From growth to cycles through beliefs

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Abstract

I present a theoretical model where the economy endogenously adopts the technological ideas of a slowly evolving technological frontier, and show that the presence of a "technological gap" between unadopted ideas and current productivity can lead to multiple equilibria and therefore the possibility that changes in beliefs can be self-fulfilling, often referred to as sunspots. In the model these sunspots take the form of beliefs about the value of adopting the new technological ideas, and unleash both a boom in aggregate quantities as well as eventual productivity growth, increasing the value of adoption and self-confirming the beliefs. Moreover, I demonstrate that the scope for these indeterminacies is a function of the steady-state growth rate of the underlying technological frontier of ideas, and that during times of low growth in ideas, the potential for indeterminacies disappears. Under this view, technology becomes important for cycles not necessarily because of sudden shifts in the technological frontier, but rather, because it defines a technological regime for the economy such that expectations about its value can produce aggregate fluctuations where in a different regime they could not.

KEYWORDS: expectations-driven business cycle, sunspot, multiple equilibria, indeterminacy, technology, news shock, intangible capital, investment-specific technical change, embodied, technological transition, technological adoption

JEL CLASSIFICATION: C62,C68,E00,E2,E3,O3,O4

1 Introduction

Can technology be important for aggregate fluctuations even if technology *shocks* are not? Much of the debate about the extent to which fluctuations are related to technology is framed in the context of theoretical models which involve sudden shocks to innovation or technology - either unanticipated or anticipated - and empirical methods that seek to identify these shocks. Yet accounts such as that of David (1990) of the larger periods that spawned the booms that many would argue to be the most likely related to technology - such as the information technology (IT) boom of the 1990's - reveal slow transformational shifts in the role of technology in the economy well before the boom periods. In the 1990's case in particular, we see a culmination of a technological revolution that began 20 years prior in the 1970's. Does viewing a boom embedded in a technological transition simply as a stochastic shift of the technological frontier unconnected to the processes driving the transition give us a complete picture of the forces driving the boom? Or rather, are these booms somehow related to the lower-frequency transformational change that preceded them?

In this paper I argue that an important feature of technological change is to define a particular *technological regime* that may influence the high and medium frequency dynamics of the economic system independent of any shocks to technology itself. I present a theoretical model where a frontier of "technological ideas" evolves gradually and without shocks, yet where the economy endogenously adopts these ideas into production at higher frequencies based on agents' self-fulfilling beliefs about the value of adoption. This frenzy of adoption then leads to a boom in aggregate quantities followed by an eventual increase in productivity growth. As a result, the rate of realization of the benefits provided by the technological frontier is a function of changes in beliefs about the value of technology as opposed to sudden shocks to technology itself as in many models in the literature. Yet the underlying nature of the frontier is critical to determining the possibility that changes in beliefs can be self-fulfilling and thus produce an adoption boom. The self-fulfilling beliefs in the model take the form of stationary sunspot equilibria associated with indeterminacies due to the presence of a "technological gap" between unadopted ideas and current productivity, and I demonstrate that the scope for these indeterminacies is a function of the steady-state growth rate of the frontier. When growth in ideas is high and a sufficient gap exists between ideas and productivity, indeterminacies can exist, whereas during times of low growth in ideas, the potential for indeterminacies diminishes and thus reduces the dimension of the state-space of possible shocks. Under this view, technology becomes important for cycles not necessarily because it produces sudden shifts in the frontier, but rather, because it defines a technological regime for the economy whereby beliefs about its value can produce aggregate fluctuations where in a different regime they could not. To the extent then that a technological transition such as the IT revolution of the 1970's-1990's spawns an extended period of growth in transformational ideas, the transition thus defines a technological regime which creates the possibility that beliefs about the technology can be self-fulfilling, providing for the existence of a belief driven boom. Thus the model is consistent with the view that a boom such as the 1990's may have resulted from sudden optimism of the benefits of the transformational ideas that had gradually evolved out of the preceding years of the IT revolution.

This "animal spirits" or self-fulfilling view of cycles is of course not new. The ideas themselves grew popular from the writing of Keynes, and through the 1980's and 1990's researchers such as Howitt and McAfee (1992), Benhabib and Farmer (1994) and Farmer and Guo (1994) formalized them into rational expectations models of aggregate fluctuations driven by beliefs. Yet the vast majority of these models exploit externalities or other structural features of the economy that are arguably structurally-ever-present in goods production or its connected markets, whether during a technological transition or not, and in this regard could be argued to describe "normal" business cycles as a result of random fluctuations of beliefs. Was the boom of the 1990's simply a result of a random wave of optimism unconnected to the IT revolution, or was there something different about this period that catalyzed this response? Was there something different about the boom of the 1990's compared to that from 2001-2007? I argue that from the perspective of models of aggregate fluctuations, a period such as the 1990's is in fact different because the preceding technological transition introduced structural features that *allowed* beliefs to become important in a way that they would not during a "normal" cycle. One important implication of this is that technology need not *always* be important for cycles: just as a technological transition can define a regime with a high-growth in technological ideas that enables a boom, a period where the distance between the frontier of new ideas and those ideas already in practice is low - resulting from either a stagnation in ideas growth or a previous adoption boom which exhausted further gains - could limit the role of technology in a belief-driven boom.

In representing growth due to technological transitions, I focus on that portion of growth which is ultimately driven by the collective knowledge of firms to optimally exploit the era of physical capital embodying a given technological paradigm in order to reorganize goods production. This core of embodied knowledge represents a frontier of nonrivalrous and nonexcludeable "technological ideas" that confronts the economy in a given technological era. Individual firms then increase their firm-specific productivity by implementing these publicly-available technological ideas through a costly adoption process during which they learn how to tailor and apply these general ideas to their specific production processes. Since these technological ideas relate to the use of a specific era of capital however, firms must first purchase the new capital to exploit the production ideas that the capital embodies. Physical capital as a result plays an extremely critical role in this economy beyond its standard role in goods production by acting as a conduit of technological ideas and knowledge.

Both this critical role for physical capital as well as the delayed realization of productivity benefits is motivated by a host of studies following the boom of the 1990's that link information technology capital with eventual delayed productivity gains. For example, Basu and Fernald [6] study a data-set of 40 industries in the U.S. over the period of 1986 to 2004 and find that TFP gains after mid-1990's were broad-based across industries, located primarily in information and communications technology (ICT) capital-*using* rather than *producing* industries, and that industry TFP accelerations in 2000's were positively correlated with industry ICT capital growth in mid-1990's.

It is important to note that in contrast to many approaches that attempt to establish a link between bottom-up firm/sectoral-level innovation or idea-creation and aggregate fluctuations, in this model I focus on a top-down diffusion of *existing* technological ideas onto firms' production processes. While these technological ideas would no doubt in reality originate within individual firms or sectors as part of a larger learning or innovation process as firms interact with a new technological paradigm, I assume that no single idea or innovation is material enough to impact aggregate-level fluctuations. Rather, a collection of ideas and knowledge across firms and sectors within a given technological paradigm is necessary for an aggregate impact at higher frequencies. As such, I make no attempt to model micro-level idea-creation. I simply use a deterministic growth rate of publicly available ideas as a proxy for the gradual process through which an entire economy digests a host of disparate firm-level ideas into a coherent structure of publicly-available technological knowledge that all firms may then draw upon.

To give some concrete substance to what I am referring to as "technological ideas" it is helpful to look at a specific example. One such candidate is the concept of supply chain management which was popularized in the 1990's during the IT revolution and involved the process by which firms plan and manage the flow of goods through the various stages of their businesses, from procurement to production to distribution. Most observers generally acknowledge that the IT revolution began many years prior in the mid-1970's, and over the next ten to fifteen years the primary impact of this revolution within incumbent firms in the economy was to automate or replace many individual tasks within those firms. Eventually as the late 1980's approached however, this gave way to a system of transformational ideas about how to reorganize processes within firms, between firms, and between firms and customers to exploit the connectivity, visibility and analytics provided by the the new hardware and software technologies. Importantly, these ideas were not the result of any one innovation by a firm or sector, but rather were well known and in the public domain in business schools, consultancies and within those in strategic capacities in the businesses themselves by early 1990's when the firms began to implement them, evolving slowly over time through the various efforts of those who interacted and studied the newly emerging technologies. Finally, while these ideas may have existed in the public domain, in order to implement them firms first had to purchase the necessary hardware and software infrastructure to interact with their existing capital stock.

To model the endogenous adoption process, I use a costly-adoption specification similar to Greenwood and Yorukoglu [45], Comin and Hobijn [24], and Nelson and Phelps [62] that models the movement of productivity in practice towards some limit defined by the technological frontier, with the distance between productivity in practice and that of the frontier being commonly referred to as the "technological gap". I then extend this concept to an environment of capital-specific learning where the effective frontier faced by an individual firm is endogenously-controlled by that firm as a function of its history of purchases of physical capital. This reliance on investment in new physical capital to push a firm's effective frontier forward is a critical ingredient in producing indeterminacies in the model. Starting from some arbitrary equilibrium path, a conjectured belief by the firm about an increase in return to adoption immediately drives up the value of increasing the effective frontier that it faces in adoption, which being a function of investment in physical capital means that during adoption physical capital provides an additional return to the firm beyond its direct impact in goods production. Combined with the process of costly adoption of the firm's effective frontier this pursuit of investment then interacts with the labour market resulting in shifts in both labour demand and supply that increase the return to all the firm's accumulated factor inputs. Yet since all firms ultimately confront a given frontier of established technological ideas within a given technological paradigm, with each additional unit of productivity created firms know that they are closing in on the technological limit, and thus through time firms value new adoption less and less, imposing the necessary stability on the system to return it to the balanced growth path. Thus the presence of the "technological gap" is critical to the dynamics of the model economy, since despite the high returns to investment in adoption, the exogenous technological frontier means that the economy overall faces a dynamic diseconomy of scale as it "closes the gap" on the theoretical frontier, invoking an effect similar to that illustrated by Howitt and McAfee [50], who show how counteracting multiple externalities can lead to locally stable equilibria in systems with multiple equilibria. At the core of this idea in my model is the interplay between medium to high-frequency efforts of the economy to move towards its theoretical frontier in response to changes in beliefs, and the low-frequency phenomena that move the frontier forward and thus constrain the economy's short-run expansionary efforts.

The remainder of the paper is structured as follows. In Section 2 I discuss how my approach relates to other works in the literature not discussed above. In Section 3 I describe the technological environment, outline the model, and define the equilibrium and balanced growth path. In Section 4 I investigate the role of indeterminacies in the model, illustrating the dependence of the scope for indeterminacies on the underlying growth regime, and examining the response of the model economy to sunspot shocks. Section 5 concludes.

2 Relation to existing literature

In this section I give a brief explanation of how the ideas and elements in this paper relate to other research in the literature.

In attempting to make a link between endogenous adoption and aggregate fluctuations, I am proceeding in the spirit of Comin, Gerter and Santacreu [23], who model an endogenous adoption process in response to stochastic shifts in the technology frontier. Unlike Comin et al however, I attempt to investigate the link between endogenous adoption and aggregate fluctuations in the absence of any sudden shocks to the technology frontier, and in an environment where the only form of uncertainty is changes in beliefs about the value of adoption.

By considering the relation between adoption and beliefs, I draw on ideas similar to those in implementation models such as Shleifer [68] and Francois and Lloyd-Ellis [38], [39] whereby firms must choose to implement a new technology to realize its productivity benefits. While these papers show how clustering of firm-level implementations can lead to endogenous cycles in an environment where profits related to the innovation are short-lived, I focus on the expanding role of physical capital to enable adoption, as well as the dynamic interaction of a slowly evolving technology and the constraint it places on an economy that adopts to it at varying rates driven by beliefs. Moreover, while Schleifer investigates the role of adoption during "normal" cycles where the benefits of innovation occur immediately after adoption, I focus on sub-periods where technological change alters the dynamics of the economy and produces delayed realization of benefits.

In drawing a connection between growth and cycles, I focus on a different aspect than that emphasized by researchers working with endogenous growth models, such as King and Rebelo [59], Comin and Gertler [22] and Comin [21]. These papers integrate growth and cycles based on a core of an endogenous growth model whereby temporary shocks unrelated to technology can have a permanent effect on output through the innovative activities of the firms, impacting the aggregate stock of "innovations" that ultimately lead to permanent growth. In contrast, in my model there is no endogenous growth; the aggregate stock of ideas evolves smoothly and unconnected to the business cycle, yet beliefs about the value of the technology can affect the *rate* that the economy realizes these benefits. Moreover, I focus on how a change in technological regime would cause the economy to respond differently during for example the 1970's versus the mid-1990's, as opposed to investigating the role of technology or innovation in "all" cycles. Additionally, in proposing an alternative connection between growth and medium-frequency fluctuations, I take advantage of the work of Comin and Gertler [22] and Comin [21] that presents evidence linking high and low-frequency fluctuations. I highlight, however, a direction of causation that works in reverse: whereas they describe how low-persistence shocks that produce business cycles can then lead to medium frequency fluctuations, in my model the emergence of certain growth regimes provides for the possibility of belief shocks that produce an immediate short-run burst of activity that eventually settles into a persistent response well into the medium-term frequencies.

In the sense that the role of beliefs relies on the underlying fundamentals of technology, my work is also related to the "news shock" idea spearheaded in a recent literature by Beaudry and Portier [11], and further investigated by Christiano et al [27], Jaimovich and Rebelo [52], Comin, Gertler and Santacreu [23], Gunn and Johri [46], and Dupor and Mehkari [30] [1]. Under this idea, agents act on information they receive about expected changes in future technological fundamentals. This contrasts with my model where there are no sudden shocks in the frontier. Furthermore, I am modeling self-fulfilling beliefs whereby the economy must endogenously create the growth that confirms its expectations, rather than having the realized change in productivity be independent of the actions of the agents, as in most of the models in the news literature with the exception of Comin, Gertler and Santacreu [23], and to some extent Gunn and Johri [46]. Nevertheless, the ideas from the news shock literature and the concept I highlight in this paper are complementary. I focus on the dynamics of the economy conditional on a certain underlying growth rate of ideas, and don't consider the dynamics of the transition between regimes of low or high growth in ideas. One could argue that the ideas of the news shock literature become especially important at the interface of transitions between growth regimes, as agents in the economy begin to learn about the increased growth potential and new stock of innovations of the new technological regime. In this regard, both the modeling strategies of Gunn and Johri [46] and Comin, Gertler and Santacreu [23] are especially complementary to this model. In a similar vein, my model is complementary to the ideas of Aghion and Howitt [1] who provide an endogenous rationale of why a boom may arise from a slow growth process, but nevertheless share a similar focus with the news literature in implying an increase in future productivity at a growth threshold.

Finally, I follow a long line of researchers including Benhabib and Farmer [12], Benhabib and Nishimura [13], Bennet and Farmer [14] and Farmer and Guo [32] who exploit variants of the neoclassical growth model to investigate the connection between stationary sunspot equilibria and aggregate fluctuations, and also build upon other research that examines the impact of externalities in the neoclassical growth model such as Baxter and King

[1] See also, Schmitt-Grohe and Uribe [66], Khan and Tsoukalas [56] and Barksy and Sims [4].

[9] and Cooper and Johri [25]. Extending these approaches, I focus on how structural change created by changing growth regimes changes the potential for indeterminacies. Moreover, I also introduce a new modeling mechanism whereby the mechanisms driving indeterminacies depend on the growth rate of the frontier, and therefore may exist only during certain technological regimes.

3 Model

The economy consists of an infinitely-lived representative household, a single final goods firm that nonetheless acts competitively, and a continuum of monopolistically competitive intermediate goods firms on a unit measure, each ith firm producing a differentiated good. Intermediate goods firms own both their physical and intangible capital stocks, financing their expenditures through shares sold to households.

3.1 Final goods firm

The final goods producer purchases intermediate goods $y_t(i)$ from intermediate goods firms and combines these goods into a single final good y_t according to the technology

$$Y_t = \left(\int_0^1 y_t(i)^\nu di \right)^{\frac{1}{\nu}}, \tag{1}$$

where $\nu \in (0,1)$ determines the elasticity of substitution between the intermediate goods. The producer then sells the final good into the final goods market to be used as consumption for households or investment for intermediate goods firms. Each period the producer chooses its demand for each intermediate good $y_t(i)$ by maximizing its profits given by

$$Y_t - \int_0^1 P_t(i) y_t(i) di, \tag{2}$$

where $P_t(i)$ is the relative price of the ith intermediate good $y_t(i)$ in terms of the final good y_t. The resulting optimality condition then yields a demand function for the ith good as

$$y_t(i) = P_t(i)^{\frac{1}{\nu-1}} Y_t. \tag{3}$$

3.2 Intermediate goods firms

Each ith intermediate goods firm produces differentiated output $y_t(i)$ with an associated *firm-specific productivity* $h_t(i)$. Firms increase their productivity by adopting freely-available technological ideas Ψ_t into their production process, where Ψ_t represents the technological frontier of ideas about how to organize production related to new physical capital. Since these ideas pertain to recent physical capital, the quantity of ideas that the firm can implement into production will be some function of its current and past investments in physical capital. A firm's *potential productivity* $j_t(i)$ thus represents the quantity of ideas from the frontier Ψ_t that the firm can adopt at time t based on its investment history, with the property that the more a firm invests in physical capital, the more ideas from the frontier Ψ_t the firm will be able to implement. Firms then convert their stock of potential productivity $j_t(i)$ into firm-specific productivity $h_t(i)$ through a costly learning and adoption process through which they learn to tailor the ideas to their specific production process.

Firms produce their output $y_t(i)$ according to the production function

$$y_t(i) = h_t(i)^{\epsilon}(X_t n_{yt}(i))^{\alpha}\tilde{k}_t(i)^{\theta} \tag{4}$$

where $h_t(i)$ is firm-specific productivity, X_t is exogenous labour-augmenting technical change, $n_{yt}(i)$ is labour allocated to goods production and $\tilde{k}_t(i)$ is physical capital services [2]. Capital services $\widetilde{k_t(i)}$ is defined as $\tilde{k}_t(i) = u_t(i)k_t(i)$, where $u_t(i)$ is the utilization rate of the stock of physical capital $k_t(i)$.

Firms accumulate physical capital according to

$$k_{t+1}(i) = [1 - \delta(u_t)]k_t(i) + i_t(i) \tag{5}$$

where $i_t(i)$ is investment in units of the final good, and the function $\delta(\cdot)$ is a standard time-varying cost of utilization as a convex function of the utilization rate, with properties $\delta'(\cdot) > 0$, $\delta''(\cdot) > 0$.

Firms grow their firm-specific productivity $h_t(i)$ through a process of costly learning and adoption during which they allocate labour to tailor and implement the ideas reflected in their potential productivity $j_t(i)$ into their

[2] Labour-augmenting technical change X_t is necessary for calibration to summarize the effect of other contributions to growth beyond the ideas frontier Ψ_t, but is not neccessary for the problem

specific production process. One salient feature of technological implementation/adoption processes is that it takes *time* to implement a technology. The literature has posited various reasons for this, but here I follow Comin and Hobijn [24], Greenwood and Yorukoglu [45], and Nelson and Phelps [62] in simply specifying the adoption process as a partial-adjustment equation of the form

$$h_{t+1}(i) = h_t(i) + h_t(i)\Big[1 - \frac{h_t(i)}{j_{t+1}(i)}\Big]\Phi\Big(n_{ht}(i)\Big), \tag{6}$$

where $\Phi\Big(n_{ht}(i)\Big) = \tau_0 n_{ht}(i)^{\eta}$, τ_0 is a constant, $0 < \eta \leq 1$, and n_{ht} is labour that the firm allocates to the adoption process [3].

Equation (6) has the property that if the realizations of $\Phi\Big(n_{ht}(i)\Big) < 1$, $h_t(i)$ is bounded above by $j_{t+1}(i)$, imposing a type of "limit to learning" on the firm whereby the firm cannot increase its productivity $h_{t+1}(i)$ beyond its productivity potential $j_{t+1}(i)$. In contrast with Greenwood and Yorukoglu [45], Comin and Hobijn [24] and Nelson and Phelps [62] where the bound of adoption is tied to the overall technological frontier outside of the control of the firm, here the bound $j_{t+1}(i)$ is endogenously controlled by the firm as a function of its history of investment in physical capital, acting as an "effective frontier" facing the firm. On the balanced growth path when growth in ideas is positive, $h_t(i)$ will converge to a constant gap between $h_t(i)$ and $j_t(i)$ and thus a constant gap between $h_t(i)$ and Ψ_t. In the special case of a "technologically stagnant" era where there is no growth in ideas, $h_t(i)$ will converge to Ψ_t such that the gap is zero.

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[3] One way to motivate this equation is to make a probabilistic interpretation of implementation based on approaches by Howitt and Mayer-Foulkes [49] and Comin and Gertler [22]. For example, a firm that attempts to increase its productivity from the $h_t(i)$ to the potential $j_{t+1}(i)$ through implementation is successful with probability $\omega_t(i)$, which is an endogenous function of the firm's choices. As such, the firm's expected productivity next period is $E_t(i)h_{t+1}(i) = \omega_t(i)j_{t+1}(i) + (1 - \omega_t(i))h_t$. The probability $\omega_t(i)$ that the firm is successful is an increasing function of the resources the firm directs towards adoption, and a decreasing function of the relative distance between the firm's current productivity and its potential productivity, $\frac{h_t(i)}{j_{t+1}(i)}$, such that $\omega_t(i) = \Phi\Big(n_{ht}(i)\Big)\frac{h_t(i)}{j_{t+1}(i)}$. Substituting this definition of $\omega_t(i)$ into the expression for expected productivity next period and re-arranging then yields $E_t(i)h_{t+1}(i) = h_t(i) + h_t(i)\Big[1 - \frac{h_t(i)}{j_{t+1}(i)}\Big]\Phi\Big(n_{ht}(i)\Big)$, giving the specification in (6) above in expectation. Comin and Gertler [22] and Comin, Gertler and Santacreu [23] then use the assumption that once a technology is in use, all firms have it to facilitate aggregation without expectation.

A firm's potential productivity $j_t(i)$ evolves as some function of its current and past investment history relative to the state of technological ideas Ψ_t during which these investments in physical capital are made, such that $j_{t+1}(i) = J(j_t(i), i_t(i), \Psi_t)$. To ease notation, it is helpful to define $x_t = \frac{j_t(i)}{\Psi_{t-1}}$ to represent the current *fraction* of ideas contained in the frontier Ψ that the firm *can* implement based on its current history of investment, and thus rewrite the function $J(\cdot)$ as $x_{t+1}(i) = X(x_t(i), i_t(i), \Psi_t)$.

The function $X(\cdot)$ essentially enforces the requirement that since the ideas Ψ_t relate to physical capital, the firm must invest in order to implement these ideas. Thus the function is essentially a property of the *technology* of production. In reality, the ability to implement ideas that depend on the characteristics of physical capital will be some complicated function of the history of investment purchases, to the extent that capital from different periods is not alike. For example, a firm in the 1990's that wished to implement supply chain management ideas required not only the physical infrastructure such as vehicles and warehouses necessary to operate its business, but also the necessary information technology hardware and software that enables the supply chain planning and optimization. To the extent that latter (software) requirements represent an expansion in the variety of capital goods through time relative the former (computers and before that warehouses), the function $X(\cdot)$ would require new purchases of capital to complement an accumulated stock of older capital as a prerequisite for implementing the supply chain management ideas.

To captures these feature most simply, I assume a simple log-linear specification of $X(\cdot)$ similar to that in Stadler [69] to describe the evolution of x_t through time,

$$x_{t+1}(i) = \Gamma_t x_t(i)^{\phi} i_t(i)^{\rho}, \tag{7}$$

where $\Gamma_t = \Gamma(J_t, I_t, \Psi_{t-1})$ is a scale-factor necessary for balanced growth that I will define later, and where $0 \leq \phi < 1$, $0 \leq \rho < 1$ [4]. Equation (7) simply expresses the evolution of the fraction of ideas that the firm can implement

[4] The results of the model hold under other specifications for the function $x_{t+1}(i) = X(x_t(i), i_t(i), \Psi_t)$. One such specification, symmetrical to that used for firm-specific productivity $h_t(i)$, is $x_{t+1}(i) = x_t(i)/g_t^{\Psi} + x_t(i)/g_t^{\Psi}\left[1 - x_t(i)/g_t^{\Psi}\right]\kappa_0 \frac{i_t(i)}{k_t(i)}$, where κ_0 is a constant, g_t^{Ψ} is the growth rate of Ψ, and where the firm internalizes all the variables indexed with i. As will become clear later however, the current specification in equation (7) offers the advantage of economizing the aggregate state vector under a symmetric equilibrum.

next period as a function of that fraction this period and new investment this period.

To understand this equation, it is helpful to first consider the extreme condition where $\phi = \rho = 0$: in this case the firm need not do anything to implement Ψ_t; it spills over simply as an external effect independent of the firms current and past investment. For $\rho > 0$ however, the rate that the firm's potential productivity $j_t(i)$ grows relative to the frontier of ideas Ψ_t depends on its new investment in physical capital. A value of $\phi > 0$ along with $\rho > 0$ makes $j_{t+1}(i)$ a function of both its current and past investment, such that a firm that accumulates a larger stock of new capital early on can expect higher growth in potential productivity. This essentially allows investment now to increase the future effectiveness of investment in growing j, implying a backwards compatibility of future ideas to past investment such that a firm can exploit some portion of future growth in ideas Ψ_t based on past investment.

Finally, firms purchase total labour $n_t(i)$ at wage w_t, and allocate it between goods production and adoption according to

$$n_{y_t}(i) + n_{ht}(i) = n_t(i). \tag{8}$$

Note that despite the fact that Ψ_t is freely available, it differs from the concept of TFP in a neoclassical growth model or a model with external effects in goods production in that the need to purchase physical capital in order to implement the free ideas requires that a firm *internalize* these ideas as additional benefits that may come with purchasing new capital. This has important consequences then for income distribution since it implies that in order to increase productivity some time in the future, a firm may be willing to invest more in physical capital in the present then is justified by that extra capital's direct returns in production in the present. As such, either decreasing returns to scale to physical capital and labour or a positive markup over marginal cost associated with imperfect competition are necessary to allow the firm to operate in such an environment without driving profits negative.

3.2.1 Intermediate goods firms' problem

Defining $S_t = (\Psi_t, Z_t, K_t, J_t, H_t)$ as a vector of aggregate state variables beyond the control of the firm, and $s_t(i) = \big(k_t(i), j_t(i), h_t(i)\big)$ as the corresponding vector of state variables under the control of the firm, each intermediate goods producer solves the recursive problem

$$V(S_t, s_t(i)) = max_{n_y(i),n_h(i),i_t(i),u_t(i),s'(i)} \Big\{ d_t(i) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} V\Big(S_{t+1}, s_{t+1}(i)\Big) \Big\} \tag{9}$$

subject to (5), (7) and (6), where $d_t(i) = P_t(i)y_t(i) - w_t(S_t)n_t(i) - i_t(i) = Y_t^{1-\nu} y_t(i)^\nu - w_t(S_t)n_t(i) - i_t(i)$ is the firm's dividend, and $\beta E_t \frac{\lambda_{t+1}}{\lambda_t}$ is the household owner's stochastic discount factor, yielding the optimal policy rules $k'(i) = k(S_t, s_t(i))$, $j'(i) = j(S_t, s_t(i))$, $h'(i) = h(S_t, s_t(i))$, $n_y(i) = n_y(S_t, s_t(i))$, $n_h(i) = n_h(S_t, s_t(i))$, $i(i) = i(S_t, s_t(i))$ and $u(i) = u(S_t, s_t(i))$. Letting $q_{kt}(i)$, $q_{jt}(i)$ and $q_{ht}(i)$ be the Lagrange multipliers on firm i's physical capital (k), firm-specific productivity potential (j) and firm-specific productivity (h) accumulation equations respectively, the firm's first-order conditions are as follows:

$$w_t = \nu\alpha \frac{P_t(i)y_t(i)}{n_{yt}(i)} \tag{10}$$

$$w_t = q_{ht}(i)h_t(i)\Big[1 - \frac{h_t(i)}{j_{t+1}(i)}\Big]\Phi'\Big(n_{ht}(i)\Big) \tag{11}$$

$$1 = q_{kt}(i) + q_{jt}(i)\rho\frac{j_{t+1}(i)}{i_t(i)} \tag{12}$$

$$q_{kt}(i)\delta'(u_t(i))k_t(i) = \nu\theta\frac{P_t(i)y_t(i)}{u_t(i)} \tag{13}$$

$$q_{jt}(i) = q_{ht}(i)\left(\frac{h_t(i)}{j_{t+1}(i)}\right)^2 \Phi(n_{ht}(i)) + E_t\Big\{\beta\frac{\lambda_{t+1}}{\lambda_t} q_{jt+1}(i)\phi\frac{j_{t+2}(i)}{j_{t+1}(i)}\Big\}, \tag{14}$$

$$q_{kt}(i) = E_t\Big\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\nu\theta\frac{P_{t+1}(i)y_{t+1}(i)}{k_{t+1}(i)} + q_{kt+1}(i)\left[1 - \delta(u_{t+1}(i))\right]\right)\Big\} \tag{15}$$

$$q_{ht}(i) = E_t\Big\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\nu\epsilon\frac{P_{t+1}(i)y_{t+1}(i)}{h_{t+1}(i)} + q_{ht+1}(i)\Big[1 + \left(1 - \frac{2h_{t+1}(i)}{j_{t+2}(i)}\right)\Phi\Big(n_{ht+1}(i)\Big)\Big]\right)\Big\}. \tag{16}$$

Equations (10) and (11) are the firm's y-hours and h-hours first-order conditions respectively, and show that the firm allocates labour between goods production and adoption to equalize the marginal products of labour in each use. Note in (11) that the gap $[1-\frac{h_t(i)}{j_{t+1}(i)}]$ increases the technical effectiveness of hours in adoption, whereas the shadow value $q_{ht}(i)$ expresses the marginal value of adoption in terms of the firm's output. Combining these two equations along with the firm's hours allocation constraint (8) yields an expression for the firm's total labour demand,

$$w_t = \frac{1}{n_t(i)}\Big(\nu\alpha y_t(i) + q_{ht}(i)h_t(i)\big[1 - \frac{h_t(i)}{j_{t+1}(i)}\big]\eta\Phi\big(n_{ht}(i)\big)\Big) \tag{17}$$

where it is clear that both the shadow value q_{ht} and the gap $[1-\frac{h_t}{j_{t+1}}]$ act as shift-factors for the firm's labour demand curve in wage-hours space. Given the dependence of the gap $[1-\frac{h_t(i)}{j_{t+1}(i)}]$ on the existence of growth in aggregate J_t due to positive trend growth in Ψ_t, the magnitude of this shift factor will be a function of the underlying technological environment. At the extreme in a technologically stagnant era this gap will go to zero, such that the second term in (17) drops out and and the firm's labour demand collapses to that of the standard one-sector neoclassical model.

The firm's investment first-order condition (12) shows that in determining investment, the firm considers both its benefit in adding to the physical capital stock as well as its potential contribution in developing a productivity potential that will eventually lead to productivity improvements, illustrating the by-product nature of investment in this economy. The firm values each of these two effects according to their respective shadow prices.

The firm's utilization first-order condition (13) shows simply that in choosing u, the firm equates the marginal product of u in goods production to the marginal cost of adjusting utilization, where the marginal cost reflects both physical depreciation of physical capital, and the relative value of physical capital in terms of the consumption good.

Equation (14) describes the firm's optimal choice of its potential productivity next period, j_{t+1}. Being a necessary input into producing firm-specific productivity, h_t, the first term on the right shows how the value of potential productivity j_t is a function of the value of new firm-specific knowledge. The final term captures the contribution of the additional j in raising future j.

Both the physical capital and firm-specific productivity first-order conditions (15) and (16) relate the respective forward-looking shadow prices to the sum of the stochastically-discounted future marginal products of each of these factors in goods production.

It is important to note that the impact of the technology related to adoption exists only while the technological gap is positive. When Ψ growth and hence J growth is high, the firm can invest in new capital, increasing it's potential productivity, j_t, and thus maintaining a positive gap $[1 - \frac{h_t(i)}{j_{t+1}(i)}]$ such that it can transfer labour out of goods production into adoption to increase productivity. Yet during periods of low-growth this effect is diminished, and during technological stagnation it is eliminated: without growth in Ψ_t, no amount of investment can increase the firm's potential productivity j_t, and thus labour transferred out of goods production into adoption is useless.

3.3 Household

The household side of the model is standard so I discuss it briefly. The representative household has preferences defined over sequences of consumption C_t and leisure L_t with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \tag{18}$$

where β is the household's subjective discount factor, and the period utility function $u(C_t, L_t) = \frac{1}{1-\sigma}\{[C_t v(L_t)]^{1-\sigma} - 1\}$ is of the class of preferences described in King, Plosser and Rebelo [58]. The household's budget constraint is given by

$$C_t + \int_0^1 v_t(i) A_{t+1}(i) di = w_t N_t + \int_0^1 [d_t(i) + v_t(i)] A_t(i) di, \tag{19}$$

where $v_t(i) = v_i(S_t)$ is the price of firm i's share, $A_t(i)$ the household's holdings of shares of firm i, $w_t = w(S_t)$ is the wage, N_t is hours-worked, and S_t is a vector of aggregate state variables.

Finally, each period, the household is endowed with one unit of time that it allocates between leisure and hours-worked according to

$$N_t + L_t = 1. \tag{20}$$

The household solves the recursive problem $V(A_t, S) = max_{C,N,L,A'}\{u(C_t, L_t) + \beta E_t V(A_{t+1}, S_{t+1})\}$ subject to (19) and (20), where A_t represents a vector of the portfolio of firms' shares, yielding the policy rules $A' = A(A, S)$, $C' = C(A, S)$ and $L_t = L(A, S)$. Letting λ_t by the Lagrange multiplier on (19), the firm's first-order conditions are

$$u_c(C_t, L_t) = \lambda_t \tag{21}$$

$$u_l(C_t, L_t) = \lambda_t w_t \tag{22}$$

$$v_t(i) = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[d_{t+1}(i) + v_{t+1}(i) \right] \right\} \quad \forall i. \tag{23}$$

3.4 Aggregate technology

To approximate what in reality may be a gradual rate of change of publicly-available technological ideas Ψ_t, I assume that Ψ_t grows deterministically as

$$\Psi_t = \Psi_{t-1} g_{\Psi}^{era}, \tag{24}$$

where the growth rate g_Ψ is relatively constant over a "technological era" but over the long run will change and thus represent a structural shift in the economy, represented by the index "era" [5] [6]. For example, Fernald [35] detects evidence of a break in the trend of average labour productivity growth such that average growth during from the 1950's to early 1970's, and after the mid-1990's was much higher than it was in between these periods. In the context of my model, I interpret these high-growth regimes as periods of structural change that increase the deterministic growth rate of the frontier from its value $g_\Psi^L \approx 1$ in the low-growth period to some value $g_\Psi^H > 1$ in the high-growth period, as an approximation to what in reality may be a gradual acceleration of the technological frontier. As such, in this model, a period such as the 1990's that yielded transformational technical change over a relatively short ten-year period would be represented by a structural shift in the growth rate parameter g_Ψ at some point prior to this period. It is important to note however that my argument in the paper does not rely on the *timing* of this structural change. For example, I am not implying that belief-driven fluctuations occur simultaneously with this structural change. Rather, the *existence* of a period of sufficiently positive growth in ideas simply opens up the possibility that a belief-driven fluctuation may occur.

[5] Alternatively one could model Ψ_t with a stochastic trend where its growth rate follows a highly-persistent process subject to low-variance shocks, such that Ψ_t evolves gradually over time. As will become apparent later however, I use a linear approximation in my analysis, and thus what becomes most important for the model dynamics I consider is only the steady-state drift portion of the growth rate, even though in reality the non-linear dynamics may be important.

[6] It would be relatively straightforward to endogenize the growth rate of the theoretical frontier Ψ_t using the various mechanisms in the endogenous growth literature, however it would complicate the model without necessarily further illuminating my central point.

In order to properly calibrate the model to average long-run growth rates over high and low growth periods, I also include deterministic labour-augmenting technical change X_t in the model to represent "other" sources of growth, where X_t follows the process

$$X_t = X_{t-1} g_X \tag{25}$$

and where the growth rate g_X is constant over "high" and "low" technological eras.

3.5 Equilibrium

I define the aggregate quantities $K_t = \int_0^1 k_t(i)di$, $J_t = \int_0^1 j_t(i)di$, $H_t = \int_0^1 h_t(i)di$, $N_t^f = \int_0^1 n_t(i)di$, $I_t = \int_0^1 i_t(i)di$ and $A_t^f = \int_0^1 a_t(i)di$ associated with the intermediate goods producers. The market clearing conditions in the model economy for the labour market, stock market and goods market as then as follows:

$$N_t = N_t^f \tag{26}$$

$$Y_t = C_t + I_t \tag{27}$$

$$A_t^f = 1 = A_t, \tag{28}$$

where the left-hand and right-hand sides of each of the equalities are the supply and demand sides respectively in the markets for each quantity.

A rational expectations equilibrium for this economy is then a collection of policies for households $a' = a(S_t)$, $n = n(S_t, a_t)$, $l = l(S_t, a_t)$, policies for intermediate goods firms $k'(i) = k(S_t, s_t(i)$, $j'(i) = j(S_t, s_t(i))$, $h'(i) = h(S_t, s_t(i))$, $n_y(i) = n_y(S_t, s_t(i))$, $n_h(i) = n_h(S_t, s_t(i))$, $i(i) = i(S_t, s_t(i))$ for $i \in [0, 1]$, policies for the final goods producer $y = y(S_t)$, $y(i) = y(i)(S_t)$, price systems $w(S_t)$, $v_i(S_t) \quad \forall i$, and aggregate laws of motion $K = K(S)$, $J = J(S)$, and $H = H(S)$, such that: (i) households solve their problem (ii) intermediate goods producers solve their problems; (iii) the final goods producer solves its problem; (iv) the markets in equations (26), (27) and (28) clear, and; (v) a fixed point such that the individual firm's policy rules confirm the aggregate laws of motion.

I consider a symmetric equilibrium where $p_t(i) = p_t$, $y_t(i) = y_t$, $n_{yt}(i) = n_{yt}$, $n_{ht}(i) = n_{ht}$, $n_t(i) = n_t$, $k_t(i) = k_t$, $j_t(i) = j_t$, $h_t(i) = h_t$ and $i_t(i) = i_t$. Substituting into the definitions of the aggregate quantities associated with the intermediate firms then implies that $k_t = K_t$, $j_t = J_t$, $h_t = H_t$, $n_{yt} = N_{yt}$, $n_{ht} = N_{ht}$, $n_= N_t$, $i_t = I_t$ and $u_t = u_t$.

I define the scale factor $\Gamma_t = (\frac{J_t}{\Psi_{t-1}})^{1-\phi}/I_t^{\rho}$ so that under a symmetrical equilibrium, the intermediate goods firms's x accumulation equation (7) reduces to $\frac{J_{t+1}}{J_t} = \frac{\Psi_t}{\Psi_{t-1}}$, such that J grows at the same rate as Ψ. As such, under the resulting transformed stationary system, we can effectively remove J_t from the equilibrium system, reducing the endogenous states down to the stationary forms of K_t and H_t. Nevertheless, despite there being no endogenous movement in J_t, the conditions imposed under the intermediate goods firm's $j_t(i)$ accumulation equation (7) continue to influence the firm's behaviour through its first-order conditions [7].

Substituting $y_t(i) = y_t$ into the final goods aggregate technology (1) yields the condition $y_t = Y_t$. Recognizing that under perfect competition the final goods firm's profits will be zero then implies that $p_t(i) = p_t = 1$. Finally, substituting $y_t = Y_t$, $h_t = H_t$, $n_{yt} = N_{yt}$ and $k_t = K_t$ into the ith intermediate goods firm's production function (4) yields the aggregate production function

$$Y_t = H_t^{\epsilon}(X_t N_{yt})^{\alpha}\tilde{K}_t^{\theta}. \tag{29}$$

In a symmetrical equilibrium, all firm's shadow prices of k, h and j will be equivalent. To represent this in equilibrium system I redefine these internal prices in aggregate in terms of household utility as $\mu_t = q_{kt}\lambda_t$, $\Upsilon_t = q_{ht}\lambda_t$ and $\zeta_t = q_{jt}\lambda_t$. The resulting equilibrium dynamic system is represented by the following system of equations:

$$Y_t = H_t^{\epsilon}(X_t N_{yt})^{\alpha}(u_t K_t)^{\theta} \tag{30}$$

$$N_{y_t} + N_{ht} = N_t. \tag{31}$$

$$K_{t+1} = (1-\delta_k)K_t + Z_t I_t \tag{32}$$

$$H_{t+1} = H_t + H_t[1 - \frac{H_t}{J_{t+1}}]\tau N_{ht}^{\eta} \tag{33}$$

$$w_t = \nu\alpha\frac{Y_t}{N_{yt}} \tag{34}$$

[7] The model results are not dependent on limiting the endogenous role of J_t in this manner. As I indicated in an earlier footnote, the results of the model hold for other specifications of $x_{t+1}(i) = X(x_t(i), i_t(i), \Psi_t)$, and under the alternate specification J_t is an endogenous state variable in both the non-stationary and stationary system.

$$w_t = \frac{\Upsilon_t}{\lambda_t} H_t[1 - \frac{H_t}{J_{t+1}}]\eta\tau N_{ht}^{\eta-1} \tag{35}$$

$$\lambda_t = \mu_t + \zeta_t \rho \frac{J_{t+1}}{I_t} \tag{36}$$

$$\frac{\mu_t}{\lambda_t}\delta'(u_t)K_t = \nu\theta\frac{Y_t}{u_t} \tag{37}$$

$$\mu_t = \beta E_t \Big\{ \lambda_{t+1}\nu\ \theta\frac{Y_{t+1}}{K_{t+1}} + \mu_{t+1}\left(1 - \delta_k\right) \Big\} \tag{38}$$

$$\Upsilon_t = \beta E_t \Big\{ \lambda_{t+1}\nu\epsilon\frac{Y_{t+1}}{H_{t+1}} + \Upsilon_{t+1}\Big[1 + \left(1 - 2\frac{H_{t+1}}{J_{t+2}}\right)\tau N_{ht+1}^{\eta}\Big]\Big\} \tag{39}$$

$$\zeta_t = \Upsilon_t \left(\frac{H_t}{J_{t+1}}\right)^2 \tau N_{ht}^{\eta} + \beta E_t \zeta_{t+1}\phi\frac{J_{t+2}}{J_{t+1}}. \tag{40}$$

$$\frac{J_{t+1}}{J_t} = \frac{\Psi_t}{\Psi_{t-1}} \tag{41}$$

$$C_t^{-\sigma}\upsilon(L_t)^{1-\sigma} = \lambda_t \tag{42}$$

$$C_t^{1-\sigma}\upsilon(L_t)^{-\sigma}\upsilon'(L_t) = \lambda_t w_t \tag{43}$$

$$C_t + I_t = Y_t. \tag{44}$$

It is important to realize that when there is no growth in Ψ, the system essentially reduces down to the neoclassical growth model where H_t is fixed through time. This can be seen in the above system by substituting $\Psi_{t+1} = \Psi_t = J_{t+1} = J_t = H_t = H_{t+1}$ into the above system.

For reference later in the discussion of the results, I also defined equilibrium observed total factor productivity (TFP) as

$$TFP_t = Y_t - \alpha N_t - (1-\alpha)u_t - (1-\alpha)K_t \tag{45}$$

according to the standard definition that uses total labour (as opposed to N_{yt}), assumes constant returns to labour and physical capital, and as well controls for the variable contribution of capacity utilization.

3.6 Balanced growth path and steady state

I define a balanced growth path for this economy whereby N_t, N_{yt}, N_{ht} and u_t are constant, and the other endogenous variables inherit trends as some function of the trend in X_t and Ψ_t. The equilibrium system implies that C, I, Y, D, w, v and K contain trend $X_t^Y = \Psi_t^{\frac{\theta}{1-\theta}} X_t^{\frac{\alpha}{1-\theta}}$, λ_t contains trend $1/X_t^\sigma$, J_t contains trend $X_t^J = \Psi_t$ and H_t contains trend $X_t^H = \Psi_t$. On the balanced growth path, the growth rates are then $g^y = \frac{X_{t+1}^Y}{X_t^Y}$, and $g^\Psi = \frac{\Psi_{t+1}}{\Psi_t}$ and $g^x = \frac{X_{t+1}}{X_t}$ for all t.

I then perform the following transformation such that each resulting variable is stationary on the balanced growth path: $\tilde{C}_t = \frac{C_t}{X_t^Y}$, $\tilde{I}_t = \frac{I_t}{X_t^Y}$, $\tilde{Y}_t = \frac{Y_t}{X_t^Y}$, ...etc., $\tilde{K}_t = \frac{K_t}{X_{t-1}^K}$, $\tilde{J}_t = \frac{J_t}{X_{t-1}^J}$, $\tilde{H}_t = \frac{H_t}{X_{t-1}^H}$, $\tilde{\lambda}_t = \lambda_t {X_t^Y}^\sigma$, $\tilde{\mu}_t = \mu_t \frac{X_t^K}{{X_t^Y}^{1-\sigma}}$, $\tilde{\zeta}_t = \zeta_t \frac{X_t^J}{{X_t^Y}^{1-\sigma}}$, and $\tilde{\Upsilon}_t = \Upsilon_t \frac{X_t^H}{{X_t^Y}^{1-\sigma}}$. Under this transformation, the stationary system now has just two endogenous state variables, $\tilde{K}_t$ and $\tilde{H}_t$. Finally, the resulting stationary system contains a unique non-stochastic steady state.

4 Examining the role of self-fulfilling beliefs

In this section I explore the properties of the model under parameterizations that produce indeterminacies such that sunspot expectation shocks can produce fluctuations in the absence of any shocks to technology. I first describe my solution method and baseline parameterization for a "high-growth" period based on quarterly data, and then investigate how the potential for indeterminacies varies with the underlying growth rate of Ψ. Finally I illustrate the impulse response of the model economy to the sunspot shocks.

4.1 Solution method

I first linearize the model around the non-stochastic state state, resulting in a first-order linear system of the form

$$E_t \mathcal{S}_{t+1} = A \mathcal{S}_t + B \epsilon_t \tag{46}$$

where $\mathcal{S}_t = [\hat{k}_t, \hat{h}_t, \hat{\mu}_t, \hat{\Upsilon}_t, \hat{\zeta}_t,]'$, and $\epsilon_t = [0, 0, 0, 0, 0]'$ such that there is no external source of uncertainty. Hats above variables denote %-deviations from steady state.

The linear system (46) contains two predetermined endogenous states (k,h) and three forward-looking non-predetermined co-states (μ,Υ,ζ). The system will exhibit saddle-path stability if the number of eigenvalues of the matrix A outside of the unit circle is equal to the number of forward-looking non-predetermined variables, and will display indeterminacy if the number of eigenvalues of A lying outside the unit circle is less than the number of forward-looking non-predetermined variables.

To analyze the response of the system to intrinsic uncertainty, I follow the approach of Farmer [31] and replace the expectations of a variable with the variable less the expectational error, so that now in this case (46) re-writes as,

$$\mathcal{S}_{t+1} = A\mathcal{S}_t + B\varepsilon_t \tag{47}$$

where ε_t is now defined as $\varepsilon_t = [0, 0, w_t^{\mu}, w_t^{\Upsilon}, w_t^{\zeta}]'$, where $w_t^{\mu} = E_t\mu_{t+1} - \mu_{t+1}$, $w_t^{\Upsilon} = E_t\Upsilon_{t+1} - \Upsilon_{t+1}$ and $w_t^{\zeta} = E_t\zeta_{t+1} - \zeta_{t+1}$ are the one-step ahead forecast errors on the Lagrange multipliers. Note also that by definition the expectational error of a predetermined variable is zero yielding the two zeros in ε_t.

For the parameterizations that I consider that yield indeterminacy, the matrix A has one less root outside the unit-circle than forward-looking variables, leaving two forward-looking variables with unstable roots. Thus under indeterminacy we can interpret the expectational errors above as iid sunspot shocks. I then diagonalize the system and iterate out the two remaining unstable roots as in a saddle-path solution, yielding a restriction on (47) that relates the two unstable forward-looking variables to the stable variables. Similar to the multi-sector model of Benhabib and Nishimura [13], the three sunspot shocks cannot be chosen independently since there is a joint restriction imposed on them from iterating out the two unstable roots.

After solving out the unstable roots, the system reduces down to

$$\tilde{\mathcal{S}}_{t+1} = \tilde{\mathcal{A}}\tilde{\mathcal{S}}_t + \tilde{\mathcal{B}}\tilde{\varepsilon}_t \tag{48}$$

where now $\tilde{\mathcal{S}}_t = [\hat{k}_t, \hat{h}_t, \hat{\Upsilon}_t]'$, and $\tilde{\varepsilon}_t = [0, 0, e_t^{\Upsilon}]'$ and where e_t^{Υ} is an iid sunspot shock to Υ, the value of H. Note now that all the roots of $\tilde{\mathcal{A}}$ are inside the unit circle, and the system is a Markovian stable process such that any value of e_t^{Υ} will set the system on a stable path that eventually returns to steady state.

These sunspot expectational shocks involve the Lagrange multipliers of K, J, and H which by definition measure the marginal value of these predetermined states. Thus in the context of the model one can interpret an expectational shock to Υ as a self-fulfilling belief by the agents about the value adoption since a marginal change in H constitutes a marginal adoption by the firm of the technology of the frontier.

4.2 Parameterization

In this section I detail an illustrative calibration for a "high-growth" period that features positive growth in the frontier of technological ideas. Where possible I assign values to parameters using restrictions on the model steady-state with values established in the literature.

I approximate what may in reality be a gradual increase in the rate of ideas-growth and associated dynamic expansion of the technological gap as a structural break in the steady-state growth rate of the parameter Ψ, creating an additional source of growth beyond the "other factors" contained within the labour-augmenting growth factor X_t. In the data this break in Ψ would *eventually* show up as a structural break in measured average productivity as firms adopt and develop firm-specific productivity. Thus I exploit existing empirical analysis by others of structural breaks in average labour productivity in post-war U.S. data to calibrate the magnitude of the growth rate of Ψ [8]. It is important to note however that my model implies that the *timing* of the structural break in Ψ need not necessarily coincide with an observed structural break in average labour productivity, since in the theoretical model there is an implementation and adoption phase that delays productivity gains. Moreover, since the rate of change of adoption and therefore realized productivity is a function of the exogenous beliefs of agents, even if the gap is large, unless firms are optimistic about its value, they may not pursue adoption with enough fervour to produce concentrated productivity growth.

Fernald (2007) finds break-dates in average labour productivity in the early-1970s and the mid-1990's, separating data productivity series into 'high" periods (from the 1950's to the early 1970's and after mid-1990's) and "low" periods (from 1970's to mid-1990's) with growth rates of approximately 3.25% and 1.5% respectively. For the purposes of illustration, I assume that the other source of growth in the model - labour augmenting growth - is constant throughout the entire post-war period, and then that growth in technological ideas makes up the difference between "high" and "low" growth regimes

[8] See Fernald [35] and Kahn and Rich [57].

with growth rates of 2.5% and 1.5% respectively [9], such that I interpret the "high" regimes as periods where the "technological gap" opens up. Given these break-dates identified by Fernald, my interpretation of breaks related to the gap is consistent with the empirical evidence of Cummins and Violante [28], who estimate a Nelson-Phelps "technological gap" style adoption model, and find that the gap increased from the mid-1950's to the early 1970's, and from the mid-1990's until the turn of the century.

Since during the "low" regime I assume that Ψ is near-stagnant, during this regime the growth rate of output g_y is related to the growth rate of labour-augmenting technical change g_x by $g_y^{low} = g_x^{\frac{\alpha}{1-\theta}}$, which given the quarterly growth rate of output $g_y^{low} = 1.015^{.25}$, determines g_x. During the "high" regime, the growth rate of output is related to both sources of growth by $g_y^{high} = g_\Psi^{\frac{1}{\alpha}} g_x$, yielding $g_\Psi = \left(\frac{g_y^{high}}{g_X}\right)^\alpha$, which given the quarterly growth rate of output $g_y^{high} = 1.025^{.25}$, determines g^Ψ. Both of these expressions are dependent on a parameterization for α and θ which I will determine below.

In the model, the overall labour share in output S_N is a function of labour in both the Y and H uses, such that

$$S_N = S_{N_y} + S_{N_h}, \tag{49}$$

where $S_{N_y} = \nu\alpha$ and S_{N_h} are the Y and H labour shares in output respectively. Comin and Hobijn (2010) calibrate the cost of *R&D* using a result from Corrado, Hulten and Sichel [26] that investment in R&D in the US corporate investment sector is approximately 5.7% of corporate income, which is analogous to S_{N_h} in this model. Using a value of $S_{N_h} = 0.057$ from Comin and Hobijn, and setting the overall labour share S_N to a 0.70 during the "high" period yields a labour share in goods production of $S_{N_y} = (0.7 - 0.057) = 0.643$. During the "low" period, since there is no growth in Ψ, S_{N_h} will then approach zero, implying that the overall labour share in the economy decreases from 0.7 to 0.643. Note however that the labour share in goods production S_{N_y} remains constant at 0.643 over both the "high" and 'low" growth periods.

[9] There is much evidence that the rate of growth of investment-specific technological change accelerated in the 1990's, such as that of Cummins and Violante [28]. Incorporating investment-specific technical change (ISTC) into the model and allowing for a structural break in ISTC for the "high" regime however does not materially impact my results, and therefore I neglect it for simplicity

To parameterize the shares of factors in goods production I first assume that the firm-specific productivity H acts as a capital-augmenting growth factor, implying that $\epsilon = \theta$. While this suggests possible increasing returns to N_y, K and H in goods production, it should not be directly interpreted as an indicator of the short-run point-in-time returns to scale of the production of Y_t as in models with contemporaneous externalities such as Baxter and King [9] or Benhabib and Farmer [12], since in this model H_t is not a function of contemporaneous (internal or external) goods-labour and/or physical capital. Instead, H_t in this context acts like a dynamic complementarity that changes marginal cost over time, in the vein of that estimated by Cooper and Johri [25]. Moreover, the impact of this dynamic complementary may only be temporary, since only during regimes with positive growth in the ideas frontier does H grow; during regimes where there is no growth in Ψ, H is constant and the production function will act as a standard production function with constant or decreasing returns to labour and capital. Nevertheless it is important to note that the results of the model are not dependent on the assumption of $\alpha + \theta + \epsilon > 1$. An alternative parameterization featuring constant returns to N_y, K and H such that $\alpha + \theta + \epsilon = 1$ still yields indeterminacies, however, it is not clear that this parameterization would make economic sense in this model since it would imply significant decreasing returns in production during times of no growth in Ψ and thus H is fixed.

Next to specify the curvature ν on the final goods aggregator and the degree of returns to scale to N_y and K in intermediate goods production I use values similar to Kehoe [3] who model technological transition featuring a intermediate goods production function with inputs of intangible capital, labour and physical capital and with decreasing returns to labour and physical capital. I set $\nu = 0.95$, implying a markup of 5.3%, and $\alpha + \theta = 0.95$. In comparison, Atkeson and Kehoe use 0.9 and 0.95 for the analogous quantities, the former of which implies a markup of 11% in their model. As Atkeson and Kehoe discuss, their markup of 11% is consistent with evidence in Basu and Fernald [5] and others, and decreasing returns of 0.95 is consistent with a wide range of empirical work that finds estimates in the rage of 0.9 to 1. Using these values then yields $\alpha = S_{N_y}/\nu = 0.643/0.95 = 0.6768$, and $\theta = \epsilon = 0.2732$.

Having determined α, we can now determine g_x and g_Ψ using the expressions from earlier, yielding $g_x = 1.0040$ and $g^\Psi = 1.0065$. Given these values, we can then determine the parameters in the H-accumulation equation using the expression for S_{N_h}, $S_{N_h} = \nu\theta\eta \frac{g^{\Psi}-1}{\frac{g^{\Psi}}{\beta g y^{1-\sigma}} - (1+[1-2\frac{H}{Jg^{\Psi}}]\Phi(N_h))}$, where in steady-state $\frac{H}{Hg^{\Psi}} = \frac{1-\Phi(N_h)-g^{\Psi}}{\Phi(N_h)}$. Since the "gap" nature of the H-accumulation equation provides for significant implicit decreasing returns to scale as the firm closes its gap, for the baseline parameterization I assume no curvature on N_h, setting $\eta = 1$, noting that providing for curvature on N_h such as letting $\eta = \alpha$ (to equate the labour intensity in the two uses of labour) does not dramatically impact the results, given a constant calibration for S_{N_h}.

For the convex cost of capacity utilization, my solution method requires that I need only specify the elasticity of marginal depreciation to utilization, $\epsilon_u = \frac{\delta''(u)}{\delta'(u)}u$, which I set to 0.56 based on the estimates of Burnside and Eichenbaum [15], and the steady state value of depreciation $\delta(u_{ss}) = \delta_k$, which I set to the standard value of 0.025.

To promote comovement, I use preferences not separable in consumption and leisure of the form used by King and Rebelo ([60]) where the the stand-in representative agent has the preference specification

$$u(C_t, L_t) = \frac{1}{1-\sigma}\left\{C_t^{1-\sigma}\upsilon(L_t)^{1-\sigma} - 1\right\} \tag{50}$$

where $\upsilon(L_t) = \left[\left(\frac{1-L_t}{H}\right)\upsilon_1^{*\frac{1-\sigma}{\sigma}} + \left(1 - \frac{(1-L_t)}{H}\right)\upsilon_2^{*\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}}$, and where H is the fixed shift length, and υ_1^* and υ_2^* are constants representing the leisure component of utility of the underlying employed group (who work H hours) and unemployed group (who work zero hours) respectively. Basu and Kimball [8] empirically investigate the general class of King, Plosser and Rebelo [58] preferences not additively separable in consumption and leisure and find estimates of the *labour held constant elasticity of intertemporal substitution in consumption* of 0.5-0.67 during the sample period 1982 to 1999, larger than the near-zero values of the intertemporal elasticity of consumption estimated by Hall [47] that assumed no non-separabilities in consumption and leisure. During the sample period 1949 to 1982 they estimated this quantity to be not significantly different from zero, in line with the results of Hall [47]. Thus to represent both these periods, I choose a value of the labour held constant elasticity of intertemporal substitution in consumption of 0.25, which in this model is equal to $1/\sigma$, implying $\sigma = 4$, in the range of the value of $\sigma = 3$ used by King and Rebelo KR00 in an illustration of these preferences. I then set the average household's share of time allocated to market work N_{ss} to 0.3, and the average household's subjective discount factor β to 0.9934.

The remaining parameters ϕ and ρ in the firm's j-accumulation equation control the dependency of adoption of physical capital, and have a very significant impact on the region of indeterminacy in addition to impacting the steady-state K-Y ratio and therefore equilibrium profit share. The steady state K-Y ratio is given by the expression

$$\frac{k}{y} = \frac{\nu\theta}{1/[\beta g^{y1-\sigma} - (1-\delta_k)/g^k]} + \left(\frac{\tilde{\zeta}}{\tilde{\Upsilon}}\right)\left(\frac{\tilde{\Upsilon}h}{\tilde{\lambda}y}\right)\rho\frac{1/h}{i/k} \tag{51}$$

where $\frac{\tilde{\zeta}}{\tilde{\Upsilon}} = \frac{\left(h/g^h\right)^2}{1-\beta g^{y1-\sigma}\phi}$, $\frac{\tilde{\Upsilon}h}{\tilde{\lambda}y} = \frac{\nu\theta}{1/[\beta g^{y1-\sigma} - (1+(1-h/g^h)\Phi(N_h))/g^h}$ and $\frac{i}{k} = 1-(1-\delta_k)/g^k$. Note that in (51), the first term on the right-hand side is the standard expression for the K-Y ratio based on the contribution of K to the production of Y. The second term however reflects the additional contributions to the steady-state capital stock as a result of the firms' internalizing the additional benefit of purchasing physical capital to grow productivity, beyond that of the marginal production of capital in goods production. All else equal, this second term is increasing in both ϕ and ρ. Since the equilibrium profit (or dividend) share is given by $\frac{D}{Y} = 1 - S_N - \frac{I}{K}\frac{K}{Y}$, we can pin down a combination of ϕ and ρ based on a plausible steady-state profit share through the effect on the K-Y ratio for a given S_N. In this model, the profit share is related to the important quantity $\frac{wN}{C}$ by $\frac{wN}{C} = \frac{S_N}{1-\frac{I}{Y}} = \frac{S_N}{S_N + \frac{D}{Y}}$. The quantity $\frac{wN}{C}$ is important because it is readily observable in the data, as discussed at length by Farmer and Ohanian [33] and Basu and Kimball [8], and moreover provides a link to the non-separable preference specification in this model through the relation $\frac{v'(L)}{v(L)}L = \frac{wN}{C}$. Farmer and Ohanian estimate this quantity to be 0.97 over the period 1929 to 1988, which in this model thus implies a steady state profit share of $\frac{D}{Y} = S_N\frac{(1-\frac{wN}{C})}{\frac{wN}{C}} = 0.0216$. For illustration, I report the impulse-response simulations for two different combinations of these two parameters: (ϕ, ρ) = (0.8,0.85) and (ϕ, ρ) = (0.9,0.45), yielding profit shares in the "high" growth period of 1.9% and 2.5% respectively.

4.3 Characterization of indeterminacy

In this section I numerically characterize indeterminacy in the model in terms of the parameters ϕ and ρ, as well as the dependence of the scope for indeterminacy on the growth rate of the ideas frontier g^{Ψ}.

4.3.1 Dependence of indeterminacies on ϕ and ρ

Using the baseline parameterization I solve the model for each combination of ϕ and ρ on a 100x100 grid ranging from 0 to 1 for each of these parameters, determining the stability properties of the system for each combination. Figure 1 shows the results of this exercise. Recall that ρ captures the extent to which a given firm must itself invest in new capital contemporaneously to exploit the new ideas, and ϕ the extent to which a firm's past purchases of new capital allow it to exploit new ideas for a give level of new investment. Interestingly, the results from the figure imply then that each firm needs to "do something" purposefully to drive indeterminacy, in the form of actively purchasing the new capital which allows it to exploit the new ideas, rather than simply just receiving a costless spillover externality independent of its own investment actions. Indeed at the extreme case discussed earlier where $\phi = 0$ and ρ - whereby the firm can adopt new ideas without investing - the system is completely determinant. The role of physical capital as an enabler of ideas in this economy is thus a critical ingredient for generating indeterminacies.

4.3.2 Dependence of indeterminacies on steady-state growth rate of the ideas frontier, g^{Ψ}

I now attempt to characterize the relation between the scope for indeterminacies in the model economy and the underlying steady state growth rate of Ψ, and thus by doing so suggest that the potential for adoption booms fueled by self-fulfilling "animal spirits" is dependent upon the growth rate of technological ideas.

Starting with the baseline parameterization, I vary the value of the growth rate parameter g^{Ψ}, keeping constant the remainder of the baseline parameters, and for each different growth rate g^{Ψ} I solve the model for each combination of ϕ and ρ on a 100x100 grid as in the previous exercise, determining the properties of the system for each combination of g^{Ψ}, ϕ and ρ.

Figure 2 shows the results of this exercise, plotting the resulting properties of determinacy as a function of combinations of ϕ and ρ for 5 different vertical "slices" of g^{Ψ} (ie each slice essentially repeats the exercise of Figure 1 for a different g^{Ψ}).

Importantly, note that as the underlying growth rate of the embodied frontier g^{Ψ} decreases, the scope for indeterminacy decreases, to the point where it disappears as the parameterization approaches very low growth rates and the limit with technological stagnation of $g^{\Psi} = 1$, where the system is completely saddle-path stable for all combinations of ϕ and ρ.

Recall that the "size" of the steady-state technological gap $(\Psi - H)$ on the balanced growth path varies positively with the underlying growth rate of Ψ. Intuitively, as the model approaches very low growth rates, the gap becomes small enough such that given a set of beliefs about the value of the K and H, the additional benefit provided by investment in physical capital as an enabler of knowledge diminishes as the falling gap reduces the technical effectiveness of labour in adoption.

4.4 Response to iid sunspot shocks

Figure 3 shows impulse response functions relative to trend of the model economy with $(\phi, \rho) = (0.9.0.45)$ to a 1% iid sunspot shock on Υ_t, the Lagrange multiplier on J, interpreted as a belief shock about the value of adoption. Firstly, note immediately in period 1 that Υ_t rises by the amount of the shock, reflecting the change in value from the belief shock. These optimistic beliefs about the value of adoption then lead to an increase in demand for the two primary inputs into adoption - investment in physical capital and labour allocated to adoption - increasing the rate of adoption of technological ideas and as a consequence producing an immediate jump in aggregate consumption, investment, hours-worked in total and in Y-hours and H-hours, as well as a drop in the real wage. Following the initial adoption frenzy, both firm-specific productivity H and TFP increase gradually with a lag. Note that the initial boom also leads to an initial drop in TFP as a result of firms reducing their allocation of total labour in goods production and increasing their labour allocation in non-production adoption activities. Both the initial drop in TFP when investment surges and the eventual delayed increase in TFP is consistent with the empirical results of Basu and Fernald [6] who in addition to finding a positive correlation with lagged ICT investment also find a negative correlation of TFP with contemporaneous investment. The drop in the real wage prior to the eventual delayed increase in TFP is also consistent with the findings of McGrattan and Prescott [61] in their study of the boom of the 1990s. Finally, note that after the initial short-run dynamics of consumption, investment and hours in the first several quarters, these variables display significant persistence, staying above trend well into the range generally associated with medium-frequency fluctuations, driven by the slow and delayed increase in H and K. This property is consistent with a potential link between high-frequency fluctuations associated with business cycles and medium-frequency fluctuations per the "medium-term" cycles phenomena discussed by researchers such as Comin and Gertler [22].

It is important to understand that the impulse responses only show movement relative to trend, and that the existence of the technological gap is a key driver of the dynamics of the model economy. Inspecting the aggregate H accumulation equation (33) shows that as long $\frac{H_t}{J_{t+1}}\Phi(N_{ht}) < 1$, the value of H in levels that *includes* trend cannot decrease. Although relative to the trend growth the dynamics of H is temporary, in the non-detrended economy these movements represent *permanent* increases in H. While this increase represent gains that the economy would have *eventually realized anyway* under the counterfactual situation *without* a belief shock, the effects of the belief shock work to produce a concentrated period whereby the economy "captures" these productivity gains at a faster rate that it would have in the absence of a change in beliefs. Moreover, recalling that on the balanced growth path the economy converges to a "constant gap" between H and Ψ, these temporary bursts of activity represent the economy temporarily "narrowing" the gap smaller than the value that is consistent with balanced growth. As a result, eventually the forces that push the economy back to its balanced growth equilibrium decrease the endogenous rate of H accumulation while the economy "waits" for the slowly growing Ψ frontier to "catch up" such that the technological gap widens and again returns back to constant value consistent with the balanced growth path. Thus realized growth slowdowns naturally follow realized growth spurts in this economy, as the endogenous forces of adoption interact with the constraints of the slowly moving theoretical frontier.

What produces the self-fulling effect in this economy? From the perspective of the firm, its beliefs about an increase in the value of firm-specific productivity Υ and thus the returns to H leads to a desire to increase H, which in turn leads to an increase in demand for the two primary inputs into H: firm-specific productivity potential J, and H-hours N_h. This is evidenced by the effect of the sudden rise Υ in both the firm's j_{t+1} first-order condition and h-hours first-order condition, which for convenience I re-state, this time using the multipliers in terms of household utility,

$$\zeta_t(i) = \Upsilon_t(i)\left(\frac{h_t(i)}{j_{t+1}(i)}\right)^2 \Phi(n_{ht}(i)) + E_t\Big\{\zeta_{t+1}(i)\phi\frac{j_{t+2}(i)}{j_{t+1}(i)}\Big\} \tag{52}$$

$$w_t = \frac{\Upsilon_t(i)}{\lambda_t}(i)h_t(i)\Big[1 - \frac{h_t(i)}{j_{t+1}(i)}\Big]\Phi'\Big(n_{ht}(i)\Big). \quad (53)$$

I will consider the effects in these two first-order conditions in turn.

First, recalling that the firms adoption process is bounded by its potential productivity j, there are high returns to the firm to increasing the upper bound of productivity, and therefore the increase in Υ immediately leads to a large increase in the value of potential knowledge ζ. This effect can be seen in the j_{t+1} first-order condition (52), where all else equal, this rise in Υ in (52) causes the value of potential knowledge ζ to rise, essentially working through a relative price margin similar to an effect described in Benhabib and Nishimura [13]. For a given value of physical capital μ - which reflects the future expected returns to physical capital in goods production through the k_{t+1} first-order condition (15) - this rise in the value of potential knowledge ζ thus causes the firm to increase investment to reduce the marginal production of investment in potential knowledge, $\rho\frac{j_{t+1}}{i_t}$ through the investment first-order condition,

$$\lambda_t = \mu_t(i) + \zeta_t(i)\rho\frac{j_{t+1}(i)}{i_t(i)}, \quad (54)$$

reflecting the fact that the firm must purchase new capital to increase its productivity potential.

Since the additional investment must come at the expense of consumption and the household wishes to smooth consumption over time, the marginal utility of consumption and thus λ_t increase both now and in the future, driving up the real interest rate and increasing the return to K and H in future periods, again through a similar relative price effect, thus contributing to the rise in μ_t and partially supporting the conjectured belief about Υ_t. Moreover, this rise in λ_t both now and in the future also has the effect of keeping ζ_t high in future periods through the effect of the investment first-order condition in future periods, since all else equal an increase in investment would tend to lower μ_t in future periods. This has the important result of amplifying the initial increase in ζ_t since from the firm's j_{t+1} first-order condition (52) the value of potential productivity in the present also depends on the value of j to future j-growth. This effect is proportional to the parameter ϕ however, and small values of ϕ in effect act like a large depreciation of j and can thus limit the rise in ζ. Consistent with the numerical evaluation of indeterminacy earlier, this effect underscores the necessity of relative high values of ϕ for indeterminacy [10].

Now turning to the labour channel, the effect of the initial rise Υ in the H-hours first-order condition (53) increases the firm's demand for labour in adoption, n_h. Since there is no shift in the productivity of goods production in the initial period however, the marginal product of N_h in goods production doesn't shift initially and therefore the firm's total labour demand shifts out in an attempt to satisfy the increase in N_h though additional labour [11]. In tandem with this shift in labour demand, the high cost of current consumption caused by the investment opportunities produces an increased willingness of the household to substitute out of current leisure, creating a shift in labour supply and lowering the real wage in the initial period. The net effect in the labour market is a rise in total hours, an effect which is amplified by capacity utilization through the impact of the increase in N_h on the marginal product of utilization. This labour market effect then continues into subsequent periods through the propagation effects discussed earlier, and is further amplified by the gradual rise in K and H which increase labour demand further.

As a result, both the relative price effects through investment and rapid expansion in labour allow the future marginal products of K and H to rise despite "investment" in these factors rising also, thus confirming the original conjectured beliefs.

Importantly however, the sink-dynamics of the stationary sunspot equilibrium require not just a self-reinforcing return, but also a channel of stability that pulls the system back to steady-state and keeps it off the explosive path. The effect of the technological gap in this model provides a critical role in this regard. To see this, it is helpful re-state the firm's h_{t+1} first-order condition,

$$\Upsilon_t(i) = \beta E_t \Big\{ \lambda_{t+1} \nu \theta \frac{P_{t+1}(i) y_{t+1}(i)}{h_{t+1}(i)} + \Upsilon_{t+1}(i) \Big[1 + \left(1 - \frac{2h_{t+1}(i)}{j_{t+2}(i)} \right) \Phi\Big(n_{ht+1}(i) \Big) \Big] \Big\}. \quad (55)$$

Note that the last term $\left(1 - \frac{2h_{t+1}(i)}{j_{t+2}(i)}\right)$ varies through time as a result of changes in the technological gap. As the firm grows its productivity h over

[10] Note that endogenous growth in J would also serve to amplify the increase in ζ, but since under the current specification for J-growth J does not vary independent of Ψ in equilibrium, this margin is shut down under this specification.

[11] Again under an alternate specification, growth in J independent of Ψ would increases the size of the gap $\left[1 - \frac{h_t(i)}{j_{t+1}(i)}\right]$, further increasing the demand for N_h, but this margin is shut down under the current specification.

time, it narrows the gap between current productivity and the technological frontier, thereby reducing future growth in productivity. Dynamically narrowing the gap thereby gradually reduces the benefit of adoption, reducing Υ_t over time. Both the combined effect of the narrowing gap and the reduction in Υ_t over time then reduces the amount of labour the firm allocates to adoption, and the reduction in Υ_t over time reduces the value of new investment, gradually pulling the system back to steady state.

Since firms must invest in new capital to reap the productivity gains, the channels through which a belief shock in this economy impacts aggregate quantities shares similarities with a broader class of "investment shock" models that affect the marginal efficiency of investment, including those describing investment specific technical change, credit and capital installation shocks, such as in Greenwood et al [43], Fisher [36], Primiceri et al [64] and Justiniano et al [54]. These models all describe a variation of a shock that drives a wedge into the household's Euler equation, making current consumption expensive as the household seeks to increase investment. While not shown in Figure 3, in this model the effects working through the investment first-order condition produce an initial drop in the relative price of physical capital in terms of consumption, $q_t^k = \frac{\mu_t}{\lambda_t}$, in response to the belief shock. Thus the model produces endogenous movement in q_t^k which is consistent with findings in this literature regarding countercyclical movements in the relative price of installed capital as a result of changes in the marginal efficiency of investment an/or investment specific technical change. This literature also typically finds that these shocks imply negative co-movement between consumption and investment in versions of the models that stay close to the neoclassical core. Without preferences with non-separabilities in consumption and leisure, this model would suffer the same co-movement issues. With non-separable preferences however, the marginal utility of consumption is increasing in hours worked, and therefore the rise in labour through the interactions in the labour market cause a similar rise in consumption.

It is important to note that capacity utilization plays subordinate role in this economy, and is not key to driving indeterminacy [12]. To see the effect of utilization, we can re-state the firm's utilization first-order condition as

$$\frac{\mu_t(i)}{\lambda_t(i)}\delta'(u_t(i))k_t(i) = \nu\theta\frac{P_t(i)y_t(i)}{u_t(i)}. \tag{56}$$

[12] Variants of the model with either extremely high costs of utilization or no utilization at all produce very similar regions of indeterminacy.

Both the expansion of labour that increases the marginal product of utilization, and the drop in $\frac{\mu_t}{\lambda_t}$ that reduces the cost of utilization increase the rate of utilization in response to the belief shock, amplifying the expansion of output in the early periods.

Figure 4 shows the response of the model economy to the same shock as Figure 3, this time for $(\phi, \rho) = (0.8, 0.85)$. As is clear from the graph, the response of the model economy for this combination of (ϕ, ρ) is very similar that that in Figure 3.

5 Conclusion

In this paper I argue that the technological frontier need not undergo sudden shifts to influence the dynamics of the economy in the short and medium run. I present a theoretical model where the technological frontier moves slowly and without shocks, yet where the economy adopts to this frontier endogenously at higher frequencies based on agents' beliefs about the actions of others. As a result, the rate of realization of the benefits provided by the technological frontier is independent of shocks to that frontier. Yet the underlying growth rate of the frontier is critical to determining the possibility that expectations can in fact influence the dynamics through its impact on indeterminacies. Thus I am ultimately providing an argument of how technology may be important not through technology shocks, but because it establishes a technological regime for the economy that either enables or inhibits expectations to play a role.

This result has a number of interesting implications. First, the argument highlights the need to properly account for structural change, not just in empirical methods, but also in theoretical models that often form the basis for the emphasis in these methods. Empirical researchers have certainly attempted to control for the slower-moving forces that change state of the system through structural change, and in fact recent work by Canova et al [23] and Fernald [35] shows that correctly controlling for the long (yet stationary) cycles in hours-worked that appear in the data is critical to obtaining an unbiased assessment of the response of hours-worked to neutral technology shocks. Yet the argument I am making suggests that obtaining a full account of the impact on technology and aggregate fluctuations may go beyond that just associated with identifying shocks that have a long-run impact on productivity. In particular, in my model structural change from changing growth regimes allows for the influence of belief shocks, thereby not only altering the conditional response of variables to other shocks, but also changing the state space of the shocks themselves. Moreover, since in the model long-run growth is deterministic, these belief shocks *do not* alter the permanent long-run level of labour-productivity; they just change the *rate* that the economy realizes these fundamental changes. As such, empirically-identified shocks that are deemed unrelated to technology may in be fact be related to beliefs about the technological regime, masquerading as "normal" temporary disturbances.

Second, the technological regime change that I highlight has the potential to create not just a role for unobserved beliefs to influence dynamics, but also the potential to influence the response to other shocks, such as monetary policy or credit, such that the structural change itself becomes at least as important as the exogenous shock. This idea of structural change in technology runs parallel with the discussions of causality related to the response of regime change in monetary policy and its reaction to exogenous shocks. John Cochrane [20] suggests that the answer to the question "What exogenous shocks account for business cycle fluctuations" has "more limited applications than is usually recognized", and goes on to propose an example where oil price shocks have a small effect on the economy, yet trigger a severe response of monetary policymakers that produces a recession. Did the oil shock cause the recession? In the context of my model, this property supports the intuitive notion that the response of the economy to a given shock depends on the state of the system - in this case the technological state - and implies that the dynamic effect of a given shock may not necessarily be stable over time, a result that has strong implications for empirical identifications.

Third, while I am modeling endogenous adoption, I am not modeling endogenous growth, and the dependence of the endogenous adoption on the technological state means that adoption and R&D activities will be different during "normal times" versus technological transition. This property thus allows the model to break any stable link between R&D effort and productivity realization, a property held by many endogenous growth models linked to the business cycles, freeing it from the criticism of Jones [53] of weak evidence between R&D effort and productivity.

Fourth, the model contains the implication that "bad" shocks such as contractionary credit shocks unrelated to technology in the midst of a technological era don't permanently impact the level of output following the recession; they only "delay" the eventual realization of the benefit of the slowly evolving technological frontier. This contrasts with models where the aggregate growth in aggregate ideas that gives rise to technological change is connected to high-frequency business cycle forces, and where recessionary forces therefore reduce the rate of ideas generated, permanently reducing the level of output. Econometric evidence provided by Beaudry and Koop [9] suggests that recessions don't permanently impact the level of output. Moreover, a corollary of this in my model is that the rate of growth following non-technological recessions *may* increase not just due to increasing the utilization of resources that were under-utilized during the recession, but also because the economy moves further below its technological potential, increasing the upside benefit of closing the technological gap [13]. In this sense, whereas a belief-driven boom would allow the economy to "pull-forward" technological benefits, a contraction would cause it to "push back" technological benefits.

Finally, since the additional returns to capital that drive indeterminacies may exist only temporarily, it poses implications for empirical methods that seek to evaluate the plausibility of models with indeterminacies by determining whether industry data exhibit the degree of returns and scale and externalities used by the sunspot literature. While a particular growth regime may in the aggregate last upwards of 10-15 years such as in the 1990's, in a given industry the effects may be more concentrated in time. Furthermore, the returns to scale may not be as evident in the variation of output as they are in eventual productivity increases resulting from purchases of "new era" capital. This proposition is particularly interesting in light of the empirical evidence found by Basu and Fernald [6] between industry capital use and eventual productivity increases. As such, the model underscores the importance of controlling for structural change related to capital transitions in industry-level regressions seeking to determine plausible degrees of returns to scale.

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Figure 1: Dependency of indeterminacy region on Φ and ρ for baseline parameterization

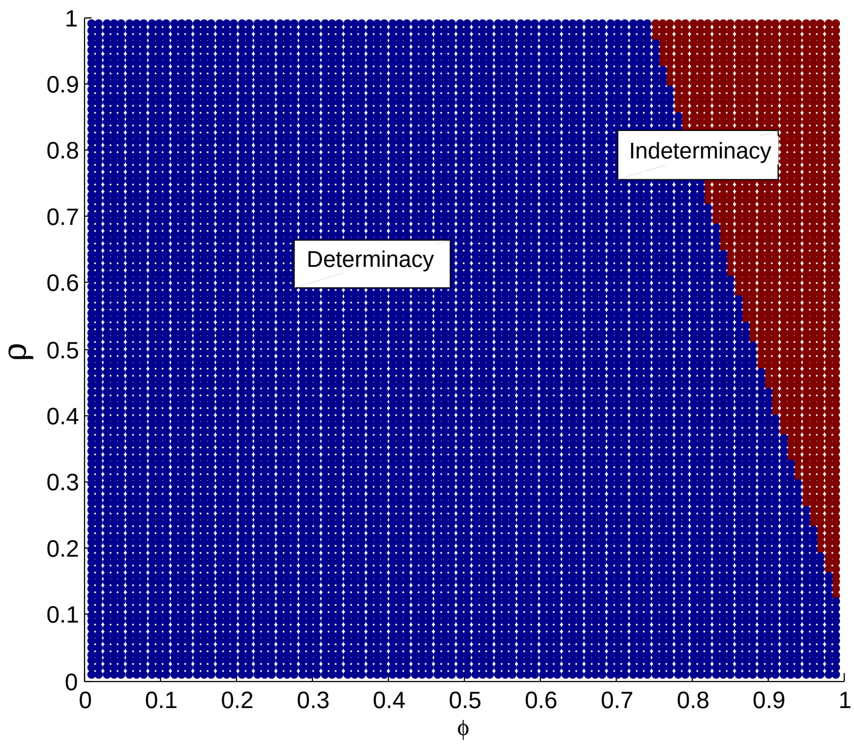


Figure 2: Dependency of region of indeterminacy on growth rate of ideas frontier g_Ψ

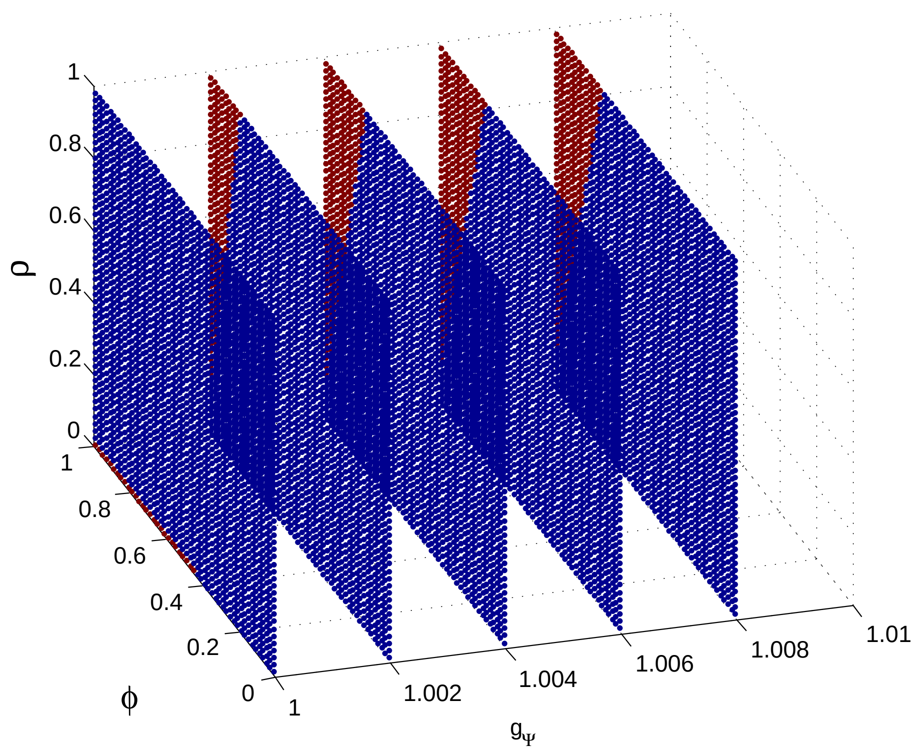
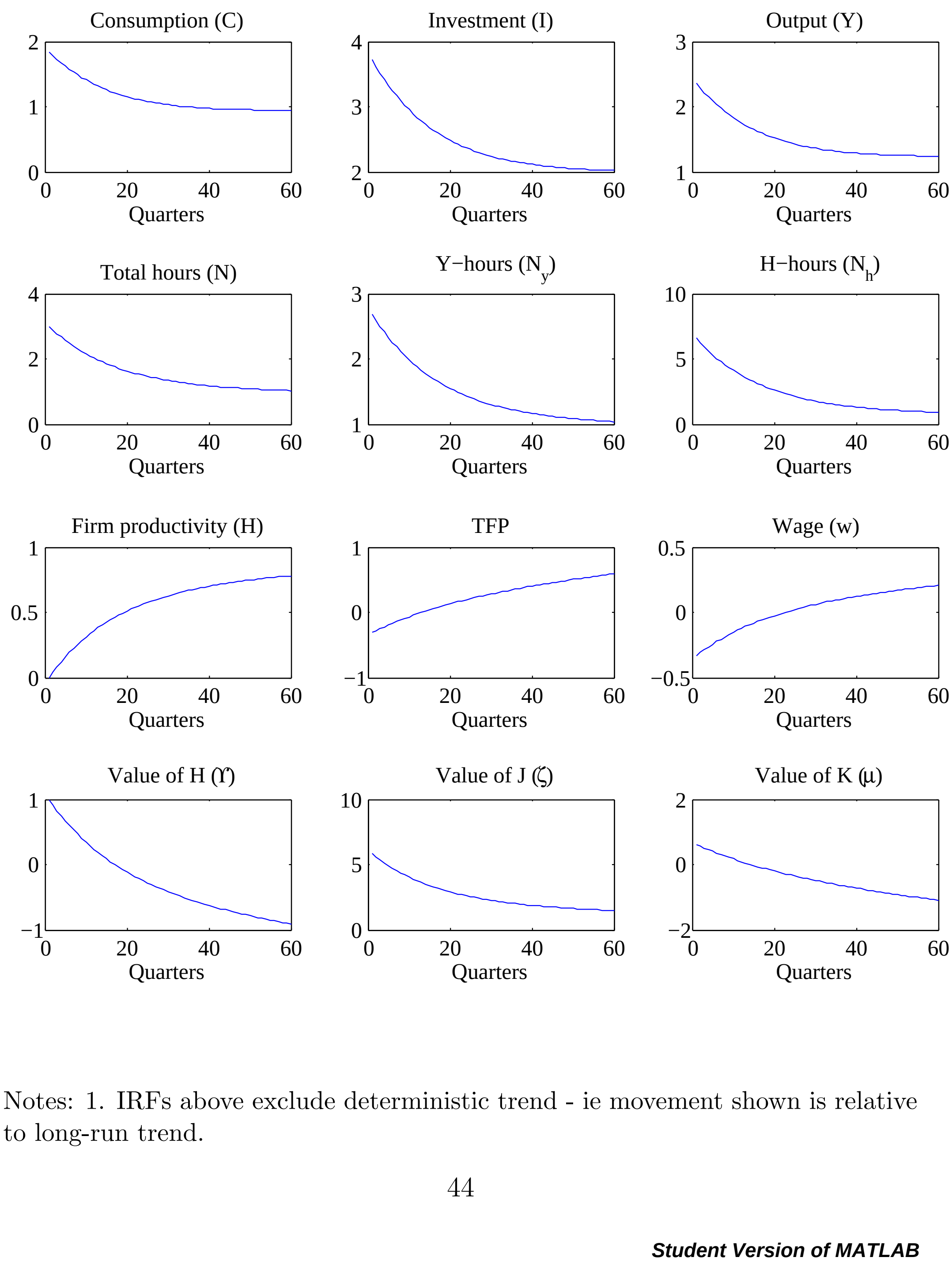
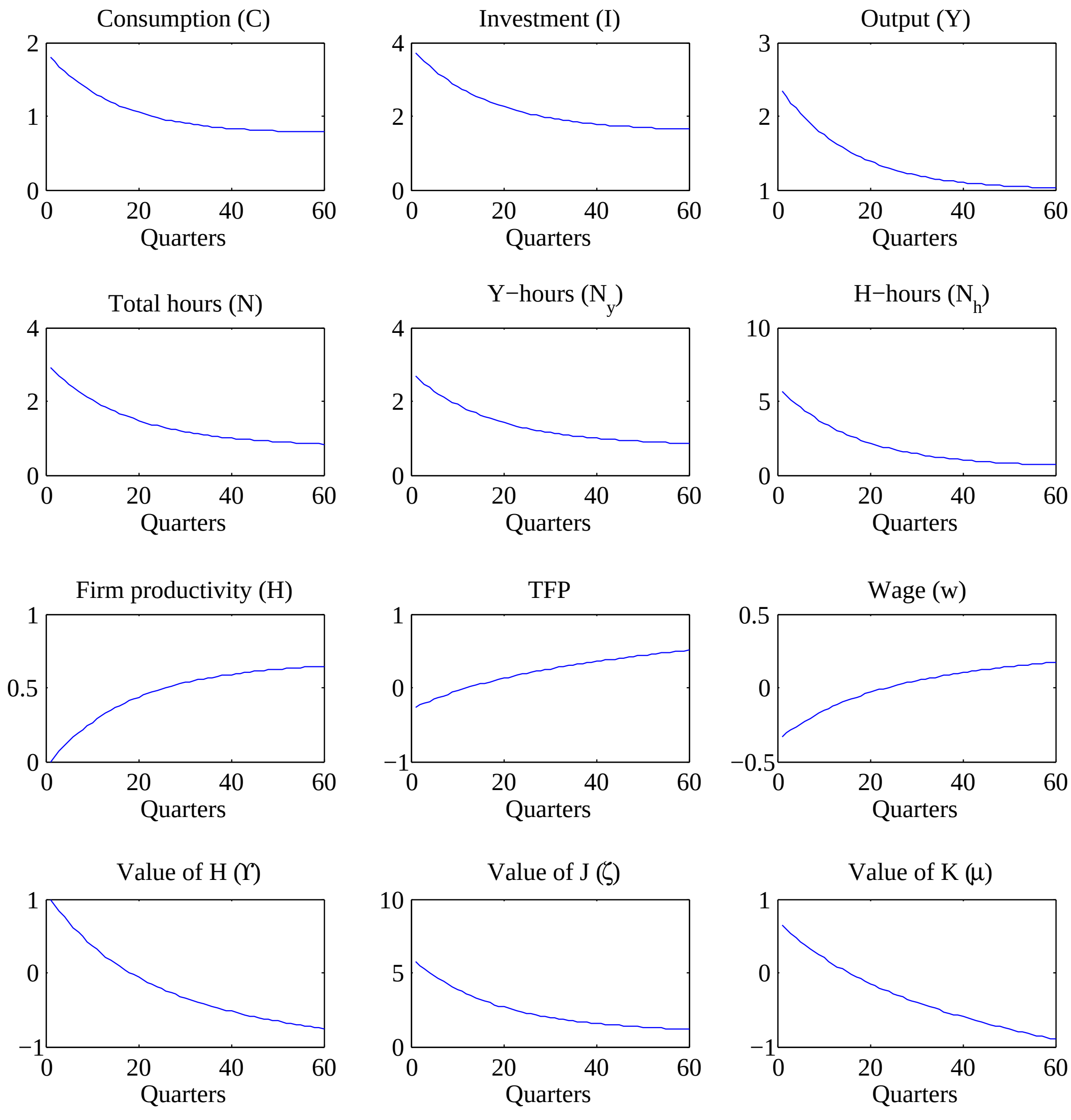


Figure 3: Response to iid sunspot shock about value of H, relative to trend: $\phi = 0.9$; $\rho = 0.45$



Notes: 1. IRFs above exclude deterministic trend - ie movement shown is relative to long-run trend.

Figure 4: Response to iid sunspot shock about value of H, relative to trend: $\phi = 0.8$; $\rho = 0.85$



Notes: 1. IRFs above exclude deterministic trend - ie movement shown is relative to long-run trend.