale with the same probability. More generally, what factors influence the value of $x(i)$? Essentially, the difference across firms in the value of winning the sale: as shown before, firms lower their prices to the extent of their incremental value of winning a sale; the firm that has the most to win will be the most aggressive, thus increasing the likelihood of a sale. The value of winning a sale may be decomposed into (a) the immediate incremental benefit from market leadership, $\Theta(i+1) - \Theta(i)$ or $\Theta(i) - \Theta(i-1)$ as the case may be; and (b) the discounted future value from market leadership, $v(i+1) - v(i)$ or $v(i) - v(i-1)$, as the case may be.

■ **Equilibrium.** Equations (9) and (11) define a Markov equilibrium, where I note that $w(i)$ is given by (6). Given the values of $v(i)$ and $x(i)$, prices $p(i)$ and sales probabilities $q(i)$ are given by (7) and (3), respectively.

Many of the results in the next sections pertain to the limit case when $\delta \to 0$. These results are based on the following existence and uniqueness result:

Proposition 1. *There exists a unique equilibrium in the neighborhood of $\delta = 0$. Moreover, equilibrium values are continuous in δ.*

The proof of this and subsequent results may be found in the Appendix.

■ **Stationary distribution of market shares.** Given the assumption that $\Phi(\cdot)$ has full support (part (iii) of Assumption 1), $q(i) \in (0, 1)\ \forall i$, that is, there are no corner solutions in the pricing stage. It follows that the Markov process of market shares is ergodic and I can compute the stationary distribution over states. This is given by the (transposed) vector m that solves $m\,M = m$. Since the process is question is a "birth-and-death" process, whereby the state only moves to adjacent states, I can directly compute the stationary distribution of market shares:

Lemma 1. *The stationary distribution $m(i)$ is recursively determined by*

$$m(i) = m(0) \prod_{k=1}^{i} \frac{q(i-1)}{1-q(i)} \cdot \frac{\eta - i + 1}{i}$$

where

$$m(0) = \left(1 + \sum_{i=1}^{\eta} \prod_{k=1}^{i} \frac{q(i-1)}{1-q(i)} \cdot \frac{\eta - i + 1}{i} \right)^{-1}$$

■ **Generalized Number 1 effects.** Strictly speaking, firm i is a market leader if and only if $i > j$. However, in order to allow for a more general and less "discontinuous" setting, I consider the possibly of a more gradated $\Theta(i)$ mapping. Specifically, for a positive integer κ, I define three critical states

$$\begin{aligned} \bar{\imath} &\equiv \frac{\eta}{2} \\ i' &\equiv \frac{\eta - \kappa}{2} \\ i'' &\equiv \frac{\eta + \kappa}{2} \end{aligned}$$

I will say that firm i is a market leader if $i > i''$ and a market follower if $i < i'$. The extreme $\kappa = 0$ corresponds to the case when a firm is a market leader even if its market share advantage is minimal.

Consistently with this notion of market leadership, I assume that, in each period, firm i receives a payoff $p(i) + \Theta(i)$ if it makes a sale and $\Theta(i)$ if it does not, where

$$\Theta(i) = \begin{cases} 0 & \text{if } \; i < i' \\ \Big(i - i' \Big) \; \frac{\theta}{\kappa} & \text{if } \; i' \le i \le i'' \\ \theta & \text{if } \; i > i'' \end{cases} \tag{14}$$

Unless otherwise noted, I will assume that $\kappa > 0$.[5] The function $\Theta(i)$ is illustrated in the top left panel of Figure 2. Notice that $\kappa = i'' - i'$ is the range of transition states between being a market follower and being a market leader.

3. Price wars and mobility barriers

I cannot find a general analytical closed form solution for the model's equilibrium. However, I can characterize the equilibrium when $\delta = 0$; and, by Proposition 1, in the neighborhood of $\delta = 0$ the equilibrium values take on values close to the limit case $\delta = 0$.

5. If $\kappa = 0$, then the above expression is not defined for the intermediate case, and so I define $\Theta(\bar{\imath}) = \theta/2$.

Proposition 2. *There exists a unique equilibrium in the neighborhood of $\delta = 0$. Moreover,*

$$\lim_{\delta \to 0} \; p(i) = \begin{cases} \frac{1}{2\,\phi(0)} - \frac{\theta}{\kappa} & \text{if } \; i' < i < i'' \\ \frac{1}{2\,\phi(0)} - \frac{i''}{\eta}\,\frac{\theta}{\kappa} & \text{if } \; i \in \{i', i''\} \\ \frac{1}{2\,\phi(0)} & \text{otherwise} \end{cases}$$
$$\lim_{\delta \to 0} \; q(i) = \frac{1}{2}$$
$$\lim_{\delta \to 0} \; m(i) = \frac{\eta!}{i!\,(\eta - i)!\,2^\eta}$$

The limiting stationary distribution is maximal at $\eta/2$.

In words, when firms market shares are close to each other they engage in a price war for market leadership, whereby both firms decrease price by up to θ/κ. This is similar to the idea underlying the Bertrand trap (Cabral and Villas-Boas, 2005): the potential gain from being a market leader is competed away through pricing. As a result, the probability of making a sale is uniform at $\frac{1}{2}$. This implies that market share dynamics follow a straightforward reversion to the mean process: smaller firms increase their market share on average, whereas larger firms decrease their market share on average. This is particularly bad for profits because it implies a constant tendency to go back to a price war.

The lighter lines in Figure 2 illustrates this situation. The top right panel depicts the equilibrium price function. (Since the equilibrium is symmetric, this is only a function of the state, not of the firm's identity.) The middle panels show the probability of making a sale (left), as well as the stationary distribution of market shares. Finally, the bottom panels show the value function for managers (left panel) as well as for shareholders (right).

In each panel I depict two vertical lines. These correspond to the kinks in the $\Theta(i)$ mapping, that is, the states at which a firm (gradually) becomes a market share leader. Specifically, I define the set of states $\{i', ..., i''\}$ as the *price war region*.

Beginning with the price mapping, we see that prices are set at a constant level when the state is outside the price war region. Inside the price war region, that is, when $i \in \{i', ..., i''\}$, firm prices drop by θ/κ, which is the change in firm value from moving up one unit in the state space. The left middle panel (lighter line) shows that the probability of a sale is flat at $\frac{1}{2}$. This is because, whenever there is a price war, both firms decrease price by exactly the same amount (both have the same to gain from making a sale). Accordingly, the stationary distribution of market shares is a simple multinomial centered around $\eta/2$ (that is, around 50% market share).

The left bottom panels (lighter line) show that the firm's value function is increasing in i. A first, one might expect the Bertrand trap to apply here (Cabral and Villas-Boas, 2005); that is, one might expect price competition to destroy the extra value given by market leadership. However, the extent of the price war is given by the *slope* of the value function (what firms have to gain from moving up in the state space). This marginal loss from aggressive pricing is less than the infra-marginal gain given by $\Theta(i)$, the benefit from being a price leader. Finally, with respect to shareholder value (the right bottom panel) the situation is different: since shareholders do not care for market leadership per se, Number 1 effects are only bad news: they lead to price wars, which in turn destroy shareholder value.

Figure 2
Equilibrium when $\delta = 0$ (lighter lines) and $\delta = \frac{3}{4}$ (darker lines), where $\theta = 2$ and $\kappa = 10$

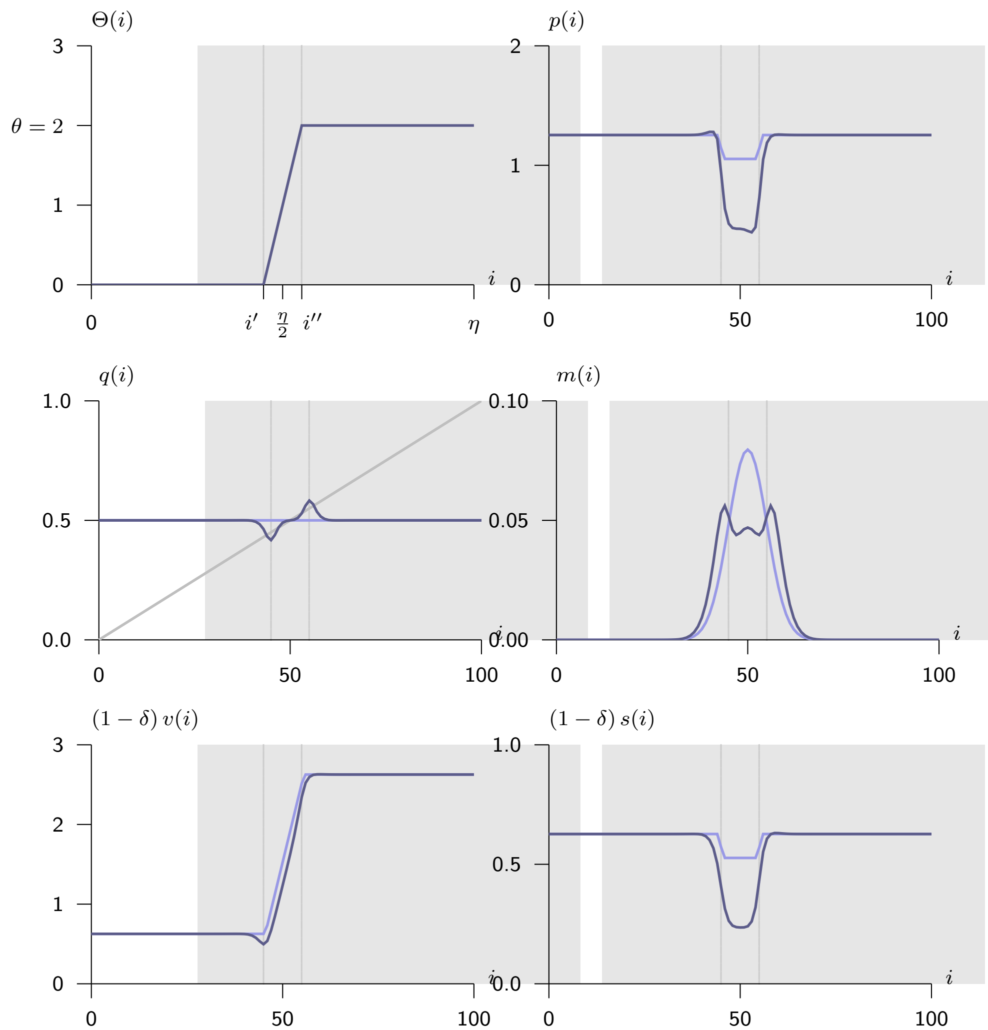
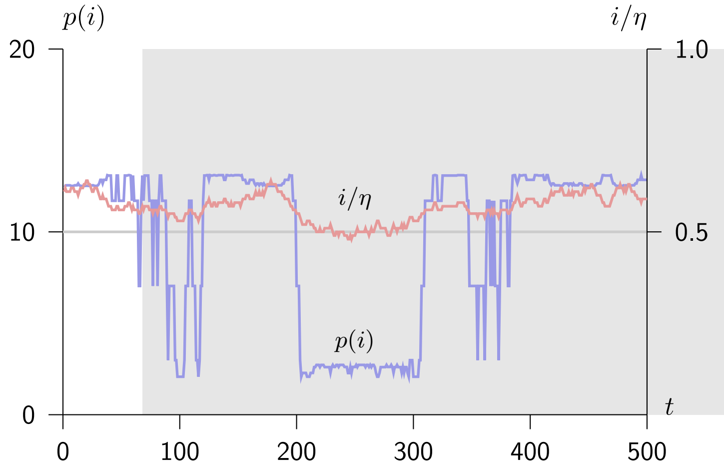


Figure 3
Price and market share dynamics ($\delta = 0.7$, $\theta = 20$, $\kappa = 10$)



■ **Higher values of δ.** Proposition 2 considers the limit case when $\delta \to 0$. From Proposition 1, I know that the system's behavior is continuous around $\delta = 0$, that is, the limit case $\delta \to 0$ is a good indication of what happens for low values of δ. For high values of δ, I cannot find a closed-form analytical solution. However, I can solve the model numerically.

The dark lines in Figure 2 show the model's solution for $\delta = \frac{3}{4}$ (keeping the other parameters fixed: $\theta = 2$, $\kappa = 10$). The top right panel shows that the price "trench" becomes deeper as the value of δ increases. This is intuitive: to the extent that firms discount the future, what they have to gain from moving up in the state space is more than the short-term gain $\Theta(i + 1) - \Theta(i)$. Since firms discount their current price by the discounted future value of winning a sale, this leads to lower prices in the "price war" region. The deeper price mapping implies a corresponding deeper shareholder value function: to the extent that the manager's pricing behavior is motivated by market leadership, it is actually a good thing that managers behave myopically.

Although the size and shape of the price function "trench" changes as the value of δ changes, Proposition 2 and numerical simulation for different values of δ suggest that the pattern of low prices when firms market shares are close to each other is fairly robust. I will next develop the implications of this result for price dynamics.

■ **Price wars.** Proposition 2 suggests that firms engage in price wars when the state space is close to the symmetric state. I now examine the dynamic implication of the $p(i)$ and $q(i)$ equilibrium mappings. Figure 3 illustrates the dynamics of price and market shares by showing the results of a simulation of the model when $\delta = \frac{3}{4}$, $\theta = 2$, and $\kappa = 10$. The red line represents firm i's market share; and the blue line, firm i's price. (Firm j's is not shown; it is almost identical to firm j's in every period.) As the figure suggests, periods of high stable prices alternate with periods of low prices.

According to my model, a price war is a period of significantly lower prices that takes place when the firms' market shares are close enough, specifically, when $i - j \leq \kappa$. As can be seen in Figure 3, price wars take place when the leader's market share drops to close to 50%. When that happens, firm i has a lot to lose from further lowering its market share, whereas firm j also increases the value from winning additional customers. This causes a price war to take place. To the extent that δ and θ are high, firm i will "normally" prevail and its market share reverts to a high level, thus re-establishing market "peace."

Figure 3 shows one of many possible equilibrium outcomes. The occurrence and duration of price wars depends on the particular realization of the two sources of model randomness: consumer preferences at birth and the time of death. I can however derive properties of the average duration of price wars. So as to better understand the notion of time in the model, I denote by ν the churn rate, that is, the percentage of consumers who switch sellers in each period of calendar time (which may be different from model period).

Proposition 3. *In the limit as $\eta \to \infty$, the average duration of a price war converges to β/ν, where $\beta \equiv \kappa/\eta$ is the width of the price war band and ν is the churn rate at $\bar{\imath}$.*

For example, suppose that in a given wireless telecommunications duopoly 2% of the customers change operators every quarter. Suppose moreover that firms turn from laggards to leaders as their market share varies form 45 to 55%. Then the average duration of a price war would be $.10/.02 = 5$ quarters.

■ **Mobility barriers.** The solutions with $\delta = 0$ and $\delta > 0$ look qualitatively similar in various respects, namely in the property that prices drop when firms market shares are close to each other. However, upon closer inspection important differences become apparent as well. First, unlike the $\delta = 0$ case, the pricing function is no longer symmetric about $\eta/2$ when $\delta > 0$. In particular, just outside the price war region, we now notice that the large firm's price is lower, whereas the smaller firm's price is higher. The intuition for this asymmetry is that, as market shares get closer to each other, the first firm to hit the high-slope portion of the value function is the large firm. The reason is that, due to the "Bertrand trap effect," the value function's continuation portion corresponds to the case when a firm loses he current sale, that is, looks at a firm's continuation value either in the current state or in the lower adjacent state. In other words, before the small firm perceives the potential gain from increasing its market share, the large firm perceives the potential loss from decreasing its market share. This makes the large firm more aggressive and increases its probability of making a sale.

In other words, the models shows that the leading firm becomes very aggressive when its market share approaches i''; which in turn implies that the probability of a sale by a leader increases when the leader's market share drops to close to i''; which implies that, most of the time, the system lies at an asymmetric state, where one firms is larger and the other firm smaller — for a long period of time.

This property provides a novel approach to the idea of barriers to mobility. In the previous literature (Caves and Porter, 1977), mobility barriers referred to a firm's access to a different industry segment. In the present context, we say there is a mobility barrier that prevents a small firm from becoming large. This mobility barrier is given by the credible threat of a price war by the large firm should the small firm's market share get sufficiently close to the large firm's market share.

4. Extensions

In this section, I consider several extensions to my basic framework. First, I consider alternative functional forms of $\Theta(i)$ and conclude that the main qualitative results remain unchanged. Second, I consider the case when installed base matters, that is, the case when each firm's per-period revenues are a function of its market share (in addition to the number 1 effect). Finally, I consider the case when being number 1 affects the buyers' utility, rather than the firms'.

■ **Installed base effects.** Suppose that a firm's per-period profit is given by $p(i) + \Theta(i) + \Psi(i)$ (or simply $\Theta(i)+\Psi(i)$) if it fails to make a sale). Specifically, suppose that $\Psi(i) = \psi\, i/\eta$, that is, in addition to sales to the "active" consumer and the benefits from market share leadership, each firm receives revenue proportional to its market share. The idea is that there exist some aftermarkets (e.g., servicing equipment) that provide firms with a certain revenue per customer. What effect does this have on equilibrium pricing and market share dynamics?

Proposition 4. *Consider an extension of the base model where firm i earns an extra per period payoff $\Psi(i) = \psi\, i/\eta$.*

- *$q(i)$ and $m(i)$ are invariant with respect to ψ*
- *$p(i)$ declines uniformly by ψ/η*
- *$v(i)$ and $s(i)$ increase by $i\,\psi/\eta/(1-\delta)$*

Figure 4 illustrates Proposition 4. It depicts the various equilibrium mappings for two cases: the case when $\Psi(i) = 0$ (lighter lines) and the case when $\Psi(i) = \psi\, i/\eta$ (darker lines) (assuming $\psi = 1$, $\theta = 2$, $\delta = 0$, $\kappa = 10$). As can be seen, installed-base effects create an additional reason for firms to lower prices. However, differently from Number 1 effects, installed-base effects work uniformly across the entire state space. That is, to the extent that $\Psi(i)$ is linear, it implies a uniform decrease in prices. Note also that both $v(i)$ and $s(i)$ increase as a result of installed base effects (unlike the effect of $\Theta(i)$, which only affects managers' value, not shareholders' value).

■ **Alternative Number 1 utility functions.** Figure 5 depicts equilibrium values for alternative $\Theta(i)$ mappings, specifically: (a) as in previous simulations but with $\kappa = 2$ (light lines); (b) as before but with $i' = 50$ (medium dark lines); (c) $\Theta(i) = \theta/\big(1+\exp\big(-(i-\bar{\imath})\big)\big)$ (darkest lines). As can be seen, the main qualitative features of the previous simulations remain valid for these alternative Number 1 effect functions.

■ **Consumers enjoy market leaders.** According to Hermann (2009),

> Some companies use their world market leadership as an advertising message. For example, Wanzl, the worldwide leader for shopping carts, says, "The size of a world market leader creates security." Being the biggest, the first or the best has always been an effective advertising message.

This quote suggests that, in addition to managerial utility, market leadership is also appreciated by consumers. This may result from a rational Bayesian process (Caminal and

Figure 4
Equilibrium when $\Psi(i) = 0$ (lighter lines) and $\Psi(i) = \psi\, i/\eta$ (darker lines), assuming $\psi = 1$, $\theta = 2$, $\delta = 0$, $\kappa = 10$

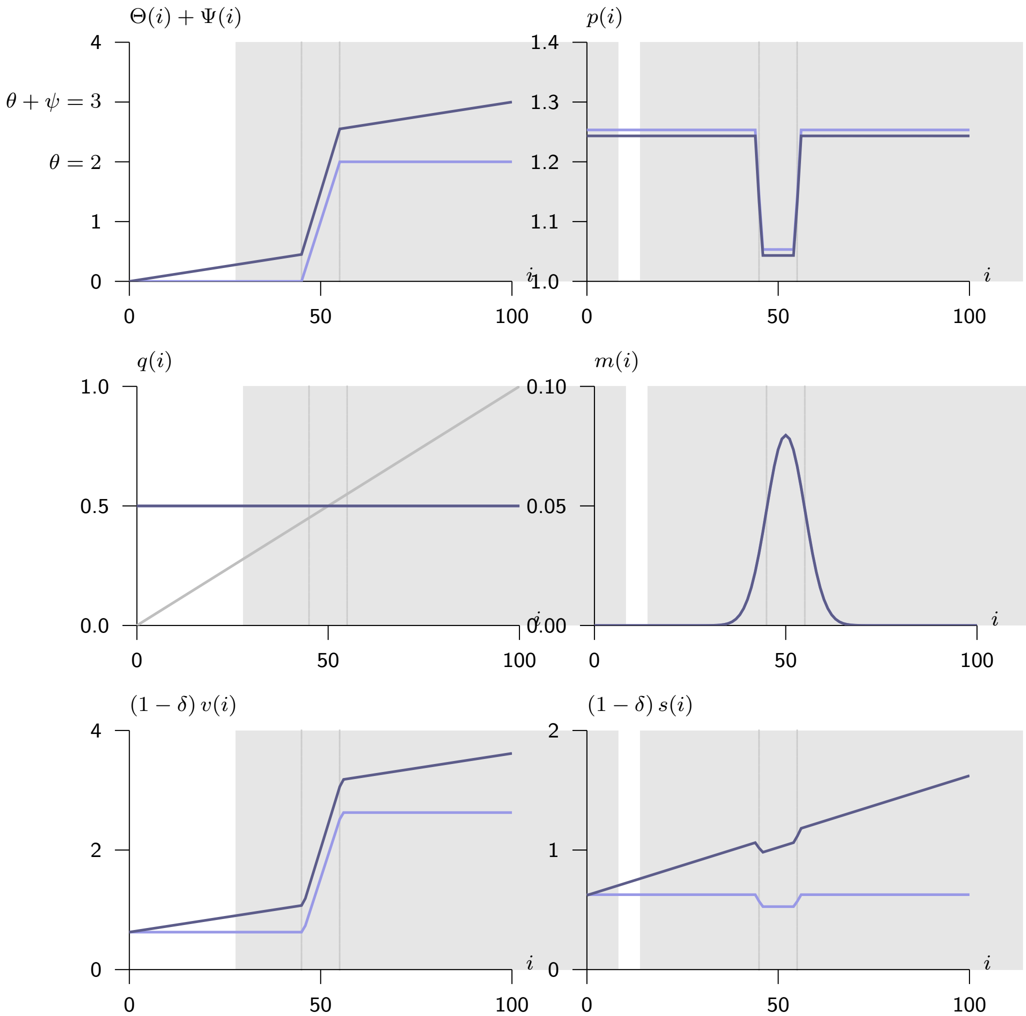
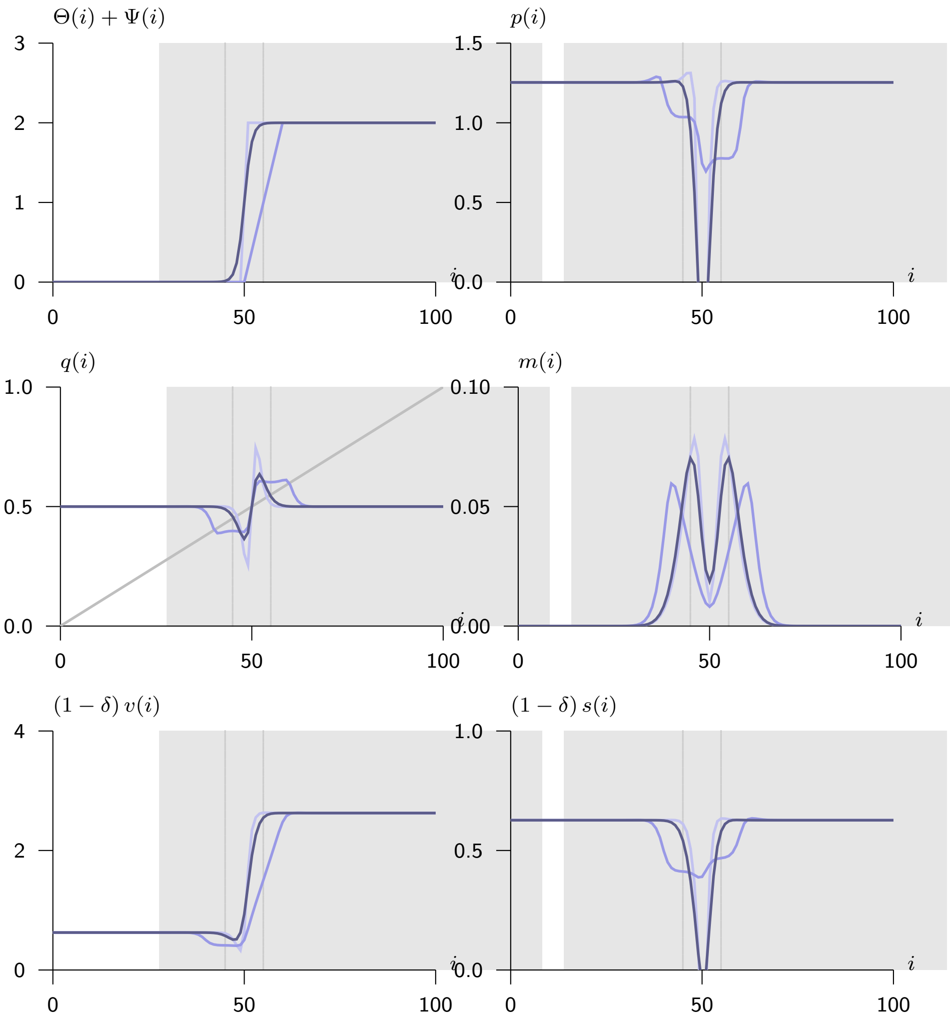


Figure 5
Alternative $\Theta(i)$ functions: (a) as in previous simulations but with $\kappa = 2$ (light lines); (b) as before but with $i' = 50$ (medium dark lines); (c) $\Theta(i) = \theta/(1 + \exp\big(-(i - \bar{\imath})\big)$ (darkest lines). (Other parameters: $\theta = 2$, $\delta = 0$, $\kappa = 10$



Vives, 1996) or simply because it's less risky to buy from a market leader: as the saying goes, "no one ever got fired for buying IBM" (when IBM was a clear market leader).[6]

In this section, I assume consumers benefit from purchasing from the market leader. Specifically, I assume an additional term in consumer utility from buying from firm i, $\Lambda(i)$. Analogously to the case of firm benefit from market leadership, I assume $\Lambda(i)$ is given by

$$\Lambda(i) = \begin{cases} 0 & \text{if } \; i < i' \\ \left(i - i'\right) \frac{\lambda}{\kappa} & \text{if } \; i' \le i \le i'' \\ \lambda & \text{if } \; i > i'' \end{cases} \tag{15}$$

At state i, an active consumer chooses firm i if and only if,

$$\zeta_i + \Lambda(i) - p(i) > \zeta_j + \Lambda(j) - p(j) \tag{16}$$

or simply

$$\xi_i \equiv \zeta_i - \zeta_j > x(i)$$

where

$$x(i) \equiv p(i) - p(j) - \Lambda(i) + \Lambda(j) \tag{17}$$

As before, my focus is on the equilibrium price function, as well as the resulting probability of a sales and stationary distribution of market shares. Suppose that $\Theta(i) = 0$, that is, firms do not derive any special direct benefit from being the market leader.

Proposition 5. *There exists a unique equilibrium in the neighborhood of $\delta = 0$. Moreover,*

$$\lim_{\delta \to 0} \, q(i) = \begin{cases} 1 - q' & \textit{if } \; i \le i' \\ 1 - q' < q(i) < q' & \textit{if } \; i' < i < i'' \\ q' & \textit{if } \; i \ge i'' \end{cases}$$

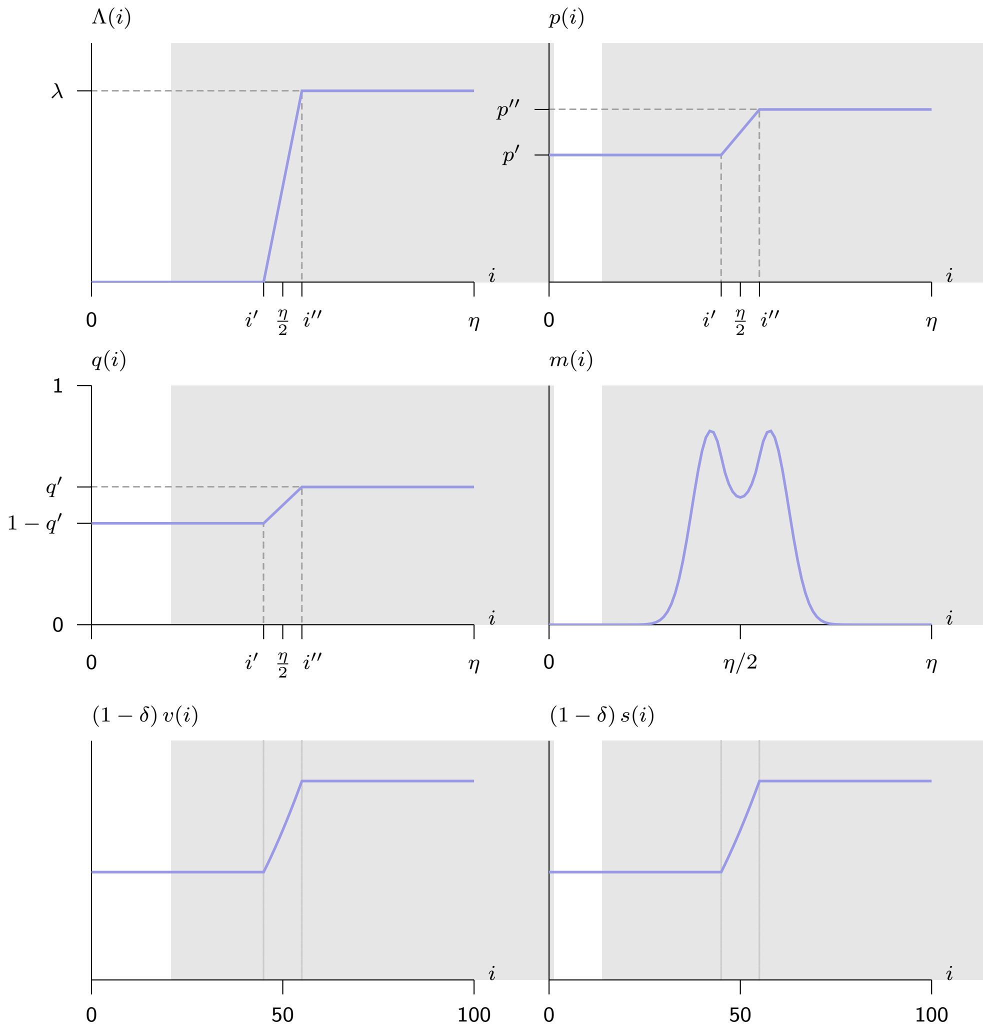
$$\lim_{\delta \to 0} \, p(i) = \begin{cases} p' & \textit{if } \; i \le i' \\ p' < p(i) < p'' & \textit{if } \; i' < i < i'' \\ p'' & \textit{if } \; i \ge i'' \end{cases}$$

where $q' > \frac{1}{2}$ and $p' < \frac{1}{2\,\phi(0)} < p''$. Finally, if λ is large enough then the stationary distribution of market shares is bimodal.

Figure 6 illustrates Proposition 5. The price function no longer exhibits the "trenchy" pattern observed Proposition 2. Instead, the market leader, enjoying a preference in the eyes of the consumer, is able to price higher than the laggard. Despite a higher price, the leader sells with a higher probability than the laggard, as shown in the bottom left panel of Figure 6. Finally, as the bottom right panel shows, this results in a bi-modal stationary distribution of market shares. The idea is that the consumer's preference for market leaders

6. Additional examples include the online page for Jim Maloof Realtor: "We sell the most homes ... giving you a clear and measured advantage;" or that of a Polish shoe retailer: "Customer satisfaction is the measure of our success. We are the market leader on the Polish footwear retailing." A particularly important case of leadership leading to consumer preference is given by online listings: for example, according to a study by Optify, the top item on a Google search receives a 36.4 click-through rate, whereas the second and third items receive on 8.9 and 1.5%, respectively.

Figure 6
Equilibrium when $\lambda > 0, \theta = 0$

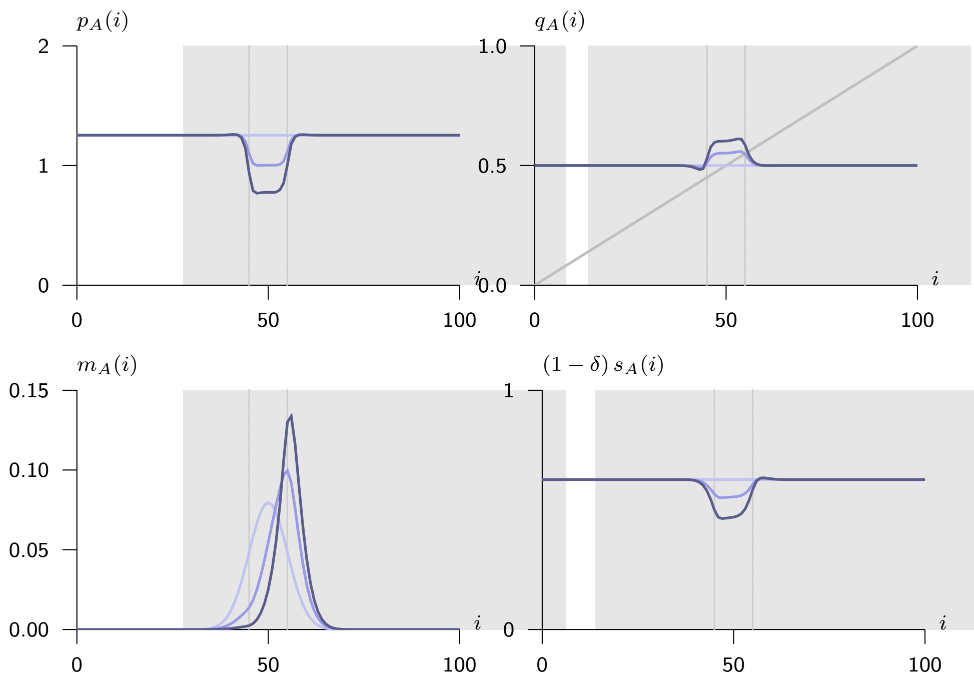


creates a self-reinforcing process whereby a leader, even if it enjoys a small lead, is able to sell with probability greater than 50% and thus cement its lead. If that lead becomes very large, then reversion to the mean dominates and the leader reduces its market share in expected terms. Together this implies a stationary distribution with modes strictly between 0 and 50% and between 50 and 100%, respectively.

Notice finally that, since $\Theta(i) = 0$, there is no divergence between the firm's and the shareholders' value: $v(i) = s(i)$. Both functions are increasing in market share: to the extent that consumers enjoy buying from the market leader, the market leader is able to set a higher price and sell with higher probability.

In sum, Proposition 5, together with Proposition 2, suggests that the effect of "the importance of being Number 1" depends on whether it's buyers or sellers who care about relative firm position.

Figure 7
Effect of changing θ_A unilaterally from 0 to 1 to 2 (increasingly dark lines) with $\Psi(i) = 0$, $\theta_B = 0, \delta = 0.75, \lambda = 0, \kappa = 10$)



5. Corporate culture and firm value

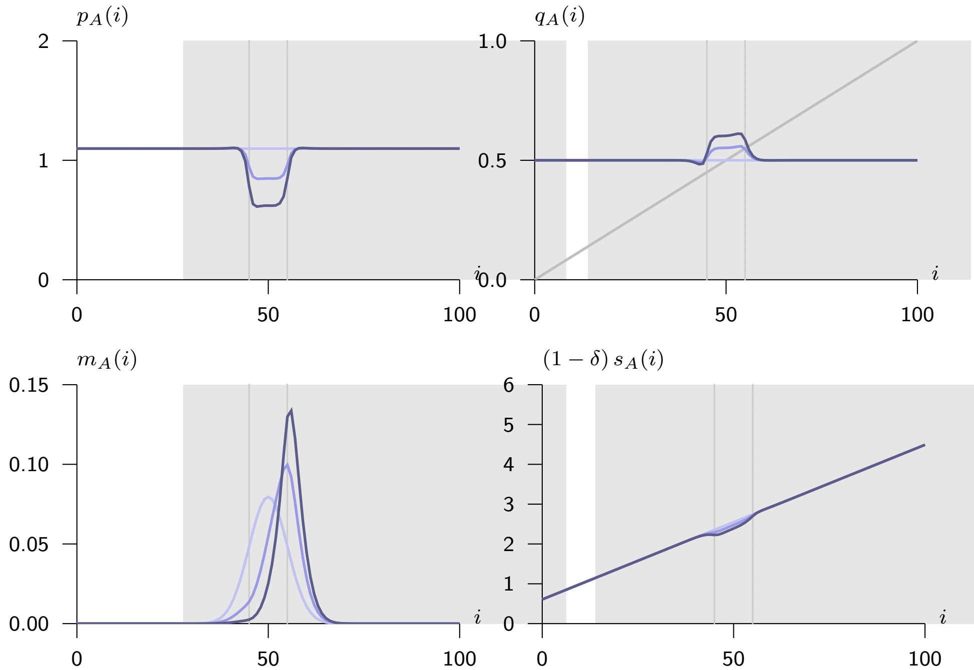
In 2012, GM recovered the position of global market share leader. According to CNN,

> GM had held onto market share and its No. 1 rank by cutting prices on cars to the point where they were unprofitable. Bob Lutz, former vice chairman of GM, said worrying about its market share rank did the company more harm than good. “There is absolutely nothing to be gained by being the world’s biggest,” he said. “I tried to tell them to say, no, it’s not our objective to be No. 1. But they just couldn’t do it.”

Suppose that being No. 1 adds no value to shareholders, as Bob Lutz suggests. Still, a tantalizing question is: does GM’s culture of wanting to be a market share leader increase shareholder value *in equilibrium*?

In order to address this question, I need to consider the effect of one firm *unilaterally* changing the value of its $\Theta(i)$ function, specifically the value of θ_i. Figure 7 shows the effect of changing θ from 0 to 1 to 2 when there are no installed-base effects (that is, $\Psi(i) = 0$). The two bottom panels show that there are two effects to take into consideration in terms of shareholder value. First, as the right panel shows, the greater θ the greater the dip in shareholder value when market shares are close to 50%. This is thrust of Bob Lutz’s point regarding GM: “worrying about its market share rank did the company more harm than good.” Against this effect, we should note that, as the left panel shows, the stationary distribution of market shares becomes more favorable to firm A. To the extent

Figure 8
Effect of changing θ_A unilaterally from 0 to 1 to 2 (increasingly dark lines) with $\Psi(i) = 4\,i/\eta$, $\theta_B = 0, \delta = 0.75, \lambda = 0, \kappa = 10$)



that shareholder value increases in θ for high states (e.g., for $i = i''$), the shift in probability should favor firm A. However, the increase in $s(i)$ driven by Number 1 effects is very small (it can hardly be seen in the right panel in Figure 7). It would seem that Mr Lutz was right.

Suppose however that shareholder value increases in market share. This is illustrated in Figure 8. Now the shareholder value function is increasing, that is, even shareholders prefer to own a large firm rather than a small firm. In this case, the shift in the stationary distribution has a big positive effect on average shareholder value: basically, an increase in θ_A makes firm A bigger on average, which is good for shareholders. The direct negative effect of a higher θ_A is still present: price wars destroy value. However, it's possible that the positive effect of shifting probability mass more than outweigh the negative effect of decreasing prices around $\bar{\imath}$.

In summary, the effect of a unilateral increase in θ_A on average shareholder value depends on the shape of shareholder value when $\theta_A = 0$. If shareholder value is relatively flat with respect to market share, then a Number 1 corporate culture leads to a decrease in average shareholder value. If shareholder value is sufficiently increasing in a firm's market share, then a Number 1 corporate culture leads to an increase in average shareholder value.

Let π_k be firm k's average value in the steady-state, that is,

$$\pi_k = \sum_i m(i)\,v(i)$$

The following result formalizes the above intuition:

Figure 9
θ_i choice metagame with $\psi(i) = 0$ (left panel) and $\psi(i) = 4\,i/\eta$ (right panel) (other parameters: $\delta = 0.75, \kappa = 10$)

		θ_B		
		0	1	2
θ_A	0	0.63 0.63	0.59 0.56	0.58 0.54
	1	0.56 0.59	0.51 0.51	0.50 0.48
	2	0.54 0.58	0.48 0.50	0.44 0.44

		θ_B		
		0	1	2
θ_A	0	2.55 2.55	2.65 2.35	2.72 2.24
	1	2.35 2.65	2.43 2.43	2.59 2.25
	2	2.24 2.72	2.25 2.59	2.36 2.36

Conjecture 1. *Suppose that $\theta_B = 0$. There exist positive values δ', θ', ψ' such that, if $\delta < \delta'$ and $\theta_A < \theta'$, then π_A is increasing in θ_A if and only if $\psi > \psi'$.*

Obviously, two can play the same game. The natural next step is to analyze the "meta-game" played by firms A and B when both can choose their value of π_i and the payoffs are given by π_i, average shareholder value according to the stationary distribution of market shares.

Figure 9 shows two versions of this game. The left-side game corresponds to the case when there are no installed-base effects (as in Figure 8), so that the shareholder value function is relatively flat with respect to market share. The right-hand game corresponds to the case when there are installed-base effects, so that the shareholder value function increases with market share.

With no installed-base effects, a value maximization corporate culture is a dominant strategy. In other words, shareholders who want to maximize firm value should instruct managers to maximize firm value. With strong installed base effects, however, the dominant strategy is to create a Number 1 corporate culture. In fact, the meta-game has the structure of a prisoner's dilemma: although it is a dominant strategy to choose a high value of θ, both firms would be better off if both firms chose a low value of θ.

More generally Proposition 2 and Conjecture 1 imply the following result:

Corollary 1. *Consider a meta-game whereby firms simultaneously choose θ_i ($i = A, B$) from the set $[0, \theta]$. There exist positive values δ', θ', ψ' such that, if $\delta < \delta'$, $\theta < \theta'$ and $\psi > \psi'$ then the game has the nature of a prisoner's dilemma.*

Finally, notice that, even if the game has the nature of a prisoner's dilemma — so that average value along the steady state is lower with $\theta > 0$ — this does not preclude the possibility that $s(i)$ be increasing for some states. In fact, as mentioned earlier, when $\delta > 0$ and just outside the price war region, the large firm's price is lower, whereas the smaller firm's price is higher, when $\theta > 0$. The intuition for this asymmetry is that, as market shares get closer to each other, the first firm to hit the high-slope portion of the value function is the large firm. The reason is that, due to the "Bertrand trap effect," the value function's continuation portion corresponds to the case when a firm loses he current sale, that is, looks at a firm's continuation value either in the current state or in the lower adjacent state. In other words, before the small firm perceives the potential gain from increasing its market share, the large firm perceives the potential loss from decreasing its market share. This makes the large firm more aggressive and increases its probability of making a sale. It also implies that the small firm's value function is actually *decreasing* in market share as we enter the price war region.

This asymmetry in prices and probability of sale has some important effects. First, since the small firm's value function is decreasing in market share, it has an incentive to price higher than it does in the region when the value function is flat. This softening of price competition benefits the large firm at states just above the price war region. In fact, not only is the value function $v(i)$ increasing in θ but so is the shareholder value function $s(i)$.

In sum, this suggests that the large firm's shareholders benefit from Number 1 effects: not directly, but indirectly. I next show a result that makes this point analytically. For simplicity, I consider the case when $\kappa = 0$, so that a firm gets the full benefit from market leadership even if market share differences are very small.

Proposition 6. *Suppose that $\kappa = 0$. There exists a δ' such that, if $\delta < \delta'$ and $\phi(0) < \frac{1}{2}$, then shareholder value in state $\bar{\imath} + 2$ is increasing in θ.*

Notice that, methodologically, Proposition 2 is very different from Proposition 6. The former corresponds to the limit $\delta = 0$, whereas the latter looks at variations as δ moves away from 0. The former is a simple limit argument, whereas the latter requires a Taylor expansion around $\delta = 0$. Specifically, in the limit when $\delta = 0$, shareholder value function is invariant with respect to δ; however, in the neighborhood of $\delta = 0$ (and for $\delta > 0$), shareholder value function at state $\bar{\imath} + 2$ is strictly increasing in θ.

6. Conclusion

Many firms seem to play a disproportionate weight on the goal of being market share leaders. At first, such goal seems contrary to shareholder value maximization. In addition to "behavioral" explanations, my analysis suggests that, in some cases, it may be the shareholders' dominant strategy to create a corporate culture that places a disproportionate weight on the goal of being market share leaders.

I am by no means the first to suggest that committing to a course of action that departs from profit maximization may increase a firm's payoff. In these situations, a crucial issue is whether players have the power to commit to an ex-post sub-optimal course of action. For example, complex contracts may be difficult to observe or verify — and are subject to renegotiation. In this sense, the goal of being Number 1 seems compelling because it is simple — and simplicity is an important factor for credibility.

Appendix

Proof of Proposition 1: To be completed ■

Proof of Lemma 1: The game I consider has the structure of a "birth and death" Markov process (Kelly, 1979, Section 1.3); that is, from any given state i the only transitions to consider are to the neighboring states: $M(i,k) = 0$ if $|\, i-k\,| > 1$. These processes are recursive (Kelly, 1979, Lemma 1.5). It follows that they are also stationary. Recursiveness also implies that detailed balance holds (Kelly, 1979, Theorem 1.3), namely

$$M(i-1,i)\, m(i-1) = M(i,i-1)\, m(i) \tag{18}$$

The value of $M(i-1,i)$ corresponds to Nature selecting as an active consumer one of the consumers with the firm that currently has $\eta - i + 1$ consumers; and that agent switching to the other firm (that is, the firm currently having $i-1$ consumers). This happens with probability

$$M(i-1,i) = \frac{\eta - i + 1}{\eta}\, q(i-1)$$

Similarly,

$$M(i,i-1) = \frac{i}{\eta}\Big(1 - q(i)\Big)$$

Equation (18) allows me to compute the stationary distribution recursively. Given $m(0)$, we have

$$m(i) = m(0) \prod_{k=1}^{i} \frac{M(i-1,i)}{M(i,i-1)} = m(0) \prod_{k=1}^{i} \frac{q(i-1)}{1-q(i)} \cdot \frac{\eta - i + 1}{i}$$

Since $\sum_{k=0}^{\eta} m(k) = 1$,

$$m(0) = \left(1 + \sum_{i=1}^{\eta} \prod_{k=1}^{i} \frac{q(i-1)}{1-q(i)} \cdot \frac{\eta - i + 1}{i}\right)^{-1}$$

Equation (18) also implies that $m(i) > m(i-1)$ if and only if

$$\frac{\eta - i + 1}{\eta}\, q(i-1) > \frac{i}{\eta}\Big(1 - q(i)\Big)$$

By a similar argument, $m(i) > m(i+1)$ if and only if

$$\frac{\eta - i}{\eta}\, q(i) < \frac{i+1}{\eta}\Big(1 - q(i+1)\Big)$$

which concludes the proof. ■

Proof of Proposition 2: Suppose that $\delta = 0$. Then (6) becomes

$$w(i) = \Theta(i+1) - \Theta(i)$$

Moreover, since $\Lambda(i) = 0$, (13) becomes

$$\Gamma(i) = -\frac{i}{\eta}\Big(\Theta(i) - \Theta(i-1) - \Theta(j+1) + \Theta(j)\Big) - \frac{j}{\eta}\Big(\Theta(i+1) - \Theta(i) - \Theta(j) + \Theta(j-1)\Big)$$

It can be seen that, for all values of i, $\Gamma(i) = 0$. It then follows from (12) that $x(i) = 0$ and $q(i) = \frac{1}{2}$, for all i.

From (7), we get

$$p(i) = \frac{1}{2\,\phi(0)} - \frac{i}{\eta}\,\Big(\Theta(i) - \Theta(i-1)\Big) - \frac{j}{\eta}\,\Big(\Theta(i+1) - \Theta(i)\Big)$$

Substituting (14) for $\Theta(i)$, we get the values in the proposition.

I next turn to the stationary distribution of market shares. Since $\lim_{\delta\to 0}\; q(i) = \frac{1}{2}$, Lemma 1 implies that, in the limit as $\delta \to 1$,

$$m(i) = m(0)\;\prod_{k=0}^{i} \frac{\eta - i + 1}{i} = \frac{\eta!}{i!\,(\eta-i)!}$$

where

$$m(0) = \left(1 + \sum_{i=1}^{\eta}\;\frac{\eta!}{i!\,(\eta-i)!}\right)^{-1} = \left(\sum_{i=0}^{\eta}\;\frac{\eta!}{i!\,(\eta-i)!}\right)^{-1} = 2^{-\eta}$$

which leads to the expression for $m(i)$ in the result. Finally, setting $q(i-1) = q(i) = q(i+1) = \frac{1}{2}$, the Lemma 1 conditions for $m\big(\eta/2\big)$ to be greater than its neighbors become

$$\Big(\eta - \frac{\eta}{2} + 1\Big)\;\frac{1}{2} > \frac{\eta}{2}\left(1 - \frac{1}{2}\right) \qquad \text{and} \qquad \Big(\eta - \frac{\eta}{2}\Big)\;\frac{1}{2} < \Big(\frac{\eta}{2} + 1\Big)\left(1 - \frac{1}{2}\right)$$

both of which are equivalent to $\eta + 2 > \eta$. ■

Proof of Proposition 6: Define, for a generic variable x,

$$x^\circ \equiv x\,\big|_{\,\delta\,=\,0} \qquad\qquad \dot{x} \equiv \frac{\partial x}{\partial \delta}\,\Big|_{\,\delta\,=\,0}$$

Notice that $x^\circ(i) = w^\circ(i) = 0$; $q^\circ(i) = \frac{1}{2}$; $p^\circ(i) = p^\circ = \frac{1}{2\,\phi(0)}$; and $s^\circ(i) = s^\circ = \frac{1}{4\,\phi(0)}$. From (5) I get

$$v^\circ(i) = \frac{1}{4\,\phi(0)} + \frac{i}{\eta}\;\Theta(i-1) + \frac{j}{\eta}\;\Theta(i)$$

and thus

$$v^\circ(i) = \frac{1}{4\,\phi(0)} + \begin{cases} 0 & \text{if } \; i < \bar{\imath} \\ \frac{1}{4}\;\theta & \text{if } \; i = \bar{\imath} \\ \Big(\frac{3}{4} + \frac{1}{2\,\eta}\Big)\,\theta & \text{if } \; i = \bar{\imath} + 1 \\ \theta & \text{if } \; i > \bar{\imath} + 1 \end{cases} \tag{19}$$

From (6), I get

$$w^\circ(i) = \Theta(i+1) - \Theta(i)$$

and thus

$$w^\circ(i) = \begin{cases} 0 & \text{if } \; i < \bar{\imath} - 1 \\ \frac{1}{2}\;\theta & \text{if } \; i = \bar{\imath} - 1, \bar{\imath} \\ 0 & \text{if } \; i > \bar{\imath} \end{cases}$$

Moreover
$$\dot{w}(i) = v^{\circ}(i+1) - v^{\circ}(i)$$
Substituting (19) for v°, I get
$$\dot{w}(i) = \begin{cases} 0 & \text{if } \; i < \bar{\imath} - 1 \\ \frac{1}{4}\,\theta & \text{if } \; i = \bar{\imath} - 1 \\ \left(\frac{1}{2} + \frac{1}{2\,\eta}\right)\theta & \text{if } \; i = \bar{\imath} \\ \left(\frac{1}{4} - \frac{1}{2\,\eta}\right)\theta & \text{if } \; i = \bar{\imath} + 1 \\ 0 & \text{if } \; i > \bar{\imath} + 1 \end{cases}$$
From (11), I get
$$3\,\dot{x}(i) = -\frac{i}{\eta}\,\Big(\dot{w}(i-1) - \dot{w}(j)\Big) - \frac{j}{\eta}\,\Big(\dot{w}(i) - \dot{w}(j-1)\Big)$$
Specifically,
$$\begin{aligned} 3\,\dot{x}(\bar{\imath}+2) &= -\frac{\bar{\imath}+2}{\eta}\,\Big(\dot{w}(\bar{\imath}+1) - \dot{w}(\bar{\imath}-2)\Big) - \frac{\bar{\imath}-2}{\eta}\,\Big(\dot{w}(\bar{\imath}+2) - \dot{w}(\bar{\imath}-3)\Big) \\ &= -\frac{\bar{\imath}+2}{\eta}\;\dot{w}(\bar{\imath}+1) \\ &= -\frac{\bar{\imath}+2}{\eta}\left(\frac{1}{4} - \frac{1}{2\,\eta}\right)\theta \end{aligned}$$
From (10), I get
$$\begin{aligned} s^{\circ}(i) &= q^{\circ}(i)\,p^{\circ}(i) = \tfrac{1}{2}\,p^{\circ}(i) \\ \dot{s}(i) &= \dot{q}(i)\,p^{\circ}(i) + q^{\circ}(i)\,\dot{p}(i) \end{aligned} \tag{20}$$
Moreover, from (3) it follows that $\dot{q}(i) = -\dot{x}(i)$.

From (7), we get
$$\dot{p}(i) = -\frac{1}{2\,\phi(0)}\;\dot{x}(i) - \frac{i}{\eta}\;\dot{w}(i-1) - \frac{j}{\eta}\;\dot{w}(i)$$
Specifically,
$$\dot{p}(\bar{\imath}+2) = -\frac{1}{2\,\phi(0)}\;\dot{x}(\bar{\imath}+2) - \frac{\bar{\imath}+2}{\eta}\;\dot{w}(\bar{\imath}+1) - \frac{\bar{\imath}-2}{\eta}\;\dot{w}(\bar{\imath}+2)$$
Substituting into (20), I get
$$\begin{aligned} \dot{s}(\bar{\imath}+2) &= \dot{q}(\bar{\imath}+2)\,p^{\circ}(\bar{\imath}+2) + q^{\circ}(\bar{\imath}+2)\,\dot{p}(\bar{\imath}+2) \\ &= -\dot{x}(\bar{\imath}+2)\,p^{\circ}(\bar{\imath}+2) + q^{\circ}(\bar{\imath}+2)\left(-\frac{1}{2\,\phi(0)}\;\dot{x}(\bar{\imath}+2) - \frac{\bar{\imath}+2}{\eta}\;\dot{w}(\bar{\imath}+1)\right) \\ &= \frac{1}{2}\left(\frac{1}{2\,\phi(0)}\;\frac{\bar{\imath}+2}{\eta}\;\dot{w}(\bar{\imath}+1) - \frac{\bar{\imath}+2}{\eta}\;\dot{w}(\bar{\imath}+1)\right) \\ &= \frac{1}{2}\left(\frac{1}{2\,\phi(0)} - 1\right)\frac{\bar{\imath}+2}{\eta}\;\dot{w}(\bar{\imath}+1) \end{aligned}$$

and the result follows. ■

Proof of Proposition 3: Define a price war as the event that the system is in state i such that $i' \le i \le i''$. Let $T(i)$ be the expected duration of the state's stay in this set given that the current state i is in this set. Then the average duration of a price war is given by $T(i') = T(i'')$, since i' and i'' are the "entry states" that initiate a price war.

Given that we have a "birth-and-death" stochastic process, we may write

$$\begin{aligned} T(i) = 1 &+ \frac{i}{\eta}\, q(\eta - i)\, T(i-1) + \frac{\eta - i}{\eta}\, q(i)\, T(i+1) \\ &+ \left(1 - \frac{i}{\eta}\, q(\eta - i) - \frac{\eta - i}{\eta}\, q(i)\, T(i+1)\right) T(i) \end{aligned} \tag{21}$$

The solution to this difference equation is given by

$$\begin{aligned} T(i) &= T(i+1) + z(i) \\ z(i) &= \frac{1 + \frac{i}{\eta}\, q(\eta - i)\, z(i-1)}{\frac{\eta - i}{\eta}\, q(i)} \end{aligned} \tag{22}$$

There are two boundary conditions. First note that, for $i = \bar{\imath}$, the above "birth-and-death" equation implies that

$$T(\bar{\imath}) = 2 + T(\bar{\imath} + 1)$$

where I use the fact that, by symmetry, $T(\bar{\imath} - 1) = T(\bar{\imath} + 1)$. This implies that

$$z(\bar{\imath}) = 2 \tag{23}$$

Moreover, by definition we have $T(i'' + 1) = 0$. It follows that

$$T(i'') = z(i'') \tag{24}$$

where $z(i'')$ is obtained from the recursion (22) and the boundary condition (23).

As $\eta \to \infty$ and for a finite κ, (eq:diffeqsolution) converges to

$$z(i) = 4 + z(i-1)$$

Together with (24), this implies that the average duration of a price war converges to $4\,(i'' - \bar{\imath}) = 2\,\kappa$. Let ψ be the number of model periods per calendar period. At $\bar{\imath}$, the churn rate (fraction of buyers who switch sellers) is given by $1/(2\,\eta)$ per model period. This implies that, around $\bar{\imath}$, the churn rate is given by $\nu = \psi/(2\,\eta)$. Let $\beta = \kappa/\eta$ be the range of market shares corresponding to a price war. Then the average duration of a price war, in calendar periods, is given by

$$2\,\kappa/\psi = 2\,(\beta\,\eta)/(\nu\,2\,\eta) = \beta/\nu$$

I next consider the time spent in "peace" states. Note that the equation (21) applies to length of stay in peace states just as well as in price war states. It follows that the solution (22) also applies. The relevant boundary condition is now given by

$$T(0) = \frac{1}{q(0)}\, T(1)$$

which implies

$$z(0) = \frac{1}{q(0)} \tag{25}$$

The average length of a peace period is then given by $z(i')$, where $z(i')$ is obtained from the recursion (22) and the boundary condition (25). To be completed ■

Proof of Proposition 5: Suppose that $\delta = 0$. Sine $\Theta(i) = 0$, (6) becomes

$$w(i) = 0$$

It follows that (13) becomes

$$\Gamma(i) = -\Lambda(i) + \Lambda(j)$$

From (15), it follows that

$$\Gamma(i) = \begin{cases} \lambda & \text{if } \; i \le i' \\ -\lambda < \Gamma(i) < \lambda & \text{if } \; i' < i < i'' \\ -\lambda & \text{if } \; i \ge i'' \end{cases}$$

Let $x(\Gamma)$ be the solution to

$$x = \frac{1 - 2\,\Phi(x)}{\phi(x)} - \Gamma$$

From the proof of Proposition 1, $x(\Gamma)$ is increasing. Let $\bar{x}$ be the solution corresponding to $\Gamma = -\lambda$ and let $q' \equiv 1 - \Phi(\bar{x})$. The results for $q(i)$ and $p(i)$ then follow.

Finally, from Lemma 1, we know that

$$M(i-1,i)\,m(i-1) = M(i,i-1)\,m(i)$$

As $\lambda \to \infty$, $q(\eta/2-1) \to 0$ and $q(\eta/2-1) \to 1$. This implies that $M(\eta/2-1,\eta/2) \to 0$, whereas $M(i,i-1) = \frac{1}{4}$ for all values of λ. This in turn implies that $m(i) < m(i-1)$ for λ large enough. ■

References

Al-Najjar, Nabil, Sandeep Baliga, and David Besanko (2008), "Market Forces Meet Behavioral Biases: Cost Misallocation and Irrational Pricing," *Rand Journal of Economics* **39** (1), 214–317.

Andreoni, James, and B Douglas Bernheim (2009), "Social Image and the 50-50 Norm: A Theoretical and Experimental Analysis of Audience Effects," *Econometrica* **77** (5), 1607–1636.

Baumol, William J. (1962), "On the Theory of Expansion of the Firm," *American Economic Review* **52** (5), 1078–1087.

Cabral, Luís (2011), "Dynamic Price Competition with Network Effects," *Review of Economic Studies* **78**, 83–111.

Cabral, Luís, and Michael H Riordan (1994), "The Learning Curve, Market Dominance and Predatory Pricing," *Econometrica* **62**, 1115–1140.

Caminal, Ramon, and Xavier Vives (1996), "Why Market Shares Matter: An Information-Based Theory," *Rand Journal of Economics* **27**, 2. 221–239

Caves, Richard E, and Michael E Porter (1977), "From Entry Barriers to Mobility Barriers: Conjectural Decisions and Contrived Deterrence to New Competition," *Quarterly Journal of Economics* **91** (2), 241–262.

Ellison, Glenn (2006), "Bounded Rationality in Industrial Organization," in Blundell, Newey and Persson (Eds), *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress*, Cambridge University Press.

Fershtman, Chaim, and Kenneth Judd (1987), "Equilibrium Incentives in Oligopoly," *American Economic Review* **77**, 927–940.

Kelly, Frank P (1979), *Reversibility and Stochastic Networks*, Chichester: Wiley.

Klemperer, Paul (1989), "Price Wars Caused by Switching Costs," *Review of Economic Studies* **56** (3), 405–420.

Lazear, Edward, and Sherwin Rosen (1981), "Rank-Order Tournaments as Optimum Labor Contracts," *Journal of Political Economy* **89**, 841–864.

Shapiro, Carl (1989), "Theories of Oligopoly Behavior," in Schmalensee andWillig (Eds), *Handbook of Industrial Organization*, Volume 1. Amsterdam: North Holland.

Vickers, John (1985), "Delegation and the Theory of the Firm," *Economic Journal* **95**, 138–147.

Webpages consulted

http://online.wsj.com/article/SB10001424052702304371504577406830992897136.html
http://www.zdnet.com/blog/btl/ibm-reclaims-server-market-share-revenue-crown-in-q4-says-gartner/45334
http://money.cnn.com/2012/01/19/autos/gm_global_sales_leader/index.htm
http://www.zdnet.com/blog/btl/sap-ceo-we-are-not-arrogant-we-are-the-market-leader/4921
http://newsonsap.blogspot.com/2007_04_01_archive.html
http://www.ipo-group.com/Insights/Advantages_of_the_Market_Leade/advantages_of_the_market_leade.html
http://www.maloofrealty.com/about/88-market-share-leaders
http://ng2.pl/en/encomp/117-firma/hasla-promocyjne/391-we-are-the-market-leader-on-the-polish-footwear-retailing
http://www.brafton.com/news/top-google-organic-search-results-get-high-ctrs-800489801