Loss Aversion, Survival and Asset Prices

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Abstract

Do loss averse investors influence asset prices in the long run? In an economy with heterogeneous investors those who are loss-averse can influence long run asset prices only if they survive, and its not obvious that they can survive in the presence of investors who do not exhibit loss aversion. This paper addresses these issues in a dynamic asset market model in which arbitrageurs have Epstein-Zin preferences. Our analysis shows that if loss aversion is the only difference in investors' preferences, then for empirically relevant parameter values, loss averse investors will be driven out of the market and thus they do not affect prices in the long run. The selection process may be slow in terms of wealth shares; but it can be effective in terms of price impacts, because of endogenous withdrawal by loss averse investors from the stock market. We also show that if investors have differing elasticities of intertemporal substitution or time patience parameters, loss averse investors can survive and affect prices in the long run.

Key words: loss aversion; Epstein-Zin preferences; market selection; asset pricing

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1 Introduction

The behavior of individuals in experiments is sometimes inconsistent with those individuals being well described as expected utility maximizers with correct expectations. Similarly, aggregate outcomes in asset markets are sometimes inconsistent with the predictions derived from (correct) expected utility maximizers interacting in a well functioning market. These observations have motivated the study of various alternative decision theories. One particularly interesting alternative theory studied in the recent behavioral finance literature is loss aversion which is a salient feature of prospect theory. Researchers have found that loss aversion helps to explain many financial phenomena, including the high mean, excess volatility and predictability of stock returns (e.g., Barberis, Huang and Santos, 2001); the value effect (Barberis and Huang, 2001); and, the GARCH effect in stock returns (McQueen and Vorkink, 2004).

Studies of the impact of loss aversion on markets are typically conducted in representative agent frameworks in which there is only one investor or equivalently all investors are identical.[1] This research has produced valuable insights into the potential for loss aversion to explain asset market puzzles, but it has a serious limitation. In particular, there is no trade, as there is no one to trade with in the economies studied in this literature. Its not just the absence of trade that is troubling, rather its whether the trade that would occur between heterogeneous individuals would dampen or even eliminate the impact of loss-averse traders on market outcomes. Recent research on market selection in economies composed of expected utility maximizers with heterogeneous beliefs (Sandroni (2000), Blume and Easley (2006)) shows that selection forces are important and cannot be ignored. This literature shows that under reasonable conditions, traders with incorrect beliefs are driven out of the market by those with correct beliefs, and as a result asset prices are eventually correct. That

[1] E.g., Benartzi and Thaler (1995). Barberis and Huang (2001, 2007, 2009), Barberis, Huang and Santos (2001), McQueen and Vorkink (2004), Grünea and Semmler (2008).

is, aggregate outcomes converge to those that would be predicted by well functioning markets composed only of expected utility maximizers with correct expectations. If investors who are not loss-averse and who have correct beliefs also drive loss-averse investors out of the market, and do so quickly, then loss-averse investors do not affect long run prices. This concern led Barberis and Huang (2009, p 1567) to caution that one should interpret the equity premium obtained in their representative agent model as “an upper bound on the equity premium that we would obtain in a more realistic heterogeneous agent economy.”

In this paper we answer the question of whether loss averse investors can survive and influence long run prices. We analyze a heterogeneous agent economy with two (classes of) investors and two tradable assets—a risk-free bond and a risky stock. Both investors have recursive preferences. The first investor, labeled the *EZ-investor*, has Epstein-Zin preferences (Epstein and Zin, 1989) and she represents “rational investors” or “arbitrageurs”. We use Epstein-Zin preferences in order to make this rational investor directly comparable with our second, loss-averse investor. The second investor is called the *LA-investor*, and he has a recursive preference representation proposed by Barberis and Huang (2007, 2009).[2] The LA-investor departs from the EZ-investor in the way he evaluates his investment in the stock market: he derives utility from investing in the market both indirectly, via its contribution to his lifetime consumption, and directly, via its resulting fluctuations in his financial wealth, and he is more sensitive to losses than to gains (loss aversion).

We have two main results. First, if investors only differ in whether they are loss averse or not, the LA-investor will be driven out of the market and will have no impact on asset prices in the long run for economies with empirically relevant parameter values (see Section 4). The selection process is slow in terms of wealth shares. For example, in calibrated economies, after 100 years, the LA-investor, on average, retains more than half of his initial wealth share. However, the selection mechanism is effective in terms of price impacts, as the

[2] Throughout this paper, we will use “she”/“her” to refer to the EZ-investor and use “he”/“him” to refer to the LA-investor.

LA-investor optimally chooses not to purchase the risky asset, leaving the equity premium to be determined by the EZ-investor.

Second, loss aversion, the elasticity of intertemporal substitution parameter (EIS henceforth) and the time-patience parameter all matter for survival. In particular, a LA-investor with a larger EIS parameter or time-patience parameter can survive in the presence of an EZ-investor (see Section 5). We find that small differences in these parameters determining intertemporal behavior can easily offset the negative effects of loss aversion. For instance, in a calibrated economy, a difference in the time-patience parameter of two percent can result in the long run dominance of the LA-investor. This second set of results quantifies the effect of the difference in investors' preferences on survival and asset prices.

The first result — that if loss aversion is the only difference in investors' preferences, the LA-investor vanishes — is driven primarily by the endogenous difference in investors' equilibrium portfolio choices. Previous studies on market selection among expected utility maximizers show that *when the saving rate is fixed*, the closer an investor's utility is to log utility, the higher is his/her expected wealth growth rate (De Long, Shleifer, Summers, and Waldman, 1991; Blume and Easley, 1992). Our analysis shows that this insight holds for recursive preferences in a general equilibrium setting. Under empirically plausible parameter values, the EZ-investor is more risk averse than the log utility, but the nature of loss aversion makes the LA-investor act as if he is even more risk averse than the EZ-investor, and therefore further from the log utility investor. Thus, the LA-investor vanishes. This result is proven in Subsection 4.2.

Although the intuition for this first result comes from the previous literature, the analysis is nonetheless complex because of the dynamic portfolio choice and savings decisions that our investors face. In particular, the result does not follow immediately from the previous literature as loss aversion causes the LA-investor's *saving behavior* to be endogenously different from the EZ-investor's, which might affect the LA-investor's survival prospects. Subsection 4.3 demonstrates that this loss-aversion-induced difference in savings cannot overcome the

LA-investor's disadvantage from his portfolio choice. Whether the LA-investor saves more or less than the EZ-investor depends on the common value of their EIS. When the common EIS is greater than one, the intertemporal substitution effect is the dominant force determining the investor's saving behavior. The presence of loss aversion makes the LA-investor's future prospects less attractive relative to those of the EZ-investor, thereby causing him to save less, which in turn further hurts his survival prospects. When the common EIS is less than one, the income effect dominates, and because the presence of loss aversion reduces the LA-investor's future prospects, this income effect implies that he consumes less and thus saves more than the EZ-investor. However, the difference in their savings rates declines with the wealth share controlled by the EZ-investor. This follows from the fact that as the EZ-investor controls more wealth, her lower saving rate raises the risk-free rate, which in turn increases the current consumption of the LA-investor, as the riskless asset is his primary investment vehicle (this in turn follows from the kink at his preferences). As a result, when the LA-investor's wealth erodes because of his adverse portfolio decisions, his saving rate decreases as well, which further drags down his wealth accumulation.

The second result — that the LA-investor can survive if he has a larger EIS parameter or time-patience parameter than the EZ-investor — follows from the endogenous difference in the investors' saving behaviors. The intuition is straightforward: When the LA-investor has a larger EIS parameter or time-patience parameter, his saving rate is larger than that of the EZ-investor. This favors his long run survival. For example, in a calibrated economy, when the EZ-investor's EIS takes a value of 0.5, it is sufficient for the LA-investor to have an EIS of 0.7 to dominate the market, as this difference in EIS generates a difference in saving rate of almost two percent. Similarly, the LA-investor's disadvantage (for survival) induced by his portfolio decisions can be overturned if his time-patience parameter increases by two percent. This result echoes Yan (2008) which shows that in a dynamic model populated with CRRA investors, a slight difference in the patience parameter makes it possible for an investor with incorrect beliefs to dominate the market, even if his beliefs persistently and substantially differ from the truth.

The remainder of this section reviews the relevant literature relevant. Section 2 outlines the model, and Section 3 characterizes the equilibrium. Section 4 demonstrates the implications for survival and price impact of loss aversion when it is the only difference in investors' preferences. Section 5 discusses its implications for survival when investors also have different EIS parameters or time-patience parameters. Section 6 concludes. The appendix provides the first-order conditions characterizing investors' decisions for the case of unit EIS and the details of the numerical algorithm.

Literature

This paper contributes to two strands of literature. The first is the market selection literature, which studies what types of investors survive and have a price impact in a dynamic economy. So far, this literature has primarily focused on selection over beliefs and not over preferences.[3] Although the idea of market selection dates back to the early 1950s (Alchian, 1950; Friedman, 1953), rigorous analysis of this idea has only recently been done. De Long, Shleifer, Summers, and Waldman (1991) are the first who cast doubts on the idea of market selection. They rely on partial equilibrium analysis and show that investors with incorrect beliefs can survive. Blume and Easley (1992) show that incorrect beliefs can be an advantage for survival in models with endogenous asset prices but exogenous savings decisions. Sandroni (2000), Blume and Easley (2006) and Yan (2008) endogenize both savings and portfolio decisions and show that only investors with beliefs closest to the objective probabilities will survive in economies with bounded aggregate endowments. Kogan, Ross, Wang, and Westerfield (2009) demonstrate that in economies with unbounded endowments, investors with incorrect beliefs may survive.

Investors in all of the above models have time-separable utility functions. Borovička

[3] One exception is Condie (2008), which analyzes the market selection problem for an economy populated with ambiguity averse investors and expected utility investors.

(2009) has recently studied the belief-selection problem in an economy with Epstein-Zin preferences and found that agents with distorted beliefs are not driven out of the market for an empirically relevant range of parameters. Other studies on market selection consider issues related to incomplete markets (Coury and Sciubba, 2005; Sandroni, 2005; Blume and Easley, 2006; Gallmeyer and Hollifield, 2008; Cao, 2009), imperfect competition (Palomino, 1996; Kyle and Wang, 1997), comparison of trading rules (Blume and Easley, 1992; Amir, Evstigneev, Hens and Schenk-Hoppé, 2005; Böhm and Wenzelburger, 2005), and asymmetric information and learning (Mailath and Sandroni, 2003; Sciubba, 2005; Cogley and Sargent, 2009). Instead of studying belief selection, this paper analyzes preference selection in frictionless and complete market economies, and it is the first study on the market selection problem between loss aversion and Epstein-Zin preferences.

The second strand of related literature considers the role of loss aversion in determining trading behavior, asset prices and trading volumes. Loss aversion, that investors are more sensitive to reductions in the value of their financial wealth than to gains, is a key feature of prospect theory which was introduced by Kahneman and Tversky (1979). Berkelaar, Kouwenberg and Post (2004), Gomes (2005) and Kyle, Ouyang and Xiong (2006) study the optimal portfolio choice problem under loss aversion. Benartzi and Thaler (1995) were the first to use loss aversion to explain the equity premium puzzle. Barberis, Huang and Santos (2001) extend Benartzi and Thaler's setting to a dynamic model and find that combining loss aversion and the "house-money effect" helps to explain the behavior of the aggregate stock market. Barberis and Huang (2001) find that loss aversion is also useful in understanding the value effect in the cross-section of stock returns. Grünea and Semmler (2008) study a production economy and find that a model incorporating loss aversion can match data much better than pure consumption-based asset-pricing models. McQueen and Vorkink (2004) show that loss aversion helps to explain the asymmetric GARCH properties of stock returns. Barberis and Huang (2007, 2009) propose a preference specification that incorporates both loss aversion and narrow framing and study its applications in portfolio choice and asset pricing.

All of the above-mentioned asset-pricing models are conducted in a representative agent framework. Gomes (2005), Gabaix (2007) and Berkelaar and Kouwenberg (2009) explore the interaction between loss averse investors and expected utility maximizers. However, all three studies have a finite horizon model and are therefore unable to answer the question of whether loss averse investors survive and affect prices in the long run.

2 The Model

We analyze a pure exchange economy with one perishable consumption good, which is the numeraire. Time is discrete and lasts forever: $t = 0, 1, 2, ...$ There are two assets — a risk-free bond and a risky stock. The bond is in zero net supply and earns a gross interest rate of $R_{f,t}$ between time t and $t+1$. The stock is a claim to a stream of the consumption good represented by the dividend sequence $\{D_t\}_{t=0}^{\infty}$. It is in limited supply (normalized to 1) and is traded in a competitive market at price P_t. Let $f_t = \frac{P_t}{D_t}$ and $R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$ be the price-dividend ratio at time t and the gross return on the stock between time t and $t+1$, respectively.

The dividend growth rate $\theta_{t+1} \triangleq \frac{D_{t+1}}{D_t}$ is *i.i.d.* over time and follows a distribution given by

$$\theta_{t+1} = \begin{cases} \theta_H, & \text{with probability } \pi_H, \\ \theta_L, & \text{with probability } \pi_L, \end{cases} \tag{1}$$

with $0 < \theta_L < \theta_H$, $0 < \pi_H < 1$ and $\pi_L = 1 - \pi_H$. We use a binomial distribution for the dividend growth rate process so that the two tradable assets induce a dynamically complete financial market. This ensures that our results on survival are driven by the difference in investors' preferences and not by any assumed incompleteness in the financial-market structure. The market structure is important as whether the market-selection argument is valid depends crucially on the completeness of financial markets (see, among others, Blume and Easley, 2006; Cao, 2009).

We follow the literature in assuming that the aggregate consumption and aggregate dividends are equal.[4] Under this assumption, even a representative agent economy with loss-aversion preferences cannot match the historical equity premium,[5] as the equilibrium stock returns are not volatile enough to induce the loss averse investor to abandon the stock market. We have extend our analysis to a three-asset setting which is capable of generating the historical equity premium via a combination of loss aversion and narrow framing, and have found that all our main results hold in this extended model. To focus on our selection results in the most transparent setting, this extension is not reported in the paper (the results are available upon request).

The economy is populated by two (classes of) investors, who are distinguished by their preferences. The first investor, labeled the *EZ-investor*, derives utility from intertemporal consumption plans according to Epstein-Zin preferences (Epstein and Zin, 1989). The second investor, labeled the *LA-investor*, is the investor emphasized in the behavioral finance literature, see Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), Barberis, Huang and Thaler (2006) and Barberis and Huang (2007, 2009). This investor gets utility not only from consumption but also from fluctuations in the value of his stock holdings, and he is loss averse over these fluctuations.

We use the preference specification developed by Barberis and Huang (2007, 2009) to describe the LA-investor's preferences. According to this specification, the EZ-investor's preference is simply a degenerate case of the LA-investor's preference, where the parameter controlling the term related to loss aversion is set to be zero. Thus, this preference specification allows us to isolate the impact of loss aversion on the LA-investor's wealth dynamics (survival) and asset prices.

[4]For consumption-based models, see, among others, Lucas (1978) and Mehra and Prescott (1985); for models studying loss aversion, see, among others, Gomes (2005) and Berkelaar and Kouwenberg (2009).

[5]See the first economy studied by Baberis, Huang and Santos, 2001, and Subsection 4.1 below.

We choose Epstein-Zin preferences to represent arbitrageurs for two reasons. First, Epstein-Zin preferences allow us to separate the risk-aversion parameter and the elasticity of intertemporal substitution parameter. These two parameters presumably have very different roles in determining investors' survival prospects, as the existing market-selection literature suggests that portfolio decisions, which are more related to risk aversion, and saving behaviors, which are more related to EIS, affect survival in different ways. Second, Epstein-Zin preferences deserve more serious investigation on their own, as the recent literature has shown that Epstein-Zin preferences help to explain many salient features of the financial market.[6]

Given that the LA-investor's preference nests the EZ-investor's preference, we write a uniform preference formulation for both investors as follows. The time t utility of investor i (=EZ, LA) is given by

$$U_{i,t} = H_i \left[C_{i,t}, \mu_i \left(U_{i,t+1} | I_t \right) + b_i E_t \left[v \left(G_{i,t+1} \right) \right] \right], , \tag{2}$$

where $b_{EZ} = 0$ and $b_{LA} \geq 0$. Here, $H_i(\cdot, \cdot)$ is the aggregator function, which combines current consumption $C_{i,t}$ and the certainty equivalent of future utility to generate current utility $U_{i,t}$. It takes the form

$$H_i(C, X) = \begin{cases} \left[(1-\beta_i) C^{\rho_i} + \beta_i X^{\rho_i} \right]^{1/\rho_i}, & \text{if } 0 \neq \rho_i < 1, \\ C^{1-\beta_i} X^{\beta_i}, & \text{if } \rho_i = 0, \end{cases} \tag{3}$$

where $0 < \beta_i < 1$ is investor i's time patience parameter. Parameter ρ_i determines the investor's elasticity of intertemporal substitution: $EIS_i = 1/(1-\rho_i)$.

Function $\mu_i(U_{i,t+1}|I_t)$ is the certainty equivalent of the random future utility $U_{i,t+1}$ con-

[6] See Tallarini (2000), Bansal and Yaron (2004), Campanale, Castro, and Clementi (2007), Uhlig (2007), Gomes and Michaelides (2008), and Guvenen (forthcoming), among others.

ditional on time t information I_t, and it has the form

$$\mu_i\left(U|I_t\right) = \begin{cases} [E_t(U^{\zeta_i})]^{1/\zeta_i}, & \text{if } 0 \neq \zeta_i < 1, \\ \exp[E_t(\log\left(U\right)], & \text{if } \zeta_i = 0, \end{cases} \tag{4}$$

where $E_t\left(\cdot\right) \equiv E\left(\cdot|I_t\right)$ is the expectation operator conditional on information I_t and where parameter ζ_i determines the investor's risk attitude toward aggregate future utility, as the implied parameter $RA_i = 1-\zeta_i$ is the investor's relative risk aversion coefficient. We assume that the investors have correct beliefs so that we can focus on the effects of differences in loss aversion.

Up to this point, the investor's preference is entirely standard. What is non-standard is that a new term, $b_iE_t\left[v\left(G_{i,t+1}\right)\right]$, is added to the second argument of $H_i\left(\cdot,\cdot\right)$, allowing the investor to get utility directly from the performance of investing in the stock. This term captures the non-consumption utility that the agent derives directly from the specific gamble of investing in the stock rather than just indirectly via this gamble's contribution to the next period's wealth and the resulting consumption; the latter effect has already been captured by the certainty equivalent function, $\mu_i\left(U_{i,t+1}|I_t\right)$. To ease exposition, we refer to this new term as *loss aversion utility*, and its components — parameter b_i, argument $G_{i,t+1}$, and function $v\left(\cdot\right)$ — are further specified as follows.

First, parameter b_i determines the relative importance of the loss aversion utility term in the investor's preference. For the EZ-investor, $b_{EZ} = 0$, meaning that she derives no direct utility from financial wealth fluctuations. For the LA-investor, $b_{LA} > 0$, meaning that, to a certain extent, his utility depends on the outcome of his stock investment over and above what that outcome implies for total wealth.

Second, variable $G_{i,t+1}$ defines the gamble that investor i is taking by investing in the risky stock. Specifically, let $W_{i,t}$ be investor i's wealth at the beginning of time t, and let $s_{i,t}$ be the fraction of post-consumption wealth allocated to the stock. Then this investment portfolio provides the investor with a gamble represented by

$$G_{i,t+1} = s_{i,t} \left(W_{i,t} - C_{i,t}\right) \left(R_{t+1} - R_{f,t}\right), \tag{5}$$

that is, the amount invested in the stock, $s_{i,t} \left(W_{i,t} - C_{i,t}\right)$, multiplied by its return in excess of the risk-free rate, $R_{t+1} - R_{f,t}$. As is standard in the literature (e.g., Barberis and Huang, 2001, 2007, 2009; Gomes, 2005; Barberis and Xiong, 2009), the risk-free rate, $R_{f,t}$, is assumed to be the "reference point" determining whether a particular outcome is treated as a gain or a loss: as long as $s_{i,t} > 0$, the stock's return is only counted as a gain (loss) if it is larger (smaller) than the risk-free rate.

Finally, function $v(\cdot)$ determines how the investor evaluates gains and losses. We follow Barberis and Huang (2007, 2009) in assuming that $v(\cdot)$ is a piecewise-linear function:

$$v(G) = \begin{cases} G, & \text{if } G \geq 0, \\ \lambda G, & \text{if } G < 0, \end{cases} \tag{6}$$

with $\lambda > 1$. This function assigns positive utility to gains and negative utility to losses. More importantly, it assigns greater negative utility to losses than positive utilities to gains of the same magnitude. This feature is known as "loss aversion" in the literature, and it is the behavioral bias that the LA-investor exhibits. Parameter λ controls the degree of loss aversion: a one-dollar loss brings the investor $\lambda > 1$ units of negative non-consumption utility, while a one-dollar gain brings him only one unit of positive non-consumption utility.

To summarize, the economy is characterized by the following two group of exogenous parameters: (i) technology parameters: θ_H, θ_L, π_H and π_L; and (ii) preference parameters: b_{LA}, λ, $\{\beta_i, \rho_i, \zeta_i\}_{i=EZ,LA}$. The technology is defined by equation (1), and the preferences are defined by equations (2)-(6).

3 Equilibrium

We consider Markov equilibria in which price-dividend ratios, the risk-free rate, and the optimal consumption and portfolio decisions are all functions of a state variable and in which the state variable evolves according to a Markov process. The Markov state variable ω_t is the LA-investor's wealth as a fraction of aggregate wealth:

$$\omega_t = \frac{W_{LA,t}}{W_{LA,t} + W_{EZ,t}}. \tag{7}$$

Intuitively, ω_t captures the state of the economy, because it determines the strength of the pricing impact of the LA-investor's trading behavior. The reason that we can summarize the state with a single variable is that the preferences of investors are homogeneous in wealth. A Markov equilibrium is formally defined as follows.

Definition 1 *An equilibrium consists of (i) a stationary price-dividend ratio function,* $f : [0,1] \to \mathbb{R}_{++}$*, (ii) a risk-free rate function,* $R_f : [0,1] \to \mathbb{R}_{++}$*, (iii) a pair of consumption propensity functions,*[7] $\alpha_{LA} : [0,1] \to [0,1]$ *and* $\alpha_{EZ} : [0,1] \to [0,1]$*, (iv) a pair of stock investment policies,* $s_{LA} : [0,1] \to \mathbb{R}$ *and* $s_{EZ} : [0,1] \to \mathbb{R}$*, and (v) a transition function of the state variable,* $\omega : [0,1] \times \{\theta_H, \theta_L\} \to [0,1]$*, such that*
(i) the consumption policy functions and the portfolio policy functions maximize investors' preferences given the distribution of the equilibrium return processes;
(ii) good and securities markets clear; and
(iii) the transition function of the state variable is generated by investors' optimal decisions and the exogenous consumption growth rate process (i.e., equation [1]).

We next go through investors' decision problems and the market clearing conditions to construct such an equilibrium.

[7] Consumption propensity is the ratio of consumption over wealth.

3.1 Investors' Decisions

Investor i chooses consumption $C_{i,t}$ and the fraction of post-consumption wealth allocated to the stock $s_{i,t}$ to maximize

$$U_{i,t} = H_i \left[C_{i,t}, \mu_i \left(U_{i,t+1} | I_t\right) + b_i E_t \left[v \left(G_{i,t+1}\right)\right]\right]$$

subject to the definition of capital gains/losses in stock investment

$$G_{i,t+1} = s_{i,t} \left(W_{i,t} - C_{i,t}\right) \left(R_{t+1} - R_{f,t}\right)$$

and to the standard budget constraint

$$W_{i,t+1} = \left(W_{i,t} - C_{i,t}\right) M_{i,t+1},$$

where

$$M_{i,t+1} = R_{f,t} + s_{i,t} \left(R_{t+1} - R_{f,t}\right) \tag{8}$$

is the gross return on the investor's portfolio, and functions $H_i(\cdot,\cdot)$, $\mu_i(\cdot)$, and $v(\cdot)$ are given by equations (3), (4), and (6), respectively.

For brevity, we only derive the first-order conditions characterizing the investor's optimal decisions for the case of a non-unit EIS (i.e., for the case of $\rho_i \neq 0$ in the aggregator function $H_i(\cdot,\cdot)$). The first-order conditions for the case of a unit EIS are relegated to Appendix A.

The Bellman equation of the investor's problem is

$$\begin{aligned} U_{i,t} &\equiv J_i \left(W_{i,t}, I_t\right) \\ &= \max_{C_{i,t}, s_{i,t}} \left[(1-\beta_i) C_{i,t}^{\rho_i} + \beta_i \left[\mu_i \left(J \left(W_{i,t+1}, I_{t+1}\right) | I_t\right) + b_i E_t \left(v \left(G_{i,t+1}\right)\right)\right]^{\rho_i}\right]^{1/\rho_i}. \end{aligned}$$

Because functions $H_i(\cdot,\cdot)$, $\mu_i(\cdot)$, and $v(\cdot)$ are all homogeneous of degree one, the indirect value function $J_i\left(W_{i,t}, I_t\right)$ is also homogeneous of degree one:

$$J_i\left(W_{i,t}, I_t\right) = A_i\left(I_t\right) W_{i,t} \equiv A_{i,t} W_{i,t}.$$

Therefore,

$$A_{i,t} W_{i,t} = \max_{C_{i,t}, s_{i,t}} \left[(1-\beta_i) C_{i,t}^{\rho_i} + \beta_i \left(W_{i,t} - C_{i,t}\right)^{\rho_i} \left[\begin{array}{c} \mu_i\left(A_{i,t+1} M_{i,t+1} | I_t\right) \\ +b_i E_t\left[v\left(s_{i,t}\left(R_{t+1} - R_{f,t}\right)\right)\right] \end{array} \right]^{\rho_i} \right]^{1/\rho_i},$$

which implies that the consumption and portfolio decisions are separable.

In particular, the portfolio decision is determined by

$$B_{i,t}^* = \max_{s_{i,t}} \left[\mu_i\left(A_{i,t+1} M_{i,t+1} | I_t\right) + b_i E_t\left[v\left(s_{i,t}\left(R_{t+1} - R_{f,t}\right)\right)\right]\right], \tag{9}$$

and after defining the consumption propensity as

$$\alpha_{i,t} = C_{i,t} / W_{i,t},$$

the consumption decision is made based on

$$A_{i,t} = \max_{\alpha_{i,t}} \left[(1-\beta_i)\alpha_{i,t}^{\rho_i} + \beta_i\left(1 - \alpha_{i,t}\right)^{\rho_i}\left(B_{i,t}^*\right)^{\rho_i}\right]^{1/\rho_i}. \tag{10}$$

The first-order condition for optimal consumption propensity $\alpha_{i,t}^*$ is[8]

$$B_{i,t}^* = \left(\frac{1-\beta_i}{\beta_i}\right)^{1/\rho_i} \left(\frac{\alpha_{i,t}^*}{1-\alpha_{i,t}^*}\right)^{1-1/\rho_i}. \tag{11}$$

[8] All of the first-order conditions of the investor's problem are both necessary and sufficient, as the objective functions are concave.

Combining equations (10) and (11) delivers

$$A_{i,t} = (1-\beta_i)^{1/\rho_i} \left(\alpha^*_{i,t}\right)^{1-1/\rho_i},$$

which, by the recursive structure, in turn implies

$$A_{i,t+1} = (1-\beta_i)^{1/\rho_i} \left(\alpha^*_{i,t+1}\right)^{1-1/\rho_i}. \tag{12}$$

Substituting equations (11) and (12) into equation (9) gives the following single program, which summarizes the investor's consumption and portfolio decisions:

$$\left(\frac{1-\beta_i}{\beta_i}\right)^{1/\rho_i} \left(\frac{\alpha^*_{i,t}}{1-\alpha^*_{i,t}}\right)^{1-1/\rho_i} = \max_{s_{i,t}} \left\{ \begin{array}{c} \mu_i \left[(1-\beta_i)^{1/\rho_i} \left(\alpha^*_{i,t+1}\right)^{1-1/\rho_i} M_{i,t+1} | I_t\right] \\ +b_i E_t \left[v\left(s_{i,t}\left(R_{t+1} - R_{f,t}\right)\right)\right] \end{array} \right\}. \tag{13}$$

As a consequence, solving the investor's partial-equilibrium problem boils down to solving a fixed-point problem defined by the first-order condition and the value function of the above maximization problem. Specifically, in the Markov equilibrium, the investor's consumption policy and investment policy are both functions of the state variable ω_t. The first-order condition and the value function of program (13) thus form a system of two equations with these two unknown functions $s_i(\cdot)$ and $\alpha_i(\cdot)$. Given the equilibrium asset return processes (R_{t+1} and $R_{f,t}$), these partial equilibrium optimal policies can be computed from this system.

It needs certain carefulness to derive the first-order conditions for the portfolio choice, as the utility function, $v(\cdot)$, the function that the investor uses to evaluate gains/losses, is not differentiable everywhere but instead has a kink at the origin. As will become clear in the subsequent analysis, it is this non-differentiability at the origin that is responsible for the non-participation of the LA-investor in the stock market. Formally, the optimal stock investment $s^*_{i,t}$ is characterized by the following conditions:[9]

$$\begin{aligned}
&FOC_{i,+} \\
&\equiv (1-\beta_i)^{1/\rho_i}\left[E_t\left(\alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i}M_{i,t+1}^{\zeta_i}\right)\right]^{1/\zeta_i-1}E_t\left[\alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i}M_{i,t+1}^{\zeta_i-1}\left(R_{t+1}-R_{f,t}\right)\right] \\
&\quad +b_iE_t\left[v\left(R_{t+1}-R_{f,t}\right)\right] \\
&=0,\ \text{for } s^*_{i,t}>0,
\end{aligned} \tag{14}$$

$$\begin{aligned}
&FOC_{i,-} \\
&\equiv (1-\beta_i)^{1/\rho_i}\left[E_t\left(\alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i}M_{i,t+1}^{\zeta_i}\right)\right]^{1/\zeta_i-1}E_t\left[\alpha_{i,t+1}^{(1-1/\rho_i)\zeta_i}M_{i,t+1}^{\zeta_i-1}\left(R_{t+1}-R_{f,t}\right)\right] \\
&\quad -b_iE_t\left[v\left(R_{f,t}-R_{t+1}\right)\right] \\
&=0,\ \text{for } s^*_{i,t}<0,
\end{aligned} \tag{15}$$

$$FOC_{i,+}\leq 0 \text{ and } FOC_{i,-}\geq 0,\ \text{for } s^*_{i,t}=0. \tag{16}$$

In particular, as for the EZ-investor, the expressions of $FOC_{i,+}$ and $FOC_{i,-}$ are the same because $b_{EZ}=0$. Therefore, her first-order conditions are reduced to the following equation:

$$E_t\left[\alpha_{EZ,t+1}^{(1-1/\rho_{EZ})\zeta_{EZ}}M_{EZ,t+1}^{\zeta_{EZ}-1}\left(R_{t+1}-R_{f,t}\right)\right]=0. \tag{17}$$

3.2 Stock Prices and Wealth Dynamics

In this subsection, we rely on market-clearing conditions to derive the expression of price-dividend ratios $f_t\equiv P_t/D_t$ and the evolution of the state variable ω_t.

[9] To be precise, the conditions apply to the case of a non-unit risk aversion, i.e., they are true when $RA_i\neq 1$ or $\zeta_i\neq 0$ in the certainty-equivalent function $\mu(\cdot)$. As for the case of a unit risk aversion, simply replace the first terms with $(1-\beta_i)^{1/\rho_i}\,e^{(1-1/\rho_i)E_t\log(\alpha^*_{i,t+1})+E_t\log(M_{i,t+1})}E_t\left(\frac{R_{t+1}-R_{f,t}}{M_{i,t+1}}\right)$, which can be obtained from the limiting formula, $\lim_{\zeta_i\to 0}\left[E_t\left(x^{\zeta_i}\right)\right]^{1/\zeta_i}=e^{E_t[\log(x)]}$.

The good market-clearing condition is

$$C_{EZ,t} + C_{LA,t} = D_t. \tag{18}$$

Using the definition of consumption propensity, we can express the consumption levels as products of consumption propensity functions and individual wealth levels:

$$C_{EZ,t} = \alpha_{EZ}(\omega_t) W_{EZ,t} \text{ and } C_{LA,t} = \alpha_{LA}(\omega_t) W_{LA,t}.$$

Then, substituting the above expressions into the good-market clearing condition gives

$$\alpha_{EZ}(\omega_t) W_{EZ,t} + \alpha_{LA}(\omega_t) W_{LA,t} = D_t. \tag{19}$$

Let $W_t = W_{EZ,t} + W_{LA,t}$ be the aggregate wealth of the whole economy at time t. Recall that the definition of ω_t in equation (7) implies that $W_{EZ,t} = (1-\omega_t) W_t$ and $W_{LA,t} = \omega_t W_t$. Therefore, equation (19) becomes

$$[\alpha_{EZ}(\omega_t)(1-\omega_t) + \alpha_{LA}(\omega_t)\omega_t] W_t = D_t,$$

which implies

$$W_t = \frac{D_t}{\alpha_{EZ}(\omega_t)(1-\omega_t) + \alpha_{LA}(\omega_t)\omega_t}. \tag{20}$$

Because the bond is zero net supply, and the stock has a net supply of one share, the aggregate economy wealth is also equal to the stock price plus its dividend:

$$W_t = P_t + D_t. \tag{21}$$

Combining equations (20) and (21) gives the price-dividend ratio function:

$$\begin{aligned} f(\omega_t) &= \frac{(1-\omega_t)\,\alpha_{EZ}(\omega_t)}{(1-\omega_t)\,\alpha_{EZ}(\omega_t)+\omega_t\alpha_{LA}(\omega_t)}\frac{1-\alpha_{EZ}(\omega_t)}{\alpha_{EZ}(\omega_t)} \\ &\quad +\frac{\omega_t\alpha_{LA}(\omega_t)}{(1-\omega_t)\,\alpha_{EZ}(\omega_t)+\omega_t\alpha_{LA}(\omega_t)}\frac{1-\alpha_{LA}(\omega_t)}{\alpha_{LA}(\omega_t)}. \end{aligned} \tag{22}$$

Equation (22) says that the price-dividend ratios in the heterogeneous agent economy are equal to a weighted average of two terms: $\frac{1-\alpha_{EZ}(\omega_t)}{\alpha_{EZ}(\omega_t)}$ and $\frac{1-\alpha_{LA}(\omega_t)}{\alpha_{LA}(\omega_t)}$. In fact, the expressions of these two terms correspond to the price-dividend ratios in the representative agent economies populated only by the EZ-investor and by the LA-investor, respectively.[10] So, roughly speaking, the price-dividend ratios in a heterogeneous economy is the weighted average of the price-dividend ratios in representative agent economies, although the weight is not simply the wealth share but is instead a rather complicated expression related to the wealth share and investors' optimal consumption policies.

Given the price-dividend ratio function $f_t = f(\omega_t)$ and the Markov structure of the state variable evolution $\omega_{t+1} = \omega(\omega_t, \theta_{t+1})$, the distribution of stock returns R_{t+1} also has a Markov structure and is determined by

$$R(\omega_t, \theta_{t+1}) \equiv R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t} = \frac{P_{t+1}/D_{t+1}+1}{P_t/D_t}\frac{D_{t+1}}{D_t} = \frac{f(\omega(\omega_t,\theta_{t+1}))+1}{f(\omega_t)}\theta_{t+1}. \tag{23}$$

We now turn to examine how the state variable, ω_t, evolves over time. The gross return to the LA-investor's optimal portfolio is

$$\begin{aligned} M_{LA}(\omega_t, \theta_{t+1}) \equiv M_{LA,t+1} &= R_{f,t} + s^*_{LA,t}(R_{t+1} - R_{f,t}) \\ &= R_f(\omega_t) + s_{LA}(\omega_t)\left[R(\omega_t,\theta_{t+1}) - R_f(\omega_t)\right]. \end{aligned} \tag{24}$$

[10] To see this, note that, in a representative agent economy, the agent holds the whole share of the stock and consumes the entire dividend, which means that $\alpha_{i,t}W_{i,t} = \alpha_{i,t}(P_t + D_t) = D_t$ and thus $P_t/D_t = (1-\alpha_{i,t})/\alpha_{i,t}$.

Therefore, the LA-investor's next period wealth is

$$\begin{aligned} W_{LA,t+1} &= \left[1-\alpha_{LA}\left(\omega_t\right)\right] W_{LA,t} M_{LA}\left(\omega_t, \theta_{t+1}\right) \\ &= \frac{\left[1-\alpha_{LA}\left(\omega_t\right)\right] \omega_t M_{LA}\left(\omega_t, \theta_{t+1}\right)}{\alpha_{EZ}\left(\omega_t\right)\left(1-\omega_t\right)+\alpha_{LA}\left(\omega_t\right)\omega_t} D_t, \end{aligned} \tag{25}$$

where the second equation follows from $W_{LA,t} = \omega_t W_t$ and equation (20).

Applying equation (20) one period forward gives

$$W_{t+1} = \frac{D_{t+1}}{\alpha_{EZ}\left(\omega_{t+1}\right)\left(1-\omega_{t+1}\right)+\alpha_{LA}\left(\omega_{t+1}\right)\omega_{t+1}}. \tag{26}$$

Combining equations (25) and (26) and recalling the definition of $\omega_{t+1} = \frac{W_{LA,t+1}}{W_{t+1}}$ and $\theta_{t+1} = \frac{D_{t+1}}{D_t}$, we have

$$\omega_{t+1} = \frac{\left[1-\alpha_{LA}\left(\omega_t\right)\right] \omega_t M_{LA}\left(\omega_t, \theta_{t+1}\right)\left[\alpha_{EZ}\left(\omega_{t+1}\right)\left(1-\omega_{t+1}\right)+\alpha_{LA}\left(\omega_{t+1}\right)\omega_{t+1}\right]}{\left[\alpha_{EZ}\left(\omega_t\right)\left(1-\omega_t\right)+\alpha_{LA}\left(\omega_t\right)\omega_t\right]\theta_{t+1}}, \tag{27}$$

which implicitly determines the evolution of ω_t: $\omega_{t+1} = \omega\left(\omega_t, \theta_{t+1}\right)$.

Finally, substituting $W_{EZ,t} = \left(1-\omega_t\right) W_t$, $W_{LA,t} = \omega_t W_t$ and equation (20) into the stock-market clearing condition,

$$P_t = s^*_{EZ,t}\left(1-\alpha^*_{EZ,t}\right) W_{EZ,t} + s^*_{LA,t}\left(1-\alpha^*_{LA,t}\right) W_{LA,t},$$

we link investors' policy functions to the price-dividend ratio function as follows:

$$f\left(\omega_t\right) = \frac{s_{EZ}\left(\omega_t\right)\left[1-\alpha_{EZ}\left(\omega_t\right)\right]\left(1-\omega_t\right)+s_{LA}\left(\omega_t\right)\left[1-\alpha_{LA}\left(\omega_t\right)\right]\omega_t}{\alpha_{EZ}\left(\omega_t\right)\left(1-\omega_t\right)+\alpha_{LA}\left(\omega_t\right)\omega_t}. \tag{28}$$

To summarize, computing the equilibrium is involved with solving the seven unknown functions, $f(\cdot)$, $R_f(\cdot)$, $\alpha_{LA}(\cdot)$, $\alpha_{EZ}(\cdot)$, $s_{LA}(\cdot)$, $s_{EZ}(\cdot)$, and $\omega(\cdot,\cdot)$ from the system formed by equations (13)-(16), (22)-(24), (27) and (28). This system consists of seven independent equations: two value functions (equation [13]), two first-order conditions (equations [14]-[16]), two market clearing conditions (equations [22] and [28]), and a state variable evolution function (equation [27]). Equations (23) and (24) are intermediate steps for calculating the wealth dynamics.

Two remarks are in order. First, although the market is complete in the present project, the standard Pareto efficiency technique commonly used in the market-selection literature (e.g., Blume and Easley, 2006; Yan, 2008; Borovička, 2009; Kogan, Ross, Wang and Westerfield, 2009) cannot be applied here, as the LA-investor's preference depends not only on the intertemporal consumption plans but also on the endogenous stock return process *per se*, thereby making it necessary to explicitly solve the equilibrium. We therefore develop an algorithm based on Kubler and Schmedders (2003) to compute the Markov equilibrium and use simulations to analyze the survival and price impact of the LA-investor. The details of the algorithm are delegated to Appendix B.

Second, our analysis ignores the issue of the existence and uniqueness of the equilibrium. As is well-known in the literature, it is hard to establish the general results on the existence and uniqueness of the equilibria in heterogeneous agent models. Therefore, in the present paper, we simply start the analysis under the assumption that an equilibrium exists and use numerical methods to find this equilibrium. Rigorously speaking, a numerical method can never find the exact equilibrium; what it finds, if any, is the "ϵ-equilibrium" defined by Kubler and Schmedders (2003), who interpret the computed ϵ-equilibrium as an approximate equilibrium of some other economy with endowments and preferences that are close to those in the original economy.

4 Implications of Loss Aversion for Survival and Price Impacts

In this section, we first analyze the representative agent economies, that is, economies populated by homogeneous investors (see Subsection 4.1). This analysis serves two purposes. First, it verifies the result that loss aversion raises equity premiums, which is well-known in the literature (e.g., Benartzi and Thaler, 1995; Barberis, Huang and Santos, 2001). Second, it provides a useful springboard for our analysis of the heterogeneous agent economies, as it helps to develop the intuition for how loss aversion changes an investor's investment and saving behaviors.

We then move to the more realistic economies populated by both the EZ-investor and the LA-investor and apply the algorithm in Appendix B to numerically compute the equilibrium price functions, $f(\cdot)$, $R_f(\cdot)$, policy functions, $\alpha_{LA}(\cdot)$, $\alpha_{EZ}(\cdot)$, $s_{LA}(\cdot)$, $s_{EZ}(\cdot)$, and the state variable transition function, $\omega(\cdot,\cdot)$. We use simulations to show how loss aversion affects the investor's survival and pricing impact via portfolio decisions in Subsection 4.2 and via saving behaviors in Subsection 4.3. To isolate the role of loss aversion in determining the investor's survival prospects, in these two subsections, we assume that both investors have otherwise identical preferences except that the LA-investor derives loss aversion utility and the EZ-investor does not.

Before solving the models, we need to calibrate the parameter values. Because we are interested in the implications of preferences, we allow the preference parameters to vary over a certain range while fixing the four technology parameters in equation (1) for all computations and simulations. We interpret one period as one year and follow Mehra and Prescott (1985) in setting $\pi_H = \pi_L = \frac{1}{2}$ so that the economy is in booms and recessions with equal probability. Based on the data spanning the 20th century, the historical mean and volatility of the log consumption growth process are 1.84% and 3.79%, respectively (see

Barberis and Huang, 2009). To match these two moments, we set $\theta_H = 1.06$ and $\theta_L = 0.98$. Table I summarizes our choice of technology parameters.

TABLE I ABOUT HERE

4.1 Representative Agent Economy

In this subsection, we assume that the EZ-investor and the LA-investor have identical preferences; that is, $\beta_{EZ} = \beta_{LA} \equiv \beta$, $\rho_{EZ} = \rho_{LA} \equiv \rho$, $\zeta_{EZ} = \zeta_{LA} \equiv \zeta$ and $b_{EZ} = b_{LA} \equiv b$. As a result, the economy is the well-studied representative agent economy.

In this case, the representative agent has to hold the stock in equilibrium, so that the first-order condition given by equation (14) with $M_{i,t+1} = R_{t+1}$ defines the optimality of the investor's investment decision. As mentioned in the discussions after equation (22), the good-market clearing condition links the price-dividend ratios f_t to the optimal consumption policy α_t as follows:

$$D_t = (1 - \alpha_t)(P_t + D_t) \Rightarrow f_t = \frac{1 - \alpha_t}{\alpha_t}. \tag{29}$$

Therefore, equations (13), (14) and (29) define a system for three unknowns: f_t, α_t and $R_{f,t}$. Given the *i.i.d.* investment opportunities, we conjecture that

$$(f_t, \alpha_t, R_{f,t}) = (f, \alpha, R_f), \forall t. \tag{30}$$

The problem can be easily solved using any non-linear solver.

Table II reports the equilibrium equity premiums, risk-free rates and consumption policies for a variety of combinations of preference parameter values. For all combinations, we hold constant the time patience parameter β and the relative risk-aversion coefficient RA: $\beta = 0.98$ and $RA = 1$ (or $\zeta = 0$). The choice of β is motivated to match the low level of the risk-free rate. When EIS is equal to one (i.e., $\rho = 0$) and when there is no loss aversion utility (i.e., $b = 0$), setting $RA = 1$ or $\zeta = 0$ reduces the investor's preference to an expected log utility, which is an important benchmark case that the market-selection literature has been focusing on (Breiman, 1961; Hakansson, 1971; De Long, Shleifer, Summers, and Waldmann, 1991; Blume and Easley, 1992).

TABLE II ABOUT HERE

Panels A, B and C correspond to different values of EIS: $EIS = 1$ ($\rho = 0$), $EIS = 0.5$ ($\rho = -1$) and $EIS = 1.5$ ($\rho = 1/3$). To check the role of loss aversion, in each panel, parameter b, which controls the relative importance of the loss aversion utility in the investor's preference, is set at three different values: $b = 0$, $b = 0.02$ and $b = 0.2$, and parameter λ, which determines the degree of loss aversion, is set at two values: $\lambda = 2.25$ and $\lambda = 3$. When $b = 0$, the investor's preference does not exhibit loss aversion, and this economy has been well studied in the literature (e.g., Weil, 1989). When $b > 0$, the investor's preference exhibits loss aversion; such an economy is the focus of behavioral finance, such as Benartzi and Thaler (1995), Barberis, Huang and Santos (2001), and Barberis and Huang (2007, 2009). In particular, for both positive values of b in the table, 0.02 and 0.2, the investor's attitudes to independent *large* monetary gambles are sensible in the sense that both parameterizations of the investor's preference satisfy Barberis and Huang's condition L (2007, p 217); moreover, when $b = 0.02$, the investor's attitude to independent *small* monetary gambles is also sensible; that is, the parameterization corresponding to $b = 0.02$ satisfies Barberis and Huang's condition S (2007, p 219).[11] The choice of $\lambda = 2.25$ is based on the estimation of Tversky and Kahneman (1992).

[11] The literature cares about investors' attitudes to independent monetary gambles, as it was, in part, the difficulty that researchers encountered in reconciling the equity premium with these attitudes that launched the equity premium literature in the first place. Barberis and Huang's (2007) condition L is: "An individual with wealth of \$75,000 should not pay a premium higher than \$15,000 to avoid a 50:50 chance of losing \$25,000 or gaining the same amount." Their condition S is: "An individual with wealth of \$75,000 should not pay a premium higher than \$40 to avoid a 50:50 chance of losing \$250 or gaining the same amount."

Three notable patterns show up in Table II. The first pattern regards the equity premium. In all three panels, when $b = 0$, that is, when loss aversion is absent in the investor's preference, the equity premium is quite small (0.15%) relative to its historical value (6%), which is the well-known equity premium puzzle. Once loss aversion is introduced, the equity premiums are raised significantly. Say, when $b = 0.2$ and $\lambda = 3$, the model can generate an equity premium as high as 1.88%, which is more than ten times the equity premium corresponding to an economy populated by only EZ-investors. The increased equity premiums still fall short of the empirical value, as in our model, the stock is a claim to the *smooth* aggregate consumption process, and, as a result of the constant equilibrium price-dividend ratios (see equation [30]), the stock returns are not volatile enough to cause the loss averse investor to be scared of holding the stock.[12] As mentioned before in Section 2, this mismatch between the model-generated equity premium and the historical equity premium does not have any impact on our analysis. What really matters is that the LA-investor is more reluctant to hold the stock than the EZ-investor, which is also an assumption maintained in the behavioral finance studies relying on loss aversion to explain the equity premium puzzle.

The second pattern concerns the risk-free rate R_f. In all three panels, the risk-free rate decreases with b and λ.[13] This occurs because as the investor is more concerned about fluctuations in the value of his financial wealth and as he is more loss averse, he is more inclined to allocate wealth to the safe asset to avoid the potential painful losses associated with the risky asset. This suggests that in a heterogeneous agent economy populated by both the LA-investor and the EZ-investor, the bond is more attractive to the former than to the latter. When parameters b and λ are fixed, the risk-free rate also decreases with the magnitude of EIS: in a growing economy, a higher EIS makes the investor more likely to

[12] Barberis, Huang and Santos (2001) also study the pricing impact of loss aversion in a representative agent economy with dividends equal to consumption, and they report an equity premium of 1.26% as the relative risk aversion coefficient is equal to 1 (see the top part of their Table II), which is close to the equity premium generated in our model (1.4% when $b = 0.2$ and $\lambda = 2.25$).

[13] To be rigorous, the negative relationship between R_f and λ holds only when $b > 0$.

save, thereby depressing the interest rate.

The third pattern is about the consumption policy. When EIS is equal to one, the investor's saving ratio is optimally chosen to be equal to the time patience parameter, β. Therefore, in Panel A, the optimal consumption propensity α is independent of parameters b and λ. However, when EIS is different from 1, α varies with b and λ: α decreases (increases) with b and λ when EIS is less (greater) than 1 in Panel B (Panel C). As is standard in the portfolio choice problem for recursive preferences, two forces — the income effect and the substitution effect — are at play here. The asymmetric treatment of losses from gains in the loss aversion utility tends to lower the value, measured in utility terms, of the investor's future investment opportunities; that is, a higher b or λ tends to yield a lower B_t^* in equation (9). This lowered B_t^* has two effects on current consumption: it lowers consumption propensity through the income effect but raises consumption propensity through the substitution effect. When EIS is below 1, the income effect dominates, so that α decreases with b and λ; when EIS is above 1, the substitution effect dominates, and the dependence of α on b and λ reverses as a result. The different responses of α to b and λ in different cases of EIS suggest that how loss aversion affects the LA-investor's survival might depend on whether EIS is greater than or smaller than 1, as the literature suggests that saving behavior is a key determinant on survival. This will be examined in Subsection 4.3.

4.2 EIS=1: Portfolio Selection

In this subsection, we study the heterogeneous agent economy and fix EIS at 1, so that both investors optimally choose to have a constant consumption-wealth ratio: $\alpha_{i,t}^* = 1 - \beta_i$, for i =EZ, LA. We assume that the preferences of both investors are otherwise identical except that the LA-investor derives loss aversion utility, while the EZ-investor does not. So, except that $b_{LA} > 0$, $b_{EZ} = 0$, all other parameters are the same across investors: $\beta_{EZ} = \beta_{LA} \equiv \beta$, $\rho_{EZ} = \rho_{LA} \equiv \rho$ and $\zeta_{EZ} = \zeta_{LA} \equiv \zeta$. The assumption of a common time-patience parameter implies that both investors have the same endogenous saving rate. The focus of this subsection is therefore essentially how loss aversion changes the LA-investor's portfolio decision, which in turn affects the LA-investor's long run survival and his pricing impacts in a complete financial market.

4.2.1 Survival

We follow the market selection literature, such as Yan (2008) and Kogan, Ross, Wang and Westerfield (2009), in defining the "extinction","survival" and "dominance" of the LA-investor in terms of his wealth shares as follows.

Definition 2 *The LA-investor is said to become extinct or vanish if*

$$\lim_{t\to\infty} \omega(t) = 0, \ a.s.;$$

to survive if extinction does not occur; and to dominate the market if

$$\lim_{t\to\infty} \omega(t) = 1, \ a.s..$$

Our subsequent analysis suggests that the LA-investor will vanish via the channel of portfolio decisions if the presence of loss aversion in his preference causes him to be further from the log investor in terms of risk attitude than the EZ-investor. We further show that empirically relevant parameter values typically lead to this result, although the process is slow.

To illustrate how the LA-investor's wealth shares (ω_t) evolve over time, Table III reports their distributions at times t = 50, 100, and 1000 years when the LA-investor has initial wealth shares of $\omega_0 = 0.5$ and both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (panel B). To understand the role of loss aversion in determining the investor's survival prospects, we report the results for various values of b_{LA}, which determines the relative importance of loss aversion utility in the investor's overall preference, and of λ, which controls the degree of loss aversion. The technology parameters are fixed at the values in Table I and the time patience parameters are $\beta_{EZ} = \beta_{LA} = 0.98$.

TABLE III ABOUT HERE

Each entry in Table III has three elements corresponding, respectively, to the 5%, 50%, and 95% percentiles of the distributions of ω_t. Say, the first entry means that when $RA_{EZ} = RA_{EZ} = 1$, $b_{LA} = 0.02$ and $\lambda = 1$, starting from $\omega_0 = 0.5$, after 50 years, the LA-investor's wealth shares will be lower than 0.4645, 0.4980, or 0.5334 with probabilities of 5%, 50%, and 95%, respectively. These quantiles are obtained from simulations. We first use the algorithm described in Appendix B to solve the equilibrium state transition function $\omega(\cdot, \cdot)$ and then use it to simulate $N = 5000$ economies. For each economy, we make $T = 1000$ independent draws of θ_{t+1} from the distribution described in equation (1) to simulate a time series $\{\theta_{t+1}\}_{t=1}^{T}$. We then use the solved function $\omega(\cdot, \cdot)$ to calculate the next-period state ω_{t+1}. Finally the quantiles of ω_t are estimated from the 5,000 simulated sample paths at time t. In the following discussion, we focus on the 50% quantiles or the second element of each entry.

The results in Table III suggest that the insight in De Long, Shleifer, Summers, and Waldman (1991) and Blume and Easley (1992) holds for recursive preferences in a general equilibrium setting: the rate at which an investor's wealth grows depends on how close his/her preference is to log utility, i.e., how close his/her coefficient of relative risk aversion is to one. In Panel A of Table III, the EZ-investor is the log investor, since $RA_{EZ} = EIS_{EZ} = 1$. We can see that for all combinations of parameters b_{LA} and λ, the LA-investor's wealth shares shrink as time passes. This suggests that in an economy populated with the log investor and the LA-investor, it is always the log investor who accumulates wealth at a faster rate.

In Panel B of Table III, we change the relative risk aversion coefficient of both investors from 1 to 3, so that the EZ-investor is no longer the log investor. In this case, we observe that the LA-investor sometimes survives, while at other times, he vanishes: when $\lambda = 1$, the LA-investor's wealth shares increase over time, while when $\lambda = 1.5$, 2.25, or 3, his wealth shares decrease over time. When $RA_{EZ} = 3$, the EZ-investor is more risk averse than the log investor. If λ is close to 1, the loss aversion utility in the LA-investor's preference is close to the expected gains/losses (i.e., $b_{LA}E_t\left[v\left(G_{LA,t+1}\right)\right] \approx b_{LA}E_t\left(G_{LA,t+1}\right)$). This causes the LA-investor's preference *as if* to be generated from combining the EZ-investor's preference and the risk neutral investor's preference, leading the LA-investor to hold portfolios corresponding to a greater risk tolerance. Therefore, the LA-investor can potentially be closer to the log investor in terms of risk attitude than the EZ-investor, which explains his survival for the case of $\lambda = 1$. On the other hand, if λ is much greater than 1, as we believe is likely, loss aversion penalizes losses much stronger than it rewards gains, making the LA-investor reluctant to invest in the volatile stocks. This causes the LA-investor to mimic an investor who is more risk averse, and hence further from log utility, than the EZ-investor. Therefore, the LA-investor vanishes in the long run.

Table IV verifies the above intuitions. The first row of both panels shows the value of the equity premium, EP_{LA}, in the representative agent economy populated with the LA-investor when the model parameters except b_{LA} and λ take the same values as those in Panel B of Table III. The second and third rows compare EP_{LA} to $EP_{\log}$ and EP_{EZ}, the equity premiums emerging from two different representative agent economies populated with the log investor and with the EZ-investor, respectively. This comparison helps to determine whether the LA-investor is closer to the log investor than the EZ-investor, and hence whether he survives. For instance, when $\lambda = 1.5$, 2.25, or 3, we have $EP_{LA} > EP_{EZ} > EP_{\log}$, which suggests that both the LA-investor and the EZ-investor are more risk averse than the log investor and that the LA-investor is more risk averse than the EZ-investor. Thus, the EZ-investor's preference is closer to the log utility and the LA-investor vanishes. In contrast, when $\lambda = 1$, if $b_{LA} = 0.02$, we have $EP_{EZ} > EP_{LA} > EP_{\log}$, suggesting that the EZ-investor is more risk averse than the LA-investor, who is in turn more risk averse than the log investor. As a result, the LA-investor is closer to the log investor and survives. When $\lambda = 1$ and $b_{LA} = 0.2$, we have $EP_{EZ} > EP_{\log} > EP_{LA}$, and in this case, we cannot determine whose preference is closer to log utility, because the EZ-investor is more risk averse than the log investor, while the LA-investor is more risk tolerant than the log investor.

TABLE IV ABOUT HERE

Another message conveyed by Table III regards the speed of the market selection process. In terms of wealth shares, the process is slow.[14] For example, for those parameterizations with $\lambda > 1$ considered in Table III, after 50 years, on a typical sample path, the LA-investor loses less than 30% of his initial wealth share; even after 100 years, he still has more than half of his initial wealth share. This is consistent with Yan (2008) who shows that it takes hundreds of years for an investor with incorrect beliefs to lose half of his wealth share in an economy populated with heterogeneous expected utility maximizers.

4.2.2 Price impact

The effectiveness of the market selection mechanism should be judged by how the price impacts and not the wealth shares of the LA-investor change over time, as what one really cares about is whether the behavior of asset prices can largely be captured by models without the LA-investor.

We use the ratio of the conditional equity premium in the heterogeneous agent economy to that in the representative agent economy with only the EZ-investor to capture the price

[14] One might argue that if we measure time at a frequency higher than annual, say, if we interpret one period as one day, then the selection process might be quicker. However, doing so also requires us to recalibrate the consumption process at a higher frequency, and after this recalibration, we find that the results do not change.

impact of the LA-investor at state ω_t; that is,

$$\text{price impact of loss aversion at state } \omega_t = EP_t/EP_{EZ},$$

where $EP_t = E\left(R_{t+1} - R_{f,t}|\omega_t\right)$ and EP_{EZ} is the equity premium in an otherwise identical economy but populated with only the EZ-investor. Although we focus on the implication of loss aversion for equity premiums, we believe that our intuitions can be extended to its implications for other dimensions of asset prices. For example, Barberis, Huang and Santos (2001) rely on dynamic loss aversion (i.e., a combination of loss aversion and "the house money effect") to generate excess volatility of stock returns in a representative agent economy. If loss aversion does not affect equity premiums in the heterogeneous agent economy in the long run, there is no reason to believe that introducing "the house money effect" will produce excess volatility in such an economy.

TABLE V ABOUT HERE

Table V reports the dynamics of EP_t/EP_{EZ} in a variety of economies. The parameters take the same values as those in Table III. Unlike Table III, Table V does not report results for $\lambda = 1$ or 1.5: the case of $\lambda = 1$ is not empirically plausible, while the results in the case of $\lambda = 1.5$ are similar to those in cases of $\lambda = 2.25$ or 3.[15] In addition, unlike Table III where the LA-investor starts with half of the aggregate wealth, Table V assumes that the LA-investor's initial wealth shares can be either $\omega_0 = 0.5$ or $\omega_0 = 0.9$, because, as will become clear shortly, the dynamics patterns of the price impact are slightly different in these two scenarios. For each economy of Table V, in the parenthesis after the value of λ, we report the equity premiums in percentages, EP_{LA} and EP_{EZ}, in the representative agent economies with the LA-investor and with the EZ-investor, respectively. Then, the first row presents

[15] In Table III, we are interested in these two cases, because we want to understand whether the insight on survival in Blume and Easley (1992) can be extended to the recursive preferences.

the value of EP_t/EP_{EZ} at date 0, and the second to the fourth rows present the 5%, 50% and 95% quantiles of EP_t/EP_{EZ} at dates $t = 50$, 100 and 1000, respectively. The quantiles of EP_t/EP_{EZ} are obtained using simulations in the same way as the quantiles of ω_t are obtained in Table III.

We observe that in all cases, as time passes, the LA-investor gradually loses his impact on the equity premiums, because the medians of EP_t/EP_{EZ} gradually decline. For example, in the first economy of Panel (A1), when $\omega_0 = 0.5$, EP_t/EP_{EZ} starts with a value of around 2, after 50 years, it declines to 1.82, and after 100 years, it further drops to 1.68. This result is not surprising given that, according to Table III, the LA-investor's wealth shares shrink over time for parameterizations considered in Table V.

In terms of price impacts, the market selection mechanism can be quite effective in the following two senses. First, in the heterogeneous agent economy, the LA-investor might optimally choose not to participate in the stock market, so that the conditional equity premium is determined by the first-order condition of the EZ-investor and not directly affected by loss aversion, leading to a substantially lower equity premium. This occurs when ω_t is not very high, that is, when the EZ-investor controls a large fraction of the aggregate wealth and her trading lowers the conditional equity premium and makes the stock not attractive to the LA-investor. For example, in the second economy of Panel (A1), EP_{LA}/EP_{EZ} is around 8; that is, in representative agent economies, loss aversion raises equity premiums by as much as eight times the value produced by Epstein-Zin preferences. However, in the heterogeneous agent economy, when $\omega_0 = 0.5$, EP_t/EP_{EZ} drops to a value around 2. So, introducing heterogeneity significantly reduces the equity premium. This is consistent with Chapman and Polkovnichenko (2009) who show that the risk premium is sensitive to ignoring heterogeneity in a static model in which investors exhibit first-order risk aversion.

Second, even when ω_0 is high and the price impact of loss aversion is initially big, it is likely that a small drop in wealth shares leads to a large drop in price impact. For instance, in the second economy of Panel (A1), when $\omega_0 = 0.9$, EP_t/EP_{EZ} is around 7 at time

0. However, after 50 years, it drops sharply to 3.41. Figure 1 displays EP_t/EP_{EZ} in the left panel and the LA-investor's investment policy function $s_{LA,t}$ in the right panel for this economy. There is a kink in the price impact function, EP_t/EP_{EZ}, in the left panel, and a decline in ω_t leads to the largest drop of EP_t/EP_{EZ} at the kink. Comparing Panel (a) with Panel (b), it can be seen that the location of the kink of EP_t/EP_{EZ} is determined by the level of the wealth share at which the LA-investor starts to buy the stock. So, if the LA-investor's initial wealth share happens to be around the level at which he is just willing to purchase the stock, then a small decline of his wealth share will cause him to switch to holding the risk-free asset only, which lowers EP_t/EP_{EZ} sharply.

FIGURE 1 ABOUT HERE

To summarize, for the unit-EIS case, if the LA-investor differs from the EZ-investor only in terms of loss aversion utility, that is, except for $b_{LA} > 0$ and $b_{EZ} = 0$, the other preference parameters are identical across investors, then for empirically relevant values (i.e., when $\lambda = 1.5$, 2.25 and 3 in Table III), the LA-investor vanishes in the long run, and, as a result, so does his price impact. The market selection process is slow in terms of wealth shares, but in terms of price impact, the selection mechanism can be effective. In the following analysis, we will focus only on the dynamics of wealth shares, as the relationship between the wealth share dynamics and the pricing impact dynamics obtained in this subsection is still valid in the case of a non-unit EIS.

4.3 EIS$\neq$1: Saving Behavior

The analysis in Subsection 4.1 suggests that when EIS is not equal to 1, loss aversion can change the investor's saving behavior, which might affect the investor's wealth accumulation and survival prospects. In this subsection, we investigate this possibility in the heterogeneous agent economy. Again, we assume that the preferences of both investors are identical except that $b_{LA} > 0$ and $b_{EZ} = 0$.

When $EIS_{EZ} = EIS_{LA} > 1$, the intuition in the representative agent economies implies that the LA-investor consumes more than the EZ-investor, which hurts his survival prospects. Because the previous subsection shows that in the absence of different saving behaviors, the LA-investor already loses to the EZ-investor; then, this extra force coming from saving should cause the LA-investor to vanish at a faster speed. This is indeed the case, as verified by Figure 2, which plots the distributions of ω_t in years 50, 100, and 1000, for the case of $EIS_{EZ} = EIS_{LA} = 1.5$, when each investor has half of the total wealth at time 0. The distributions are obtained in the same way as in Table III, and the technology parameters are fixed at the values in Table I, while the other preference parameters are $\beta_{EZ} = \beta_{LA} = 0.98$, $RA_{EZ} = RA_{LA} = 1$, $\lambda = 2.25$ and $b_{LA} = 0.02$. Indeed, the p.d.f.s of ω_t shift to the left as time passes, suggesting that the LA-investor is losing his wealth share over time.

FIGURE 2 ABOUT HERE

When $EIS_{EZ} = EIS_{LA} < 1$, the analysis in Subsection 4.1 suggests that the LA-investor saves more than the EZ-investor, which favors his survival. As a result, two forces are at play here: portfolio decisions are against the LA-investor's wealth accumulation, while consumption decisions benefit it. It thus becomes nontrivial to explore whether the saving force is strong enough to reverse the result in the previous subsection. Table VI presents the distributions of ω_t and EP_t/EP_{EZ} in Panels A and B for the case of $EIS_{EZ} = EIS_{LA} = 0.5$. The other parameter values are identical to those used in Table IV. We here assume that both investors have a relative risk aversion coefficient of 1, because the key insight in Subsection 4.2 is that the role of loss aversion is making the LA-investor more risk averse than the EZ-investor, and thus the result will be insensitive to the choice of the relative risk aversion coefficient, as long as the LA-investor is maintained to be more cautious in buying the stock. Comparing Table VI with Tables III and V, one can find that the results are almost identical, implying that in calibrated economies, the difference in saving behaviors induced by the common small EIS shared by both investors is not large enough to help the LA-investor survive in the long run.

TABLE VI ABOUT HERE

To better understand the effect of savings, Figure 3 plots the consumption policies for both investors for the parameter configuration considered in the first economy of Panel (A2) of Table VI ($b_{LA} = 0.2$ and $\lambda = 2.25$). Figure 3 shows that, although the difference in the endogenous saving rates of the two investors can be large, with a maximum of 3.5 percent achieved at $\omega_t = 1$, this difference shrinks sharply, as the LA-investor loses wealth over time. This weakens the effect of the saving difference on changing the LA-investor's survival prospect, as once his wealth-eroding investment positions start to reduce his wealth share, he also saves less, making his situation even worse in terms of wealth accumulation. This explains why the difference in the endogenous saving behavior cannot overcome the disadvantage coming from the portfolio positions of the LA-investor in terms of wealth accumulation.

FIGURE 3 ABOUT HERE

In sum, for the non-unit EIS case, if the LA-investor differs from the EZ-investor only in a way such that he derives loss aversion utility, then for empirically relevant parameter values, he will lose his wealth share in the long run, and his price impacts diminish along the way.

5 Multi-Dimensional Heterogeneity in Preferences

So far, our analysis has assumed that the LA-investor and the EZ-investor are different only in one dimension: the LA-investor derives loss aversion utility, while the EZ-investor does not. However, it is highly likely that they are also different in other dimensions. This raises the question of how robust the result that the LA-investor vanishes in calibrated economies is to the introduction of additional differences in the investors' preferences.

Before examining the effect of the multi-dimensional heterogeneity of preferences on survival, we briefly discuss what kind of heterogeneity might be plausible in reality. In principle, on top of loss aversion utility, investors can be different in the following three dimensions: risk aversion (parameter ζ), EIS (parameter ρ), and time preference (parameter β). Risk aversion might not be a good candidate, as the very reason why the literature introduces loss aversion is to increase the LA-investor's risk aversion, which serves to generate a high equity premium. Therefore, to make the analysis empirically relevant, any perturbation of the risk aversion parameter ζ should not reverse the order of the investors' risk attitude and would not change the survival prospects of the LA-investor.

However, in the existing literature, researchers have not reached a consensus regarding the "reasonable" value for the EIS or the time discount rate. Some studies estimate the EIS to be well above 1 (e.g., Hansen and Singleton, 1982; Attanasio and Weber, 1989; Guvenen, 2001; Vissing-Jorgensen, 2002), while others estimate it to be well below 1 (e.g., Hall, 1988; Epstein and Zin, 1991; Campbell, 1999).[16] Similarly, the calibrations of the time-patience parameter β are widely dispersed, ranging from 0.89 (Campbell and Cochrane, 1999) to 1.1 (Brennan and Xia, 2001). Therefore, in the following, we investigate the effect of a differing EIS or a differing time-patience parameter on the LA-investor's survival.

[16] See Guvenen (2006) for a comprehensive review of the empirical evidence on the heterogeneity in the EIS across the population.

5.1 Different EIS Parameters

To set the deck against the EZ-investor, we need to assume that the LA-investor has a larger EIS than the EZ-investor, so that he would save more than the EZ-investor in a growing economy. Specifically, we set $EIS_{EZ} = 0.5$, and increase EIS_{LA} to see when the LA-investor will survive for an economy with the technology parameters fixed at the values in Table I and the other preference parameters fixed at $\beta_{EZ} = \beta_{LA} = 0.98$, $RA_{EZ} = RA_{LA} = 1$, $\lambda = 2.25$ and $b_{LA} = 0.02$. It turns out that when $EIS_{LA} = 0.7$, the LA-investor starts to dominate the economy. The result is driven by the different saving behaviors induced by the different EIS. Figure 4 displays the dynamics of ω_t in Panel (a) and the consumption policies in Panel (b) assuming that $\omega_0 = 0.5$. Panel (a) shows that the p.d.f.s shift to the right as time passes, suggesting that the LA-investor tends to dominate the market in the long run. Panel (b) shows that the difference in the consumption ratios does not drop much when ω_t declines from intermediate levels of ω_t, because when ω_t decreases, the EZ-investor consumes more as a result of a strong income effect of the raised risk-free rate. Therefore, when the wealth share of the LA-investor declines due to his portfolio decisions, his advantage in terms of saving behavior will help him.

FIGURE 4 ABOUT HERE

We have also tried other perturbations, and the result is qualitatively similar, although if we start from a larger value of EIS_{EZ}, a larger difference between EIS_{LA} and EIS_{EZ} is needed to make the LA-investor survive in the long run. For instance, when the EZ-investor has an EIS of 0.8, the critical level for EIS_{LA} increases to 1.5. This is because when EIS_{EZ} is large, the EZ-investor's saving rate has already been very high in a growing economy, leaving very little room for the LA-investor to improve. Whether the differences in investors' EIS parameters are "reasonable" is an empirical issue and is subjective. The point of the present paper is to provide a framework that can be used to analyze under what conditions the LA-investor survives and his pricing impact persists.

5.2 Different Time Patience Parameter β

We conduct a similar exercise as in examining the effect of different EIS parameters. Specifically, we set $\beta_{LA} = 0.98$ and decrease β_{EZ} to examine when the LA-investor dominates the market in the long run. The survival result is very sensitive to the time discount rate: a slight difference in β as small as two percent can overturn the effect of the LA-investor's portfolio decisions on his survival prospects. To illustrate this sensitivity, we set $EIS_{EZ} = EIS_{LA} = 1.5$, which means that the deck is set against the LA-investor, as he would consume more than the EZ-investor if they had a common β. Other preference parameters are fixed at $RA_{EZ} = RA_{LA} = 1$, $\lambda = 2.25$ and $b_{LA} = 0.02$, and the technology parameters are fixed at the values in Table I. The result is robust to different relative risk aversions. Figure 5 depicts the dynamics of the distributions of ω_t and investors' consumption propensities when the LA-investor has half of the total wealth at time 0. Panel (a) shows that, as time passes, the p.d.f.s of ω_t shift to the right, suggesting that the LA-investor is accumulating wealth at a faster rate than the EZ-investor. Panel (b) displays the large difference in the endogenous saving ratios induced by the time-patience parameter. The minimum of this difference is 1.88%, and the maximum is 2.86%. These large magnitudes account for the LA-investor's eventual prosperity.

FIGURE 5 ABOUT HERE

6 Concluding Remarks

This paper studies the survival and price impact of loss averse investors in a financial economy in which "arbitrageurs" have Epstein-Zin preferences. We find that if the LA-investor differs from the EZ-investor only in the way of deriving loss aversion utility, then the LA-investor will be driven out of the market and thus will have no effect on long run asset prices for an empirically relevant range of parameters. In the short run, the selection process is slow in terms of wealth shares, but it can be effective in terms of price impacts. Once additional heterogeneity is recognized in investors' preferences, for example, when they are heterogeneous with respect to their EIS or time-patience parameters, the LA-investor can survive because of the different equilibrium saving behaviors induced by this new heterogeneity. This paper thus helps us to understand under what conditions loss aversion can affect asset prices in a dynamic financial market. Empirical studies are needed to examine whether and to what extent the real investors who exhibit loss aversion are different from those who do not in terms of their EIS or time-patience parameters, which in turn, with the help of the our framework, is useful in determining the validity of the statement made in the representative agent models in behavioral finance.

We conclude by discussing the robustness of our results and some generalizations of the model developed above. As we mentioned in the main text, our results are robust to the introduction of narrow-framing, and although our focus on the price impact of loss aversion has been limited to equity premiums, we believe that the intuitions can arguably apply to other dimensions of asset prices, such as the excess volatility studied in Barberis, Huang and Santos (2001) and the GARCH effect in stock returns studied in McQueen and Vorkink (2004), by introducing additional features, such as the "house money effect" in Barberis, Huang and Santos (2001) and "state-dependent sensitivity to news" in McQueen and Vorkink (2004), into the LA-investor's preferences.

The analysis in our previous sections has been conducted under the technology parameter configuration specified in Table I. We have verified that most of our results remain unchanged under different parameter values, as long as the volatility of the consumption growth rate is not unreasonably high (say, lower than 20% on an annual basis), given that loss aversion is one form of first-order risk aversion and thus has its largest impact on investor's risk attitude for gambles of small or intermediate volatilities.

Our model allows for many possible generalizations. First, it has assumed a complete market structure. The survival result might be very different in an incomplete market, as suggested by the existing literature (e.g., Blume and Easley, 2006; Cao, 2009). It would thus be interesting to compute how market incompleteness would change the results. Second, this paper focuses only on the loss aversion feature of prospect theory and ignores its two other features, namely, diminishing sensitivity and probability weighting. The literature has shown that both features help to explain certain financial phenomena. For example, Li and Yang (2008) show that diminishing sensitivity can generate price momentum, while Barberis and Huang (2008) argue that probability weighting leads to the overpricing of positively skewed securities. It would also be interesting to examine the survival and price impacts of an investor whose preference has all three features of prospect theory. Third, the loss -averse investors are homogeneous in our model. It is likely that even loss averse investors are heterogeneous in a number of ways: in the degree of sensitivity to losses (parameter λ), in the relative importance of loss aversion utility in their preferences (parameter b_i)[17] or in the reference levels that determine their gains/losses. We believe incorporating this heterogeneity would not change our results dramatically. Finally, in our model, both investors live and trade for ever. In reality, it is likely that investors voluntarily enter and exit the economies for some exogenous reasons, such as life cycles. We can embed our current setting into an overlapping generation model and assume that in each period, investors with different levels of loss aversion bias are replenished with some probability. We leave all these interesting questions for future research.

[17] In fact, the difference between the LA-investor and the EZ-investor in our model can be viewed as an extreme case of differing b_i: $b_{EZ} = 0$ and $b_{LA} > 0$.

Appendix

A. First-Order Conditions for the Case of EIS=1

This appendix derives the conditions that define the investor's optimal decisions when the EIS takes the value of 1 ($\rho_i = 0$). In this case, the aggregator function has the Cobb-Douglas form:

$$H_i(C, X) = C^{1-\beta_i} X^{\beta_i}.$$

The Bellman equation becomes

$$\begin{aligned} A_{i,t} W_{i,t} &= \max_{C_{i,t}, s_{i,t}} C_{i,t}^{1-\beta_i} \left[\mu_i \left[J_i \left(W_{i,t+1}, I_{t+1} \right) | I_t \right] + b_i E_t \left[v \left(G_{i,t+1} \right) \right] \right]^{\beta_i} \\ &= W_{i,t} \max_{\alpha_{i,t}, s_{i,t}} \alpha_{i,t}^{1-\beta_i} \left(1 - \alpha_{i,t} \right)^{\beta_i} \left[\mu_i \left[A_{i,t+1} M_{i,t+1} | I_t \right] + b_i E_t \left[v \left(s_{i,t} \left(R_{t+1} - R_{f,t} \right) \right) \right] \right]^{\beta_i} \\ &= W_{i,t} \max_{\alpha_{i,t}} \alpha_{i,t}^{1-\beta_i} \left(1 - \alpha_{i,t} \right)^{\beta_i} \max_{s_{i,t}} \left[\mu_i \left(A_{i,t+1} M_{i,t+1} | I_t \right) + b_i E_t \left[v \left(s_{i,t} \left(R_{t+1} - R_{f,t} \right) \right) \right] \right]^{\beta_i}. \end{aligned}$$

Therefore, the optimal consumption policy can be explicitly solved:

$$\max_{\alpha_{i,t}} \alpha_{i,t}^{1-\beta_i} \left(1 - \alpha_{i,t} \right)^{\beta_i} \Rightarrow \alpha_{i,t}^* = 1 - \beta_i. \tag{31}$$

As a result,

$$A_{i,t} = \left(1 - \beta_i \right)^{1-\beta_i} \beta_i^{\beta_i} \max_{s_{i,t}} \left[\mu_i \left(A_{i,t+1} M_{i,t+1} | I_t \right) + b_i E_t \left[v \left(s_{i,t} \left(R_{t+1} - R_{f,t} \right) \right) \right] \right]^{\beta_i}. \tag{32}$$

The partial equilibrium problem is therefore summarized by the above equation, which involves solving the optimal investment-decision function, $s_i(\cdot)$, and the value function, $A_i(\cdot)$. So, relative to the case of a non-unit EIS, one can avoid numerically solving the investor's consumption policy, as it is given by equation (31), but he needs to numerically solve the investor's indirect value function using equation (32).

The first-order conditions to the portfolio choice problem are

$$
\begin{aligned}
FOC_{i,+} &= \left[E_t(A_{i,t+1}^{\zeta_i} M_{i,t+1}^{\zeta_i})\right]^{1/\zeta_i - 1} E_t[A_{i,t+1}^{\zeta_i} M_{i,t+1}^{\zeta_i - 1} (R_{t+1} - R_{f,t})] + b_i E_t \left[v \left(R_{t+1} - R_{f,t}\right)\right] \\
&= 0, \text{ for } s_{i,t}^* > 0,
\end{aligned}
$$

$$
\begin{aligned}
FOC_{i,-} &= \left[E_t(A_{i,t+1}^{\zeta_i} M_{i,t+1}^{\zeta_i})\right]^{1/\zeta_i - 1} E_t[A_{i,t+1}^{\zeta_i} M_{i,t+1}^{\zeta_i - 1} (R_{t+1} - R_{f,t})] - b_i E_t \left[v \left(R_{f,t} - R_{t+1}\right)\right] \\
&= 0, \text{ for } s_{i,t}^* < 0,
\end{aligned}
$$

$$
FOC_{i,+} \leq 0 \text{ and } FOC_{i,-} \geq 0, \text{ for } s_{i,t}^* = 0.
$$

In particular, for the EZ-investor, $b_{EZ} = 0$, and the above first-order conditions boil down to

$$
E_t[A_{EZ,t+1}^{\zeta_{EZ}} M_{EZ,t+1}^{\zeta_{EZ} - 1} (R_{t+1} - R_{f,t})] = 0.
$$

B. Numerical Algorithm

This appendix sketches the procedure used to numerically solve the model. I focus on the non-unit EIS case ($\rho_i \neq 0$), and the solution procedure for the unit EIS case is slightly different. The algorithm is developed based on Kubler and Schmedders (2003) and is summarized as follows.

Step 0: Define a finite grid on [0,1]. Choose two continuous functions, $\alpha_{EZ}^0(\cdot)$ and $\alpha_{LA}^0(\cdot)$, as initials for the investors' consumption policy functions. These initials define the initial for the price-dividend ratio function, $f^0(\cdot)$, through equation (22). Then on each grid point ω_t, go through steps 1-4.

Step 1: Given functions $\alpha_{EZ}^n(\cdot)$ and $\alpha_{LA}^n(\cdot)$, suppose that the LA-investor allocates nothing on the stock; that is, $s_{LA}^{n+1}(\omega_t) = 0$. Then use both investors' value functions, equation (13), the EZ-investor's first-order condition, equation (17), and the state transition functions, equation (27), to solve five unknowns: $\alpha^*_{EZ,t}$, $\alpha^*_{LA,t}$, $R_{f,t}$, $\omega_{t+1,H}$, $\omega_{t+1,L}$, where $\omega_{t+1,H}$ and $\omega_{t+1,L}$ are the next-period wealth shares when $\theta_{t+1} = \theta_H$ and θ_L, respectively.

Step 2: Plug the solved $\alpha^*_{LA,t}$, $R_{f,t}$, $\omega_{t+1,H}$ and $\omega_{t+1,L}$ into equations (14) and (15) to get $FOC_{LA,+}$ and $FOC_{LA,-}$. If $FOC_{LA,+} \leq 0$ and $FOC_{LA,-} \geq 0$, then set $\alpha^{n+1}_{EZ}(\omega_t) = \alpha^*_{EZ,t}$ and $\alpha^{n+1}_{LA}(\omega_t) = \alpha^*_{LA,t}$. If $FOC_{LA,+} > 0$, then go to Step 3; otherwise, go to Step 4.

Step 3: Use both investors' value functions, equation (13), the EZ-investor's first-order equation, (17), the LA-investor's first-order condition for a positive investment, equation (14), and the state transition functions, equation (27), to solve six unknowns: $\alpha^*_{EZ,t}$, $\alpha^*_{LA,t}$, $R_{f,t}$, $\omega_{t+1,H}$, $\omega_{t+1,L}$, $s^*_{LA,t}$. Set $\alpha^{n+1}_{EZ}(\omega_t) = \alpha^*_{EZ,t}$ and $\alpha^{n+1}_{LA}(\omega_t) = \alpha^*_{LA,t}$.

Step 4: Use both investors' value functions, equation (13), the EZ-investor's first-order equation, (17), the LA-investor's first-order condition for a negative investment, equation (15), and the state transition functions, equation (27), to solve six unknowns: $\alpha^*_{EZ,t}$, $\alpha^*_{LA,t}$, $R_{f,t}$, $\omega_{t+1,H}$, $\omega_{t+1,L}$, $s^*_{LA,t}$. Set $\alpha^{n+1}_{EZ}(\omega_t) = \alpha^*_{EZ,t}$ and $\alpha^{n+1}_{LA}(\omega_t) = \alpha^*_{LA,t}$.

Step 5: Check whether the following stop criterion is satisfied:

$$\max_{\omega_t} \left\| \left(\alpha^{n+1}_{EZ}(\cdot), \alpha^{n+1}_{LA}(\cdot), f^{n+1}(\cdot)\right) - \left(\alpha^{n}_{EZ}(\cdot), \alpha^{n}_{LA}(\cdot), f^{n}(\cdot)\right) \right\| < \tau,$$

where τ is an error tolerance. If yes, then the algorithm terminates, and the next step is to set the consumption and investment policy functions and the risk-free rate function as those solved in the last round. Otherwise, increase n by 1 and go to Step 1.

In the implementation of the algorithm, I divide $[0, 1]$ into 150 grid points and set the tolerance level at 10^{-7}. Kubler and Schmedders (2003) provide a method to assess the accuracy of a candidate solution by computing the maximal relative error in Euler equations. In my computations, the maximum errors lie below 10^{-6}, suggesting that the algorithm produces reasonably accurate solutions.

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Table I Technology Parameter Values

This table reports the technology parameter values used in the computation of equilibria. The calibration takes one period to be one year. The consumption growth rate parameters θ_H and θ_L are calibrated to match the historical mean (1.84%) and volatility (3.79%) of the log consumption growth rate.

Parameters	π_H	π_L	θ_H	θ_L
Values	0.5	0.5	1.0579	0.98069

Table II Asset Prices and Consumption Policies in Representative Agent Economies

This table reports the equilibrium equity premiums (EP), risk-free rates (R_f) and consumption policies (α) in percentages, assuming that investors are identical in preferences, so that $\beta_{EZ} = \beta_{LA} \equiv \beta$, $EIS_{EZ} = EIS_{LA} \equiv EIS$ (or $\rho_{EZ} = \rho_{LA} \equiv \rho$), $RA_{EZ} = RA_{LA} \equiv RA$ (or $\zeta_{EZ} = \zeta_{LA} \equiv \zeta$) and $b_{EZ} = b_{LA} \equiv b$. The equity premiums are calculated as EP = E(R_{t+1} - R_f). For all combinations, β =0.98 and RA=1. The technology parameters are fixed at the values in Table I. Panels A, B and C correspond to different values of EIS: EIS=1 (ρ=0), EIS=0.5 (ρ=-1) and EIS=1.5 (ρ=1/3). Parameter b controls the relative importance of loss aversion utility in the investor's preferences. Parameter λ controls the degree of loss aversion.

Panel A: *EIS* = 1

	λ = 2.25			λ = 3		
	EP (%)	R_f -1 (%)	α (%)	EP (%)	R_f -1 (%)	α (%)
b = 0	0.15	3.86	2.00	0.15	3.86	2.00
b = 0.02	0.79	3.22	2.00	1.19	2.82	2.00
b = 0.2	1.41	2.60	2.00	1.88	2.13	2.00

Panel B: *EIS* = 0.5

	λ = 2.25			λ = 3		
	EP (%)	R_f -1 (%)	α (%)	EP (%)	R_f -1 (%)	α (%)
b = 0	0.15	5.79	3.79	0.15	5.79	3.79
b = 0.02	0.67	4.76	3.31	1.02	4.04	2.98
b = 0.2	1.39	3.29	2.63	1.87	2.32	2.16

Panel C: *EIS* = 1.5

	λ = 2.25			λ = 3		
	EP (%)	R_f -1 (%)	α (%)	EP (%)	R_f -1 (%)	α (%)
b = 0	0.15	3.23	1.40	0.15	3.23	1.40
b = 0.02	0.85	2.76	1.62	1.25	2.50	1.75
b = 0.2	1.42	2.38	1.80	1.88	2.08	1.94

Table III Survival of the LA-Investor: EIS = 1

This table reports the distributions of the LA-investor's wealth shares (ω_t) at times t=50, 100, and 1000 years when the LA-investor has initial wealth shares of ω_0=0.5 and both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (panel B). Both investors have a unit EIS: $EIS_{EZ} = EIS_{LA} = 1$. They have the same time patience parameter: $\beta_{EZ} = \beta_{LA} = 0.98$. The technology parameters are fixed at the values in Table I. Each entry in this table has three elements, corresponding, respectively, to the 5%, 50%, and 95% percentiles of the distributions of ω_t. The quantiles are estimated from the 5000 simulated sample paths at time t.

Panel A: 5%, 50%, and 95% percentiles of ω_t when $RA_{EZ} = RA_{LA} = 1$ ($\zeta_{EZ} = \zeta_{LA} = 0$)

(A1) $b_{LA} = 0.02$

	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2.25$	$\lambda = 3$
t=50	[0.4645 0.4980 0.5334]	[0.3054 0.4777 0.6573]	[0.3022 0.4523 0.6677]	[0.3022 0.4523 0.6674]
t=100	[0.4447 0.5013 0.5509]	[0.2311 0.4224 0.7107]	[0.2296 0.4059 0.6928]	[0.2296 0.4058 0.6836]
t=1000	[0.3240 0.4789 0.6356]	[0.0259 0.1511 0.7772]	[0.0246 0.1282 0.4603]	[0.0243 0.1269 0.4523]

(A2) $b_{LA} = 0.2$

	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2.25$	$\lambda = 3$
t=50	[0.3447 0.4776 0.6144]	[0.3022 0.4523 0.6677]	[0.3022 0.4523 0.6672]	[0.3022 0.4523 0.6672]
t=100	[0.2530 0.4906 0.6638]	[0.2296 0.4059 0.6947]	[0.2296 0.4058 0.6826]	[0.2296 0.4058 0.6815]
t=1000	[0.0005 0.2035 0.8079]	[0.0246 0.1283 0.4637]	[0.0243 0.1267 0.4506]	[0.0243 0.1266 0.4494]

Panel B: 5%, 50%, and 95% percentiles of ω_t when $RA_{EZ} = RA_{LA} = 3$ ($\zeta_{EZ} = \zeta_{LA} = -2$)

(B1) $b_{LA} = 0.02$

	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2.25$	$\lambda = 3$
t=50	[0.4734 **0.5058** 0.5441]	[0.4479 0.4879 0.5209]	[0.2556 0.3964 0.5499]	[0.2464 0.3633 0.5351]
t=100	[0.4634 **0.5212** 0.5715]	[0.4123 0.4657 0.5249]	[0.1646 0.2867 0.5138]	[0.1590 0.2755 0.4671]
t=1000	[0.5177 **0.6659** 0.7851]	[0.1104 0.2146 0.3605]	[0.0014 0.0083 0.0429]	[0.0013 0.0079 0.0414]

(B2) $b_{LA} = 0.2$

	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2.25$	$\lambda = 3$
t=50	[0.3633 0.4954 0.6283]	[0.2781 0.4320 0.5492]	[0.2464 0.3635 0.5357]	[0.2464 0.3635 0.5361]
t=100	[0.2954 **0.5242** 0.6864]	[0.1771 0.3176 0.5407]	[0.1590 0.2756 0.4662]	[0.1590 0.2756 0.4670]
t=1000	[0.1073 **0.6121** 0.8982]	[0.0015 0.0090 0.0474]	[0.0013 0.0079 0.0414]	[0.0013 0.0079 0.0414]

Table IV Risk Attitude and Survival

This table illustrates the relationship between the risk attitude and the survival prospects of the LA-investor. The variables EP_{LA}, EP_{log} and EP_{EZ} are equity premiums in the representative agent economies populated with the LA-investor, the log investor, and the EZ-investor, respectively. The technology parameters are fixed at the values in Table I. The other preference parameters are set at the following values: $EIS_{EZ} = EIS_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$ and $RA_{EZ} = RA_{LA} = 3$. "Y" and "N" represent "Yes" and "No" respectively.

Panel A: $b_{LA} = 0.02$

	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2.25$	$\lambda = 3$
EP_{LA} (%)	0.32	0.57	0.95	1.31
$EP_{LA} > EP_{log}$ (=0.15%)	Y	Y	Y	Y
$EP_{LA} > EP_{EZ}$ (=0.45%)	N	Y	Y	Y
Is LA closer to log?	Y	N	N	N
Does LA survive?	Y	N	N	N

Panel B: $b_{LA} = 0.2$

	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2.25$	$\lambda = 3$
EP_{LA} (%)	0.10	0.74	1.43	1.89
$EP_{LA} > EP_{log}$ (=0.15%)	N	Y	Y	Y
$EP_{LA} > EP_{EZ}$ (=0.45%)	N	Y	Y	Y
Is LA closer to log?	?	N	N	N
Does LA survive?	Y	N	N	N

Table V Price Impact of the LA-Investor: EIS = 1

This table reports the distributions of the LA-investor's impact on asset prices at times t=50, 100, and 1000 years when the LA-investor has initial wealth shares of ω_0=0.5 or ω_0=0.9 and both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (panel B). The price impact is measured by EP_t/EP_{EZ}, the ratio of the conditional equity premium in the heterogeneous agent economy to the equity premium in the economy with only the EZ-investor. Both investors have a unit EIS ($EIS_{EZ} = EIS_{LA} = 1$) and they have the same time patience parameter ($\beta_{EZ} = \beta_{LA} = 0.98$). The technology parameters in both economies are fixed at the values in Table I. Each entry in this table associated with $t > 0$ has three elements, corresponding, respectively, to the 5%, 50%, and 95% percentiles of the distributions of EP_t/EP_{EZ}. The quantiles are estimated from the 5000 simulated sample paths at time t. The variables EP_{LA} and EP_{EZ} are equity premiums in the representative agent economies populated with the LA-investor and the EZ-investor, respectively.

Table V Price Impact of the LA-Investor: EIS = 1 (Cont.)

Panel A: 5%, 50%, and 95% percentiles of (EP_t/EP_{EZ}) when RA_{EZ} = RA_{LA} = 1 ($\zeta_{EZ} = \zeta_{LA} = 0$)

(A1) $b_{LA} = 0.02$

	$\lambda = 2.25$ ($EP_{LA} = 0.79$, $EP_{EZ} = 0.15$)		$\lambda = 3$ ($EP_{LA} = 1.19$, $EP_{EZ} = 0.15$)	
	$\omega_0 = 0.5$	$\omega_0 = 0.9$	$\omega_0 = 0.5$	$\omega_0 = 0.9$
t = 0	1.9933	4.9583	1.9933	6.9932
t=50	[1.4317 1.8219 2.9837]	[2.1311 4.6164 5.1976]	[1.4317 1.8219 2.9813]	[1.9438 3.4092 7.4518]
t=100	[1.2973 1.6804 3.2219]	[1.6444 2.9883 5.2150]	[1.2973 1.6803 3.1304]	[1.5579 2.4239 7.2660]
t=1000	[1.0251 1.1468 1.8487]	[1.0348 1.1889 2.0776]	[1.0248 1.1451 1.8220]	[1.0320 1.1722 1.9227]

(A2) $b_{LA} = 0.2$

	$\lambda = 2.25$ ($EP_{LA} = 1.41$, $EP_{EZ} = 0.15$)		$\lambda = 3$ ($EP_{LA} = 1.88$, $EP_{EZ} = 0.15$)	
	$\omega_0 = 0.5$	$\omega_0 = 0.9$	$\omega_0 = 0.5$	$\omega_0 = 0.9$
t = 0	1.9933	8.9629	1.9933	8.9590
t=50	[1.4317 1.8219 2.9790]	[1.9008 3.0970 9.2290]	[1.4317 1.8219 2.9790]	[1.8853 2.9820 8.7371]
t=100	[1.2973 1.6803 3.1202]	[1.5380 2.3034 7.1630]	[1.2973 1.6803 3.1100]	[1.5254 2.2575 5.3833]
t=1000	[1.0248 1.1448 1.8164]	[1.0311 1.1681 1.8986]	[1.0248 1.1446 1.8122]	[1.0309 1.1665 1.8940]

Panel B: 5%, 50%, and 95% percentiles of (EP_t/EP_{EZ}) when RA_{EZ} = RA_{LA} = 3 ($\zeta_{EZ} = \zeta_{LA} = -2$)

(B1) $b_{LA} = 0.02$

	$\lambda = 2.25$ ($EP_{LA} = 0.95$, $EP_{EZ} = 0.45$)		$\lambda = 3$ ($EP_{LA} = 1.31$, $EP_{EZ} = 0.45$)	
	$\omega_0 = 0.5$	$\omega_0 = 0.9$	$\omega_0 = 0.5$	$\omega_0 = 0.9$
t = 0	1.7327	2.0746	1.8145	2.8444
t=50	[1.3181 1.5890 1.7885]	[1.9902 2.0500 2.0802]	[1.3031 1.5029 1.9186]	[1.8671 2.6595 2.8328]
t=100	[1.1877 1.3686 1.7487]	[1.8554 1.9984 2.0723]	[1.1803 1.3487 1.7291]	[1.4031 1.8451 2.7346]
t=1000	[1.0013 1.0082 1.0439]	[1.0038 1.0205 1.0952]	[1.0013 1.0079 1.0424]	[1.0020 1.0113 1.0557]

(B2) $b_{LA} = 0.2$

	$\lambda = 2.25$ ($EP_{LA} = 1.43$, $EP_{EZ} = 0.45$)		$\lambda = 3$ ($EP_{LA} = 1.89$, $EP_{EZ} = 0.45$)	
	$\omega_0 = 0.5$	$\omega_0 = 0.9$	$\omega_0 = 0.5$	$\omega_0 = 0.9$
t = 0	1.8083	3.1782	1.8057	4.1599
t=50	[1.3032 1.5029 1.9085]	[1.6659 2.3577 3.1695]	[1.3032 1.5028 1.9044]	[1.5447 1.9470 2.9489]
t=100	[1.1803 1.3488 1.7235]	[1.3367 1.6581 2.7276]	[1.1804 1.3488 1.7240]	[1.2887 1.5476 2.1180]
t=1000	[1.0013 1.0079 1.0424]	[1.0018 1.0105 1.0525]	[1.0013 1.0079 1.0424]	[1.0017 1.0098 1.0501]

Table VI Survival and Price Impact of the LA-Investor: EIS = 0.5

This table reports the distributions of the LA-investor's wealth shares (ω_t) and price impacts (EP_t/EP_{EZ}) at times t=50, 100, and 1000 years when the LA-investor has initial wealth shares of ω_0=0.5 and both investors have a relative risk aversion coefficient of 1 (Panel A) or 3 (panel B). Both investors have a common EIS: $EIS_{EZ} = EIS_{LA} = 0.5$. They have the same time patience parameter: $\beta_{EZ} = \beta_{LA} = 0.98$. The technology parameters are fixed at the values in Table I. Each entry in this table associated with $t > 0$ has three elements, corresponding, respectively, to the 5%, 50%, and 95% percentiles of the distributions of ω_t and EP_t/EP_{EZ}. The quantiles are estimated from the 5000 simulated sample paths at time t.

Panel A: 5%, 50%, and 95% percentiles of ω_t when $RA_{EZ} = RA_{LA} = 1$ ($\zeta_{EZ} = \zeta_{LA} = 0$)

	(A1) $b_{LA} = 0.02$		(A2) $b_{LA} = 0.2$	
	$\lambda = 2.25$	$\lambda = 3$	$\lambda = 2.25$	$\lambda = 3$
$t = 0$	0.50	0.50	0.50	0.50
$t=50$	[0.3148 0.4775 0.6894]	[0.3148 0.4774 0.6908]	[0.3147 0.4772 0.6916]	[0.3147 0.4772 0.6919]
$t=100$	[0.2447 0.4343 0.7601]	[0.2446 0.4343 0.7457]	[0.2446 0.4342 0.7448]	[0.2446 0.4342 0.7444]
$t=1000$	[0.0379 0.2028 0.8938]	[0.0376 0.1961 0.6673]	[0.0376 0.1921 0.6215]	[0.0374 0.1910 0.6112]

Panel B: 5%, 50%, and 95% percentiles of (EP_t/EP_{EZ}) when $RA_{EZ} = RA_{LA} = 1$ ($\zeta_{EZ} = \zeta_{LA} = 0$)

	(B1) $b_{LA} = 0.02$		(B2) $b_{LA} = 0.2$	
	$\lambda = 2.25$	$\lambda = 3$	$\lambda = 2.25$	$\lambda = 3$
$t = 0$	1.8874	1.8941	1.8984	1.9002
$t=50$	[1.4269 1.8184 2.6928]	[1.4270 1.8226 2.7720]	[1.4271 1.8253 2.8295]	[1.4270 1.8264 2.8534]
$t=100$	[1.3041 1.6964 3.0570]	[1.3040 1.6986 3.1374]	[1.3040 1.6998 3.2264]	[1.3039 1.7003 3.2581]
$t=1000$	[1.0378 1.2400 3.4570]	[1.0375 1.2303 2.6328]	[1.0374 1.2246 2.4118]	[1.0372 1.2231 2.3672]

Figure 1 Price Impact and Stock Market Participation

Figure 1 graphs the price impact (EP_t/EP_{EZ}) and the stock investment policy ($S_{LA,t}$) of the LA-investor. The technology parameters are fixed at the values in Table I. The preference parameters are $EIS_{EZ} = EIS_{LA} = 1$, $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$, λ=3 and b_{LA}=0.02.

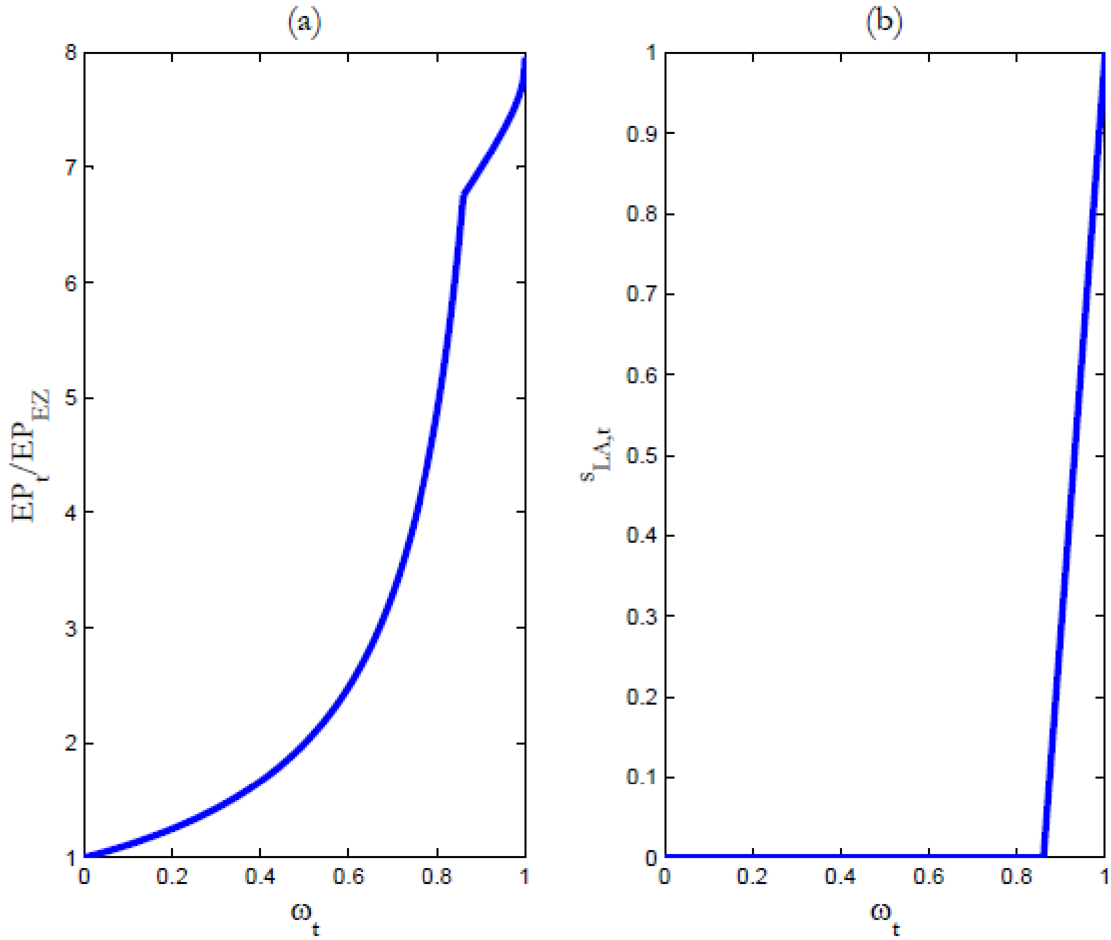

Figure 2 Survival of the LA-Investor when $EIS_{EZ} = EIS_{LA} = 1.5$

Figure 2 graphs the probability density functions (p.d.f.s) of the LA-investor's wealth shares (ω_t) at times t=50,100,1000 when $EIS_{EZ} = EIS_{LA}$ =1.5. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, each investor has half of the aggregate wealth; that is, ω_0=0.5. The technology parameters are fixed at the values from Table I. The other preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$, λ=2.25 and b_{LA}=0.02.

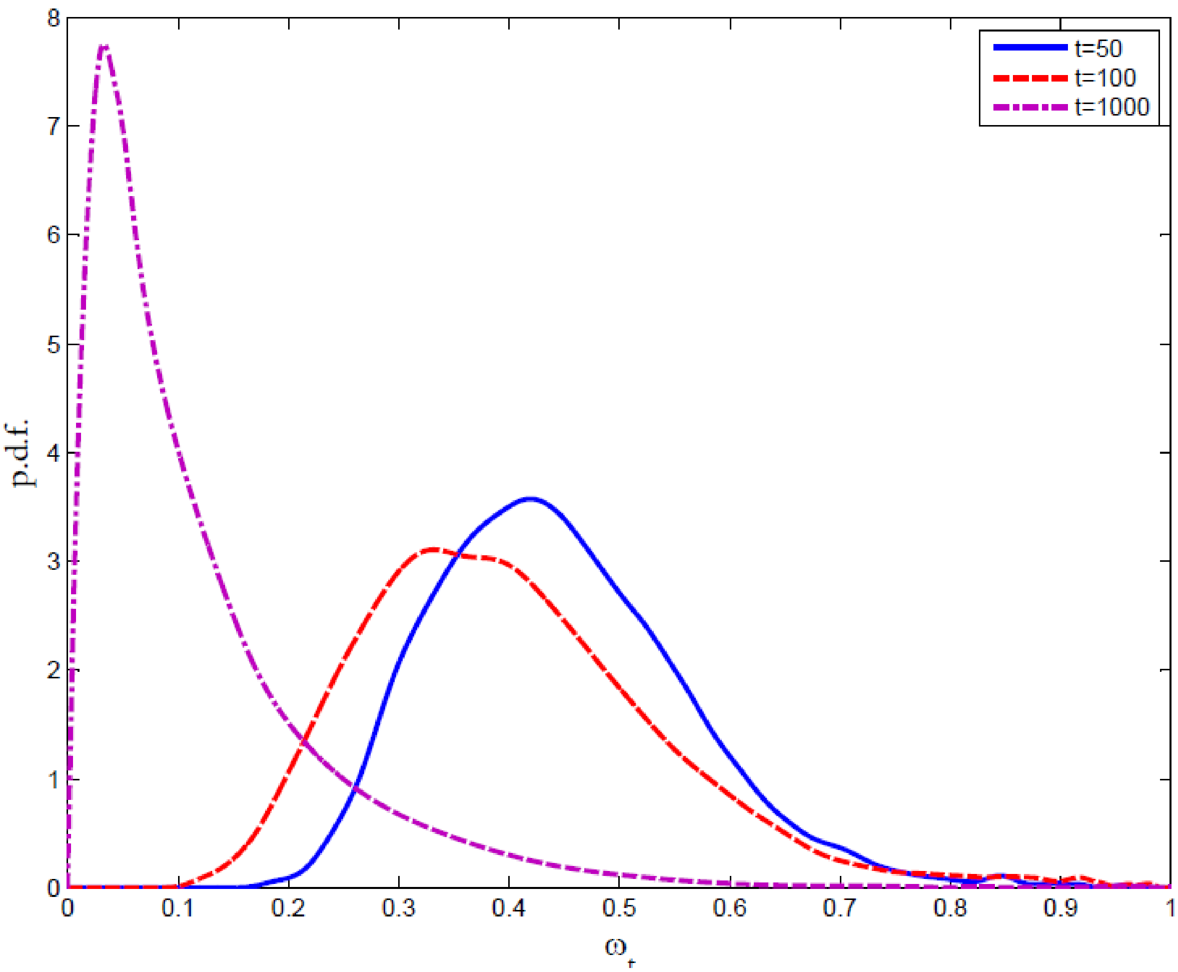

Figure 3 Consumption Policies when EIS_{EZ} = EIS_{LA} = 0.5

Figure 3 depicts the consumption policies of both investors when $EIS_{EZ} = EIS_{LA} = 0.5$. The other preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$, $\lambda=2.25$ and $b_{LA}=0.2$. The technology parameters are fixed at the values in Table I.

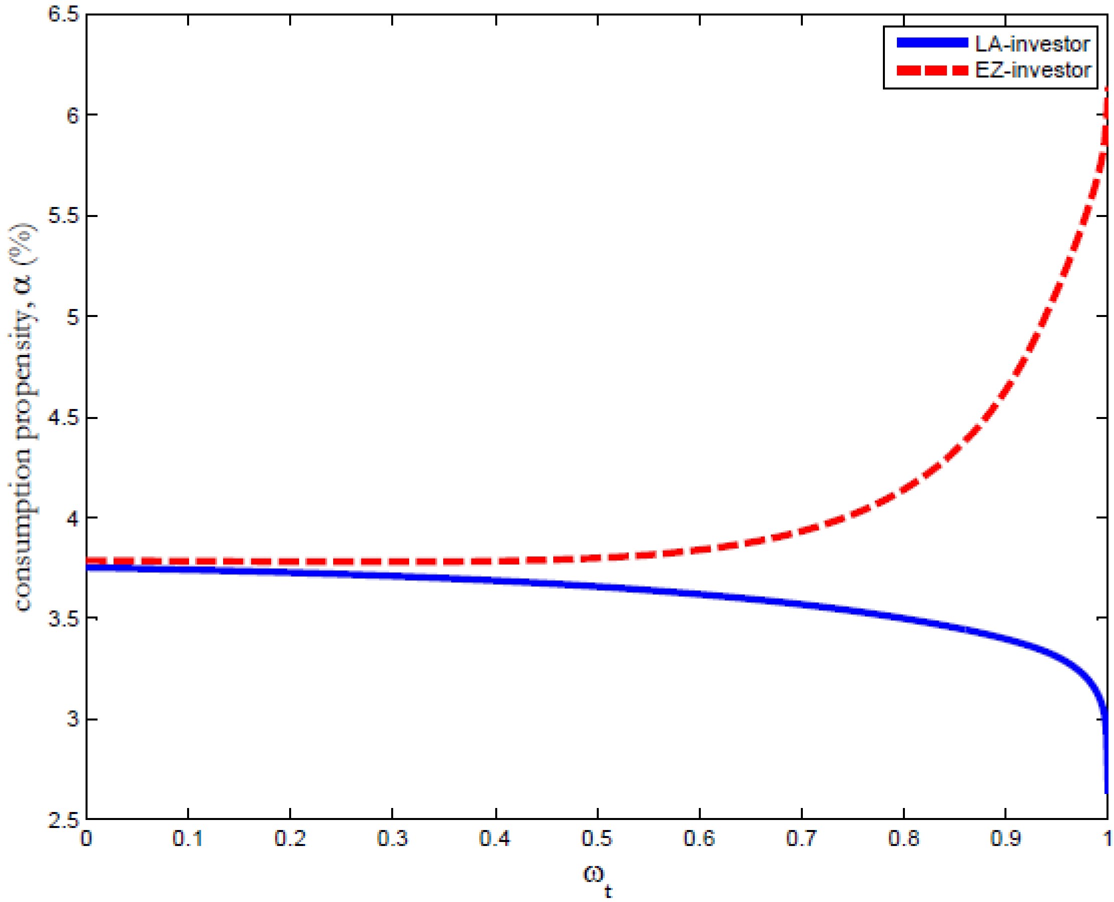

Figure 4 Survival and Consumption Policies when $EIS_{EZ} < EIS_{LA}$

Figure 4 depicts the probability density functions (p.d.f.s) of the LA-investor's wealth shares (ω_t) at times t = 50,100,1000, as well as the consumption policies of both investors when EIS_{EZ} =0.5 and EIS_{LA} =0.7. The other preference parameters are $RA_{EZ} = RA_{LA} = 1$, $\beta_{EZ} = \beta_{LA} = 0.98$, λ=2.25 and b_{LA}=0.02.. The technology parameters are fixed at the values in Table I. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, each investor has half of the aggregate wealth; that is, ω_0=0.5.

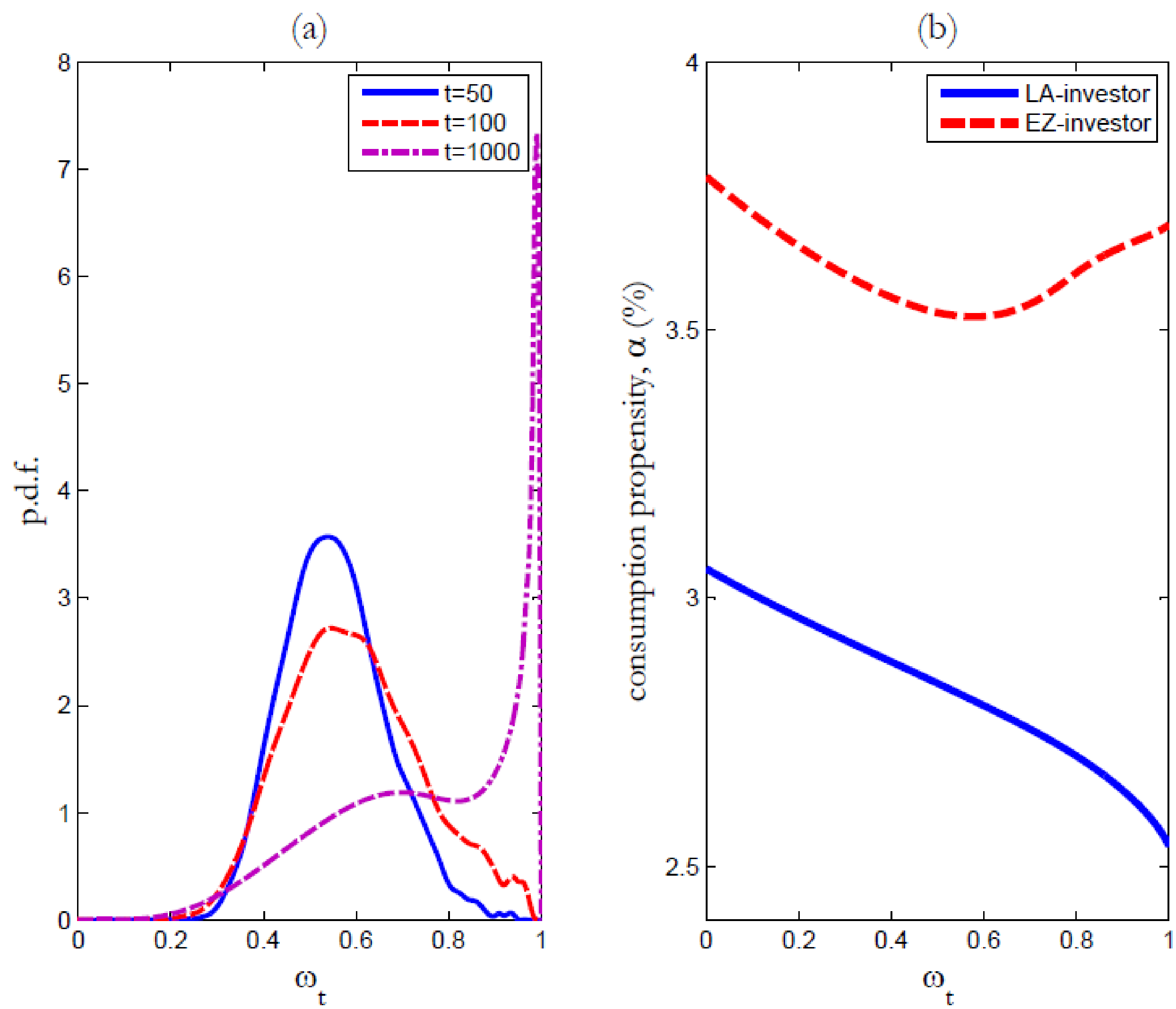

Figure 5 Survival and Consumption Policies when $\beta_{EZ} < \beta_{LA}$

Figure 5 depicts the probability density functions (p.d.f.s) of the LA-investor's wealth shares (ω_t) at times t=50,100,1000, as well as the consumption policies of both investors when β_{EZ} =0.96 and β_{LA} = 0.98. The other preference parameters are EIS_{EZ} =EIS_{LA} =1.5, RA_{EZ} = RA_{LA} = 1, λ=2.25 and b_{LA} =0.02. The technology parameters are fixed at the values in Table I. The p.d.f.s are estimated non-parametrically from 5000 simulated data. At time 0, each investor has half of the aggregate wealth; that is, ω_0=0.5.