Monopoly Differential Pricing and Welfare*

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Abstract. A firm often desires to charge different prices for its product to distinct consumer groups based on their different demands or marginal costs of service. When only costs differ, monopoly differential pricing generally raises both consumer and total welfare, compared to uniform pricing. Total welfare rises due to the output reallocation and quantity change effects: The pass-through from marginal cost to monopoly price dictates that at least one of these two effects must be positive (and dominate if the other is not), provided that demand satisfies a minor curvature condition. Consumers gain in aggregate, because to reallocate output the firm must vary prices, creating price dispersion that entails no upward bias in average price. We also contrast these findings with results under classic third-degree price discrimination, and provide sufficient conditions for beneficial differential pricing when both demand and cost differ.

Keywords: price discrimination, differential pricing, cost-based pricing.

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1. INTRODUCTION

A firm often wishes to charge different prices for its product to distinct consumer groups based on their different demands or marginal costs of service. What are the welfare properties of such differential pricing, as opposed to offering a uniform price to all consumers? An extensive literature, dating to Pigou (1920) and advanced recently by Aguirre, Cowan and Vickers (2010) and Cowan (2012), analyzes monopoly differential pricing when consumer groups differ only in demand elasticities—classic (third-degree) price discrimination. There is virtually no welfare analysis, however, of differential pricing when costs of service differ. Yet the issue has significant policy relevance: in important cases firms are constrained to price uniformly despite cost differences, as with geographically averaged pricing in traditional utility industries or gender-neutral pricing in insurance. (We elaborate on these and other examples shortly.) While common intuition may suggest that cost-based pricing unambiguously enhances welfare, the answer is not immediate under market power due to the latitude in pass-through from marginal cost to price.[1]

This paper presents a formal welfare analysis of monopoly differential pricing. Our focus is on understanding the welfare properties of (purely) cost-based pricing, but we also contrast the results to those under classic price discrimination and consider the mixed case with both demand and cost differences, thereby providing a comprehensive and unified treatment of the problem. To facilitate the comparison with classic price discrimination, we adopt the standard setting of that literature: under uniform pricing the firm serves two consumer groups or markets, and moving to differential pricing raises price in one market but lowers it in the other. The extant literature suggests that price discrimination (except where it opens new markets) is tilted against aggregate consumer welfare and is more likely to reduce than to increase even overall welfare, which includes the firm's profit. By contrast, we find that cost-based pricing raises aggregate consumer welfare under very broad demand conditions, and increases total welfare still more generally.

[1] Pass-through by firms with market power was first analyzed by Cournot (1838), and is shown by Weyl and Fabinger (2013) to be a powerful analytical device in numerous applications.

That cost-based pricing can *potentially* increase total welfare is obvious: With two markets having the same demand, under uniform pricing the high-cost market will have a lower price-cost margin, and hence reallocating *some* output to the low-cost market will yield cost savings that improve total welfare.[2] But this argument does not answer whether total welfare will be higher under *profit-maximizing* monopoly differential pricing. In fact, if the pass-through rate exceeds one, as with constant-elasticity demand, the monopolist may shift excessive output when changing from uniform to cost-based pricing, worsening the output allocation; and, in addition, it is also possible that the change in price regime results in a reduction in total output.

Nevertheless, decomposing the welfare changes under differential pricing into the reallocation and the output effects, we show that for all demand functions that satisfy a mild curvature condition, at least one of these two effects must be positive (and will dominate if the other is not) under cost-based pricing.[3] This is because the profit-maximizing pass through naturally connects these two effects: the reallocation effect can be negative only if the pass-through rate is larger than 1, but then the demand must be highly convex so that the price dispersion induced by a mean-preserving spread in marginal cost will result in a (large) increase in total output. The curvature condition, which ensures the positive effect to dominate when the other is not, holds widely for all demands for which the pass-through rate does not increase too fast, including those with any constant pass-through rate (such as constant elasticity demands).[4] Therefore, differential pricing will indeed generally increase

[2] Indeed, uniform pricing under different marginal costs entails discrimination as commonly defined, since the (zero) price difference does not reflect cost differences.

[3] For these demands, it then follows immediately that cost-based pricing increases welfare even if total output falls. (See also Bertoletti 2009, who focuses on the case of linear demands for which switching to cost-based pricing does not change total output, as we later discuss in Section 3.) In contrast, under classic price discrimination an expansion of total output is necessary but not sufficient for overall welfare to rise (Schmalensee 1981; Varian 1985; Schwartz 1990).

[4] Thus, as our analysis will further elucidate, both the level of the pass-through rate and its direction/speed of change will play key roles in determining the welfare comparisons between the pricing regimes. Equivalently, these key roles are played by both the level and the direction/speed of changes in the curvature of the (inverse) demand function.

total welfare, and our analysis illuminates the underlying economic logic for this result.

Aggregate consumer welfare also is shown to rise with cost-based pricing under broad conditions, including cases where output falls, with a curvature condition that is qualitatively similar to, albeit tighter than, the condition for total welfare. The mechanism is subtle, since the cost savings do not come from a decrease in the firm's marginal cost but from output reallocation, which does not benefit consumers directly. The reason consumers benefit is that in order to reallocate output the firm must vary its prices and consumers gain from the resulting price dispersion by purchasing more in the market where price falls and less where price rises. Importantly, this cost-motivated price dispersion does not entail an upward bias in the weighted-average price across markets—in sharp contrast to classic price discrimination.

As we alluded earlier, there is a wide range of industries in which government policy, private contracts, or customer perceptions have constrained firms to price uniformly across customer groups that impose different costs. "Universal service" regulation in the U.S. requires local telephone companies to set uniform rates for all residential customers across large geographic areas within which the costs of connecting premises can vary widely depending on customer density (Nuechterlein and Weiser 2007). Similar geographic averaging provisions apply in other utility industries such as electricity, water, and postal service, in the U.S. and elsewhere.[5] Gender-neutral rules provide another illustration. For instance, the European Court of Justice ruled that effective December 21, 2012, premiums or benefits for certain private insurance and private pension services in EU member states may not differ based on gender (ECJ 2011). Yet costs vary with gender, though the ranking differs across services so that price uniformity will favor women in some cases and disfavor them in others (Hyde and Evans 2011).[6] While such uniform-price mandates may reflect social

[5]These examples involve regulated industries, but are relevant to our analysis of a profit-maximizing monopolist insofar as price levels are not always tightly regulated, only the price structure (uniform versus differential) is. Under tight regulation, welfare-maximizing prices follow the familiar Ramsey principles—they increase with marginal cost and decrease with demand elasticity. For further analysis of third-degree price discrimination under regulation see, for example, Armstrong and Vickers (1991).

[6]The cost of providing a given yearly pension benefit (annuity) is higher for women since their life

goals, it is nevertheless important to understand their welfare implications.

Uniform price constraints have also arisen in payment card networks, such as Amex, Mastercard and Visa, which imposed no-surcharge rules barring merchants from surcharging when customers pay with a card instead of cash (Prager et al. 2009). Another broad class of examples involves resistance to add-on pricing. Sellers commonly offer a base good and optional add-on services that can only be consumed with the base good (Ellison 2005): airlines sell a ticket (the base good) and offer options such as checking a bag; hotels offer a room and extras such as phone service. Importantly, some consumers take the optional items while others do not. If an all-inclusive price is charged (bundled pricing), this represents uniform pricing across consumer groups that impose different costs depending on whether they use the add-ons or not. Pricing the add-ons separately can be used to implement cost-based pricing. At the same time, such unbundled pricing is often controversial because the add-on prices may substantially exceed the incremental costs and be partly motivated by demand differences across the customer groups—add-on pricing may implement indirect price discrimination.[7] Our analysis of differential pricing when demand and cost both differ across consumer groups can shed light on the welfare effects of these and other business practices.

The remainder of the paper is organized as follows. In Section 2, we characterize the

expectancy is longer than for men, so requiring equal benefits should lower the annuity for men and raise it for women. But the cost of providing women life insurance at a given age is lower than for men (due to women's longer life expectancy), so requiring equal premiums should harm women, and similarly for car insurance since women on average are safer drivers than men. Reportedly, the average pension annuity in the U.K. has fallen for men and risen for women since the EU's directive took effect, while the average car insurance premium has fallen for men and risen for women (Wall 2013).

[7] The above examples involve sales to final users, on which our analysis will focus. Uniformity constraints also arise in sales of inputs to competing firms, either explicitly or because cost differences are hard to verify. For instance, the U.S. Robinson-Patman Act, which prohibits price discrimination where it may substantially reduce competition among input purchasers, allows the seller to offer a cost-justification defense for different prices, e.g., that some buyers order in larger volume or perform more wholesale functions that relieve the seller of certain costs. But for many years the standard of proof demanded for cost differences was stringent, leading manufacturers to charge uniform prices to highly diverse distributors (Schwartz 1986).

monopolist's uniform and differential prices, provide bounds on the welfare changes due to a switch in pricing regimes, and present the total welfare decompositions that help understand subsequent results. Section 3 provides the analysis under linear demands, where the contrast of welfare effects between cost- and demand-based pricing is most transparent, and obtains the necessary and sufficient conditions for differential pricing to raise welfare when markets differ in both demands and marginal costs (Proposition 1). Section 4 conducts the general analysis of cost-based pricing, for broad demand functions that are equally elastic. We establish the sufficient condition for consumer welfare to rise (Proposition 2) and for total welfare to rise (Proposition 3), and contrast these results with those under classic price discrimination. Examples are presented where cost-based pricing enhances welfare despite a negative reallocation or output effect. Section 5 extends the analysis to situations where markets also differ in the elasticities of general demand functions, providing sufficient conditions for beneficial differential pricing (Proposition 4). Sections 4 and 5 also contain examples in which differential pricing reduces consumer and total welfare if the respective sufficient conditions are violated. Section 6 concludes.

2. PRICING REGIMES AND WELFARE BOUNDS

Consider two markets, H and L, with strictly decreasing demand functions $q^H(p)$, $q^L(p)$ and inverse demands $p^H(q)$, $p^L(q)$. When not necessary, we omit the superscripts in these functions. The markets can be supplied at constant marginal costs c_H and c_L, with $c_H \geq c_L$.

Denote the prices in the two markets by p_H and p_L. Profits in the two markets are

$$\pi^i(p_i) = (p_i - c_i)\, q^i(p_i), \text{ for } i = H, L,$$

and $\pi^i(p_i)$ is assumed to be strictly concave for the relevant ranges of prices. (As observed in fn. 4, this assumption is not innocuous.)

Under differential pricing, maximum profit in each market is achieved when $p_i = p_i^*$, where p_i^* satisfies

$$\pi^{i\prime} = q^i(p_i^*) + (p_i^* - c_i)\, q^{i\prime}(p_i^*) = 0.$$

We assume $p_H^* > p_L^*$.[8] In Robinson's (1933) taxonomy, H is the "strong" market while L is the "weak", though we allow the prices to differ (also) for cost reasons.

If the firm is constrained to charge a uniform price, we assume parameter values are such that both markets will be served (obtain positive outputs) at the optimal uniform price.[9] That price, $\bar{p}$, solves

$$\pi^{H\prime}(\bar{p}) + \pi^{L\prime}(\bar{p}) = 0.$$

The strict concavity of $\pi^i(p)$ and $p_H^* > p_L^*$ implies that $p_H^* > \bar{p} > p_L^*$, $\pi^{H\prime}(\bar{p}) > 0$, and $\pi^{L\prime}(\bar{p}) < 0$.[10] Let $\Delta p_L \equiv p_L^* - \bar{p} < 0$ and $\Delta p_H \equiv p_H^* - \bar{p} > 0$. Also, let $\Delta q_L = q^L(p_L^*) - q^L(\bar{p}) \equiv q_L^* - \bar{q}_L > 0$ and $\Delta q_H = q^H(p_H^*) - q^H(\bar{p}) \equiv q_H^* - \bar{q}_H < 0$.

Aggregate consumer surplus across the two markets, which we take as the measure of consumer welfare, is

$$S^* = \int_{p_H^*}^{\infty} q^H(p)\,dp + \int_{p_L^*}^{\infty} q^L(p)\,dp \tag{1}$$

under differential pricing. Aggregate consumer surplus under uniform pricing, $\bar{S}$, is obtained by replacing q_i^* with $\bar{p}$ in (1). The change in consumer surplus due to differential pricing is

$$\Delta S \equiv S^* - \overline{S} = \int_{p_L^*}^{\bar{p}} q^L(p)\,dp - \int_{\bar{p}}^{p_H^*} q^H(p)\,dp, \tag{2}$$

which, together with $p_H^* > \bar{p} > p_L^*$, $\Delta p_L < 0$ and $\Delta p_H > 0$, immediately implies the following lower and upper bounds for ΔS:

$$-\bar{q}_L \Delta p_L - \bar{q}_H \Delta p_H < \Delta S < -q_L^* \Delta p_L - q_H^* \Delta p_H. \tag{3}$$

That is, the change in consumer surplus is bounded below by the sum of price changes multiplied by outputs at the original (uniform) price, and is bounded above by the sum

[8] That is, price is higher in the market with (weakly) higher marginal cost. The assumption implies that cost is strictly higher in market H if demands in the two markets have the same price elasticity. In Section 3 we discuss briefly the alternative case where the market with the less elastic demand has the lower cost.

[9] When one market is not served under uniform pricing, discrimination may add new markets and can then yield a Pareto improvement (Hausman and MacKie-Mason 1988).

[10] The traditional assumption that price discrimination will move prices in opposite directions can fail if the profit function in at least one of the separate markets is not concave (Nahata et al. 1990, Malueg 1992), or if demand in each market also depends on the price in the other market (Layson 1998).

of price changes multiplied by outputs at the new (differential) prices. The result below follows immediately from (3) and provides sufficient conditions for differential pricing to raise or lower aggregate consumer surplus:

Lemma 1 *(i)* $\Delta S > 0$ *if* $\bar{q}_L \Delta p_L + \bar{q}_H \Delta p_H \leq 0$, *and (ii)* $\Delta S < 0$ *if* $q_L^* \Delta p_L + q_H^* \Delta p_H \geq 0$.

To see the intuition for part (i), suppose $\bar{q}_L \Delta p_L + \bar{q}_H \Delta p_H = 0$. If both demand curves were vertical at the initial quantities, consumers' gain in market L would exactly offset the loss in market H. But since demands are downward-sloping, consumers in market L gain more than $\bar{q}_L \Delta p_L$ by increasing the quantity purchased while consumers in H mitigate their loss by decreasing their quantity. These quantity adjustments imply $\Delta S > 0$. If $\bar{q}_L \Delta p_L + \bar{q}_H \Delta p_H < 0$, then $\Delta S > 0$ even before considering the quantity adjustments. A similar argument explains part (ii), because if the price changes are weighted by the *new* quantities, $q_L^* \Delta p_L$ will *overstate* the gain in L while $q_H^* \Delta p_H$ will *understate* the loss in H.

The condition in Lemma 1(i) for consumer surplus to rise also can be expressed as

$$\Delta S > 0 \text{ if } \left(\frac{\bar{q}_L}{\bar{q}_L + \bar{q}_H} \right) p_L^* + \left(\frac{\bar{q}_H}{\bar{q}_L + \bar{q}_H} \right) p_H^* \leq \bar{p}. \tag{4}$$

That is, differential pricing raises aggregate consumer surplus if the average of the new prices weighted by each market's share of the initial total output is no higher than the initial uniform price. This formulation highlights an important principle: Increased price dispersion that does not raise the weighted average price (weighted by the initial outputs) will benefit consumers overall, because they can advantageously adjust the quantities purchased.[11]

Now consider total welfare, the sum of consumer surplus and profit: $W = S + \Pi$. Since differential pricing increases profit (by revealed preference), total welfare must rise if consumer surplus does not fall; but if consumer surplus falls the change in welfare is ambiguous. It will be useful also to analyze welfare directly as total willingness to pay minus cost. Under differential pricing

$$W^* = \int_0^{q_L^*} \left[p^L(q) - c_L \right] dq + \int_0^{q_H^*} \left[p^H(q) - c_H \right] dq. \tag{5}$$

[11] This point, which follows simply from the negative slope of demand curves (convexity of the indirect utility function), dates back to Waugh (1944) and was also used in Newbery and Stiglitz (1981).

Welfare under uniform pricing, $\overline{W}$, is obtained by replacing q_L^* and q_H^* in W^* with $\bar{q}_L$ and $\bar{q}_H$. The change in total welfare from moving to differential pricing is

$$\Delta W = W^* - \overline{W} = \int_{\bar{q}_L}^{q_L^*} \left[p^L(q) - c_L\right] dq + \int_{\bar{q}_H}^{q_H^*} \left[p^H(q) - c_H\right] dq, \tag{6}$$

which, together with $\Delta q_L = q_L^* - \bar{q}_L > 0$ and $\Delta q_H = q_H^* - \bar{q}_H < 0$, immediately implies the following lower and upper bounds for ΔW:

$$(p_L^* - c_L)\,\Delta q_L + (p_H^* - c_H)\,\Delta q_H < \Delta W < (\bar{p} - c_L)\,\Delta q_L + (\bar{p} - c_H)\,\Delta q_H. \tag{7}$$

That is, the change in welfare is bounded below by the sum of the output changes weighted by the price-cost margins at the new (differential) prices; and it is bounded above by the sum of output changes weighted instead by the markups at the original (uniform) price.[12] From (7), we immediately have the following sufficient conditions for differential pricing to raise or lower total welfare:

Lemma 2 *(i)* $\Delta W > 0$ *if* $(p_L^* - c_L)\,\Delta q_L + (p_H^* - c_H)\,\Delta q_H \geq 0$, *and (ii)* $\Delta W < 0$ *if* $(\bar{p} - c_L)\,\Delta q_L + (\bar{p} - c_H)\,\Delta q_H \leq 0$.

As with Lemma 1, these results arise because demands are negatively sloped.

The insight from the price discrimination literature, that discrimination reduces welfare if total output does not increase, obtains as a special case of Lemma 2(ii) when $c_H = c_L$. When costs differ ($c_L < c_H$), part (i) of Lemma 2 implies:

Remark 1 *If differential pricing does not reduce total output compared to uniform pricing* ($\Delta q_L \geq -\Delta q_H > 0$), *then total welfare increases if the price-cost margin under differential pricing is weakly greater in the lower-cost than in the higher-cost market* ($p_L^* - c_L \geq p_H^* - c_H$).

Intuitively, the absolute price-cost margin (i.e., the marginal social value of output) under uniform pricing is higher in the lower-cost market L than in H ($\overline{p} - c_L > \overline{p} - c_H$), so welfare can be increased by reallocating some output to market L. Differential pricing induces

[12] Varian (1985) provides a similar expression for the case where marginal costs are equal.

such a reallocation, and if the margin in L remains no lower than in H then the entire reallocation is beneficial, hence welfare must increase if total output does not fall.

To distinguish the effects of output reallocation and a change in total output, we use the mean value theorem to rewrite (6) as

$$\Delta W = \left[p^L(\xi_L) - c_L\right] \Delta q_L + \left[p^H(\xi_H) - c_H\right] \Delta q_H,$$

where $\xi_L \in (\bar{q}_L, q_L^*)$ and $\xi_H \in (q_H^*, \bar{q}_H)$ are constants, with $p^L(\xi_L) < \bar{p}$ and $p^H(\xi_H) > \bar{p}$ representing the average willingness to pay in market L and market H, respectively. Let $\Delta q \equiv \Delta q_L + \Delta q_H$. Then, with $\Delta q_H = \Delta q - \Delta q_L$, we have the following decomposition of the welfare change due to differential pricing:

$$\Delta W = \underbrace{\left[p^L(\xi_L) - p^H(\xi_H)\right] \Delta q_L}_{\text{consumption misallocation}} + \underbrace{(c_H - c_L) \Delta q_L}_{\text{cost saving}} + \underbrace{\left[p^H(\xi_H) - c_H\right] \Delta q}_{\text{output effect}}, \tag{8}$$

where the first term is negative and represents the reduction in consumers' total value from reallocating output between markets starting at the efficient allocation under uniform pricing, the second term is positive and represents the cost savings from the same output reallocation to the lower-cost market, and the last term is the welfare effect from the change in total output (which takes the sign of Δq since price exceeds marginal cost).[13]

We can combine the first two terms in (8) and call it the (output) reallocation effect, as opposed to the (change in) output effect:

$$\Delta W = \underbrace{\left[(p^L(\xi_L) - c_L) - (p^H(\xi_H) - c_H)\right] \Delta q_L}_{\text{reallocation effect}} + \underbrace{\left[p^H(\xi_H) - c_H\right] \Delta q}_{\text{output effect}}. \tag{9}$$

When output does not decrease ($\Delta q \geq 0$), differential pricing increases welfare if the *average* value net of cost of the reallocated output is higher in market L: $p^L(\xi_L) - c_L > p^H(\xi_H) - c_H$. This is a weaker condition than $p_L^* - c_L \geq p_H^* - c_H$ in Remark 1 (since $p^L(\xi_L) > p_L^*$ and $p^H(\xi_H) > p_H^*$), but the latter condition may be more observable.

[13] Alternatively, one can use the output change in market H and write $\Delta W = -\left[p^L(\xi_L) - p^H(\xi_H)\right] \Delta q_H - (c_H - c_L) \Delta q_H + \left[p^L(\xi_L) - c_L\right] \Delta q$. Our decompositions are similar in spirit to expression (3) of Aguirre, Cowan and Vickers (2010), except that they consider infinitesimal changes in the allowable price difference and assume equal marginal costs hence no cost savings.

3. LINEAR DEMANDS

The case of linear demands highlights a sharp contrast between the welfare effects of classic price discrimination versus cost-based differential pricing. Relative to uniform pricing, classic price discrimination lowers consumer surplus and total welfare, whereas differential pricing that is motivated solely by cost differences will raise both.

Suppose that

$$p^i(q) = a_i - b_i q, \text{ where } a_i > c_i \text{ for } i = H, L.$$

Note that the demand elasticity in market i equals $p/(a_i - p)$, which depends only on the "choke price" a_i (the vertical intercept) and not on the slope. Under differential pricing,

$$p_i^* = \frac{a_i + c_i}{2}; \quad q_i^* = \frac{a_i - c_i}{2b_i}; \quad \pi_i^* = \frac{(a_i - c_i)^2}{4b_i},$$

and $p_H^* > p_L^*$ requires that $(a_H - a_L) + (c_H - c_L) > 0$. Under uniform pricing, provided that both markets are served:

$$\bar{p} = \frac{(a_H + c_H)\, b_L + (a_L + c_L)\, b_H}{2\,(b_L + b_H)}; \quad \bar{q}_i = \frac{1}{b_i}\left[a_i - \frac{(a_H + c_H)\, b_L + (a_L + c_L)\, b_H}{2\,(b_L + b_H)}\right].$$

It follows that

$$q_H^* - \bar{q}_H = -\left(q_L^* - \bar{q}_L\right) = -\frac{a_H - a_L + c_H - c_L}{2\,(b_H + b_L)} < 0.$$

Pigou (1920) proved this equal outputs result for linear demands when marginal cost depends only on the level of total output and not its allocation between markets. We showed that the result holds also when marginal costs across markets are different but constant:

Remark 2 *If both markets have linear demands, constant but possibly different marginal costs, and would be served under uniform pricing, then total output will be the same under uniform or differential pricing.*

We now can readily compare the change in welfare moving from uniform to differential pricing in two polar cases: (i) the classic price discrimination scenario where demand elasticities differ but costs are equal ($a_H > a_L$, but $c_H = c_L$), versus (ii) equal demand elasticities but different costs ($a_H = a_L$, but $c_H > c_L$).

Total Welfare. Since differential pricing leaves total output unchanged, the change in welfare is determined by the reallocation effect. When only demand elasticities differ, the reallocation effect is harmful since uniform pricing allocates output optimally while differential pricing misallocates consumption (see (8)). When only costs differ, uniform pricing misallocates output by under-supplying the lower-cost market L where the price-cost margin is higher ($\overline{p} - c_L > \overline{p} - c_H$). Differential pricing reallocates output to market L, and with linear demands the margin remains higher in market L at the new prices ($p_L^* - c_L = (a - c_L)/2 > (a - c_H)/2 = p_H^* - c_H$), implying from Remark 1 that welfare rises.

Consumer Surplus. When only demand elasticities differ (i.e., $a_H > a_L$ but $c_L = c_H$), moving to differential pricing causes the sum of the price changes weighted by the *new* outputs to be positive,

$$q_L^* \Delta p_L + q_H^* \Delta p_H = -\frac{(a_H - a_L)(a_L - a_H)}{4(b_H + b_L)} > 0.$$

So from Lemma 1(*ii*), consumer surplus falls. By contrast, when only costs differ ($a_H = a_L$, but $c_H > c_L$),

$$\bar{q}_H \Delta p_H + \bar{q}_L \Delta p_L = \frac{(a_H - a_L + c_H - c_L)(a_H - a_L)}{2(b_H + b_L)} = 0,$$

so by Lemma 1(*i*), consumer surplus rises: the sum of the price changes weighted by the *initial* outputs is zero, hence the weighted average price equals the initial uniform price and consumers gain due to the price dispersion (recall (4)).

The property that classic price discrimination creates an upward bias in the (weighted) average price extends beyond linear demands, as we will discuss in Section 4. There, we also show that cost-based pricing generally does not have this bias. When markets differ only in costs, differential pricing will not raise average price for the two markets if the pass-through rate is non-increasing, which holds for many common demand functions.

In the general case where both demand elasticities and costs may differ, from (2):

$$\Delta S = \frac{(a_H - a_L + c_H - c_L)\left[(c_H - c_L) - 3(a_H - a_L)\right]}{8(b_H + b_L)}. \tag{10}$$

It follows that

$$\Delta S > 0 \text{ if } a_H - a_L < \frac{c_H - c_L}{3}, \text{ and } \Delta S < 0 \text{ if } a_H - a_L > \frac{c_H - c_L}{3}. \tag{11}$$

Furthermore, since $\Delta\Pi = \frac{(a_H - c_H)^2}{4b_H} + \frac{(a_L - c_L)^2}{4b_L} - \bar{\pi}(\bar{p}) = \frac{(a_H - a_L + c_H - c_L)^2}{4(b_H + b_L)}$, we have

$$\Delta W = \Delta S + \Delta\Pi = \frac{(a_H - a_L + c_H - c_L)\left[3(c_H - c_L) - (a_H - a_L)\right]}{8(b_H + b_L)}. \tag{12}$$

Therefore,

$$\Delta W > 0 \text{ if } a_H - a_L < 3(c_H - c_L), \text{ and } \Delta W < 0 \text{ if } a_H - a_L > 3(c_H - c_L). \tag{13}$$

Recalling that a_i is the choke price in market i, we summarize the above results as follows:

Proposition 1 *If both markets have linear demands, a move from uniform to differential pricing has the following effects. (i) Total welfare increases (decreases) if the difference between markets in their choke prices is lower (higher) than three times the difference in costs* $(a_H - a_L < (>)\ 3(c_H - c_L))$*.(ii) Consumer surplus increases (decreases) if the difference in choke prices is lower (higher) than one third of the cost difference* $(a_H - a_L < (>)\ \frac{c_H - c_L}{3})$*.*

Thus, differential pricing is beneficial when the difference in demand elasticities (that motivates classic third-degree price discrimination) is not too large relative to the difference in costs. The condition for welfare to rise is less stringent than for consumer surplus since differential pricing raises profit, so $\Delta S \geq 0$ implies $\Delta W > 0$ but not conversely.[14]

Bertoletti (2009) also analyzes linear demands and considers n (≥ 2) markets. If higher-cost markets have (weakly) lower elasticities of demand at the uniform price as in our case, he shows that differential pricing reduces average cost by shifting output to the lower-cost markets—the cost-savings effect in our decomposition (8).[15] Although he does not analyze

[14] The condition for welfare to rise implies that the difference in margins between markets under differential pricing is less than under uniform pricing, $(p_H^* - c_H) - (p_L^* - c_L) < (\bar{p} - c_L) - (\bar{p} - c_H) = c_H - c_L$: the output reallocation to market H does not create a greater (but opposite) discrepancy in margins. The condition for consumer surplus to rise can be shown to imply that the weighted-average price under differential pricing is not sufficiently higher than the uniform price to outweigh consumers' gain from the price dispersion.

[15] He also provides sufficient conditions for total welfare and consumer surplus to rise based on Laspeyre or Paasche price variations in some special cases (as opposed to our necessary and sufficient conditions based on parameter values for the two markets).

the case where higher-cost markets have higher demand elasticities, in the two-markets case the effects can be understood from the logic of our Proposition 1 and decomposition (8). Continue assuming $a_H > a_L$, so demand is less elastic in market H, but suppose $c_H < c_L$. If the cost difference is small relative to $a_H - a_L$, differential pricing will raise price in market H and lower price in L, thereby shifting output from the lower-cost market and increasing the distortion, so total welfare and consumer surplus fall. However, if the cost difference is large enough ($a_H - a_L < c_L - c_H$), price will fall in market H and rise in L, yielding cost savings. From (10) and (12), consumer and total welfare will both rise.

4. EQUALLY ELASTIC DEMANDS

This section and the next extend the analysis beyond linear demand functions. For constant marginal cost c, the monopolist's profit under demand $q(p)$ is $\pi = q[p(q) - c]$. The monopoly price $p^*(c)$ satisfies $p(q) + qp'(q) - c = 0$. Let $\eta(p) = -pq'(p)/q$ be the price elasticity of demand (in absolute value); let $q^* = q(p^*(c))$; and let $\sigma = -qp''(q)/p'(q)$ be the curvature (the elasticity of the slope) of inverse demand, which takes the sign of $p''(q)$.

The pass-through rate from marginal cost to the monopoly price also will prove useful. As noted by Bulow and Pfleiderer (1983), the pass-through rate equals the ratio of the slope of inverse demand to that of marginal revenue. Thus,

$$p^{*\prime}(c) = \frac{p'(q^*)}{2p'(q^*) + q^* p''(q^*)} = \frac{1}{2 - \sigma(q^*)} > 0, \tag{14}$$

where we maintain the standard assumption that the marginal revenue curve is downward-sloping, so that $2p'(q) + qp''(q) < 0$ and hence $2 - \sigma(q) > 0$.[16] Therefore,

$$p^{*\prime\prime}(c) = \frac{\sigma'(q^*)}{[2 - \sigma(q^*)]^2} q'(p^*) p^{*\prime}(c) \leq 0 \tag{15}$$

if and only if

$$\sigma'(q) \geq 0. \tag{16}$$

[16] For future reference, note that $p^{*\prime}(c) < (=)$ or $> 1/2$ as $\sigma(q^*) < (=)$ or $>$ 0, i.e., depending on whether inverse demand is concave, linear or convex.

That is, the pass-through rate from marginal cost to the monopoly price will be non-increasing in marginal cost if and only if the curvature of the inverse demand is not decreasing in output (inverse demand is not less convex or more concave at higher q).

The curvature σ is non-decreasing for many common demand functions, including those with constant pass-through rates (Bulow and Pfleiderer, 1983): (i) $p = a - bq^{\delta}$ for $\delta > 0$, which reduces to linear demand if $\delta = 1$, with pass-through rate $p^{*\prime}(c) = 1/(1+\delta) \in (0,1)$; (ii) constant-elasticity demand functions $p = \beta q^{-1/\eta}$ for $\beta > 0$, $\eta > 1$, hence $p^{*\prime}(c) = \eta/(\eta - 1) > 1$; and (iii) $p = a - b\ln q$ for $a, b > 0$ and $q < \exp(a/b)$, which reduces to exponential demand ($q = e^{-\alpha p}$) if $a = 0$ and $\alpha = 1/b$, with pass-through rate $p^{*\prime}(c) = 1$.

To isolate the role of pure cost differences, this section abstracts from classic price discrimination incentives by considering demand functions in the two markets that have equal elasticities at any common price. This requires that demands be proportional to each other, which we express as $q^L(p) = \lambda q(p)$ and $q^H(p) = (1-\lambda) q(p)$ so that $q^L = \frac{\lambda}{1-\lambda} q^H$, for $\lambda \in (0,1)$.[17] A natural interpretation is that all consumers have identical demands $q(p)$ while λ and $(1-\lambda)$ are the shares of all consumers represented by market L and H, respectively. The function $q(p)$ can take a general form.

With proportional demands the monopolist's differential prices are given by the same function $p^*(c)$ but evaluated at the different costs: $p_L^* \equiv p^*(c_L)$, $p_H^* \equiv p^*(c_H)$. Let $\bar{c} \equiv \lambda c_L + (1-\lambda) c_H$. The optimal uniform price $\overline{p}$ maximizes $\pi(p) = \lambda(p - c_L)q(p) + (1-\lambda)(p - c_H)q(p) = [p - \bar{c}]q(p)$. Thus, $\overline{p} \equiv p^*(\bar{c})$: the monopolist chooses its uniform price as though its marginal cost in both markets were $\bar{c}$, the average of the actual marginal costs weighted by each market's share of all consumers. It follows that $\lambda p_L^* + (1-\lambda) p_H^* \leq \overline{p}$, or differential pricing does not raise average price for the two market, if $p^*(c)$ is concave ($p^{*\prime\prime}(c) \leq 0$), i.e., if the pass-through rate is non-increasing.

Proportional demands further imply that aggregate consumer surplus at any pair of prices

[17] For two demand functions $q = f(p)$ and $q = g(p)$, equal elasticities at any common price p imply $f'(p)p/f(p) = g'(p)p/g(p)$, hence $d(\ln f(p) - \ln g(p))/dp = 0$, so $\ln f(p) - \ln g(p)$ is constant, implying $f(p)/g(p)$ is constant (demands must be proportional). Conversely, proportional demands obviously have the same elasticities at any common price.

(p_L, p_H) equals $\lambda S(p_L) + (1-\lambda) S(p_H)$, i.e., consumer surplus in each market is obtained using the common function $S(p) \equiv \int_p^\infty q(x)\, dx$, but evaluated at that market's price and weighted by its share of consumers. Then, when $p^*(c)$ is concave (or if $\sigma'(q) \geq 0$ from (16)),

$$\begin{aligned} S^* &= \lambda S(p_L^*) + (1-\lambda) S(p_H^*) \\ &> S(\lambda p_L^* + (1-\lambda) p_H^*) \quad \text{(since } S(p) \text{ is convex)} \\ &\geq S(\bar{p}) \ \ \text{(since } \lambda p_L^* + (1-\lambda) p_H^* \leq \overline{p} \text{ by the concavity of } p^*(c)\text{)}. \end{aligned}$$

That is, when $\sigma'(q) \geq 0$ or the pass-through rate is non-increasing, which ensures that average price is not higher under differential than under uniform pricing, the price dispersion caused by differential pricing must raise consumer welfare.

Even if differential pricing raises the average price somewhat, as occurs when $\sigma'(q) < 0$ (hence $p^{*\prime\prime}(c) > 0$), consumer welfare will still increase due to the gain from price dispersion if $\sigma(q)$ does not decrease too fast, specifically

$$\sigma'(q) > -\frac{2-\sigma(q)}{q}, \tag{A1}$$

where the right hand side is negative since $2 - \sigma(q) > 0$ from (14).

Proposition 2 *Assume $q^L(p) = \lambda q(p)$ and $q^H(p) = (1-\lambda) q(p)$ for $\lambda \in (0,1)$. If (A1) holds, differential pricing increases consumer surplus relative to uniform pricing.*

Proof. First, we show that, if and only if (A1) holds, aggregate consumer surplus is a strictly convex function of constant marginal cost c. With demand $q(p)$, aggregate consumer surplus under $p^*(c)$ is

$$s(c) \equiv S(p^*(c)) = \int_{p^*(c)}^\infty q(x)\, dx.$$

Thus, $s'(c) = -q(p^*(c))\, p^{*\prime}(c)$ and

$$s''(c) = -q'(p^*(c)) \left[p^{*\prime}(c)\right]^2 - q(p^*(c))\, p^{*\prime\prime}(c).$$

Using the expressions for $p^{*\prime}(c)$ and $p^{*\prime\prime}(c)$ from (14) and (15), we have $s''(c) > 0$ if and only if (A1) holds.

Second, consumer surplus under differential pricing (S^*) and under uniform pricing ($\bar{S}$) are ranked as follows:

$$\begin{aligned} S^* &= \lambda S\left(p_L^*\right) + (1-\lambda) S\left(p_H^*\right) \\ &= \lambda s\left(c_L\right) + (1-\lambda) s\left(c_H\right) \\ &> s\left(\lambda c_L + (1-\lambda) c_H\right) \text{ (by the convexity of } s(c)) \\ &= S\left(p^*\left(\bar{c}\right)\right) = \bar{S}. \end{aligned}$$

■

We note that (A1) is satisfied by numerous demand functions, including those for which the pass-through rate is constant or decreasing. It is a fairly tight sufficient condition for differential pricing to raise consumer surplus, in the sense that it is the necessary *and* sufficient condition for consumer surplus as a function of constant marginal cost, $s(c)$, to be strictly convex.[18]

Condition (A1) can be equivalently stated as $\eta > p^*(c)\, p^{*\prime\prime}(c) / \left[p^{*\prime}(c)\right]^2$, the assumption on the pass-through rate made in Cowan (2012, p. 335). Cowan (2012) analyzes price changes due to pure price discrimination as if there were counterfactual changes in marginal costs.[19] Under the pass-through rate assumption or, equivalently, our (A1), he shows that discriminatory pricing will increase aggregate consumer surplus if, evaluated at the uniform price, the ratio of pass-through rate to price elasticity of demand is no lower in market L than in H (Cowan's Proposition 1(*i*)).[20] This condition turns out to be rather restrictive. Cowan notes: "The set of demand functions whose shape alone implies that [consumer]

[18] If $s''(c)$ has a consistent sign over the relevant range of c, then (A1) will also be the necessary condition for differential pricing to increase consumer welfare, but since in general $s''(c)$ may not have a consistent sign, (A1) is sufficient but may not be necessary.

[19] The analogy holds because the monopolist's uniform price $\overline{p}$ would be its optimal price for each market if, counterfactually, it faced different costs in the two markets: $\widehat{c}_H = MR_H(q_H(\overline{p})) < MR_L(q_L(\overline{p})) = \widehat{c}_L$ instead of the common marginal cost c. Whereas under differential pricing the monopolist sets prices based on c and the different demand elasticities.

[20] Intuitively, differing elasticities create a bias for discrimination to raise the average price. In order to offset this bias the demand curvatures must be such that the monopolist has a stronger incentive to cut price in the market where its virtual marginal cost fell than to raise price in the other market.

surplus is higher with discrimination is small. The surprise, perhaps, is that it is non-empty."[21] In contrast, when differential pricing is motivated solely by different costs, our Proposition 2 shows that (A1) *alone* is sufficient for consumer welfare to rise.[22]

The reason why differential pricing is more favorable for consumer surplus when it is based on different costs than on different demand elasticities (classic price discrimination) stems from the effects on the weighted average price. Since consumers would gain from pure price dispersion, Cowan's finding that classic price discrimination tends to reduce consumer surplus implies that discrimination tends to raise the average price. We showed this explicitly for linear demands, but the upward price bias under elasticity-based pricing is more general, as discussed next.

Recall that the monopolist's optimal uniform price $\bar{p}$ satisfies $0 < \pi^{H\prime}(\bar{p}) = -\pi^{L\prime}(\bar{p})$ or

$$\bar{q}_H \left(1 - \frac{\bar{p}-c}{\bar{p}}\eta_H(\bar{p})\right) = \bar{q}_L \left(\frac{\bar{p}-c}{\bar{p}}\eta_L(\bar{p}) - 1\right),$$

with $\frac{\bar{p}-c}{\bar{p}}\eta_H(\bar{p}) < 1 < \frac{\bar{p}-c}{\bar{p}}\eta_L(\bar{p})$. Under discrimination, the firm raises p_H until $\frac{p_H^*-c}{p_H^*}\eta_H = 1$ and lowers p_L until $\frac{p_L^*-c}{p_L^*}\eta_L = 1$. Since $q + (p-c)\,q' \gtreqless 0$ if $p \lesseqgtr p^*(c)$, we have

$$\frac{d\left(\frac{p-c}{p}\eta\right)}{dp} = \frac{-\left[q' + (p-c)\,q''\right]q + (p-c)\left[q'\right]^2}{q^2} \lesseqgtr -\frac{2q' + (p-c)\,q''}{q} \text{ if } p \lesseqgtr p^*(c),$$

where $-\frac{2q'+(p-c)q''}{q} > 0$ from the concavity of $\pi(p)$. Therefore, $\frac{p_L-c}{p_L}\eta_L$ monotonically increases in p_L for $p_L \in [p_L^*, \bar{p}]$; and while $\frac{p_H-c}{p_H}\eta_H$ also tends to increase in p, it increases at a slower rate (when it does) for $p_H \in [\bar{p}, p_H^*]$. Consequently, starting from $p_H = \bar{p} = p_L$, the same absolute change in $\frac{p_H-c}{p_H}\eta_H$ and $\frac{p_L-c}{p_L}\eta_L$,would require p_H to rise by more than the fall in p_L (i.e., there is a natural bias for $\Delta p_H > -\Delta p_L$).[23] Thus, if $\overline{q}_H = \overline{q}_L$, we immediately have $\overline{q}_H\Delta\, p_H + \overline{q}_L\Delta\, p_L > 0$: discrimination raises the weighted average price. If $\overline{q}_H > \overline{q}_L$,

[21] Specifically, his sufficient condition is only satisfied by two demand functions: logit demands with pass-through above one half, and demand based on the Extreme Value distribution (Cowan, pp. 340-1).

[22] The contrast between cost-based versus elasticity-based pricing is also seen from Cowan's Proposition 1(*ii*) which provides sufficient conditions for consumer surplus to fall. One such case is concave demands in both markets with the same pass-through rate (Cowan, p. 339). That case falls within our Proposition 2, hence consumer surplus would rise when differential pricing is motivated purely by different costs.

[23] This is obviously true for constant elasticities $\eta_H < \eta_L$, but our argument holds more generally.

the rise in p_H may be less than the fall in p_L, but the former is weighted by a larger output, so again the weighted average price is likely to rise (e.g., with parallel linear demands the absolute price changes are equal but price rises in the market where output is larger).

By contrast, there is no tendency for the average price to rise when differential pricing is purely cost-based, as we saw for any constant pass-through demand function. While the cost savings do not benefit consumers directly, to attain these cost savings the firm varies its prices without an upward bias, and consumers gain from the resulting price dispersion.

Total welfare increases with differential pricing more often than does consumer surplus since welfare includes profits which necessarily rise. At first glance it is not surprising that welfare increases with differential pricing when only costs differ: uniform pricing then creates a misallocation since $\bar{p} - c_L > \bar{p} - c_H$, hence reallocating *some* output to market L is beneficial. And if $p^{*\prime}(c) \leq 1$, then $p_H^* - p_L^* = \int_{c_L}^{c_H} p^{*\prime}(c)\, dc \leq \int_{c_L}^{c_H} dc = c_H - c_L$, hence from (9) the reallocation effect is positive. But if $p^{*\prime}(c) > 1$, then $p_L^* - c_L < p_H^* - c_H$, and differential pricing can *worsen* the output allocation (as we show later in an example). Nevertheless, our result below shows that cost-based differential pricing indeed increases total welfare quite generally. The result uses the following sufficient condition:

$$\sigma'(q) \geq -\frac{[3 - \sigma(q)][2 - \sigma(q)]}{q}, \tag{A1'}$$

where $3 - \sigma(q) > 1$ since $2 - \sigma(q) > 0$ from (14). Note that condition (A1') relaxes (A1). Condition (A1') is the necessary and sufficient condition for welfare to be a strictly convex function of marginal cost (as (A1) is for consumer surplus), yielding the next result whose proof is similar to that of Proposition 2 and therefore relegated to the Appendix.

Proposition 3 *Assume $q^L(p) = \lambda q(p)$ and $q^H(p) = (1 - \lambda) q(p)$ for $\lambda \in (0, 1)$. If (A1') holds, differential pricing increases total welfare.*

Since (A1') allows any constant pass-through rate, including much larger than 1, it encompasses cases where differential pricing creates a severe output misallocation. Further, it is also possible that differential pricing reduces total output. What then prevents welfare from falling? Recall that the pass-through rate is given by $p^{*\prime}(c) = \frac{1}{2 - \sigma(q^*)}$. When the pass-through rate does not exceed 1, the reallocation effect is positive and would outweigh a negative output effect should it arise. The reallocation effect *may* be negative if the pass-through exceeds 1, but this is possible only when the inverse demand curve is highly convex, with $\sigma > 1$ for $p^{*\prime}(c) > 1$, which in turn induces a larger output under differential pricing.[24] In short, the reallocation and the output effects are naturally connected through the profit-maximizing pass-through rate, so that in general at least one effect will be positive and, under (A1'), will dominate the other effect if the latter is negative.

It is instructive to compare the above result for cost-based differential pricing to the analysis of classic price discrimination by Aguirre, Cowan and Vickers (2010, ACV). They assume an increasing ratio condition (IRC): $z(p) = (p-c) / \left[2 - \frac{p-c}{p}\eta\sigma\right]$ strictly increases. ACV then show that price discrimination *reduces* welfare if the direct demand function in the strong market (our H) is at least as convex as in the weak market at the uniform price (ACV, Proposition 1). One can verify that $z'(p) > 0$ is equivalent to

$$\sigma'(q) < \frac{1}{-q'}\left[\frac{2 - \frac{p-c}{p}\eta\sigma}{p-c} + \frac{d\left(\frac{p-c}{p}\eta\right)}{dp}\sigma\right]\frac{1}{\frac{p-c}{p}\eta},$$

which, provided $\frac{d\left(\frac{p-c}{p}\eta\right)}{dp} \geq 0$, is satisfied if $\sigma'(q)$ is not too positive.[25] Therefore, the IRC condition in ACV and our (A1') both can be satisfied if $\sigma(q)$ neither increases nor decreases

[24]Recall from footnote 16 that $p(\cdot)$ is strictly convex if $p^{*\prime}(c) > 1/2$, or $\sigma > 0$. Thus, when $\sigma > 1$, $\lambda p(q_L^*) + (1-\lambda)p(q_H^*) > p(\lambda q_L^* + (1-\lambda)q_H^*)$. Therefore, if $p(\bar{q}) \geq \lambda p(q_L^*) + (1-\lambda)p(q_H^*))$, or even if $p(\bar{q})$ is somewhat lower, we will have $p(\bar{q}) > p(\lambda q_L^* + (1-\lambda)q_H^*)$, or $\bar{q} < \lambda q_L^* + (1-\lambda)q_H^*$. Intuitively, when $\sigma > 1$, starting from $\bar{p}$, output will rise by more with a price decrease that it will fall with a price increase. Similar logic underlies the finding in the price discrimination literature that discrimination can increase total output only if demand is less convex in the market where price rises (i.e., where demand is less elastic at the uniform price) than in the other market (Robinson 1933, Shih, Mai and Liu 1988, ACV 2010). Differing convexity is needed there to compensate for the fact that elasticity-based pricing is biased to raise the (output-weighted) average price across the markets. Malueg (1993) shows how concavity or convexity of demands in the two markets yield bounds on the percentage changein welfare moving from uniform pricing to discrimination.

[25]From ACV, condition $z'(p) > 0$ holds for a large number of common demand functions, including linear, constant-elasticity, and exponential. IRC neither implies nor is implied by our (A1').

too fast, which encompasses the important class of demand functions with constant σ. For these demand functions differential pricing that is purely cost based will *increase* welfare.[26]

The remainder of this section presents examples that further illustrate the channels through which differential pricing affects overall welfare and consumer surplus.

Example 1 *(Differential pricing reduces output but raises consumer and total welfare.) Suppose* $p = a - bq^\delta$, *with* $q = \left(\frac{a-p}{b}\right)^{1/\delta}$ *and* $\delta > 1$. *For* $c < a$, *we have* $p^*(c) = a - \frac{a-c}{\delta+1}$, $q^*(c) = \left(\frac{1}{b}\frac{a-c}{\delta+1}\right)^{1/\delta}$, *so* $q^*(c)$ *is strictly concave when* $\delta > 1$. *Hence*

$$\Delta q = (q_L^* + q_H^*) - (\bar{q}_L + \bar{q}_H) = \lambda q^*(c_L) + (1-\lambda) q^*(c_H) - q^*(\lambda c_L + (1-\lambda) c_H) < 0,$$

so differential pricing reduces total output. However, this demand function satisfies (A1). Thus, differential pricing increases consumer surplus and, hence, also total welfare.

Consumer surplus increases here because the weighted-average price is equal to the uniform price (since $p^{*\prime\prime}(c) = 0$) and the pure price dispersion benefits consumers. Welfare increases due to the reallocation effect, which is beneficial since the pass-through rate is less than one, $p^{*\prime}(c) = 1/(\delta+1)$, and in this case dominates the negative output effect.[27]

In the next example, differential pricing worsens output allocation, but the output expansion is large enough to dominate the negative reallocation effect.

Example 2 *(Differential pricing worsens allocation but raises consumer and total welfare.) Consider constant-elasticity demands:* $q^H \equiv q^L = p^{-\eta}$. *Then* $p^*(c) = c\frac{\eta}{\eta-1}$. *Suppose* $c_L = 0.1$, $c_H = 0.3$, $\eta = \frac{5}{4}$. *Then* $\bar{p} = 1$, $\bar{q}_L = \bar{q}_H = 1$; $p_L^* = 0.5$, $q_L^* = 2.3784$; $p_H^* = 1.5$, $q_H^* = 0.6024$. *Furthermore, from (6) and (9),* $p^L(\xi_L) = 0.6863$, $p^H(\xi_H) = 1.2122$, *and*

[26] ACV's Proposition 2 shows that under the IRC, welfare is *higher* with discrimination if the discriminatory prices are not far apart and the inverse demand function in the weak market is locally more convex than that in the strong market. (ACV's Proposition 5 shows that discrimination also increases welfare if σ is constant and larger than 1, under some additional conditions.) Our Proposition 3 shows that differential pricing motivated solely by cost differences increases welfare also when market demands have the *same* curvature.

[27] The reallocation is beneficial for any $\delta > 0$. If $\delta \leq 1$ (instead of > 1 as assumed thus far), then differential pricing would not lower total output, and the two effects would reinforce each other to increase total welfare.

hence $p^L(\xi_L) - c_L < p^H(\xi_H) - c_H$, *worsening the output allocation. But this demand satisfies (A1), so differential pricing raises both consumer and total welfare. Consumer welfare increases due to the price dispersion (the weighted average price is unchanged). Total welfare rises despite the negative and large reallocation effect (*$p^L(\xi_L) - c_L = 0.5863$, *while* $p^H(\xi_H) - c_H = 0.9122$*) since the latter is dominated by the output expansion.*

For proportional demands, although unusual, there are cases where (A1) does not hold and differential pricing *reduces* consumer surplus, as in the example below. (Additional examples with other demand functions are also available.)

Example 3 *(Differential pricing reduces consumer welfare.) Assume* $c_L = 0$, $c_H = 0.5$, $\lambda = 1/2$, *and logit demand* $q^L = \frac{1}{1+e^{p-a}} = q^H$; $p^L = a - \ln\frac{q}{1-q} = p^H$. *Let* $a = 8$. *Then* $p_L^* = 6.327$, $p_H^* = 6.409$, $\bar{p} = 6.367$; $q_L^* = 0.842$, $q_H^* = 0.831$, $\bar{q} = 0.837$. *Differential pricing now raises the average price and lowers output. Consumer welfare decreases:* $\Delta S = -8.59 \times 10^{-4}$; *but total welfare increases:* $\Delta W = 4.87 \times 10^{-4}$. *Notice that in this example, (A1) is violated when* $q > 0.5$, *but (A1') is satisfied for* $q < 1$ *(which is always true).*

We have not found examples where differential pricing reduces total welfare for demand functions that are everywhere differentiable. However, if demand is a step function so that its derivative does not exist at the kink, then it is possible that $W^* < \bar{W}$, as we show in the example below, where $p_i^* = \arg\max_p \pi^i(p)$ and $\bar{p} = \arg\max_p \left[\pi^H(p) + \pi^L(p)\right]$.

Example 4 *(Differential pricing reduces total welfare.) Assume* $c_L = 0.6$, $c_H = 1.4$, $\lambda = 1/2$, *and demand*

$$q^L = q^H = \frac{1}{2}\begin{cases} (2 - 0.5p) & if \quad 0 \le p \le 2 \\ (3 - p) & if \quad 2 < p \le 3 \end{cases}.$$

Then, $p_L^* = 2$, $p_H^* = 2.2$, $q_L^* = 0.5$, $q_H^* = 0.4$; $\bar{p} = 2$, $\bar{q}_L = 0.5 = \bar{q}_H$; *and* $\Delta W = -0.07$. *Notice that (A1') is not satisfied when* $p = 2$, *where the demand has a kink so that no derivative exists.*

In example 4, due to the kink in the demand function which makes the demand function concave, switching to differential pricing does not increase sales in the low-cost market but reduces sales in the high-cost market. Consequently, differential pricing reduces total welfare.

5. GENERAL DEMANDS

When demand is linear in both markets Proposition 1 showed that if the cost difference is sufficiently large relative to the demand difference, differential pricing will increase both total welfare and consumer surplus. It is not clear whether this result would extend to general demands, because as the cost difference grows the average price under differential pricing may rise faster than that under uniform pricing (as shown later in Example 6). To address the mixed case where there are differences both in general demand functions and in costs, we develop an alternative analytical approach that more clearly disentangles their roles, and use it to derive sufficient conditions for differential pricing to be beneficial.

Without loss of generality, let

$$c_H = c + t, \qquad c_L = c - t.$$

Then, $c_H - c_L = 2t$, which increases in t, and $c_H = c_L$ when $t = 0$. Thus, c is the average of the marginal costs and t measures the cost differential. For $i = H, L$, the monopoly price under differential pricing $p_i(t)$ satisfies $\pi^{i\prime}(p_i(t)) = 0$, from which we obtain:

$$p_H'(t) = \frac{q^{H\prime}(p_H(t))}{\pi^{H\prime\prime}(p_H(t))} = \frac{1}{2 + \frac{[p_H(t) - c_H]}{p_H(t)} p_H(t) \frac{q^{H\prime\prime}(p_H(t))}{q^{H\prime}(p_H(t))}}.$$

Since $\alpha \equiv -pq''/q'$ is the curvature of the direct demand function, $\frac{p^*(c)-c}{p^*(c)} = \frac{1}{\eta(p^*(c))}$, and $\sigma = \alpha/\eta$, we have, with $dc_L/dt = -dc_H/dt = -1$:

$$p_H'(t) = \frac{1}{2 - \sigma^H(q^H(\cdot))} > 0; \qquad p_L'(t) = -\frac{1}{2 - \sigma^L(q^L(\cdot))} < 0, \tag{17}$$

where $2 - \sigma^i\left(q^i(p_i(t))\right) > 0$ from (14).

Let $\bar{p}(t)$ be the monopoly uniform price, which solves $\pi^{H\prime}(\bar{p}(t)) + \pi^{L\prime}(\bar{p}(t)) = 0$, from which we obtain

$$\bar{p}'(t) = \frac{q^{L\prime}(\bar{p}(t)) - q^{H\prime}(\bar{p}(t))}{-\pi^{H\prime\prime}(\bar{p}(t)) - \pi^{L\prime\prime}(\bar{p}(t))}. \tag{18}$$

Thus $\bar{p}'(t) \geq (<)\ 0$ if $q^{L\prime}(\bar{p}(t)) \geq (<)\ q^{H\prime}(\bar{p}(t))$. Intuitively, an increase in the cost difference t leads the monopolist to raise the output-mix ratio q_L/q_H. This requires increasing the uniform price if q^L is steeper than q^H and lowering price if q^H is steeper. Define

$$\phi^i(q) \equiv \frac{q}{2-\sigma^i(q)}. \tag{19}$$

Then $\phi^{i\prime}(q) > 0$ for $i = H, L$ if and only if (A1) holds.

From (1), (17) and (19), the change in consumer welfare under differential pricing due to a marginal change in t is

$$\begin{aligned} S^{*\prime}(t) &= -q^L(p_L(t))\,p_L'(t) - q^H(p_H(t))\,p_H'(t) \\ &= \phi^L\left(q^L(p_L(t))\right) - \phi^H\left(q^H(p_H(t))\right). \end{aligned} \tag{20}$$

Under uniform pricing,

$$\overline{S}'(t) = -\left[q^L(\bar{p}(t)) + q^H(\bar{p}(t))\right]\bar{p}'(t), \tag{21}$$

which takes the opposite sign of $\bar{p}'(t)$ as determined by the relative slopes of the demand functions in (18). From (2), the difference in the changes of consumer welfare due to a marginal increase in t under the two pricing regimes, is equal to

$$\Delta S'(t) = S^{*\prime}(t) - \overline{S}'(t). \tag{22}$$

Consumer welfare will increase faster under differential than under uniform pricing with a marginal increase in t if $\Delta S'(t) > 0$, and, for any given $t > 0$, consumer welfare will be higher under differential pricing if $\Delta S(t) > 0$.

The result below provides two alternative sufficient conditions for consumer and total welfare to be higher under differential than under uniform pricing, encompassing the alternative cases where an increase in cost dispersion (t) raises or lowers the uniform price:

Proposition 4 *Suppose that (A1) holds and there exists some $\hat{t} \geq 0$ such that $\Delta S(\hat{t}) \geq 0$. Then, for $t > \hat{t}$, differential pricing increases consumer and total welfare if either (i) $\bar{p}'(t) \geq 0$ and $S^{*\prime}(\hat{t}) \geq 0$, or (ii) $\bar{p}'(t) \leq 0$, $\bar{p}''(t) \geq 0$, and $\Delta S'(\hat{t}) \geq 0$.*

Proof. First, for all $t \geq 0$, $S^*(t)$ is strictly convex:

$$S^{*\prime\prime}(t) = \phi^{L\prime}\left(q^L(p(t))\right) q^{L\prime}(p_L(t))\, p_L'(t) - \phi^{H\prime}\left(q^H(p_H(t))\right) q^{H\prime}(p_H(t))\, p_H'(t) > 0,$$

because $\phi^{i\prime}\left(q^L(p(t))\right) > 0$ from (A1), $q^{i\prime}(\cdot) < 0$, and $p_L'(t) < 0$ but $p_H'(t) > 0$ from (17). Next, for part (i), if $\bar{p}'(t) \geq 0$ and $S^{*\prime}\left(\hat{t}\right) \geq 0$, then from (22), for $t > \hat{t}$:

$$\Delta S'(t) \geq S^{*\prime}(t) > S^{*\prime}\left(\hat{t}\right) \geq 0,$$

which, together with $\Delta S\left(\hat{t}\right) \geq 0$, implies $\Delta S(t) > 0$ for all $t > \hat{t}$. Since $\Delta S(t) > 0$ implies $\Delta W(t) > 0$, this proves (i). Finally, if, for $t > \hat{t}$, $\bar{p}'(t) \leq 0$ and $\bar{p}''(t) \geq 0$, then using $\Delta S''(t) = S^{*\prime\prime}(t) - \overline{S}''(t)$ we obtain

$$\Delta S''(t) = S^{*\prime\prime}(t) + \left[q^{L\prime}(\bar{p}(t)) + q^{H\prime}(\bar{p}(t))\right] \bar{p}'(t) + \left[q^L(\bar{p}(t)) + q^H(\bar{p}(t))\right] \bar{p}''(t) > 0,$$

which, together with $\Delta S\left(\hat{t}\right) \geq 0$ and $\Delta S'\left(\hat{t}\right) \geq 0$, proves part (ii). ■

The sufficient conditions for differential pricing to benefit consumers under general demands include (A1), as with proportional demands, and either of the two additional conditions whose roles are as follows. Under condition (i), with $\bar{p}'(t) \geq 0$, under uniform pricing a marginal increase in t does not reduce price (and hence does not increase consumers surplus). Moreover, with $S^{*\prime}\left(\hat{t}\right) \geq 0$, consumer surplus increases in t at some $\hat{t} \geq 0$ under differential pricing, and (A1) further ensures that it will increase at an increasing rate. Hence, if consumer surplus is not too much lower under differential pricing with no cost difference, which is ensured by the assumption that there exists a $\hat{t}$ such that $\Delta S\left(\hat{t}\right) \geq 0$, then consumer surplus will be higher under differential pricing if the cost difference is sufficiently large $(t > \hat{t})$.

Even when $\bar{p}'(t) < 0$, so the uniform price falls with greater cost dispersion, consumer welfare can still be higher under differential pricing if $\bar{p}(t)$ does not fall too fast (i.e., $\bar{p}''(t) \geq 0$) while under differential pricing consumer welfare increases with cost dispersion fast enough (which is ensured by (A1) and $\Delta S'\left(\hat{t}\right) \geq 0$). Together with $S\left(\hat{t}\right) \geq 0$, condition (ii) then provides the alternative sufficient condition for welfare-improving differential pricing.

The demand conditions in Proposition 4 can be satisfied in numerous settings, even where classic price discrimination $(c_H = c_L)$ would reduce consumer welfare, as in many of the cases identified in ACV's Proposition 1. For instance, the linear demands case of Section 3 is covered by Proposition 4.[28] Example 6 in the Appendix shows that Proposition 4 also applies when $q^H(p)$ is an affine transformation of $q^L(p) \equiv q(p)$: $q^H(p) = a + bq^L(p)$, where $a \geq 0$, $b > 0$ and $q(p)$ satisfies (A1). (If $a = 0$, this reduces to proportional demands as in Section 4 .)

The example below applies Proposition 4 to a setting where q^H is neither an affine transformation of q^L nor linear. In this example, consumer welfare is lower under differential pricing when there is no cost difference ($t = 0$), but is higher when the cost difference is large enough.

Example 5 *Suppose that* $q^L = \frac{2}{3}(1-p)$, $q^H = (1-p)^{1/2}$, $c = 0.4$, $t \in [0,\ 0.3]$. *Then,* $p_L(t) = \frac{1.4-t}{2}$, $p_H(t) = \frac{2.4+t}{3}$, *and both markets are served under uniform pricing. When* $t = 0$, $\bar{p} = 0.770\,12$ *and* $\Delta S = -0.0015 < 0$, *so that classic price discrimination* ($c_H = c_L$) *reduces consumer welfare. However, since* $\sigma^L = 0$ *and* $\sigma^H = -1$, *(A1) is satisfied. Furthermore, (i) holds with* $\hat{t} = 0.1632$. *Therefore* $\Delta S > 0$ *for all* $t > \hat{t} = 0.1632$. *As expected, differential pricing increases total welfare for an even larger set of parameter values. In fact, in this example* $\Delta W > 0$ *for all* $t \geq 0$.

The next example shows that if the conditions in Proposition 4 are not met, differential pricing can reduce total welfare, hence also consumer surplus, even as $c_H - c_L$ becomes arbitrarily large (subject to the constraint that both markets will still be served under uniform pricing).

Example 6 *(Differential pricing reduces welfare for any cost difference.) Suppose that demands are* $q^L = 2(1-p)$ *and* $q^H = e^{-2p}$, *with corresponding marginal costs* c_L *and* c_H. *Then, under differential pricing,* $p_L^* = 0.5 + \frac{c_L}{2}$; $p_H^* = 0.5 + c_H$. *Notice that condition (i) in*

[28] Recall that $q^H = \frac{a_H - p}{b_H}$ and $q_L = \frac{a_L - p}{b_L}$, with $a_H > a_L$. When $b_L \geq b_H$, (A1) and (i) are satisfied, with $\hat{t} = \frac{3}{2}(a_H - a_L)$. When $b_L < b_H$, (A1) and (ii) are satisfied. Thus, if $t > \hat{t}$—implying $(c_H - c_L) > 3(a_H - a_L)$, the condition in part *(ii)* of Proposition 1—then differential pricing increases consumer welfare, even though for linear demands classic price discrimination reduces consumer welfare.

Proposition 4 is violated here since $\bar{p} \geq 0.5$ *and*

$$q^{L\prime}(p) = -2 < q^{H\prime}(p) = -2e^{-2p} \text{ for all } p \geq 0.5.$$

Thus, under uniform pricing $\bar{p}$ *would fall as the cost difference rises if average cost were kept constant. This force causes total welfare to be lower under differential than under uniform pricing. Table 1 illustrates this, where for convenience we have fixed* $c_L = 0$ *and considered increasing values of* c_H *(so that* $\bar{p}$ *increases, but less so than* $(p_L^* + p_H^*)/2$*, as average cost rises). For the entire range of parameter values in which both markets are served under uniform pricing (*$c_H \in (0, 0.539]$*), differential pricing reduces total welfare*[29]*:*

Table 1. $c_L = 0,\ p_L^* = 0.5,\ p_H^* = 0.5 + c_H,\ \Delta q = q^* - \bar{q}$

c_H	0.1	0.2	0.3	0.4	0.5	0.539
$\bar{p}$	0.515 1	0.529 6	0.543 3	0.556 5	0.5691	0.573 9
$\frac{p_L^*+p_H^*}{2}$	0.5500	0.6000	0.6500	0.7000	0.7500	0.769 5
Δq	−0.0255	−0.0409	−0.0489	−0.0503	−0.0469	−0.044
ΔW	−0.010	−0.011	−0.004	−0.044	−0.03 7	−0.034

Interestingly, in Example 6 the allocation of output is efficient under differential pricing (but not under uniform), since the markups are equal in the two markets: $p_H^* - c_H = p_L^* - c_L = 0.5$. However, average price under differential pricing $(0.5 + c_H/2)$ exceeds the uniform price $\bar{p}$ for all values of c_H and output is lower, which reduces welfare despite the improved allocation. By contrast, differential pricing improved welfare in Example 1 that exhibited pure cost differences, even though output fell there as well (but the average price equaled the uniform price for all cost values). The added incentive to raise average price under differential pricing when demand elasticities differ causes a stronger negative output effect here that outweighs the improved allocation.

[29] This implies that there exists no $\hat{t} \geq 0$ such that $\Delta S(\hat{t}) > 0$. Thus neither of the sufficient conditions in Proposition 4, (i) or (ii), can be applied.

6. CONCLUSION

Prevailing economic analysis of third-degree price discrimination by an unregulated monopolist paints an ambivalent picture of its welfare effects relative to uniform pricing. In order for overall welfare to rise total output must expand, and without specific knowledge of the shapes of demand curves the literature yields no presumption about the change in output unless discrimination leads the firm to serve additional markets. Moreover, since discrimination raises profits, an increase in overall welfare is necessary but not sufficient for aggregate consumer surplus to rise.

This paper showed that judging differential pricing through the lens of classic price discrimination understates its beneficial role when price differences are motivated at least in part by differences in the costs of serving various markets. Differential pricing then saves costs by reallocating output to lower-cost markets, and benefits consumers in the aggregate under broad demand conditions by creating price dispersion which—unlike classic price discrimination—does not come with a systematic bias for average price to rise.

Our analysis formalizes the intuition that price uniformity mandated in pursuit of social goals likely comes at a cost to aggregate consumer welfare. It also cautions against hostility in unregulated settings to differential pricing that is plausibly cost based, such as the common and growing practice of add-on pricing that unbundles the pricing of various elements from the price of the base good. An important extension would be to analyze whether and how the beneficial aspects of differential pricing under different costs might extend beyond monopoly to imperfect competition, building on the analyses of oligopoly price discrimination (e.g., Stole 2007).

APPENDIX

Proof of Proposition 3. First, we show that if and only if (A1') holds, total welfare is a strictly convex function of constant marginal cost c. Total welfare under $p^*(c)$ is

$$w(c) \equiv W(p^*(c)) = \int_0^{q(p^*(c))} [p(x) - c]\, dx.$$

Thus, $w'(c) = [p^*(c) - c]\, q'(p^*(c))\, p^{*\prime}(c) - q(p^*(c))$. From the first-order condition for $p^*(c)$, we have $[p^*(c) - c]\, q'(p^*(c)) = -q(p^*(c))$. Hence

$$w'(c) = -q(p^*(c))\, p^{*\prime}(c) - q(p^*(c)) = -q(p^*(c)) \left[p^{*\prime}(c) + 1\right],$$

$$w''(c) = -q'(p^*(c))\, p^{*\prime}(c) \left[\frac{1}{2 - \sigma(q^*)} + 1\right] - q(p^*(c)) \frac{\sigma'(q^*)}{[2 - \sigma(q^*)]^2} q'(p^*)\, p^{*\prime}(c).$$

Therefore, $w''(c) > 0$ if and only if

$$3 - \sigma(q^*) + q(p^*(c)) \frac{\sigma'(q^*)}{2 - \sigma(q^*)} > 0,$$

or if and only if (A1') holds. Next,

$$\begin{aligned} W^* &= \lambda W(p^*(c_L)) + (1 - \lambda) W(p^*(c_H)) \\ &= \lambda w(c_L) + (1 - \lambda) w(c_H) \\ &> w(\lambda c_L + (1 - \lambda) c_H) \ \text{(by the convexity of } w(c)) \\ &= W(p^*(\bar{c})) = \bar{W}. \end{aligned}$$

■

Example 7 *(Applying Proposition 4 to cases where $q^H(p)$ is an affine transformation of $q^L(p) \equiv q(p)$) Suppose that $q^H(p) = a + bq(p)$, where $a \geq 0$, $b > 0$ and $q(p)$ satisfies (A1). When $b \geq 1$, then $q^{L\prime} \geq q^{H\prime}$ (demand is steeper in market L). If in addition both a and b are not too large, there will exist some $\hat{t}$ such that $\Delta S(\hat{t}) \geq 0$ and $q(p_L(\hat{t})) \geq a + bq(p_H(\hat{t}))$ (which ensures $S^{*\prime}(t) \geq 0$). Hence condition (i) is met. When $b < 1$, we have $q^{L\prime} < q^{H\prime}$ and $\bar{p}(t) = p^*(\tilde{c})$, where $p^*(\tilde{c})$ is the optimal price under marginal cost $\tilde{c} = \left(c - \frac{1-b}{1+b}t\right)$ and demand $\tilde{q} = \frac{a}{1+b} + q(p)$. Hence, from (14), $\bar{p}'(t) = -\frac{(1-b)}{1+b} p^{*\prime}(\tilde{c})$, and*

$\bar{p}''(t) = \left(\frac{1-b}{1+b}\right)^2 p^{*\prime\prime}(\tilde{c}) \geq 0$ *if* $p^{*\prime\prime}(\tilde{c}) \geq 0$. *Therefore, if* $\Delta S(0)$ *and* $\Delta S'(0)$ *are not too negative, which would be true if* a *is not too high, condition (ii) will be satisfied when* t *is large. Summarizing: for* $q^H(p) = a + bq^L(p)$, *suppose that (A1) holds and* a *is not too high. Differential pricing increases consumer and total welfare when the cost difference is large enough if either (i)* $b \geq 1$ *but not too high or (ii)* $b < 1$ *and the pass-through rate is non-decreasing.*

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