"Gesell" Tax and Efficiency of Monetary Exchange

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Abstract

A periodic "Gesell Tax" on money holdings as a way to overcome the zero-lower-bound on nominal interest rates is studied in a framework where money is essential. For this purpose, I characterize the efficiency properties of taxing money in a full-fledged macroeconomic business cycle model of the third-generation of monetary search models. Both, inflation and "Gesell taxes" maximize steady state capital stock, output, consumption, investment and welfare at moderate levels. The Friedman rule is sub-optimal, unless accompanied by a moderate 'Gesell tax'. In a recession scenario a Gesell tax speeds up the recovery in a similar way as a large fiscal stimulus but avoids "crowding out" of private consumption and investment.

Key words: monetary search-theory, negative interest rates, Gesell tax, capital formation, DSGE model

JEL Classification: D83, E19, E32, E49

1 Introduction

In the light of the experience of the Japanese economy in the 90's of the last century economists started to rethink the importance of the zero-lower bound on nominal interest rates that limits the powers of a Central Bank to stimulate the economy through interest rate policy. The related liqiudity trap hypothesis reaches back to Keynes' General Theory and its later interpretations claiming a zero interest rate elasticity of money demand when the nominal interest rate reaches low values such that any increment of the money supply is absorbed by a higher money demand, i.e. by the public willing to demand these additional money holdings without any change in the interest rate. In such a situation a standard expansionary monetary policy aiming at lowering interest rates by an increased money supply in order to foster investment and consumption does not work, since interest rates do not react to the increased money supply. The liquidity trap hypothesis was resurrected by Krugman [20],[21] [1] claiming that in low inflation episodes with low short term interest rates the zero-lower bound on nominal interest rates might get binding giving rise to such a liquidity trap behaviour. Thereafter, a new literature on how to engineer negative nominal interest rates to overcome this lower bound started with the work of Buiter and Panigirtzoglou [6], [7], Goodfriend [13] and Buiter [5]. They propose a periodic tax on money holdings to increase the opportunity cost of not lending money such that lenders have incentives to part with their liquid money balances even at negative interest rates.[2]

The original proposal of this tax stems from the writings of Silvio Gesell [12] and was thought to overcome a market failure stemming from the superiority of money as a store of value compared to real assets that have higher carrying costs: In Gesell's view, money holders can excert their economic power by forcing borrowers to pay a *basic interest* (german:'Ur-Zins') and the tax on money aims at eliminating this advantage of moneyholders and therefore at reducing the riskless nominal interest rate, which will reach 0 in the long run. When accumulation of capital does not run anymore into the lock of positive nominal interest rates this will lead to the termination of capital rents, and the market economy will be free of "capitalism". This is

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[1]See Buiter and Paniggirtzoglou [7] for more references.

[2]Implementability issues of such a tax are discussed - apart from these references - in Mankiw [24] and Ilgmann and Menner [15] and are therefore not dealt with in this article.

basically the same argument as Keynes' "euthanasia of the rentiers"[3]. Thus, Gesell hinted at social reform of the capitalist system, maintaining property rights and the market system, but eliminating undue capital rents (effortless income) in the long run by monetary and land reform.[4]

My intention is far from trying to give a complete exposition, interpretation and critical evaluation of Gesell's ideas[5]. However, the idea of inefficiencies in the current monetary system might be an interesting aspect in the discussion of the 'pros' and 'cons' of the application of a Gesell tax to achieve negative interest rates and to overcome the lower-bound restriction on interest rate policies. The recent macroeconomic literature on negative interest rates, and the papers in this literature dealing with Gesell taxes evaluated their implications in Walrasian dynamic stochastic general equilibrium models, where there is no inefficiency in monetary exchange, but on the other hand money is totally inneccessary for the allocation of resources.

However, there is another class of models building on the seminal work of Kiyotaki and Wright [16], [17] that use search theory for the micro-foundation of money.[6] The common feature of monetary search models in this tradition is tha they set up an environment where money eases bilateral trade in overcoming the problem of an 'absence of double coincidence of wants'. Hence,

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[3]"Though this state of affairs (just about enough return to cover cost of capital replacement) would be quite compatible with some measure of individualism, yet it would mean the euthanasia of the rentier, and consequently, the euthanasia of the cumulative oppressive power of the capitalist to exploit the scarcity value of capital " (Keynes' General Theory. pp. 375-376).

[4]Fisher [11] also embraced Gesell's idea of money taxes since he believed that this would temporarily increase local commerce and employment in the depressed economy where means of payments were scarce, so he fostered the "Stamp Scrip Movement" that extended to 500 communities experimenting with issuance of local currencies prone to money taxes in the U.S. and Canada. But he rejected Gesell's theory, including the advise of a tax rate of 5-6%, and proposed a self-liquidating tax rate on new issued local currencies, i.e. a annual tax rate of 104%. While these experiments failed, there existed some successful implementations of Gesell's proposals at the local level in Wörgl (Austria) and Schwanenkirchen (Germany) that caught much attention at the time and received many visitors, including Fisher himself, but the experiments where stopped by the monetary authorities. See Blanc [3].

[5]See, e.g., in this respect the articles (and the references therein) of Ilgmann [14] who discusses Gesell's theory of interest and its connection to J. M. Keynes' General Theory, and of Ilgmann and Menner [15] who give a review of Silvio Gesell's academic reception and discuss current proposals on negative nominal interest rates and 'Gesell taxes'.

[6]See for example Rupert et al.[29], chapter 4, and Shi [35] for extensive overviews over the literature based on the search-theoretic approach.

money plays an essential role in the sense that some of the allocations achievable in a monetary equilibrium cannot be achieved in an equilibrium without money.[7] In terms of efficiency monetary equilibria with positive value of money improve on barter equilibria but can still suffer from inefficiencies.

The literature distinguishes now three generations of search models of money, each of them dealing in a different way with the the high degree of heterogeneity of agents that arises through the pairwise exchange of goods which is generating non-degenerate distributions of goods inventories and money holdings[8]. Early search models limited the state space by assuming indivisible money and indivisible goods such that an agent could hold only 1 unit of money or 1 unit of goods and trade took place at a constant price 1. A second generation of models endogeneized prices by allowing for divisible quantities and bargaining about how much goods to exchange for 1 unit of money. In both set-ups indivisible money inhibits the study of money growth. Third generation search models have come around the need to impose this restrictive assumption and allow therefore to study directly money growth and inflation.

An interesting feature of these search models is that the exchange process can be inefficiently low because of too little search effort of buyers. Li [22] was the first to point out the externalities that can arise with endogenous search effort of buyers in a first generation model: Since search is costly, buyers compare their search costs with the private gains from search, rather than considering the social gains and costs of a higher search intensity. In sufficiently productive economies, there is a positive search externality that leads to a lower aggregate number of transactions relative to the social optimum. The author proposes a 'tax on money' to deal with this inefficiency and claims that the welfare improvement 'emanates directly from the ability of such policies to increase search efforts and the aggregate rate of transactions. That is, the search externality provides a role for government in subsidizing search activity through taxing "nonsearch"', (Li [22] p. 938). This resembles

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[7]Kocherlakota [18] establishes that necessary conditions for the essentiality of money are the lack of complete memory and of full commitment to future actions. The latter follows from the usual assumption of random-matching and rules out the use of credit, while the former inhibits the use of punishments to trigger gift-giving equilibria. See also Corbae, Temzelides and Wright [10] for models with directed search where money remains essential as long as long as agents are restricted to one bilateral trade per period.

[8]The use of simulation methods to keep track of these distributions is very cumbersome. See Molico [26] and Molico and Zhang [27].

Gesell's [12] idea of taxing the hoarding of money to incentivate the spending or lending of money balances[9]. The tax on money is modelled as a random expropriation of a unit of money. It is then interpreted as a "proxy for inflation", which is generally thought to have the same consequences as the money tax in this model: increase in the cost of holding money, crowding out of real commodities through seignorage revenue and reduction of real money balances. Thus, the author conjectures the optimality of inflation in more general models that would allow for positive money growth rates[10].

This paper studies the macroeconomic effects of money taxes a la Gesell in a full-fledged business cycle model that builds on the third-generation search-theoretic monetary model of Shi [32]. Long run steady states of various macroeconomic variables depend on the combination of money growth rates and Gesell taxes: search intensity and velocity increase with both, inflation and Gesell taxes. At moderate levels of in‡ation, a Gesell tax can still increase steady state output and capital and has positive effects on consumption, investment and welfare. The 'Friedman rule' that equalizes the growth rate of money to the discount factor is feasible only if accompanied by a Gesell tax, and the welfare maximizing tax rate under the Friedman rule is of 5-6% annualized. A temporary implementation of Gesell taxes has qualitatively similar effects as a temporary shock to money growth with hump-shape impulse responses of output, employment, capital and velocity and short-run increases in sales, consumption and investment. Although a permanent implementation of Gesell taxes leads to negative responses of employment and output in the short run, when parting from a steady state, this policy can speed up the recovery out of a recession scenario in a similar way as a large fiscal stimulus program, and more so than if only implemented

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[9]Most monetary search models deal only with the spending aspect and do not treat the possibility of lending idle balances.

[10]This conjecture is studied in Liu et. al [23] under the label "hot potato effect". They also provide some empirical evidence of the link between inflation and velocity. Notably, there is no mentioning of the tax on money as a *policy proposal* to engineer negative interest rates. Instead, they discuss money taxes only as a *proxy for inflation*. When extending their analysis to a third generation search model in the tradition of Lagos and Wright [19], the authors focus only on the effects of inflation, since their environment now allows to study positive money growth rates directly. Therefore, it is no exageration to claim that the two different literatures on money taxes - (1) as a means to *proxy for inflation* and to *overcome inefficiencies* in the monetary exchange process and (2) the *Gesell tax* proposal to achieve *negative nominal interest rates* - have been totally unconnected up to the present.

temporarily. It can be concluded that a permanent Gesell tax, since it is efficiency improving, is preferable to a temporary use to overcome a zero-bound on nominal interest rates as usually suggested.

The remainder of the paper is organized as follows: Section 2 presents the model. Section 3 discusses the effects of inflation and of the Gesell tax on steady states. Section 4 considers the dynamic transition path after a permanent or transitory introduction of the Gesell tax starting from steady state or from a recession scenario, respectively. Section 6 concludes.

2 "Gesell" Taxes in a Third Generation Monetary Search Model

Instead of analyzing the role of Gesell taxes in a framework of alternating decentralized and centralized goods markets as in Lagos and Wright [19] I focus here on the alternative framework of Shi's [32] representative agent model. The reason is mainly the lack of persistence that decisions in the decentralized markets tend to have when agents can undo many imbalances easily in the following centralized market. On the contrary, the model of Shi [32] and its extension of Menner [25], features a persistent propagation of shocks to money growth through a "search-intensity - inventory-deployment" feedback which seems worth while to explore when dealing with Gesell taxes. The analysis of Gesell taxes in a Lagos and Wright [19] type of model is left for future research.

2.1 The Model Environment

This section presents an extended version of the search-theoretic monetary model developped in Menner [25] on the base of Shi [32]. The difference to the former model lies in a different treatment of adjustment costs in the investment process, in allowing for external habit persistance on behalf of the consumers, and in the incorporation of a government with balanced budget that spends in goods and imposes lump-sum taxes and a tax on money. In this model economy there are two search frictions: costly labor search and costly search for consumption goods. The economy is populated by a continuum of households with measure one, denoted by H. Each household produces a distinct good with labor and capital as inputs to production. Each good $h \in H$ is storable as inventories only by its producer, and it can

be installed as capital only by all other households, that means, producers cannot use their own product as capital. Each household $h \in H$ produces good h and wants to consume a subset of goods that is different from its own product. This induces a need for exchange before consumption or investment can take place. In the absence of a centralized market with a Walrasian auctioneer households have to search for trading partners with the desired goods. Generally, there will be no double-coincidence of wants and anonimity prevents credit arrangements as a mean to overcome this problem.

The literature following Kiyotaki and Wright [16], [17] showed that in random search models under certain parametrizations fiat money gets valuable and is the only medium of exchange. Since we are not interested in the direct competition of indirect barter and money, fiat money is assumed to be required in each transaction.[11]

The production function is assumed to be neoclassical. The employment of factors of production evolves over time in the following way: A fraction δ_n of the currently employed workers is fired at the end of each period. New workers have to be hired through a costly search process and get productive in the next period. Households invest part of their purchased goods to augment their capital stock in the next period. To do so they have to pay a quadratic installation cost. Each period a fraction δ_k of the capital stock depreciates.

The matching in the goods market between sellers and buyers and in the labor market between producers and unemployed is assumed to be random. Hence, individual agents face idiosyncratic risk. As a consequence, money holdings, capital stocks and inventories differ across agents, as well as the number of people employed.

To avoid the need of tracking the distributions of money holdings, capital stocks, inventories, and employment, it is assumed that the decision unit - the household - is itself a continuum of different agents. The members of the household share the bought consumption goods and regard the household's utility as the common objective. Wage payment regardless of whether the

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[11]In principle the presence of inventories and capital could prevent fiat money from being essential. Commodity money is prevented under the assumption that goods can be stored only by their producers or as installed capital, but claims on capital or inventories could still circulate as medium of exchange. One can rule out this possibility either through the assumption that this claims are prone to easy counterfeiting while money is not (see Arouba et al. [2]) or through the specification of the matching process such that the probability of meeting again a trading partner or someone who has traded with him is zero so that claims cannot be redeemed (see Aliprantis et al. [1]).

firms had a suitable match in the goods market is made possible by resource sharing of firms within a household. Inventory holdings, the capital stock as well as the employees for the next period are shared among the firms of a household, too. Under these assumptions there is no idiosyncratic risk anymore due to the random matching process.

The household consists of five groups: one group of members enjoys leisure while the other four groups are active in markets: Entrepreneurs (set A_p with measure a_p), unemployed (A_u, measure u) workers (A_{nt}, measure $a_p n_t$), and buyers (A_b, measure a_b). The values of a_p, u and a_b are assumed to be constant, while the number of workers per firm n_t may vary over time. An entrepreneur consists of two agents: a producer and a seller. A producer in household h hires workers from other households to produce good h, which is sold by the seller. A worker inelastically supplies one unit of labor each period to other households' firms. A buyer searches with search intensity $s > 0$ to buy the household's desired good. The sellers' search intensity is set to 1. Thus, we focus only on the effect of monetary policy on buyers' search intensity. Let $B = a_b/a_p$ be the buyers/sellers ratio. In the following a hat on a variable indicates that the household takes this variable and all its future values as given when making the decisions at t.

The matching process is specified as follows. The total number of matches in the goods market is given by the matching function:

$$g\left(\hat{s}\right) \equiv z_1 \left(a_b \hat{s}\right)^\alpha \left(a_p\right)^{1-\alpha}, \qquad \alpha \in (0,1).$$

By normalizing $z \equiv z_1 B^{\alpha-1}$ the matching rate per unit of search intensity is $g_b\left(\hat{s}\right) \equiv z\hat{s}^{\alpha-1}$, so that a buyer finds a desirable seller at a rate sg_b, and a seller meets a desirable buyer at a rate $g_s\left(\hat{s}\right) \equiv zB\hat{s}^\alpha$. Thus, the measure of the set of buyers with suitable matches, A_{b*}, is $sg_b a_b$ and that of sellers with suitable matches, A_{p*}, is $g_s a_p$.[12]

Each buyer j having found a seller $-j$ with his desired good exchanges $\hat{m}_t\left(j\right)$ units of money for $\hat{q}_t\left(-j\right)$ units of good $-j$, which implies a price of good $-j$ in this match of $\hat{P}_t\left(j\right) = \hat{m}_t\left(j\right)/\hat{q}_t\left(-j\right)$.[13] Bought goods are brought back to the household where they are shared for consumption c_t and investment x_t. Unsold goods are brought back to the household and stored as inventories that depreciate at a rate δ_i. Next period, before the remaining inventories are distributed among sellers, the government buys an

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[12] The notation $*$ stands for agents that are suitably matched in the current period.

[13] The notation $-j$ stands for an agent with whom agent j is matched.

exogenously specified amount of goods Gov_{t+1} from each household paying for it at the end of the period. We denote Gov_t/a_p as gov_t. So, after production each seller j holds $i_{t+1}(j) + f(n_{t+1}(j), K_{t+1}) - gov_{t+1}$ units of goods on stock.

Households value private consumption through the instantaneous utility function $U(c_t) = (c_t - \zeta \tilde{c}_{t-1})^{1-RA}/(1-RA)$, with CRRA parameter RA, external habit ζ and $\tilde{c}_{t-1}$ is last period's average consumption. They value government expenditure through the utility function: $U(Gov_t) = \omega_g \log(Gov_t)$, where ω_g is a non-negative weight less than one.

Each producer j can create vacancies $v_t(j)$ with a cost of $\Upsilon(v_t(j))$. Unemployed workers have to search for a job and they do this by supplying one unit of search effort inelastically. A worker supplies inelastically one unit of labor each period and receives a wage $\hat{W}(j)$ in units of money. There is an exogenous constant job separation rate δ_n. The matching function in the labor market is linearly homogeneous. The number of matches between firms and unemployed workers is given by $(a_p \hat{v})^A (u)^{1-A}$ and the number of matches per vacancy is $\mu(\hat{v}) \equiv (a_p \hat{v}/u)^{A-1}$.

2.2   The Households' Decisions

Households decide at the beginning of each period about their buyers' search intensity s_t, consumption c_t, investment x_t, and the number of vacancies per firm v_t, as well as next period's total capital stock K_{t+1}, employment per firm n_{t+1}, and the amount of 'fiat' money M_{t+1} and inventories i_{t+1} to be carried into period $t+1$. Imposing symmetry within a household each member of a group is assigned the same stocks and decision rules. Each buyer receives $m_{t+1} = M_{t+1}/a_b$ units of money and each firm a capital stock $k_{t+1} = K_{t+1}/a_p$.

An individual firm's production technology is assumed to be Cobb-Douglas:

$$f^i(n,k) = F_0 n^{e_f} k^{1-e_f}, \qquad \text{where} \ \ e_f < 1,$$

or, more conveniently, in terms of aggregate capital K: $f(n_t, K_t) \equiv f^i\left(n_t, \frac{K_t}{a_p}\right)$.

In their decisions households take the sequence of terms of trade and wages $\left\{\hat{q}_t, \hat{m}_t, \hat{W}_t\right\}_{t \geq 0}$ as given, as well as $\{M_0, K_0, i_0, n_0\}$. Since both buyers and sellers have a positive surplus from trade, it is optimal for households to choose $M_{t+1}, K_{t+1,} n_{t+1}$ and i_{t+1} such that in period $t+1$ every buyer carries the required amount of money $\hat{m}_{t+1}$ and that every seller carries $\hat{q}_{t+1}$ units of good h. The assumptions $M_0 \geq \hat{m}_0 a_b$ and $i_0 + f(n_0, K_0) \geq \hat{q}_0$ ensure that buyers and sellers do so also in period 0.

Households choose the sequence $\Gamma_h \equiv \{c_t, x_t, s_t, v_t, M_{t+1}, K_{t+1}, i_{t+1}, n_{t+1}\}_{t \geq 0}$ to maximize their expected lifetime utility over an infinite time horizon:

$$\max_{\Gamma_h} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(c_t) + U^G(Gov_t) - \int_{A_{n_t}} \varphi dj - \int_{A_b} \phi(s_t(j)) \, dj - \int_{A_p} \Upsilon(v_t(j)) \, dj \right] \right\}$$
(PH)

subject to the following constraints:

$$c_t + x_t + \frac{b}{2} \left(\frac{x_t}{K_t} - \delta_k \right)^2 K_t \leq \int_{A_{bt*}} \hat{q}_t(-j) \, dj,$$
(1)

$$\frac{M_{t+1}}{a_b} \geq \hat{m}_{t+1}(j), \qquad \forall j \in A_{bt+1*},$$
(2)

$$i_{t+1}(j) + f(n_{t+1}(j), K_{t+1}) - gov_t \geq \hat{q}_{t+1}(j), \qquad \forall j \in A_{pt+1*},$$
(3)

$$(1 - \delta_k)K_t + x_t \geq K_{t+1},$$
(4)

$$
\begin{aligned}
M_{t+1} \leq \ & (1 - \tau_t^m) \left[M_t - \int_{A_{bt*}} \hat{m}_t(j) \, dj + \int_{A_{n_t^s}} \hat{P}_t \hat{W}_t(-j) \, dj \right.\\
& \left. + \int_{A_{pt*}} \hat{m}_t(-j) \, dj - \hat{P}_t \int_{A_p} \hat{W}_t(j) n_t(j) \, dj \right] + \hat{P}_t Gov_t - T_t,
\end{aligned}
$$
(5)

$$\int_{A_p} \left[(1 - \delta_n) n_t(j) + v_t(j) \mu(\hat{v}_t) - n_{t+1}(j) \right] dj \geq 0,$$
(6)

$$(1 - \delta_i) \left[\int_{A_p} \left[i_t(j) + f(n_t(j), K_t) - gov_t \right] dj - \int_{A_{pt*}} \hat{q}_t(j) \, dj \right] \geq \int_{A_p} i_{t+1}(j) \, dj.$$
(7)

Constraint (1) states that the household's consumption and investment plus the quadratic investment cost has to be bought by buyers which successfully meet a trading partner and are endowed with sufficient money for the purchase of $\hat{q}_t$ goods each. Condition (2) represents a minimum money

holdings constraint for each suitably matched buyer in period $t+1$, while (3) is a similar trading restriction for suitably matched sellers: in period $t+1$ each needs a sufficient stock of inventory and newly produced goods to satisfy customer's demand. Expression (4) is the usual capital accumulation equation. The law of motion of money balances (5) states that money holdings at the beginning of period $t+1$ are no larger than the *after-money-tax* end-of-period t money holdings (brackets) plus the receipts of Government spending minus lump-sum taxes T_t. Here, τ_t^m is the "Gesell" tax on money holdings and the bracket consists of the beginning-of-period t minus the money spent plus wages earned and cash receipts from firms. Expression (6) indicates that a household cannot allocate more workers of other households to its firms in period $t+1$ than those who worked there in period t and have not quitted plus the newly hired workers. Finally, expression (7) states that inventories in period $t+1$ consist of the fraction of the excess supply of goods in period t which has not depreciated. The firing rate δ_n and the depreciation rates of inventories, δ_i, and capital, δ_k, are assumed constant.

It is convenient to denote by ω_{Mt} the shadow price of money at the beginning of period $t+1$ (M_{t+1}), measured in terms of period-t utility. Then ω_{Mt} is the multiplier of (5). Similarly, let ω_{Kt}, ω_{it} and ω_{nt} be the shadow prices of capital, inventory and workers at the beginning of period $t+1$, all measured in terms of period-t utility. Thus, ω_{Kt}, ω_{nt} and ω_{it} are the multipliers of (4), (6) and (7). Also, let $\Lambda_{t+1}, \omega_{qt+1}$, be the multipliers of (2) and (3), respectively, both measured in terms of period-$t+1$ utility.

2.3   Government

The government finances its expenditures through seignorage, Gesell taxes and lump-sum taxes which leads to the following budget constraint:

$$(\gamma_t - 1)M_t + \tau_t^m M_t + T_t = \hat{P}_t Gov_t. \tag{8}$$

where γ_t is the gross growth rate of money. Gesell taxes, hence, help finance government expenditures. In a more realistic setup where lump-sum transfers are not feasible, the proceeds of the Gesell tax could be used to reduce distortionary taxes.

2.4 Terms of Trade

2.4.1 Goods Market

In order to determine the terms of trade in each match and the associated price $P = m/q$, each agent is interpreted as an identity of a small measure Δ. First, the terms of trade contingent on Δ are calculated, then take the limit $\Delta \to 0$. This procedure is necessary because the contribution of a match to the households' utility is negligible when agents are negligible in a household.

When a seller from household h meets a buyer of household $-h$, they trade $q\Delta$ units of goods against $\bar{m}\Delta$ units of money. These terms of trade lead to the following surpluses in the two agents' households:[14]

The seller's surplus: $\quad \omega_{Mt}(1 - \tau_t^m)\bar{m}_t\Delta - [(1 - \delta_i)\,\omega_{it} + \omega_{qt}]\,q_t\Delta$.

The buyer's surplus:
$$U\left(\bar{c}_t + \zeta q_t\Delta\right) - U\left(\bar{c}_t\right) + (1 - \zeta)q_t\Delta\bar{\omega}_{xt}$$
$$- \left((1 - \tau_t^m)\bar{\omega}_{Mt} + \bar{\Lambda}_t\right)\bar{m}_t\Delta,$$

where $\bar{\omega}_{xt} = \omega_{Kt} \,/\, \left[1 + b\left(\frac{x_t}{K_t} - \delta_k\right)\right] = U'\left(c_t\right)$ is the value of an additional unit of investment and ζ denotes the fraction of q_t which is consumed.[15] Nash-bargaining with equal weights and taking the limit $\Delta \to 0$ implies

$$\omega_{qt} = (1 - \tau_t^m)\omega_t - (1 - \delta_i)\,\omega_{it}, \tag{9}$$
$$\bar{\lambda}_t = U'\left(\bar{c}_t\right) - (1 - \tau_t^m)\bar{\omega}_t. \tag{10}$$

with $\bar{\omega}_t \equiv P_t\bar{\omega}_{Mt}$, and $\bar{\lambda}_t \equiv P_t\bar{\Lambda}_t$.

2.4.2 Wage Bargaining

The firm's surplus from hiring a new worker is given by:

$$[\omega_{nt} - \beta\left(1 - \delta_n\right)\omega_{nt+1}]\,\Delta = \omega_{it}\left(f\left(n_{t+1} + \Delta, K_{t+1}\right) - f\left(n_{t+1}, K_{t+1}\right)\right) - \beta\omega_{t+1}W_{t+1}\Delta(1 - \tau_t^m).$$

The extra utility for a household when an additional member is working is:

$$\beta\left((1 - \tau)\bar{\omega}_{t+1}W_{t+1} - \varphi\right)\Delta.$$

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[14]Symbols with a bar refer to variables of household $-h$.

[15]Note that the latter equality represents the FOC for x_t (see Appendix A.1, eq. (19)).

The bargaining outcome is the wage rate that maximizes the weighted Nash product of the two agent's surpluses, with weight $\sigma \in (0, 1)$. After taking the limit $\Delta \to 0$, the bargained wage rate is:

$$W_{t+1} = \sigma \frac{\varphi}{\bar{\omega}_{t+1}(1 - \tau_t^m)} + (1 - \sigma) \frac{\omega_{it} f_n (n_{t+1}, K_{t+1})}{\beta \omega_{t+1}(1 - \tau_t^m)}. \tag{11}$$

2.5 Equilibrium

A symmetric search equilibrium can be defined as in Menner [25] and the equations determining equilibrium are derived in Appendix A.1.

Of particular interest with respect to "Gesell taxes" are the following two optimality conditions (Eq. (14) and (18), evaluated at equilibrium):

Money Holding : $\omega_{Mt} = \beta E_t \left\{ (1 - \tau_t^m)\omega_{Mt+1} + g_b(s_{t+1})s_{t+1}\Lambda_{t+1} \right\}$ ((14'))

Search Intensity : $\Phi'(s_t) = g_b(s_t) \left[U'(c_t) q_t - \omega_{Mt} m_t (1 - \tau_t^m) \right]$ ((18'))

Equation (14') tells us that the value of holding an additional unit of money at the beginning of period $t + 1$ is reduced by a positive tax on money, since only a fraction $(1 - \tau_t^m)$ will survive to period $t + 2$. The second part of the above sum reflects the value of money in relaxing the transaction constraint 2 during period $t + 1$.

Expression (18') balances the utility cost of an additional unit of buyer's search intensity with its benefits: the utility gain of exchanging money for goods in the event of being matched with a suitable seller.

Here, it can be seen how the Gesell tax incentivates search by increasing the utility gain of exchange: buyers that give up m_t units of money loose ω_{Mt} utils of only a fraccion $(1 - \tau_t^m)$ of m_t since only this fraction survives taxation of money balances at the end of the period. In other words, buyers try to 'dump' the depreciating money in the goods market, and to manage to do this they search harder for potential trading partners.

2.6 Nominal Interest Rates and the Friedman Rule

Definition 1 *The nominal interest rate is given by the inverse of the price of a bond that would deliver 1 unit of money next period:*

$$R_t = \frac{\omega_{Mt}}{\beta E_t \{\omega_{Mt+1}\}} \tag{12}$$

Lemma 2 *From (14') it follows that the nominal interest rate can be expressed as:*

$$R_t = \frac{\omega_{Mt}}{\beta E_t \{\omega_{Mt+1}\}} = (1 - \tau_t^m) + g_b(s_{t+1})s_{t+1}\frac{E_t \{\lambda_{t+1}\}}{E_t \{\omega_{t+1}\}}, \tag{13}$$

Proposition 3 *The Gesell tax allows for equilibria with negative nominal interest rates.*

Proof: *By use of $P = m/q$ and $\omega_t = P\omega_{Mt}$ rewrite (18') as $\Phi'(s_t) = g_b(s_t)q_t [U'(c_t) - \omega_t(1 - \tau_t^m)]$ which by (10) reads: $\Phi'(s_t) = g_b(s_t)q_t\lambda_t$. To have positive search effort and positive consumption in equilibrium in all periods we must have positive gains from searching, implying $\lambda_t > 0 \; \forall t$. By (13) this is equivalent to having $R_t - (1 - \tau_t^m) > 0$, or $R_t - 1 > - \tau_t^m$, i.e. the net interest rate $> - \tau_t^m$.*

Corollary 4 *Without a Gesell tax, only positive nominal interest rates are compatible with monetary equilibrium.*

Proof: *Set $\tau_t^m = 0$ in the proof of Proposition 1.*

Corollary 5 *A monetary equilibrium exists under the "Friedman rule" ($R = 1$) if and only if there is a Gesell tax ($\tau_t^m > 0$)*

Proof: *Set $R = 1$ in the proof of Proposition 1.*

The last Corollary gives us a very important result. In all models where money is not essential the Friedman rule of engineering a constant deflation by setting the money growth rate equal to the discount factor is optimal, since it eliminates the inefficiency of not having enough cash on hand when the opportunity cost of holding cash (a positive nominal interest rate) is causing agents to economize on their cash holdings. This results holds true also in search theoretic environments with a fixed number of buyers and exogenous search intensity.

However, when search intensity is endogenous as in the present model, there enters another inefficiency, i.e. the search externality, that asks for a higher incentive for buyers to search of for more buyers to enter the market. At the Friedman rule buyers have no incentive at all to search, so monetary equilibrium fails to exist. If a monetary authority would like to engineer the Friedman rule it would have to give buyers an additional incentive to search which could be a positive Gesell tax on money. This reduces the value of hoarding money in comparison to spending it and leads therefore to a positive gain of trade. The present model presents therefore a knife-edge case (the Friedman rule) where monetary equilibrium fails to exist and a remedy to make the Friedman rule equilibrium feasible: the Gesell tax. Both aspects have been overlooked so far in the "traditional" macroeconomic models without a microfoundation of money.

3 Steady State Effects of Different Gesell Taxes and Inflation Rates

Numerical methods are used to evaluate the steady state effects of different levels of money taxes and money growth rates (and hence inflation rates). To this extent we have to calculate the steady state conditional on calibrated parameters and on tax rates on money and money growth rates. The steady state relations and the calibration procedure are presented in the Appendix, the chosen parameter values are listed in Table 1. Unless specified otherwise parameter values proposed by Shi [32] are used. The introduction of capital formation adds two more parameters: the depreciation rate of capital δ_k (set to 2.5% quarterly) and the adjustment cost parameter b (set to 17.8).

To study the effect of different inflation rates and tax rates on money we use a grid going from money growth rates equal the Friedman rule up to approximately 7% of annual inflation and Gesell taxes from 0 to 9% annually. The grid includes the Friedman rule with no Gesell tax, but this grid point (-0.02,0) has no monetary equilibrium. All other grid points are consistent with monetary equilibrium.

Figure 1 shows the steady state levels of output, capital, investment, consumption, sales and welfare, and in continuation search intensity, velocity, employment and final good inventories over the whole grid and allows to visualize the effects of inflation and Gesell taxes in a comprehensive way:

First, for the first six variables one can detect moreless diagonal regions (bold contour lines) where the steady state values achieve a maximum. The lower the inflation rate the more of a Gesell tax is necessary to achieve the maximum values, so in the absence of Gesell taxes some level of inflation is optimal, while a Friedman rule regime is optimal only when accompanied by Gesell taxes in the range of 4-6% annual[16]. Interestingly, a zero-inflation policy is best combined with a Gesell tax of around 5% under the present model specification and parametrization[17]. This surprisingly coincides with Gesell's proposed policy: price stability (zero-inflation) combined with an annualized tax rate on money holdings of 5.2% (0.1% a week) or 6% (0.5% a month) respectively[18]. Second, very high values of either inflation or the Gesell tax are counterproductive[19]. Third, steady state search intensity and velocitiy grow monotonically with both inflation and Gesell taxes. Forth, both, inflation and the Gesell tax lower the level of unsold goods stored as inventories as a consequence of the higher search intensity. The implied avoidance of losses through depreciation helps economizing on working hours and increases welfare by a reduction in disutility of working[20]. This effect dominates the effects on utility of consumption goods and on disutility of search and vacancy posting. For zero or low levels of the weight of government spending on HH's utility ω_g this means that welfare is monotonically

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[16]The welfare function is calculated for a weight $\omega_g = 0.5$. See the next paragraph for a discussion.

[17]Remember that in a model with menu costs, price dispersion or sticky prices a zero-inflation policy would be optimal. In addition it would avoid signal extraction problems of the Lucas' island model type, since individual price changes are perfectly identifiable as such in a zero inflation environment. Possibly it's also easier to anchor inflation expectations to zero rather than moderate inflation.

[18]Gesell's focus was to provide a monetary environment where price stability is achievable and money keeps circulating although the hoarding of money is not punished by the inflation tax or high interest rates. It is therefore not comprehensible that he was classified as an "inflationist".

[19]This motivates the fighting of high inflation by monetary authorities, as well shed's some light on probable causes of the failure of Stamp Scrip experiments in the U.S. and Canada that followed Fisher's [11] proposal of annual taxes on money holding of 104% (2% weekly): According to Fisher the scrips where in general accepted by workers that were paid for community work but then immediately ended in the hand of merchants and stopped circulating.

[20]This effect on goods inventories surprisingly coincides with the predictions of Gesell's theory that a tax on money holdings makes buyers spend their money more rapidly (or lend it to people in need for a means of exchange at low interest rates), thus reducing merchants' inventories and saving on storage costs.

increasing in inflation and 'Gesell taxes' (see Figure 2), which is not very plausible as approximation to the real world. A sufficiently positive valuation of public expenditure makes people care also about the level of output and compensates for the higher disutility of work. I consider this as the more realistic case. Fifth, a negative effect of both measures is the increase in the price level or the decrease of real money balances in steady state. The latter is a consequence of the inefficiency Friedman wanted to account for. But nonetheless the search-enhancing effect of taxing money overcompensates the reduction in sales per trade match, thus increasing aggregate consumption.

4   Dynamic Effects of 'Gesell' Taxes

The dynamics of this model cannot be examined analytically, so the model is log-linearized and solved by standard techniques.[21]

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To be revised.....

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In the following I present the dynamic impulse responses to a temporary and a permanent of Gesell taxes. Moreover I study a "demand-driven recession" scenario characterized by very low interest rates that might hit the zero-lower bound, where expansionary monetary policy would be ineffective because of a liquidity trap. In this scenario I compare the effectiveness of Gesell taxes with a monetary expansion made possible through Gesell taxes and with the current US fiscal stimulus plan stipulated in the American Rescue and Recovery Act (ARRA)[22].

Figure 3 shows the impulse responses of the model economy after a temporary introduction of a Gesell tax, modelled as an autoregressive shock with autocorrelation parameter $\rho_{ges} = 0.5$. These impulse responses are qualitatively similar to the ones after a shock to the money growth rate already documented in Menner [24]. The internal propagation mechanism of the model, though, is here kick-started by the loss of the shadow value of money due to the Gesell tax and not due expected in‡ation. The increase in search-intensity leads to higher sales and depleted inventories, output is

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[21]Here, the methodology and programs of Uhlig [38] are used.

[22]I follow here Cogan et. al.[7] who study government spending multipliers in a New-Keynesian Macro-Model.

predetermined and cannot react immediately to the increased valuation of inventories, so goods supply falls short of steady state values in the next period keeping search-intensity and sales high. This feedback mechanism leads to positive sales revenues for various periods and that makes hiring profitable. Output rises not only because of higher employment but also through the higher capital stock that is accumulated through increased investment of households who want to smooth consumption by saving and investing more. Overall we see hump shaped responses of output and employment, as well as of velocity, and damped oscillatory responses of consumption, investment, sales and in‡ation.

Since we know already that a permanant implementation of Gesell taxes can be welfare improving, this is the more interesting case and it is shown in Figure 4. Sales, consumption, investment and inflation are now permanently above steady state and decline monotonically after strong impact effects that are larger by an order of magnitude. Search intensity increases permanently by about 20%, velocity constantly rises up to about 17% above steady state, and inventories and goods supply decrease permanantly by about 30% and 17% respectively. The strong increase in investment lets capital rise a lot. Output and employment however behave quite differently. Although output ends up in a new steady state above the old steady state, there is a temporary decrease in output. This is because of the strong reduction in inventories, that saves on depreciation costs, and the higher search intensity that provides the households with more consumption and investment goods although the firms produce temporarily less and individual sellers hold less goods supply. Once the optimal inventory level is reached, output rebounds to fill the gap of depreciated inventories and to keep up with higher demand. Since sales revenues fall below steady state soon for a long period there is no incentive to hire workers and the higher stock of capital makes workers obsolete for the production of the desired output. As we know from above, in the long run employment can be reduced even though output and consumption are increased. This results basically from the substitution of capital for workers.

In light of these results, should we then fear that an introduction of permanent Gesell taxes in bad times, e.g in a "great recession" where interest rates are hitting the zero bound, is counter-productive in the sense that it temporary lowers output and employment - at a time where a stimulus of output and employment would be more than appropriate?

This conjecture is not quite correct, as it makes a difference whether a policy measure is introduced when the economy is in steady state, or if ag-

gregate activity is below steady state because of a series of negative shocks[23]

To see this I consider a "recession scenario" that is characterized by a temporary shock to the weight of consumption in the HH's utility - a preference shock in short - and a temporary shock to the productivity of investment expenditures in increasing the capital stock. Figure 5 visualizes the output and employment responses to these shocks when there is no policy intervention, together with the time series of shocks that generate the "demand-driven recession". Then 3 policy scenarios are shown: A permanent introduction of a 6% annualized Gesell tax, a government spending stimulus quantitatively and qualitatively similar to the current U.S. fiscal stimulas programme recollected in the American Rescue and Recovery Act of 2009[24], and a monetary expansion made possible through the implementation of the above Gesell tax to overcome the liquidity trap.The latter is therefore modeled as an autoregressive shock to money growth combined with a Gesell tax like the one in the first scenario.

The impulse responses for the four different scenarios are collected in Figure 6. As we see there are 4 quite distinct ways out of a recessionary scenario. Taking the laissez-faire response as the benchmark (black line) we can see that the fiscal stimulus programme (red line) can considerably shorten the recession, boost employment quickly and strongly, but crowds out significantly private consumption and investment. Hence, the capital stock shrinks considerably and this must be compensated by higher employment that stays high above steady state for a long time. Since working creates disutility this adds negatively to the loss in welfare through reduced private consumption. Only when government spending enters the utility function of the households higher government spending compensates partly for these losses. When the stimulus ceases the economy enters a new - but less deep - recession. Gesell taxes alone (blue line) or in combination with a monetary expansion (green line) can avoid this W-shaped recession. The employment

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[23]See also Christiano et al. [9] for a related argument, that government spending multipliers are higher when the economy hits the zero bound of nominal interest rates as a result of adverse shocks.

[24]Cogan et al [8] calculate a fiscal expenditure measure composed of a weighted average of US Federal Government expenditure and transfers to states as stipulated in the Act. For the simulations, instead of treating each period's fiscal impulse as an exogenous shock, I take Uhlig's [39] AR(2) approximation of the whole time series of fiscal impulses. This reflects the fact that agents are not taken by surprise by the future Government spending but foresee its whole shape when making decisions.

responses avoid overshooting since there is no strong deterioration of physical capital in the medium and long run. There is no "crowding out" effect. On the contrary, the higher search intensity increases sales, consumption and also investment which restores the capital stock more quickly. An expensionary monetary policy considerably reduces the output and employment drop in the trough of the recession. This is because it boosts search-intensity and sales even more and helps avoiding the large drops in consumption and investment.

Overall, we can state that a Gesell tax alone can shorten the recession by a similar amount of time as expensive fiscal stimulus programmes. Although steady state output is reached about 2 quarters later this policy avoids the fall back into a double dip recession and leads to a permanent increase of output in the long run. Crowding out effects can be avoided and therefore there is no need to compensate through more labor for the reduced capital stock. But Gesell taxes get fully effective when they are combined with expansionary monetary policy - which is the raison d'etre of this policy instrument in the minds of the recent proponents of Gesell taxes as a measure to overcome the limitations of monetary policy at the zero lower bound on interest rates.

Now, when one repeats the experiment with temporary Gesell taxes instead, as shown in Figure 7, the results are quite similar for the first quarters, but then we see dips in consumption, investment and output when the taxes phase out. Moreover all variables will finally converge to the old steady state, so there is no long-run efficiency gain. Generally, there seems to be no trade-off between short-run and long-run effects, since the way out of the recession is quite independent of the duration of the Gesell tax.

5   Conclusion

We have seen first that the monetary search literature establishes a role for taxing money in order to achieve higher search intensity and therefore to overcome the suboptimal level of transactions stemming from a positive search externality. This goes beyond the other aim of these taxes to achieve negative nominal interest rates. In particular in a third generation search model a situation of low inflation that could lead to a lower zero bound problem in case of big adverse shocks is characterized by high unsold inventories, low level of transactions, low consumption and investment and low capital and output per worker. When discussing the pros and cons of a Gesell tax vs a higher level of steady state inflation in order to prevent situations with

liquidity traps and lower zero bound problems, Buiter and Panigirtzoglou [7] balance the administrative costs of a Gesell tax with the "shoeleather costs" and "menu costs" of higher inflation rates.

This paper argues that the benefits of Gesell taxes are not only the avoidance of a liquidity trap but also the increase in efficiency that would be achievable only by very high inflation. Especially in low inflation economies and at the Friedman rule these Gesell taxes are very effective.Finally, we have seen that a permanent implementation of Gesell taxes in this model environment is preferable to a merely temporary use as suggested by recent proposers of taxes on money to avoid or escape liquidity traps.

Some of these results might be very well model specific, so robustness has to be checked in other models where money is essential before making policy recommendations. Especially models with more elaborate asset markets, a banking sector, different currencies and the possibility of barter trade would be interesting to achieve deeper insights of the effects of Gesell taxes on different monetary assets, on currency substitution, velocity, interest rates and lending, as well as the limits to the tax rates beyond which the monetary equilibrium would cease to exist and only barter trades take place. So, hopefully the present work contributes to stimulate further research on this policy instrument, that has been ignored or forgotten for many decades and only recently found its way into formal economic analysis.

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Appendix

A.1 Equilibrium Conditions

Necessary conditions for an optimum are the FOCs (with respect to M_{t+1}, i_{t+1}, n_{t+1}, K_{t+1}, s_t, x_t, and v_t):

$$\omega_{Mt} = \beta E_t \left\{ (1 - \tau_t^m)\omega_{Mt+1} + g_b(\hat{s}_{t+1})s_{t+1}\Lambda_{t+1} \right\}, \tag{14}$$

$$\omega_{it} = \beta E_t \left\{ g_s(\hat{s}_{t+1})\omega_{qt+1} + (1 - \delta_i)\omega_{it+1} \right\}, \tag{15}$$

$$\omega_{nt} = E_t \left\{ \beta (1 - \delta_n) \omega_{nt+1} - \beta(1 - \tau_t^m)\omega_{Mt+1}\hat{P}_{t+1}\hat{W}_{t+1} + \omega_{it}f_n (n_{t+1}, K_{t+1}) \right\}, \tag{16}$$

$$\omega_{Kt} = E_t \left\{ \beta(1 - \delta_k)\omega_{Kt+1} + \beta U'(c_{t+1}) \frac{b}{2} \left(\left(\frac{x_{t+1}}{K_{t+1}} \right)^2 - \delta_k^2 \right) + a_p \omega_{it} f_k (n_{t+1}, K_{t+1}) \right\}, \tag{17}$$

$$\Phi'(s_t) = g_b(\hat{s}_t) \left[U'(c_t) \hat{q}_t - \omega_{Mt} \hat{m}_t (1 - \tau_t^m) \right], \tag{18}$$

$$U'(c_t) \left[1 + b \left(\frac{x_t}{K_t} - \delta_k \right) \right] = \omega_{Kt}, \tag{19}$$

$$\omega_{nt} = \Upsilon'(v) / \mu, \tag{20}$$

with the slackness conditions associated with (2) and (3):

$$\Lambda_{t+1} \left[\frac{M_{t+1}}{a_b} - \hat{m}_{t+1} \right] = 0, \qquad \forall j \in A_{bt+1^*}, \tag{21}$$

$$\omega_{qt} \left[i_{t+1} + f (n_{t+1}, K_{t+1}) - \hat{q}_{t+1} \right] = 0, \qquad \forall j \in A_{pt+1^*}, \tag{22}$$

and the transversality equation:

$$\lim_{t \to \infty} \beta^{-t} E_t \left\{ \omega_{Kt} K_t \right\} = 0. \tag{23}$$

The optimality conditions, the laws of motion for capital, money balances, employment and inventories (4) - (7), the resource constraint (1) and the trading constraints (2) and (3) determine the solution to this decision problem once the terms of trade are specified, i.e. equations (9), (10) and (11) hold, and equilibrium conditions are imposed.

Considering symmetric equilibria where $\lambda > 0$ and $\omega_q > 0$, which turns out to be fulfilled around the considered steady states, and following similar steps as in Menner [25] we get a system of static equations:

$$k\left(v_t\right) = \Upsilon'\left(v_t\right)/\mu\left(v_t\right), \tag{24}$$

$$q_t = i_t + f(n_t, K_t) - gov_t, \tag{25}$$

$$x_t = a_p B z s_t^\alpha q_t - c_t - \frac{b}{2}\left(\frac{x_t}{K_t} - \delta_k\right)^2 K_t, \tag{26}$$

$$U'\left(c_t\right)\left[1 + b\left(\frac{x_t}{K_t} - \delta_k\right)\right] = \omega_{Kt}, \tag{27}$$

$$s_t^{1-\alpha}\Phi'\left(s_t\right) = z q_t\left[U'\left(c_t\right) - \omega_t(1 - \tau_t^m)\right], \tag{28}$$

where (24) is a convenient definition and the other equations jointly determine $\{q_t, c_t, x_t, s_t\}$ as functions of the states $\{n_t, i_t, K_t\}$ and the costates $\{\omega_t, \omega_{Kt}\}$.

Substituting above expressions into (6) - (18) we get the dynamic system:

$$n_{t+1} = \left(1 - \delta_n\right) n_t + v_t \mu_t\left(v\right), \tag{29}$$

$$i_{t+1} = \left(1 - \delta_i\right)\left(i_t + f(n_t, K_t) - gov_t - B z s_t^\alpha q_t\right), \tag{30}$$

$$K_{t+1} = (1 - \delta_k)K_t + x_t, \tag{31}$$

$$\omega_t = \beta E\left\{\frac{q_{t+1}}{\gamma_t q_t}\left((1 - \tau_t^m)\omega_{t+1} + z s_{t+1}^\alpha\left[U'\left(c_{t+1}\right) - (1 - \tau_t^m)\omega_{t+1}\right]\right)\right\}, \tag{32}$$

$$k\left(v_t\right) = E\left\{\beta\left(1 - \delta_n\right) k\left(v_{t+1}\right) + \sigma\left[\omega_{it} f_n(n_{t+1}, K_{t+1}) - \beta\varphi\right]\right\}, \tag{33}$$

$$\omega_{it} = \beta E\left\{(1 - \delta_i)\omega_{it+1} + B z s_t^\alpha\left(\omega_{t+1}(1 - \tau_t^m) - (1 - \delta_i)\omega_{it+1}\right)\right\}, \tag{34}$$

$$\omega_{Kt} = E\left\{\beta\left[(1 - \delta_k)\omega_{Kt+1} + \frac{b}{2}\left(\left(\frac{x_{t+1}}{K_{t+1}}\right)^2 - \delta_k^2\right) U'\left(c_{t+1}\right)\right] + a_p\omega_{it} f_k\left(n_{t+1}, K_{t+1}\right)\right\}, \tag{35}$$

where (29) - (31) are the laws of motion of the state variables $\{n_t, i_t, K_t\}$ and the others are expectational equations for the jump variables $\{\omega_t, v_t, \omega_{it}, \omega_{Kt}\}$.

A.2 Steady State

The equilibrium equations (24)-(35) imply the following steady state relations with a recursive structure:

$$i^* = \frac{(1 - d_i)(1 - Bzs^{*\alpha})\left(f(n^*, K^*) - gov^*\right)}{1 - (1 - d_i)(1 - Bzs^{*\alpha})}, \tag{36}$$

$$x^* = \delta_k K^*, \tag{37}$$

$$\omega_K^* = U'(c^*), \tag{38}$$

$$q^* = \frac{(f(n^*, K^*) - gov^*)}{1 - (1 - d_i)(1 - Bzs^{*\alpha})}, \tag{39}$$

$$v^* \mu(v^*) = \delta_n n^*, \tag{40}$$

$$\omega_i^* = \frac{\beta Bzs^{*\alpha}\omega^*(1 - \tau^*)}{\left(1 - \beta\left(1 - \delta_i\right)\left(1 - Bzs^{*\alpha}\right)\right)}, \tag{41}$$

$$s^{*^{1-\alpha}}\Phi'(s^*) = zq^*\left[U'(c^*) - \omega^*(1 - \tau^*)\right], \tag{42}$$

$$c^* = a_b Bzs^{*\alpha}\frac{(f(n^*, K^*) - gov^*)}{1 - (1 - d_i)(1 - Bzs^{*\alpha})} - \delta_k K^*, \tag{43}$$

$$U'(c^*) = \left(\frac{\gamma - \beta(1 - \tau^*)\left(1 - zs^{*\alpha}\right)}{\beta zs^{*\alpha}}\right)\omega^*, \tag{44}$$

$$2\Upsilon_0 v^*/\mu(v^*)\frac{(1 - \beta(1 - \delta_n))}{\sigma\beta} = \frac{Bzs^{*\alpha}\omega^*(1 - \tau^*)}{(1 - \beta(1 - \delta_i)(1 - Bzs^{*\alpha}))}f_n(n^*, K^*) - \varphi, \tag{45}$$

$$U'(c^*) = \frac{a_p f_K(n^*, K^*)}{1 - \beta(1 - \delta_K)}\frac{\beta Bzs^{*\alpha}\omega^*(1 - \tau^*)}{(1 - \beta(1 - \delta_i)(1 - Bzs^{*\alpha}))}. \tag{46}$$

A.3 Calibration

For sake of comparability the values of all parameters that were exogenously specified by Shi [32] are kept the same and shown in the 2 first rows of Table 1. The last row is calibrated in the following way:

As in Shi [32], the disutilities of search and vacancies are assumed to be:

$$\phi(s) = \varphi(\varphi_0 s)^{1+1/e_\phi}, \quad \Upsilon(v) = \Upsilon_0 v^2. \tag{47}$$

Further, assume that the production function has the form:

$$f(n, K) = F_0 n^{e_f} \left(\frac{K}{a_p}\right)^{1-e_f}.$$

Total factor productivity is normalized to one: $F_0 = 1$. Define $F = \frac{1}{a_p^{1-e_f}}$ to express production per firm as $f(n, K) = F n^{e_f} K^{1-e_f}$.

The capital depreciation rate δ_k is assumed to be 2.5% quarterly. The adjustment cost parameter b is calibrated such that the semi-elasticity of investment with respect to real asset returns (Tobin's Q) is 2.25 .[25] If $\Phi(I/K)$ denotes the adjustment cost function, then $Q \equiv 1/\Phi'(I/K)$ can be interpreted as Tobin's Q:

$$(semi\_\varepsilon)_{IQ} = -\frac{\delta Q \cdot I}{\delta I} = \frac{\Phi''}{(\Phi')^2}\frac{I^*}{K^*} = \frac{1}{b\frac{I^*}{K^*}} \Rightarrow b = \frac{1}{2.25 \cdot \delta_k} = 17.8 .$$

Consistent with Wang and Shi [40] β is set to 0.995, and the steady state value of velocity is chosen to be 1.7.. The same procedure as used in Shi [32] where a value of 1 was chosen instead, implies a value of $z = 0.1959$ and $B = 0.3723$.

The remaining parameters are functions of steady state values of the model and are determined endogenously as follows:

Equations (36)-(41) give $(i^*, x^*, \omega_K^*, q^*, v^*, \omega_i^*)$ as functions of $(K^*, c^*, \omega^*, s^*, n^*)$. Further, equations (42) - (46) involve only $(K^*, c^*, \omega^*, s^*, n^*)$.

Steady state employment n^* is normalized to 100. Steady state search intensity s^* is calibrated as in Shi [32] by $a_b s^* = 0.116 * 0.3 a_p (1 + n^*)$ by quantifying shopping time as 11,6 % of working time which is 30% of agent's discretionary time. Wage bargaining determines the wage rate according to (11):

$$W^* = \sigma \frac{\varphi}{\omega^*(1-\tau)} + (1-\sigma)\frac{zBs^{*\alpha}}{1 - \beta(1-\delta_i)(1 - Bzs^{*\alpha})(1-\tau)} f'(n^*, k^*). \tag{48}$$

Using (48) and the definition of the labor income share $LIS \equiv W^* n^*/f$, φ can be expressed by:

$$\varphi = \left(\frac{(1-\tau^*)LIS}{e_f} - \frac{(1-\sigma)zBs^{*\alpha}}{1 - \beta(1-\delta_i)(1 - Bzs^{*\alpha})}\right)\frac{f'(n^*, k^*)\omega^*}{\sigma}. \tag{49}$$

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[25] See Neiss and Pappa [28].

Define further the hiring cost relative to labor cost as:

$$HC \equiv \Upsilon_0 v^{*2}/(\omega^* W^* n^*). \tag{50}$$

Hence

$$\Upsilon_0 = \omega^* W^* n^* HC/v^{*2} = \omega^* LIS \ f^* HC/v^{*2}, \tag{51}$$

and by making use of (40)

$$k(v^*) \equiv \Upsilon'(v^*)/\mu(v^*) = 2\Upsilon_0 v^*/\mu(v^*) = 2\omega^* LIS \ f^* HC/(\delta_n n^*). \tag{52}$$

Substituting (49) and (52) into (45) one can solve for e_f :

$$e_f = LIS\left((1-\tau^*)\beta + \frac{2HC\left(1-\beta\left(1-\delta_n\right)\right)}{\delta_n}\right)\frac{1-\beta\left(1-\delta_i\right)\left(1-Bzs^{*\alpha}\right)}{\beta Bzs^{*\alpha}}. \tag{53}$$

With the hiring cost ratio set to 2% this determines e_f.

The steady state equations (43) to (44) and (46) can be used to determine the parameter φ_0, as well as the steady state values K^*, c^* and ω^*. K^* is required to solve (46) , with $U'(c^*)$ given by (44), and ω^* cancels out. Plugging in the functional form of the production function one gets:

$$\left(\frac{\gamma - \beta\left(1-\tau^*\right)\left(1-zs^{*\alpha}\right)}{\beta zs^{*\alpha}}\right) = \frac{a_p\left(1-e_f\right)\cdot F\cdot\left(\frac{K^*}{n^*}\right)^{-e_f}}{1-\beta\left(1-\delta_K\right)}\frac{\beta Bzs^{*\alpha}(1-\tau^*)}{\left(1-\beta\left(1-\delta_i\right)\left(1-Bzs^{*\alpha}\right)\right)},$$

which can be solved for K^*. Once K^* is determined, c^* follows from (43) and ω^* from (44). Now ω_i^* and q^* are given by (39) and (41) and φ can be calculated by (49). Finally, φ_0 is determined through (42) and (47) and Υ_0 is given by (51).

The resulting parameter values are shown in Table 1.

Table 1   Parameter Values

RA	e_Φ	δ_i	δ_n	a_p	α	A	σ	u	γ
2	2	0.0072	0.06	0.0069	0.8	0.6	0.7	0.0447	$0.995 - 1.018$

| δ_k	b	β	B	z	Υ_0	φ_0	φ	e_f	τ^*
0.025	17.8	0.995	0.3723	0.1959	0.0371	0.0973	2.418	0.6576	$0 - 0.023$

A.4 Graphs

Figure 1: Steady State Effects of Gesell Tax and Inflation
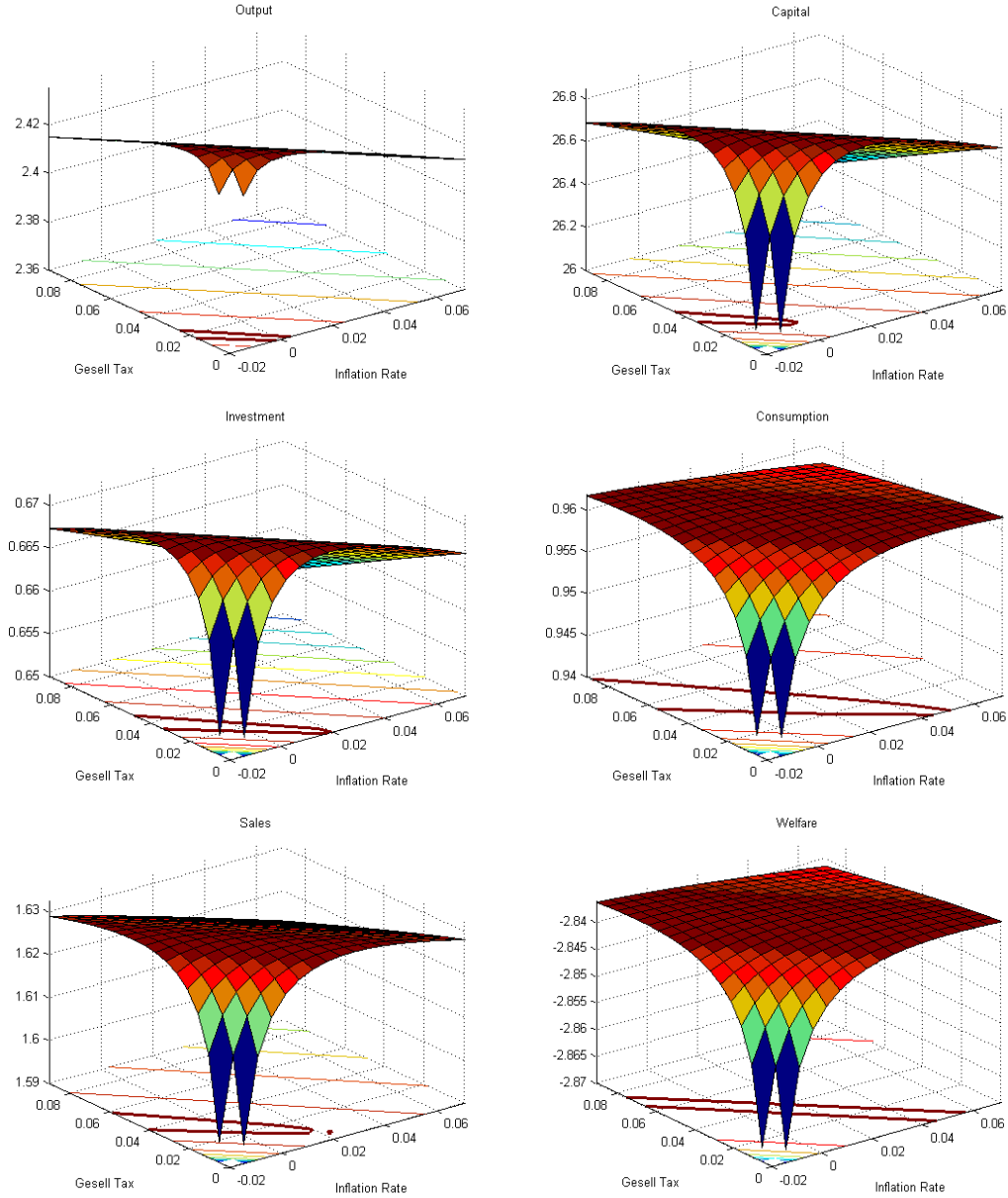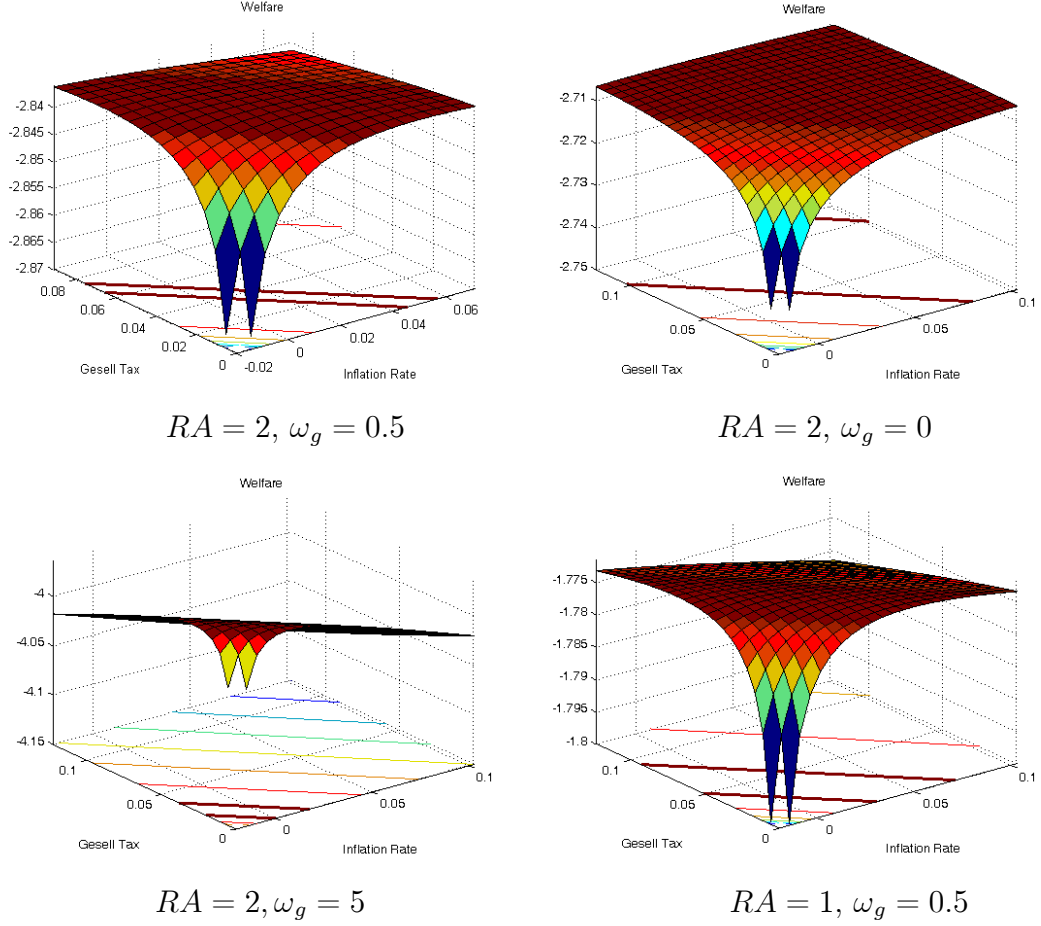

Steady state values implied by different rates of inflation and "Gesell taxes" ($\beta = 0.995$). _For welfare calculations I use a weight of 0.33 to government expenditure and 1 to private consumption.

**Figure 1:   Steady State Effects of Gesell Tax and Inflation -
continued**
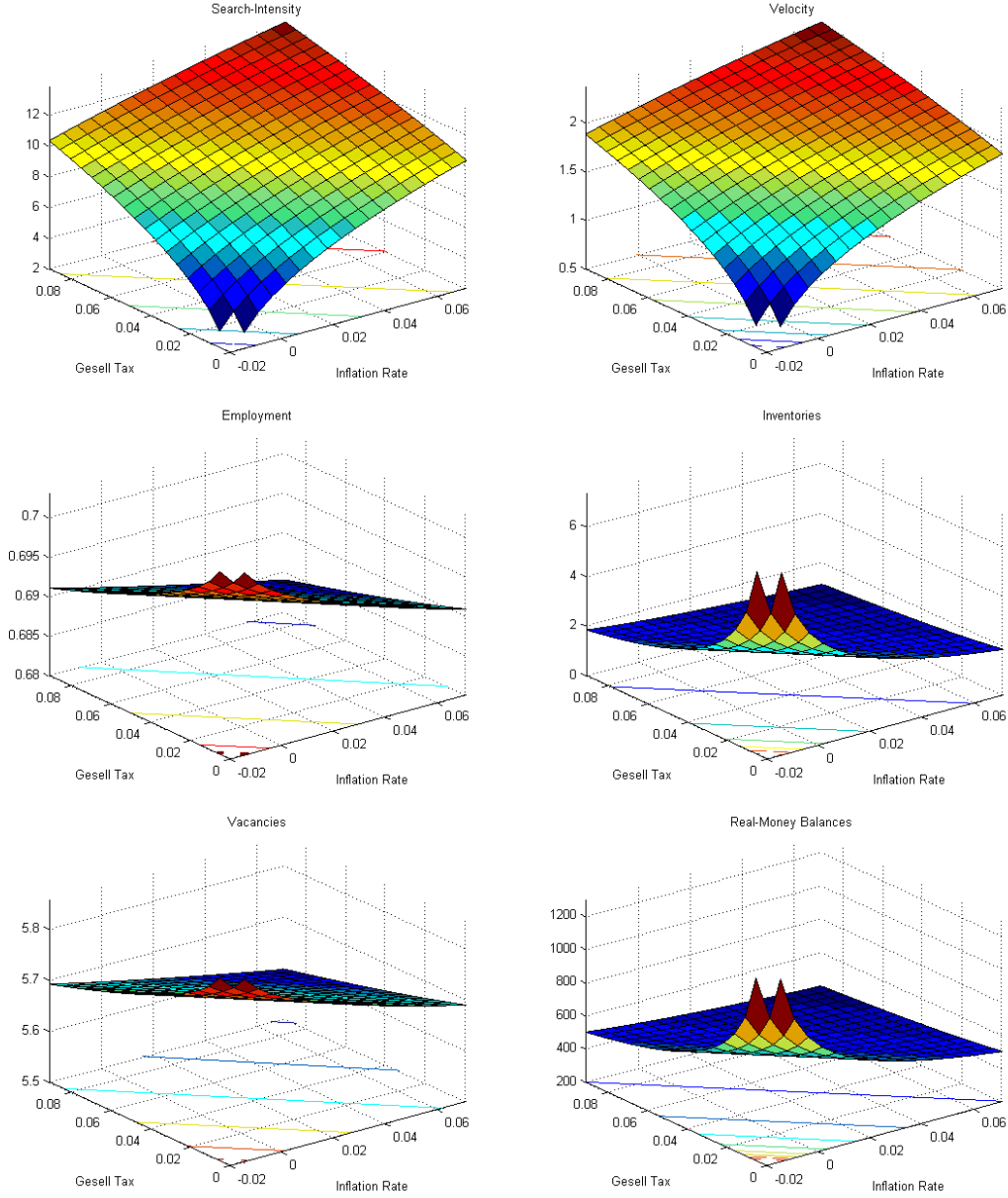
Steady state values implied by different rates of inflation and "Gesell taxes" ($\beta= 0.995$).

Figure 2: Robustness Checks: Welfare

$RA = 2, \omega_g = 0.5$

$RA = 2, \omega_g = 0$

$RA = 2, \omega_g = 5$

$RA = 1, \omega_g = 0.5$

Steady state values implied by different rates of inflation and "Gesell taxes" (ß = 0.995). For welfare calculations I use a weight !g to government expenditure and 1 to private consumption. RA is constant relative risk aversion.

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