Trade, Teams and Multiple Goods

Greg Spanos

March 15, 2011

Abstract

This paper attempts to investigate the relationship between income inequality, international trade and communication costs. More specifically, this paper extends Garicano and Rossi-Hansberg (2004) to two goods and two countries. The main findings of the paper are that under certain assumptions a competitive equilibrium exists in a closed and open economy. Moreover, in such an equilibrium there is positive sorting, the earnings functions are convex and the sets of managers, self-employed and workers are connected. In addition, in the open economy, identical workers located in different countries do not always earn the same income, the country with a more knowledgeable population specializes in the production of the good produced in teams, while the other country specializes in the good produced by self-employed workers. Furthermore, this paper finds that whether international trade increases or decreases income inequality depends on the cost of communication between managers and workers.

1 Introduction

Within the past 30 years three facts have been observed. First, the costs of communication have been decreasing over time. Since production in firms requires the coordination of several tasks, this implies that the costs of producing goods have also decreased over time, thereby making firms more efficient in production. Second, trade between developed and developing countries has increased. In most traditional models of international, such as the

Heckscher-Ohlin model, this would imply that income inequality in the developed country should increase, while income inequality in the developing country should decrease. This conclusion, however, is at odds with the third fact, that within developed and developing countries income inequality has actually increased.

This paper attempts to reconcile these three facts. More specifically, this paper attempts to investigate the relationship between income inequality, international trade and communication costs by extending Garicano and Rossi-Hansberg (2004) to two goods and two countries. The main features of the model are that production requires time and knowledge, agents are heterogeneous in ability, and they can specialize in production or problem-solving. Also, the economy contains two sectors. In one sector agents are self-employed and produce alone, while in the other agents produce in teams. The latter are composed of a manager, who focuses on solving problems, and one or more workers, who specialize in production.

The main findings of the paper are that under certain assumptions a competitive equilibrium exists in a closed and open economy. Moreover, in such an equilibrium there is positive sorting, the earnings functions are convex and the sets of managers, self-employed and workers are connected. In addition, in the open economy, identical workers located in different countries do not always earn the same income, the country with a more knowledgeable population specializes in the production of the good produced in teams, while the other country specializes in the good produced by self-employed workers. Furthermore, this paper finds that whether international trade increases or decreases income inequality depends on the cost of communication between managers and workers.

The paper will proceed as follows. Section 2 presents the closed economy version of the model. Section 3 defines a closed-economy competitive equilibrium, and shows that it exists. Section 4 discusses how the equilibrium changes, with the model's exogenous parameters. Section 5 presents the open-economy model and discusses some comparative static results. And finally, section 6 concludes. All proofs, graphs and tables are relegated to the appendix.

2 The Model

Two goods are produced in this economy, labeled x_1 and x_2. Good 1 is produced in teams, which are composed of a manager and a group of workers, while good 2 is produced by self-employed workers. Production requires knowledge; that is, in order to produce a unit of good 1 or 2, a problem needs to be solved. Let Ω be the set of possible problems an agent can encounter while producing, and be equal to $[0, 1]$. A worker encounters a problem ω with frequency $f(\omega)$ and cumulative distribution $F(\omega)$.

In every country workers draw their knowledge, z, from a cumulative distribution of knowledge $G(z)$, with support $[0, \alpha]$, where $\alpha \leq 1$. The boundary point α represents the maximum amount of knowledge an agent can possess. [1] Agents with knowledge z, are able to solve all problems in the set $A = [0, z]$. Thus, an agent with knowledge z, can solve problem ω and produces a unit of output, if and only if $\omega \in A$.

2.1 Consumer's Problem

Consumers have Cobb-Douglas utility over goods x_1 and x_2. Labor income derives from producing in a team, managing a team, or self-employment. Production workers earn a wage $w(z)$, managers collect rents from team's production $R(z)$, and self-employed earn the profits from production $p_2 F(z)$. [2]

An agent with skill level z faces the following budget constrained utility maximization problem, in which consumption expenditures equal income earned from his occupation of choice:

$$\begin{gathered} \max_{\{x_1, x_2\}} x_1^{\beta} x_2^{1-\beta} \\ \text{s.t} \quad p_1 x_1 + p_2 x_2 = E(z), \qquad E(z) = \max \{w(z), p_2 F(z), R(z)\} \end{gathered} \tag{1}$$

The Cobb-Douglas structure of utility provides the following optimal consumption bundles as a function of income and prices: $x_1 = \beta \frac{E(z)}{p_1}$ and $x_2 = (1 - \beta) \frac{E(z)}{p_2}$.

[1] If α is less than 1, then the most knowledgeable agent will not be able to solve every problem.

[2] The section below discusses production of goods in detail.

2.2 Production

Good x_1 is produced in teams. Teams are composed of a manager and a group of workers. All agents are endowed with 1 unit of time. A worker spends his time producing. When he produces he encounters a problem ω from his or his manager's set of solvable problems. If his ability is such that $\omega \in A$, then he solves the problem and produces a unit of output. If $\omega \notin A$, he asks his manager for the solution. In turn, the manager spends a fraction h of his time communicating with his worker. If $\omega \in A$ for the manager, the manager communicates the solution to his worker who then proceeds to produce a unit of the good, at a cost of manager time h.

Teams are composed of one manager and one or more workers. The exact size of a team is determined endogenously. Production in a team composed of n workers and a manager with ability z_m is $nF(z_m)$. A manager chooses a team of workers to maximize his rents $[p_1 F(z_m) - w(z_p)]$, where $w(z_p)$ is the wage earned by workers with ability z_p. [3] However, the manager is constrained in the number of workers he can form a team because the total time he spends communicating his knowledge to his workers cannot exceed 1. Thus the manager's optimization problem has the following form:

$$\begin{aligned} R(z_m; w) = \max_{\{z_p, n\}} \; & n \left[p_1 F(z_m) - w(z_p)\right] \\ \text{s.t} \qquad & hn \left[1 - F(z_p)\right] = 1 \end{aligned} \tag{2}$$

The manager's optimization problem indicates that there are positive complementarities between the knowledge of workers and managers. Since more knowledgeable workers can solve a large fraction of problems, they require help less often, thereby allowing their manager to supervise more workers and increasing the team's total output. Also, since a team's output is dependent on the manager's ability, workers matched with more knowledgeable managers will be able to produce more often, increasing the output from the match.

The first-order condition to problem (2) is

[3] Managers only hire workers of a single ability. See Antras et. al. (2006) for details.

$$w^{'}(z_p) = \frac{\left[p_1 F(z_m) - w(z_p)\right] F^{'}(z_p)}{1 - F(z_p)}, \qquad \forall z_p \leq z_1, \tag{3}$$

where z_1 is the ability of most knowledgeable worker. In a similar manner, define z_2 as the ability of the least knowledgeable manager.[4]

Good x_2 is produced by agents who are neither managers nor do they work for managers. [5] Since they are not members of a team, they cannot use the knowledge of other agents to produce. A self-employed agent with ability z will produce $F(z)$ units of good x_2 and earn $p_2F(z)$. This is because they can solve and produce a proportion z of the problems they encounter.

In an equilibrium, the earnings function must be continuous on $[0, 1]$, and differentiable at all values of z except at the threshold values z_1 and z_2. [6] Since each agent chooses the occupation with the highest earnings, and agents can always choose to become self-employed, to ensure continuity of the earnings function, the marginal agent z_1 must be indifferent between being a worker and remaining self-employed, and the marginal agent z_2 must be indifferent between being a manager and being self-employed. Thus, the earnings of the marginal worker z_1 must satisfy the condition $w(z_1) = p_2F(z_1)$. Similarly, the earnings of the marginal manager must satisfy the condition $R(z_2) = p_2F(z_2)$.

2.3 Market Clearing Conditions

In equilibrium all three markets must clear. In this model there are three markets, the labor market, the market for good x_1 and the market for x_2. Let $m(z_p)$ represent the skill level of a manager that is matched to a worker with ability z_p. In the appendix it is shown that in an equilibrium $m(z_p)$ must exhibit positive sorting, and is therefore invertible. The labor market clearing condition has the following form:

[4]More formally, as will be shown below, the equilibrium will be characterized by values z_1 and z_2 such that all agents with knowledge between $[0, z_1]$ will choose to be production workers, all agents with knowledge between $[z_1, z_2]$ will be self-employed, and all agents with ability between $[z_2, \alpha]$ will choose to be managers.

[5]Self-employed agents also possess 1 unit of time.

[6]If the earnings function is not continuous at points z_1 and z_2, agents marginally below or above will wish to deviate from their occupation. Differentiability of wage and rent function is required from the manager's optimization problem.

$$\int_{m(0)}^{m(z_p)} n(m^{-1}(z))g(z)dz = \int_0^{z_p} g(z)dz, \qquad \forall z_p \leq z_1 \tag{4}$$

where $m^{-1}(z)$ is the ability of a worker matched to a manager with ability z. The left-hand side of equation (4) represents the demand for workers by managers, while the right-hand side represents the supply of workers. Since equation (4) holds for values of $z_p \leq z_1$, we can substitute for $n(m^{-1}(z))$ and derive it with respect to z_p obtain

$$m'(z_p) = \frac{h\left[1 - F(z_p)\right]g(z_p)}{g(m(z_p))} \tag{5}$$

Equation (3), along with the conditions that the least knowledgeable manager is matched with the worst worker (i.e. $m(0) = z_2$), and the most knowledgeable manager is teamed up with the best worker (i.e. $m(z_1) = \alpha$) determine the assignment function $m(z)$.[7]

In equilibrium the aggregate demand for goods x_1 and x_2 must equal their aggregate supply. Agent z wants to consume $\beta\frac{E(z)}{p_1}$ units of x_1 and $(1-\beta)\frac{E(z)}{p_2}$ units of x_2. Every team led by a manager of ability z produces $n(m^{-1}(z))F(z)$ units of good 1, and every self-employed worker with ability z produces $F(z)$ units of good 2. Therefore, the goods market clearing conditions for x_1 and x_2, respectively, are

$$\frac{\beta}{p_1}\int_0^{\alpha} E(z)g(z)dz = \int_0^{z_1} m(z)g(z)dz, \tag{6}$$

$$\frac{(1-\beta)}{p_2}\int_0^{\alpha} E(z)g(z)dz = \int_{z_1}^{z_2} F(z)g(z)dz. \tag{7}$$

In this economy agents' income is derived from production. As a result, aggregated income is determined endogenously and is equal to the total value of production as shown explicitly in equation (8):

$$\int_0^{\alpha} E(z)g(z)dz = p_1\int_0^{z_1} m(z)g(z)dz + p_2\int_{z_1}^{z_2} F(z)g(z)dz \tag{8}$$

[7]These conditions follow from the fact that $m(z_p)$ exhibits positive sorting.

3 Equilibrium

The following is a definition of a competitive equilibrium:

Definition 1 *A competitive equilibrium consists of a wage function $w(z)$, a rent function $R(z)$, an assignment function $m(z)$, a set of prices $\{p_1, p_2\}$ and a pair of thresholds $\{z_1, z_2\}$ such that the following conditions hold:*

i. Agents maximize utility

ii. Managers maximize rents

iii. All markets clear

Condition i implies that every agent chooses the occupation that maximizes his income, and demand for goods 1 and 2 satisfy (1). Condition ii implies managers' decisions solve (2) and managers at α are not willing to hire workers slightly above z_1. [8] Condition iii implies that equations (6) and (7) are satisfied, $m(0) = z_2$, and $m(z_1) = \alpha$.

To simplify the analysis, assume that $G(z)$ and $F(z)$ are uniformly distributed over their respective domains. Then, from (5) and the boundary condition $m(0) = z_2$, the assignment function will be equal to

$$m(z) = z_2 + h\left[z - \frac{z^2}{2}\right], \qquad \forall z \leq z_1.$$

Substituting this expression into (3) and imposing the boundary condition $w(z_1) = p_2 F(z_1)$ yields the wage equation

$$w(z) = p_1 z_2 - \sigma(1 - z) + 1/2 p_1 h z^2, \qquad \forall z \leq z_1$$

where $\sigma = \frac{p_1 z_2 + 1/2 p_1 h z_1^2 - p_2 z_1}{1 - z_1}$. [9]

We can also obtain simplified expressions for the supply of goods 1 and 2 respectively:

[8] As shown in the appendix, this is prevented from happening as long as $w^{'}(z_1) < p_2 F^{'}(z_1)$.

[9] From the equilibrium condition $R(z_2) = p_2 F(z_2)$ it follows that $\sigma = p_2 z_2 h > 0$.

$$\int_0^{z_1} m(z)g(z)dz = \frac{1}{6\alpha}\left[6z_2 z_1 - hz_1^3 + 3hz_1^2\right],$$

$$\int_{z_1}^{z_2} F(z)g(z)dz = \frac{1}{2\alpha}\left[z_2^2 - z_1^2\right].$$

Proposition (2) states that an equilibrium exists under the assumption that $G(z)$ and $F(z)$ are uniformly distributed.

Proposition 2 *Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. For any values of α and $h \in \left[0, \overline{h}\right]$ there exists a competitive equilibrium. Moreover, in such an equilibrium, there is positive sorting, the earnings function is convex and the sets of managers, self-employed and workers are connected.*

Proof. See Appendix. ■

Condition ii of the definition of an equilibrium implies that managers at α must not be willing to hire workers slightly above z_1. For very high values of h this is not always the case. The manager's problem indicates that a team's size, output and revenues, are inversely related to the costs of communication. For a given value of h, managers can always increase their revenues by employing more knowledgeable agents at the cost of higher wages. In an equilibrium, it must be the case that the increase in labor costs is greater than the increase in revenues; otherwise, managers would not be maximizing their profits. However, when h is very high, managers with ability α will have an incentive to deviate from their match and employ more knowledgeable agents, because their profits will increase; that is, their revenues will increase by more than the cost of labor.

Since h is the fraction of time a manager spends communicating with a worker, it cannot be greater than 1. As is discussed in the appendix, condition ii is equivalent to the restriction $h < \frac{1}{z_1+z_2}$. Therefore, these two restrictions imply that $h < min\left\{\frac{1}{z_1+z_2}, 1\right\}$. Since there does not exist a closed form solution for the equilibrium values of z_1 and z_2, it is not possible to obtain an expression for $\frac{1}{z_1+z_2}$ as function of the parameters of the model. Hence, it is impossible to accurately determine an upper bound on h. One restrictive assumption is for h to be less than the smallest value $\frac{1}{z_1+z_2}$ can undertake in the domain $[0,\alpha]\,X\,[0,\alpha]$, that is $h<\overline{h}=\min\left\{\frac{1}{2\alpha},1\right\}$. This assumption guarantees that an equilibrium exist for all values of h in the interval $\left[0,\overline{h}\right]$. [10] Figure 1 below present the equilibrium earnings function for an economy with parameters $h=0.4$ and $\alpha=1$.

4 Comparative Statics

Assume consumers value goods 1 and 2 equally (i.e. $\beta=0.5$). This section analyzes comparative statics in a closed economy. First, we ask how a change in the distribution of worker abilities affect the equilibrium. Second, we ask how a change in the communication costs, h, affect the economy.

Proposition 3 *Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. An increase in α has the following effect: (i.) The maximum ability of a worker, and the minimum ability of a self-employed worker, z_1, increases (ii.) The minimum ability of a manager, and the maximum ability of a self-employed workers, z_2, increases.*

Proof. See Appendix. ■

Increasing α decreases the density $\frac{1}{\alpha}$ and changes the distribution of worker abilities. As α increases, holding everything else constant, more agents will choose to become workers, increasing z_1. As a result, the most knowledgeable managers will now be able to manage larger teams, increasing the aggregate output of good 1. However the increase in aggregate output decreases the relative price of good 1, which reduces the earnings of the existing set of managers, and forces the least knowledgeable of them to exit the industry for self-employment.

Simulations, presented in Table 1, provide additional information on how a change in the distribution of worker abilities affects the equilibrium. First, notice that the mass of workers, self-employed workers and managers, increases with α. This along with the fact

[10] Simulations of the model suggest that h can be as high as 0.92.

that the minimum and maximum ability of the set of self-employed workers increase, imply that the aggregate output of good 2 increases with α. The sets of managers and workers are also larger, implying that there are more teams in the economy. Proposition (3) indicates that the minimum ability of a manager increases, and so it follows that the aggregate output of good 1 also increases with α. Second, notice that the relative price of good 1, p_1/p_2 decreases as α increases. Since the relative price of good 1 is equal to the ratio of the aggregate supplies of goods 2 and 1, it follows that aggregate output of good 1 increases by more than the aggregate output of good 2. [11]

Proposition 4 *Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. A decrease in h has the following effect: (i.) The maximum ability of a worker, and the minimum ability of a self-employed worker, z_1, increases (ii.) The minimum ability of a manager, and the maximum ability of a self-employed workers, z_2, increases.*

Proof. See Appendix. ■

The intuition behind proposition (4) follows from the fact that as communication costs decrease, managers can spread their knowledge across more workers, and as a result the team size increases. Since the output of all existing teams increases, it follows that the aggregate output of good 1 increases as well. However the increase in aggregate output depresses the relative price p_1/p_2, reducing the rents of the existing managers and forcing the least knowledgeable to exit the industry for self-employment. In turn, the mass of self-employed workers increases, which also increases the aggregate output of good 2, increases the relative price p_1/p_2 and forces the least qualified self-employed workers to exit the industry and work in a team.

Table 1 also provides information on how a change in communication costs affects the equilibrium. First, notice that the mass of self-employed workers moves in the opposite direction of h. This along with the fact that the minimum and maximum ability of the set of self-employed workers increase, imply that the aggregate output of good 2 increases as

[11] The fact that the relative price of good 1 is equal to the ratio of the aggregate supplies of goods 2 and 1 follow directly from equation (3') of the appendix.

communication costs fall. Second, notice that the mass of workers also moves in the opposite direction of h, while the mass of managers decreases with h consistent with proposition (4). However, as Table 1 indicates, when communication costs decrease the output of good 1 increases consistent with the intuition in the previous paragraph. Third, notice that the relative price of good 1, p_1/p_2 decreases with h, and therefore aggregate output of good 1 increases by more than the aggregate output of good 2. The corollary below indicates how a change in h affects the assignment of the initial set of workers and managers.

Corollary 5 *Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. Let z_1 be the upper bound on the initial set of workers. Let $z_2^{'}$ refer to the lower bound on the set of managers at $h^{'} < h$. Then for a decrease in h there exists an η such that:*

i. All existing workers from $[0, \eta]$ get matched to more knowledgeable managers

ii. All existing workers from $[\eta, z_1]$ get matched to less knowledgeable managers

iii. All remaining managers from $\left[z_2^{'}, m(\eta)\right]$ get matched to less knowledgeable workers

iv. All remaining managers from $[m(\eta), \alpha]$ get matched to more knowledgeable workers

Proof. See Appendix. ■

Holding α constant at 1, Figure 2 compares the distribution of earnings under different communication costs. First notice that the earnings of all workers increase as h is reduced from 0.4 to 0.1, while the earnings of all self-employed workers remain the same. [12] The previous discussion indicated that the relative price of good 1 decreases as communication costs decrease, and so all workers, and all agents who remained self-employed, are better off from a decrease in h. Second, notice that not all remaining managers earn more when communication costs decrease. The rents earned by the most knowledgeable managers rise with a decrease in h, while the least knowledgeable managers have their earnings reduced. For some of them, the welfare loss from a drop in their earnings is greater than the welfare

[12]This simply follows from the fact that the earnings of a self-employed worker with ability z is simply $p_2 z$ and p_2 is normalized to 1.

gain from a decrease in the relative price of good 1. As a result, all managers who had their earnings improve are better off from a decrease in the costs of communication, while there are some managers that are worse off. Third, notice that the managers that switched industries had their earnings decline the most. For some of them, the welfare loss from a drop in their earnings again outweighs the welfare gains from a decrease relative prices. Therefore, we can conclude from this discussion that a decrease in communication costs is not Pareto improving.

5 Open Economy

In this section, we extend the model to an open-economy environment. Assume that there are 2 countries, labeled Home and Foreign. [13] Both countries have populations of the same size. [14] Assume that Home has a more knowledgeable population than Foreign. For simplicity, we allow the domain of $G(z)$ to be $[0, 1]$ and the domain of $G^*(z)$ to be $[0, \alpha]$, for some $\alpha < 1$.

Goods are traded in international markets however, unlike in Antras, et al. (2006), managers are able to form teams only with workers in their own country. [15] Assume that the costs of communicating with a worker are constant across both countries. The decision problems encountered by consumers and managers are thus identical to those from previous sections. The goods market clearing conditions, however, are no longer the same. Since each good is traded in international markets, its price is determined by international demand and supply. Therefore, the goods market clearing conditions for x_1 and x_2, respectively, are

$$\frac{\beta}{p_1}\left[\int_0^{\alpha} E(z)g(z)dz + \int_0^1 E(z)g(z)dz\right] = \int_0^{z_1} m(z)g(z)dz + \int_0^{z_1^*} m(z)g(z)dz, \tag{9}$$

$$\frac{(1-\beta)}{p_1}\left[\int_0^{\alpha} E(z)g(z)dz + \int_0^1 E(z)g(z)dz\right] = \int_{z_1}^{z_2} F(z)g(z)dz + \int_{z_1^*}^{z_2^*} F(z)g(z)dz. \tag{10}$$

[13]Variable referring to the foreign country will a indexed by *

[14]This condition ensures that trade patterns are the result of differences in the distribution of abilities between the two countries.

[15]This condition restricts the analysis to studying how communication costs affect trade patterns between countries.

The definition of an equilibrium is similar to that of a closed-economy, however, it now takes into account that there are two countries. It is presented below:

Definition 6 *A competitive open-economy equilibrium consists of a set of wage functions* $\{w(z), w^*(z)\}$*, a set of rent functions* $\{R(z), R^*(z)\}$*, a set of assignment functions* $\{m(z), m^*(z)\}$*, a set of prices* $\{p_1, p_2\}$ *and set of a pair of thresholds* $\{(z_1, z_2), (z_1^*, z_2^*)\}$ *such that the following conditions hold:*

i. Agents in both countries maximize utility

ii. Managers in both countries maximize rents

iii. All markets clear

The proposition below indicates that an open-economy equilibrium exists. It is very similar to its closed-economy counterpart, however, it has the additional claim that factor price equalization does not hold. Specifically, two workers of the same ability level but who reside in different countries, will generally not earn the same wage. This is because, these two workers will not be matched to a manager of the same ability. In fact, the worker in the Home country will team up with a more knowledgeable manager than the worker in the Foreign country.

Proposition 7 *Let* $G(z)$*,* $G^*(z)$ *and* $F(z)$ *be uniformly distributed over their domains. For any values of* α*,* $h \in \left[0, \overline{h}\right]$ *there exists a competitive equilibrium. Moreover, in such an equilibrium, there is positive sorting, the earnings functions are convex, factor price equalization does not hold, and the sets of managers, self-employed and workers are connected.*

Proof. See Appendix. ■

As in the closed-economy, h has be to restricted in the interval $\left[0, \overline{h}\right]$. Since there are two economies, there are three upper bound restrictions on h, $h < \overline{h} = \min\left\{\frac{1}{z_1^*+z_2^*}, \frac{1}{z_1+z_2}, 1\right\}$. The first two restrictions are derived from condition ii of the definition of an equilibrium, and the third results from the fact that a manager, in either country, cannot spend more than 1 unit of his time communicating with his workers (i.e. $h \leq 1$). Since there does not exist a closed form solution for the occupational choice variables $\{z_1^*, z_2^*, z_1, z_2\}$, it is impossible to obtain a closed form expression for $\overline{h}$. One assumption is for h to be less than the smallest value $\frac{1}{z_1^*+z_2^*}$ and $\frac{1}{z_1+z_2}$ can undertake in their respective domains, that is $h < \overline{h} = \min\left\{\frac{1}{2\alpha}, \frac{1}{2}, 1\right\}$. Since $\alpha < 1$, $\frac{1}{2}$ is always smaller than $\frac{1}{2\alpha}$, and $\overline{h}$ simplifies to: $\overline{h} = \min\left\{\frac{1}{2}, 1\right\}$. [16]

The following proposition indicates how trade affects both economies:

Proposition 8 *Let $G(z)$, $G^*(z)$ and $F(z)$ be uniformly distributed over their domains. For any values of α, $h \in \left[0, \overline{h}\right]$ the following hold: (i) the relative price of good 1 in an open economy, is between Home and Foreign's autarkic relative price (ii.) In Home, z_1 increases while z_2 decreases, while in Foreign the opposite takes place (iii.) In Home, production of good 1 increases and production of good 2 decreases, while in Foreign the opposite takes place.*

Proof. See Appendix. ■

Since in a closed economy the relative price p_1/p_2 falls as α rises, and by assumption Home's population is more knowledgeable than Foreign's, in autarky the relative price of good 1 is higher in the Foreign country. With the opening of trade, Home producers of good 1 can thus increase their earnings by exporting to Foreign consumers. Similarly, the relative price of good 2 will be higher in the Home economy, and so Foreign producers of good 2 can increase their earnings by exporting to Home consumers. Therefore, Home will export and expand its production good 1, while Foreign will export and expand its production of good 2. In equilibrium, this process ensures that the relative price of good 1 is the same in both countries.

International trade also has an effect on occupational thresholds. Since with trade it becomes more profitable for agents in Home to produce good 1, there is an influx of managers and workers into the industry. As a result, the ability of the most knowledgeable worker, z_1, increases and the ability of the least knowledgeable manager, z_2, decreases with trade. In contrast, in the Foreign country the opposite effect takes place, the ability of the most knowledgeable worker, z_1^*, decreases, and the ability of the least knowledgeable manager, z_2^*, increases with trade. Furthermore, as the following corollary indicates, international trade also affects the assignment of workers and managers.

[16] Simulations of the model suggest that h can be as high as 0.65

Corollary 9 *Let $G(z)$, $G^*(z)$ and $F(z)$ be uniformly distributed over their domains. Then the following hold:*

i. In Home, all agents who were workers in autarky are matched to less knowledgeable managers

ii. In Home, all agents who were managers in autarky are matched to more knowledgeable workers

iii. In Foreign, all agents who were workers in autarky are matched to more knowledgeable managers

iv. In Foreign, all agents who were managers in autarky are matched to less knowledgeable workers

Proof. See Appendix. ∎

The corollary above indicates that all agents in the Home country who were managers in autarky are assigned to more knowledgeable workers. From the managers' budget constraint, it follows that $n(m^{-1}(z_m))$ increases. In words, all agents in Home who were managers under autarky are assigned to more knowledgeable and larger teams. Therefore, as the following proposition indicates, in the Home country, in every team that is headed by an agent who was a manager under autarky, the average output per person increases. [17]

Proposition 10 *Let $G(z)$, $G^*(z)$ and $F(z)$ be uniformly distributed over their domains. Under trade, in the Home country the productivity of all agents who were managers in autarky increases. In the Foreign country, the productivity of all agents who were managers in autarky decreases.*

Proof. See Appendix. ∎

Until now, we have discussed how trade leads to a reallocation of resources within countries. However, since in each country trade determines the relative price p_1/p_2, the earnings

[17] More specifically, in a team headed by a manager of ability z, the average output per person is equal to $z\frac{n(m^{-1}(z))}{n(m^{-1}(z))+1}$.

of the factors of production are also impacted. More specifically, as the proposition below indicates, in the Home country, the earnings of all agents who become workers and managers increase, whereas in the Foreign country, the opposite takes place. In addition, since a self-employed worker of ability z earns z, the earnings of all agents who were self-employed in autarky, and remain self-employed under trade, remain the same.

Proposition 11 *Let $G(z)$, $G^*(z)$ and $F(z)$ be uniformly distributed over their domains. Under trade, in the Home country the earnings of all workers increase, and the earnings of all managers increase. In the Foreign country the opposite takes place. The earnings of all agents who remain self-employed are unaffected by trade.*

Proof. See Appendix. ■

Furthermore, evidence from simulations suggests that communication costs, h, impact the open economy equilibrium. First, communication costs affect the magnitude of the change in the occupational thresholds in both Home and Foreign. As Table 2 indicates, for $\alpha = 0.8$ when h is low, the ability of the least knowledgeable manager in Home is higher than when h is high.[18] Similarly, when h is low, the ability of the most knowledgeable worker in Home increases. A similar effect is observed in Foreign. Therefore, a low h dampens the movement of the occupational threshold z_2, and amplifies the movement of the occupational threshold z_1 in both countries. Second, as Table 4 indicates, the effect of international trade on earnings inequality varies with communication costs. For instance, when h is equal to 0.1, the inequality between the earnings of the most knowledgeable manager and the least knowledgeable worker in the economy decrease from trade, whereas when h is equal to 0.5 the opposite occurs. [19] Similarly, in the South, when h is 0.1 the difference in the earnings of the most knowledgeable manager and least knowledgeable worker decreases with trade, whereas when h is 0.5 the opposite takes place.

[18]In Tables 2 and 3, h is equal to 0.1 or 0.5. Although this is outside of the bounds for h, when h equals 0.5 an equilibrium exists in both the open and closed economies.

[19]Inequality here is measured as the difference in earnings between two agents.

6 Conclusion

In conclusion, this paper attempts to investigate how communication costs and trade affect earnings inequality. In order to accomplish this it extends Garicano and Rossi-Hansberg (2006) to two goods and two countries. The main findings of the paper are that under certain assumptions a competitive equilibrium exists in a closed and open economy. Moreover, in such an equilibrium there is positive sorting, the earnings functions are convex and the sets of managers, self-employed and workers are connected. In addition, in the open economy, identical workers located in different countries do not always earn the same income, the country with a more knowledgeable population specializes in the production of the good produced in teams, while the other country specializes in the good produced by self-employed workers. Furthermore, this paper finds that whether international trade increases or decreases income inequality depends on the cost of communication between managers and workers.

One limitation of this paper, is that it does not allow for the cost of communication to vary across countries. Integrating this feature in the present model would provide an insight into how differences in technology influence the patterns of trade. Furthermore, in such a setting, one can also investigate what happens when the less efficient technology converges to the more advanced one. Another drawback of the present model is the assumption that the distribution of knowledge and problems are uniform. Although this assumption simplifies the analysis, it limits the robustness of the findings. A question of interest is how the results change when the distribution of knowledge and the distribution of problems have another form.

References

[1] P. Antras, L. Garicano, E. Rossi-Hansberg. 2006.“Offshoring in a Knowledge Economy ” Quarterly Journal of Economics 121(1): 31-77.

[2] P. Antras, L. Garicano, E. Rossi-Hansberg. 2008. “Organizing Offshoring: Middle Managers and Communication Costs ” in E. Helpman, D. Marin, T. Vierdier eds., The Organization of Firms in a Global Economy, Cambridge, US: Harvard University Press: 311-339.

[3] P. Antras, E. Rossi-Hansberg. 2009.“Organizations and Trade ” Annual Review of Economics: 43-64.

[4] R. C Feenstra, G. H. Hanson. 2003. “Global Production Sharing and Rising Inequality: A Survey of Trade and Wages ” in E. K. Choi and J. Harrigan eds., Handbook of International Trade, Oxford, UK: Basil-Blackwell: 146-1850.

[5] L. Garicano. 2000. “Hierarchies and the Organization of Knowledge in Production ” Journal of Political Economics 108(5): 874-904.

[6] L. Garicano, E. Rossi-Hansberg. 2004. “Inequality and the Organization of Knowledge ” American Economic Review, Papers and Proceedings: 197-202.

[7] L. Garicano, E. Rossi-Hansberg. 2006. “Organization and Inequality in a Knowledge Economy” Quarterly Journal of Economics 121(4): 1383-1435.

[8] G. M. Grossman, G. Maggi. 2000. “Diversity and Trade ” American Economic Review 90:1255-1275

[9] R. E. Lucas. 1978. “On the Size Distribution of Firms ” Bell Journal of Economics 9(2): 508-523.

[10] M. Kremer, E. Maskin. 1996. “Wage Inequality and Segregation by Skill ” Working Paper., NBER.

[11] M. Kremer, E. Masking. 2006. “Globalization and Inequality ” Working Paper., Harvard University Press.

7 Appendix

Proposition: An equilibrium in this economy has positive sorting.

Proof. The argument in this section is similar to the first part of the proof of Theorem 1 in Antras et. al (2006). Let $R(z_m, z_p)$ denote the rents of a manager of ability z_m assigned workers of ability z_p. The equilibrium is characterized by an assignment function $m(z)$ such that $z_m = m(z_p)$. Since managers are maximizing rents, it follows that

$$\frac{\partial R(z_m, z_p)}{\partial z_p} = 0.$$

From the expression above, we can find an expression for $\frac{\partial z_m}{\partial z_p}$. Namely,

$$\frac{\partial z_m}{\partial z_p} = -\frac{\partial^2 R(z_m, z_p)/\partial z_p^2}{\partial^2 R(z_m, z_p)/\partial z_p \partial z_m}.$$

Because the manager is solving a maximization problem, the numerator is negative. Also, the denominator is positive since from

$$\frac{\partial R(z_m, z_p)}{\partial z_m} = \frac{p_1}{h\left[1 - z_p\right]},$$

it follows that

$$\frac{\partial^2 R(z_m, z_p)}{\partial z_p \partial z_m} = \frac{p_1}{h\left[1 - z_p\right]^2} > 0.$$

Therefore, the equilibrium has positive sorting. ■

Proposition 2: Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. For any values of α and $h \in \left[0, \overline{h}\right]$ there exists a competitive equilibrium. Moreover, in such an equilibrium, there is positive sorting, the earnings function is convex and the sets of managers and workers are connected.

Proof.

To show that an equilibrium exists, the following conditions must be satisfied:

i. The sets of managers, self-employed and workers are connected

ii. Agents do not want to deviate from their occupational choices.

iii. There exists an equilibrium.

i.) The sets are connected.

To show that the sets of workers, self-employed and managers are connected, begin by assuming that this is not the case. Suppose that the sets of managers, self-employed and workers is disconnected. That is $W = [a_1, a_2] \cup [a_4, a_5]$, $S = [a_2, a_3] \cup [a_5, a_6]$ and $M = [a_3, a_4] \cup [a_6, a_7]$. For each interval $[a_1, a_4]$ and $[a_4, a_7]$ solve the agents' problem under the restriction that teams can be formed only with members in the same interval. Then it follows in the interval $[a_1, a_4]$ that $m(a_1) = a_3$, $m(a_2) = a_4$. Similarly in the interval $[a_4, a_7]$ it follows that $m(a_4) = a_6$, $m(a_5) = a_7$.

For this setup to be an equilibrium, the earnings function has to be continuous. Thus at a_4 it must be the case that $R_{14}(a_4)$ and $w_{47}(a_4)$ are equal. If it is also the case that at a_4:

$$\lim_{\downarrow a_4} \frac{\partial R_{14}(z)}{\partial(z)} > \lim_{\uparrow a_4} \frac{\partial w_{47}(z)}{\partial(z)}$$

then manager a_6 will like to hire $a_4 - \epsilon$.

Since demand and supply in this model are homogeneous of degree zero, the assignment function is independent of prices, and equation $\sigma = p_2 z_2 h$ is also independent of prices, the equilibrium bundles in the economy are not affected by proportional price changes. Further, in any equilibrium either

$$\tfrac{p_1}{p_2} > 1 \qquad \text{or} \qquad \tfrac{p_1}{p_2} \leq 1$$

Consider the case $\frac{p_1}{p_2} \leq 1$. Then normalize p_1 to 1. Since $w_{14}(a_2) = a_2 p_2$, $[F(z)p_2]^{'} = p_2 \geq 1$ and $R_{14}^{'}(z) = \frac{1}{h[1-m^{-1}(z)]}$, it follows that $R_{14}(a_4) > a_4$. Therefore, since $a_1 < a_2 < a_3 < a_4 < a_5 < a_6 < 1$

$$w_{47}^{'}(a_4) = \frac{a_6 - w_{47}(a_4)}{1 - a_4} = \frac{a_6 - R_{14}(a_4)}{1 - a_4} < \frac{a_6 - a_4}{1 - a_4} < 1$$

Consider the case $\frac{p_1}{p_2} < 1$. Then normalize p_2 to 1. Using a similar argument as above, it follows that $R_{14}(a_4) > a_4$. Then the following is also true:

$$w_{47}^{'}(a_4) = \frac{p_1 a_6 - w_{47}(a_4)}{1-a_4} = \frac{a_6 - R_{14}(a_4)}{1-a_4} < \frac{p_1}{h\left[1-a_2\right]} = R_{14}^{'}(a_4)$$

Substituting the expression for $R_{14}(a_4)$, the inequality amounts to showing that

$$p_1 a_6 \left[1-a_2\right] h < p_1 - w_{14}(a_2) = p_1 - p_2 a_2$$

Which follows since $p_1 a_6 h < p_1 < p_1 \frac{1-a_2/p_1}{1-a_2}$.

Now consider manager a_6. If he were to hire $a_4 - \epsilon$ he would earn

$$\Pi(a_6, a_4-\epsilon) = \frac{p_1 a_6 - R_{14}(a_4-\epsilon)}{h\left[1-(a_4-\epsilon)\right]}$$

Since $R_{14}(a_4) = w_{47}(a_4)$ and $w_{47}^{'}(a_4) = \frac{p_1 a_6 - w_{47}(a_4)}{1-a_4}$

$$\lim_{\epsilon \to 0} \frac{\partial \Pi(a_6, a_4-\epsilon)}{\partial \epsilon} = \frac{R_{14}^{'}(a_4) - w_{47}^{'}(a_4)}{h\left[1-a_4\right]} > 0$$

manager a_6 would increase his earnings if he hires $a_4 - \epsilon$. Therefore, he has an incentive to deviate. Thus, in an equilibrium the sets of managers, workers and self-employed must be connected.

ii.) Agents do not want to deviate from their occupational choices.

First, one must show that managers with knowledge α do not want to hire workers with ability greater than z_1. Without loss of generality, normalize p_2 to 1. A manager with ability α may decide to hire a worker with ability $z_1 + \epsilon$ at wages $F(z_1+\epsilon)$. Then he would earn

$$\Pi(\alpha, z_1+\epsilon) = \frac{p_1 \alpha - F(z_1+\epsilon)}{h\left[1-(z_1+\epsilon)\right]}$$

And since $F(z_1) = w(z_1)$ and $w^{'}(z_1) = \frac{p_1 \alpha - w(z_1)}{1-z_1}$

$$\lim_{\epsilon \to 0} \frac{\partial \Pi(\alpha, z_1+\epsilon)}{\partial \epsilon} = \frac{F^{'}(z_1) - w^{'}(z_1)}{h\left[1-z_1\right]} > 0$$

as long as $F^{'}(z_1) > w^{'}(z_1)$. At z_1, this is equivalent to $\sigma + z_1 h = (z_2+z_1)h < 1$. Points z_1 and z_2 are endogenously determined in the model and are therefore a function of h, however since there does not exist a closed form solution to z_1 and z_2, to ensure that an equilibrium exists we impose the condition that h is less than the largest value the ratio $\frac{1}{z_1+z_2}$ can take. In the domain $[0,\alpha]\,X\,[0,\alpha]$, the largest value $\frac{1}{z_1+z_2}$ can ever have is $\frac{1}{2\alpha}$. Thus $h < min\left\{\frac{1}{2\alpha},1\right\}$.

Similarly, at point z_2, it must the case that condition $F^{'}(z_2) < R^{'}(z_2)$. With some manipulation, one can show that it is always satisfied.

iii.) There exists an equilibrium.

To show that an equilibrium exists, one has to show that the following system of equations has a solution.

$$h\left[z_1 - \frac{z_1^2}{2}\right] + z_2 = \alpha \tag{1}$$

$$p_1 z_2 + 1/2 p_1 h z_1^2 - p_2 z_1 = p_2 z_2 h(1 - z_1) \tag{2}$$

$$\frac{\beta}{p_1}\int_0^\alpha E(z)g(z)dz = \int_0^{z_1} m(z)g(z)dz \tag{3}$$

$$\frac{1-\beta}{p_2}\int_0^\alpha E(z)g(z)dz = \int_{z_1}^{z_2} F(z)g(z)dz \tag{4}$$

$$\int_0^\alpha E(z)g(z)dz = p_1\int_0^{z_1} m(z)g(z)dz + p_2\int_{z_1}^{z_2} F(z)g(z)dz \tag{5}$$

Equation (1) is the condition $m(z_1) = \alpha$, while equation (2) results from $R(z_2) = p_2 F(z_2)$. Equations (3) and (4) describe the goods market clearing conditions, and equation (5) describes the fact total income equals total expenditures.

STEP 1: Rewrite the above system of five equations and five unknowns into a system of two equations and two unknowns.

From equation (5), the term $\int_0^\alpha E(z)g(z)dz$ can substituted into equations (3) and (4) yielding:

$$\beta\frac{p_2}{p_1}\int_{z_1}^{z_2} F(z)g(z)dz = (1-\beta)\int_0^{z_1} m(z)g(z)dz \tag{3'}$$

$$(1-\beta)\frac{p_1}{p_2}\int_0^{z_1} m(z)g(z)dz = \beta\int_{z_1}^{z_2} F(z)g(z)dz \tag{4'}$$

Clearly, equations (3') and (4') are identical to one another. Further, substituting for p_1 from (3') into (2), integrating the expressions and rearranging the terms yields:

$$(1-\beta)\left[6z_2z_1 - h{z_1}^3 + 1/2hz_1^2\right]\left[z_2h(1-z_1) + z_1\right] = 3\beta\left[z_2^2 - z_1^2\right]\left[z_2 + 1/2hz_1^2\right] \tag{2'}$$

Therefore, the system has been reduced to the following two equations in two unknowns:

$$h\left[z_1 - \frac{z_1^2}{2\bar{z}}\right] + z_2 = \alpha \qquad (1)$$

$$(1-\beta)\left[6z_2z_1 - h{z_1}^3 + 1/2hz_1^2\right]\left[z_2h(1-z_1)+z_1\right] = 3\beta\left[z_2^2 - z_1^2\right]\left[z_2 + 1/2hz_1^2\right] \quad (2')$$

The domain of interest is $[0,\alpha]\,X\,[0,\alpha]$. Equation (1) can be rewritten in the form of z_2 as a function of z_1. Equation (2'), however, represents a specific isoline of a function $y = f(z_1, z_2)$ in the $\Re^2$ plane. Therefore both functions have a graphical representation in a standard two-dimensional plot.

STEP 2: Show that at the point $z_1 = 0$ equation (1) is above equation (2').
Evaluate equations (1) and (2') at the point $z_1 = 0$. Equation (1) yields the following expression $z_2 = \alpha$ while equation (2') yields the expression $3\beta {z_2}^2 = 0$. The latter expression has one solution $z_2 = 0$.

STEP 3: Show that at the point $z_1 = \alpha$ equation (2') is above equation (1).
Evaluate equation (1) at the point $z_1 = \alpha$. Equation (1) provides the solution $z_2 = \alpha - \frac{h}{2}\alpha$ which is smaller than α. For equation (2'), notice than in the relevant domain $[0,\alpha]\,X\Re_{\geq 0}$, the left hand side is positive, and the right hand side will be positive if and only if the following condition holds :

$$z_2^2 - z_1^2 > 0$$

Thus it follows that at $z_1 = \alpha$ equation (2') is above equation (1).

STEP 4: Show that equation (1) is decreasing in the interval $[0,\alpha]$.
Totally differentiating equation (1) yields

$$h\left[1 - z_1\right]dz_1 + dz_2 = 0$$

Therefore $\frac{dz_2}{dz_1} = -h\left[1 - z_1\right]$ is negative.

STEP 5: Show that equation (2') is increasing in the interval $[0,\alpha]$.
Totally differentiating equation (2') yields

$$dz_2\,[A] + dz_1\,[B] = 0$$

where

$A = (1-\beta)6z_1(z_2h(1-z_1)+z_1) + (1-\beta)(6z_2z_1 - hz_1^3 + 3hz_1^2)h(1-z_1) - 3\beta(z_2 + 1/2hz_1^2)2z_2 - 3\beta(z_2^2 - z_1^2)$

$B = (1-\beta)(6z_2 - 3hz_1^3 + 6hz_1)(z_2h(1-z_1)+z_1) + (1-\beta)(6z_2z_1 - hz_1^3 + 3hz_1^2)(1-hz_2) + 3\beta(z_2 + 1/2hz_1^2)2z_1 - 3\beta(z_2^2 - z_1^2)hz_1$

Claim 1: A is negative.

Proof:

First note that:

$$(1-\beta)(6z_2z_1 - hz_1^3 + 3hz_1^2)h(1-z_1)1/2hz_1^2 < (1-\beta)(6z_2z_1 - hz_1^3 + 3hz_1^2)z_1$$

With some manipulation, one can show that:

$$(1-\beta)6z_1(z_2h(1-z_1)+z_1)(z_2 + 1/2hz_1^2) < 6\beta(z_2 + 1/2hz_1^2)^2z_2$$

Then it follows that:

$$\begin{gathered}(1-\beta)6z_1(z_2h(1-z_1)+z_1)(z_2+1/2hz_1^2) + (1-\beta)(6z_2z_1 - hz_1^3 + 3hz_1^2)h(1-z_1)(z_2+1/2hz_1^2) < \\ 3\beta(z_2+1/2hz_1^2)^2 2z_2 + (1-\beta)(6z_2z_1 - hz_1{}^3 + 1/2hz_1^2)(z_2h(1-z_1)+z_1)\end{gathered}$$

This expression is equivalent to:

$$\begin{gathered}(1-\beta)6z_1(z_2h(1-z_1)+z_1) + (1-\beta)(6z_2z_1 - hz_1^3 + 3hz_1^2)h(1-z_1) < \\ 3\beta(z_2+1/2hz_1^2)2z_2 + \tfrac{(1-\beta)(6z_2z_1 - hz_1{}^3 + 1/2hz_1^2)(z_2h(1-z_1)+z_1)}{(z_2+1/2hz_1^2)}\end{gathered}$$

And using the fact that equation (2') holds with equality, the expression above can be rewritten as

$$(1-\beta)6z_1(z_2h(1-z_1)+z_1) + (1-\beta)(6z_2z_1 - hz_1^3 + 3hz_1^2)h(1-z_1) < 3\beta(z_2+1/2hz_1^2)2z_2 + 3\beta(z_2^2 - z_1^2)$$

Thus A is negative.

Claim 2: B is positive.

Proof:

First notice that

$$(6z_2 - 3hz_1^2 + 6hz_1)(z_2h(1-z_1) + z_1)(z_2 + 1/2hz_1^2) > (6z_2z_1 - hz_1^3 + 3hz_1^2)(z_2h(1-z_1) + z_1)hz_1$$

Then it follows that

$$(1-\beta)(6z_2-3hz_1^2+6hz_1)(z_2h(1-z_1)+z_1)+3\beta(z_2+1/2hz_1^2)2z_1+(1-\beta)(6z_2-hz_1^3+3hz_1^2)(1-hz_2) > (1-\beta)hz_1\frac{(6z_2z_1-hz_1^3+3hz_1^2)(z_2h(1-z_1)+z_2)}{z_2+1/2hz_1^2}$$

Using the fact that equation (2') holds with equality, the expression above can be rewritten as

$$(1-\beta)(6z_2-3hz_1^2+6hz_1)(z_2h(1-z_1)+z_1)+3\beta(z_2+1/2hz_1^2)2z_1+(1-\beta)(6z_2-hz_1^3+3hz_1^2)(1-hz_2) > 3\beta(z_2^2 - z_1^2)hz_1$$

Thus B is positive.

Therefore, from Claims 1 and 2, we can conclude that for equation (2') $\frac{dz_2}{dz_1} = -\frac{B}{A} > 0$. Therefore, since equation (1) is above equation (2') when $z_1 = 0$, equation (2') is above equation (1) when $z_1 = \alpha$, and equation (1) is decreasing over the interval $[0, \alpha]$, while equation (2') is increasing over the same interval, equations (1) and (2') intersect once over the domain $[0, \alpha]$. Thus, the proposition is true.

iv.) The earnings function is convex.

It remains to show that the earnings function is convex. Since $\forall z \leq z_1, w'(z) = \sigma + hz$ the wage function is convex, and since $\forall z \in [z_1, z_2], F(z) = z$ the earnings function of self-employed workers is also convex, and since $\forall z \geq z_2, R'(z) = \frac{p_1}{h[1-m^{-1}(z)]}$ and there is positive sorting, the rent function is also convex. Therefore the earnings function is convex.

v.) An equilibrium exhibits positive sorting

This result follow directly from the previous proposition. ■

Proposition 3: Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. An increase in α has the following effect: (i.) The maximum ability of a worker, and the minimum ability of a self-employed worker, z_1, increases (ii.) The minimum ability of a manager, and the maximum ability of a self-employed workers, z_2, increases.

Proof.

The proof in proposition (2) showed that the equilibrium is characterized by the two equations:

$$h\left[z_1 - \frac{z_1^2}{2\bar{z}}\right] + z_2 = \alpha \qquad (1)$$
$$(1-\beta)\left[6z_2z_1 - hz_1{}^3 + 1/2hz_1^2\right]\left[z_2h(1-z_1)+z_1\right] = 3\beta\left[z_2^2 - z_1^2\right]\left[z_2 + 1/2hz_1^2\right] \quad (2')$$

From the equation (1) it follows that

$$\frac{dz_1}{d\alpha} = \frac{1}{h\,(1-z_1) + \frac{dz_2}{dz_1}}.$$

Since from equation (2') it follows that $\frac{dz_2}{dz_1} > 0$, the expression above is positive. Furthermore, since $\frac{dz_2}{d\alpha} = \frac{dz_2}{dz_1}\frac{dz_1}{d\alpha}$, $\frac{dz_2}{d\alpha}$ is positive because $\frac{dz_2}{dz_1} > 0$ and $\frac{dz_1}{d\alpha} > 0$. ■

Proposition 4: Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. A decrease in h has the following effect: (i.) The maximum ability of a worker, and the minimum ability of a self-employed worker, z_1, increases (ii.) The minimum ability of a manager, and the maximum ability of a self-employed workers, z_2, increases.

Proof. Let $\beta = 0.5$. Then the equilibrium is characterized by the two equations:

$$h\left[z_1 - \frac{z_1^2}{2\bar{z}}\right] + z_2 = \alpha \qquad (1)$$
$$\left[6z_2z_1 - hz_1{}^3 + 1/2hz_1^2\right]\left[z_2h(1-z_1)+z_1\right] = 3\left[z_2^2 - z_1^2\right]\left[z_2 + 1/2hz_1^2\right] \quad (2')$$

Taking total derivates with from equation (1) and (2') and setting $d\alpha = 0$, yields the following expressions:

$$[D]\,dh + [E]\,dz_1 + dz_2 = 0 \qquad (1')$$
$$[B]\,dz_1 + [A]\,dz_2 + [C]\,dh = 0 \qquad (2'')$$

where,

$[A] = 6z_1(z_2h(1-z_1)+z_1) + (6z_2z_1 - hz_1^3 + 3hz_1^2)h(1-z_1) - 6(z_2 + 1/2hz_1^2)z_2 + 3(z_2^2 - z_1^2)$,

$[B] =$

$(6z_2 - 3hz_1^2 + 6hz_1)(z_2h(1-z_1)+z_1) + 6(z_2 + 1/2hz_1^2)z_1 + (6z_2 - hz_1^3 + 3hz_1^2)(1-hz_2) - 3(z_2^2 - z_1^2)$,

$[C] = (z_2h(1-z_1)+z_1)(3z_1^2 - z_1^3) + (6z_2z_1 - hz_1^3 + 3hz_1^2)(1-z_1)z_2 - 3/2(z_2^2 - z_1^2)z_1^2$,

$[D] = z_1(1 - \frac{z_1}{2})$,

$[E] = (1-z_1)h$.

It is straighforward to see that $[D] > 0$ and $[E] > 0$. From the proof of proposition (2), we know that $[A] < 0$ and $[B] > 0$. Furthermore, with some manipulation one can also show that $[C] > 0$.

From equation (1') it follows that

$$\frac{dz_2}{dh} = -[D] - [E]\frac{dz_1}{dh}.$$

Substituting that expression into equation (2") yields

$$\frac{dz_1}{dh} = \frac{-[C]+[A][D]}{[B]-[A][E]},$$

which is negative. Therefore, from the two expressions above it follows that $\frac{dz_2}{dh} < 0$, if and only if $[E][C] < [D][B]$. Indeed, with some manipulation one can show that this inequality is true. ■

Corollary 5: Let $G(z)$ and $F(z)$ be uniformly distributed over their domains. Let z_1 refer to the upper bound on the initial set of workers. Let $z_2^{'}$ refer to the lower bound on the new set of managers. Then for a decrease in h there exists an η such that:

i. All existing workers from $[0, \eta]$ get matched to more knowledgeable managers

ii. All existing workers from $[\eta, z_1]$ get matched to less knowledgeable managers

iii. All remaining managers from $\left[z_2^{'}, m(\eta)\right]$ get matched to less knowledgeable workers

iv. All remaining managers from $[m(\eta), \alpha]$ get matched to more knowledgeable workers

Proof.

Let $h^{'} < h$. Then we know that $z_2 < z_2^{'}$ and $z_1 < z_1^{'}$. In order to determine how workers' assignment is affected by a change in communication costs, we want to compare $m^{'}(z)$ with $m(z)$ in the following manner:

$$\begin{aligned} m(z) - m^{'}(z) &= h\left(z - \frac{z^2}{2}\right) + z_2 - h^{'}\left(z - \frac{z^2}{2}\right) - z_2^{'} \\ &= \left(h - h^{'}\right)\left(z - \frac{z^2}{2}\right) + \left(z_2 - z_2^{'}\right) \end{aligned}$$

At $z=0$, $m(z)-m^{'}(z)=z_2-z_2^{'}<0$.
At $z=z_1$, $m(z)-m^{'}(z)=\alpha-h^{'}\left(z_1-\frac{z_1^2}{2}\right)-z_2^{'}>0$. This follows from the fact that $z_1<z_1^{'}$ and at $z=z_1^{'}$, $m^{'}(z'_1)=\alpha$, and $\frac{dm^{'}(z)}{dz}=h^{'}(1-z)>0$.
Since $h>h^{'}$, it follows that $m(z)-m^{'}(z)$ is increasing. That is,

$$\frac{d(m(z)-m^{'}(z))}{dz}=(h-h^{'})(1-z)>0.$$

By the Intermediate Value Theorem, there exists a $z=\eta$ such that $m(\eta)-m^{'}(\eta)=0$. Hence for all $z\in[0,\eta)$, it follows that $m(z)-m^{'}(z)<0$, and workers get assigned to more knowledgeable managers when communication costs are $h^{'}$. At $z=\eta$, it follows that $m(\eta)-m^{'}(\eta)<0$, and workers get assigned to managers of the same ability. And, for all $z\in(\eta,z_1]$, it follows $m(z)-m^{'}(z)>0$, and workers get assigned to less knowledgeable managers when communication costs are $h^{'}$. Therefore, statements i and ii are true.
Since the matching function is monotonic and invertible, statement iii and iv follow directly from i and ii. ■

Proposition 7: Let $G(z)$, $G^{*}(z)$ and $F(z)$ be uniformly distributed over their domains. For any values of α, $h\in\left[0,\overline{h}\right]$ there exists a competitive equilibrium. Moreover, in such an equilibrium, there is positive sorting, the earnings functions are convex, factor price equalization does not hold, and the sets of managers, self-employed and workers are connected.

Proof.

To show that an equilibrium exists in an open economy setting, the following conditions must be satisfied:

i. In each country, the sets of managers, self-employed and workers are connected

ii. Agents do not want to deviate from their occupational choices.

iii. There exists an equilibrium.

The proofs for conditions i and ii are identical to the closed-economy framework. Further, to show that an equilibrium exists, one has to show that the following system of equations has a solution.

$$h\left[z_1^* - \frac{z_1^{*2}}{2}\right] + z_2^* = \alpha \tag{11}$$

$$p_1 z_2^* + 1/2 p_1 h z_1^{*2} - p_2 z_1^* = p_2 z_2^* h(1 - z_1^*) \tag{12}$$

$$h\left[z_1 - \frac{z_1^2}{2}\right] + z_2 = 1 \tag{13}$$

$$p_1 z_2 + 1/2 p_1 h z_1^2 - p_2 z_1 = p_2 z_2 h(1 - z_1) \tag{14}$$

$$\frac{\beta}{p_1}\left[\int_0^\alpha E(z)g^*(z)dz + \int_0^1 E(z)g(z)dz\right] = \int_0^{z_1^*} m(z)g^*(z)dz + \int_0^{z_1} m(z)g(z)dz \tag{15}$$

$$\frac{1-\beta}{p_2}\left[\int_0^\alpha E(z)g^*(z)dz + \int_0^1 E(z)g(z)dz\right] = \int_{z_1^*}^{z_2^*} F(z)g^*(z)dz + \int_{z_1}^{z_2} F(z)g(z)dz \tag{16}$$

$$\int_0^\alpha E(z)g^*(z)dz = p_1 \int_0^{z_1^*} m(z)g^*(z)dz + p_2 \int_{z_1^*}^{z_2^*} F(z)g^*(z)dz \tag{17}$$

$$\int_0^1 E(z)g(z)dz = p_1 \int_0^{z_1} m(z)g(z)dz + p_2 \int_{z_1}^{z_2} F(z)g(z)dz \tag{18}$$

Equations (11) and (13) describe the condition that in each country, the most knowledgeable worker is matched to the most knowledgeable manager, while equations (12) and (14) result from $R(z_2^*) = p_2 F(z_2^*)$ and $R(z_2) = p_2 F(z_2)$, respectively. Equations (15) and (16) describe the goods market clearing conditions, and equations (16) and (17) describe the fact total income equals total expenditures in the foreign and domestic country, respectively.

STEP 1: Rewrite the above system of eight equations and eight unknowns into a system of two equations and two unknowns.

Substituting equations (17) and (18) into (15) or (16) yields the expression

$$\beta\frac{p_2}{p_1}\left[\int_{z_1^*}^{z_2^*} F(z)g^*(z)dz + \int_{z_1}^{z_2} F(z)g(z)dz\right] = (1-\beta)\left[\int_0^{z_1^*} m(z)g^*(z)dz + \int_0^{z_1} m(z)g(z)dz\right]. \tag{19}$$

Equations (11) and (13) can be rewritten in the form of z_2^* as a function of z_1^*, and z_2 as a function of z_1. Furthermore, isolating $\frac{p_2}{p_1}$ in equation (19) and substituting it into (12) and (14) yields the two equations

$$\frac{\beta}{1-\beta}\frac{B}{A} = \frac{z_2^*(z_1^*)h(1-z_1^*)+z_1^*}{z_2^*(z_1^*)+1/2hz_1^{*2}}, \tag{20}$$

$$\frac{\beta}{1-\beta}\frac{B}{A} = \frac{z_2(z_1)h(1-z_1)+z_1}{z_2(z_1)+1/2hz_1^2}, \tag{21}$$

where $B = \int_{z_1^*}^{z_2^*(z_1^*)} F(z)g^*(z)dz + \int_{z_1}^{z_2(z_1)} F(z)g(z)dz = \frac{1}{2\alpha}\left[z_2^*(z_1^*)^2 - z_1^{*2}\right] + \frac{1}{2}\left[z_2(z_1)^2 - {z_1}^2\right]$ and $A = \int_0^{z_1^*} m(z)g^*(z)dz + \int_0^{z_1} m(z)g(z)dz = \frac{1}{6\alpha}\left[6z_2^*(z_1^*)z_1^* - hz_1^{*3} + 3hz_1^{*2}\right] + \frac{1}{6}\left[6z_2(z_1)z_1 - h{z_1}^3 + 3h{z_1}^2\right]$, with z_2^* and z_2 are written as functions of z_1^* and z_1. Equations (20) and (21) provide a system of 2 equations in 2 unknowns, z_1^* and z_1. The domain of interest is $[0,\alpha]X[0,1]$. Equation (20) represents a specific isoline of a function $y = f_1(z_1^*, z_1)$ in the $\Re^2$ plane. Similarly, equation (21) also represents a specific isoline of a different function $y = f_2(z_1^*, z_1)$ in the $\Re^2$ plane. Therefore, equations (20) and (21) can be thought of as functions of three variables (z_1^*, z_1, y)

$$f_1(z_1^*, z_1) = \frac{\beta}{1-\beta}\frac{B}{A} - \frac{z_2^*(z_1^*)h(1-z_1^*)+z_1^*}{z_2^*(z_1^*)+1/2hz_1^{*2}} \tag{20}$$

$$f_2(z_1^*, z_1) = \frac{\beta}{1-\beta}\frac{B}{A} - \frac{z_2(z_1)h(1-z_1)+z_1}{z_2(z_1)+1/2hz_1^2} \tag{21}$$

evaluated at $y = 0$.

STEP 2: Show that at the point $z_1^* = 0$ equation (20) is above equation (21).

Let $z_1^* = 0$. Then $z_2^* = \alpha$, and from equation (20) it follows that $\frac{\beta}{1-\beta}\frac{B}{A} = h$. Substituting this into (21) yields:

$$f_1(0, z_1) = h - \frac{z_2(z_1)h(1-z_1) + z_1}{z_2(z_1) + 1/2hz_1{}^2}.$$

Since $h < \frac{1}{z_1+z_2(z_1)}$, and $\frac{1}{z_1+z_2(z_1)}(1/2hz_1 + z_2(z_1)) < 1$ it follows that

$$1/2h^2z_1 + hz_2(z_1) < 1.$$

As a result,

$$f_1(0, z_1) = h - \frac{z_2(z_1)h(1-z_1) + z_1}{z_2(z_1) + 1/2hz_1{}^2} < 0,$$

which implies that equation (20) is above equation (21) when $z_1^* = 0$.

STEP 3: Show that at the point $z_1 = 0$ equation (21) is above equation (20).

Let $z_1 = 0$. Then $z_2 = 1$, and from equation (21) it follows that $\frac{\beta}{1-\beta}\frac{B}{A} = h$. Substituting this into (20) yields:

$$f_1(z_1^*, 0) = h - \frac{z_2^*(z_1^*)h(1-z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2hz_1^{*2}}.$$

Since $h < \frac{1}{z_1^*+z_2^*(z_1^*)}$, and $\frac{1}{z_1^*+z_2^*(z_1^*)}(1/2hz_1^* + z_2^*(z_1^*)) < 1$ it follows that

$$1/2h^2z_1^* + hz_2^*(z_1^*) < 1.$$

As a result,

$$f_1(z_1^*, 0) = h - \frac{z_2^*(z_1^*)h(1-z_1^*) + z_1^*}{z_2^*(z_1^*) + 1/2hz_1^{*2}} < 0,$$

which implies that equation (21) is above equation (20) when $z_1 = 0$.

From steps 3 and 4, and the fact that equations (20) and (21) are continuous, it follows that there exists at least one point where the equations intersect. Therefore, there exists at least one equilibrium.

STEP 4: Show that the equilibrium is unique.

From equation (20) it follows that

$$\frac{dz_1}{dz_1^*} = -\frac{\frac{\partial}{\partial z_1^*}\left(\frac{\beta}{1-\beta}\frac{B}{A}\right) - \frac{\partial}{\partial z_1^*}\left(\frac{z_2^*(z_1^*)h(1-z_1^*)+z_1^*}{z_2^*(z_1^*)+1/2hz_1^{*2}}\right)}{\frac{\partial}{\partial z_1}\left(\frac{\beta}{1-\beta}\frac{B}{A}\right)}. \tag{22}$$

From equation (21) it follows that

$$\frac{dz_1}{dz_1^*} = -\frac{\frac{\partial}{\partial z_1^*}\left(\frac{\beta}{1-\beta}\frac{B}{A}\right)}{\frac{\partial}{\partial z_1}\left(\frac{\beta}{1-\beta}\frac{B}{A}\right) - \frac{\partial}{\partial z_1}\left(\frac{z_2(z_1)h(1-z_1)+z_1}{z_2^*(z_1)+1/2hz_1^{2}}\right)}. \tag{23}$$

When equations (20) and (21) intersect, the expression $\frac{\beta}{1-\beta}\frac{B}{A}$ has the same value in both equations. As a result, its partial derivatives with respect to z_1^* and z_1 will also be equal in equations (20) and (21).

Claim 1: $\frac{\partial}{\partial z_1{}^*}\left(\frac{\beta}{1-\beta}\frac{B}{A}\right) < 0$ and $\frac{\partial}{\partial z_1}\left(\frac{\beta}{1-\beta}\frac{B}{A}\right) < 0$.

Proof:
The partial derivative of $\frac{\beta}{1-\beta}\frac{B}{A}$ with respect to z_1^* is equal to:

$$\frac{\partial}{\partial z_1^*}\left(\frac{\beta}{1-\beta}\frac{B}{A}\right) = \frac{\beta}{1-\beta}\frac{\frac{1}{\alpha}\left[\frac{dz_2^*(z_1^*)}{dz_1^*}-z_1^*\right]A - \frac{1}{6\alpha}\left[6\frac{dz_2^*(z_1^*)}{dz_1^*}z_1^*+6z_2^*(z_1^*)-3hz_1^{*2}+6hz_1^*\right]B}{A^2}.$$

From equation (11) it follows that $\frac{dz_2^*(z_1^*)}{dz_1^*} = -h(1-z_1^*) < 0$, and since $A > 0$ the first term in the fraction above is negative. Since $B > 0$, and substituting $\frac{dz_2^*(z_1^*)}{dz_1^*} = -h(1-z_1^*)$ into $\left[6\frac{dz_2^*(z_1^*)}{dz_1^*}z_1^* + 6z_2^*(z_1^*) - 3hz_1^{*2} + 6hz_1^*\right] A$ and canceling similar terms, we obtain the expression $\left[3hz_1^{*2} + 6z_2^*(z_1^*)\right] B$, which is positive. Therefore, $\frac{\partial}{\partial z_1{}^*}\left(\frac{\beta}{1-\beta}\frac{B}{A}\right) < 0$. A similar argument shows that $\frac{\partial}{\partial z_1}\left(\frac{\beta}{1-\beta}\frac{B}{A}\right) < 0$. Hence the claim is true.

Claim 2: $\frac{\partial}{\partial z_1^*}\left(\frac{z_2^*(z_1^*)h(1-z_1^*)+z_1^*}{z_2^*(z_1^*)+1/2hz_1^{*2}}\right) > 0$ and $\frac{\partial}{\partial z_1}\left(\frac{z_2(z_1)h(1-z_1)+z_1}{z_2^*(z_1)+1/2hz_1^{2}}\right) > 0$.

Proof:
The partial derivative of $\frac{\partial}{\partial z_1^*}\left(\frac{z_2^*(z_1^*)h(1-z_1^*)+z_1^*}{z_2^*(z_1^*)+1/2hz_1^{*2}}\right)$ with respect to z_1^* is equal to:

$$\frac{\partial}{\partial z_1^*}\left(\frac{z_2^*(z_1^*)h(1-z_1^*)+z_1^*}{z_2^*(z_1^*)+1/2hz_1^{*2}}\right) = \frac{\left[\frac{dz_2^*(z_1^*)}{dz_1^*}h(1-z_1^*)+1-z_2^*(z_1^*)h\right]\left[z_2^*(z_1^*)+1/2hz_1^{*2}\right]-\left[\frac{dz_2^*(z_1^*)}{dz_1^*}+hz_1^*\right]\left[z_2^*(z_1^*)h(1-z_1^*)+z_1^*\right]}{\left[z_2^*(z_1^*)+1/2hz_1^{*2}\right]^2}.$$

First note that since $h < \frac{1}{z_2^*+z_1^*}$, it follows that $z_2^* z_1^* > 2hz_2^* {z_1^*}^2$. Second, note that $z_2^* z_1^* > {z_1^*}^2$, and that $h(1-z_1^*)z_1^* > h^2 z_2^* z_1^*(1-z_1^*)$. Therefore, it follows from these three inequalities that:

$$\left[\frac{dz_2^*(z_1^*)}{dz_1^*}h(1-z_1^*)+1-z_2^*(z_1^*)h\right]\left[z_2^*(z_1^*)+1/2h{z_1^*}^2\right] > \left[\frac{dz_2^*(z_1^*)}{dz_1^*}+hz_1^*\right]\left[z_2^*(z_1^*)h(1-z_1^*)+z_1^*\right].$$

Thus, $\frac{\partial}{\partial z_1^*}\left(\frac{z_2^*(z_1^*)h(1-z_1^*)+z_1^*}{z_2^*(z_1^*)+1/2h{z_1^*}^2}\right) > 0$. A similar argument shows that $\frac{\partial}{\partial z_1}\left(\frac{z_2(z_1)h(1-z_1)+z_1}{z_2^*(z_1)+1/2h{z_1}^2}\right) > 0$. Hence the claim is true.

From the two claims above, along with expressions (22) and (23), we can conclude that when equations (20) and (21) intersect in the domain $[0,\alpha]X[0,1]$, equation (20) is always steeper than equation (21). Therefore, these equations intersect only once, and so the equilibrium is unique.

The steps required to show that the earnings functions are convex, and the equilibrium exhibits positive sorting in Home and Foreign, are the same as in the closed economy. Therefore, it remains to show that factor price equalization does not hold. The earnings of workers with ability $z_1^* = 0$ and $z_1 = 0$ are:

$$w^*(0) = p_1 z_2^* - \sigma^* = (p_1 + h)z_2^*$$

$\mathrm{w}(0) = \mathrm{p}_1 z_2 - \sigma = (p_1 + h)z_2$

Since z_2 is not equal to z_2^*, it follows that their earnings are not equal. Hence, factor price equalization does not hold. ■

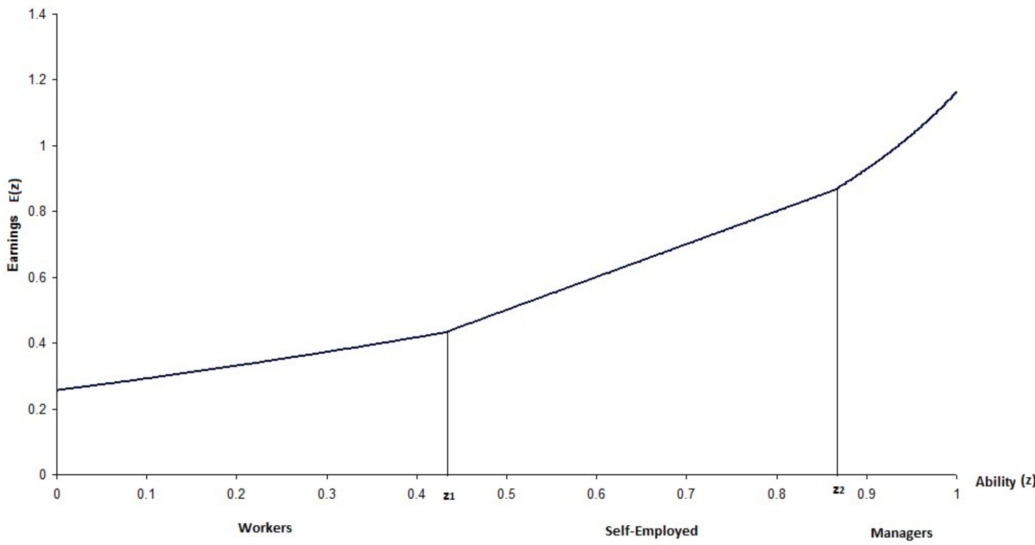

Figure 1: Population Earnings for $h = 0.4$ and $\alpha = 1$.

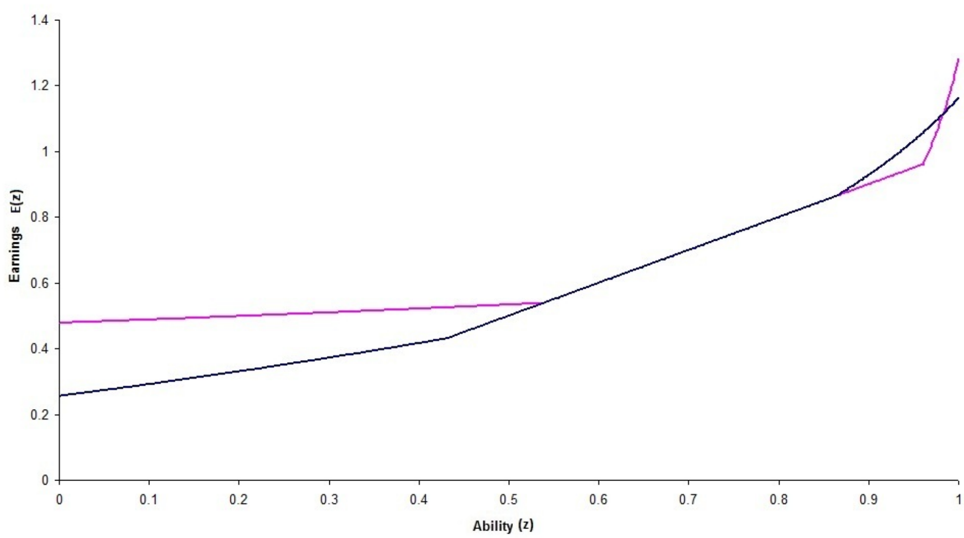

Figure 2: Population Earnings for $h = 0.4$ (blue) vs $h = 0.1$ (pink) and $\alpha = 1$.

h=0.1	z_2	Mass of workers z_1	Mass of self-emp workers z_2-z_1	Mass of managers α-z_2	p_1/p_2	Agg. output y^1	Agg output y^2
0.2	0.1902	0.1034	0.0868	0.0098	0.6315	0.1009	0.0637
0.4	0.3813	0.2088	0.1725	0.0187	0.6232	0.2041	0.1272
0.6	0.5734	0.3164	0.2569	0.0266	0.6149	0.3098	0.1905
0.8	0.7665	0.4263	0.3402	0.0335	0.6064	0.4182	0.2536
1	0.9606	0.5386	0.4221	0.0394	0.5978	0.5293	0.3164

h=0.4	z_2	Mass of workers z_1	Mass of self-emp workers z_2-z_1	Mass of managers α-z_2	p_1/p_2	Agg. output y^1	Agg output y^2
0.2	0.1705	0.0768	0.0937	0.0295	0.8139	0.0712	0.0579
0.4	0.3419	0.1575	0.1844	0.0581	0.7863	0.1464	0.1151
0.6	0.5146	0.2429	0.2717	0.0854	0.7575	0.2264	0.1715
0.8	0.6888	0.3336	0.3552	0.1112	0.7275	0.3120	0.2270
1	0.8649	0.4305	0.4343	0.1351	0.6961	0.4041	0.2813

h=0.5	z_2	Mass of workers z_1	Mass of self-emp workers z_2-z_1	Mass of managers α-z_2	p_1/p_2	Agg. output y^1	Agg output y^2
0.2	0.1661	0.0702	0.0959	0.0339	0.8810	0.0643	0.0567
0.4	0.3330	0.1444	0.1886	0.0670	0.8483	0.1327	0.1125
0.6	0.5008	0.2233	0.2775	0.0992	0.8141	0.2057	0.1674
0.8	0.6698	0.3077	0.3621	0.1302	0.7783	0.2842	0.2212
1	0.8404	0.3987	0.4417	0.1596	0.7406	0.3695	0.2737

h=0.9	z_2	Mass of workers z_1	Mass of self-emp workers z_2-z_1	Mass of managers α-z_2	p_1/p_2	Agg. output y^1	Agg output y^2
0.2	0.1551	0.0513	0.1038	0.0449	1.1755	0.0455	0.0535
0.4	0.3099	0.1057	0.2041	0.0901	1.1278	0.0940	0.1061
0.6	0.4645	0.1640	0.3005	0.1355	1.0775	0.1460	0.1574
0.8	0.6190	0.2269	0.3920	0.1810	1.0244	0.2023	0.2073
1	0.7734	0.2955	0.4779	0.2266	0.9678	0.2639	0.2554

Table 1: Effect of Changing h and α on the closed economy

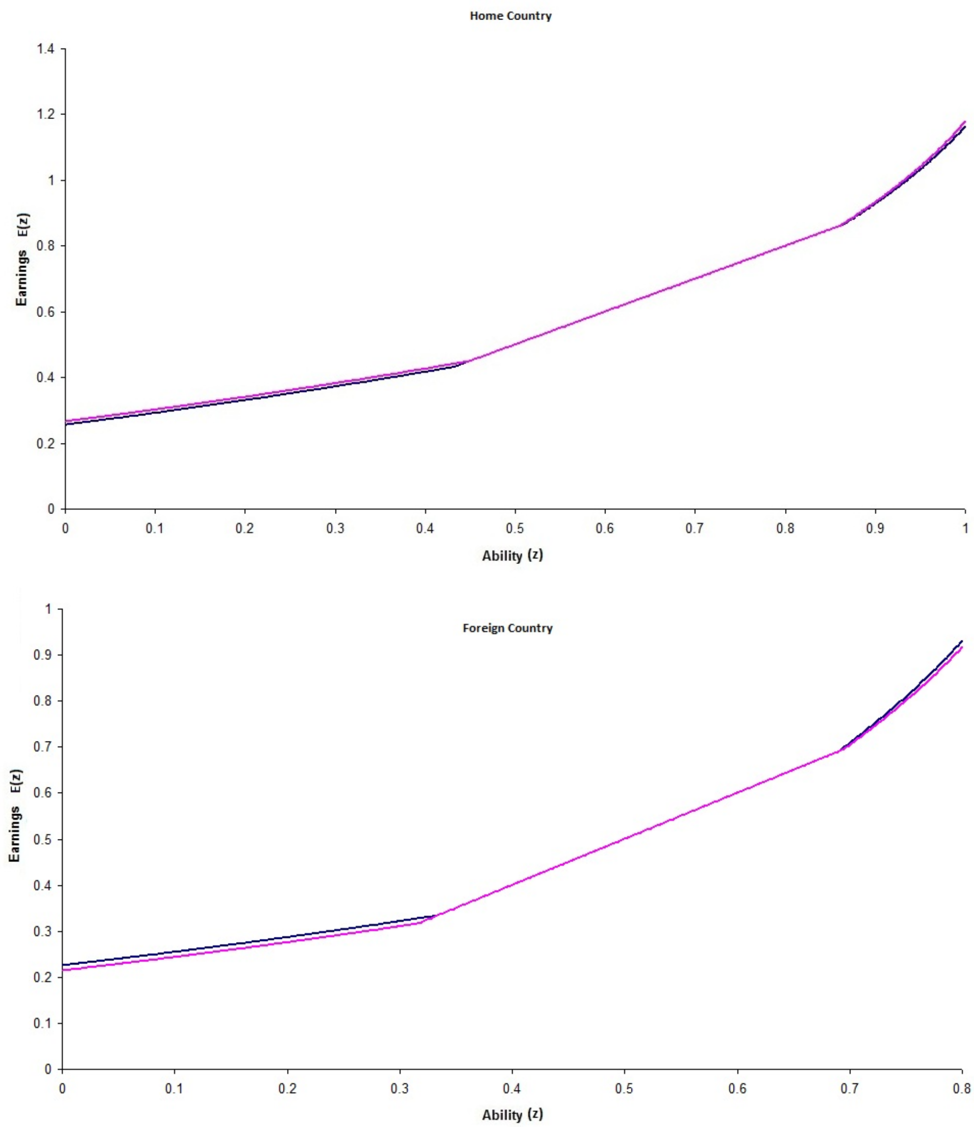

Figure 3: Closed Economy Population Earnings (blue) vs. Open Economy Population Earnings (pink) for $h = 0.4$ and $\alpha = 0.8$.

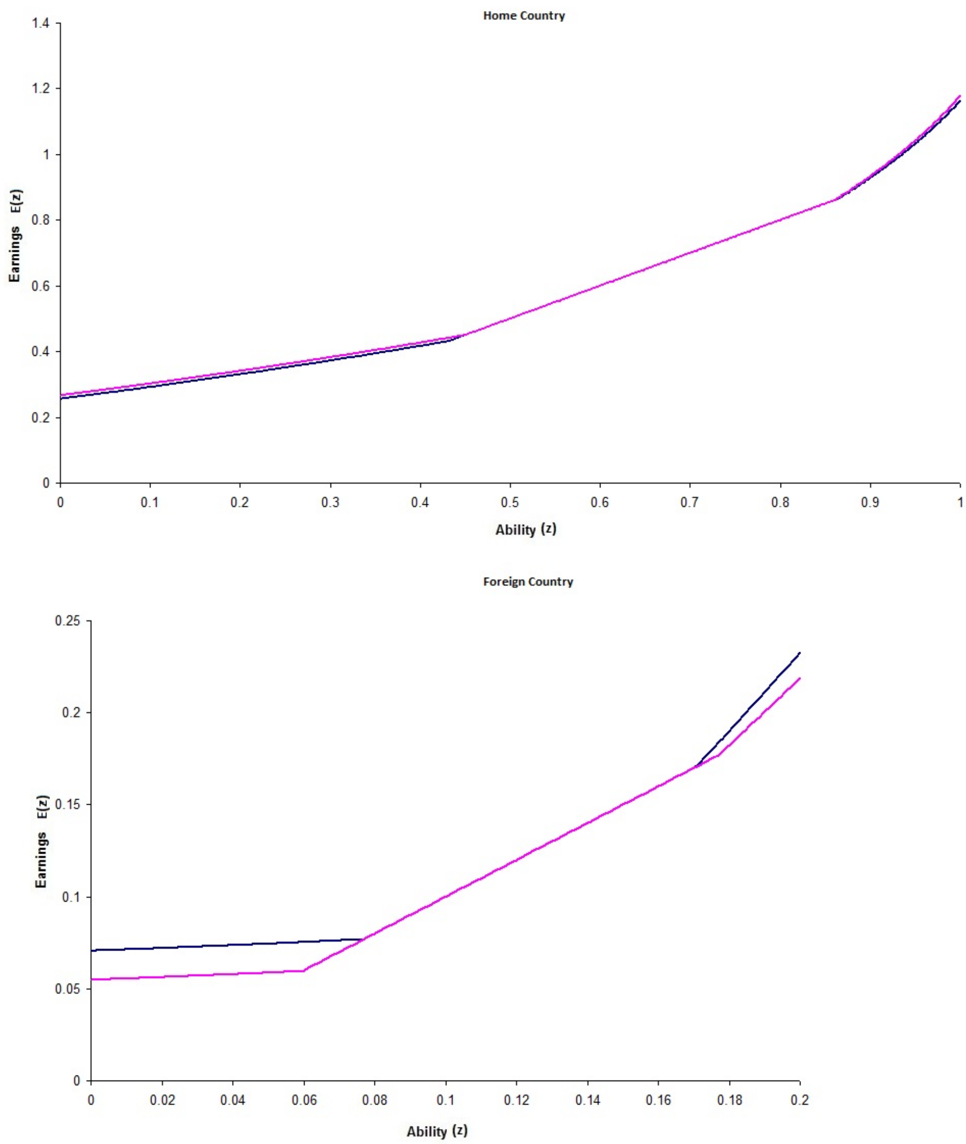

Figure 4: Closed Economy Population Earnings (blue) vs. Open Economy Population Earnings (pink) for $h = 0.4$ and $\alpha = 0.2$.

Home	Autarky		Open Economy	
h	z_1	z_2	z_1	z_2
0.1	0.538	0.960	0.542	0.980
0.5	0.398	0.840	0.422	0.836
Foreign	Autarky		Open Economy	
h	z^*_1	z^*_2	z^*_1	z^*_2
0.1	0.426	0.766	0.422	0.767
0.5	0.308	0.669	0.285	0.678

Table 2: Effect of Changing h in an open economy with $\alpha = 0.8$

	Measures of Inequality		
Foreign	Autarky		
h	Manager-Worker	Worker-Worker	Manager-Manager
0.5	0.7243	0.1213	0.2412
0.1	0.6315	0.0382	0.2526
Foreign	Open Economy		
h	Manager-Worker	Worker-Worker	Manager-Manager
0.5	0.5969	0.1248	0.0247
0.1	0.6329	0.0377	0.251
Home	Autarky		
h	Manager-Worker	Worker-Worker	Manager-Manager
0.5	0.9169	0.1985	0.3102
0.1	0.8015	0.0605	0.327
Home	Open Economy		
h	Manager-Worker	Worker-Worker	Manager-Manager
0.5	0.935	0.21	0.3121
0.1	0.5579	0.062	0.0698

Table 3: Measures of Inequality with $\alpha = 0.8$

- Manager-Worker inequality is measured as the difference in earnings of the most knowledgeable manager with the least knowledgeable worker.
- Worker-Worker inequality is measured as the difference in earnings of the most knowledgeable worker with the least knowledgeable worker.
- Manager-Manager inequality is measured as the difference in earnings of the most knowledgeable manager with the least knowledgeable manager.