Information Contagion in Coelection Environments: Theory and Evidence from Entry and Exit of Senators*

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Abstract

It is well-known that the electorate in midterm elections is more ideologically extreme than the electorate in presidential elections; yet, surprisingly, we find that United States senators that are elected in midterm elections are consistently more ideologically moderate than those first elected in presidential elections. Furthermore, senators who are ousted or retire from office around presidential elections are significantly more ideologically moderate than those who exit around midterm elections. We propose a theory in which the presence of party labels enables voters to rationally update their beliefs about candidates across contemporaneous races for office. Wide support for a candidate in one race aids marginal candidates from the same party in other races. Our model generates predictions that are consistent with our new findings as well as a broad set of phenomena from the literature and suggests that unbiased public signals, such as party labels, may have unexpected effects on the aggregation of private information and preferences. Our empirical findings illustrate that simple elements of institutional design may not be outcome-neutral and may profoundly impact the extent to which duly-elected representatives reflect their constituents' preferences.

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1 Introduction

The laws governing the timing of races for public office vary across electoral systems. While predetermined term lengths for elected federal offices fix the timing of general elections in the United States, the number of simultaneously contested offices in a given election cycle is variable. Specifically, the office of president is contested every four years, whereas congressional races are held every two years. The presence of an additional concurrent race for office during a presidential election cycle is what sets it apart from midterm elections. We set out to answer the following question: can this institutional detail have a systematic and long-term effect on the type of candidates that prevail?

This paper presents strong empirical evidence that the presence of a presidential race for office affects the manner in which citizen preferences are aggregated. Further, we show that electoral outcomes are biased in a particular and counterintuitive way. Our empirical results suggest that the two electoral environments—midterm and presidential elections—aggregate the preferences of citizens differently: we find that senators first elected during presidential elections are more ideologically extreme than senators first elected during midterm elections. Conversely, we find that senators who are ousted or retire from the Senate around presidential elections are more ideologically moderate than those who exit around midterm elections. We also find that extremism is highly associated with party loyalty. Thus, while empirical evidence suggests that the electorate is relatively moderate in presidential elections as compared to midterm elections, this electoral environment returns a more ideologically extreme and party-disciplined Senate overall.

These results have theoretical implications for models of voting and electoral competition. Moreover, demonstrating that midterm and presidential elections systematically produce different outcomes may have implications for politics and policymaking in general, and the timing of elections and ballot initiatives in particular. Awareness of such persistent differences may aid lawmakers in establishing electoral rules by bringing attention to the possible significance of holding multiple races for office at the same time.

There are two main reasons for holding concurrent races for office. First, since the variable costs of running an election—hiring monitors and polling staff, printing ballots, delivering equipment, securing polling locations—do not vary much with the number of offices being contested, the marginal cost of holding an additional race for office is relatively low. The fixed costs, however, can be substantial. A recent special election for a single congressional seat in Illinois was estimated to cost over \$3.5 million, or \$33 per vote.[1]

[1] F.N. D'Alessio, *The Associated Press State & Local Wire*, March 4, 2009, http://www.huffingtonpost.com/2009/03/05/board-wants-mail-in-speci_n_172157.html.

With all 435 seats in the House and one-third of the 100 seats in the Senate contested every general-election year, the possible savings are considerable. A second reason is the cost reduction for prospective voters. Holding races for different offices simultaneously decreases the time and effort required for citizens to turn out and vote, and mitigates the cost of acquiring information about policy issues, parties and candidates.

While the direct effects of holding concurrent races for office are unambiguously positive, there are a variety of indirect effects that make the overall net benefit of such elections less obvious. The behavior of voters, candidates, parties, the media and other players may differ systematically between presidential and midterm elections. Citizens may have strategic and informational concerns that alter their voting behavior in each environment. In response, parties may implement their own sets of changes. For example, differences in fundraising between the two environments can affect the candidate selection process. If changes in voter or party behavior are widespread, they can lead to persistent differences in electoral outcomes.[2]

Moreover, since the voting behavior of citizens and the electoral decisions of candidates are linked, it may not be feasible to determine from where such differences might stem: the inclusion of an ideologically appealing candidate may encourage citizens to turn out and vote. Similarly, a particular distribution of citizen preferences may motivate a candidate of a particular bent to run for office. However, because we utilize the exogenous oscillation between presidential and midterm elections in the United States, our findings do not suffer from this simultaneity problem. Although there is significant variation in voter turnout between midterm and presidential elections, the preferences of the citizenry as well as the electoral institutions and the rules governing these elections remain broadly the same. In both midterm and presidential elections, congressional offices are contested by two major parties and outcomes are determined by plurality rule. Consequently, we are able to recover the marginal effect of a presidential race for office on features that are present in both electoral environments.

We propose a model in which the presence of party labels in an environment of incomplete information produces a contagion effect across contemporaneous races for office. The presence of so-called "presidential coattails" alters the range of electorally viable positions in down-ticket races and, therefore, alters the expected type of winner and loser. Because coattails are only present in presidential election years, they constitute a significant structural difference between midterm and presidential elections. In the aggregate,

[2] An evaluation of the normative implications of such differences in the aggregation of preferences depends on the underlying mechanism that generates them and the degree to and manner in which they differ. We suggest a possible mechanism, but leave out discussion of normative concerns.

this contagion generates electoral outcomes that are consistent with our empirical findings about both voter behavior and electoral outcomes.

In our model, candidates from the same party share a common general ideology, but differ in some very specific aspects of their respective ideologies due to idiosyncratic discrepancies and local electoral conditions.[3] Citizens have prior beliefs about party platforms, but do not know the exact party positions. They may observe candidate positions in some races for office but not in all. Those who observe candidate positions in only a subset of races for office can update their beliefs about parties and candidates in other races. This phenomenon introduces a rational contagion effect, which alters the competitive landscape of races in which citizens do not observe candidate positions. A relatively more moderate candidate in the observed race creates a coattail effect for other members of her party running for office. This enables relatively more extreme and less electorally viable candidates to win. Conversely, a relatively more extreme candidate reduces the range of electorally attainable positions for his party's ticket.

Our empirical and theoretical results suggest that previous treatment of the seat-voteshare relationship and the effect of coattails on down-ticket performance may understate the impact that simultaneous elections have on one another. Our work raises policy questions about the timing of elections and ballot initiatives and the role of party labels in elections for offices and may have particular relevance for offices that have little to do with or are designed to be removed from ideological contamination, such as school boards and judicial elections.

We next summarize the motivating institutional facts and our empirical findings. In Section 3, we present a simple model that broadly captures our theory and in Section 4 we present our formal model in detail. In Section 5, we derive the main results of our model as applied to individual and aggregate voter behavior and electoral outcomes followed by an empirical evaluation of the main implications of our theory. We conclude with a discussion of our results and other possible explanations for the empirical regularities and briefly consider the implications of our work on models of voting and party competition.

2 Empirical Motivation

A common belief and desirable result is that institutions, such as American elections, in which a plurality rule determines who wins office generate outcomes that reflect the preferences of the median voter. There are three well-known facts about American elections that

[3] We do not model parties or their candidate selection process directly. For examples of such models, see Snyder and Ting (2002) and Caillaud and Tirole (2002).

we use to construct an empirical test for this belief.

2.1 Three Facts About American Elections

Federal elections in the United States are held on the first Tuesday following the first Monday in November during even-numbered years. Members of the House of Representatives, senators, and presidents serve terms of two, six and four years, respectively. Although all scheduled elections occur during even-numbered years, the staggered terms of the presidency and congressional offices in the United States create two electoral cycles: midterm elections and presidential elections. During midterm elections, the entire House of Representatives and one-third of the seats in the Senate are contested, and, during presidential-election years, in addition to congressional races, the presidency is contested. These two electoral environments generate two cohorts of senators: those who are first elected to the Senate concurrently with a president and those who are first elected in midterm elections, between presidential-election years. The existence of these two election cycles is an artifact of the variation in term lengths between senators and presidents.

A well-known empirical regularity related to the cyclical nature of midterm and presidential elections is the systematic oscillation in voter turnout. Turnout consistently varies between midterm elections (low turnout) and presidential elections (high turnout). The electorate is significantly larger in presidential elections than in midterm elections. The average turnout over the last forty-year period is 58.1 percent in presidential elections and 42.4 percent in midterm elections.[4]

Finally, the link between turnout and ideology is well-established in the literature. In Palfrey and Poole (1987), the likelihood to vote is highly (positively) correlated with the ideological extremism of the voter. Leighley and Nagler (2007) find additional evidence that voters are more strongly partisan than non-voters. Two recent working papers lend further evidence by highlighting the ideological differences between voters and citizens in general. In a recent working paper, Shor (2009) estimates citizen preferences in a left-right (liberal-conservative) ideological space and uncovers a unimodal distribution, whereas Herron and Bafumi (2007) estimate voting preferences and find a bimodal distribution with low density about the mean. Thus, this literature strongly suggests that, given a fixed population, the ideological extremism of an electorate is likely to decrease in its size.

Given these facts, one concern might be that midterm elections produce extreme out-

[4]We employ nationwide general-election turnout data from 1968 to 2008. There are two common measures of voter turnout: Voter Age Population (VAP) and Voter Eligible Population (VEP). We use VAP rates; however, a similar pattern emerges for VEP rates. McDonald and Popkin (2001) for years 1968-2000 and McDonald (2009) for years 2002-2008.

comes because turnout is light. While there are varying opinions on why this might be true, our working hypothesis was that a more bimodal (i.e., more ideologically extreme) distribution of preferences is likely to generate more variance about the median voter and thus a more bimodal distribution of electoral outcomes. Thus, by design, we expect the distribution of electoral outcomes to correspond to the distribution of voter preferences that generate them. However, much to our surprise, we can soundly reject this conjecture in the case of United States senators. In fact, we find the opposite to be true and the evidence to be very strong. In what follows, we summarize our main empirical findings with reference to Halberstam and Montagnes (2009a).[5]

2.2 Our (Surprising) Findings

Our dataset consists of all senators who faced a midterm or presidential election for the first time between 1968 to present. Over the course of the data, 221 new senators are elected and 137 incumbents exit the Senate. To proxy for senator ideology we present our results using the first dimension of Poole and Rosenthal's DW-NOMINATE scores. These scores are the most widely-used and robust measures of ideology. The first dimension most closely corresponds to the liberal (left)-conservative (right) ideology space and accounts for the vast majority of variation in voter behavior during our period of study. The scores lie between -1 and +1 where a higher score is associated with a more conservative voting behavior. For example, in the 109th Congress, Ted Kennedy's score was -0.56, John McCain's was 0.374, whereas Arlen Specter's was 0.081.[6]

If we segment the Senate by party, and then further divide senators by the electoral environment in which they first won office, we arrive at four distinct groups. One is the cohort of Democratic senators first elected to the Senate during midterm elections (the Democratic midterm-entry cohort). Another is the cohort of Democratic senators first elected to the Senate during presidential elections (the Democratic presidential-entry cohort). The final two, similarly obtained, cohorts are the Republican midterm-entry cohort and the Republican presidential-entry cohort.

For each of the four entry cohorts, we compute the average DW-NOMINATE scores for every congress in our data.[7] In Figure 1, we plot these averages.

[5] In Halberstam and Montagnes (2009a), we describe in detail our data and results, and subject our empirical findings to a series of robustness checks, which include alternative measures of ideology and different data selection and weighting criteria. We also control for various electoral concerns and local effects.

[6] For a detailed description of the data see Appendix.

[7] DW-NOMINATE scores are reported once every two years, corresponding to the length of a congress.

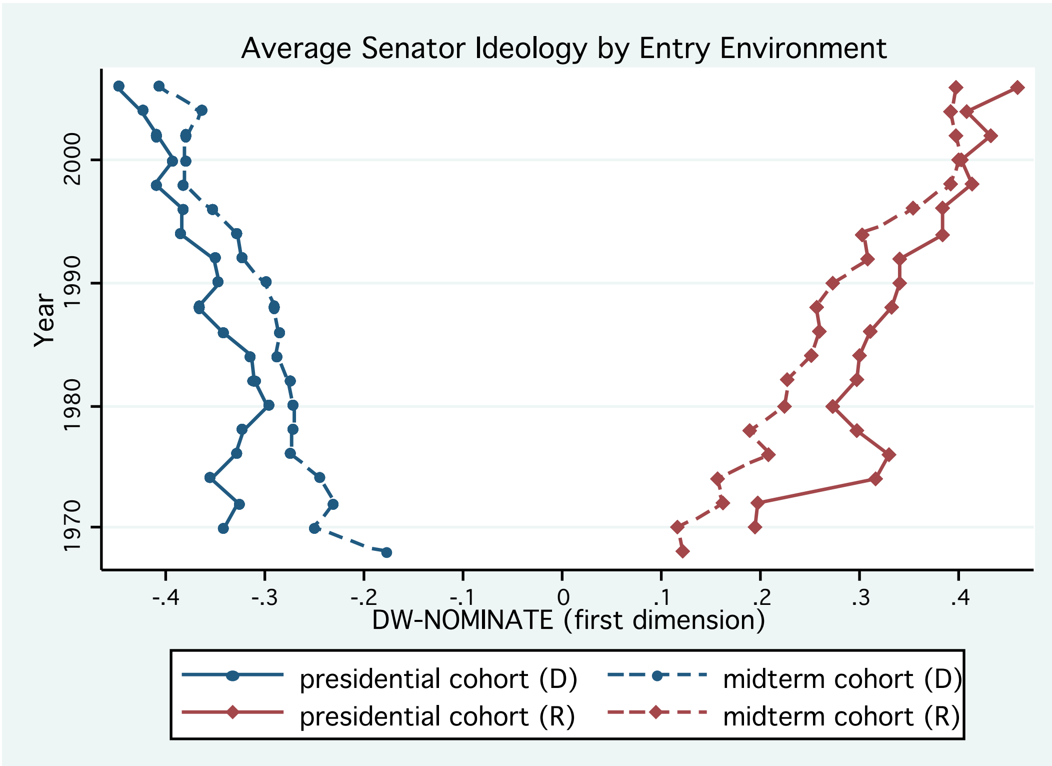


Figure 1: Ideologies of Entering Senators

The results are striking. The ideologies of presidential-entry cohorts are consistently more extreme than the ideologies of midterm-entry cohorts. The set of points associated with the average DW-NOMINATE scores of presidential-entry Democrats (Republicans) are persistently more negative (positive) than the set of points associated with the average DW-NOMINATE scores of midterm-entry Democrats (Republicans). The interparty ideology gap for presidential-entry senators diverges more than the interparty ideology gap for midterm-entry senators. In other words, for both Democrats and Republicans on average, the voting behavior of a senator who is first elected to the Senate in a presidential election is more ideologically extreme than that of a senator who is first elected to the Senate in a midterm election.[8]

We establish a similar, but distinct, effect related to exit. As in senator entry, we divide the Senate into cohorts by party and exit environment into midterm- and presidential-exit cohorts. In our analysis of exit environments, the pattern that arises is even more pronounced than the one observed for entry environments. In Figure 2, we note that

[8]Another pattern, which is not the subject of this paper, that emerges in Figure 1 is well-documented in the literature: over the past 40 years, Democrats have become relatively more liberal and Republicans more conservative, the overall effect being increasing ideological polarization in Congress.

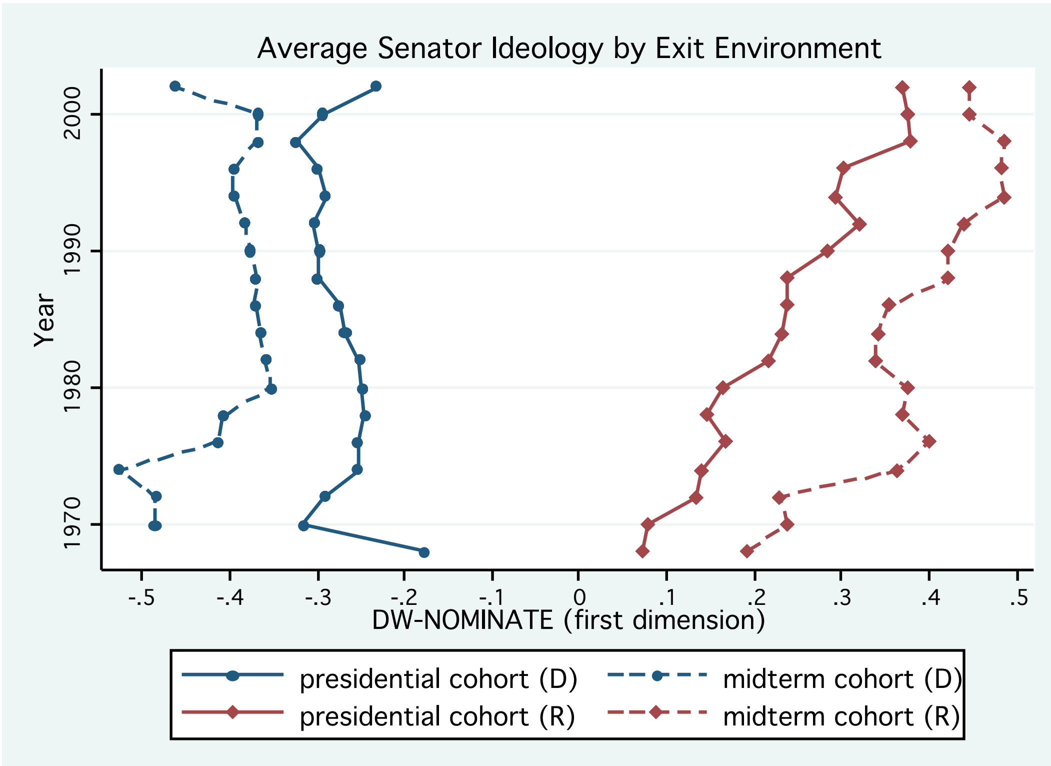


Figure 2: Ideologies of Leaving Incumbents

incumbents who retire or are ousted during presidential elections tend to be significantly more moderate than those who exit during midterm elections. Here, too, the pattern is persistent throughout the data.[9]

An alternative approach to present our findings is depicted in Figure 3 . In this figure, we plot the distribution of DW-NOMINATE scores for senators by entry and exit environments using a kernel-density estimation. It appears that the distribution of senator ideology is inversely related to the preferences that generated them: the distribution of presidential-entry (-exit) senator ideology is more (less) bimodal than the distribution of midterm-entry (-exit) senator ideology.

One way to get a sense of the magnitude of these differences is to compare the interparty ideological differences across electoral environments. In our period of analysis, the average difference between Democrats and Republicans in DW-NOMINATE scores is roughly 0.65. Relative to the interparty ideological difference, the presidential entry cohorts are 11 percent ideologically further apart and the midterm-entry cohorts are 11 percent ideologically closer

[9]In fact, in Halberstam and Montagnes (2009a), we show that that ideological differences observed in exit are consistently greater than the ideological differences observed in entry.

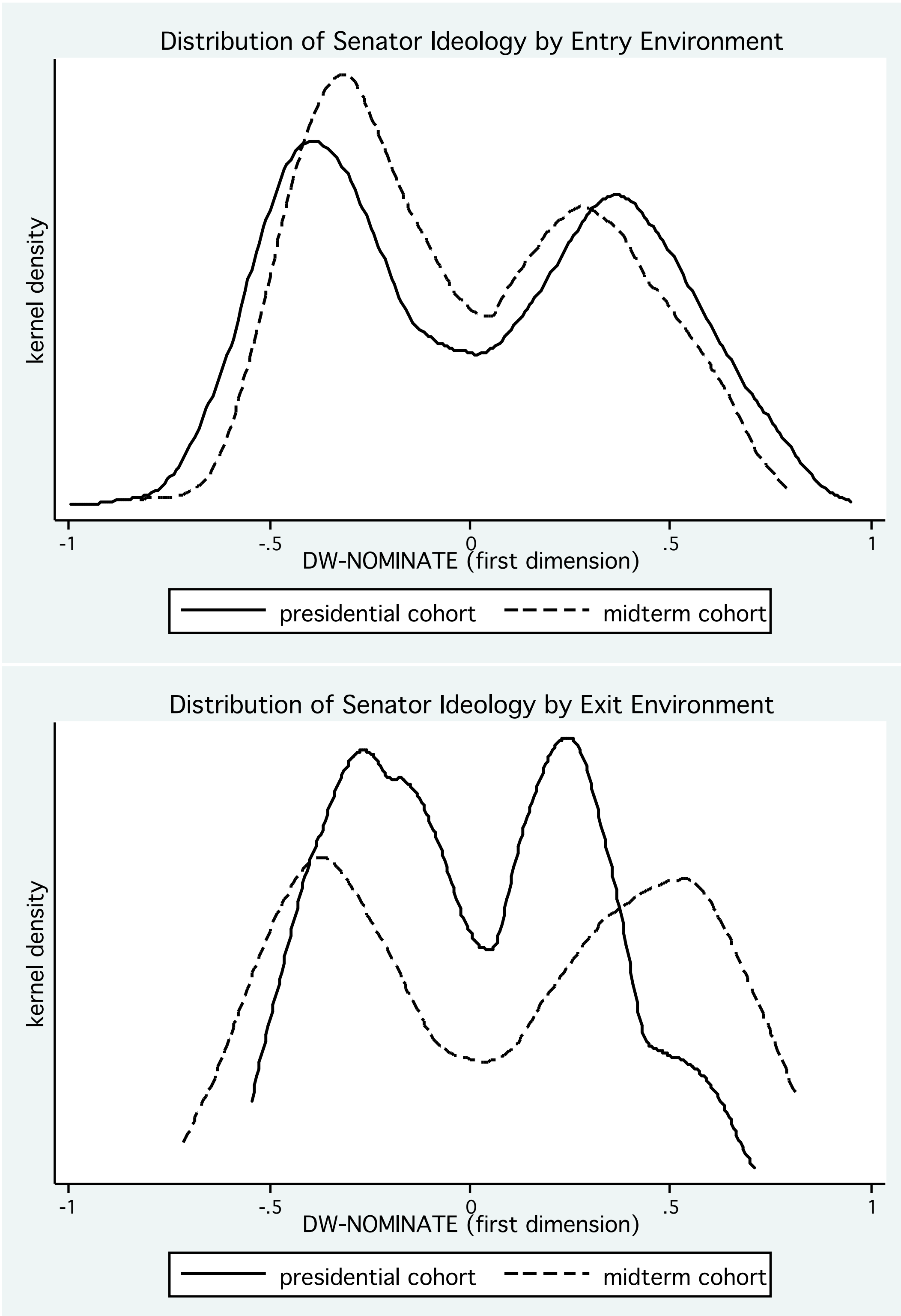


Figure 3: Distribution of Senator Ideology

together. Overall, the ideological interparty average distance between the presidential entry cohorts is 25 percent greater than between midterm-entry cohorts, whereas midterm-exit cohorts are 63 percent more ideologically differentiated than presidential-exit cohorts.

Finally, we must emphasize that DW-NOMINATE scores correspond to real-world voting records of senators. The differences in the ideology of senators will, thus, be reflected in their voting records and ultimately in the types of bills passed by Congress. To illustrate the connection, we provide in Table 1 the breakdown of one important roll call vote chosen from the Americans for Democratic Action (ADA) set of critical roll calls from 1984. The vote was on an amendment to a bill that appropriates $700 million in federal grants to states to provide health benefits to the long-term unemployed. The amendment was rejected by 57 to 39 votes. With 14 Democrats joining 43 Republicans in voting nay, and 11 Republicans joining 28 Democrats in voting yay, voting on this amendment was not entirely partisan. The total number of senators voting against the majority of their party was 25 out of 96 or about 26 percent (11 out of 54 or 20 percent for Republicans and 14 out of 42 or 33 percent for Democrats). However, if we restrict our attention to the set of senators in our data and look at the differences between electoral environments, we find that the moderation of the midterm-entry cohort is reflected in a much greater propensity to cross party lines and vote against the majority of the party. For Democrats, a 53 percent majority of the midterm-entry cohort voted with the Republican majority, whereas only a 19 percent minority of the presidential-entry cohort did the same. Similarly, for the Republicans, close to one-quarter of the midterm-entry cohort voted with the Democrats, while over 80 percent of the presidential-entry cohort voted party-line. We find a similar pattern when we tabulate the votes on this bill with respect to one's exit environment. Consistent with our broader results, the midterm-exit cohorts are significantly less likely than presidential-exit cohorts to vote independently of their party. Among Democrats, nearly half the presidential-exit cohort crossed party lines, while only 22 percent of the midterm-exit cohort crossed. Although counterfactual predictions about the type of legislation produced and passed by the Senate are difficult to make, since the agenda of the Senate is endogenous to the particular character of the Senate, this example illustrates that a Senate comprised of moderates from the midterm-entry cohort would be more likely to have cross-partisan voting patterns if faced with the same set of voting decisions.

2.3 Coattails

Our findings suggest that holding concurrent races for office is not outcome-neutral. The literature on multiple election environments primarily focuses on the impact of presidential coattails on down-ticket wins and individual ticket-splitting. Research on the coattail ef-

Table 1: Example Roll Call

	Entry				Exit			
	Midterm		**Presidential**		**Midterm**		**Presidential**	
Vote	Yay	Nay	Yay	Nay	Yay	Nay	Yay	Nay
Democrats	8	9	13	3	7	2	8	7
Republicans	5	16	5	22	3	15	4	18

Source: 1984 Americans for Democratic Action, Senate Vote 3

fect—the relationship between the popularity of a presidential candidate and the winning prospects of other candidates from the same party—attempts to identify the seat-vote relationship or identify differences in the electorate. (See Besley and Preston (2007), Campbell (1993) and Coate and Knight (2007) for a review of this literature.) For example, Campbell (1993) has examined the differences in the size and composition of the voting population in midterm and presidential elections, and the success of the president's party in each environment. However, previous studies have failed to look at the types of candidates elected in each environment. Our results suggest that presidential coattails or other mechanisms might have consequences beyond success for the president's party and may affect the type of senators elected from both parties as reflected in their ideological positions and voting behavior.

In addition to the patterns of voting behavior presented thus far, there are also systematic outcome differences between the presidential and midterm election environments. Perhaps the most widely known facts about outcomes in presidential and midterm elections are the associated surge and decline in support for the president's party in congressional elections. Many scholars have studied the relationship between presidential voteshares and the change in the share of congressional seats captured by the president's party following a presidential or midterm election. In all but a handful of cases, in presidential elections, the president's party experiences a surge that supports other candidates from the president's party, resulting in an increase of its share in Congress. Conversely, in midterm elections, the president's party typically experiences a decline. In both the House and the Senate, the presidential surge and midterm decline is significant. On average, from 1968 to present, the president's party lost 2.6 seats in the Senate and 18.3 seats in the House in midterm elections and gained back 1.8 seats in the Senate and 6.2 seats in the House in presidential elections.[10] This phenomenon has motivated a variety of theories that explore the differences between

[10]Source: *The American Presidency Project*, November 13, 2009, http://www.presidency.ucsb.edu/data/mid-term_elections.php.

midterm and presidential elections (Campbell (1960), Campbell (1991), Campbell (1997), Tufte (1975), Kernell (1977), Erikson (1988), Alesina and Rosenthal (1989), Alesina and Rosenthal (1995) and Alesina and Rosenthal (1996)).

Our objective is to present a theory that not only explains our new findings but that accounts for these and other known facts from the literature, such as coattails.

3 A Simple Model of Coattails

The most basic insight to our theory is that midterm elections aggregate preferences as we expect: the candidate whose position is closest to the preferred position of the median voter wins office. On the other hand, in presidential elections, voter uncertainty introduces errors and occasionally the "wrong" candidate is elected (i.e., the candidate farther away from the median voter's preferred position). Thus, in expectation, outcomes generated in presidential elections are more ideologically extreme than outcomes generated in midterm elections. This theory can be illustrated with a simple model of information contagion. The presence of party labels in elections enables citizens to form informational linkages across contemporaneous races and introduces bias in voting behavior and subsequent electoral outcomes. A popular candidate in one race can support a marginal candidate from the same party in another.

Suppose there are two races for office denoted by p and s and that each office is contested by two parties, L and R. There are two election cycles, midterm and presidential elections; in presidential elections both offices are contested while in midterm elections only office s is contested. Let the policy space be the interval $[-1, 1]$ and C_q^r denote the policy position in this space of a candidate from party q in race for office r. For simplicity assume that candidates from party L are drawn uniformly from the interval $[-1, 0]$. Similarly, candidates from party R are drawn uniformly from the interval $[0, 1]$. The selection of candidates is independent from one another.

Citizens have different preferred positions in the policy space, which are drawn uniformly from the interval $[-1, 1]$. Conditional on voting, a citizen votes for the candidate whose position is closest to his own preferred position. In particular, if a citizen's preferred position is to the left of $\frac{C_L^r+C_R^r}{2}$ he votes for party L's candidate; otherwise, he votes for party R's candidate.

There are two types of citizens: those who observe candidate positions (type A) and those who observe candidate positions only in race p (type B). Let the proportion of type B citizens in the population be $\delta > 0$. Type A always turns out and votes; type B participates in presidential elections but not in midterm elections. In presidential elections, type B votes for his preferred candidate in race p, and votes for the same party in race s.

In midterm elections, a candidate from party L in race s with policy position C^s_L wins office if and only if

$$\frac{C^s_L + C^s_R}{2} > 0.$$

In words, party L wins if the midpoint of candidate positions is to the right of the median voter's preferred position. For any given draw of C^s_R, the probability that party L wins is C^s_R. Party L's unconditional probability of winning is one half.

In presidential elections, the winner in race s will also depend on race p. Let π be the proportion of type B citizens who vote in race p for party L's candidate.[11] Then party L's candidate in race s wins if and only if

$$\frac{C^s_L + C^s_R}{2} > (1 - 2\pi)\,\frac{\delta}{1-\delta}.$$

Thus, for any given draw of C^s_R, the probability that party L wins is $C^s_R - (1-2\pi)\,\frac{2\delta}{1-\delta}$, which is increasing in π. In particular, in presidential elections, a candidate in race s is more likely to win than not if a majority votes for his party in race p. For a given δ and C^s_R, let the conditional probability that party L's candidate in race s wins when $\pi > \frac{1}{2}$ be equal to $\gamma(\delta, R) > \frac{1}{2}$.[12]

We can now express party L's expected winning position in race s both for midterm and presidential elections. In a midterm election the expected position, $E_M\left[C^s_L|win\right]$, is $-\frac{C^s_R}{2}$ whereas in a presidential election $E_P\left[C^s_L|win\right] = -\frac{C^s_R}{2} + (1-2\pi)\,\frac{\delta}{1-\delta}$, which is decreasing in π. Thus, greater support in race p results in more extreme outcomes in race s.

Finally, we can compare the expected winning positions for party L in midterm and presidential elections. In a presidential election $E_P\left[C^s_L|win\right]$ can be rewritten as

$$E_P\left[C^s_L|win, \pi > \frac{1}{2}\right] Prob\left(\pi > \frac{1}{2}|win\right) + E_P\left[C^s_L|win, \pi \leq \frac{1}{2}\right] Prob\left(\pi \leq \frac{1}{2}|win\right),$$

which, given our assumptions, is equal to

$$\left[-\frac{C^s_R}{2} - \frac{\delta}{6(1-\delta)}\right]\gamma(\delta, R) + \left[-\frac{C^s_R}{2} + \frac{\delta}{6(1-\delta)}\right](1 - \gamma(\delta, R)).$$

[11] Given our assumptions, $\pi \equiv \frac{\frac{C^p_L + C^p_R}{2} + 1}{2}$.

[12] Given our assumptions, $\gamma(\delta, R) = C^s_R + \frac{2\delta}{3(1-\delta)}$. In this example, we consider only nondegenerate cases (i.e., $C^s_R < 1$).

Since $\gamma(\delta, R) > \frac{1}{2}$ we have that

$$E_P [C_L^s | win] < E_M [C_L^s | win],$$

which illustrates how presidential coattails can bias outcomes toward more extreme positions relative to outcomes in midterm elections.

This simple model generates more turnout and a less informed and more moderate electorate in presidential elections, all of which is consistent with the data. It also accounts for presidential surge and decline and our new empirical findings. In our formal model we provide a more rational and robust framework for parties and voter behavior. In particular, we focus on the mechanism that enables rational information contagion across races for office.

Our formal model accounts for additional phenomena as well, such as "roll-off". While turnout regularly increases during presidential-election years, not all voters cast their votes in all races for office in a given election. For example, many voters choose to vote for a presidential candidate (the up-ticket race), but abstain from voting for candidates in senatorial, house or other non-presidential races (the down-ticket races). This phenomenon presents a puzzle for many models of voting, however, our theory generates behavior that is both rational and consistent with it.

4 Formal Model

We have shown that midterm and presidential electoral environments are distinguished by significant, systematic differences in both voting behavior and electoral outcomes. In this section, we present a model that explains these variations. The key assumptions of our model are that voter utility depends on information, and that party labels have informational value.[13] The presence of multiple races at a single time generates more information for voters, but also introduces information contagion among the races. As we will demonstrate, this contagion generates significant, but predictable, differences in voting behavior, which in turn generate differences in aggregate behavior and outcomes that we observed in Section 2.

In order to focus on elections with systematic variation in the number of races contested, we model voting behavior in statewide races for the Senate in both the midterm and presidential electoral environments.[14]

[13]For recent work documenting the interaction between information and spatial voting, see Jesse (2008) and (2009).

[14]An analysis of the strategic interaction of party-platform choice across multiple heterogeneous districts

4.1 Policy Space and Parties

Let the policy space be $P \subseteq \mathbb{R}$ and let $s \in \{1, 2, 3, ..., 50\}$ denote one of the 50 states. In each race there are candidates from two parties, L and R, running for office. Depending on the election cycle, there are one or two races for office.[15] We denote presidential and senatorial candidates with superscripts p and s, respectively.

Let $C_q^r \in P$ denote the position of a candidate from party $q \in \{L, R\}$ running for office $r \in \{p, s\}$. In a given race for office, let[16]

$$C_L^r \quad \leq \quad C_R^r \quad \forall r \in \{p, s\}\,. \tag{1}$$

Let $M^r \equiv \frac{C_L^r + C_R^r}{2}$ be the ideological midpoint of candidate positions running in the race for office r.[17]

In our model, senatorial and presidential candidates from the same party share a degree of ideological similarity.[18] Let M^p be the midpoint of candidates in a presidential race, and

(as in our model) is beyond the scope of this paper. For analysis of such issues, see Austen-Smith (1981) and Callander (2005).

[15]Some states may not include a race for Senate during a given presidential election cycle. We categorize a Senate race as occurring during presidential elections if it is held concurrently with the presidential race, and deem it a midterm race otherwise.

[16]Empirically, we observe that Democrat senators are almost always to the left of Republican senators; however, our assumption is made for state-level races, and in our data the assumption is always true when we observe senators from both parties in one state.

[17]As we will see shortly, the underlying preferences of voters in our model are a form of proximity preferences. Accordingly, given our assumption about candidate ordering in the policy space, a sufficient statistic for a voter in any given race is the midpoint of candidate positions. Modeling only the midpoints of party competition has several advantages. The first is tractability; by not modeling the underlying party competition process, it is much easier to aggregate the underlying decision process of citizens. Additionally, the updating process between races that citizens employ and comparative statics on the relevant model primals will be transparent. Second, this approach highlights the robustness of our results to a variety of models of party competition. Many spatial models of party competition will result in some distribution of candidate positions (including degenerate distributions). These distributions of candidate positions will in turn generate a distribution of midpoints, as in our model. While making a distributional assumption about candidate midpoints is benign, the same cannot be said about the distribution of candidates. For empirical evidence that corresponds to our model of candidate selection, see Ansolabehere, Snyder and Steward (2001).

[18]While we model this congruence in a particular parametric manner for tractability, the essential assumption we need to make is that there is some common element in ideological positioning. There are a variety of mechanisms that may account for such congruence of candidate positions. From a candidate choice perspective, Snyder and Ting (2002) argue that candidates with similar ideological perspectives may join parties to reduce information costs for voters. Joining a party is costly if the party presents a different ideological position than the candidate. It imposes ideological consistency across party members and allows the party label to convey information to voters. For other work that makes this argument, see Cox and McCubbins (1993) and Aldrich (1995). Alternatively, the candidate-selection process, fundraising, and the behavior of party elites may serve to generate common ideological positions among candidates from the same party. See Gerber and Morton (1998) and Besley and Case (2003) for a discussion of the candidate-selection process.

M^s be the midpoint of candidates in a senatorial race. Equations (2) and (3) describe the model of midpoints in presidential and senatorial races, respectively.[19]

$$\underbrace{M^p}_{\substack{\text{Presidential Race}\\\text{Midpoint}}} = \underbrace{\Omega}_{\substack{\text{Party}\\\text{Midpoint}}} + \underbrace{\epsilon^p}_{\substack{\text{Presidential Race}\\\text{Idiosyncratic Effect}}} \tag{2}$$

$$\underbrace{M^s}_{\substack{\text{Senatorial Race}\\\text{Midpoint}}} = \underbrace{\Theta^s}_{\substack{\text{State}\\\text{Fixed Effect}}} + \underbrace{\Omega}_{\substack{\text{Party}\\\text{Midpoint}}} + \underbrace{\epsilon^s}_{\substack{\text{Senatorial Race}\\\text{Idiosyncratic Effect}}} \tag{3}$$

Let the midpoint of parties, Ω, be fixed but otherwise unknown. Additionally, let the idiosyncratic race-specific effects be independently and normally distributed with mean zero and variance σ_r^2 for $r \in \{p, s\}$.[20] The state fixed effect is assumed to be non-stochastic. Then we have that

$$M^r \sim N\left(\mu_r, \sigma_r^2\right) \quad \forall r \in \{p, s\}, \tag{4}$$

where $\mu_s = \Theta^s + \Omega$ and $\mu_p = \Omega$. Notice, however, that candidate midpoints in races s and p are independent from one another.

4.2 Citizens

Citizen Preferences

Our model of citizen microfoundations is one in which uncertainty and ideological proximity preferences interact to drive both turnout and spatial voting decisions.[21] The utility

[19]We implicitly assume that parties are playing a simultaneous-move game. This assumption founded upon the sequential nature of candidate selection, entry and primaries at the state level.

[20]We have chosen to have the midpoints of candidate position be normally distributed around a competition mean. The choice is made in order to make the updating process for voters clear and tractable. Our results do not depend substantially on these assumptions and are robust to a variety of underlying models of party competition.

[21]The main objective of our model is to explain voting behavior and regular differences in outcomes between midterm and presidential elections; thus, we are less concerned with the particular details of the microfoundations of preferences and party competition. Nonetheless, we face a trade off when studying large elections: the tractability and clarity characteristic of modeling agents in a continuum comes at the cost of providing little motivation for strategic rational citizens to turn out and vote in any race for office since each citizen's likelihood of being pivotal is, essentially, zero. Since we focus on two-candidate races for office, however, we note that an equilibrium and its outcomes when voters vote sincerely is equivalent to one of the equilibria that exists when they vote strategically; sincere voting in a two-party election is equivalent to the unique Nash equilibrium in weakly undominated strategies. As we are modeling the two-party competition of the United States, our results are not compromised by focusing on underlying proximity preferences and voting. For a recent example of a model where voters have preferences over actions and not outcomes, see Feddersen, Gailmard and Sandroni (2009).

framework we use is inspired by Degan and Merlo (2007) and is one in which a citizen benefits from voting in a given race but is subjected to an ex ante cost of voting for the "wrong" candidate, that is, voting for the candidate whose position is not closest to her own.[22] In an environment in which there is no uncertainty about candidate positions, all citizens vote for the "right" candidate and obtain the benefit of voting. However, in our setting, these preferences have great appeal as they directly incorporate the role of different levels of information.[23]

Let $I_s \subseteq \mathbb{R}$ be the set of citizens in state s. We specify for each citizen $i \in I_s$ a corresponding ideal point $y^i \in P_s \subseteq P$, which is citizen i's most preferred policy position. The preferences of voters in state s are distributed symmetrically and unimodally with full support over P_s around the state preference mean, μ_s, with the corresponding cumulative distribution function, F_s.[24] Let $u_i(x)$ denote citizen i's disutility from voting for a candidate with policy position x. We assume that citizens incur symmetric disutility from voting for candidates with policy positions equally diverging from their own.[25] Formally, u_i achieves a global maximum at y^i, is strictly decreasing away from y^i and $u_i(x) = u_i\left(2y^i - x\right)$ for any x.

We next assume that each citizen i can receive a benefit $b \in \left(0, \frac{1}{2}\right)$ for voting in a particular race for office.[26] This can be thought of as the utility of doing one's civic duty or the right to boast about one's participation in the democratic process. In a case in which there is no uncertainty, there is no associated cost of voting and each citizen votes for the

[22] We are able to generate the same individual behavior and aggregate results using different utility specifications, such as the ambiguity-aversion preference framework developed by Ghirardato and Katz (2006). In Degan and Merlo (2007), however, the authors test their model using individual-level voting data. They estimate a structural model of voter choice employing a version of the voter preferences used here and find that their estimated model is able to replicate the observed levels of abstention, selective abstention, split-ticket voting, and straight-ticket voting. While we focus on aggregate voting behavior and crucially expand the analysis to electoral outcomes, this provides strong empirical support for the value of our preference specification in a model of uncertainty and spatial preferences. For a discussion of the psychological motivations behind these preferences see Degan and Merlo (2007).

[23] These preferences also have an interesting interpretation in light of the swing voter's curse (Feddersen and Pesendorfer (1996)). The cost of a voter's uncertainty as to which candidate's position is closest to hers can be thought of as a psychological proxy for a voter's strategic concerns. Higher uncertainty corresponds with a greater probability of making a mistake. As a voter's preferences become more extreme, the voter is less likely to make a mistake by voting for one party or the other and, therefore, becomes a partisan voter.

[24] Our assumptions about the distribution of citizens are similar to those made elsewhere in the literature. See, for example, Callander (2005).

[25] The actual functional form of these preferences is not essential to our model. Our result will hold if we employ any form of symmetric-loss preferences. More generally, our results hold for any single-peaked preference but at the expense of clarity and tractability.

[26] See Riker and Ordeshook (1968) for a conical model in which voters derive an intrinsic benefit from voting. A large literature has also modeled the benefit associated with voting, see Tullock (1971), Brennan and Buchanan (1984), Brennan and Lomasky (1993), Schuessler (2000), Feddersen and Sandroni (2006a,b) and Feddersen, Gailmard and Sandroni (2009).

candidate who shares a policy position that is closest to her own.

Henceforth, we present the model with respect to party L. The derivations and results symmetrically apply to party R. Given our setup, citizen i votes for C_L^r in race for office r if and only if

$$u_i\left(C_L^r\right) \quad \geq \quad u_i\left(C_R^r\right) \tag{5}$$

and obtains a benefit of voting, b.[27] Equivalently, the condition above can be rewritten as:

$$y^i \leq \frac{C_L^r + C_R^r}{2} \equiv M^r. \tag{6}$$

Note that the only information a citizen considers when deciding for whom to vote is the midpoint of candidate positions. A citizen is concerned with the *relative* ideological position of a candidate rather than the candidate's absolute position.

Incomplete Information

We now introduce an environment in which, *a priori*, not all citizens are perfectly informed about the policy positions of candidates. Let Δ_i^r denote the information set ("beliefs") that citizen i has about the ideological midpoint of candidate positions in race r and let $G_i^r\left(M^r\right)$ denote the subjective distribution that represents citizen i's beliefs in race r.[28] If citizen i has no uncertainty about the ideological midpoint in race r, then $\Delta_i^r = M^r$, in which case the distribution representing this information set, $G_i^r\left(M^r\right)$, is degenerate; otherwise, $\Delta_i^r = G_i^r\left(M^r\right)$. In this case, uncertainty can produce a non-zero ex ante psychological cost of voting.

Let $c_i\left(q^r; y^i, \Delta_i^r\right)$ denote citizen i's psychological cost associated with voting for the candidate from party q in race r with information Δ_i^r. Specifically, citizen i's cost of voting for a candidate from party L in race for office r is:

$$c_i\left(L^r; y^i, \Delta_i^r\right) = \int\limits_{\{M^r : M^r < y^i\}} dG_i^r\left(M^r\right) = Prob\left(M^r < y^i\right). \tag{7}$$

This expression is a sum of all states in which voting for the candidate from party R is the better choice weighted by the subjective probability of the state. Overall, we arrive at a closed-form solution that bears some appeal: citizen i's cost of voting for the candidate

[27] We resolve ties in favor of party L.

[28] Citizens are aware of the underlying party conditions and labels; thus, the subjective beliefs about candidate positions correspond with their actual distribution.

from party L is the probability of making a "mistake," which occurs when the candidate from party R shares a closer position with citizen i than the candidate from party L. Since this probability ranges from zero to one and the benefit from voting is less than one half, there are two decisions each citizen must face: whether to vote in a given race and for whom.[29]

Citizen Classification

We segment citizens into two classes differentiated by the degree to which they are informed about races p and s.[30] Let $\Delta_i = \{\Delta_i^p, \Delta_i^s\}$ denote citizen i's information about the ideological midpoints of candidates in both races. Accordingly, each citizen in a given state, s, is classified in one of the following two ways:[31]

1. Fully Informed Citizen (FIC): a citizen who observes the ideological midpoints of candidate positions in both the presidential and senatorial races for office. For citizen i in this group, $\Delta_i = \{M^p, M^s\}$

2. Partially Informed Citizen (PIC): a citizen who observes the ideological midpoint of candidate positions in the presidential race only. For citizen i in this group, $\Delta_i = \{M^p, G_i^s(M^s)\}$

Let citizens' beliefs about the unknown party midpoint, Ω, be represented by a normal distribution with mean Ω and variance σ_ω^2. Thus, in a midterm election, the distribution that represents a PIC's beliefs over the senatorial race midpoint is

$$M^s \sim N\left(\mu_s, \sigma_s^2 + \sigma_\omega^2\right). \tag{8}$$

Since the senatorial and presidential races share a common party element, in a presidential election, citizens can update their priors about the senatorial race using information from

[29] A strategic interpretation of these preferences is that as the possibility of making a mistake decreases, then regardless of pivot probabilities, a voter is more likely to vote for her preferred party.

[30] Although we now assume that the degree to which a citizen is informed is uncorrelated with her preferences, this assumption can be relaxed and is not essential for generating our results. To the degree that preferences are correlated with how well-informed a citizen is about candidate positions, empirical evidence indicates that ideological extremism is associated with more-informed citizens, in which case our key aggregate results would be even more pronounced. See Palfrey and Poole (1987).

[31] Two classes of citizens that we do not model are those who are uninformed about both races, $\Delta_i = \{G_i^p(M^p), G_i^s(M^s)\}$, and those who observe ideological positions in the senatorial race but not in the presidential race, $\Delta_i = \{G_i^p(M^p), M^s\}$. The first type of citizen does not alter her behavior in the two electoral environments and is not sensitive to the realized positions of candidates. The second type of citizen is empirically less relevant, given the evidence about cyclical turnout in the U.S. and that more information is conveyed about the presidential race than a senatorial one. Including this type of citizen increases the complexity of the model without contributing much to our understanding of the mechanism.

the presidential race. A citizen's updated prior over Ω, conditional upon observing the presidential-race midpoint, m^p, is

$$(\Omega|M^p = m^p) \sim N\left(\Omega + (m^p - \mu_p)\frac{\sigma_\omega}{\sigma_p}\rho_{\omega,p}, \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)\right), \tag{9}$$

where $\rho_{\omega,p}$ is the correlation coefficient between the prior distribution over Ω and the presidential-candidate midpoint, M^p.[32] Citizens observing candidate positions in one race will use Bayes' rule to update their priors about candidate positions in unobserved races. Since candidates from the same party in races p and s are linked by their party labels, in each race for office, citizens use either party labels or their observations of races to make turnout and voting decisions. Consequently, in a presidential election, a PIC's updated prior over the senatorial race midpoint is

$$(M^s|M^p = m^p) \sim N\left(\mu_s + (m^p - \mu_p)\frac{\sigma_\omega}{\sigma_p}\rho_{\omega,p}, \sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)\right). \tag{10}$$

Given our specification of preferences, a few results are immediate. First, FICs always observe candidate positions, turn out in both races and vote for the candidates whose positions are closest to their own. Second, PICs always turn out in the presidential race, use their proximity preferences to vote, then update their beliefs about the party midpoint and consequent senatorial-candidate positions and accordingly decide whether and for whom to vote using party labels.[33]

Citizen Choices and Objective

Denote citizen i's turnout decision in race r by $t_i^r \in \{0,1\}$, where if she decides to vote in race r, ($t_i^r = 1$), and if she abstains, ($t_i^r = 0$), and let the ballot she casts be $v_i^r \in \{L^r, R^r\}$. Given these specifications, citizen i solves the following optimization problem:

$$\underset{t_i^r \in\{0,1\}, v_i^r \in\{L^r,R^r\}}{Max} t_i^r\left[b - c_i\left(v_i^r; y^i, \Delta_i^r\right)\right]. \tag{11}$$

Citizen Behavior

Each citizen's voting and turnout decisions can be solved using backward induction. A citizen first chooses her preferred candidate, and then decides whether the benefit of voting outweighs the cost of voting for her preferred candidate. Solving for the program, we derive

[32] Notice that $\rho_{\omega,p} = \frac{\sigma_\omega}{\sqrt{\sigma_\omega^2+\sigma_p^2}} \geq 0$.

[33] Theoretically, there may exist other groups of voters, namely, those who observe senatorial candidates but not presidential ones and who would employ up-ticket inferences.

for each citizen i a conditional voting rule and a turnout rule as follows:

Lemma 1: Voting Rule

Conditional on voting in race r, vote for party L's candidate if and only if

$$Prob\left(M^r < y^i\right) \quad \leq \quad \frac{1}{2} \tag{12}$$

and for party R's candidate otherwise.

Proof: See Appendix.

Lemma 2: Turnout Rule

Turn out to vote in race r if and only if

$$Min\left\{Prob\left(M^r < y^i\right), Prob\left(M^r \geq y^i\right)\right\} \quad < \quad b \tag{13}$$

and abstain otherwise.[34]

Proof: See Appendix.

Note that the voting rule is independent of whether a citizen decides to actually turn out and vote; it just specifies that, conditional on voting in race r, a citizen should cast her vote for the candidate from the party whose associated cost incurred by voting is less than one-half. It follows that since the sum of the costs of voting for the candidates from parties L and R in race r is equal to one if the cost associated with voting for one candidate is less than one-half then the cost of voting for the other is greater than one-half. Thus, conditional on voting in the race for office r, a citizen votes for the party whose candidate's position, she expects, is likeliest to be closet to her own.

Since the cost of voting is less than half for only one candidate and the benefit of voting is no greater than half, the Turnout Rule implies a cutoff position at which a citizen is indifferent about whether to obtain the benefit of voting and incur the cost associated with voting for her preferred party's candidate or not turn out to vote in race r at all.

[34] We assume that turnout occurs if the benefit of voting strictly outweighs the cost.

4.3 Electoral Rule

In each senatorial race, the electoral winner is decided by a plurality rule. In the context of our model, this implies that the candidate who garners the highest share of the combined PICs' and FICs' votes will be the winner.[35] In particular, let δ be the proportion of PIC in a given state, s. Let $\Pi_{L,r}$ be party L's voteshare in race r and $\pi_{L,r}^{PIC}$ and $\pi_{L,r}^{FIC}$ be party L's voteshares in race r among PICs and FICs, respectively. Then under the plurality electoral rule, party L's candidate wins office if and only if

$$\Pi_{L,r} \equiv \delta\pi_{L,r}^{PIC} + (1-\delta)\,\pi_{L,r}^{FIC} \geq \frac{1}{2} \tag{14}$$

.

5 Results

We derive the results related to differences in observed aggregate behavior and outcomes between midterm and presidential elections. The presence of PICs and their reliance on prior beliefs and partially informative signals introduces bias in voting that skews outcomes away from the median voter.

5.1 Individual Citizen Behavior

Citizens' decisions whether to turn out and, conditional on turning out, for whom to vote in senatorial races depend on their expectations and uncertainty about candidate positions. Citizens differ not only in their preferences, but also with respect to the information they have about candidates. Electoral environments present citizens with differing quality of information. In midterm elections, citizens can only observe candidates in the race of interest, but in presidential elections an additional signal in the position of presidential candidates is available to citizens. In order to highlight the role of information and electoral environments, we present the behavior of citizens by information levels in each electoral environment.

[35] We resolve ties in favor of party L; our results are not sensitive to this assumption. For simplicity, we model only two-party competition, so the plurality rule will simply be the majority rule. In fact, two states, Georgia and Louisiana, employ an absolute majority rule for senatorial races, and hold runoffs if no candidate garners more than half the votes. The empirical regularities presented earlier are strengthened by excluding these two states from the analysis, but they are included for completeness.

Fully Informed Citizens

As FICs are not uncertain about candidate positions and do not rely on priors and signals to shape their voting decisions, their behavior is not systematically biased in any election. Nonetheless, FICs play an important role in determining outcomes. Thus, before turning to the behavior of PICs, we characterize the turnout and voting behavior of FICs in Proposition 1.

Proposition 1: *A FIC turns out to vote in every senatorial race; in a given senatorial race, a FIC, i, votes for party L's senatorial candidate if and only if*

$$y^i \quad \leq \quad m^s \tag{15}$$

and for party R's senatorial candidate otherwise.

Proof: Follows directly from the definition of a FIC i's information set $\Delta_i = \{M^p, M^s\}$ and Lemmas 1 and 2.

The behavior rules governing conditional voting and turnout for FICs do not vary by electoral environment. In both electoral environments, the cost of voting for FICs is zero and, thus, given any positive benefit to voting, FICs will turn out. Thus, the set of FICs turning out to vote is constant. This suggests that the variation in turnout will come from the set of PICs. Their voting decision depends solely on the realized position of candidates and does not incorporate their prior (*ex ante*) beliefs. While FICs might exhibit an *ex post* bias in voting for one party over another, they will always vote for the candidate that is closest to them in ideology regardless of party labels. It should be noted, that because senate candidates differ from presidential candidates for both local and idiosyncratic reasons, FICs may exhibit both ticket-splitting and straight-ticket voting.[36]

Partially Informed Citizens

Unlike that of FICs, the turnout and voting behavior of PICs varies according to electoral environment. For this reason, we consider the turnout and conditional voting behavior of PICs separately, but compare them across electoral environments. Proposition 2 characterizes the zone of abstention for PICs in each environment.

[36] The voting behavior of FICs may be described as that of swing voters, as they rely upon the observed candidate position to make their voting decision and not on party labels. This behavior contrasts that of a completely uninformed voter who relies solely on party labels.

Proposition 2: *In a midterm election, a PIC, i, turns out to vote in the senatoral race if and only if*

$$y^i \notin \left[\mu_s \pm \Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2}\right] \tag{16}$$

and abstains otherwise; in a presidential election, a PIC, i, turns out to vote in the senatorial race if and only if

$$y^i \notin \left[\mu_s + (m^p - \mu_p)\frac{\sigma_\omega}{\sigma_p}\rho_{\omega,p} \pm \Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1 - \rho_{\omega,p}^2\right)}\right] \tag{17}$$

and abstains otherwise.

Proof: See Appendix.

PICs choose to abstain when the uncertainty over which party's candidate is closer to them in ideological position makes voting too costly.[37] This cost of voting results in a zone of abstention centered around the expected midpoint of candidate positions.[38] The size of this zone of abstention depends on the level of uncertainty about party midpoints and the amount of benefit that PICs derive from voting.[39] As the uncertainty increases or the benefit of voting decreases, the range of abstention increases on both sides.

In presidential elections, the effect on citizens of observing presidential candidates is twofold. First, their overall uncertainty about the positions of senatorial candidates is reduced, and their zone of abstention shrinks. Second, the center of the zone of abstention can move depending on the realization of presidential candidate positions. As will be discussed later, the overall effect is that the turnout of PICs increases during presidential elections relative to midterm elections.[40]

[37]Note, that this is different from risk consideration, where for almost all voters one of the choices is in expectation closer and, thus, less risky.

[38]PICs find it too costly to vote when their ideal points lie near the expected midpoint of the candidates.

[39]Even if voters have a strict preference in expectation for one of the two choices, they still may not turn out to vote.

[40]A general point about roll-off can be made at this point. For our population of PICs, the position of candidates in the presidential race is certain and, thus, all PICs are expected to turn out for the presidential race. Note, however, that even during presidential elections, the zone of abstention for the senatorial race has a positive measure. This implies that there will be a measurable amount of roll-off. However, we also show that in some instances there is a degree of roll-on, or higher turnout in down-ticket races than in up-ticket races. To generate this effect in our model, we would need to assume that there are voters who observe candidate positions in senate races, but not in presidential races. While we have assumed these voters away for simplicity, it might be reasonable to assume a positive measure of these voters in practice. Note that, in order to generate a positive roll-on, these voters do not need to be of greater measure than

Having described the turnout decisions of PICs, we now characterize their conditional voting decisions in Proposition 3.

Proposition 3: *In a midterm election, conditional on turning out, a PIC, i, votes for party L's senatorial candidate if and only if*

$$y^i \quad \leq \quad E\left[M^s\right] = \mu_s \tag{18}$$

and votes for party R's candidate otherwise; in a presidential election, conditional on turning out, a PIC, i, votes for party L's senatorial candidate if and only if

$$y^i \quad \leq \quad E\left[M^s | M^p = m^p\right] = \mu_s + \left(m^p - \mu_p\right) \frac{\sigma_\omega}{\sigma_p} \rho_{\omega,p} \tag{19}$$

and votes for party R's candidate otherwise.

Proof: See Appendix.

The conditional voting behavior of PICs is simply to vote for the candidate whose position is in expectation closest to their ideal position.[41] In midterm elections, this means that the conditional voting rule for PICs incorporates only their priors about party competition. During presidential elections, conditional voting for PICs incorporates both their prior beliefs about the positions of Senate candidates and their observations of realized presidential-candidate positions, with the degree of updating depending on the amount of correlation and the degree of variance. This updating can introduce bias in the conditional voting decisions of PICs, something that we explore in more detail in the following section.[42]

5.2 Aggregation of Citizen Behavior

In the following sections, we discuss the overall differences in turnout by electoral environment and then turn to the question of a coattail effect generated by rational informational contagion. Recall that in race p FICs and PICs behave identically: both types of citizens observe the realized presidential-race midpoint and use their proximity preferences to decide for whom to vote. Moreover, since there is no uncertainty about the location of the mid-

our set of PICs.

[41] Since our distribution is continuous, indifference occurs with a probability of zero. Nonetheless, our voting rule assigns indifferent voters to party L. For voters with incomplete information, indifference will also imply that they abstain. Thus, indifference in senatorial races is only relevant for the behavior of FICs.

[42] Note that much like FICs, PICs can exhibit both ticket-splitting and straight-ticket voting.

point, everyone turns out to vote and gains a benefit b. As a result, $\Pi_{L,p} = \pi_{L.p}^{FIC} = \pi_{L,p}^{PIC}$. Since any variation in behavior will occur in race s, we focus on the differences in voting and turnout decisions in senatorial races.

Turnout

Turnout decisions of FICs are the same in both electoral environments–they always turn out to vote. All citizens participate in the presidential race. Any difference in turnout occurs between midterm and presidential elections in the senatorial race and is generated by the behavior of PICs; thus, we can focus our attention on comparisons of the zone of abstention between the two environments. We present our analysis for a given realization of midpoints, $M^p = m^p$ and $M^s = m^s$, where we assume that the senatorial midpoint is identical in both electoral environments.

Recall that in midterm elections, the range of PICs who choose to abstain is a connected set of length:

$$-2\Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2} \tag{20}$$

which is entered around μ_s; while in the presidential elections, the zone of abstention is of length:

$$-2\Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1 - \rho_{\omega,p}^2\right)} \tag{21}$$

centered around $\mu_s + (m^p - \mu_p)\frac{\sigma_\omega}{\sigma_p}\rho_{\omega,p}$; thus, for any non-zero correlation, the zone of abstention will be narrower in presidential elections than in midterm elections, implying a higher turnout for any given midpoint. Additionally, since μ_s is the median of a unimodal symmetric distribution of preferences, shifting a fixed range of abstention decreases the mass of citizens abstaining overall. This result, combined with a strictly smaller zone of abstention, implies increased turnout.

Thus, the necessary condition for increased turnout is the presence of contagion across races for office. Once this condition is satisfied, the correlation and the difference between m^p and μ_p have a complementary effect on turnout. This result is summarized in Proposition 4.

Proposition 4: *Aggregate turnout in presidential elections is strictly greater than in midterm elections if and only if $\rho_{\omega,p} \neq 0$.*

Proof: See Appendix.

Proposition 4 demonstrates the connection between electoral environments and the aggregate variation in voter turnout between these two environments. Our model is also consistent with the observation of positive levels of roll-off.

Candidate Voteshares

The conditional voting decisions of FICs in the senatorial races for office are independent of the realization of the presidential-candidate idiosyncratic effect. Focusing on PICs, we establish a relationship between the voteshare of party L's presidential candidate and party L's senatorial candidate conditional on a realization of party midpoints.

Consider a particular realization of $M^p = m^p$. We can rewrite the realized presidential effect, $\epsilon^p = e$, in terms of the fixed and unknown party effect and the realized midpoint of presidential candidates, such that $e = m^p - \Omega$. The presidential voteshare among PICs for party L is now a function of the realization of the idiosyncratic presidential effect of the two candidates. Let $\pi_{L,p}^{PIC}(e)$ be the presidential voteshare for party L among PICs conditional on the realization of m^p. The following equation establishes the strictly increasing relationship between $\pi_{L,p}^{PIC}(e)$ (and implicitly $\Pi_{L,p}$) and e:

$$\pi_{L,p}^{PIC}(e) = F_s(m^p) = F_s(\Omega + e) \tag{22}$$

Similarly let $\pi_{L,s}^{PIC}(e)$ be the senatorial voteshare for party L among PICs conditional on the realization of party positions.[43] Note that this is a function of the presidential idiosyncratic realization and not of the senatorial one. Let $\zeta_r(\Omega + e)$ be the fraction among PICs who choose to turn out and vote in race for office r as a function of the presidential race idiosyncratic effect, conditional on the realization of $M^p = m^p$.[44] We then have:

$$\pi_{L,s}^{PIC}(e) = \frac{F_s\left(\mu_s + (\Omega + e - \mu_p)\frac{\sigma_\omega}{\sigma_p}\rho_{\omega,p} + \Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1 - \rho_{\omega,p}^2\right)}\right)}{\zeta_s(\Omega + e)}. \tag{23}$$

Note that since $\pi_{L,p}^{PIC}(e)$ and $\pi_{L,s}^{PIC}(e)$ are both strictly increasing in e, among PICs the relationship between the presidential voteshare and the senatorial voteshare is positive.[45]

[43] Unless mentioned otherwise, we will refer to $\pi_{L,r}$ as the voteshare for party L in race r among PICs associated with a presidential idiosyncratic draw, e.

[44] In a presidential race, $\zeta_p(e) = \zeta_p = 1$.

[45] Let $A(\Omega + e) \equiv F_s\left(\mu_s + (\Omega + e - \mu_p)\frac{\sigma_\omega}{\sigma_p}\rho_{\omega,p} + \Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1 - \rho_{\omega,p}^2\right)}\right)$ and $B(\Omega + e) \equiv 1 - F_s\left(\mu_s + (\Omega + e - \mu_p)\frac{\sigma_\omega}{\sigma_p}\rho_{\omega,p} - \Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1 - \rho_{\omega,p}^2\right)}\right)$. We can now rewrite $\pi_{L,s}$ in terms of A and

This relationship is the result of information contagion across races p and s. Consistent with the use of the term in the literature, we call this relationship the coattail effect as it is independent of the actual realizations of the party platform and is generated solely by good or bad (for party L) draws of the idiosyncratic characteristics of presidential candidates. Among PICs, favorable idiosyncratic draws in the presidential race are associated with greater support for presidential and senatorial candidates. We characterize this contagion result in Proposition 5 and its corollary.

Proposition 5: *Let e' and e^*, such that $e' > e^*$, be two distinct realizations of ϵ^p, then $\pi_{L,p}^{PIC}(e') > \pi_{L,p}^{PIC}(e^*)$ and $\pi_{L,s}^{PIC}(e') > \pi_{L,s}^{PIC}(e^*)$.*

Proof: Follows directly from the derivations of $\pi_{L,p}^{PIC}$ and $\pi_{L,s}^{PIC}$.

Recall that since FICs and PICs behave identically in race p, we have that $\Pi_{L,p} = \pi_{L,p}^{FIC} = \pi_{L,p}^{PIC}$. Thus, Proposition 5 implicitly establishes a strictly increasing relationship between party L's presidential voteshare, $\Pi_{L,p}$, and its senatorial voteshare, $\pi_{L,s}^{PIC}$. We can then condition $\pi_{L,s}^{PIC}$ on $\Pi_{L,p}$, which is an observable quantity. In particular, since F_s has full support, we can rewrite $\pi_{L,s}^{PIC}$ as a function of $\Pi_{L,p}$ as follows:

$$\pi_{L,s}^{PIC}(\Pi_{L,p}) = \frac{F_s\left(\mu_s + \left(F_s^{-1}(\Pi_{L,p}) - \mu_p\right)\frac{\sigma_\omega}{\sigma_p}\rho_{\omega,p} + \Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}\right)}{\zeta_s\left(F_s^{-1}(\Pi_{L,p})\right)}. \quad (24)$$

The dependence of senatorial race voteshares on presidential race voteshares is summarized below.

Corollary: *For any pair $\Pi'_{L,p}, \Pi_{L,p} \in [0,1]$, such that $\Pi'_{L,p} > \Pi_{L,p}$, we have that $\pi_{L,s}^{PIC}\left(\Pi'_{L,p}\right) > \pi_{L,s}^{PIC}(\Pi_{L,p})$.*

Proof: Follows directly from Proposition 5 and definition of $\pi_{L,s}^{PIC}(\Pi_{L,p})$ above.

Proposition 5 and its corollary are the key results that link observed differences in aggregate voting behavior to consistent and systematic differences in outcomes bias. The

B. That is, $\pi_{L,s} = \frac{A}{A+B}$ where $\zeta_s = A + B$. Since F_s has full support we can divide through by A and get that $\pi_{L,s} = \frac{1}{1+\frac{B}{A}}$. Since A is strictly increasing in e and B is strictly decreasing in e, then $\frac{B}{A}$ is strictly decreasing in e and, thus, $\pi_{L,s}$ is strictly increasing in e.

presence of informational coattail voting will create a systematically biased electoral landscape, which in turn affects the type and party of candidates elected to office. We explore these outcome effects in the next section.

5.3 Electoral Outcomes in the Senatorial Race for Office

Without loss of generality, we focus on party L to demonstrate the results for aggregate electoral outcomes of candidates in the senatorial race for office in state s. The following results will hold symmetrically for party R.

Our model of candidate selection contains an idiosyncratic stochastic component. Thus, for a fixed population of citizens, the electoral outcomes of senatorial races will not be deterministic, and our results will be presented as expectations. Before presenting our main propositions, we present a set of general results for the midterm and presidential electoral environments.

Midterm Elections

In a midterm election, a senatorial candidate from party L in state s with senatorial race midpoint $M^s = m^s$ wins office if and only if

$$(1-\delta) F_s (m^s) + \delta \pi_{L,s}^{PIC} \quad \geq \tfrac{1}{2} \tag{25}$$

The first term on the LHS is associated with the measure of FICs whose ideal point is to the left of the candidate-position midpoint (and who will vote for the candidate from party L) weighted by their proportion in the overall population in state s. The second term relates to the measure of PICs in the population who vote for the senatorial candidate from party L. Since the proportion of PICs' votes is split equally between the candidates from both parties independent of the realized ideological midpoint in the senatorial race, that measure is equal to one-half.[46] Overall, this condition simplifies to

$$m^s \geq F_s^{-1}\left(\frac{1}{2}\right) = \mu_s, \tag{26}$$

which means that party L's candidate wins if the realized senatorial-race midpoint is to the right of the median citizen's ideal point in state s. Thus, the probability that a candidate

[46] Uniformed voters provide no advantage for any given candidate since they vote with equal proportions for both candidates.

from party L wins when $M^s = m^s$ is

$$Prob\left(win_L^s | M^s = m^s\right) \quad = \quad \begin{cases} 1 & m^s \geq \mu_s \\ 0 & otherwise. \end{cases} \tag{27}$$

Note that FICs determine which candidate will win. Given their proximity preferences, if the senatorial-race midpoint is to the right of the median FIC's preference, then the candidate from party L wins office. Formally, the range of midpoints that result in a victory for party L's senatorial candidate is $[\mu_s, \infty)$.

Presidential Elections

Recall that when $\rho_{\omega,p} = 0$, the behavior of PICs in presidential elections is equivalent to their behavior in midterm elections and there is no contagion across races for office during presidential elections. Now, instead, suppose that $\rho_{\omega,p} > 0$, but that $\epsilon^p = 0$. While there is no bias in voting, PICs have less uncertainty about candidate positions and turnout increases. As $\rho_{\omega,p}$ increases and ϵ^p remains zero, the number of citizens induced to turn out and vote for each party increases evenly and the proportion of PICs voting for each party remains the same (i.e., $\pi_{L,s}^{PIC}(0) = \pi_{R,s}^{PIC}(0) = \frac{1}{2}$). Thus, if the realized presidential idiosyncratic error is identical to its expectation, then the votes of PICs in the senatorial race are split equally; however, more PICs turn out to vote relative to midterm elections.[47]

Now, suppose that $\rho_{\omega,p} > 0$ and that $\epsilon^p \neq 0$; the sign of the realized error will determine which senatorial candidate will benefit from a built-in advantage passed down from the presidential race for office. We will focus the following comparative statics on the presidential idiosyncratic error while conditioning on a fixed and positive level of correlation.

In a presidential election with $\epsilon^p = e$, a senatorial candidate from party L in state s with senatorial race midpoint $M^s = m^s$ wins office if and only if

$$(1-\delta)F_s\left(m^s\right) + \delta\pi_{L,s}^{PIC}\left(\Pi_{L,p}\right) \quad \geq \quad \frac{1}{2}. \tag{28}$$

As before, the first term on the LHS of equation 28 is the measure of FICs who vote for the candidate from party L weighted by their measure in the population, while the second

[47] Essentially, an increase in $\rho_{\omega,p}$ signifies that PICs have more information when facing the senatorial race for office, which in turn reduces their ex-ante probability of voting for the wrong candidate *ceteris paribus*.

term is the weighted measure of PICs who vote for party L, respectively. The inequality simplifies to[48]

$$m_s \geq F_s^{-1}\left[\frac{1}{2}+\frac{\delta}{2\left(1-\delta\right)}\left(1-2\pi_{L,s}^{PIC}\left(\Pi_{L,p}\right)\right)\right] \equiv \Lambda\left(\Pi_{L,p}\right). \tag{29}$$

Thus, the probability that a candidate from party L wins when $M^s = m^s$ is:

$$Prob\left(win_L^s | M^s = m^s\right) = \begin{cases} 1 & \text{if } m_s \geq \Lambda\left(\Pi_{L,p}\right) \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

Since $\Pi_{L,p}$ is strictly increasing in e and $\Lambda\left(\Pi_{L,p}\right)$ is strictly decreasing in $\Pi_{L,p}$, we have that the leftward bound of winnable positions for party L's candidate strictly decreases in e. Suppose there is a positive draw, such that $e > 0$, then $\mu_s > \Lambda\left(\Pi_{L,p}\right)$. Consequently, the range of ideological midpoints that result in a win by party L's candidate in the senatorial race contains the corresponding range derived for midterm elections. As before, the range of midpoints that result in a win by party L in a midterm election is $[\mu_s, \infty)$, whereas in a presidential election the range is $[\Lambda\left(\Pi_{L,p}\right), \infty)$. The relationship between the presidential race idiosyncratic error and attainable positions in the senatorial race for party L in midterm and presidential elections is described below:

$$\left(\mu_s, \infty\right) \subsetneq \left(\Lambda\left(\Pi_{L,p}\right), \infty\right) \iff e > 0 \tag{31}$$

and

$$\left(\mu_s, \infty\right) \supsetneq \left(\Lambda\left(\Pi_{L,p}\right), \infty\right) \iff e < 0. \tag{32}$$

Thus, in presidential elections, as the support for the presidential candidate of party L

[48]Note that since $\pi_{L,s}\left(e\right) \in [0,1]$,

$$\Lambda\left(\Pi_{L,p}\right) \quad \in \quad \left[F_S^{-1}\left(\frac{1}{2}-\frac{\delta}{2\left(1-\delta\right)}\right), F_S^{-1}\left(\frac{1}{2}+\frac{\delta}{2\left(1-\delta\right)}\right)\right].$$

Also, note that for $\rho_{\omega,p} > 0$,

$$\frac{\partial \Lambda\left(\Pi_{L,p}\right)}{\partial \Pi_{L,p}} \quad < \quad 0$$

and that

$$\Lambda\left(\Pi_{L,p}\right) \quad = \quad \mu_s \iff e = 0.$$

in which case this result boils down to the baseline midterm-election win condition (though turnout increases).

increases, the range of electorally viable positions for that party's senatorial candidate increases. This increased range of electoral viability occurs as a larger set of PICs are induced to vote for a senatorial candidate independent of the realized idiosyncratic ideologies of the candidates. This result is summarized in Proposition 6.

Proposition 6: *For any pair* $\Pi'_{L,p}, \Pi_{L,p} \in [0,1]$, *such that* $\Pi'_{L,p} > \Pi_{L,p}$, *we have that* $(\Lambda(\Pi_{L,p}), \infty) \subsetneq \left(\Lambda\left(\Pi'_{L,p}\right), \infty\right)$.

Proof: Follows from the derivations above.

Proposition 6 establishes the main mechanism by which electoral bias is introduced into observed outcomes. The observation of presidential candidate positions is informative and it is rational for PICs to condition their turnout and voting decisions upon it; however, any generic realization of the idiosyncratic error in the presidential race introduces bias into the behavior of PICs, which in turn alters the electoral landscape.

We now present our final two key results. The first accounts for the well-known presidential surge phenomenon and the second for our new findings. Proposition 6 states that greater support for a party's presidential candidate is associated with a wider range of winnable positions for their senatorial candidate. In turn, a wider range of positions that a party's candidate can take makes it more likely that it will win office. The following proposition establishes this result.

Proposition 7: *Let* win_L^s $(lose_L^s)$ *denote a win (loss) for party L in race s. Then for any pair* $\Pi'_{L,p}, \Pi_{L,p} \in [0,1]$, *such that* $\Pi'_{L,p} > \Pi_{L,p}$, *we have that*

$$Prob\left(win_L^s | \Pi'_{L,p}\right) > Prob\left(win_L^s | \Pi_{L,p}\right)$$

and

$$Prob\left(lose_L^s | \Pi'_{L,p}\right) < Prob\left(lose_L^s | \Pi_{L,p}\right).$$

Proof: See Appendix.

Finally, we present our result that connects the electoral environment to the expected positions of winning candidates and losing incumbents. The previous proposition states that greater support for a presidential candidate aids other candidates from the same party. In particular, a marginal senatorial candidate who most likely would lose can win in the presence of sufficient support for his party's presidential candidate. Similarly, a relatively moderate senatorial candidate can lose if the support for his party's presidential candidate it poor. Thus, the expected ideological extremism of winning senatorial candidates increases in the support for their party's presidential candidate, while the converse it true for losers. This relationship is summarized in Proposition 8.

Proposition 8: *For any pair* $\Pi'_{L,p}, \Pi_{L,p} \in [0,1]$, *such that* $\Pi'_{L,p} > \Pi_{L,p}$, *we have that*

$$E\left[C^s_L | win^s_L, \Pi'_{L,p}\right] < E\left[C^s_L | win^s_L, \Pi_{L,p})\right]$$

and

$$E\left[C^s_L | lose^s_L, \Pi_{L,p}\right] > E\left[C^s_L | lose^s_L, \Pi'_{L,p}\right].$$

Proof: See Appendix.

Sketch of Proof: By Proposition 6, the range of winning midpoints for party L's senatorial candidate is $[\Lambda(\Pi_{L,p}), \infty)$. If we consider a fixed position for party R's senatorial candidate, $C^s_R = c$, we can rewrite this range in terms of party L's senatorial candidate positions as $[2\Lambda(\Pi_{L,p}) - c, c]$. A greater presidential voteshare $\Pi'_{L,p}$ implies a more pronounced coattail effect and wider range of viable midpoints. Since $\Lambda\left(\Pi'_{L,p}\right) < \Lambda\left(\Pi_{L,p}\right)$, the overall range of winning positions for party L's senatorial candidate strictly increases on the leftward boundary and remains constant on the rightward boundary:

$$[2\Lambda(\Pi_{L,p}) - c, c] \subsetneq \left[2\Lambda(\Pi'_{L,p}) - c, c\right].$$

As the range of possible positions for party L's senatorial candidates range over this entire support, the expected position of those who win with greater presidential support will be strictly more negative. The logic of the proof is similar for expected positions of losers. Poor support for party L's presidential candidate implies an anti-coattail effect. This increases the range of losing positions for party L's senatorial candidates on the rightward boundary, which, in turn, increases the expected position of party L's losing candidates.

Propositions 6, 7 and 8 connect our observations about the presidential electoral landscape to differences in expected outcomes. For entry, candidates from a party benefiting from a positive coattail effect are presented with an advantageous electoral environment.

As a result, more of these candidates, particularly the more-extreme candidates, are able to win elections. This increases the expected ideological extremism relative to an environment without coattails, such as a midterm election. Conversely, negative coattails handicap candidates and increase the importance of ideological moderation and FICs' votes to win office. Our results for exit are analogous. Positive coattails allow relatively more extreme candidates to stay in office, but negative coattails stymie moderate candidates relative to midterm elections. Overall, since a candidate is more likely to win in the presence of a favorable coattail, the presidential electoral environment will return a more polarized Senate relative to the midterm electoral environment.

These results also connect our model to traditional accounts of coattails. While previous models focus on the relationship between presidential support to party success down-ticket, we suggest that such studies may overlook the broader implications of such support. Not only is there a party effect, but the type of candidates that prevail from both parties is affected by coattails.

We also provide microfoundations for the coattail effect that can easily be related to characteristics of political competition and the electorate. Coattails arise in our model due to informational cues and proximity preferences. If party discipline increases, then the degree to which voters rely on party labels increases and ticket-splitting becomes less common. Similarly, as the proportion of PICs in the electorate increases, the effect of informational contagion on outcomes increases. In our model, a larger proportion of voters who rely on party labels results in greater coattail swings and more extreme outcomes.

6 Empirical Coattail Effect

Our model indicates that the presidential electoral environment returns a more extreme and polarized Senate than the midterm electoral environment. Figure 4 demonstrates that increased support for a presidential candidate is associated with ideological extremism of senators from the same party. Restricting our attention to senators who enter during a presidential-election cycle, we classify each senator in our dataset by her party identification and the voteshare decile of the Democratic presidential candidate who ran for office at the time of her entry. Following this classification, we calculate the mean DW-NOMINATE score for all senators that fall into each of these decile groups. The notable observation is that, for each party, the average DW-NOMINATE score is strictly decreasing in the Democratic presidential voteshare. That is, Democrats become more extreme and Republicans more moderate, as predicted by our model.

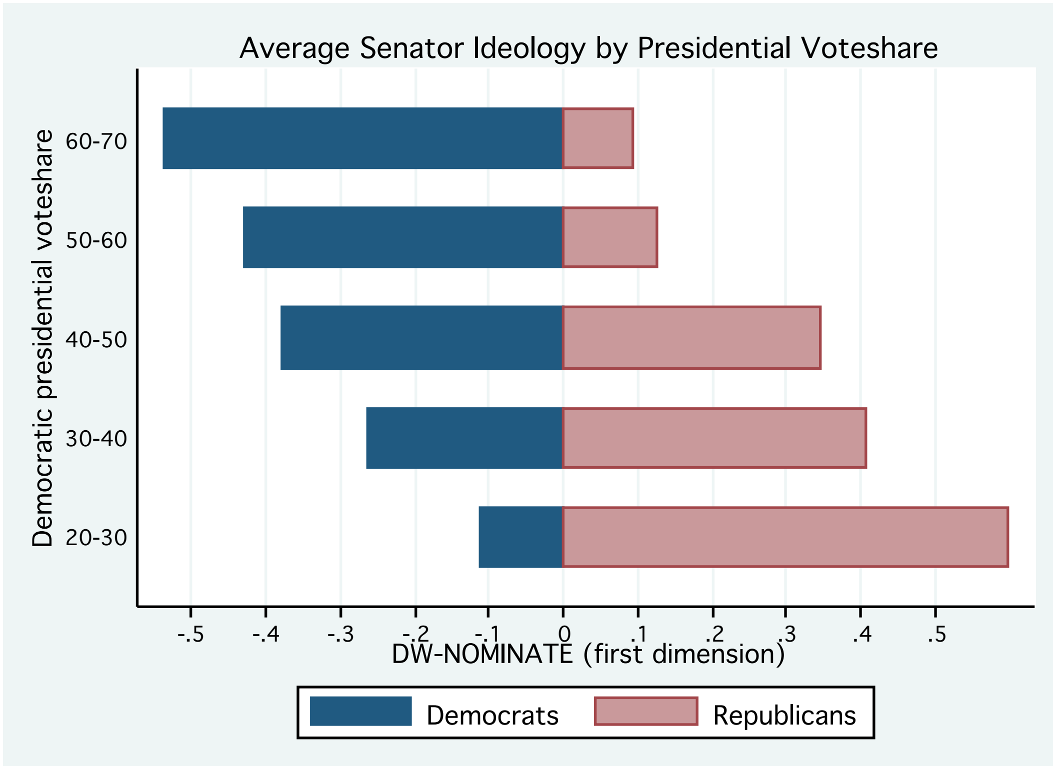


Figure 4: Senator Ideology and Presidential Support

In our model, presidential support is positively related with the latent variable that pins down the coattail effect: the realized idiosyncratic error in the presidential race. We use this relationship to show that the expected position of a senator elected in a presidential election becomes more extreme as her party's presidential candidate garners more votes. While consistent with our model's prediction, the ideological preferences of voters in a given state may also account for the observed phenomenon in Figure 4. For example, a Democratic presidential candidate is likely to generate more support in a liberal-leaning state, which in turn is likely to elect more liberal senators. To address this concern, we develop a measure of expected presidential support and use realized presidential support to derive the coattail effect: the gap between expected and realized presidential support.

To purge presidential support at the state level from the ideological preferences of voters we employ two separate methodologies. The first is used by Aldrich et al. (2008), albeit for a different purpose, in which he estimates a linear model that relates historical state-level presidential support for the Democratic candidate to its demographics. We then use the postestimation residual–the difference between the realized and predicted presidential support–as our measure of coattails. In the second we use the difference between the state-level realized Democratic presidential voteshare and its trailing average. In the Appendix, we report regression results in which we relate senators' DW-NOMINATE scores to the estimated coattail effect in their entry while controlling for state characteristics. In Table 3, we use the residual from the Aldrich demographics model as a measure of coattails and in Table 4 we use the difference from the average Democratic presidential support. Since the degree to which a citizen is informed about candidate positions is likely to interact with certain characteristics, we also interact the coattail estimates with state demographics.[49] Consistent with our model, the regression results indicate that unexpected Democratic presidential support is associated with more ideologically liberal senators.

Overall, these results suggest that information contagion plays a significant role in the senatorial race for office and its outcomes irrespective of the ideological preferences of voters in a given state.

7 Conclusion

This paper presents strong evidence that senators first elected during presidential-election years are ideologically more extreme than their counterparts first elected during midterm elections. In addition to this result, we find even stronger evidence suggesting that the environment in which a senator leaves office is also correlated with her policy positions in the Senate. Here, we find that senators who are more moderate leave office during presidential-election years, and more extreme senators exit during midterm elections. Together, these two facts suggest that the presidential-election season returns a more extreme and polarized Senate.

In the United States, elections for office are rarely held in isolation. Instead, many offices are contested simultaneously, and candidates across the ticket are linked through party identification. At the same time, because acquiring information is costly, voters possess limited information about candidates. The degree to which voters are informed about candidates is expected to vary by office. For example, greater media coverage of tickets for higher-level office may reduce the cost of information acquisition for that particular race, but may do so to a lesser degree for other races.

Formal literature on the interaction between contemporaneous elections is sparse. With the important exception of Alesina and Rosenthal (1989), (1995) and (1996), most of the theoretical modeling has been informal. This literature has tended to view down-ticket voting in light of presidential politics, and has been preoccupied with midterm decline. There are two broad themes in this literature. The first views midterm elections as a

[49] For more work on this topic, see Halberstam and Montagnes (2009b).

reversion to the mean in terms of presidential support. For example, in the surge and decline models (see Angus Campbell (1960) and James Campbell (1991)), the major difference between the two electorates is the presence of presidential partisan voters. Thus, midterm elections are distinguished from presidential elections by their lack of voters in support of the president's party. Conversely, another strand of the literature characterizes midterm elections by the presence of voters who vote against the president's party (see Erikson (1988), Kernell (1977) and Tufte (1975)). Both approaches, however, fail to provide an account of why we might observe consistent differences in ideology by electoral environment across parties and over time.

There are three main reasons why these models are ill-suited for explaining the facts we uncover in Halberstam and Montagnes (2009a). First, they do not directly consider down-ticket races, but instead focus on the effect of presidential politics on voter decision-making. Second, the models are purely partisan and do not consider races in light of a spatial setting. Third, it is difficult to incorporate spatial competition into models that connect voting decisions in Congress to the perception of the presidency.

One model that accounts for the effects of multiple simultaneous elections in a spatial setting is Alesina and Rosenthal's balanced government model. In this model, voters attempt to balance the policy produced by Congress and the president by electing a divided government. This produces a more moderate policy outcome, which better reflects the preferences of voters. Our results on electoral entry and exit environments are difficult to reconcile with such a model, unless voters are systematically electing senators who are extreme in the opposite direction of the presidential preference. Furthermore, presidential candidates preferred by such split-ticket voters would have to be sufficiently ideologically extreme such that the balancing senators from the opposing party would necessarily be even more extreme than their midterm counterparts. In fact, our results on party matching and ideology suggests that the opposite is true: states that vote for Democratic presidential candidates elect more-liberal candidates during presidential elections then during midterm elections.

Previous research has suggested that an important role of parties in elections is to serve as a cue or brand that conveys information about candidate positions. In a recent series of papers, Stephen Jessee has found strong support for both spatial voting and the importance of information in making correct spatial choices. In this paper, we have examined the aggregate effect of party labels in the context of incomplete information and citizens' distaste for making mistakes. Our first substantive result establishes that the presence of these labels creates informational contagion between races for office. Since candidates from the same party share ideological characteristics, citizens make rational inferences about candidate positions in one race based upon their observations in another. This informational contagion creates a coattail effect between races, due to which an electorally advantageous draw of presidential candidates from one party creates an electoral advantage for the same party on the down-ticket senatorial race.

Our second theoretical contribution solidifies the role of this coattail effect in the context of a spatial model of electoral competition. We demonstrate that, beyond the effect on a senatorial candidate's prospect of winning, positive presidential coattails affect the expected positions of winners and losers. By swaying a portion of the electorate towards one party, the coattail effect alters the range of electorally viable positions for parties. This result suggests that previous literature may understate the importance of coattails on electoral outcomes.

If party labels introduce informational contagion between races, then policy makers may want to consider the role and implications of party labels in other contexts, such as judicial and local elections. In addition to the partisan effects of party labels, our model implies that their presence in presidential elections generates a less informed electorate relative to midterm elections. This finding warrants further research on the availability of such cues, their effects on the type of information being processed by the electorate, and the resulting consequences.

Finally, our results suggest that the timing of multiple elections has significant effects on the type of senator elected as reflected by her voting behavior. The normative implications of such a result depend on the model of policy formation that one employs. However, an awareness of the result might inform debates over policy issues such as the timing of elections for multiple offices. When studying electoral institutions, the temptation is to look at elections in isolation. Our results caution against that approach.

References

[1] Abramowitz, Alan I. 1988. "Explaining Senate Election Outcomes." American Political Science Review 82(2): 385-403.

[2] Aldrich, John, Michael Brady, Scott de Marchi, Ian McDonald, David Rohde and Michael Tofias. 2008. "Party and Constituency in the U.S. Senate, 1933-2004." in Why Not Parties? Nathan W. Monroe, Jason M. Roberts, and David Rhode, eds., University of Chicago Press.

[3] Aldrich, JH. 1995. Why parties?: The University of Chicago Press.

[4] Alesina, Alberto, and Howard Rosenthal. 1989. "Partisan Cycles in Congressional Elections and the Macroeconomy." American Political Science Review 83(2): 373-398.

[5] Alesina, Alberto and Howard Rosenthal. 1995. Partisan Politics, Divided Government, and the Economy. Cambridge University Press. Cambridge.

[6] Alesina, Alberto, and Howard Rosenthal. 1996. "A Theory of Divided Government." Econometrica 64(6): 1311-1341.

[7] Ansolabehere, Stephen, James M. Snyder and Charles Stewart. 2001. "Candidate Positioning in U.S. House Elections." American Journal of Political Science 45(1): 136-159.

[8] Austen-Smith, D. 1981. "Party policy and campaign costs in a multi-constituency model of electoral competition." Public Choice 37(3):389-402.

[9] Bafumi, Joseph and Michael Herron. 2007. "Preference Aggregation, Representation, and Elected American Political Institutions." Dartmouth University. Working paper. http://www.dartmouth.edu/~herron/RepInstitutions.pdf

[10] Besley, T., and A. Case. 2003. "Political institutions and policy choices: evidence from the United States." Journal of Economic Literature 41(1):7-73.

[11] Besley, Timothy and Ian Preston. 2007. "Electoral Bias and Policy Choice: Theory and Evidence." Quarterly Journal of Economics 122(4): 1473-1510.

[12] Brennan, G., and J.M. Buchanan. 1984. "Voter choice." American Behavioral Scientist 28(2):185-201.

[13] Burden, Barry C., Gregory A. Caldeira and Tim Groseclose. 2000. "Measuring the Ideologies of U.S. Senators: The Song Remains the Same." Legislative Studies Quarterly 25(2): 237-258.

[14] Caillaud, B., and J. Tirole. 2002. "Parties as Political Intermediaries*." Quarterly Journal of Economics 117(4):1453-89.

[15] Callander, S. 2005. "Electoral competition in heterogeneous districts." Journal of Political Economy 113 (5):1116-45.

[16] Callander, Steven and Catherine H. Wilson. 2007. “Turnout, Polarization, and Duverger’s Law.” Journal of Politics 69(4): 1047-1056.

[17] Campbell, Angus. 1960. “Surge and Decline: A Study of Electoral Change.” Public Opinion Quarterly 24(3): 397-418.

[18] Campbell, James E. 1991. “The Presidential Surge and its Midterm Decline in Congressional Elections, 1868-1988.” Journal of Politics 53(2): 477-487.

[19] Campbell, James E. 1993. The Presidential Pulse of Congressional Elections. 2nd ed. University Press of Kentucky.

[20] Coate, Stephen and Brian Knight. 2007. “Socially Optimal Districting: A Theoretical and Empirical Exploration.” Quarterly Journal of Economics 122(4): 1409-1471.

[21] Cox, G.W., and M.D. McCubbins. 1993. Legislative leviathan: Party government in the House: Univ of California Pr.

[22] Degan, A., and A. Merlo. 2007. "Do voters vote sincerely?", University of Pennsylvania Working Paper http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1133809

[23] Diermeier, Daniel, Michael Keane and Antonio Merlo. 2005. “A Political Economy Model of Congressional Careers.” American Economic Review 95(1): 347-373.

[24] Erikson, Robert S. 1988. “The Puzzle of Midterm Loss.” Journal of Politics 50(4): 1011-1029.

[25] Feddersen, T., S. Gailmard, and A Sandroni. 2009. "Moral bias in large elections: theory and experimental evidence." American Political Science Review 103(02):175-92.

[26] Feddersen, T., and A. Sandroni. 2006a. "A theory of participation in elections." American Economic Review 96(4):1271-82.

[27] Feddersen, T., and A. Sandroni. 2006b. "Ethical voters and costly information acquisition." Quarterly Journal of Political Science 1(3):287-311.

[28] Feddersen, T., and W. Pesendorfer. 1996. "The swing voter's curse." The American economic review 86(3):408-24.

[29] Gelman, Andrew et al. 2008. Red State, Blue State, Rich State, Poor State: Why Americans Vote the Way They Do. Princeton University Press.

[30] Gerber, E.R., and R.B. Morton. 1998. "Primary election systems and representation." Journal of Law Economics and Organization 14(2):304-24

[31] Ghirardato, P., and J.N. Katz. 2006. "Indecision Theory: Weight of Evidence and Voting Behavior." Journal of Public Economic Theory 8(3):379.

[32] Halberstam, Yosh and B. Pablo Montagnes. 2009a. "Consistent Biases in Electoral Environments: Evidence from Entry and Exit of Senators." Northwestern University. Working Paper.

[33] Halberstam, Yosh and B. Pablo Montagnes. 2009b. "Party Labels and Information: The Implications of Contagion in Coelection Environments." Northwestern University. Working Paper.

[34] Jessee, Stephen A. 2008. "Partisan Bias, Political Information and Spatial Voting in the 2008 Presidential Election.", University of Texas Working Paper https://webspace.utexas.edu/sjessee/www/research-files/spatial08JOP.pdf

[35] Jessee, Stephen A. 2009. "Spatial Voting in the 2004 Presidential Election." American Political Science Review 103(01):59-81.

[36] Kernell, Samuel. 1977. "Presidential Popularity and Negative Voting: An Alternative Explanation of the Midterm Congressional Decline of the President's Party." American Political Science Review 71(1): 44-66.

[37] Leighley, Jan E. and Jonathan Nagler. 2007. "Who Votes Now? And Does It Matter?" presented at the Annual Meeting of the Midwest Political Science Association, April, 2007.

[38] McDonald, M.P. 2009. "Turnout 1980-2008." http://elections.gmu.edu/voter_turnout.htm

[39] McDonald, M.P., and S.L. Popkin. 2002. "The myth of the vanishing voter." American Political Science Review 95(04):963-74.

[40] Palfrey, Thomas R. and Keith T. Poole. 1987. "The Relationship between Information, Ideology, and Voting Behavior." American Journal of Political Science 31(3): 511-530.

[41] Patty, J.W. 2006. "Loss aversion, presidential responsibility, and midterm congressional elections." Electoral Studies 25(2):227-47.

[42] Poole, Keith T. and Howard Rosenthal. 2000. Congress: A Political-Economic History of Roll Call Voting. Oxford University Press, USA.

[43] Riker, W.H., and PC Ordeshook. 1968. "A Theory of the Calculus of Voting." The American Political Science Review 62(1):25-42.

[44] Schuessler, A.A. 2000. "Expressive voting." Rationality and Society 12(1):87.

[45] Shor, Boris. 2009. "All Together Now: Putting Congress, State Legislatures, and Individuals in a Common Ideological Space" University of Chicago. Working paper. http://home.uchicago.edu/~bshor/research/ccap.pdf

[46] Snyder Jr., J.M., and M.M. Ting. 2002. "An informational rationale for political parties." American Journal of Political Science 46(1):90-110.

[47] Tufte, Edward R. 1975. "Determinants of the Outcomes of Midterm Congressional Elections." American Political Science Review 69(3): 812-826.

[48] Tullock, G. 1971. "Public decisions as public goods." The Journal of Political Economy 79(4):913-8.

Data

Our dataset comprises senators who were elected to the senate from 1966 to present.[50] For each senator we gather biographical information from the CQ Congressional Collection. We combine these data with Poole and Rosenthal's DW-NOMINATE dataset. Using information about the senators' entry and exit environments and party identification, we employ a non-parametric methodology to compare the DW-NOMINATE scores of senators first elected during presidential elections to their midterm counterparts. We perform this comparison for each congress in our 40 years of data, and run an identical analysis in which we use a senator's exit environment as the classification criterion.

Senate Composition

Our dataset consists of all senators who faced federal elections for the first time between 1966 and 2006. For each senator we gather biographical information from the CQ Congressional Collection. These data include party affiliation as well as the starting and ending dates for service in the Senate. We use these dates to construct two classification variables: the first indicates the entry environment of each senator and the second, if applicable, the exit environment. We exclude senators who were appointed to fill a vacated seat, unless they were subsequently elected during regular federal elections. In our analysis of exit electoral environments, we do not distinguish between incumbent senators who lose and those who do not seek reelection; however, we exclude senators who leave office due to death or who leave office before the end of their term.[51] Finally, we preclude from our analysis senators who were not affiliated with the Democratic or Republican parties during both their entry into and, if applicable, exit from the Senate.

In Table 2, we summarize the data by party identification with respect to our classification criteria.[52] Since our data on exiting senators is right-censored, we observe a greater

[50]This period is characterized by the solidification of two major parties following the decline of Southern Democrats that ensued the ratification of the Voting Rights Act in 1965 and the signing of the Civil Rights Act of 1964. See Gelman, Park, Shor, Bafumi and Cortina (2008) for further reading. To check the robustness of our results, in Halberstam and Montagnes (2009a), we extend our analysis to include senators who were elected before 1966.

[51]This is done with the strategic concern of the senator in mind and in light of our focus on the effects of electoral environments. If a senator anticipates an unfavorable electoral environment, her best course of action might be to step down rather than seek reelection. (See Diermeier, Keane and Merlo (2005) for more on this topic.) This classification will present an interpretive problem only if there is a systemic difference in the retirement decisions of senators facing different electoral environments that does not depend on how favorable an environment is but, nonetheless, varies with it. For example, if senators have a strong preference for serving one term or one and three terms, their retirement decision will be correlated with their electoral environment. However, having performed appropriate testing, we know this not to be the case.

[52]Senators who switch parties during their service are coded using their initial party affiliation for entry

Table 2: Senate Electoral Composition

	Democrats		**Republicans**	
	Entry	Exit	Entry	Exit
Midterm	45	21	54	40
Presidential	58	35	64	41
Total	103	56	118	81

number of new senators who enter the Senate than the number of incumbents who leave. Over the course of the data, 221 new senators are elected and 137 incumbents exit the Senate.

In Figure 6a, we display for each congress in our data the composition of the Senate with respect to the entry classification. For example, of 100 senators in the 109th congress, 93 were elected during regular federal elections in 1966 or later, 55 were elected in a presidential election, and the rest in a midterm election. The remaining seven senators were either elected before 1966, appointed or specially elected and did not subsequently compete in regular federal elections or were affiliated with a third party.[53] Analogous to the description on entry, in Figure 6b, we display the exit decomposition of the Senate.

DW-NOMINATE Scores

As a measure of a senator's ideology we use Poole and Rosenthal's DW-NOMINATE scores.[54] The data employed for estimating these scores consists of (nearly) all individual roll call votes in United States congressional history. DW-NOMINATE scores are estimates derived from a dynamic weighted nominal three-step estimation procedure, which was created by Poole and Rosenthal in the 1990s. An iterative Maximum Likelihood estimation is employed to recover each legislator's ideal point and roll call midpoints of a spatial model in a random utility framework. The points are placed in a common space and constrained to lie within a unit hyperspace. The point estimates are robust to concerns about strategic voting, logrolling and time-variant ideal points.[55] We employ the

and final party affiliation for exit.

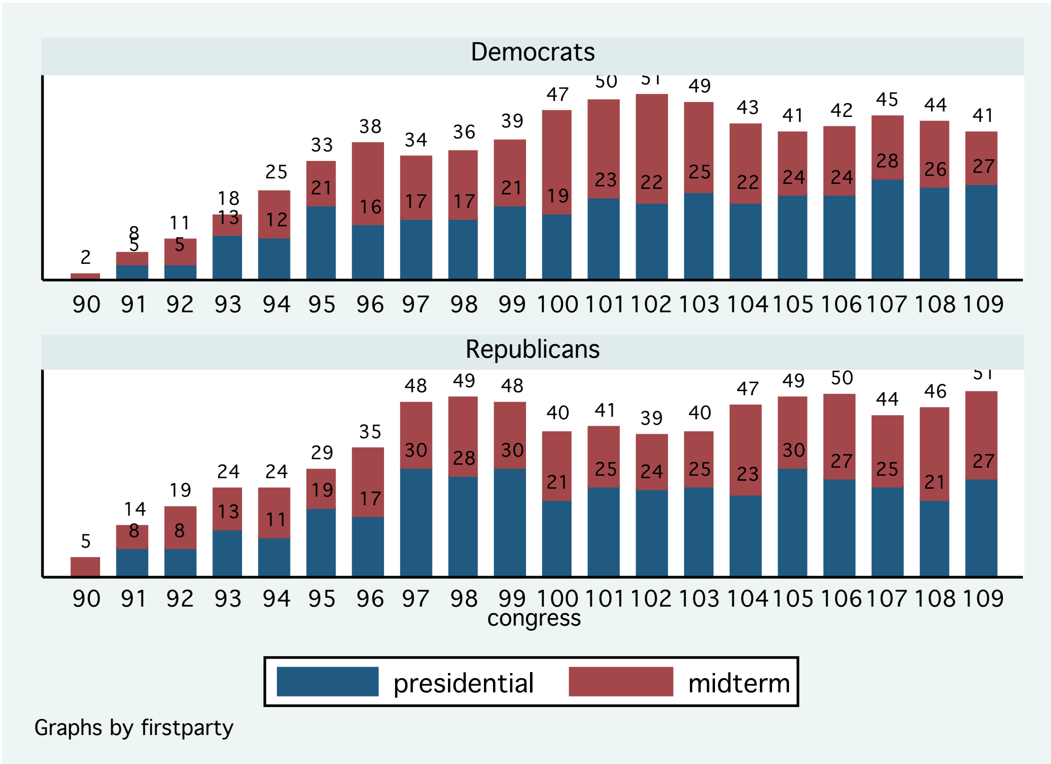
[53]Ted Kennedy and Robert Byrd were elected before 1966 and are still serving in the Senate, there was one Independent senator and four appointed senators who did not face federal elections.

[54]In our robustness checks in the Appendix we employ alternative ideology measures and achieve similar results. For further reading see Poole and Rosenthal (2000).

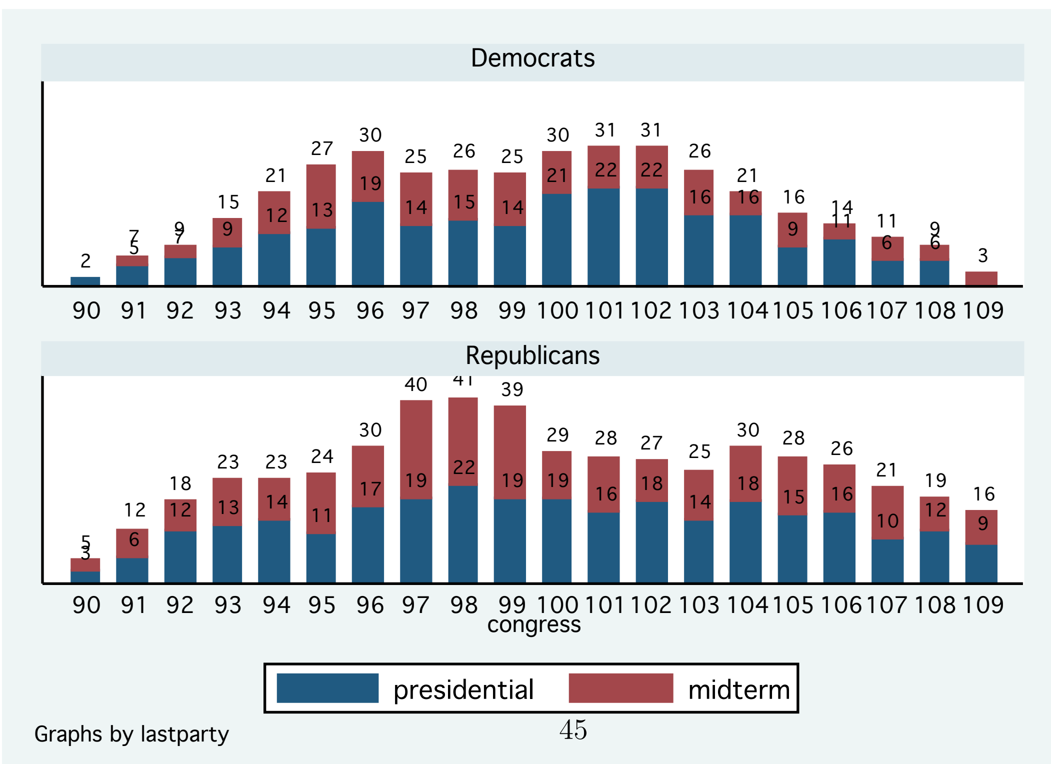
[55]

In Table **??** in the Appendix we employ W-NOMINATE scores, which are a static version of DW-NOMINATE scores, and rule out a potential concern that the persistence of our results is driven by uncommon voting behavior in a particular year.

Figure 5: Senate Composition by Environment



(a) Entry Environment



(b) Exit Environment

first dimension of DW-NOMINATE scores, which captures the ideology of senators in the liberal-conservative (or left-right) space; a higher score is associated with a more conservative voting record. For example, in the 109th congress, the Senate voted on 645 roll calls; the average DW-NOMINATE score for a Democrat was -0.428, and for a Republican, 0.458.

The dynamic weighting of roll calls in the DW-NOMINATE estimation procedure affords the scores cardinality. In other words, while information on scores alone cannot indicate the exact number of roll calls on which one senator voted differently from another, increasing disparity between their DW-NOMINATE scores suggests that the underlying voting records that generated them are increasingly different. In this paper, we focus on differences in electoral outcomes. For our purposes, DW-NOMINATE scores are particularly useful as they are derived from realized voting records of senators and not ideological preferences reported by them or other surveyed groups.[56] Importantly, the use of DW-NOMINATE scores allows us to connect directly electoral environments to the spatial model framework that is central to theories of electoral competition and voting.

We merge the DW-NOMINATE data with the CQ Congressional Collection data. An observation in our dataset consists of time variant and invariant variables. A senator's biographic information as well as her original entry environment and (if applicable) her exit environment, do not vary by congress; however, the measure of a senator's ideology does vary by congress and is a function of her voting behavior. For a given senator, the time variant components are congress number, length in office, upcoming and past electoral environments, and DW-NOMINATE scores; the time invariant variables are party labels, dummies for entry electoral environments, dummies for exit electoral environments (if applicable), characteristics of the entry environment, and starting and ending years (if applicable).

[56]Several alternatives to DW-NOMINATE scores that are occasionally used in empirical research, most notably ADA scores, are produced by partisan lobby groups and rely on voter behavior on a particular set of votes that each group chooses.

The Coattail Effect

Table 3: Coattails Model: Residual from Demographics Regression

Fixed Effects: DW-NOMINATE (first dimension)

Variables	(1) Full Tenure	(2) First Congress	(3) First Term
Trailing average presidential voteshares	20.94***	16.09	20.25***
	(3.699)	(10.22)	(4.625)
Residual from demographics regression	-20.86***	-15.89	-20.06***
	(3.634)	(10.06)	(4.553)
Difference* Proportion age 65 and over	1.615	1.207	0.880
	(3.500)	(10.71)	(4.849)
Difference* Area o State	-7.21e-06***	-7.70e-06	-4.64e-06
	(2.69e-06)	(7.76e-06)	(3.51e-06)
Difference*Log of per-capita income	2.079***	1.575	1.986***
	(0.377)	(1.051)	(0.476)
Democrat	-0.685***	-0.635***	-0.640***
	(0.0119)	(0.0378)	(0.0171)
Proportion of population age 65 and over	16.28***	12.50	15.72***
	(3.337)	(9.146)	(4.141)
Proportion of population black	-0.697	2.123	1.408
	(1.250)	(3.652)	(1.653)
Area of state in square miles	3.29e-06**	3.30e-06	2.75e-06
	(1.59e-06)	(4.59e-06)	(2.08e-06)
Proportion of population in urban areas	4.102***	3.011	3.939***
	(0.714)	(1.959)	(0.887)
Log of per-capita income	0.562***	0.413*	0.527***
	(0.0778)	(0.219)	(0.0993)
Log of total population (interpolated)	0.210***	0.233	0.294***
	(0.0494)	(0.159)	(0.0719)
Proportion of population in farming	-2.061	-1.677	-0.819
	(1.409)	(4.181)	(1.893)
Proportion of population in finance	-69.57***	-50.84	-63.44***
	(12.23)	(33.74)	(15.28)
Proportion of population foreign born	-7.081***	-5.082	-7.181***
	(1.134)	(3.235)	(1.465)
Proportion of population govt. worker	37.68***	29.96	37.42***
	(7.133)	(19.75)	(8.940)
Proportion of population in manufacturing	16.88***	12.91*	15.79***
	(2.556)	(6.962)	(3.152)
Population density per square mile	-0.00177***	-0.00212	-0.00269***
	(0.000626)	(0.00177)	(0.000803)
Constant	-24.18***	-19.64**	-24.70***
	(3.470)	(9.441)	(4.274)
Observations	800	116	348
Number of stateid	49	49	49

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

Table 4: Coattails Model: Difference from Average Presidential Support

Fixed Effects: DW-NOMINATE (first dimension)			
	(1)	(2)	(3)
Variables	Full Tenure	First Congress	First Term
Trailing average presidential voteshares	0.0227	0.155	0.145
	(0.212)	(0.662)	(0.298)
Difference from trailing average	-24.78***	-18.67*	-24.40***
	(3.819)	(10.84)	(4.883)
Difference* Proportion age 65 and over	1.543	1.864	1.557
	(3.469)	(10.54)	(4.749)
Difference* Area o State	-6.13e-06**	-6.97e-06	-3.83e-06
	(2.60e-06)	(7.59e-06)	(3.42e-06)
Difference*Log of per-capita income	2.466***	1.840	2.407***
	(0.393)	(1.118)	(0.504)
Democrat	-0.682***	-0.630***	-0.634***
	(0.0118)	(0.0377)	(0.0170)
Proportion of population age 65 and over	-2.063**	-1.372	-1.791
	(0.938)	(2.727)	(1.229)
Proportion of population black	5.010***	6.392**	6.821***
	(1.007)	(2.817)	(1.269)
Area of state in square miles	3.19e-06**	3.22e-06	2.61e-06
	(1.58e-06)	(4.57e-06)	(2.06e-06)
Proportion of population in urban areas	0.134	-0.0106	0.127
	(0.103)	(0.330)	(0.149)
Log of per-capita income	0.164***	0.117	0.152**
	(0.0498)	(0.150)	(0.0674)
Log of total population (interpolated)	0.330***	0.327*	0.416***
	(0.0537)	(0.167)	(0.0754)
Proportion of population in farming	-6.115***	-4.837	-4.878***
	(1.398)	(4.140)	(1.866)
Proportion of population in finance	-9.277***	-5.168	-6.099*
	(2.580)	(7.802)	(3.516)
Proportion of population foreign born	-1.806***	-1.172	-2.275***
	(0.471)	(1.520)	(0.685)
Proportion of population govt. worker	-1.886	-0.241	-0.672
	(1.154)	(3.456)	(1.557)
Proportion of population in manufacturing	3.979***	3.063	3.381***
	(0.924)	(2.732)	(1.231)
Population density per square mile	0.00145***	0.000395	0.000487
	(0.000271)	(0.000942)	(0.000424)
Constant	-6.802***	-6.520**	-8.182***
	(1.048)	(3.199)	(1.442)
Observations	800	116	348
Number of stateid	49	49	49

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

Proofs

Lemma 1:

($\Rightarrow$) If $Prob\left(M^r < y^i\right) < \frac{1}{2}$, then $Prob\left(M^r > y^i\right) < \frac{1}{2}$. Therefore, $c_i\left(L^r; y^i, \Delta_i^r\right) < \frac{1}{2}$ and $c_i\left(R^r; y^i, \Delta_i^r\right) > \frac{1}{2}$, thus, $c_i\left(L^r; y^i, \Delta_i^r\right) < c_i\left(R^r; y^i, \Delta_i^r\right)$. This implies that conditional on voting $b - c_i\left(v_i^r; y^i, \Delta_i^r\right)$ is maximized at $v_i^r = L^r$.

($\Leftarrow$) If $b - c_i\left(v_i^r; y^i, \Delta_i^r\right)$ is maximized at $v_i^r = L^r$, then $c_i\left(L^r; y^i, \Delta_i^r\right) < c_i\left(R^r; y^i, \Delta_i^r\right)$. Since $c_i\left(L^r; y^i, \Delta_i^r\right) = 1 - c_i\left(R^r; y^i, \Delta_i^r\right)$ and $c_i\left(v_i^r; y^i, \Delta_i^r\right) \in [0,1]$, we know that $c_i\left(L^r; y^i, \Delta_i^r\right) < \frac{1}{2}$ and $c_i\left(R^r; y^i, \Delta_i^r\right) > \frac{1}{2}$, which implies that $Prob\left(M^r < y^i\right) < \frac{1}{2}$.

Q.E.D.

Lemma 2:

($\Rightarrow$)If $Min\left\{Prob\left(M^r < y^i\right), Prob\left(M^r > y^i\right)\right\} < b$, then either $Prob\left(M^r > y^i\right) < b$ (and $Prob\left(M^r < y^i\right) > b$) or $Prob\left(M^r < y^i\right) < b$(and $Prob\left(M^r > y^i\right) > b$). WLOG, assume $Prob\left(M^r < y^i\right) < b$. Therefore, $c_i\left(L^r; y^i, \Delta_i^r\right) < b$ and $b - c_i\left(L^r; y^i, \Delta_i^r\right) > 0$. This implies that $t_i^r\left[b - c_i\left(L^r; y^i, \Delta_i^r\right)\right]$ is maximized at $t_i^r = 1$.

($\Leftarrow$)WLOG, we consider the case $v_i^r = L^r$. If $t_i^r\left[b - c_i\left(L^r; y^i, \Delta_i^r\right)\right]$ is maximized at $t_i^r = 1$, then $b - c_i\left(L^r; y^i, \Delta_i^r\right) > 0$. That implies that $c_i\left(L^r; y^i, \Delta_i^r\right) < b$, which in turn implies that $Prob\left(M^r < y^i\right) < b$. If $Prob\left(M^r < y^i\right) < b$, then clearly $Min\left\{Prob\left(M^r < y^i\right), Prob\left(M^r > y^i\right)\right\} < b$.

Q.E.D.

Proposition 2:

Midterm Elections:

($\Rightarrow$) It follows from Lemma 2 that a PIC, i, turns out if (a) $Prob\left(M^s < y^i\right) < b \iff \Phi\left(\frac{y^i - \mu_s}{\sigma_s}\right) < b \iff y^i < \Phi^{-1}(b)\,\sigma_s + \mu_s$ (since $\Phi()$ is continuous and strictly increasing) or if (b) $Prob\left(M^s > y^i\right) < b \iff 1 - \Phi\left(\frac{y^i - \mu_s}{\sigma_s}\right) < b \iff \Phi\left(\frac{\mu_s - y^i}{\sigma_s}\right) < b$ (since $\Phi(x) = 1 - \Phi(-x)$) $\iff y^i > -\Phi^{-1}(b)\,\sigma_s + \mu_s$.

($\Leftarrow$) Suppose PIC, i, does not turn out and

$$y^i \notin \left[\Phi^{-1}(b)\,\sigma_s + \mu_s, -\Phi^{-1}(b)\,\sigma_s + \mu_s\right].$$

By Lemma 2 it must be that if citizen i does not turn out that $\mathbb{P}\left(M^s < y^i\right) > b$ and $\mathbb{P}\left(M^s > y^i\right) > b$. But then according to the derivation above $y^i > \Phi^{-1}(b)\,\sigma_s + \mu_s$ and $y^i < -\Phi^{-1}(b)\,\sigma_s + \mu_s$. Contradiction. Q.E.D.

Presidential Elections:

($\Rightarrow$) For a given realization $M^p = m^p$, It follows from Lemma 2 that a PIC, i, turns out if (a) $Prob\left(M^s < y^i | M^p = m^p\right) < b \iff \Phi\left(\frac{y^i - \mu_s - \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)}{\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}}\right) < b \iff y^i < \Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)} + \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)$ or if (b) $Prob\left(M^s > y^i | M^p = m^p\right) < b \iff 1 - \Phi\left(\frac{y^i - \mu_s - \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)}{\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}}\right) < b \iff \Phi\left(\frac{\mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right) - y^i}{\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}}\right) < b \iff y^i > -\Phi^{-1}(b)\sqrt{\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}} + \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)$. Q.E.D.

($\Leftarrow$) Suppose PIC i does not turn out and

$$y^i \notin \left[\mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right) \pm \Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}\right]$$

By Lemma 2 it must be that if citizen i does not turn out that $Prob\left(M^s < y^i\right) > b$ and $Prob\left(M^s > y^i\right) > b$. But then according to the derivation above $y^i > \Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)} + \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)$ and $y^i < -\Phi^{-1}(b)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)} + \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)$. Contradiction.

Q.E.D.

Proposition 3:
Midterm Elections:

($\Rightarrow$) It follows from Lemma 1 that a PIC, i, votes for party L if and only if $Prob\left(M^s < y^i\right) < \frac{1}{2} \iff \Phi\left(\frac{y^i - \mu_s}{\sigma_s}\right) < \frac{1}{2} \iff y^i < \Phi^{-1}\left(\frac{1}{2}\right)\sigma_s + \mu_s = \mu_s = E\left[M^s\right]$, since $\Phi^{-1}\left(\frac{1}{2}\right) = 0$.

($\Leftarrow$) Suppose PIC i votes for party R and $y^i < \mu_s$. By Lemma 2 it must be that $Prob\left(M^s > y^i\right) < \frac{1}{2}$. But then by the derivation above $y^i > \mu_s$. Contradiction.

Presidential Elections:

($\Rightarrow$) It follows from Lemma 1 that a PIC, i, votes for party L if and only if $Prob\left(M^s < y^i | M^p = m^p\right) < \frac{1}{2} \iff \Phi\left(\frac{y^i - \mu_s - \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)}{\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}}\right) < \frac{1}{2} \iff y^i < \Phi^{-1}\left(\frac{1}{2}\right)\sqrt{\sigma_s^2 + \sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)} + \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right) = \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right) = E\left[M^s | M^p = m^p\right]$.

($\Leftarrow$) Suppose PIC i votes for party R and $y^i < \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)$. By Lemma 2 it must be that $Prob\left(M^s > y^i\right) < \frac{1}{2}$. But then by the derivation above $y^i > \mu_s + \sigma_\omega \rho_{\omega,p}\left(\frac{m^p - \mu_p}{\sigma_p}\right)$. Contradiction.

Q.E.D.

Proposition 4:

By Proposition 1, all FICs turn out to vote in every senatorial race; thus, changes in turnout are generated by PICs only.

($\Rightarrow$) Suppose that turn out strictly increases in presidential elections but $\rho_{\omega,p}=0$. Let $T_m = \left[\Phi^{-1}(b)\,\sigma_s+\mu_s, -\Phi^{-1}(b)\,\sigma_s+\mu_s\right]$ and $T_p=\left[\mu_s+\sigma_\omega\rho_{\omega,p}\left(\frac{m^p-\mu_p}{\sigma_p}\right)\pm\Phi^{-1}(b)\sqrt{\sigma_s^2+\sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}\right]$ be the zones of abstention in midterm and presidential elections, respectively. Then greater turnout implies that $\delta\int\limits_{I\backslash T_p} f_s(e)\,de > \delta\int\limits_{I\backslash T_m} f_s(e)\,de$. But $T_p=T_m$ since $\rho_{\omega,p}=0$. Contradiction.

($\Leftarrow$) Suppose $\rho_{\omega,p}\neq 0$ and turnout decreases in presidential elections. Since by Proposition 1 all FICs turn out to vote in every senatorial race, it must be that

$$\delta\int\limits_{I\backslash T_p} f_s(e)\,de < \delta\int\limits_{I\backslash T_m} f_s(e)\,de \tag{33}$$

Lemma (*): For some $d>0$, let $A\equiv\{[a,b]\in\mathbb{R}:\ b>a,\ b-a=d\}$ and $x=\left[\mu_s-\frac{d}{2},\mu_s+\frac{d}{2}\right]$. Then given our assumptions about F_s, for any $x'\in A:\ x'\neq x,\quad \int\limits_x f_s(e)\,de>\int\limits_{x'} f_s(e)\,de$.

Given Lemma (*), for a given $\rho_{\omega,p}$ and for any $m^p\neq\mu_p$, $\int\limits_{I\backslash T_p(\mu_p)} f_s(e)\,de<\int\limits_{I\backslash T_p(m^p)} f_s(e)\,de$; thus, it suffices to show that inequality (33) is violated for $T_p=T_p(\mu_p)$. Note that $T_p(\mu_p)=\left[\mu_s+\Phi^{-1}(b)\sqrt{\sigma_s^2+\sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)},\mu_s-\Phi^{-1}(b)\sqrt{\sigma_s^2+\sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}\right]$. Then for any $\rho_{\omega,p}\neq 0$, $\sqrt{\sigma_s^2+\sigma_\omega^2\left(1-\rho_{\omega,p}^2\right)}\leq\sigma_s\Longrightarrow T_p(\mu_p)\subseteq T_m\Longrightarrow\int\limits_{I\backslash T_p(\mu_p)} f_s(e)\,de>\int\limits_{I\backslash T_m} f_s(e)\,de$. Contradiction.

Q.E.D.

Proposition 7:

1. Need to show:$Prob\left(win_L^s|\Pi'_{L,p}\right)>Prob\left(win_L^s|\Pi_{L,p}\right)$

Note that $Prob\left(win_L^s|\Pi'_{L,p}\right)=Prob\left(M^s\in\left[\Lambda\left(\Pi'_{L,p}\right),\infty\right)\right)$. Since Λ is strictly decreasing in $\Pi_{L,p}$, $\left[\Lambda\left(\Pi_{L,p}\right),\infty\right)\subsetneq\left[\Lambda\left(\Pi'_{L,p}\right),\infty\right)$. This implies that $Prob\left(M^s\in\left[\Lambda\left(\Pi'_{L,p}\right),\infty\right)\right)>Prob\left(M^s\in\left[\Lambda\left(\Pi_{L,p}\right),\infty\right)\right)$.

2. Need to show: $Prob\left(lose_L^s|\Pi'_{L,p}\right)<Prob\left(lose_L^s|\Pi_{L,p}\right)$

Note that $Prob\left(lose_L^s|\Pi'_{L,p}\right)=Prob\left(M^s\in\left(-\infty,\Lambda\left(\Pi'_{L,p}\right)\right)\right)$. Since Λ is strictly decreasing in $\Pi_{L,p}$, $\left(-\infty,\Lambda\left(\Pi'_{L,p}\right)\right)\subsetneq\left(-\infty,\Lambda\left(\Pi_{L,p}\right)\right)$. This implies that $Prob\left(M^s\in\left(-\infty,\Lambda\left(\Pi_{L,p}\right)\right)\right)>Prob\left(M^s\in\left(-\infty,\Lambda\left(\Pi'_{L,p}\right)\right)\right)$.

Q.E.D.

Proposition 8:

1. Need to show:$E\left[C_L^s|win_L^s,\Pi'_{L,p}\right]<E\left[C_L^s|win_L^s,\Pi_{L,p}\right)]$

Notice that from equation ((30)) we have that $E\left[C_L^s|win_L^s,\Pi_{L,p}\right)]=E\left[C_L^s|M^s\in\left(\Lambda\left(\Pi_{L,p}\right),\infty\right)\right]$. We can rewrite the interval of winnable midpoints in terms of the function Λ and consider an arbitrary draw of $C_R^s=c$. Then we have that $E\left[C_L^s|M^s\in\left[\Lambda\left(\Pi_{L,p}\right),\infty\right),C_R^s=c\right]=E\left[C_L^s|C_L^s\in\left[2\Lambda\left(\Pi_{L,p}\right)-c,c\right]\right]$. Note that since Λ is strictly decreasing in $\Pi_{L,p}$, $\Lambda\left(\Pi_{L,p}\right)>\Lambda\left(\Pi'_{L,p}\right)$. We need to show $E\left[C_L^s|C_L^s\in\left[2\Lambda\left(\Pi'_{L,p}\right)-c,c\right]\right]<E\left[C_L^s|C_L^s\in\left[2\Lambda\left(\Pi_{L,p}\right)-c,c\right]\right]$. By law of iterated expectations, we have

$$E\left[C_L^s|C_L^s\in\left[2\Lambda\left(\Pi'_{L,p}\right)-c,c\right]\right]=$$

$$E\left[C_L^s|C_L^s \in \left[2\Lambda\left(\Pi'_{L,p}\right)-c, 2\Lambda\left(\Pi_{L,p}\right)-c\right]\right] \cdot$$
$$P\left[C_L^s \in \left[2\Lambda\left(\Pi'_{L,p}\right)-c, 2\Lambda\left(\Pi_{L,p}\right)-c\right]|C_L^s \in \left[2\Lambda\left(\Pi'_{L,p}\right)-c, c\right]\right] +$$

$$E\left[C_L^s|C_L^s \in \left(2\Lambda\left(\Pi_{L,p}\right)-c, c\right]\right] \cdot$$
$$P\left[C_L^s \in \left(2\Lambda\left(\Pi_{L,p}\right)-c, c\right]|C_L^s \in \left[2\Lambda\left(\Pi'_{L,p}\right)-c, c\right]\right]$$
$$< E\left[C_L^s|C_L^s \in \left[2\Lambda\left(\Pi_{L,p}\right)-c, c\right)\right]$$

Since $E\left[C_L^s|C_L^s \in \left(2\Lambda\left(\Pi_{L,p}\right)-c, c\right]\right] > E\left[C_L^s|C_L^s \in \left[2\Lambda\left(\Pi'_{L,p}\right)-c, 2\Lambda\left(\Pi_{L,p}\right)-c\right]\right]$.

Q.E.D.

2. Need to show: $E\left[C_L^s|lose_L^s, \Pi_{L,p}\right] > E\left[C_L^s|lose_L^s, \Pi'_{L,p}\right]$

Notice that from equation ((30)) we have that $E\left[C_L^s|lose_L^s, \Pi_{L,p}\right] = E\left[C_L^s|M^s \in \left(-\infty, \Lambda\left(\Pi_{L,p}\right)\right)\right]$. We can rewrite the interval of winnable midpoints in terms of the function Λ and consider an arbitrary draw of $C_R^s = c$. Then we have that $E\left[C_L^s|M^s \in \left(-\infty, \Lambda\left(\Pi_{L,p}\right)\right), C_R^s = c\right] = E\left[C_L^s|C_L^s \in \left(-\infty, 2\Lambda\left(\Pi_{L,p}\right)-c\right)\right]$. We need to show $E\left[C_L^s|C_L^s \in \left(-\infty, 2\Lambda\left(\Pi'_{L,p}\right)-c\right)\right] < E\left[C_L^s|C_L^s \in \left(-\infty, 2\Lambda\left(\Pi_{L,p}\right)-c\right)\right]$. By law of iterated expectations, we have

$$E\left[C_L^s|C_L^s \in \left(-\infty, 2\Lambda\left(\Pi_{L,p}\right)-c\right)\right] =$$

$$E\left[C_L^s|C_L^s \in \left(2\Lambda\left(\Pi'_{L,p}\right)-c, 2\Lambda\left(\Pi_{L,p}\right)-c\right)\right] \cdot$$
$$P\left[C_L^s \in \left(2\Lambda\left(\Pi'_{L,p}\right)-c, 2\Lambda\left(\Pi_{L,p}\right)-c\right)|C_L^s \in \left(-\infty, 2\Lambda\left(\Pi_{L,p}\right)-c\right)\right] +$$

$$E\left[C_L^s|C_L^s \in \left(-\infty, 2\Lambda\left(\Pi'_{L,p}\right)-c\right]\right] \cdot$$
$$P\left[C_L^s \in \left(-\infty, 2\Lambda\left(\Pi'_{L,p}\right)-c\right]|C_L^s \in \left(-\infty, 2\Lambda\left(\Pi_{L,p}\right)-c\right)\right]$$
$$\geq E\left[C_L^s|C_L^s \in \left(-\infty, 2\Lambda\left(\Pi'_{L,p}\right)-c\right)\right] \quad .$$

Q.E.D.