Transactions and Settlement in a Model of Unsecured Credit [Job Market Paper]

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Abstract

I study the terms of credit in a decentralized market where sellers are willing to repeatedly finance the purchases of buyers by extending direct credit. An important aspect of this credit arrangement is the rule for the periodic settlement of debt within the enduring relationship. Under full commitment and complete information, the equilibrium credit contract that a lender (seller) offers to a borrower (buyer) is such that the terms of credit for the current transaction do not affect those for future transactions. This does not seem to be consistent with observed behavior in credit markets. If a lender is asymmetrically informed with respect to a borrower's ability to settle his debt and both parties cannot fully commit to their promises, the optimal provision of incentives by a lender creates a critical interplay between the terms of credit for the current transaction and those for future transactions. The equilibrium credit contract specifies delayed settlement by the borrower in some circumstances, which is a mechanism through which the lender obtains more favorable terms of credit for future transactions. This property of the optimal contract reproduces observed characteristics of credit markets such as revolving credit.

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1 Introduction

Many relationships in the credit market consist of long-term bilateral arrangements. For instance, in many circumstances we observe a retailer who is willing to repeatedly finance the purchases of his customers by extending direct credit. Within these relationships, there is a role for credit in current and future transactions, which generates an agreement between the two parties with respect to the periodic settlement of debt. One remarkable characteristic of the market for unsecured loans is the practice of revolving credit. According to the Federal Reserve Board's Survey of Consumer Finances, the proportion of families carrying a balance on their credit cards was 41.6% in 2007. The median balance was $7,300. Another observed characteristic is that the terms of credit for a consumer who usually pays off a small fraction of her balance with a lender are less favorable than those for a consumer who carries a balance but at least sometimes pays it off in full. The interest rate charged on the former was on average 1.0% higher than that charged on the latter in 1995, 1.2% in 1998, and 3.0% in 2001. This suggests that the terms of credit are variable and depend on a consumer's history of repayments to a lender.

From a borrower's point of view, the practice of revolving credit seems to be consistent with consumption smoothing. From a lender's point of view, it is not clear that this practice can be part of an optimal contractual arrangement, especially because in this market lenders usually post the terms of the contract. This raises the following question. What are the properties of a lender's optimal contract in the market for unsecured loans? I show that the practice of revolving credit arises as part of an optimal contractual arrangement between a lender and a borrower. This property of a lender's contract is a mechanism through which a lender obtains more favorable terms of credit within the relationship. The repayment schedule for a borrower becomes state-contingent as the result of optimal intertemporal allocation of resources by a risk-neutral lender. Finally, I show that asymmetric information with respect to a borrower's ability to repay his loan is a critical friction for reproducing some observed characteristics of the market for unsecured loans.

I construct a model in which exchange is bilateral and credit arrangements facilitate transactions. I characterize an equilibrium in which a unique, market-determined credit contract is offered to each borrower at the first date with the property that a lender and a borrower engage in a long-term relationship, in spite of the limited ability of both parties to fully commit to the contract. An important feature of the model is that the settlement process within the relationship involves a friction, which takes the form of an unobservable preference shock that affects a borrower's ability to repay his loan. The market contract specifies the loan amounts to which a borrower is entitled in each transaction round and his repayment schedule in each settlement round. The ensuing arrangement resembles a retail credit market in which lenders post competitive contracts to attract borrowers.

The analysis in this paper relies on dynamic contracting to explain observed behavior in the market for unsecured loans. One of the novelties of the analysis is that it characterizes the competitive, bilateral credit contract under one-sided asymmetric information and two-sided limited commitment. Specifically, a borrower can defect from the contract at any moment, and a lender cannot commit to deliver a contract that results in a payoff for her that is lower than that associated with autarky. This is quite different from the analyses in Phelan (1995) and Krueger and Uhlig (2006), which essentially assume some degree of commitment by a lender.[1] Another important aspect of the model is that it explicitly accounts for the flow of payments associated with credit contracts, which makes it suitable to the study of repayment behavior in the market for unsecured loans.[2]

This paper emphasizes the role of settlement in long-term credit arrangements as a mechanism that critically affects the terms of credit for future transactions. As a useful benchmark, I characterize the equilibrium allocation under full commitment and complete information. In this case, the equilibrium credit contract that a lender offers to a borrower is such that the terms of credit for the current transaction do not affect those for future transactions. This does not seem to be consistent with observed behavior in credit markets. If a lender is asymmetrically informed with respect to a borrower's ability to settle his debt and both parties cannot fully commit to their promises, the optimal provision of incentives by a lender creates a critical interplay between the terms of credit for the current transaction and those for future transactions. The equilibrium credit contract specifies delayed settlement by the borrower in some circumstances, which in turn results in more favorable terms of credit for the lender in future transactions. This property of the optimal contract reproduces observed characteristics of the market for unsecured loans such as the practice of revolving credit.

The model in this paper relates to decentralized models of credit, such as

[1] The analysis in Krueger and Uhlig (2006) assumes that the consumer's income is publicly observable.

[2] Recent papers that emphasize the role of credit in decentralized exchange and analyze the properties of the ensuing flow of payments include Nosal and Rocheteau (2006) and Koeppl, Monnet, and Temzelides (2008).

Diamond (1990), Temzelides and Williamson (2001), Nosal and Rocheteau (2006), and Koeppl, Monnet, and Temzelides (2008), as opposed to centralized models of credit, such as Kehoe and Levine (1993) and Alvarez and Jermann (2000). The model also builds on search-theoretic models of money, such as Shi (1997) and Lagos and Wright (2005). However, I depart from these models by weakening the assumption that agents cannot engage in enduring relationships. Finally, the analysis in this paper is consistent with the so-called endogenously incomplete markets approach – see Sleet (2008) – where trading arrangements are derived from primitive frictions instead of assumed. Important papers in this literature include Green (1987), Thomas and Worrall (1990), Atkeson and Lucas (1992, 1995), Kocherlakota (1996), and Aiyagari and Williamson (1999).

2 The Model

Time is discrete and continues forever, and each period has two subperiods. There are two types of agents, referred to as borrowers and lenders. In the first subperiod, a lender is able to produce the unique perishable consumption good but does not want to consume it, and a borrower wants to consume it but cannot produce it. In the second subperiod, both agents are able to produce and want to consume the settlement good, which is also perishable and cannot be stored. This generates a double coincidence of wants and, for this reason, I refer to the first subperiod as the transaction stage and to the second subperiod as the settlement stage. The types (borrower and lender) refer to the agent's role in the transaction stage. The production technology allows each agent to produce one unit of either good with one unit of labor. Each agent receives an endowment of $h > 0$ units of time in each subperiod.

A lender's utility in period t is given by $-q_t^l + x_t^l - y_t^l$, where q_t^l is production of the consumption good in the transaction stage and x_t^l and y_t^l are consumption and production of the settlement good in the settlement stage, respectively. A borrower's momentary utility from consuming q_t^b units of the consumption good in the transaction stage is given by $u\left(q_t^b\right)$. Assume that $u : \mathbb{R}_+ \to D \subset \mathbb{R}$ is increasing, strictly concave, and continuously differentiable. Let H denote the inverse of u, and let $w^a \equiv u(0)$ denote the value associated with autarky. In the settlement stage, the marginal utility of leisure γ_t for a borrower is an idiosyncratic shock, with $\gamma_t \in \Gamma = \{1, \bar{\gamma}\}$ and $1 < \bar{\gamma}$. Assume that γ_t is independently and identically distributed over time, and the associated distribution function assigns probability $\pi \in (0, 1)$ to the higher realization $\bar{\gamma}$. Each borrower learns his preference shock at the beginning of the settlement stage, which is privately observed, and his preferences are given by $x_t^b - \gamma_t y_t^b$, where x_t^b and y_t^b are consumption and production of the settlement good, respectively. Finally, let $\beta \in (0, 1)$ be the common discount factor over periods.

Suppose that there is a large number of borrowers and lenders, with the set of lenders sufficiently large. Another feature of the model is that there is a cost $k > 0$ in terms of the consumption good for a lender to post a credit contract, which specifies consumption and production by each party as a function of the available information. All contracts are publicly observable once they are posted, and each lender can have at most one borrower. Once a borrower and a lender meet, either one of them can walk away from the meeting at any moment. Finally, notice that there are gains from trade since the lender can produce the consumption good for the borrower in the first subperiod (transaction stage) and the borrower can produce the settlement good for the lender in the second subperiod (settlement stage). An important feature of the model is that, with probability π, the borrower will have high marginal utility of leisure in the second subperiod, in which case it will be costly for him to produce the settlement good and discharge his debt. This is equivalent to assuming that the settlement process involves a friction.

3 Equilibrium under Complete Information and Full Commitment

As a useful benchmark, let us characterize an equilibrium allocation when both lenders and borrowers can fully commit to a credit contract and a borrower's preference shock in the settlement stage is publicly observable. In this section, I show that the properties of a lender's optimal contract in this case do not reproduce some observed characteristics of the market for unsecured loans. In particular, I show that in any equilibrium allocation the terms of credit for the current transaction do not affect those for future transactions, which is quite different from what is observed in reality. As we will see, a lender's optimal contract involves a state-contingent rule for the periodic settlement of debt within a credit relationship that requires a repayment from a borrower contingent on the realization of the low marginal utility of leisure and no repayment contingent on the realization of the high marginal utility of leisure. This property derives from optimal intertemporal allocation of resources by a risk-neutral lender. However, a lender's optimal contract does not result in a credit transaction that has the same properties as those observed in the market for unsecured loans. As we have seen, an important characteristic of this market is that the dynamics of credit arrangements is such that the practice of revolving credit results in terms of credit for future transactions that are less favorable for a borrower.

Now, I carefully describe an equilibrium allocation. Notice that each lender is a retailer who produces the consumption good but does not want to consume it in the transaction stage. She seeks a borrower, who wants to consume the consumption good in the first subperiod, to trade intertemporally given that such a borrower is able to repay the loan in the second subperiod. A lender may find it optimal to continue to trade with the same borrower for many periods, developing a long-term credit relationship. Recall that a lender needs to post a contract to enter the credit market - which involves a one-shot cost for her in terms of the consumption good - and can have at most one borrower. All contracts are publicly observable, and so are the transactions in each credit relationship.

Let $\pi_t\left(\gamma^{t-1}\right)$ denote the probability of observing a particular history of preference shocks $\gamma^{t-1}=\left(\gamma_0,\gamma_1,...,\gamma_{t-1}\right)\in\Gamma^t$ for a borrower. In a symmetric equilibrium, each borrower receives a market-determined credit contract offered by a lender that gives him expected discounted utility $w_F^*\in D$, from the perspective of the contracting date. In this equilibrium, some lenders remain inactive while others post a contract and match with a borrower. When offering her own contract, each lender takes as given the contracts offered by the other lenders. The only relevant characteristic about these contracts is the expected discounted utility w_F^* that each borrower associates with them. The equilibrium is symmetric because every active lender offers the same credit contract.

Taking as given a borrower's expected discounted utility $w\in D$, a lender who decides to enter the credit market must choose a credit contract $\left\{q_t\left(\gamma^{t-1},w\right),y_t\left(\gamma^{t-1},\gamma_t,w\right)\right\}_{t=0}^{\infty}$ to offer to a borrower, where $q_t:\Gamma^t\times D\rightarrow\mathbb{R}_+$ gives the loan amount to which a borrower is entitled in the transaction stage of period t as a function of the available history of shocks and $y_t:\Gamma^{t+1}\times D\rightarrow[0,h]$ gives the repayment amount that is required in the settlement stage of period t as a function of the available history of shocks. Such a contract is chosen to minimize a lender's expected discounted cost,

$$\left(1-\beta\right)\sum_{t=0}^{\infty}\beta^t\pi_t\left(\gamma^{t-1}\right)\left[q_t\left(\gamma^{t-1},w\right)-\pi y_t\left(\gamma^{t-1},\bar{\gamma},w\right)-\left(1-\pi\right)y_t\left(\gamma^{t-1},1,w\right)\right],$$

subject to a borrower's individual rationality constraint:

$$(1-\beta)\sum_{t=0}^{\infty}\beta^{t}\pi_{t}\left(\gamma^{t-1}\right)\left\{u\left[q_{t}\left(\gamma^{t-1},w\right)\right]-\pi\bar{\gamma}y_{t}\left(\gamma^{t-1},\bar{\gamma},w\right)-(1-\pi)\,y_{t}\left(\gamma^{t-1},1,w\right)\right\}\geq w.$$

Let $\mu \geq 0$ denote the Lagrange multiplier on a borrower's individual rationality constraint. The first-order necessary and sufficient conditions for a lender's optimization problem are

$$1-\mu u'\left[q_{t}\left(\gamma^{t-1},w\right)\right]=0,$$

$$-1+\mu\bar{\gamma}\geq 0,\text{ with equality if } y_{t}\left(\gamma^{t-1},\bar{\gamma},w\right)>0,$$

$$-1+\mu\geq 0,\text{ with equality if } y_{t}\left(\gamma^{t-1},1,w\right)>0,$$

for all $t \geq 0$ and $\gamma^{t-1} \in \Gamma^{t}$. Notice that these conditions imply that $\mu = 1$,

$$q_{t}\left(\gamma^{t-1},w\right)=q^{*}, \tag{1}$$

and

$$y_{t}\left(\gamma^{t-1},\bar{\gamma},w\right)=0, \tag{2}$$

for all $t \geq 0$ and $\gamma^{t-1} \in \Gamma^{t}$, where q^{*} is the quantity satisfying $u'(q^{*}) = 1$. As a result, any sequence $\left\{q_{t}\left(\gamma^{t-1},w\right),y_{t}\left(\gamma^{t-1},\gamma_{t},w\right)\right\}_{t=0}^{\infty}$ satisfying (1), (2), and

$$(1-\beta)\sum_{t=0}^{\infty}\beta^{t}\pi_{t}\left(\gamma^{t-1}\right)y_{t}\left(\gamma^{t-1},1,w\right)=\frac{u\left(q^{*}\right)-w}{1-\pi} \tag{3}$$

is a solution to a lender's optimization problem for any given $w \in D$.

Let $C^{F}(w)$ denote the cost function associated with a lender's optimization problem. Then, we have that

$$C^{F}(w)=-\left[u\left(q^{*}\right)-q^{*}\right]+w.$$

A symmetric equilibrium is a credit contract $\left\{q_{t}\left(\gamma^{t-1},w\right),y_{t}\left(\gamma^{t-1},\gamma_{t},w\right)\right\}_{t=0}^{\infty}$ and a market utility $w_{F}^{*} \in D$ for a borrower such that *(i)* the credit contract satisfies (1)-(3) and *(ii)* w_{F}^{*} satisfies:

$$C^{F}\left(w_{F}^{*}\right)+(1-\beta)\,k=0. \tag{4}$$

Notice that w_{F}^{*} gives a borrower's expected discounted utility, from the perspective of period $t = 0$, associated with the credit contract.

Given that there is free entry of lenders, the expected discounted utility associated with the market contract needs to be such that each lender is indifferent between entering the credit market by posting a contract and remaining inactive. Condition (4) guarantees that each lender is indifferent from the perspective of period $t = 0$. Once a lender and a borrower meet, both parties can fully commit to the credit contract proposed in period $t = 0$, where w_F^* is the expected discounted utility for a borrower associated with the market contract. Notice that an equilibrium exists provided that $k > 0$ is sufficiently small. In fact, there are many symmetric equilibria given that any sequence $\left\{y_t\left(\gamma^{t-1}, \gamma_t, w\right)\right\}_{t=0}^{\infty}$ satisfying (2) and (3) is part of a lender's optimal contract.

All of these equilibria have the property that the loan amount that a borrower receives in each transaction stage is always q^*, regardless of his history of payments to a lender. Another property is that the repayment schedule in the settlement stage is such that a lender does not require a repayment from a borrower contingent on the realization of the high marginal utility of leisure – the costly state of nature for a borrower to make a payment to a lender. A third property of any equilibrium allocation is that the terms of credit for the current transaction do not affect those for future transactions. In any equilibrium, the sequence $\left\{y_t\left(\gamma^{t-1}, \gamma_t, w\right)\right\}_{t=0}^{\infty}$ satisfying (2) and (3) is defined in period $t = 0$. The terms of credit for the current transaction are given by $\left[q_t\left(\gamma^{t-1}, w\right), y_t\left(\gamma^{t-1}, \bar{\gamma}, w\right), y_t\left(\gamma^{t-1}, 1, w\right)\right]$, and we must have $q_t\left(\gamma^{t-1}, w\right) = q^*$ and $y_t\left(\gamma^{t-1}, \bar{\gamma}, w\right) = 0$ for all $t \geq 0$ and $\gamma^{t-1} \in \Gamma^t$. As a result, a borrower's payment to a lender does not affect the terms of the contract for future transactions within the credit relationship.

To clearly illustrate these properties, consider an equilibrium allocation in which each lender requires a fixed repayment in the settlement stage contingent on the realization of the high marginal utility of leisure for a borrower. Notice that $y_t\left(\gamma^{t-1}, 1, w\right) = (1-\pi)^{-1}\left[u\left(q^*\right) - w\right]$ for all $t \geq 0$ and $\gamma^{t-1} \in \Gamma^t$ is part of a solution to a lender's optimization problem. In this equilibrium, the expected return to a lender on the current transaction is given by

$$R^* \equiv \frac{q^* + (1-\beta)\, k}{q^*}.$$

This is exactly the expected return to a lender on her current loan of q^* in the transaction stage given that, in the settlement stage, she receives a repayment of $(1-\pi)^{-1}\left[q^* + (1-\beta)\, k\right]$ with probability $1-\pi$ and a repayment of zero with probability π. As mentioned before, a remarkable property of this credit contract is that the optimal rule for settlement does not affect the terms of credit for future transactions. Regardless of a borrower's history of payments to a lender, the terms of credit for the subsequent transaction are exactly the same, which means that a borrower's expected discounted utility at the beginning of each period is always w_F^*. Neither this nor any other equilibrium reproduces the observed characteristics of real-world credit markets.

4 Equilibrium under Asymmetric Information and Limited Commitment

In this section, I study the terms of credit under limited commitment and asymmetric information. As in the previous analysis, I restrict attention to a symmetric, stationary equilibrium in which each borrower receives a market-determined credit contract offered by a lender that gives him expected discounted utility $w^* \in D$, from the perspective of the contracting date. Now, each lender needs to provide incentives to induce the desired behavior by a borrower. A lender's optimal contract results in a long-term relationship from which neither party wants to deviate. As before, the expected discounted utility w^* associated with the market contract must be such that it makes each lender indifferent between entering the credit market by posting a contract and remaining inactive, from the perspective of the contracting date. As a result, some lenders post a contract and successfully match with a borrower while others do not post a contract and remain inactive.

The market contract must *always* result in an expected discounted utility for a borrower that is greater than or equal to w^*. If the market contract promises, in a given period, an expected discounted utility w' for a borrower which is less than w^*, then an inactive lender can do better by posting a new contract promising such a borrower an expected discounted utility slightly higher than w' but still lower than w^*, from the perspective of the current period. Although the currently inactive lender has to pay a cost to post the new contract - so that the borrower can actually observe it - he is better off by proceeding in this way, and so is the borrower. Given that there is free entry of lenders and limited commitment, we can have an equilibrium only if the lowest promised expected discounted utility at any moment is exactly w^*.

The equilibrium that I have just described is similar to that studied by Phelan (1995). In his model, there is no cost for a lender to post a contract but she has to pay a cost if she wants to opt out of it. In equilibrium, each lender is indifferent *ex ante* between posting a contract and remaining inactive. *Ex post* a lender who posts a contract gets an expected discounted utility that is lower than that associated with autarky. The lender does not quit because there is a sufficiently large cost that she must pay in order to effectively quit, which is equivalent to assuming that there is some degree of commitment by the lender. In this paper, each lender is also indifferent between posting a contract and remaining inactive. However, after matching with a borrower, a lender gets an expected discounted utility that is *always* greater than or equal to that associated with autarky, given that a lender cannot fully commit to any contract that gives her a continuation payoff that is lower than that associated with autarky.

4.1 Recursive Formulation of the Contracting Problem

A contract specifies in every period a transfer of the consumption good from the lender to the borrower in the transaction stage and a payment – a transfer of the settlement good from the borrower to the lender – in the settlement stage as a function of the available history of reports by the borrower. The optimal contracting problem has a recursive formulation in which we can use the borrower's expected discounted utility $w \in D$ as the state variable. The optimal contract minimizes the expected discounted cost for the lender of providing expected discounted utility w to the borrower subject to incentive compatibility. Let $C_{(w^*,\bar{w})} : [w^*, \bar{w}] \to \mathbb{R}$ denote the expected discounted cost for the lender that satisfies the following functional equation:

$$C_{(w^*,\bar{w})}(w) = \min_{\boldsymbol{\varphi} \in \Upsilon_{(w^*,\bar{w})}(w)} \left\{ \begin{array}{c} (1-\beta)\left[H(u) - \pi y_{\bar{\gamma}} - (1-\pi) y_1\right] + \\ \beta\left[\pi C_{(w^*,\bar{w})}(w_{\bar{\gamma}}) + (1-\pi) C_{(w^*,\bar{w})}(w_1)\right] \end{array} \right\}. \tag{5}$$

Here, the choices are given by $\boldsymbol{\varphi} = (u, y_{\bar{\gamma}}, y_1, w_{\bar{\gamma}}, w_1)$, where u denotes the borrower's momentary utility of consumption in the transaction stage, y_γ denotes his production in the settlement stage given that his current report is $\gamma \in \Gamma$, and w_γ denotes his promised expected discounted utility at the beginning of the following period given that his report in the current period is $\gamma \in \Gamma$. The constraint set $\Upsilon_{(w^*,\bar{w})}(w)$ consists of all $\boldsymbol{\varphi}$ in $D \times [0, h]^2 \times [w^*, \bar{w}]^2$ satisfying the borrower's individual rationality constraints,

$$-(1-\beta)\gamma y_\gamma + \beta w_\gamma \geq \beta w^*, \text{ for each } \gamma \in \{1, \bar{\gamma}\}, \tag{6}$$

the borrower's truth-telling constraints,

$$-(1-\beta)\gamma y_\gamma + \beta w_\gamma \geq -(1-\beta)\gamma y_{\gamma'} + \beta w_{\gamma'}, \text{ for } \gamma, \gamma' \in \{1, \bar{\gamma}\}, \tag{7}$$

and the promise-keeping constraint,

$$(1-\beta)\left[u - \pi\bar{\gamma} y_{\bar{\gamma}} - (1-\pi) y_1\right] + \beta\left[\pi w_{\bar{\gamma}} + (1-\pi) w_1\right] = w. \tag{8}$$

It can be shown that, for any fixed lower bound w^* and upper bound $\bar{w}$, there exists a unique continuously differentiable, strictly increasing, and strictly convex function $C_{(w^*,\bar{w})} : [w^*, \bar{w}] \to \mathbb{R}$ satisfying the functional equation (5). Let $\hat{u} : [w^*, \bar{w}] \to D$, $y : [w^*, \bar{w}] \times \Gamma \to [0, h]$, and $g : [w^*, \bar{w}] \times \Gamma \to [w^*, \bar{w}]$ denote the associated policy functions, which can be shown to be continuous and bounded.

Notice that a lender cannot commit to any contract that gives her at any moment an expected discounted utility that is lower than that associated with autarky. As a result, individual rationality for a lender requires that $C_{(w^*,\bar{w})}(w) \leq 0$ holds for all $w \in [w^*, \bar{w}]$. I show next that, for any given w^*, there exists an upper bound $\bar{w} = \bar{w}(w^*)$ on the set of expected discounted utilities that gives the highest promised expected utility to which a lender can commit to deliver given that the lowest expected utility that can be promised is w^*. As we will see later, the market utility w^* is determined endogenously and is such that it makes each lender indifferent between entering the credit market by posting a contract and remaining inactive.

Lemma 1 *For any $w^* \geq w^a$ such that $C_{(w^*,w^*)}(w^*) \leq 0$, there exists an upper bound $\bar{w}(w^*)$ such that $C_{(w^*,\bar{w}(w^*))}[\bar{w}(w^*)] = 0$.*

Proof. Let $\bar{w}_F$ denote the expected discounted utility such that the expected discounted cost of providing $\bar{w}_F$ given full information equals zero. Define the function $\tau : [w^*, \bar{w}_F] \to [w^*, \bar{w}_F]$ as follows. For any given $w \in [w^*, \bar{w}_F]$, if there is no $w' \in [w^*, w]$ such that $C_{(w^*,w)}(w') = 0$, then $\tau(w) = w^*$. Otherwise, $\tau(w)$ equals the highest point w' in $[w^*, w]$ for which $C_{(w^*,w)}(w') = 0$. Notice that $C_{(w^*,w^*)}(w^*) \leq 0$ by assumption, which implies that $\tau(w^*) = w^*$. For any other $\bar{w}$ such that $\tau(\bar{w}) = \bar{w}$, it must be that $C_{(w^*,\bar{w})}(\bar{w}) = 0$.

Now, construct a sequence $\{w_t\}_{t=0}^{\infty}$ of candidates for the upper bound $\bar{w}$ in the following way. Let $w_0 = \bar{w}_F$. We have that $C_{(w^*,w_0)}(w_0) \geq 0$, with strict inequality if either truth-telling constraint (7) binds. Also, notice that $\Upsilon_{(w^*,w^*)}(w^*) \subseteq \Upsilon_{(w^*,w_0)}(w^*)$, which implies that $C_{(w^*,w_0)}(w^*) \leq C_{(w^*,w^*)}(w^*) \leq 0$. The first inequality is strict if either truth-telling constraint binds. Continuity implies that there exists $w_1 \in [w^*, w_0]$ such that $C_{(w^*,w_0)}(w_1) = 0$. This means that $w_1 = \tau(w_0) \leq w_0$. We proceed in the same fashion to define w_2. From the fact that $C_{(w^*,w_0)} \leq C_{(w^*,w_1)}$, it follows that $C_{(w^*,w_1)}(w_1) \geq C_{(w^*,w_0)}(w_1) = 0$. Given that $\Upsilon_{(w^*,w^*)}(w^*) \subseteq \Upsilon_{(w^*,w_1)}(w^*)$, we have that $C_{(w^*,w_1)}(w^*) \leq C_{(w^*,w^*)}(w^*) \leq 0$. Again, continuity implies that there exists $w_2 \in [w^*, w_1]$ such that $C_{(w^*,w_1)}(w_2) = 0$. This means that $w_2 = \tau(w_1) \leq w_1$. Notice then that $\{w_t\}_{t=0}^{\infty}$ is a nonincreasing sequence on a closed interval. As a result, it converges to a point

w_∞ in the interval $[w^*, \bar{w}_F]$. The Theorem of the Maximum guarantees that $\phi(w) \equiv C_{(w^*,w)}(w)$ moves continuously, which implies that w_∞ is the highest fixed point of τ. **Q.E.D.** ■

To ease notation, define $C_{w^*}(w) \equiv C_{(w^*,\bar{w}(w^*))}(w)$ and $D_{w^*} \equiv [w^*, \bar{w}(w^*)]$. Given that C_{w^*} is strictly increasing, it follows that $C_{w^*}(w) \leq 0$ for all w in the set D_{w^*}. This means that, for any given lower bound w^*, D_{w^*} gives the set of promised expected discounted utilities that are actually incentive-feasible. If at least one truth-telling constraint binds, then it follows that $\bar{w}(w^*) > w^*$ for any lower bound w^* satisfying $C_{(w^*,w^*)}(w^*) \leq 0$. I show next that the truth-telling constraint for a borrower with the low marginal utility of leisure always binds. But first notice that the truth-telling constraints (7) imply that $y(w,1) \geq y(w,\bar{\gamma})$ and $g(w,1) \geq g(w,\bar{\gamma})$ for all $w \in D_{w^*}$, which means that the optimal contract needs to assign a bigger repayment in the settlement stage and at the same time to promise a higher expected discounted utility to a borrower contingent on the realization of the low marginal utility of leisure to induce truthful reporting.

Lemma 2 *The truth-telling constraint (7) for* $(\gamma, \gamma') = (1, \bar{\gamma})$ *binds for any* $w \in D_{w^*}$.

Proof. Suppose that

$$-(1-\beta) y_1 + \beta w_1 > -(1-\beta) y_{\bar{\gamma}} + \beta w_{\bar{\gamma}} \tag{9}$$

holds at the optimum. First, notice that

$$-(1-\beta) y_{\bar{\gamma}} + \beta w_{\bar{\gamma}} \geq -(1-\beta) \bar{\gamma} y_{\bar{\gamma}} + \beta w_{\bar{\gamma}} \geq \beta w^*,$$

so that

$$-(1-\beta) y_1 + \beta w_1 > \beta w^* \tag{10}$$

must hold at the optimum. Now, reduce the left-hand side of (9) and (10) by a small amount $\Delta > 0$ so that both inequalities continue to hold. Define $w_1' = w_1 - \pi\Delta$ and $w_{\bar{\gamma}}' = w_{\bar{\gamma}} + (1-\pi)\Delta$. Notice that $\pi w_{\bar{\gamma}}' + (1-\pi) w_1' = \pi w_{\bar{\gamma}} + (1-\pi) w_1$ and $w_1' - w_{\bar{\gamma}}' < w_1 - w_{\bar{\gamma}}$. The strict convexity of C_{w^*} implies that

$$\pi C_{w^*}\left(w_{\bar{\gamma}}'\right) + (1-\pi) C_{w^*}\left(w_1'\right) < \pi C_{w^*}\left(w_{\bar{\gamma}}\right) + (1-\pi) C_{w^*}\left(w_1\right),$$

so that the value of the objective function on the right-hand side of (5) falls. Since all constraints continue to be satisfied, this implies a contradiction. **Q.E.D.** ■

An immediate consequence of the previous result is that the individual rationality constraint (6) for $\gamma = 1$ and the truth-telling constraint (7) for $(\gamma, \gamma') = (\bar{\gamma}, 1)$ are slack.

4.2 Existence and Uniqueness of Stationary Equilibrium

Now, we need to ensure that there exists a market-determined expected discounted utility w^* associated with a market contract that makes each lender indifferent between posting a contract and remaining inactive. This is equivalent to showing the existence of an equilibrium.

Formally, a stationary and symmetric equilibrium consists of a cost function $C_w : D_w \to \mathbb{R}$, policy functions $\hat{u} : D_w \to D$, $y : D_w \times \Gamma \to [0, h]$, $g : D_w \times \Gamma \to D_w$, and a market utility w^* such that: *(i)* C_{w^*} satisfies (5); *(ii)* $(\hat{u}, y, g)$ are the optimal policy functions for (5); and *(iii)* w^* satisfies the free-entry condition:

$$C_{w^*}(w^*) + (1 - \beta) k = 0. \tag{11}$$

The market utility w^* gives the expected discounted utility for a borrower at the signing date. Due to limited commitment and free entry of lenders in the credit market, it is also the lower bound on the set of expected discounted utilities.

Lemma 3 *There exists a unique expected discounted utility w^* satisfying (11) provided that $k > 0$ is sufficiently small.*

Proof. First, notice that $C_{w^a}(w^a) < 0 < C_{\bar{w}_F}(\bar{w}_F)$. Suppose that $k > 0$ is sufficiently small such that $C_{w^a}(w^a) + (1 - \beta) k < 0$. Given that $\hat{\phi}(w) \equiv C_w(w)$ is continuous, there exists $w^* \in [w^a, \bar{w}_F]$ such that $\hat{\phi}(w^*) + (1 - \beta) k = 0$. To show uniqueness, define the mapping $\sigma : [w^a, \bar{w}_F] \to [w^a, \bar{w}_F]$ as follows. If $C_w + (1 - \beta) k$ is always greater than zero on $[w, \bar{w}_F]$, then $\sigma(w) = w^a$. Otherwise, $\sigma(w)$ equals the point $w' \in [w, \bar{w}_F]$ for which $C_w(w') + (1 - \beta) k = 0$. I claim that σ is a non-increasing function. To verify this claim, we need to show first that $\bar{w}(w)$ is non-increasing in w. Fix a lower bound w_0 in the set $[w^a, \bar{w}_F]$, and consider the associated upper bound $\bar{w}(w_0)$. Take another point $w_1 > w_0$ in the set $[w^a, \bar{w}(w_0)]$. Notice that $C_{(w_0, \bar{w}(w_0))} \leq C_{(w_1, \bar{w}(w_0))}$. Thus, we have that $C_{(w_1, \bar{w}(w_0))}[\bar{w}(w_0)] \geq 0$ given that $C_{(w_0, \bar{w}(w_0))}[\bar{w}(w_0)] = 0$ by the definition of $\bar{w}(w_0)$. This implies that $\bar{w}(w_1) \leq \bar{w}(w_0)$, and we conclude that $\bar{w}(w)$ is indeed non-increasing in w. The fact that $\bar{w}(w)$ is non-increasing then implies that raising the lower bound w only tightens the constraint set $\Upsilon_{(w, \bar{w}(w))}(\cdot)$. As a result, the point at which $C_w + (1-\beta)\,k$ equals zero is a non-increasing function of the lower bound w, which means that σ can have at most one fixed point. **Q.E.D.** ■

Notice that *ex ante* each lender gets zero expected discounted utility by posting a contract. *Ex post* a lender gets a higher utility, given that $C_{w^*}(w^*) < 0$. Moreover, as the contract is executed, there is no history of reports by a borrower that gives a lender an expected discounted utility that is lower than that associated with autarky. For this reason, neither a lender nor a borrower finds it optimal to renege on the credit contract.

4.3 Properties of the Optimal Contract

An important aspect of the credit arrangement described in this paper is the rule for the settlement of debt implied by the optimal credit contract. The following result shows that we obtain a state-contingent rule for settlement which is similar to the one obtained under full commitment and complete information, provided that there is enough dispersion in the preference shock.

Lemma 4 *$y(w, \bar{\gamma}) = 0$ for all $w \in D_{w^*}$ provided that*

$$H'(w^*) \geq \frac{1}{\bar{\gamma} - 1}. \tag{12}$$

Proof. Suppose that $y_{\bar{\gamma}} > 0$ holds at the optimum. We know from Lemma 2 that

$$\beta w_1 = (1-\beta)(y_1 - y_{\bar{\gamma}}) + \beta w_{\bar{\gamma}} \tag{13}$$

must hold at the optimum. Consider reducing $y_{\bar{\gamma}}$ by a small amount $\Delta > 0$ and at the same time reducing $w_{\bar{\gamma}}$ by $\Delta\beta^{-1}(1-\beta)$. Notice that the right-hand side of (13) remains unchanged. The left-hand side of the truth-telling constraint (7) for $(\gamma, \gamma') = (\bar{\gamma}, 1)$ and of the individual rationality constraint (6) for $\gamma = \bar{\gamma}$ increase by $\Delta(1-\beta)(\bar{\gamma}-1) > 0$, and the left-hand side of the promise-keeping constraint (8) increases by $\Delta\pi(1-\beta)(\bar{\gamma}-1) > 0$.

Conjecture that $u > w^*$ holds at the optimum for any w in the interior of D_{w^*}. Now, consider reducing u by $\Delta\pi(\bar{\gamma}-1)$. Define the function $f(\Delta) = H[u - \Delta\pi(\bar{\gamma}-1)] - H(u)$. Notice that $f(0) = 0$ and $f' < 0$. Also, f' is strictly increasing. Finally, we have that

$$f'(0) = -\pi(\bar{\gamma}-1)\,H'(u) < -\pi(\bar{\gamma}-1)\,H'(w^*) \leq -\pi$$

because of (12). Then, there exists $\bar{\Delta} \in (0, \infty)$ such that $f(\Delta) + \pi\Delta < 0$ for all $\Delta \in (0, \bar{\Delta})$. This means that there exists $\Delta \in (0, \bar{\Delta})$ sufficiently small such that the change in the value of the objective function on the right-hand side of (5) is given by

$$(1-\beta)\left[f(\Delta)+\pi\Delta\right]+\beta\xi(\Delta)<0,$$

where $\xi(\Delta)<0$. But this implies a contradiction. Hence, we must have $y(w,\bar{\gamma})=0$ for all w in the interior of D_{w^*} provided that (12) holds. Since $y(\cdot,\bar{\gamma}):D_{w^*}\to[0,h]$ is continuous, it follows that $y(w,\bar{\gamma})=0$ for all $w\in D_{w^*}$. **Q.E.D.** ■

Although a lender now needs to provide incentives to induce truthful reporting, she finds it optimal not to require any repayment from a borrower in the settlement stage if the latter reports the high marginal utility of leisure – the costly state of nature for a borrower to make a payment to a lender and settle his debt. This property arises due to optimal intertemporal allocation of resources by a risk-neutral lender. Notice that condition (12) holds if the higher realization $\bar{\gamma}$ for a borrower's marginal utility of leisure is sufficiently large. For instance, suppose that a borrower's preferences exhibit constant absolute risk aversion: $u(q)=-\exp(-\alpha q)$, with $\alpha>0$. Thus, a sufficient condition for obtaining (12) is $\bar{\gamma}\geq 1+\alpha$, which requires that the preference shock have enough dispersion to matter in the settlement process. For the rest of the paper, I restrict attention to contracts that satisfy condition (12). To obtain a sharp characterization of the optimal contract, we need to study the properties of the optimal payment schedule $y(w,1)$ and the optimal continuation values $g(w,\gamma)$ for each $\gamma\in\Gamma$.

We can rewrite the optimization problem on the right-hand side of (5) in the following way. Given the results in Lemma 2 and 4, the relevant constraints for the optimization problem are (13) and

$$(1-\beta)(u-y_1)+\beta w_1=w. \tag{14}$$

Substituting (13) and (14) into (5), the optimization problem now consists of choosing y_1 and w_1 to minimize:

$$(1-\beta)\left[H\left(\frac{w-\beta w_1}{1-\beta}+y_1\right)-(1-\pi)y_1\right]+\beta\left\{\begin{array}{c}\pi C_{w^*}\left[w_1-\frac{(1-\beta)}{\beta}y_1\right]+\\(1-\pi)C_{w^*}(w_1)\end{array}\right\},$$

subject to $w^*\leq w_1\leq\bar{w}(w^*)$, $0\leq y_1\leq h$, and

$$w_1-\frac{(1-\beta)}{\beta}y_1\geq w^*. \tag{15}$$

The first-order conditions for the optimal choice of y_1 are

$$H'\left[\frac{w-\beta g(w,1)}{1-\beta}+y(w,1)\right]-\left\{\pi C'_{w^*}\left[g(w,\bar{\gamma})\right]-\frac{\lambda(w)}{\beta}\right\}\geq 1-\pi, \quad (16)$$

if $y(w,1)<h$, and

$$H'\left[\frac{w-\beta g(w,1)}{1-\beta}+y(w,1)\right]-\left\{\pi C'_{w^*}\left[g(w,\bar{\gamma})\right]-\frac{\lambda(w)}{\beta}\right\}\leq 1-\pi, \quad (17)$$

if $y(w,1)>0$. The first-order condition for the optimal choice of w_1 is

$$H'\left[\frac{w-\beta g(w,1)}{1-\beta}+y(w,1)\right]\geq\left\{\begin{array}{c}\pi C'_{w^*}\left[g(w,\bar{\gamma})\right]+\\(1-\pi)C'_{w^*}\left[g(w,1)\right]-\frac{\lambda(w)}{\beta}\end{array}\right\}, \quad (18)$$

with equality if $g(w,1)<\bar{w}(w^*)$. Also, we have that

$$\lambda(w)\left[g(w,1)-\frac{(1-\beta)}{\beta}y(w,1)-w^*\right]=0, \quad (19)$$

where $\lambda(w)\geq 0$ is the Lagrange multiplier on constraint (15). Finally, the envelope condition is given by

$$C'_{w^*}(w)=H'\left[\frac{w-\beta g(w,1)}{1-\beta}+y(w,1)\right], \quad (20)$$

for any w in the interior of the set D_{w^*}.

Now, I establish some properties of the optimal continuation value $g(w,\gamma)$ for each $\gamma\in\Gamma$. These give a borrower's expected discounted utility at the beginning of the following period associated with the market contract as a function of his initially promised expected discounted utility w and his report in the settlement stage of the current period. If a borrower's expected discounted utility falls in the subsequent period relative to the current period, this means that the terms of the contract become less favorable for him - and as a result more favorable for the lender.

Lemma 5 *The function $g(w,1)$ has the following properties: (i) $g(w,1)\geq w$ for all $w\in D_{w^*}$ and (ii) $g(w,1)$ is non-decreasing.*

Proof. Suppose that $g(w,1)<w$ for some w in the interior of D_{w^*}. Given that $g(w,1)<w\leq\bar{w}(w^*)$, it must be that

$$C'_{w^*}(w)=\pi C'_{w^*}\left[g(w,\bar{\gamma})\right]+(1-\pi)C'_{w^*}\left[g(w,1)\right]-\frac{\lambda(w)}{\beta}.$$

Recall that $g(w,1) \geq g(w,\bar{\gamma})$ and that C_{w^*} is strictly convex. As a result, we have that

$$C'_{w^*}(w) < C'_{w^*}(w) - \frac{\lambda(w)}{\beta} \leq C'_{w^*}(w),$$

where the last inequality follows because $\lambda(w) \geq 0$. But this results in a contradiction. Hence, we conclude that $g(w,1) \geq w$ for all w in the interior of D_{w^*}. The fact that $g(w,1)$ is continuous implies that $g(w,1) \geq w$ holds for all $w \in D_{w^*}$ as claimed.

Now, we show that $g(w,1)$ is a non-decreasing function. Given that $y(\cdot,1): D_{w^*} \rightarrow [0,h]$ is a continuous function, the set

$$I = \{w \in D_{w^*} : y(w,1) = h\}$$

can be written as a finite union of closed intervals. Suppose that w belongs to I. Either $g(w,1)$ is constant or $g(w,1)$ is given by

$$C'_{w^*}(w) = \pi C'_{w^*}\left[g(w,1) - \frac{(1-\beta)}{\beta}h\right] + (1-\pi)\, C'_{w^*}[g(w,1)].$$

This means that $g(w,1)$ is non-decreasing on I. The set $D_{w^*} \setminus I$ can be written as a finite union of intervals. If w belongs to this set, then (16)-(18) imply that either $g(w,1) = \hat{g}$ or $g(w,1) = \bar{w}(w^*)$, where the constant $\hat{g}$ solves $C'_{w^*}(\hat{g}) = 1$. This means that $g(w,1)$ is constant on $D_{w^*} \setminus I$. **Q.E.D.** ■

Lemma 6 *The function $g(w,\bar{\gamma})$ has the following properties: (i) $g(w,\bar{\gamma}) < w$ for all $w > w^*$; (ii) $g(w^*,\bar{\gamma}) = w^*$; and (iii) there exists $\delta > 0$ such that $g(w,\bar{\gamma}) = w^*$ for all $w \in [w^*, w^* + \delta)$.*

Proof. First, notice that we must have $y(w,1) > 0$ for all $w \in D_{w^*}$. To verify this claim, suppose that $y(w,1) = 0$ for some $w \in (w^*, \bar{w}(w^*)]$. Then, we must have $g(w,1) = g(w,\bar{\gamma})$ given that (7) for $(\gamma,\gamma') = (1,\bar{\gamma})$ holds with equality and $y(w,\bar{\gamma}) \leq y(w,1)$. Moreover, either $g(w,1) = g(w,\bar{\gamma}) = \bar{w}(w^*)$ or $g(w,1) = g(w,\bar{\gamma}) < \bar{w}(w^*)$. If $g(w,1) = g(w,\bar{\gamma}) = \bar{w}(w^*)$, then (18) and (20) imply that $C'_{w^*}(w) \geq C'_{w^*}[\bar{w}(w^*)]$, which requires $w = \bar{w}(w^*)$. Otherwise, we obtain a contradiction. Then, we have that $C_{w^*}[\bar{w}(w^*)] = H[\bar{w}(w^*)] > H(w^a) = 0$, which contradicts the definition of $\bar{w}(w^*)$. Suppose now that $g(w,1) = g(w,\bar{\gamma}) < \bar{w}(w^*)$. From (18) and (20), we conclude that $g(w,1) = g(w,\bar{\gamma}) = w$. Again, we have that $C_{w^*}(w) = H(w) > H(w^a) = 0$, which implies a contradiction. Therefore, we must have $y(w,1) > 0$ for all $w \in (w^*, \bar{w}(w^*)]$. Continuity then implies that $y(w^*,1) > 0$, so that $y(w,1) > 0$ for all $w \in D_{w^*}$ as claimed. As a result, $g(w,1) > g(w,\bar{\gamma})$ for all $w \in D_{w^*}$.

Suppose that $g(w,\bar{\gamma}) \geq w$ for some $w > w^*$. From (18) and (20), we have that

$$C'_{w^*}(w) \geq \pi C'_{w^*}[g(w,\bar{\gamma})] + (1-\pi) C'_{w^*}[g(w,1)] > C'_{w^*}(w),$$

where the last inequality follows from the strict convexity of C_{w^*} and from the fact that $g(w,1) > g(w,\bar{\gamma})$. But this results in a contradiction. Hence, we must have $g(w,\bar{\gamma}) < w$ for all $w > w^*$. Since $g(w,\bar{\gamma})$ is continuous, it follows that $g(w^*,\bar{\gamma}) = w^*$.

Finally, to prove the last part, suppose that $g(w^*+\varepsilon,\bar{\gamma}) > w^*$ for all $\varepsilon > 0$. Then, (18) and (20) require that

$$C'_{w^*}(w^*+\varepsilon) \geq \pi C'_{w^*}[g(w^*+\varepsilon,\bar{\gamma})] + (1-\pi) C'_{w^*}[g(w^*+\varepsilon,1)]$$

holds for all $\varepsilon > 0$, which in turn requires that $\lim_{\varepsilon \to 0} g(w^*+\varepsilon,1) = w^*$. But this implies a contradiction. **Q.E.D.** ■

If a borrower reports the high marginal utility of leisure in the settlement stage, the terms of credit that are offered by a lender in subsequent periods are such that his expected discounted utility falls. This property of the optimal contract is very different from those obtained under full commitment and complete information. The fact that a borrower is not required to make a repayment contingent on the realization of the costly state of nature now results in a change of the terms of credit for future transactions within the relationship. This can be interpreted as delayed settlement by a borrower: a circumstance in which a borrower does not make a payment to a lender in the settlement stage – despite the fact that he has received a loan from such a lender in the transaction stage – and as a result the terms of credit for future transactions become less favorable for him. This corresponds to a circumstance in which a borrower carries a balance on his account with a lender.

On the other hand, the terms of credit either become more favorable or remain the same for a borrower if he is able to settle his debt by making a payment to a lender in the settlement stage. This means that a lender needs to promise more favorable terms of credit for future transactions within the relationship in order to induce a borrower to pay off his balance at the end of the period.

Notice that the envelope condition (20) implies that the loan amount to which a borrower is entitled in the transaction stage is strictly increasing in his promised expected discounted utility w. As we have seen, a lender does not require any repayment from a borrower when the latter reports the high marginal utility of leisure, and the optimal provision of incentives by a lender results in a lower promised expected discounted utility for a borrower at the beginning of the subsequent period. As a result, the loan amount that a borrower receives from a lender in the subsequent transaction stage shrinks, given that $H[\hat{u}(w)]$ is a strictly increasing function. This shows how the loan amount that a borrower receives in the current transaction depends on his history of payments to a lender.

Finally, notice that the expected return to a lender on the current transaction is given by

$$R(w) \equiv \frac{(1-\pi)\, y(w,1)}{H[\hat{u}(w)]},$$

which summarizes the terms of credit for the current transaction. Unlike the equilibrium under full commitment and complete information, the expected return to a lender now depends on w and fluctuates over time as a result.

4.4 Long-Run Properties

Now, I study the long-run properties of the equilibrium allocation. Specifically, I show that there exist well-behaved long-run distributions of consumption and production. Let $\Psi(D_{w^*}, \mathbf{D})$ be the space of all probability measures ψ on the measurable space $(D_{w^*}, \mathbf{D})$, where $\mathbf{D}$ is the collection of Borel subsets of D_{w^*}. Define the operator T^* on $\Psi(D_{w^*}, \mathbf{D})$ by

$$(T^*\psi)(D') = \pi \int_{\mathbf{Q}_{\bar{\gamma}}(D')} d\psi + (1-\pi) \int_{\mathbf{Q}_1(D')} d\psi, \tag{21}$$

for each $D' \in \mathbf{D}$, where, for each $\gamma \in \{1, \bar{\gamma}\}$, the set $\mathbf{Q}_\gamma(D')$ is given by

$$\mathbf{Q}_\gamma(D') = \{w \in D_{w^*} : g(w,\gamma) \in D'\}.$$

Notice that a fixed point of the operator T^* corresponds to an invariant distribution over D_{w^*}.

Lemma 7 *The operator T^* has a unique fixed point ψ^*, and for any probability measure ψ in $\Psi(D_{w^*}, \mathbf{D})$, $T^{*n}\psi$ converges to ψ^* in the total variation norm.*

Proof. Let ψ_w denote the probability measure that concentrates mass on the point w. I will show that there exist $N \geq 1$ and $\varepsilon > 0$ such that

$\left(T^{*N}\psi_w\right)(w^*) \geq \varepsilon$ for all $w \in D_{w^*}$. From Lemma 6, there exists $k > 0$ such that either $g(w, \bar{\gamma}) \leq w - k$ or $g(w, \bar{\gamma}) = w^*$ for all $w \in D_{w^*}$. Now, choose an integer $N \geq 1$ large enough so that $\bar{w}(w^*) - kN \leq w^*$. Then, the probability of moving from the point $\bar{w}(w^*)$ to the point w^* in N steps is at least π^N. Since $g(w, \bar{\gamma})$ is non-decreasing in w, such a transition to w^* is at least as probable from any other point in D_{w^*}. Thus, if $\varepsilon = \pi^N$, then the implied Markov process satisfies the hypotheses of Theorem 11.12 of Stokey, Lucas, and Prescott (1989), and the proof is complete. **Q.E.D.** ■

The existence of a non-degenerate long-run distribution derives from the fact that there is no absorbing point, which implies that the entire state space is an ergodic set. The role of limited commitment is to bound the set of promised utilities, which is necessary to obtain a non-degenerate long-run distribution. Specifically, the lower bound w^* on the set of expected discounted utility entitlements arises due to the fact that a borrower can defect from his current contract and sign with another lender at any moment. The upper bound $\bar{w}(w^*)$ is the highest expected discounted utility to which a lender can commit to deliver to a borrower given that the lowest expected discounted utility that can be promised is w^*.

5 Discussion

As we have seen, a lender's optimal contract is such that the property of delayed settlement results in less favorable terms of credit for a borrower in future transactions. I have shown that the settlement of debt within the long-term credit relationship follows a state-contingent rule that is similar to the one obtained under full commitment and complete information. However, if a lender is asymmetrically informed with respect to a borrower's ability to settle his debt, the optimal intertemporal allocation of resources creates a critical interplay between this rule for settlement and the terms of credit for future transactions within the relationship.

If a borrower reports the high marginal utility of leisure in the settlement stage, it is optimal for a lender, in terms of current and future resources, to initially reward such a borrower – by delaying settlement – and to punish him later – by specifying terms of credit for future transactions that result in a lower expected discounted utility for the borrower. This corresponds to a situation in which a borrower finds it hard to pay off his balance at the end of the month – becoming liable to a lender – and the latter obtains more favorable terms of credit for future transactions. On the other hand, if a borrower reports the low marginal utility of leisure in the settlement stage, it is optimal for a lender to collect a repayment from him today and postpone any reward to future periods. This corresponds to a situation in which a lender induces a borrower to pay off his balance at the end of the month by offering him at least the same terms of credit in future transactions. This is the optimal response of a risk-neutral lender to the fact that a borrower's ability to settle his debt at the end of the month is unobservable.

This property of the optimal contract reproduces qualitatively some observed characteristics of the market for unsecured loans. Specifically, the property of delayed settlement resembles the practice of revolving credit. As we have seen, this practice usually results in a higher interest rate for a borrower. In the framework developed in this paper, the fact that a borrower who pays off his balance with a lender in full is rewarded with more favorable terms of credit than those offered to a borrower who carries a balance is the result of an optimal contracting problem.

Koeppl, Monnet, and Temzelides (2008) find that the settlement of debt in an efficient credit system should be sufficiently frequent in the context of the model by Lagos and Wright (2005). In their analysis, there is no uncertainty in the settlement process, and the consumers always settle all of their remaining balances in the settlement stage as a result of preferences that are quasilinear with respect to leisure. Within the same framework, Andolfatto (2008) studies the implementation of constrained-efficient allocations through short-term credit arrangements – namely standard debt contracts. If the settlement process involves uncertainty and if enduring relationships are feasible, then long-term credit arrangements become essential, and asymmetric information with respect to a borrower's ability to settle his debt is critical to reproduce observed characteristics of credit markets.

6 Comparative Statics

An important parameter in the model is the cost $k > 0$ that a lender has to pay in order to post a credit contract. We have seen that, for any sufficiently small $k > 0$, there exists a unique $w^*(k)$ such that $\hat{\phi}[w^*(k)] + (1-\beta)k = 0$, where $\hat{\phi}(w) \equiv C_w(w)$. Given that $\hat{\phi}(w)$ is a continuous function, for any k' in a neighborhood of k, there exists a unique $w^*(k')$ such that $\hat{\phi}[w^*(k')] + (1-\beta)k' = 0$. Moreover, if $k' > k$, we have that $w^*(k') < w^*(k)$; if $k' < k$, we have that $w^*(k') > w^*(k)$. In the proof of Lemma 3, we have established that the upper bound $\bar{w}(w^*)$ is a non-increasing function of the lower bound w^*. Thus, we have that $D_{w^*(k)} \subseteq D_{w^*(k')}$ if $k' > k$ and that $D_{w^*(k')} \subseteq D_{w^*(k)}$ if $k' < k$. This means that a lower k results in a smaller set of expected discounted utilities.

Notice that a lower k makes each borrower better off from the perspective of the signing date because the expected discounted utility associated with the market contract increases. We can interpret $k > 0$ as the cost per customer for a lender in the market for unsecured loans. Some changes in the regulation of consumer credit are likely to affect the size of k. This means that a change in regulation that reduces the cost per customer makes each borrower better off from the perspective of the contracting date and results in a loan schedule that is less volatile.

7 Conclusion

The analysis in this paper relies on dynamic contracting to explain observed behavior in the market for unsecured loans. We have seen that a lender's optimal contract in a competitive credit market involves a state-contingent repayment schedule for a borrower that critically affects the terms of credit for future transactions within an enduring relationship. This property of the optimal contract is crucial to reproduce observed characteristics of the market for unsecured loans such as the practice of revolving credit. Asymmetric information with respect to a borrower's ability to repay his loan is a critical friction that creates an interplay between the terms of credit for the current transaction and those for future transactions.

The model could be easily extended to include other trading frictions that are important to the study of credit markets. For any quantitative analysis, it is necessary to introduce persistent shocks in the model, especially because the consumers who usually carry a balance on their credit card have a considerably higher credit card utilization rate. This suggests that shocks to the consumer finances tend to be persistent, which in turn leads to the repeated practice of revolving credit. The analysis then requires the techniques developed in Fernandes and Phelan (2000) to solve for a lender's optimal contract.

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