# Signaling to a Network of Consumers

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**Abstract**

This paper models how the structure of the communication network among consumers can influence the ability of a firm to signal its quality through a launch strategy. A firm chooses the location, number and price of sales offers in a launch stage ('today') and a mature stage ('tomorrow'); consumers who purchase today share information on quality with their network neighbors before tomorrow. Since exploiting communication is beneficial for high quality and detrimental for low quality, the firm will restrict the breadth of its launch and locate sales strategically in network locations that maximize the spread of quality information between periods. However, this general prediction includes two distinct classes of equilibria. In the analog of money burning in a traditional signaling model, the high type firm uses a costly signal whose magnitude depends on the architecture of the network, so that strategic location of launch sales is complementary to costly signaling. In a second class of equilibria, strategic location substitutes for costly signaling: the high type pools with the low type on the consumers strategically targeted today such that communication separates the types by tomorrow. Which pertains depends on consumers' beliefs about unknown products: for a refinement of the equilibrium notion such that off-equilibrium beliefs are not 'too pessimistic', the most profitable equilibria of the second class are the only equilibria of the game.

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# 1 Introduction

When confronted with a product, person or place whose quality is unknown, it is natural that we seek information from others who have some prior experience with the object of our interest. The structure of this information-seeking, however, is not random; the information-seeker's network of personal relationships and her preferred media, for example, determine the channels along which she will be able to accumulate information. The structure of these communication networks that shape information flows are therefore of interest to firms or other entities that seek to convince a population of the quality of their unknown product. For example, when a firm introduces a new product, its choice of launch strategy can plausibly depend on what it knows about the structure of the communication network among its target population.

There are surely many reasons why this may be the case. One is awareness: when a population is initially uninformed, the best way to disseminate information will depend on how that information is retransmitted among the population. This case has been studied in, for example, Galeotti and Goyal (2007), Galeotti and Goyal (2009), A. Campbell (2010) and J.D. Campbell (2011), where in general knowledge of the communication network among the population is shown to be valuable for the informer. A second motivation is persuasion: the value an agent derives from some product or action can depend on its valuation by his network neighbors, and so for the informer convincing one consumer can have recursive effects. This setting corresponds to graphical games as analyzed in Kearns, Littman, and Singh (2001), Galeotti, Goyal, Jackson, Vega-Redondo, and Yariv (2005) and Jackson and Yariv (2008). A third motivation is inference: the firm's launch strategy in relation to observable communication channels may tell a consumer something about the quality of a product. This paper presents a theoretical model of this third motivation to ask: how does the structure of communication among consumers affect the ability of a firm to signal its quality through its launch strategy?

Examples of this motivation are found in a variety of settings, with the common feature that, broadly, opening communication channels benefits good quality and damages bad quality. Movie audiences and movie producers know that critics' opinions are visible and influential, and so critics routinely infer that a movie not screened for them is bad. For a young firm or academic department to make a rich offer to a prominent worker can serve as a signal to the worker of the firm's commitment to quality, since his prominence means that a poor firm will suffer a large reputation loss if he reports to his connections the true quality of the firm. A firm's decision to use a narrow launch for a new consumer good can signal to those offered the good during the launch that the product is good, since a good product will benefit from communication between early adopters and later customers, while a bad product will suffer.

The model below captures the key features of such settings, adopting the language of the example of a firm launching a consumer product. A firm has a product of exogenous quality that is either high or low; consumers value a high quality product, but would never buy a product they knew to be of low quality and cannot verify quality before purchase. The firm chooses a launch strategy that defines in each of two periods, 'today' and 'tomorrow' (corresponding to a launch stage and a single subsequent stage), a subset of consumers to whom the product will be made available and at what price. Consumers observe the firm's launch strategy and choose whether or not to buy the product, given their belief on quality, which can come either from Bayesian updating over the firm's strategy or, tomorrow, direct observation of the experience of a network neighbor who purchased today. The model abstracts from the informing and persuading roles of communication networks; consumers knows that the product exists, and the values of 'good' or 'bad' versions of it are commonly held and fixed. This means that the model will isolate the signaling role and how it depends on the architecture of the communication network.

In this setting there are two places that separation of high and low quality products can take place: before the first round of sales, or between the two sales rounds. Together with the possibility that the two types are never separated, this admits three classes of equilibria. Of these, the class that features initial pooling followed by separation driven by consumer communication is novel to this setting. As in signaling models in general, predictions about which equilibrium will pertain depend crucially on what beliefs consumers form when observing a given strategy by the firm. Under application of the Intuitive Criterion (Cho and Kreps (1987)), equilibria from all three classes are in general possible, and the nature of surviving equilibria depends on the structure of the communication network (Theorem 1).

First consider the class with separation before the first sales round, below called *S1 equilibria*. A single equilibrium of this type survives the Intuitive Criterion. In this equilibrium, a firm with a high quality product launches the product by offering it today at zero price to a set of individuals who form a *minimum dominating set* of the graph of the network, and tomorrow offers the product to all remaining consumers at a price that reflects its high quality. A minimum dominating set of the graph is such that all consumes are either in the set or are neighbors to at least one consumer in the set. Since (i.) price today is zero in this equilibrium and (ii.) all remaining consumers will learn the true quality of the product through communication by tomorrow, those targeted today know that a firm with a low quality product cannot make positive profit with such a strategy, and so are convinced that the firm employing this strategy has a high quality product. This is because if a firm with a low quality product were to give its product away today to a minimum dominating set, all remaining consumers would learn that the product was bad before tomorrow and would shun the product at any positive price. This equilibrium is therefore the precise analog of money burning in a traditional quality signaling game. Here the channel available to burn money is the launch price and location, but the intuition is common: the high quality firm makes a choice whose cost could not be recouped by a low type that imitated it, and so under the Intuitive Criterion consumers must hold the belief that this strategy was played by firm of high type.

The implication of network structure in this S1 equilibrium is that the amount of money burning required to signal a high quality product depends on the structure of communication. The smaller is the cardinality of the minimum dominating set of the network graph (that is, the smaller the graph's *domination number*), the fewer the number of consumers who must be persuaded to adopt today in order that all consumers will learn the product's quality by tomorrow. The communication channels that benefit the high quality product are more cheaply accessed when the domination number of the graph is small, and so less costly signaling is required to convince today's targeted consumers to buy. Consider the example of a firm hiring a prominent worker: the zero price today corresponds to a high salary offer to the employee, and the minimum dominating set corresponds to the number and location in the professional network of those workers whose hiring would spread information about the firm's quality to the rest of the profession. Note also that 'price' being the mechanism for money burning need not be literal. For example, the classic Coleman, Katz, and Menzel (1966) study of direct marketing by pharmaceutical firms of new drugs to physicians finds that 'influential' physicians in the network receive disproportionate time and attention from sales representatives. Since this time and attention is costly, this strategy is consistent with that of the equilibrium here.

In the second class of equilibria, below called *S2 equilibria*, separation of types takes place after the first sales round but before the second. These equilibria are novel to this setting. In these equilibria, firms with high quality products and firms with low quality products coexist in the first sales round at a pooled price that is sufficiently low to induce those consumers targeted in the launch to buy, despite the fact that in contrast to the S1 equilibrium those consumers targeted today cannot infer with certainty that the product they are being offered is of high quality. Indeed, in S2 equilibrium separation operates solely due to communication among those targeted today and the wider market tomorrow, so that there is no money burning in the traditional sense. Similarly to the S1 case, however, network neighbors of these consumers targeted today earn through communication before tomorrow the true quality of the product, and so low quality products are driven out and the high quality producer is again able to set a price reflective of high quality for those neighboring consumers tomorrow.

This class of equilibria can in general be better for the producer of a high quality product than the S1 equilibrium that separates types today. In particular, the most profitable S2 equilibrium (the 'efficient' S2 equilibrium) for the high type firm also sees the firm target a minimum dominating set of the network graph today, but at a price consistent with the probability that an unknown product is good. In the context of the hiring example, this corresponds to the prominent employee receiving a less generous wage offer than in S1, not sufficient to convince her that the firm *must* be good, but sufficient to convince her that the firm *could* be good. If her beliefs when receiving such an offer are not 'too pessimistic', she will be willing to accept this offer. However, the fact that a low quality firm makes positive profits today in S2 equilibrium means that the Intuitive Criterion does not restrict her beliefs in this way; she is free to hold the belief that the firm making this less generous offer is bad with high probability. Thus although the high quality firm does better in S2 than S1 it is not the case that the high quality firm can be assured that such an offer will be interpreted favorably, and whether it is interpreted so will depend on what the consumer 'expected' and how they form beliefs following an unexpected offer. This formation of beliefs is of more than theoretical relevance, since the manner in which consumers form beliefs is not necessarily straightforward; for example, experimental evidence in Dawar and Sarvary (1997) suggests that consumers associate low price with low quality but also with greater intention to purchase. This increases the importance of interpreting the 'price signal' in the model as accommodating a richer array of signaling mechanisms.

The most profitable equilibrium of all for the high type firm is the efficient S2 equilibrium, but because it calls on those targeted today to buy without them being able to infer that the product is certainly good, the Intuitive Criterion is not a sufficient restriction on beliefs so that the high type firm can adopt this strategy and be assured of its success. A restriction that is sufficient is the *passive conjectures* refinement (Rubinstein (1985), Rasmusen (2001)), which requires that when consumers see an out-of-equilibrium action by the firm, they retain the population distribution of types as their belief of the firm's type, and so in a sense rules out beliefs that are 'unreasonably' pessimistic. If consumers form beliefs in such a way, the strategy in the efficient S2 equilibrium will result in acceptance by those targeted today, and since this equilibrium is that most preferred by the high type firm, the high type firm will adopt such a strategy. The efficient S2 equilibrium is thus the unique equilibrium of the game under the passive conjectures refinement (Theorem 2).

A pervasive implication of the analysis is that the location of a targeted launch in relation to observable communication channels can variously be either complementary to signaling of a product's quality or a substitute for costly signaling. By visibly making offers today to a set of consumers strategically chosen for their ability to generate network-covering communication about quality, the firm signals to these consumers that it is willing to restrict activity today, with both parties knowing that if the quality of the product is bad, such a strategy results in no business tomorrow. This in combination with a money burning action (whose magnitude depends on the number of seed locations needed to generate network-covering communication and thus on the structure of the network) serves as a signal of quality before any sales take place. However, the same strategic location decision can be associated with a temporary pooling of types on this set of consumers followed by separation solely through communication. The model suggests that which of these will pertain depends on how pessimistic consumers are when seeing the second strategy: the more skeptical are consumers, the more valuable is the complementary money burning action. This intuition is robust to a more general process by which restricting activity fosters separating word-of-mouth, but the ability to strategically target in a well-defined communication network enhances the effect by admitting the most parsimonious possible version of such a strategy.

The role of communication in general in driving inference over quality has received attention in several previous contributions. Literature on word-of-mouth effects in marketing a product generally confirm the existence and tangible impact of post-purchase communication among consumers (see, for example, Godes and Mayzlin (2004) and Godes and Mayzlin (2009)). The model's predictions are consistent with the argument in Godes and Mayzlin (2004) that “more dispersed buzz may be better than concentrated buzz”. Dispersion of launch locations across the network here is a stronger signal of high quality than a concentrated launch, since dispersion is associated with a greater volume of the irredundant word-of-mouth that benefits good products over bad. Bagwell and Riordan (1991) models a setting in which some consumers are initially informed about quality and some are initially uninformed. This, with a production cost differential between high and low quality products, admits that an initially high price can be a signal of product quality. Kennedy (1994) models the quality signaling problem for a firm that can set price, absent such a cost differential that drives the familiar single-crossing property (following Spence (1973)), and demonstrates that a low and rising price can be a signal of quality due to the word-of-mouth that will benefit a good product and hurt a bad one. Navarro (2006) models a consumer network in which quality information is transmitted across links; a firm chooses quality and price, and consumers simultaneously decide whether or not to buy, where willingness-to-pay is in equilibrium consistent with the information they will receive as a function of other players’ decisions. Godes and Mayzlin (2009) considers a firm’s decision to announce a reference program that commits it to facilitating information flows from early adopters to potential late adopters. Most relevant to the current model is the case in which consumers are uncertain about product quality, in which case committing to a reference program can function as a signal of quality.

The framework of the present model incorporates a network graph as, first, a model for the transmission of information about quality during a signaling game, and, second, as the strategy space of the firm’s launch location decision. A natural constraint on the literal interpretation of the model is that a network graph can in practice be highly complex. A rich literature in computer science has explored problems of choosing nodes in a graph to maximize the recursive spread of influence through the network (for example Domingos and Richardson (2001), Richardson and Domingos (2002), Kempe, Kleinberg, and Éva Tardos (2003)). These graph coverage problems are known to be NP-hard (Garey and Johnson (1979)). The literal application of the model presented below is therefore restricted to settings in which a relevant network is ‘small enough’ to be understandable to the decision-maker. This could mean a small number of individuals, or, if we interpret a ‘consumer’ more liberally (for example as a cluster of consumers, a geographic area or a firm), a small number of relevant decision units on the consumer side. More generally, the problem of translating the model to settings with less well-understandable networks is discussed below following the analysis.

The paper proceeds as follows. Section 2 presents the model. Section 3 identifies the equilibria of the game, and its Theorem 1 identifies those that survive the application of the Intuitive Criterion. Section 4 and its Theorem 2 identify equilibria surviving the application of the passive conjectures refinement. Sections 5 and 6 discuss relaxing some of the model’s assumptions on what consumers and firms know, discuss extensions of the framework, and conclude.

# 2 Model

Consider a dynamic game with incomplete information. A firm will encounter a set of consumers arranged in a social network, and has two periods in which to make sales offers to consumers. Each time the firm can choose a price and a subset of consumers to whom the price will be offered, and the consumers who receive a price offer can choose to accept or reject it. Between the two rounds, communication will reveal details of round 1 transactions to neighbors of those consumers who transacted. The remainder of this section will formalize this game.

### 2.0.1 Players

There is a set $C = \{1, ..., n\}$ of risk-neutral consumers. Each consumer is a node in an undirected and connected graph $(C, g)$[1], where $g$ is a real-valued $n \times n$ matrix in which $g_{ij}$ represents the relationship between consumers $i$ and $j$: $g_{ij} = 1$ if there is an edge joining $i$ and $j$ and 0 otherwise. $g_{ij} = g_{ji}$ since the graph is undirected; let $g_{ii} = 0$. Consumer $j$ is considered a "neighbor" to consumer $i$ if $g_{ij} = 1$. Since $C$ is assumed fixed, for convenience denote the graph by $g$. Let $\Gamma \subseteq C$ be some set of consumers, and let $\gamma = |\Gamma|$ be its cardinality.

The open neighborhood of consumer $i$ is $N(i) = \{j : g_{ij} = 1\}$. The open neighborhood of $\Gamma$ is $N(\Gamma) = \cup_{i \in \Gamma} N(i)$. Let $\Omega(\Gamma) = N(\Gamma) - \Gamma$; this is the set of consumers in the neighborhood of $\Gamma$ who are not themselves in $\Gamma$. For notational consistency denote $\Omega(i) \equiv N(i)$. In turn let $\omega(i) = |\Omega(i)|$ denote the number of neighbors to consumer $i$ (equivalently $i$'s degree) and $\omega(\Gamma) = |\Omega(\Gamma)|$.

A dominating set of the network graph is some $\Gamma$ such that $\Gamma \cup \Omega(\Gamma) = C$. Denote by $\Gamma^*$ a **minimum dominating set**, which is a dominating set with the lowest possible cardinality. The **domination number**, denoted $\gamma^* = |\Gamma^*|$, is the cardinality of a minimum dominating set; it is the smallest number of consumers such that marking $\gamma^*$ would result in every single consumer either being marked or having a marked neighbor. While $\gamma^*$ is unique for a given network, $\Gamma^*$ is not: there can in general be more than one minimum dominating set in a given graph. Note that since $g$ is connected, the domination number cannot exceed $\frac{1}{2}n$ (established in Ore (1962)), although it will in general be smaller. For discussion of the concepts of dominating sets, minimum dominating sets and domination numbers, see, for example, Haynes, Hedetniemi, and Slater (1998).

The example in Figure 1 illustrates these concepts, and will be valuable throughout the analysis below to illustrate equilibria.

This 'double star' network has 8 consumers in total, a domination number of 2, and an associated minimum dominating set consisting of the shaded central consumers: each consumer is either shaded or a direct neighbor of a shaded consumer. This configuration is an example of a network with a unique minimum dominating set.

There is one firm which produces a good of quality $q \in \{q_L, q_H\}$, where $q_H$ is realized with

[1] That the graph is connected is without loss of generality, since an unconnected graph would in this model be qualitatively identical to each of its connected components.

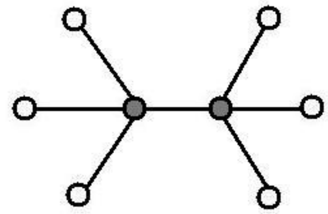

Figure 1: Double star network

probability $\alpha$. Call the expected quality $\bar{q} \equiv \alpha q_H + (1-\alpha)q_L$, where $q_L < 0 < \bar{q} < q_H$. Production costs are zero for both types of firm. This could be considered a normalization of production costs, or more literally that the goods to be sold have already been produced and the problem for the firm in this model is simply to sell them. This guarantees that either type of firm is always willing to sell at any positive price if possible and that a traditional signaling argument based on cost differential is ruled out to isolate signaling via launch strategy. Denote the type of a firm with a high quality product $H$ and the type of a firm with a low quality product $L$.

Consumers have unit demand for the good, so they seek to buy at most one unit during the game. This could be interpreted, for example, as the good being a durable, or as a good with no repeat consumption value, like a movie. This assumption could be relaxed at little cost, but maintaining it precludes repeat purchase by consumers as a justification for the firm restricting early sales.

### 2.0.2 Information

Consumers are familiar with the distribution of quality but cannot directly observe the quality of good provided by the firm. It is not possible for the firm and consumer to write contingent contracts, such as warranties, legal recourse or money back guarantees. Each consumer follows a simple purchasing rule: they will always be willing and able to purchase the good when the price does not exceed expected quality.

The price and set chosen by the firm in each round is fully observable to consumers in that round. This means that the firm's marketing strategy is public knowledge. The value of this assumption is that it allows the model to accommodate very general forms of costly signaling alongside strategies that rely solely on word-of-mouth. Section 5 demonstrates that relaxing this assumption does not affect the key qualitative results of the model.

The firm is fully informed of the whole game at all stages, including the quality of its own good and the structure of the social network. The firm's knowledge of the whole network best approximates settings in which information on the network is cheap to acquire, either because the relevant population is small or because technology to analyze the network is effective. The range of settings covered by this assumption is likely to increase as technology develops and the sophistication of online interactions continues to grow[2]. In incorporating this assumption, the model captures strategic targeting as a signaling mechanism, while at the same time implying the (intuitively identical) channel by which a narrow but untargeted launch could achieve the same results in the case when precise targeting is impossible. Again, Section 5 considers the implications of relaxing this assumption.

#### 2.0.3 Moves

The firm and consumers play a two-round market game. Let $t = 1, 2$ index the rounds. The structure of play is as follows:

**0** Nature chooses $q \in \{q_L, q_H\}$.

**1 a** The firm chooses price $p_1$ and the set $\Gamma_1$ of consumers to whom the price will be offered.

- **b** Sales round 1: consumers observe $p_1$ and $\Gamma_1$. If consumer $i$ was offered a price, she chooses to buy $(b = 1)$ or not buy $(b = 0)$.
- **c** If consumer $i$ accepts the sale, consumer $i$ and each neighbor of consumer $i$ observe the quality of the good.

**2 a** The firm chooses price $p_2$ and the set $\Gamma_2$ of consumers to whom the price will be offered.

- **b** Sales round 2: consumers observe $p_2$ and $\Gamma_2$. If consumer $i$ was offered a price, she chooses to buy $(b = 1)$ or not buy $(b = 0)$.

The strategic variables for the firm are therefore the two price and visitation policies, making a strategy for the firm as follows: $\sigma_F = \{p_1, p_2, \Gamma_1, \Gamma_2\}$, where $p_t$ specifies the price at time $t$ and $\Gamma_t$ the set of consumers to whom the product is offered at time $t$. The stage 2 strategy for the firm can depend on history (the firm's own stage 1 offer and the profile of consumer responses).

The firm can choose a policy that can vary price according to round and can specify the set of consumers to which each round's price will be offered. Stage 1c is the "communication" stage, in which any purchases made in stage 1 are observed by the neighbors of the consumer who made the purchase. Note that $\Gamma_2$ is not restricted to be disjoint with $\Gamma_1$.

A strategy for consumer $i$ who is offered a price in stage 1 is $\sigma_{1,i} = \{b|p_1, \Gamma_1\}$. A strategy for consumer $i$ who is offered a price in stage 2 also incorporates information set $I$ gained in stage 1: $\sigma_{2,i} = \{b|p_1, \Gamma_1, p_2, \Gamma_2, I\}$. In round 2, denote by $\mathcal{I}$ the set of 'informed' consumers: those who have a neighbor who purchased in round 1 and so know for sure in round 2 the quality of the product. Call the belief held by household $i$ on the firm's quality at some time $\hat{q}$.

[2]The technical burden borne by this assumption also differs in how we interpret 'consumers' in the model. For example, we could view 'consumers' as individual discussion websites, and the links between them as hyperlinks or cross-posts.

#### 2.0.4 Payoffs

The firm discounts round 2 payoffs with the discount factor $\delta$, where $\delta > \frac{\bar{q}}{q_H}$ so that all else equal the firm prefers a sale at price $q_H$ in round 2 to a sale at $\bar{q}$ in round 1. Consumers may discount at any rate; this does not materially change the game, since the simplicity of the consumer side means that the consumers' discounting serves only to define the value of the consumers' maximal willingness to pay in round 2, which is throughout taken as given.

The ex ante payoff to a consumer $i$ who buys at any point during the game at a price $p$ is as follows:

$$E(U_i) = \hat{q} - p \tag{1}$$

$\hat{q}$ is the belief held by the consumer at the time of purchase, which may have been updated from their prior held at the start of the game. The payoff to a firm of type $j$ is as follows, where the $y_t$ is the number of buyers from the firm in round $t$ and $p_t$ the price set by the firm in round $t$:

$$\pi_j = y_1 p_1 + \delta y_2 p_2 \tag{2}$$

Note that the game differs slightly from a graphical game or a network game, as defined in Jackson and Yariv (2008); here, a consumer's location in the network determines what information she sees, but neighbors' actions do not *directly* affect the payoff earned by a consumer.

### 2.1 Consumer communication

Communication is modeled as a stage between the two rounds in which neighbors of consumers who accept an offer in sales round 1 observe truly and certainly the quality of the good. Recall that quality is not a choice variable for the firm; this reveals with certainty the quality of a good that the neighbor will receive if they should accept an offer in sales round 2. Communication is not a strategic choice by the consumers, and can travel no further than one degree, to direct neighbors.

Network links are very general here. A link from consumer $i$ to another consumer could represent anything from a relationship with an acquaintance to readership of media. It may be reasonable to think of a very well connected "consumer" as a reviewer, critic or other journalist: their experience of the unknown product diffuses information to many more consumers than another individual simply talking to his friends.

The structure of communication is entirely exogenous and static. This rules out any strategic action by the firm that is designed to manipulate the information a consumer sees. In particular, real-world marketing choices that purport to recommend to a consumer items that other consumers like them have bought (especially popular among internet retailers like Amazon and Netflix) are ruled out here, although it would be possible to incorporate this into a similar model as the firm strategically choosing to create links between consumers in the network.

With communication potentially substituting for costly signaling, the single-crossing property of different types having different signaling costs, familiar since Spence (1973), can be satisfied somewhat differently in the game with communication than it is in the game without. Communication itself implies satisfaction of the single-crossing property: when consumers talk, for firms with low quality products the revelation of quality by communication is a cost, but for firms with high quality goods it is a benefit.

# 3 Equilibria

The object of interest will be perfect Bayesian equilibria in the game. The components of an equilibrium are strategies for each type of firm $\sigma_{F,j} = \{p_1, p_2, \Gamma_1, \Gamma_2\}$, $j = H, L$, strategies for each consumer $\sigma_{1,i} = \{b|p_1, \Gamma_1\}$, $\sigma_{2,i} = \{b|p_1, \Gamma_1, p_2, \Gamma_2, I\}$ and supporting beliefs.

Define the Intuitive Criterion in the context of this game:

**Definition 1.** *By the* ***Intuitive Criterion*** *(Cho and Kreps (1987)), if there exists a type of firm who could not benefit from an off-equilibrium action no matter what beliefs are held by the consumers, then the consumers' beliefs must put zero probability on that type of firm being the one which takes the off-equilibrium action, when that action is observed.*

This section will describe the equilibria in this game by class, and then apply the Intuitive Criterion. There are three broad classes:

| Round 1 | Round 2 |
|---|---|
| Separated | Separated |
| Pooled | Pooled |
| Pooled | Separated |

First, the case in which the two types are separated before the first sales round; second, the case in which the two types pool and both types make sales in both periods; third, the case in which the types pool in the first round and are separated by word-of-mouth between rounds. Of these, the third class is novel, being that class where the firm completely forgoes price signaling in favor of signaling using a visitation strategy that exploits network communication. By contrast, the other two classes of equilibria are similar in spirit to pooling and separating equilibria in a traditional single-principal, single-agent signaling game.

Where an equilibrium can be supported by more than one strategy - for example if the low type firm is indifferent between making offers which will not be accepted in equilibrium and making no offers - the description of the equilibrium is restricted to just one supporting strategy for convenience. Generally, the salient features of each equilibrium class can be supported by either pooling *or* separating strategies by the firms. Superscripts on strategic variables refer to the class of equilibria, so that, for example, $p_1^1$ refers to round 1 price ($p_1$) in the first class.

## 3.1 Equilibria surviving application of the Intuitive Criterion

### 3.1.1 Separation before round 1

There exist equilibria in this game analogous to separating equilibria in one-stage signaling games: the high type firm takes an action to distinguish itself from the low type firm before any consumers have purchased. Here this involves setting round 1 price $p_1$ low enough so as to preclude profitable imitation by the low type. This action is here analogous to money burning, for example by launching a costly uninformative advertising campaign, as a signal in a one-round signaling game. In these equilibria, the high type firm makes sales in both rounds and the low type makes no sales in either round.

**Definition 2.** *An equilibrium with **separation before round 1** (S1) features price signaling and an appropriate choice of targeting by the high type before round 1, so that no sale takes place in either round to a consumer who does not know the true type of the firm.*

An example of an S1 equilibrium (where the superscript 1 denotes this class of equilibria) is:

$$\begin{aligned}
\sigma_{F,L} &: \{p_1^1, p_2^1, \emptyset, \emptyset\} \\
\sigma_{F,H} &: \{\hat{p}, q_H, \Gamma_1^1, C - \Gamma_1^1\} \\
\sigma_{1,i} &: \{(b = 1|p_1(i) \leq \hat{p} \wedge \Gamma_1 = \Gamma_1^1), (b = 0|p_1(i) > \hat{p} \vee \Gamma_1 \neq \Gamma_1^1)\} \\
\sigma_{2,i} &: \{(b = 1)\}
\end{aligned}$$

The payoff to each type of firm is as follows:

$$\pi_{L,S1} = 0 \tag{3}$$

$$\pi_{H,S1} = \gamma_1 \hat{p} + \delta(N - \gamma_1) q_H \tag{4}$$

With supporting beliefs:

$$(Pr(low) = 1|p_1(i) \geq \hat{p} \vee \Gamma_1 \neq \Gamma_1^1) \tag{5}$$

$$(\hat{q}_2 = q|i \in \mathcal{I}), (\hat{q}_2 = q_H|i \notin \mathcal{I}) \tag{6}$$

Note the small point that in Equation 6, $\hat{q}_2 = q$ (where $q$ is true quality) since those consumers in $\mathcal{I}$ learned true quality since they have a neighbor who bought today. In this equilibrium high types offer a discount to the consumers in $\Gamma_1^1$ sufficient to both preclude profitable imitation by low types in stage 1 itself and to inform all consumers of their true type before round 2, regardless of whether the consumer actually observed true quality during the communication stage.

**Lemma 1.** *At most one high type payoff value is realized in S1 equilibria that survive the application of the Intuitive Criterion.*

The proof of this result, along with those of all others to follow, appears in the appendix. By separating types immediately, the strategy employed by the high type firm in the S1 equilibrium effectively collapses the two-stage market game to be equivalent to the traditional one-period signaling game. Lemma 1 is therefore analogous to the result in Cho and Kreps (1987) that in the two-type Spence signaling model, the only equilibrium surviving the Intuitive Criterion is the "Riley outcome", the separating equilibrium with the least amount of inefficient signaling. Here, in the class of equilibria in which separation occurs before the first sales round, only those with the least amount of inefficient signaling survive the Intuitive Criterion

What is the exact nature of the surviving S1 equilibria?

**Lemma 2.** *All S1 equilibria surviving the Intuitive Criterion feature the same high type strategy and payoff as the following:*

$$\begin{aligned}\sigma_{F,L} &: \{p_1^1, p_2^1, \emptyset, \emptyset\}\\ \sigma_{F,H} &: \{0, q_H, \Gamma^*, N-\Gamma^*\}\\ \sigma_{1,i} &: \{(b=1|p_1(i)\le 0), (b=0|p_1(i)>0)\}\\ \sigma_{2,i} &: \{(b=1|p_2(i)\le q_H), (b=0|p_2(i)>q_H)\}\end{aligned}$$

*All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:*

$$\pi_{L,S1} = 0 \quad (7)$$

$$\pi_{H,S1} = \delta(N-\gamma^*)q_H \quad (8)$$

*With appropriate supporting beliefs.*

Again, the surviving S1 equilibria take on exactly the same form as the familiar Intuitive Criterion-compatible separating equilibria in standard signaling games: the high type firm undertakes the least-cost action to distinguish itself from the low type. The least-cost action is to offer a price of 0 to consumers in $\Gamma^*$ in round 1 and to exploit the revelation of type by offering a price of $q_H$ to all other consumers. Note also that the strategy employed by the high type firm to realize S1 equilibria is unique in the case of networks with a unique minimum dominating set. As an example, recall the earlier double star network:

The surviving S1 equilibria here feature the firm giving away its product ($p_1 = 0$) to the shaded consumers; all neighbors learn true quality before round 2 and, if the quality is $q_H$, will be willing to buy at that price in round 2. This means that the round 1 strategy signals to all consumers that the product is good, but only those who receive round 1 offers are required to interpret this strategy as such; the nature of the signaling strategy means that in S1 equilibrium communication informs the consumers who didn't receive a round 1 offer that the product is good. These consumers *can* infer from the firm's round 1 strategy that the product was good, but this inference is never tested

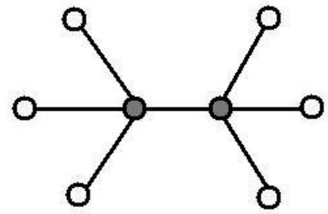

Figure 2: Double star network

directly.

To see this concretely, consider again the example of a firm hiring prominent workers, and say that the industry has two superstar individuals so that all others in the industry know at least one. The S1 equilibrium can be interpreted as follows: the firm makes a wage offer to those individuals that is sufficiently high that it would not be profitable in isolation. The individuals sees that the offers they have received are not profitable in isolation, and know that if they learn later that the firm is bad and leave the firm, all others in the industry will observe their experience and the firm's reputation will be destroyed across the whole industry. From these observations the two targeted individuals can infer that the firm making the offer must be of high quality, and are therefore willing to accept the offers. All other individuals in the industry hear that the individuals have been headhunted in this way, and can make the same inference; this inference is confirmed as time goes on and the other individuals hear and observe the superstars' experience. The firm can then offer the market wage for a high quality firm to those others, and since they now know the firm to be good they are willing to accept such an offer.

Why is the surviving S1 strategy superior for the high type firm than, say, offering a negative price to, say, just *one* of the shaded consumers sufficient to reveal type to all others—in the example, why not make a still *more* generous offer to just one of the superstars? In Figure 2.0.1 the latter strategy would require a discount sufficient to convince three of the outside consumers (who would not learn quality through communication) that the firm was of high type. Generating such inference requires a more costly price discount than it pays to the high type. In the example, some of the other individuals in the industry would have to infer the firm's quality from the offer to a superstar whose experience they would not be able to observe or hear directly. Even though these equilibria are analogous to money burning in a one-round signaling game, they operate somewhat differently here since they can exploit communication. Pricing at cost during the launch stage also demonstrates that the incentive to signal and to generate communication can be a motivation for penetration pricing that is different to the motivation in, for example, Katz and Shapiro (1986) of penetration pricing to encourage adoption of a product with network externalities in consumption.

That targeting the minimum dominating set that most efficiently generates network-spanning communication dominates a more general money burning exercise demonstrates that the architecture of the network affects the cost of costly signaling. All else equal, a network whose structure admits a minimum dominating set with smaller cardinality is associated with cheaper signaling. In the model this is reflected in a smaller number of zero-price offers in the first round, but in translating the model to real settings it need not be literally true that signaling takes place through price. For example, in a setting in which advertising expense or time-consuming sales effort (as in the Coleman, Katz, and Menzel (1966) pharmaceutical marketing study) are the money burning mechanism, it remains true that the magnitude of money burning required to signal type decreases as the breadth of launch required to generate network-spanning communication decreases.

Thus in an equilibrium with costly signaling, communication is complementary to money burning; however, since the relationship between the number of edges in a graph and the graph's domination number is not monotonic, the extent to which an expansion in the proliferation of communication channels, for example, can reduce the amount of money burning required to signal type depends on the precise architecture of the network.

As a corollary, Lemma 2 guarantees a lower bound on the payoff of the high type under the Intuitive Criterion, which in turn guarantees that no equilibrium in which no sales are made survives the Intuitive Criterion.

**Lemma 3.** *No equilibrium in which the high type receives a payoff lower than* $\pi_H = \delta(N - \gamma^*)q_H$ *survives the application of the Intuitive Criterion.*

The S1 strategy is a profitable deviation for the high type firm from any hypothetical equilibrium that gives a lower payoff; under any consumer beliefs satisfying the Intuitive Criterion such a deviation will always be accepted by consumers.

### 3.1.2 No separation

Next, consider the cases in which both types of firm make sales in both rounds, so that there is no separation of types except by word-of-mouth. This can only be equilibrium play either if a 'pooling' price - one which does not exceed average quality - is set by both types in both rounds, or if there are no sales in the second round. In either case this means that in these equilibria the high type does not attempt to exploit its distinction from a low type, even though neighbors of consumers who buy from the high type in round 1 learn of its type before round 2.

**Definition 3.** *An equilibrium with* ***no separation*** *(NS) has* $p_1 \leq \bar{q}, p_2 \leq \bar{q}$.

An example of an equilibrium of this type (where the superscript 2 denotes this class of equilibria) is:

$$
\begin{aligned}
\sigma_{F,L} &: \{p_1^2 \leq \bar{q}, p_2^2 \leq \bar{q}, \Gamma_1^2, \Gamma_2^2\} \\
\sigma_{F,H} &: \{p_1^2 \leq \bar{q}, p_2^2 \leq \bar{q}, \Gamma_1^2, \Gamma_2^2\} \\
\sigma_{1,i} &: \{(b=1|p_1(i)=p_1^2 \wedge \Gamma_1 = \Gamma_1^2), (b=0|p_1(i) \neq p_1^2 \vee \Gamma_1 \neq \Gamma_1^2)\} \\
\sigma_{2,i\in\Omega_{\Gamma_1}} &: \{(b=1|q=q_H), (b=0|q=q_L)\} \\
\sigma_{2,i\notin\Omega_{\Gamma_1}} &: \{(b=1|p_2(i) \leq \hat{q}), (b=0|p_2(i) > \hat{q})\}
\end{aligned}
$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_{L,NS} = \gamma_1^2 p_1^2 + \delta(\gamma_2^2 - \omega(\Gamma_1^2))p_2^2 \quad (9)$$

$$\pi_{H,NS} = \gamma_1^2 p_1^2 + \delta\gamma_2^2 p_2^2 \quad (10)$$

With supporting beliefs:

$$(Pr(low) = 1|p_1(i) \neq p_1^2 \vee \Gamma_1 \neq \Gamma_1^2) \quad (11)$$

$$(\hat{q}_2 = q|i \in \mathcal{I}), (\hat{q}_2 = \bar{q}|i \notin \mathcal{I}) \quad (12)$$

We can quickly rule out cases in which no offers are made in the second round. NS equilibria in which $\Gamma_2^2 = \emptyset$ do not survive the application of the Intuitive Criterion:

**Lemma 4.** *No equilibrium in which no offers are made in the second sales round survives the application of the Intuitive Criterion.*

In the surviving NS equilibria, both types of firm offer some price to a given group in round 2, and some price to a given nonempty group in round 2. The high type firm makes no attempt to exploit the revelation of information about its type.

Equilibria of this type may well survive the Intuitive Criterion, provided that the high type firm prefers making sales at $p_2$ to $\Gamma_2$ rather than making sales at $q_H$ to informed neighbors of $\Gamma_1$. Conversely, however, if the high type firm prefers to sell at $q_H$ to neighbors of $\Gamma_1$ than to sell at some $p_2$ to $\Gamma_2$, that equilibrium in this class does not survive application of the Intuitive Criterion. An example in the double star network is the following:

If consumers' beliefs are such that they believe that the firm is of low type if they observe $\gamma_1 > 1$, and if the high type firm prefers to sell to the seven unshaded consumers at $\bar{q}$ than to sell to the four neighbors of the shaded consumer at $q_H$, then a NS equilibrium exists with $\Gamma_1$ the shaded consumer, $\Gamma_2$ all others and $p_1 = p_2 = \bar{q}$. This example illustrates that the Intuitive Criterion cannot rule out consumers' pessimistic beliefs on the breadth of the firm's launch strategy since expanding the set of first round offers at positive price can be beneficial for either type of firm. In the context of the prominent worker example, perhaps all hold a belief that for a young

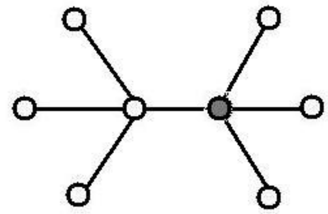

Figure 3: Double star network

firm to make middling offers to two superstars simultaneously is 'crazy', and so believe that the firm making such offers is not a good firm. The firm could play the S1 strategy and generate the associated inference, or could settle for making offers today and tomorrow at the market wage for a firm of indeterminate quality. If this strategy of pooling in both rounds is more profitable, then it survives application of the Intuitive Criterion.

#### 3.1.3 Separation between rounds

This class of equilibria exclusively uses communication rather than price signaling. In this class, firms with a high quality product make no attempt to distinguish themselves via price from low type firms before sales round 1, but instead choose to restrict round 1 sales so that their type is revealed by word-of-mouth during the communication stage. They can then exploit this in their round 2 strategy. High type firms sell in both rounds, and low type firms sell only in round 1.[3]

**Definition 4.** *An equilibrium with* ***separation between rounds*** *(S2) has a common, pooled price $p$ in round 1 for both types of firm, and a price $q_H$ in round 2 for at least the high type firm.*

An example of an S2 equilibrium (where the superscript 3 denotes this class of equilibria) is:

$$\begin{aligned}
\sigma_{F,L} &: \{p, q_H, \Gamma_1^3, N - \Gamma_1^3\} \\
\sigma_{F,H} &: \{p, q_H, \Gamma_1^3, N - \Gamma_1^3\} \\
\sigma_{1,i} &: \{(b = 1 | p_1(i) = p \wedge \Gamma_1 = \Gamma_1^3), (b = 0 | p_1(i) \neq p \vee \Gamma_1 \neq \Gamma_1^3)\} \\
\sigma_{2,i} &: \{(b = 1 | p_2(i) \leq \hat{q}), (b = 0 | p_2(i) > \hat{q})\}
\end{aligned}$$

All consumers receive an expected payoff of zero. Recall that $\omega(\Gamma_1^3)$ is the count of all consumers

[3] Note that although the firms are 'separated' by word-of-mouth and experience different round 2 outcomes, in equilibria of this class the two types of firm may choose identical strategies in round 2, since the low type firm will be indifferent between mimicking the high type strategy (thus making no sales) and setting some different strategy.

who are neighbors of consumers in $\Gamma_1^3$. The payoff to each type of firm is as follows:

$$\pi_{L,S2} = \gamma_1^3 p \tag{13}$$
$$\pi_{H,S2} = \gamma_1^3 p + \delta\omega(\Gamma_1^3) q_H \tag{14}$$

With supporting beliefs:

$$(Pr(low) = 1 | p_1(i) \neq p, \Gamma_1 \neq \Gamma_1^3) \tag{15}$$
$$(\hat{q}_2 = q | i \in \mathcal{I}), (\hat{q}_2 = \bar{q} | i \notin \mathcal{I}) \tag{16}$$

Both types of firm set a price of $p$ in sales round 1 and make this offer to those consumers who belong to the set $\Gamma_1^3$, and in sales round 2 the high type firm offers a price of $q_H$ to neighbors of $\Gamma_1^3$, that is, those consumers who learn the firm's type during the communication stage. This strategy is also that to which a high type firm could deviate to circumvent the NS equilibria, should the firm wish to do so. In the double star example from the preceding section, this deviation would constitute the high type firm playing $\Gamma_2 = \Omega(\Gamma_1)$ and $p_2 = q_H$.

This communication-driven class of equilibria helps to dictate an upper bound on the number of sales in round 1 in any equilibrium that survives the application of the Intuitive Criterion:

**Lemma 5.** *No equilibrium in which $\gamma_1 > \gamma^*$, $\gamma_2 < N - \gamma^*$ survives the application of the Intuitive Criterion.*

The proof again appears in the appendix. Note that in a deviation by the high-type firm away from a hypothetical equilibrium with $\gamma_1 > \gamma^*$, the location of offers is important: by locating offers at the minimum dominating set, consumers can be assured that if all of those offers are accepted word-of-mouth will reveal true type to the remainder of consumers. It is this that guarantees that such a deviation would not be profitable for the low type firm. As discussed in Section 2, the domination number of a connected graph never exceeds $\frac{1}{2}n$, but is in general lower. Nevertheless it is difficult to literally interpret this result, which seems to place a very loose upper bound on first round sales. This is a relic of the fact that the model is for tractability restricted to have two rounds; the more general implication of Lemma 5 is the argument that when a targeted launch is possible, the ability to strategically locate early sales to spread information about quality pushes strongly towards a launch whose breadth is capped at that needed to spread the word in a suitably 'short' time.

This result demonstrates that in such settings the number of sales made in the first round of the game is bounded above in a way that depends on the structure of the communication network. Because the firm can target sales it can visit an appropriately small set of consumers to exploit local communication, which is profitable only for the high-type firm, being the one not hurt by revelation of true quality.

### 3.2 Surviving equilibria

This result also completes the characterization of equilibria surviving the Intuitive Criterion.

**Theorem 1.** *The set of surviving equilibria consists of:*

1. *the S1 equilibria described in Lemma 2, in which the high type plays* $p_1 = 0$*,* $\Gamma_1 = \Gamma^*$*,* $p_2 = q_H$*,* $\Gamma_2 = N - \Gamma^*$*.*

2. *the set of S2 equilibria that satisfy Lemma 3 and Lemma 5, in which the high type play* $p_1 \leq \bar{q}$*,* $\gamma_1 \leq \gamma^*$*,* $p_2 = q_H$*,* $\gamma_2 = N - \gamma_1$ *and* $\pi_H > \delta(N - \gamma^*)q_H$*.*

3. *the set of NS equilibria that satisfy Lemma 3 and Lemma 5, in which the high type play* $p_1 \leq \bar{q}$*,* $\gamma_1 \leq \gamma^*$*,* $p_2 \leq \bar{q}$*,* $\Gamma_2$ *and* $\pi_H > \delta(N - \gamma^*)q_H$*.*

This follows directly from the Lemmas. It is apparent that applying the Intuitive Criterion does not select a unique equilibrium in this game; nevertheless, we can draw some concrete conclusions from Theorem 1. No equilibrium features more than $\gamma^*$ offers in sales round 1, so that the monopolist never visits more than a number of consumers equal to the domination number in any equilibrium that survives the application of the Intuitive Criterion. The lower the domination number $\gamma^*$, the more work the social network structure does to spread news of the firm's type, and so the lower the number of consumers who are targeted in round 1 as part of a signaling strategy to reveal type before round 2.

Comparing the surviving equilibria, we can see that the equilibria with separation before round 1 are less profitable to the high type firm than at least some of the S2 equilibria in which the high type pools on some group with the low type in round 1 and allows communication to signal their type to other consumers.

If the payoff to the high type firm in the surviving S1 equilibria *was* higher than in any S2 equilibrium, the S1 equilibrium will be the unique perfect Bayesian equilibrium in this game that survives the Intuitive Criterion. However, it is emphatically *not* the case that the S1 equilibrium is more profitable than any S2 equilibrium.

To see this, consider the most profitable S2 equilibrium, in which the two types of firm 'pool' on $\Gamma^*$ at the maximal pooled price, $\bar{q}$, in round 1, and the high type sells at $q_H$ in round 2 to $N - \Gamma^*$; this can be supported, for example, by the following strategies and suitable beliefs:

$$\begin{aligned}
\sigma_{F,L} &: \{\bar{q}, q_H, \Gamma^*, N - \Gamma^*\} \\
\sigma_{F,H} &: \{\bar{q}, q_H, \Gamma^*, N - \Gamma^*\} \\
\sigma_{1,i} &: \{(b = 1|p_1(i) = \bar{q}), (b = 0|p_1(i) \neq \bar{q})\} \\
\sigma_{2,i} &: \{(b = 1|p_2(i) \leq \hat{q}), (b = 0|p_2(i) > \hat{q})\}
\end{aligned}$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_L = \gamma^* \bar{q} \tag{17}$$

$$\pi_H = \gamma^* \bar{q} + \delta(N - \gamma^*) q_H \tag{18}$$

In the recurring double star example, this would constitute $\Gamma_1$ being the shaded consumers and $p_1 = \bar{q}$, followed by $\Gamma_2$ being all other consumers and $p_2 = q_H$:

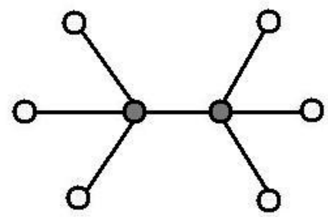

Figure 4: Double star network

In the context of the prominent worker example, this equilibrium is equivalent to the firm targeting the two superstar individuals with the market wage for a firm of unknown quality, which is a wage lower than in S1 equilibrium. The targeted individuals observe that the offers they receive could be profitable for either type of firm in isolation, but that their prominent position continues to mean that the industry as a whole will learn from them the true quality of the firm if they accept the offer. The two individuals are unable to infer for sure (by an Intuitive Criterion argument) that the firm is good, but in equilibrium they are willing to accept the offer since they believe with sufficient confidence that the firm making the offer *could* be good. As before, all other individuals go on to hear from the targeted individuals the true quality of the firm; tomorrow good firms can offer these individuals the market wage for a high quality firm, and low quality firms are driven out. A distinction with S1 is that now the communication from the early targets to the rest of the market is not simply confirming the inference all were able to make over the launch strategy, but is directly revealing information that the wider population could not directly infer.

This S2 equilibrium is clearly more profitable for the high type firm than the S1 equilibrium. The fundamental difference between the two classes of equilibria is, again, that in the between-round separation case the types are not separated in round 1, while in the pre-round 1 separation case the high type firm prices at a discount sufficient to separate before round 1. The high type thus earns positive profits in the pooled first round; it is these first-round profits that must be foregone in order to prove type in order to separate before round 1. This means that the analog of the Riley outcome in this two-round game is *not* the efficient S1 equilibria, but rather the efficient S2 equilibria. Minimizing inefficient signaling here means exploiting word-of-mouth rather than simply engaging in the least amount of inefficient price signaling.

Theorem 1 demonstrates that the Intuitive Criterion clearly admits a large class of equilibria depending on the beliefs held by consumers. In particular, this refinement cannot put a *lower* bound on $\Gamma_1$, the set of consumers to whom offers are made in round 1. In the extreme, if consumers believe that an out-of-equilibrium $\gamma_1 > 0$ must come from a low type (perhaps "a startup that immediately makes contract offers to industry veterans must be bad"), $\Gamma_1 = 0$ is admitted as a S2 or no separation equilibrium, despite the firm's clear incentive to generate word-of-mouth, if the round 2 component of the equilibrium makes it more profitable than the fallback S1 equilibria. This extreme case could be 'blamed' on the extreme pessimism of those supporting beliefs. The following section explores an equilibrium refinement that will restrict further consumers' beliefs and select a unique equilibrium in this game.

# 4 Equilibria surviving passive conjectures

The divinity criterion of Banks and Sobel (1987) offers no further help in refining the set of equilibria in this game. In particular, it is not possible to use this further refinement to select out the "inefficient" S2 equilibria, despite the intuitive appeal of the most profitable of that class. If consumer beliefs support $p_1 < \bar{q}$ or $\gamma_1 < \gamma^*$, any deviation to a higher round 1 price or a larger round 1 clientele will always be equally beneficial to both the high and low types, and thus the divinity refinement (or, indeed, the related D1 criterion) cannot restrict consumer beliefs in the face of such a deviation in a way that would rule out the original strategies as equilibrium play.

This is certainly related to the abandonment of static single-crossing in the setup of the game. The class of S2 equilibria feature pooling in round 1 and separating in round 2, and while the nature of single-crossing in this game has ruled out those S2 equilibria that do not conform to Lemma 5, it cannot be of any further help now.

Consider again the "efficient" S2 equilibrium:

$$\begin{aligned}
\sigma_{F,L} &: \{\bar{q}, q_H, \Gamma^*, N - \Gamma^*\} \\
\sigma_{F,H} &: \{\bar{q}, q_H, \Gamma^*, N - \Gamma^*\} \\
\sigma_{1,i} &: \{(b = 1 | p_1(i) = \bar{q}), (b = 0 | p_1(i) \neq \bar{q})\} \\
\sigma_{2,i} &: \{(b = 1 | p_2(i) \leq q_H), (b = 0 | p_2(i) > q_H)\}
\end{aligned}$$

All consumers receive an expected payoff of zero. The payoff to each type of firm is as follows:

$$\pi_L = \gamma^* \bar{q} \tag{19}$$

$$\pi_H = \gamma^* \bar{q} + \delta(N - \gamma^*) q_H \tag{20}$$

It is unambiguously true that this is preferred by the high type firm to the S1 equilibria, since the round 1 pooling is certainly preferred to incurring the cost of making the price discount to separate in round 1. Again, however, this is not evidence that the S1 case does not survive refinement: since any deviation from the S1 case to this S2 case is beneficial for either type, neither the Intuitive Criterion nor divinity can restrict consumer beliefs in the face of such a deviation.

What restriction *would* be sufficient to guarantee that this equilibrium was unique in the game? It would be sufficient that when consumers see an out of equilibrium action they place sufficiently high probability on it having come from a high type. One possibility would be to impose a **passive conjectures** property, as discussed in, for example, Rubinstein (1985) and Rasmusen (2001). It is defined here as follows:

**Definition 5.** ***Passive conjectures***: *When consumers see an out of equilibrium action that would be profitable for either type of firm if it was accepted, they maintain their prior beliefs, which place probability $\alpha$ on the deviation coming from a high type and probability $1-\alpha$ on the deviation coming from a low type, the true exogenous propensities of each type of firm.*

Passive conjectures can be motivated as the consumers perceiving out-of-equilibrium play as a mistake that is equally likely to be made by either type of firm, or as uninformative about type for some other reason. The key feature of passive conjectures is that when a consumer is offered the good under a price and visitation menu they did not anticipate, they are still willing to purchase the good if the unexpected price does not exceed mean product quality in the population of firms.

**Theorem 2.** *When consumers' beliefs satisfy passive conjectures, all equilibria in any network are efficient S2 equilibria.*

*Proof.* With passive conjectures, a unilateral deviation by the high type firm to the strategy specified in an efficient S2 equilibrium will result in all consumers who are offered a price in round 1 under that strategy accepting the offer: the round 1 price of $\bar{q}$ is acceptable under the assumption that consumers place probability $\alpha$ on the unknown firm being of high type. Since efficient S2 equilibria are the most profitable for the high type firm, this unilateral deviation is always attractive to the high type firm. □

In the case in which the network has a unique minimum dominating set, this equilibrium is unique, and in the case in which there are multiple minimum dominating sets the equilibria are qualitatively distinct in that they all feature the firm visiting some minimum dominating set in round 1. In order to select these "efficient" S2 cases as the unique set of equilibria in the game, it must be the case that the high type firm can deviate from a strategy with $p_1 < \bar{q}$ or $\gamma_1 < \gamma^*$ - a lower round 1 price or fewer round 1 customers - to $p_1 = \bar{q}$, $\Gamma_1 = \Gamma^*$ and have this menu be accepted by round 1 consumers. Under passive conjectures, consumers will indeed be willing to accept such a deviation, since it would be equally profitable to either type of firm and remains weakly profitable for the consumers who accept. Such a restriction is very strong, but arguably rules out 'unjustifiable' pessimism on the part of consumers. Nevertheless it is instructive to consider the conditions under which a unique equilibrium would be selected in this game.

The high type firm would, in the ideal, prefer to use the communication mechanism as a substitute for a costly action that reveals its type rather than to undertake costly signaling that complements the communication mechanism. Nevertheless, in both cases the parameters $\Gamma^*$ and $\gamma^*$ are critical throughout. Representing, respectively, the set and number of consumers who must be sold to in round 1 in order to inform all other consumers in the network of product quality before sales round 2, they represent the most profitable signaling strategy using visitation. They are therefore the analog in this signaling context of the kind of opinion leaders and key influencers that a practical 'network analysis' strategy seeks to identify in order to most successfully generate word-of-mouth in a more general context that also includes the informing and persuading roles of communication.

This shows that selection of an efficient S2 equilibrium is independent of the structure of the social network, despite the fact that the material outcome of the equilibrium varies with the network structure. The firm's discount factor and the consumers' willingness to pay for a product of unknown quality are important, however, since the analysis above assumes that the firm is sufficiently patient that a sale tomorrow at the high quality price is preferred to a sale today at the unknown quality price. If there is little difference between a high and low quality product or if the firm is sufficiently impatient, this may not be the case. With that caveat, provided we are not dealing with a singleton network, the quantitative result changes but qualitatively the solution to the problem is the same for all networks. Any equilibrium satisfying the passive conjectures is always an efficient S2 equilibrium, for any network.

This quantitative change, however, depends entirely on $\Gamma^*$ and $\gamma^*$. As discussed in relation to the S1 equilibrium, the relationship between the average number of neighbors in a given network and the domination number $\gamma^*$ is not straightforward. For example, a circle network can have a higher average number of neighbors per consumer than a star network with one central consumer yet have a larger domination number. Networks with a smaller domination number will have fewer round 1 sales in equilibrium; that is, more offers will be postponed to round 2 since fewer consumers must be 'convinced' today in order to reveal type to all tomorrow, which naturally enhances the signaling value of restricting round 1 activity.

The implication of the efficient S2 equilibrium with temporary pooling dominating the S1 equilibrium for the high type firm is that low quality products are traded in equilibrium, but then driven out.

# 5 Relaxing information assumptions

This section considers the effect of relaxing the assumptions on what consumers and firms know.

## 5.1 Consumers' knowledge of the firm's strategy

The model assumed that consumers were able to observe the firm's marketing strategy. This allows the model to cover a range of signaling (S1) equilibria alongside equilibria that separate only by word-of-mouth, without requiring an outside mechanism by which the firm can burn money. Consider instead an alternative assumption under which consumers can observe the firm's marketing strategy only in their neighborhood - that is, a consumer observes when a neighbor is targeted and the price they are offered, but does not observe anything further away.

Under this alternative assumption, Theorem 2 holds identically, up to the supporting beliefs on the efficient S2 equilibria, which now must condition only on each consumer's neighborhood. This is because under passive conjectures consumers still accept the deviation to efficient S2, and since the high type firm prefers that to any other case, no other visitation pattern will be observed from the high type in equilibrium. Lemmas 2 and 3, which identified the unique S1 equilibrium and the associated floor on high-type payoff, also holds identically up to the supporting beliefs (which in this case can be entirely pessimistic if no offer is made in the neighborhood). The deviation to the strategy of the high type firm in this equilibrium is not locally profitable for a low-type firm.

Theorem 1 no longer holds, however. In particular, the proof of Lemma 5, which demonstrates that no equilibria with more first-round sales offers than the graph's domination number, relies on consumers' inference on global information about offers. An intermediate assumption on consumers' information - for example that consumers observe the location of offers in their own neighborhood, the global number of offers, and the offered price - would be required to maintain Theorem 1.

Generally, the survival of Theorem 2 and Lemma 2 demonstrate that the qualitative features of the most relevant equilibria identified in the model are robust to coarser or less information for consumers. This is because when word-of-mouth and targeting are incorporated into the signaling framework, efficient money burning equilibria *themselves* make full use of local communication and do not force consumers to make indirect inference.

## 5.2 The firm's knowledge of the network

In the model the firm knows the precise structure of the network of consumers, which means it can address the novel question of how firms may choose to strategically locate sales in specific network locations.

This assumption could be relaxed in several ways, which suggest directions in which the current framework could be usefully extended. Most closely related to the goal of allowing firms to strategically locate sales, and least different from the current model, would be to assume that the network can be divided into regions, and assume that the firm knows something about how information percolates among regions. This perhaps corresponds to a conception of online interactions in which information flows within social websites are complex but cross-linking that captures information flows among websites is easier to track. A variation of the above model that followed this would operate similarly on a larger scale, pulling back on the 'microscope' from the consumer level to the network segment level, and could be achieved so simply as by reinterpreting nodes in the network as agglomerations of consumers rather than individuals.

Another way to relax the assumption on firm's knowledge would be to assume instead that the firm knows some summary statistics of the network but not its precise structure. Campbell (2010), Galeotti and Goyal (2007) and Galeotti and Goyal (2009) take this approach, although not in a quality signaling setting. Taking this approach would sacrifice the modeling of strategic location unless the summary statistics that the firm capture some differentiation across regions of the underlying network graph. To operationalize this would again require some concept of segmentation of the network.

Finally, we could consider that the firm may know the precise characteristics of *some* individuals in the network, perhaps able to identify those individuals with the most connections. This would admit firm strategies similar in spirit to those suggested by this model, in which early sales are located among those connected individuals. Imperfect knowledge would affect the degree to which such strategies would outperform costly signaling, and the degree of knowledge would in that case affect the firm's best strategy.

# 6 Concluding comments

This paper has studied the strategy of a firm that is introducing a product whose quality is initially hidden from consumers, but that can also exploit its knowledge of the structure of communication within its market by strategically choosing not just a price but also which consumers to offer the product to in each of two consecutive rounds.

Analogs of separating and pooling outcomes in one-round signaling games are present in this framework, but there are also equilibria in which low and high quality types pool in the first round and are separated by communication before the second round. In all equilibria, though, the fact that the firm can strategically locate sales today places an upper bound on those sales sufficient to allow communication to spread information about its type for benefit tomorrow. Visiting 'too many' consumers today can be costly tomorrow. In relation to traditional signaling results, the model therefore predicts that when a publicly observable targeted launch is possible, a firm will engage in less money-burning signaling effort.

Two separating outcomes warrant special attention. One is the precise analog of separation in a one-round game: the high quality firm takes an action that immediately reveals its type to all consumers. In this framework, this outcome involves the firm giving away its products in the first round to a set of consumers such that communication among consumers will reveal type to all others before the second round of sales. The second outcome is that in which the high quality firm sets a pooled price to that same set of consumers, forgoing immediate separation and allowing the coexistence of low type firms in the first round. The high type firm prefers the second outcome: it is more profitable to receive some positive price on the first round of sales than to give the product away.

If consumers' beliefs permit, therefore, the unique equilibrium of this game features pooling in the first round and separation in the second. Despite the common strategic location across equilibria, how consumers interpret a launch strategy is therefore crucial in determining whether the high quality firm can attain the pooling-then-separating equilibrium rather than undertaking costly signaling in conjunction with its location decision. For the informer, understanding the response of targeted consumers to the presence or lack of a money-burning signal is therefore key to understanding which launch strategy will be most appropriate.

In general comparing this model to a traditional signaling game admits the possibility that while in a setting in which a high type firm is ignorant of the structure of communication among consumers it may choose an action to separate immediately, in a setting in which the firm knows and can exploit the communication structure among consumers there exist attractive equilibria in which low quality trade is readmitted in the first round, before true quality is revealed during play. For the policymaker, more information for the firm can thus be in some sense harmful: consumers are willing to buy at the pooled price ex ante and firms make higher profits when they can pool before type is revealed, but ex post some consumers end up with a bad product. A regulator interested in minimizing low quality trade may therefore be interested in both the extent of targeting by firms and the extent of initial marketing intensity; little or no price discounting on new products, which may be considered beneficial for avoiding predation, may in fact be a sign that low quality products are being identified and weeded out less quickly than may be possible. By the same token, targeted penetration pricing can plausibly represent signaling rather than predation.

For the firm, the attractiveness of the pooling-then-separating equilibria also shows that the costs of signaling type can be lessened by incorporating knowledge of consumers' communication structure into a product launch strategy. Provided it is not outright loss-making to be temporarily indistinguishable from a low quality firm, with targeting the only cost that must be tolerated is temporary pooling. Simply allowing consumers to reveal to each other the high quality of the product could preclude the need for extravagant money burning. Although this pure result will be complicated by consumer heterogeneity and competitive pressures, the analysis presented here suggests at a minimum the value of consideration of strategic targeting as a separation mechanism during a product launch.

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# A Proofs

## A.1 Lemma 1

*Proof.* We require $\hat{p}$ such that a low-type firm imitating the high-type strategy completely would receive zero profit in expectation. The potential credible imitator would make $\gamma_1$ sales at $\hat{p}$ in round 1 and $(N - \gamma_1 - \Sigma_1)$ sales at $q_H$ in round 2, where $\Sigma_1$ is the number of neighbors to consumers in $\Gamma_1$ (since these neighbors observe the true type, they will not buy from the imitator in round 2). $\hat{p}$ must thus satisfy:

$$\pi_{L,imitate} = \gamma_1 \hat{p} + \delta(N - \gamma_1 - \Sigma_1) q_H \tag{21}$$

$$\leq 0 \tag{22}$$

$$\Rightarrow \hat{p} \leq -\frac{\delta(N - \gamma_1 - \Sigma_1) q_H}{\gamma_1} \tag{23}$$

The profit to the high type is thus:

$$\pi_H = \gamma_1 \hat{p} + \delta(N - \gamma_1) q_H \tag{24}$$

All S1 equilibria that do not maximize this profit with respect to visitation $\gamma_1$ and price $\hat{p}$ fail to survive the application of the Intuitive Criterion. By the Intuitive Criterion consumers will place zero probability on a deviation from any such equilibrium to the profit-maximizing S1 equilibrium coming from the low type firm; the deviation is profitable for the high type and will be made. □

## A.2 Lemma 2

*Proof.* Consider the S1 equilibrium in which $\gamma_1 = \gamma^*$. In that case, $\Sigma_1$, the number of neighbors to consumers in $\Gamma_1$, is equal to $N - \gamma^*$, since by the definition of $\gamma^*$ all remaining consumers are neighbors of those in $\Gamma^*$. The maximal price in that case is:

$$\hat{p} = -\frac{\delta(N - \gamma^* - \Sigma_1) q_H}{\gamma^*} \tag{25}$$

$$\hat{p} = -\frac{\delta(N - \gamma^* - (N - \gamma^*)) q_H}{\gamma^*} \tag{26}$$

$$\hat{p} = 0 \tag{27}$$

The profit to the high type is thus:

$$\pi_{H,1} = \delta(N - \gamma^*) q_H \tag{28}$$

Now consider any other S1 equilibrium, with generic $\gamma_1$. This has a maximal price:

$$\hat{p} = -\frac{\delta(N - \gamma_1 - \Sigma_{1,\gamma_1})q_H}{\gamma_1} \tag{29}$$

Which yields profit to the high type:

$$\pi_{H,2} = -\delta(N - \gamma_1 - \Sigma_{1,\gamma_1})q_H + \delta(N - \gamma_1)q_H \tag{30}$$

Now comparing the two:

$$\pi_{H,2} \quad > \quad \pi_{H,1} \tag{31}$$

$$\text{if } \Sigma_{\gamma_1} \quad > \quad N - \gamma^* \tag{32}$$

But $\Sigma_{1,\gamma_1} = N - \gamma^*$ if $\gamma_1 \geq \gamma^*$ and $\Sigma_{1,\gamma_1} < N - \gamma^*$ if $\gamma_1 < \gamma^*$, so it is never true that $\pi_{H,2} > \pi_{H,1}$. The most profitable S1 equilibrium thus features $\gamma_1 = \gamma^*$ and has profit given by:

$$\pi_H = \delta(N - \gamma^*)q_H \tag{33}$$

□

## A.3 Lemma 3

*Proof.* The S1 strategy $\sigma_{F,H} : \{0, q_H, \Gamma^*, N - \Gamma^*\}$ is profitable for the high type but not profitable for the low type firm, so when consumers see this strategy they must place zero probability on it coming from the low type (by the Intuitive Criterion). The high type earns payoff $\delta(N - \gamma^*)q_H$ by playing this strategy, so will prefer to deviate to it from any other strategy that yields a lower equilibrium payoff. □

## A.4 Lemma 4

*Proof.* Assume not, so that there exists an equilibrium with some $\Gamma_1$ and $\Gamma_2 = \emptyset$. Consider three cases:

1. Take $\Gamma_1 = N$. Consider a deviation to $\Gamma_1 = N - i$, $\Gamma_2 = i$ by the high type firm. Consumer $i$ will buy since she will learn that quality is $q_H$; this is profitable for the high type firm since $\delta > \frac{\bar{q}}{q_H}$, but not profitable for the low type firm, which would lose consumer $i$.

2. Take $\Gamma_1 \subset N$, $\Gamma_1 \neq \emptyset$. Consumers in $\Omega_{\Gamma_1}$ learn that quality is $q_H$. A deviation to $\Gamma_2 = \Omega_{\Gamma_1}$, $p_2 = q_H$ is profitable for the high type firm, but (weakly) not profitable for the low type firm since those consumers learn true quality before stage 2 and so will not buy from the low type.

3. Take $\Gamma_1 = \emptyset$. Consider a deviation to $p_1 = 0$, $\Gamma_1 \neq \emptyset$, $\Gamma_2 = \Omega_{\Gamma_1}$, $p_2 = q_H$. This is profitable for the high type firm, but (weakly) not profitable for the low type firm, which makes no profit in stage 1 and no sales in stage 2.

In each case there is a deviation to $\Gamma_2 \neq \emptyset$ that is profitable for the high type but not the low type. By the Intuitive Criterion, consumers must place zero probability on that deviation coming from the low type and so will accept the deviation's offers; this is profitable for the high type. $\Gamma_2 = \emptyset$ could not have been part of an equilibrium. □

## A.5 Lemma 5

*Proof.* Assume not, so that $\gamma_1 > \gamma^*$ in an equilibrium that survives the Intuitive Criterion. Consider two cases:

1. If consumers in $\Gamma_1$ choose not to buy in round 1 or if $p_1 < 0$, the high type firm cannot realize a higher payoff than in the equilibria of Lemma 2. By that result, the high type can profitably deviate to an S1 equilibrium, and since the low type cannot profit from such a deviation consumers place zero probability on the deviant being of low type.

2. If consumers in $\Gamma_1$ choose to buy in round 1, the high type firm can deviate to $\Gamma_1 = \Gamma^*$, $\Gamma_2 = N - \Gamma^*$, $p_2 = q_H$ and $p_1$ unchanged. This deviation is profitable for the high type firm since $\delta > \frac{\bar{q}}{q_H}$, but is not profitable for the low type since it reduces sales in round 1 and does not increase sales in round 2, since word-of-mouth reveals type to all in $N - \Gamma^*$.

$\gamma_1 > \gamma^*$ could not have been part of an equilibrium surviving the Intuitive Criterion. □