Skewed (and Risky) Businesses *

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Abstract

A risky business such as drug development and venture capital investment fails most of times but it generates an extremely high profit occasionally. For the purpose of understanding the emergence of these skewed businesses, we study a simple bandit problem in which a single decision maker learns the average return of a new business by experimentation. Differently from previous literature, we order a new business not only by its cash flow variance but also by skewness. We find that (1) a higher skewness reinforces a signal quality of favorable outcome and weakens that of unfavorable outcome and (2) a higher outside option may make a positively skewed business more attractive relative to a negatively skewed business.

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1 Introduction

Some business models such as new drug development and venture capital investment are inherently risky and undertaking of such models is justified only because they may return extremely high profit. Pisano (2006) writes that there is only about a 1-in-5,000 chance that a newly developed molecule will be turned into a commercially viable drug. However, if turned into a blockbuster drug, it may generate multi-billions in sales. By examining a sample of venture capital backed firms that experienced exit events, Cochrane (2005) finds that investment returns are highly skewed and unprofitable investments are the majority. Kaplan and Strömberg (2004) study 67 investment memoranda prepared by venture capitalists and find that 68.7% of the sample mention large market size and growth *potentials* as reasons to invest in that companies.

The emergence of these risky business models is a puzzle for the following reasons. First, the standard expected utility hypothesis states that high risk needs to be compensated by high returns. Nevertheless, evidence shows that the drug development and venture capital investments are not necessarily more profitable than other safer business models. (See Pisano (2006) for profitability of pharmaceutical firms and Kedrosky (2009) and Phalippou and Gottschalg (2008) for profitability of venture capital investments.) Second, when the profitability of a new business is not known and learnt by experimentation, a risky business is at a disadvantage, because outcome of experimentation is a noisy signal of the business quality and as a consequence experimentation becomes less profitable. (See Berevy-Meyer and Roth (2006), Bernardo and Chowdhry (2004), and Ryan and Lippmann (2003)).

The previous literature characterizes a risky business by a higher variance of return distribution alone. Nevertheless, both pharmaceutical development and VC investments have not only a higher variance of return distribution, but also a higher skewness. In this paper, we address this omission of literature and study the impact of skewness on the decision to adopt a risky business model.

Concretely, we model a single firm that can undertake a new project. The new project yields periodical cash flows. At the outset the average of cash flow is unknown. The adopted project is experimental in nature: the cash flow in each period provides information about the underlying cash flow process. The gained information is relevant as after each period the firm can decide to abandon its new project and revert to a traditional project. The firm has perfect knowledge about the cash flow process of the traditional project.

We identify two different effects that make experimentation of a risky project more valuable than experimentation of a safer project. First, roughly speaking, a positive outcome is rare in a risky project and therefore the information value that a positive outcome delivers is larger for a risky project than a safer project. Second, as a risky project rarely returns a positive outcome, a probability of "false positive " is lower for a risky project than for a safer project. When a traditional project yields a high return, cost of continuing an experimental project which happens to show a positive outcome but has a low average return is higher. As a consequence, when return to a traditional project is high, a risky project becomes more valuable relative to a safer project. We also find that, unlike the literature that only studies variance of return, if a project is riskier, the decision maker updates her belief positively over a wider range of experimental outcome.

Our model falls into the category of two-armed bandit problems. Variants of the two-armed-bandit problem have been studied in various contexts.[1] For instance, Bolton and Harris (1999) study the strategic incentives of firms to experiment. They find that there may be a complementarity between experimentation – one experiment induces another experiment. Copeland (2007) introduces public and private signals in a setting similar to Bolton and Harris. Manso (2007) studies the incentive mechanism to motivate an agent to experiment. Berk, Green and Naik (2004) study the effect of learning on the firm's idiosyncratic and systematic risks. Myers and Howe (1997) argue that a systematic component of R& D risk decreases over time, due to operating leverage. These literatures do not study the difference between risky and safer projects as this paper does.

Two papers, Bernardo and Chowdhry (2000) and Ryan and Lippman (2003) study the impact of cash flow variance on an experimental project, as we do in this paper. Their models are a continuous time and their distribution is normal distribution instead of binomial. The agent is not sure about drift of Brownian motion. Both papers find that value of the firm is negatively related with noise in firm's performance measures in the presence of learning and real options.[2]

[1]See Bergemann and Välimäki (2006) for a recent survey on bandit problems.

[2]Slow learning due to noisy cash flow is related to the observation, long known in the psychology literature, that partial reinforcement of actions slows learning, especially early learning, i.e., when subjects are inexperienced (see, e.g., Solomon Weinstock, 1958, and, for a review, see Donald Robbins, 1971). In the psychology literature, an action is said to be partially reinforced if it is rewarded only some of the time, in contrast to actions that are rewarded every time they are taken, which are said to be fully (or continuously) reinforced.

Our model is also related to literatures on real options with passive learning. Roughly speaking, under these passive learning literatures, current cash flow is unbiased estimator of future cash flow. Given that the value of a project is discounted sum of future cash flows, cash flows and the value of the underlying assets comove linearly. In particular, if cash flow is more volatile, so is the value of the underlying assets. The key result of this standard framework is that the riskier the cash flow, the higher the value of the project is, since the firm can discontinue the project if cash flow turns out to be low and can selectively participate in upside potential of the project.

In our framework, current cash flow is not unbiased estimator of future cash flow and, as a result, cash flows and the value of the underlying assets do not comove linearly. In particular, if cash flow is more volatile, the learning about the value of the underlying assets is slower and, as a consequence, the value of the project is *less volatile*. As the risky project has more volatile cash flows but less volatile value of the underlying assets, the experimental value of the risky project is lower than that of the safer project.

The organization of this paper is as follows. Section 2 introduces the model. Section 3 characterize the model with two experimental outcomes. Section 4 studies the case in which experimental outcome is continuously distributed. Section 5 concludes.

2 The Model

We study a problem of a single decision maker who can undertake one project per period over two periods.[3] Her utility is linear in her payoff and the second period payoff is discounted at factor, δ. Her objective is to maximize the discounted sum of her expected utility over two periods.

For each of the two periods, she can undertake either one of two projects – a traditional project and a experimental project. The traditional project returns average output equal to a constant, R. The distribution of the traditional project's output is well known and undertaking the traditional project will bring no new information. The experimental project's output per period, $\widetilde{x}$, is random and its density and cumulative distribution are denoted by $f(\widetilde{x})$ and $F(\widetilde{x})$, respectively. Per period output $\widetilde{x}$ is identically

An early general conclusion from this literature was that learning proceeds somewhat more rapidly and reaches a higher final training level under continuous reinforcement than under partial reinforcement (William Jenkins and Julian Stanley, 1950).

[3] Even though the unconditional expected payoff of the experimental project is negative, if she can, a decision maker may scale up the size of the experimental project in the first period, according to a similar logic to Bolton and Harris.

and independently distributed over two periods. At the outset, the decision maker does not perfectly know $f(\widetilde{x})$, but knows that it is either $\overline{f}(\widetilde{x})$ with probability θ_0 or $\underline{f}(\widetilde{x})$ with probability $1-\theta_0$. We denote $g(\widetilde{x})$ and $G(\widetilde{x})$ be unconditional density and distribution, respectively. That is,

$$\begin{aligned} G(\widetilde{x}) &= \theta_0 \overline{F}(\widetilde{x}) + (1-\theta_0)\underline{F}(\widetilde{x}) \text{ and} \\ f(\widetilde{x}) &= \theta_0 \overline{f}(\widetilde{x}) + (1-\theta_0)\underline{f}(\widetilde{x}) \end{aligned}$$

The average output of the experimental project is a constant $\overline{\Phi}$ if $f(\widetilde{x}) = \overline{f}(\widetilde{x})$ and a constant $\underline{\Phi}$ if $f(\widetilde{x}) = \underline{f}(\widetilde{x})$. We assume that

$$\underline{\Phi} < R < \overline{\Phi},$$

such that if $f(\widetilde{x})$ were perfectly known, the decision maker were to undertake the experimental project if $f(\widetilde{x}) = \overline{f}(\widetilde{x})$ and to undertake the traditional project otherwise. For convenience, we sometimes call the experimental project to be bad if $f(\widetilde{x}) = \underline{f}(\widetilde{x})$ and to be good if $f(\widetilde{x}) = \overline{f}(\widetilde{x})$.

It is possible to change the project over the two periods and the decision maker may switch from the experimental project to the traditional project if the first period outcome is low and indicates lower quality of the experimental project. To be concrete, if the experimental project is undertaken in the first period, the decision maker updates her belief about $f(\widetilde{x})$ in the Bayesian manner. If the realized output is x, then the posterior probability that $f(\widetilde{x}) = \overline{f}(\widetilde{x})$ is

$$\theta_1(x) = \left[1 + \frac{1-\theta_0}{\theta_0} \frac{\underline{f}(x)}{\overline{f}(x)}\right]^{-1}.$$

Therefore, it is optimal to undertake the experimental project in the second period if and only if

$$\theta_1(x)\overline{\Phi} + (1-\theta_1(x))\underline{\Phi} \geq R.$$

We assume the monotone likelihood property that $\theta_1(x)$ is increasing. As a result, there exists a threshold $\widehat{x}$ such that

$$\begin{aligned} \theta_1(x)\overline{\Phi} + (1-\theta_1(x))\underline{\Phi} &\geq R \text{ if } x \geq \widehat{x} \text{ and} \\ \theta_1(x)\overline{\Phi} + (1-\theta_1(x))\underline{\Phi} &< R \text{ otherwise.} \end{aligned}$$

That is, the decision maker should continue undertaking the experimental project if $x \geq \widehat{x}$ and should discontinue and switch to the traditional project, otherwise. We also define the threshold posterior in the same way as follows:

$$\widehat{\theta}_1 = \frac{R-\underline{\Phi}}{\overline{\Phi}-\underline{\Phi}}.$$

Under the optimal continuing-discontinuing decision rule described above, the decision maker's payoff if she undertakes the experimental project in the first period is

$$V = \theta_0 \overline{\Phi} + (1 - \theta_0)\, \underline{\Phi} + \delta\, (R + \pi)\,, \tag{1}$$

where

$$\begin{aligned} \pi &= \int_{-\infty}^{\infty} \max\left\{\theta_1(\widetilde{x})\, \overline{\Phi} + (1 - \theta_1(\widetilde{x})\, \underline{\Phi}) - R, 0\right\} d\overline{G}(\widetilde{x}) \\ &= \int_{\widehat{x}}^{\infty} \left(\theta_1(\widetilde{x})\, \overline{\Phi} + (1 - \theta_1(\widetilde{x})\, \underline{\Phi}) - R\right) d\overline{G}(\widetilde{x})\,. \end{aligned}$$

The first two terms ($\theta_0 \overline{\Phi} + (1 - \theta_0)\, \underline{\Phi}$) of equation (1) represent the expected payoff in the first period. The bracket multiplied by δ in equation (1) is the expected payoff in the second period and consists of two parts; R represents the baseline expected payoff and π represents the option value of experimenting in the first period. If the first period output is high and indicates that the experimental project has a higher value than the traditional project, the decision maker can continue undertaking the experimental project and take its value premium.

In what follows, we assume that θ_0, $\overline{\Phi}$, $\underline{\Phi}$ and R are all fixed when comparing different experimental projects. This assumption has two merits. First, due to this assumption, the first period profit of experimental projects, $\theta_0 \overline{\Phi} + (1 - \theta_0)\, \underline{\Phi}$, is fixed and therefore we can focus on studying the characteristics of the option value. Second, the standard real option model defines "risk" as a difference between $\overline{\Phi}$ and $\underline{\Phi}$ and has already revealed how "risk" influences the option value. Therefore, assuming that $\overline{\Phi}$ and $\underline{\Phi}$ are constant allows us to isolate our contribution from the standard real option framework.

3 Two-Output Case

To understand the basic mechanisms through which the riskiness of the experimental project affects the option value, we begin with the simplest case in which output of the experimental project takes only two values. In particular, we assume that

$$f(x) = \begin{cases} p & \text{if } x = H \\ 1 - p & \text{if } x = L \\ 0 & \text{otherwise} \end{cases}$$

and $H > x_L$. If the experimental project is good, then $p = \overline{p}$ and otherwise, $p = \underline{p}$. Therefore,

$$\underline{\Phi} = \underline{p}H + \left(1 - \underline{p}\right) L \quad \text{and} \quad \overline{\Phi} = \overline{p}H + \left(1 - \overline{p}\right) L. \tag{2}$$

The unconditional density is

$$g(x) = \begin{cases} \theta_0 \overline{p} + (1 - \theta_0)\, \underline{p} & \text{if } x = H \\ \theta_0 (1 - \overline{p}) + (1 - \theta_0) \left(1 - \underline{p}\right) & \text{if } x = L \\ 0 & \text{otherwise} \end{cases}.$$

Further, the posterior is computed as

$$\theta_1(x) = \begin{cases} \left[1 + \frac{1-\theta_0}{\theta_0} \frac{\underline{p}}{\overline{p}}\right]^{-1} & \text{if } x = H \\ \left[1 + \frac{1-\theta_0}{\theta_0} \frac{1-\underline{p}}{1-\overline{p}}\right]^{-1} & \text{if } x = L \end{cases}.$$

With the two-output assumption just described, we examine two cases. In the first case, one project has both higher second and third moments than the other project. In the second case, the variance is constant but one project has a higher third moment than the other.

3.0.1 Risky and Skewed

We are now going to fix L but vary $\overline{p}$, $\underline{p}$ and H. As $\overline{\Phi}$ and $\underline{\Phi}$ are assumed to be fixed, we can express $\underline{p}$ and H as functions of $\overline{p}$ as follows.

$$\underline{p}(\overline{p}) = \alpha \overline{p}, \tag{3}$$

where $\alpha = \left(\underline{\Phi} - L\right) / \left(\overline{\Phi} - L\right)$,[4] and

$$H(\overline{p}) = \frac{\overline{\Phi} - L}{\overline{p}} + L.$$

Obviously, H is decreasing in $\overline{p}$. That is, one project is characterized by a higher potential upside but with a lower probability of the upside than another project.

What is the impact of changing $\overline{p}$ described the above on the characteristics of the experimental project? First, conditional variances of per period

[4]Because $\overline{p}_j H_j + \left(1 - \overline{p}_j\right) L_j = \overline{p}_k H_k + \left(1 - \overline{p}_k\right) L_k$ and $\underline{p}_j H_j + \left(1 - \underline{p}_j\right) L_j = \underline{p}_k H_k + \left(1 - \underline{p}_k\right) L_k$, $\overline{p}_j (H_j - L_j) = \overline{p}_k (H_k - L_k)$ and $\underline{p}_j (H_j - L_j) = \underline{p}_k (H_k - L_k)$. Taking the ratio of both right hand sides and left hand sides give the result.

output decreases in $\overline{p}$. The variance of per period output if the project is good is $\overline{E}\left[\left(x-\overline{\Phi}\right)^2\right] = \overline{p}\left(1-\overline{p}\right)\left(H\left(\overline{p}\right)-L\right)^2$ and the variance of per period output if the project is bad is $\underline{E}\left[\left(x-\underline{\Phi}\right)^2\right] = \alpha\underline{p}\left(1-\alpha\underline{p}\right)\left(H\left(\overline{p}\right)-L\right)^2$. Again, because $\overline{\Phi}$ and $\underline{\Phi}$ are assumed to be the same for both projects and greater than L by assumption, we have

$$\frac{d\overline{E}\left[\left(x-\overline{\Phi}\right)^2\right]}{d\overline{p}}, \frac{d\underline{E}\left[\left(x-\underline{\Phi}\right)^2\right]}{d\overline{p}} < 0.^5$$

Second, changing $\overline{p}$ also impacts conditional third moments. In particular,

$$\frac{d\overline{E}\left[\left(x-\overline{\Phi}\right)^3\right]}{d\overline{p}}, \frac{d\underline{E}\left[\left(x-\overline{\Phi}\right)^3\right]}{d\overline{p}} < 0.$$

Therefore, increasing $\overline{p}$ reduces both second and third moment of $\overline{f}\left(x\right)$ and $\underline{f}(x)$. Third, with no surprise, increasing $\overline{p}$ enhances unconditional probability of obtaining H, because

$$g\left(x\right) = \left\{\begin{array}{cc} \theta_0\overline{p} + \left(1-\theta_0\right)\alpha\overline{p} & \text{if } x = H \\ \theta_0\left(1-\overline{p}\right) + \left(1-\theta_0\right)\left(1-\alpha\overline{p}\right) & \text{if } x = L \\ 0 & \text{otherwise} \end{array}\right. .$$

We are now going to examine the impact of $\overline{p}$ on the learning process and the option value. Due to equation (3),

$$\theta_1\left(H\right) = \left[1 + \frac{1-\theta_0}{\theta_0}\alpha\right]^{-1},$$

which is constant over $\overline{p}$ and

$$\theta_1\left(L\right) = \left[1 + \frac{1-\theta_0}{\theta_0}\frac{1-\alpha\overline{p}}{1-\overline{p}}\right]^{-1},$$

which is decreasing in $\overline{p}$. What is the intuition behind this belief updating process? A part of the intuition is found in the standard argument that learning is more difficult in a noisier circumstance. If $\overline{p}$ is lower, then the second moment of both $\overline{f}\left(\widetilde{x}\right)$ and $\underline{f}(\widetilde{x})$ is lower and, as a result, the first period output is a weaker signal of the project quality. As a consequence,

[5] Rearranging $pH + (1-p)L = \Phi$, we have $H - L = (\Phi - L)/p$. Substituting this equation for $H - L$ into the definition of the variance gives the result.

lower $\overline{p}$ makes realization of high output less blessing and realization of low output less cursing. Nevertheless, this standard argument is just a part of the intuition, because it does not explain why lower $\overline{p}$ does not make realization of high output less blessing. The rest of the intuition is provided in the subsection which follows, when we isolate the impact of skewness on the learning process.

To illustrate this belief updating process, we compare two experimental projects. One has $\overline{p} = \overline{p}_r$ and the other has $\overline{p} = \overline{p}_e > \overline{p}_r$. For convenience, we call the former project to be risky and the latter to be safer. Let $\theta_{1r}(x)$ and $\theta_{1e}(x)$ be the posterior function of the risky project and the safer project, respectively. Then, we can illustrate the belief updating process as a tree in Figure 1.

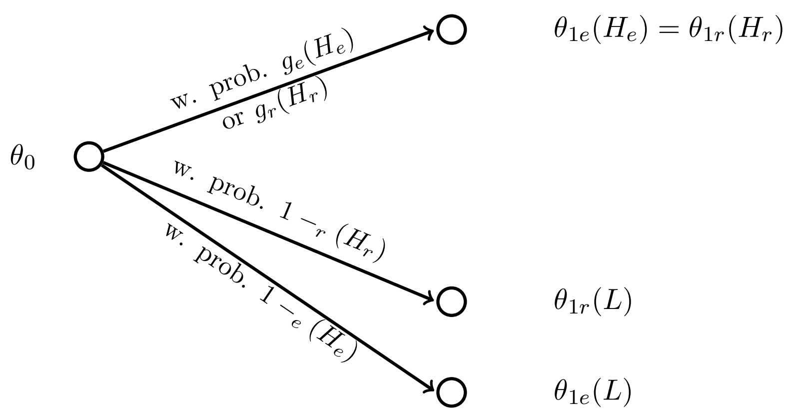

Figure 1: Learning

Given the distributional assumption described above, we are now going to study the impact of $\overline{p}$ on the option value. To prepare for presenting and proving the following proposition, we introduce some notation. We denote π_r and π_e be the option value of risky and safer project respectively. We also define $R(i)$, $i = 1, 2, 3$, implicitly by the following equations:

$$\begin{aligned}
\theta_{1e}(L)\,\overline{\Phi} + (1 - \theta_{1e}(L))\,\underline{\Phi} &= R(1) \\
\theta_{1r}(L)\,\overline{\Phi} + (1 - \theta_{1r}(L))\,\underline{\Phi} &= R(2) \text{ and} \\
\theta_{1r}(H_r)\,\overline{\Phi} + (1 - \theta_{1r}(H_r))\,\underline{\Phi} &= \theta_{1e}(H_e)\,\overline{\Phi} + (1 - \theta_{1e}(H_e))\,\underline{\Phi} = R(3).
\end{aligned}$$

Note that because $\theta_{1e}(L) < \theta_{1r}(L)$ and $\theta_{1e}(H_e) = \theta_{1r}(H_r)$, $R(1) < R(2) < R(3)$. A brief explanation about $R(i)$ is useful. When $R \leq R(1)$, the decision maker should continue the experimental project in the second period, even if the project is safer and returns L in the first period. When $R \leq R(2)$, the decision maker should continue the experimental project in the second period after observing L, only if the project is risky. When $R > R(3)$, the decision maker should no longer continue the experimental project in the second period no matter what level of output the experimental project returns in the first period.

Proposition 1 *If $R < R(1)$ or $R \geq R(3)$, then $\pi_r = \pi_e$. Otherwise, $\pi_r \leq \pi_e$.*

Proof. We separately prove this proposition for four different cases depending on R. First, suppose $R < R(1)$. Then, the option value is, for $j = e, r$

$$\begin{aligned}\pi_j &= g_j(H_j)\left(\theta_{1j}(H_j)\overline{\Phi} + (1-\theta_{1j}(H_j))\underline{\Phi} - R\right) \\ &\quad + g_j(L)\left(\theta_{1j}(L)\overline{\Phi} + (1-\theta_{1j}(L))\underline{\Phi} - R\right) \\ &= \theta_0\overline{\Phi} + (1-\theta_0)\underline{\Phi} - R,\end{aligned}$$

which does not depend on $\overline{p}$. As a result, $\pi_r = \pi_e$.

Second, suppose $R \geq R(3)$. Then, the option value is zero regardless of $\overline{p}$. As a result, $\pi_r = \pi_e$.

Third, suppose $R(1) \leq R < R(2)$. Then, the risky project should be continued in the second period regardless of the realization in the first period. Therefore,

$$\pi_r = \theta_0\overline{\Phi} + (1-\theta_0)\underline{\Phi} - R.$$

The safer project should be continued only if $x = H_e$ in the first period. Therefore,

$$\begin{aligned}\pi_e &= g_e(H_e)\left(\theta_{1e}(H_e)\overline{\Phi} + (1-\theta_{1e}(H_e))\underline{\Phi} - R\right) \\ &= \theta_0\overline{\Phi} + (1-\theta_0)\underline{\Phi} - R - \left(g_e(L)\left(\theta_{1e}(L)\overline{\Phi} + (1-\theta_{1e}(L))\underline{\Phi} - R\right)\right) \\ &> \theta_0\overline{\Phi} + (1-\theta_0)\underline{\Phi} - R = \pi_r.\end{aligned}$$

The last inequality follows because $g_e(L) > 0$ and

$$g_e(L)\left(\theta_{1e}(L)\overline{\Phi} + (1-\theta_{1e}(L))\underline{\Phi} - R\right) < 0$$

due to $R(1) \leq R$.

Fourth, suppose $R(2) \leq R < R(3)$. Then, both risky and safer project should be continued if and only if the first period out put is H. Therefore, for $j = e, r$

$$\pi_j = g_j(H_j)\left(\theta_{1j}(H_j)\overline{\Phi} + (1-\theta_{1j}(H_j))\underline{\Phi} - R\right).$$

As $\theta_{1r}(H_r) = \theta_{1e}(H_e)$,

$$\begin{aligned} & \theta_{1e}(H_e)\overline{\Phi} + (1-\theta_{1e}(H_e))\underline{\Phi} - R \\ = \;& \theta_{1r}(H_r)\overline{\Phi} + (1-\theta_{1r}(H_r))\underline{\Phi} - R \\ \geq \;& 0. \end{aligned}$$

The last inequality follows because $R(2) \leq R$. Since $g_r(H_r) < g_e(H_e)$, $\pi_r < \pi_e$. ■

Intuition behind this proposition is as follows. If $R < R(1)$, it is optimal to continue the experimental project irrespective of output in the first period. Therefore, the option value is equal to the unconditional expected payoff of the experimental project minus R – the same for both projects. If $R > R(3)$, then it is optimal to discontinue the experimental project irrespective of output in the first period.

If $R(1) < R < R(2)$, then the option value of the risky experimental project is just equal to the unconditional expected payoff the experimental project, whereas the option value of the safer experimental project consists of the unconditional expected payoff of the experimental project and the insurance value of discontinuing the experimental project when the first period output is equal to L. As a consequence, the option value of the safer project is higher than that of the risky project. Note that, for the risky project, the first period output has no information value, whereas for the safer project, it does have the information value because it affects the continuation decision in the second period. This difference stems from the standard argument that experiment bring about more information if the project output is less noisy – true for the safer project in our context.

If $R(2) < R < R(3)$, the first period output is informative and influential on the continuation decision for both risky and safer projects. Nevertheless, the probability of upside is higher for the safer project. And therefore, the value of the safer project is higher than that of the risky project. This effect has not been identified in the previous literature and we tentatively call it Yankees effect.

4 Positively Skewed versus Negatively Skewed

In the preceding example, if one experimental project has a higher third moment than another, then it also has a higher second moment than another. As a consequence, it is difficult to disentangle the impact of the second moment on the option value from that of the third moment. We now fix the second moment and vary only the third moment. In particular, we consider two experimental projects. One, which we call to be risky, has $\underline{p}= \underline{p}_r$, $\overline{p} = \overline{p}_r = \underline{p}_r + \triangle p$, $H = H_r$ and $L = L_r = H_r - \triangle x$, where $\triangle p$ and $\triangle x$ are both constant. The other project, which we call to be safe, has $\underline{p}= \underline{p}_e = 1 - \underline{p}_r$, $\overline{p} = \overline{p}_e = \underline{p}_e + \triangle p$, $H = H_e$ and $L = L_e = H_e - \triangle x$. We continue to assume that θ_0, $\overline{\Phi}$ and $\underline{\Phi}$ are all common for both risky and safer projects. We also assume that $\underline{p}_r < \underline{p}_e$.

Under this formulation, the variance of per period output for risky project and that for safer project conditioned that $p = \underline{p}$ are the same because $\underline{p}_e = 1 - \underline{p}_r$ and

$$\underline{E}\left[(x - \underline{\Phi})^2\right] = p\,(1-p)\,(\triangle x)^2 .$$

The variance conditioned that $p = \overline{p}$ is higher for the risky project than for the safer project. The third moment of $f\,(x)$ is

$$\begin{aligned} E\left[(x-\Phi)^3\right] &= p\,(H-\Phi)^3 + (1-\overline{p})\,(L-\Phi)^3 \\ &= p\,(H-(p\triangle x + L))^3 + (1-p)\,(L-(p\triangle x + L))^3 \\ &= p\,(\triangle x - p\triangle x)^3 - (1-p)\,(p\triangle x)^3 \\ &= p\,(1-p)\,(1-2p)\,\triangle x^3, \end{aligned}$$

which decreases in p. That is, the risky project is more positively skewed than the safer project.

With no surprise, the safer project has a higher unconditional probability of obtaining H, because

$$g\,(x) = \begin{cases} \theta_0\overline{p} + (1-\theta_0)\,(\overline{p} - \triangle p) & \text{if } x = H \\ \theta_0\,(1-\overline{p}) + (1-\theta_0)\,(1-\overline{p}+\triangle p) & \text{if } x = L \\ 0 & \text{otherwise} \end{cases} .$$

We are now going to examine the impact of $\overline{p}$ on the learning process and the option value. Due to equation (3),

$$\theta_1\,(H) = \left[1 + \frac{1-\theta_0}{\theta_0}\frac{\overline{p}-\triangle p}{\overline{p}}\right]^{-1},$$

and

$$\theta_1\,(L) = \left[1 + \frac{1-\theta_0}{\theta_0}\frac{1-\overline{p}+\triangle p}{1-\overline{p}}\right]^{-1}.$$

Note that $\theta_1\,(H)$ and $\theta_1\,(L)$ are both decreasing in $\overline{p}$. This result occurs because unlike the previous example the difference of $\overline{p}$ and $\underline{p}$ is now the same for both risky and safer projects. Since $\overline{p}$ is smaller for the risky project, then the ratio $\overline{p}/\underline{p}$ now becomes bigger for the risky project. As a consequence, good news (H) is more blessing for the risky project than the safer project. Similarly to the previous example, bad news (L) is more cursing for the safer project than the risky project, since the ratio $\left(1-\underline{p}\right)/\left(1-\overline{p}\right)$ is bigger for the safer project. Figure 2 summarizes this learning process.

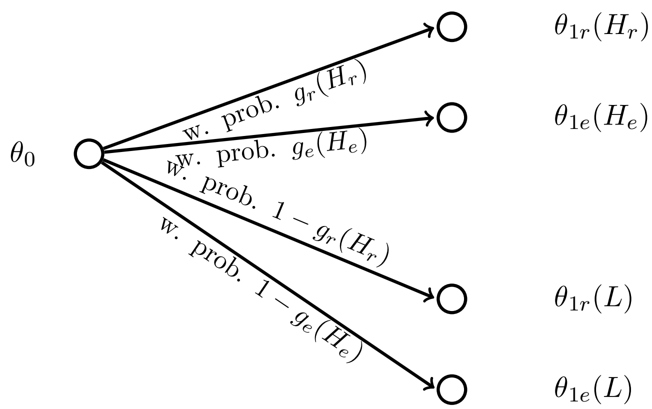

Figure 2: Learning when Projects are Ordered by Skewness Only

The former effect explains why in the previous example $\theta_1(H)$ does not increase when the second moment of f decreases and the first period output becomes more informative. In the previous example, increasing the second moment also increases the third moment. The current example shows that a higher third moment tends to increase $\theta_1(H)$. The negative effect of the second moment was offset by the positive effect of the third moment in the previous example.

Given the distributional assumption described above, we are now going to study the impact of $\overline{p}$ on the option value. To prepare for presenting and proving the following proposition, we redefine $R(i)$, $i=1,2,3,4$, implicitly by the following equations:

$$\begin{aligned}
\theta_{1e}(L_e)\,\overline{\Phi}+(1-\theta_{1e}(L_e))\,\underline{\Phi} &= R(1)\\
\theta_{1r}(L_r)\,\overline{\Phi}+(1-\theta_{1r}(L_r))\,\underline{\Phi} &= R(2)\\
\theta_{1e}(H_e)\,\overline{\Phi}+(1-\theta_{1e}(H_e))\,\underline{\Phi} &= R(3) \text{ and}\\
\theta_{1r}(H_r)\,\overline{\Phi}+(1-\theta_{1r}(H_r))\,\underline{\Phi} &= R(4).
\end{aligned}$$

Note that because $\theta_{1e}(L)<\theta_{1r}(L)$ and $\theta_{1e}(H_e)<\theta_{1r}(H_r)$, $R(1)<R(2)<R(3)<R(4)$.

Proposition 2 *There exists $R(2) < \widehat{R} < R(3)$ such that $\pi_r \geq \pi_e$ if $R \geq \widehat{R}$ and $\pi_r \leq \pi_e$ if $R < \widehat{R}$, then $\pi_r = \pi_e$. Otherwise, $\pi_r \leq \pi_e$.*

Proof. We separately prove this proposition for four different cases depending on R. First, suppose $R < R(1)$. Then, the option value is, for $j = e, r$

$$\pi_j = \theta_0 \overline{\Phi} + (1 - \theta_0) \underline{\Phi} - R,$$

which does not depend on $\overline{p}$. As a result, $\pi_r = \pi_e$.

Second, suppose $R(1) \leq R \leq R(2)$. Then, the risky project should be continued in the second period regardless of the realization in the first period. Therefore,

$$\pi_r = \theta_0 \overline{\Phi} + (1 - \theta_0) \underline{\Phi} - R.$$

The safer project should be continued only if $x = H_e$ in the first period. Therefore,

$$\begin{aligned}
\pi_e &= g_e(H_e) \left(\theta_{1e}(H_e) \overline{\Phi} + (1 - \theta_{1e}(H_e)) \underline{\Phi} - R \right) \\
&= \theta_0 \overline{\Phi} + (1 - \theta_0) \underline{\Phi} - R - \left(g_e(L) \left(\theta_{1e}(L) \overline{\Phi} + (1 - \theta_{1e}(L)) \underline{\Phi} - R \right) \right) \\
&> \theta_0 \overline{\Phi} + (1 - \theta_0) \underline{\Phi} - R = \pi_r.
\end{aligned}$$

The last inequality follows because $g_e(L) > 0$ and $g_e(L) \left(\theta_{1e}(L) \overline{\Phi} + (1 - \theta_{1j}(L)) \underline{\Phi} - R \right) < 0$ due to $R(1) \leq R$.

Third, suppose $R(3) \leq R < R(4)$. Then, $\pi_e = 0$ and

$$\pi_r = g_r(H_r) \left(\theta_{1r}(H_r) \overline{\Phi} + (1 - \theta_{1r}(H_r)) \underline{\Phi} - R \right) > 0.$$

Therefore, $\pi_e < \pi_r$.

Fourth, suppose $R(2) \leq R < R(3)$. Then, for $j = r, e$

$$\begin{aligned}
\pi_j &= g_j(H_j) \left(\theta_{1j}(H_j) \overline{\Phi} + (1 - \theta_{1j}(H_j)) \underline{\Phi} - R \right) \\
&= \theta_0 \overline{p}_j \overline{\Phi} + (1 - \theta_0) \left(\overline{p}_j - \triangle p \right) \underline{\Phi} - \left(\theta_0 \overline{p}_j + (1 - \theta_0) \left(\overline{p}_j - \triangle p \right) \right) R \\
&= \theta_0 \overline{p}_j \left(\overline{\Phi} - R \right) + (1 - \theta_0) \left(\overline{p}_j - \triangle p \right) \left(\underline{\Phi} - R \right). \qquad (4)
\end{aligned}$$

Taking the cross derivative gives

$$\frac{d^2 \pi_j}{d\overline{p}_j dR} = -1.$$

Therefore, as R increases over $R(2) \leq R < R(3)$, $\pi_r - \pi_e$ should increase. Because $\pi_r - \pi_e$ is negative at $R = R(2)$ and positive at $R = R(3)$, by continuity such R that $\pi_r = \pi_e$ should exist between $R(2)$ and $R(3)$.

Lastly, suppose $R \geq R(4)$. Then, the option value is zero regardless of $\overline{p}$. As a result, $\pi_r = \pi_e$. ■

An interesting aspect of this proposition is that when R increases between $R(2) \leq R < R(3)$, π_r decreases slowly than π_e. Intuition behind this observation is found by rewriting equation (4) as follows.

$$\pi_j = \theta_0 \overline{p}_j \left(\overline{\Phi} - R\right) + (1 - \theta_0) \underline{p}_j \left(\underline{\Phi} - R\right).$$

Roughly speaking, a benefit of the optimal strategy for $R(2) \leq R < R(3)$ is to participate in the upside, $\overline{\Phi} - R$, and a cost of the optimal strategy is a possibility of false positive, that is to lose money, $(\underline{\Phi} - R)$. As R increases, the benefit becomes smaller and the cost becomes relatively more important. The risky project has a lower probability of continuing and therefore it lowers the possibility of false positive. As a consequence, when R increases, the risky project becomes more attractive relative to the safer project.

5 Continuous Probability Distribution

In the previous section, we assumed that return takes only two points, H or L. In this section, we examine a case in which x is continuously distributed and study an impact of increasing the second and the third moments of $f(x)$ simultaneously on the option value.

We assume that $f(x)$ is log-normal and, to be concrete,

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{\left(\ln x - \left(\phi - \sigma^2/2\right)\right)^2}{2\sigma^2}\right) \text{ for } x > 0 \text{ and zero otherwise.}$$

The first, second and third central moments of this distribution e^{ϕ}, $\left(e^{\sigma^2} - 1\right) e^{2\phi}$ and $\left(e^{\sigma^2} + 2\right)\left(e^{\sigma^2} - 1\right)^2 e^{3\phi}$. Therefore, both the second and third central moments are increasing in σ^2. We call a project riskier if σ^2 is higher than the other. Similar to the setups we examined so far, the decision maker knows the value of σ^2 but only knows that the mean e^{ϕ} is equal to $\overline{\Phi}$ with probability θ_0 and $\underline{\Phi}$ with probability $1 - \theta_0$. We define $\overline{\phi} = \ln\left(\overline{\Phi}\right)$ and $\underline{\phi} = \ln\left(\underline{\Phi}\right)$. In all the figures presented in this section, we assume that $\overline{\phi} = 2$, $\underline{\phi} = 1.5$, and $\theta_0 = 0.2$.

If an experimental project returns x in the first period, then the posterior is

$$\theta_1(x) = \left(1 + \frac{(1-\theta_0)}{\theta_0} \exp\left(\frac{(\overline{\phi} - \underline{\phi})\left(\frac{\overline{\phi}+\underline{\phi}-\sigma^2}{2} - \ln x\right)}{\sigma^2}\right)\right)^{-1}.$$

A few observations are worthwhile to note. First, $\theta_1(x)$ is increasing in x (the monotone likelihood condition is satisfied) and greater than the prior θ_0 if and only if $x > \exp\left(\left(\overline{\phi} + \underline{\phi} - \sigma^2\right)/2\right)$. Second, differentiating the posterior with respect to σ^2, we have

$$\begin{aligned} & \frac{d\theta_1(x)}{d\sigma^2} \\ = \; & \frac{(1-\theta_0)}{\theta_0} \frac{\left(\overline{\phi} - \underline{\phi}\right)}{\theta_1(x)^2} \exp\left(\frac{\left(\overline{\phi} - \underline{\phi}\right)\left(\frac{\overline{\phi}+\underline{\phi}-2\sigma^2}{2} - \ln x\right)}{\sigma^2}\right) \frac{\sigma^2 + \left(\frac{\overline{\phi}+\underline{\phi}-2\sigma^2}{2} - \ln x\right)}{\sigma^4}. \end{aligned}$$

Thus,

$$\frac{d\theta_1(x)}{d\sigma^2} = \begin{cases} \geq 0, \; x \leq \exp\left(\frac{\overline{\phi}+\underline{\phi}-\sigma^2}{2} + \sigma^2\right) \\ < 0, \; x > \exp\left(\frac{\overline{\phi}+\underline{\phi}-\sigma^2}{2} + \sigma^2\right) \end{cases},$$

that is, originally very good news – $x > \exp\left(\left(\overline{\phi} + \underline{\phi} - \sigma^2\right)/2 + \sigma^2\right) > \exp\left(\left(\overline{\phi} + \underline{\phi} - \sigma^2\right)/2\right)$ – has a bigger positive impact on the posterior, and originally moderately good news and bad news – $x < \exp\left(\left(\overline{\phi} + \underline{\phi} - \sigma^2\right)/2 + \sigma^2\right)$ – has a bigger negative impact on the posterior if the project is safer. This effect arises because a smaller noise makes the first period return more informative. Third, the threshold x such that $\theta_0 = \theta_1(x)$ is decreasing in σ^2, that is, the risky project takes lower returns as a good signal than the safer project does. Note that this effect is absent if we assume $f(x)$ is symmetric. These characteristics are summarized in Figure 3.

The expected return from continuing the experimental project conditioned on x is

$$\theta_1(x)\, e^{\overline{\phi}} + (1 - \theta_1(x))\, e^{\underline{\phi}}.$$

Hence, the threshold posterior and threshold return such that the decision maker is indifferent between continuing and discontinuing the experimental project are

$$\widehat{\theta}_1 = \frac{R - e^{\underline{\phi}}}{e^{\overline{\phi}} - e^{\underline{\phi}}}$$

and

$$\widehat{x} = \exp\left(\frac{\overline{\phi}+\underline{\phi}-\sigma^2}{2} - \frac{\sigma^2}{\left(\overline{\phi}-\underline{\phi}\right)}\ln\left(\frac{e^{\overline{\phi}}-R}{R-e^{\underline{\phi}}}\frac{\theta_0}{1-\theta_0}\right)\right).$$

The threshold $\widehat{x}$ is obviously increasing in R. Differentiating $\widehat{x}$ with respect to σ^2, we have

$$\frac{d\widehat{x}}{d\sigma^2} = -\frac{1}{\left(\overline{\phi}-\underline{\phi}\right)}\left(\ln\left(\frac{e^{\overline{\phi}}-R}{R-e^{\underline{\phi}}}\frac{\theta_0}{1-\theta_0}\right) + \sigma^2\frac{\frac{1}{2}e^{\overline{\phi}}-\frac{1}{2}e^{\underline{\phi}}}{\left(e^{\overline{\phi}}-R\right)\left(R-e^{\underline{\phi}}\right)}R\right)\widehat{x},$$

which is negative if $\ln\left(\frac{e^{\overline{\mu}}-R}{R-e^{\underline{\mu}}}\frac{\theta_0}{1-\theta_0}\right) > 0$. Figure 4 is written with the assumption that $\ln\left(\frac{e^{\overline{\mu}}-R}{R-e^{\underline{\mu}}}\frac{\theta_0}{1-\theta_0}\right)$ is sufficiently high such that $\widehat{x}$ is decreasing in σ.

The probability with that the experimental project is continued is $1 - G\left(\widehat{x}\right)$ and Figure 5 describes an example of the impact of R and σ^2 on the continuation probability. For all lines, the continuation probability is higher for the risky project than for the safer project. Given that Figure 5 is drawn under the same numerical assumptions as Figure 4, this is surprising. The threshold $\widehat{x}$ increases in σ^2, but a higher σ^2 creates a fatter right side tail in f. As a consequence, the continuation probability is increasing in σ^2 under our numerical assumption.

The option value is

$$\begin{aligned}\pi &= \theta_0\int_{\widehat{x}}^{\infty}\frac{\exp\left(-\left(\ln x-\overline{\phi}+\sigma^2\right)^2/2\sigma^2\right)}{x\sigma\sqrt{2\pi}}\left(\theta_1(x)\,e^{\overline{\phi}}+(1-\theta_1(x))\,e^{\underline{\phi}}-R\right)dx \\ &\quad +(1-\theta_0)\int_{\widehat{x}}^{\infty}\frac{\exp\left(-\left(\ln x-\underline{\phi}+\sigma^2\right)^2/2\sigma^2\right)}{x\sigma\sqrt{2\pi}}\left(\theta_1(x)\,e^{\overline{\phi}}+(1-\theta_1(x))\,e^{\underline{\phi}}-R\right)dx.\end{aligned}$$

Figure 7 presents the impact of σ^2 and R on this option value. Note that the option value is already positive by the definition in equation (4). It is decreasing in R because continuing the experimental project becomes more attractive when the outside option is low. Finally, the option value is decreasing in σ^2 being consistent with the standard argument.

6 Conclusion

We study the impact of skewness on the value of experimentation when the decision maker can discontinue the experimentation after a bad outcome is observed. As skewness and variance often comove, an important consequence of higher skewness is poorer signal quality of experimental outcomes and thereby a lower option value. The impact of skewness in isolation is characterized as follows. First, a higher skewness makes the decision maker to positively update her belief under a larger set of outcome. Second, a higher skewness reinforces a signal quality of favorable outcome and weakens that of unfavorable outcome. Third, a higher outside option makes a positively skewed project more valuable relative to a negatively skewed project, because false positive becomes more harmful and a positively skewed project has a lower probability of false positive.

These results are derived under the assumption that there are two periods and one decision maker in the economy. It is interesting to extend our model to more than two periods and to the framework of group learning.

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Appendix

Impact of Riskiness in General Two Output Case

Definition 1 *An experimental project* $X^1 = (H_1, L_1)$ *is riskier than* $X_2 = (H_2, L_2)$ *if* $H_1 \geq H_2$ *and* $L_1 \leq L_2$.

In other words, our riskiness measure is defined as spread in returns.

The following theorem is a direct application of this definition.

Theorem 1 *Let* $\left(\overline{p}^j, \underline{p}^j\right)$ *be project j's* $\overline{p}$ *and* $\underline{p}$, *respectively. If* X^1 *is riskier than* X^2, *then*

$$\frac{\overline{p}^1}{\underline{p}^1} < \frac{\overline{p}^2}{\underline{p}^2}$$

and

$$\frac{1-\underline{p}^1}{1-\overline{p}^1} < \frac{1-\underline{p}^2}{1-\overline{p}^2}.$$

Proof. Let $pH + (1-p)L$ be constant. Consider a change of p, H and L such that a project becomes riskier. Therefore, $\Delta H \geq 0$ and $\Delta L \leq 0$. Taking total difference gives

$$\Delta pH + p\Delta H + \Delta p\Delta H - \Delta pL + (1-p)\Delta L - \Delta p\Delta L = 0.$$

Arranging terms gives

$$\Delta p = -\frac{(\Delta H - \Delta L)\, p + \Delta L}{H + \Delta H - L - \Delta L}. \tag{5}$$

Since $\Delta H - \Delta L \geq 0$ and the denominator of the right hand side of equation (5) are positive, Δp decreases in p. As a result, if $\overline{p} > \underline{p}$ then,

$$\frac{\overline{p} + \Delta\overline{p}}{\underline{p} + \Delta\underline{p}} < \frac{\overline{p}}{\underline{p}}$$

and

$$\frac{1 - \left(\underline{p} + \Delta\underline{p}\right)}{1 - \left(\overline{p} + \Delta\overline{p}\right)} > \frac{1-\underline{p}}{1-\overline{p}}.$$

■

Roughly speaking, this theorem implies that one experiment brings more information if the experimental project is safer. If the return is high, p is more likely to be $\overline{p}$ instead of $\underline{p}$. If the return is low, p is more likely to be $\underline{p}$ instead of $\overline{p}$. However, high return in a riskier project is relatively more likely in a bad project and low return in a riskier project is relatively more likely in a good project. As a consequence, if the experimental project is riskier, a result of experimentation is less helpful in determining the true value of $\overline{p}$.

6.1 Binary Output with Many Periods.

6.1.1 Learning

We now describe how a decision maker updates her assessment of p by observing outcomes of the experimental project. The problem is in a class of the two-armed bandit problem with a single known parameter. One can undertake the same experimental project up to $T+1$ times. We assume that outcomes of each trial are identically distributed. Let $\theta(F,S)$ be posterior probability that $p=\overline{p}$ when the project has yielded zero for F times and Y for S times. Then, applying the Bayes' rule gives

$$\theta(S,F)=\frac{(1-\overline{p})^{F}\theta_0}{(1-\overline{p})^{F}\theta_0+\alpha^{S}(1-\alpha\overline{p})^{F}(1-\theta_0)}.$$

Then, by calculation,

$$\begin{aligned}&\theta(S,F+1)-\theta(S,F)\\ =\ &-\frac{\alpha^{S}(1-\alpha\overline{p})^{F}(1-\theta_0)}{(1-\overline{p})^{F+1}\theta_0+\alpha^{S}(1-\alpha\overline{p})^{F+1}(1-\theta_0)}(\overline{p}-\alpha\overline{p})\theta(S,F)<0.\end{aligned}$$

and

$$\begin{aligned}&\theta(S+1,F)-\theta(S,F)\\ =\ &\frac{\alpha^{S}(1-\alpha\overline{p})^{F}(1-\theta_0)}{\left((1-\overline{p})^{F}\theta_0+\alpha^{S+1}(1-\alpha\overline{p})^{F}(1-\theta_0)\right)}(1-\alpha)\theta(S,F)>0.\end{aligned}$$

That is, the posterior declines when additional failure is observed and increases when additional success is observed.

The following two lemmas describe the basic difference of learning between a risky project and a safer project.

Lemma 1 *If $F\geq 1$, then*

$$\begin{aligned}\frac{d\theta(S,F)}{d\overline{p}}&=-\frac{(1-\alpha)\alpha^{S}F(1-\overline{p})^{F-1}(1-\alpha\overline{p})^{F-1}\theta_0(1-\theta_0)}{\left[(1-\overline{p})^{F}\theta_0+\alpha^{S}(1-\alpha\overline{p})^{F}(1-\theta_0)\right]^2}\\ &=-\frac{(1-\alpha)F}{(1-\overline{p})(1-\alpha\overline{p})}\theta(S,F)(1-\theta(S,F))<0.\end{aligned}$$

Otherwise, that is, $F = 0$*, then*

$$\theta(S,0) = \frac{\theta_0}{\theta_0 + \alpha^S(1-\theta_0)}$$

and therefore the posterior does not depend on the riskiness of the project.

Proof. By algebra. ■

This lemma implies that given the same history, the riskier the project, the higher the posterior. Intuition behind this lemma is as follows. If the project is riskier and fails more likely, when a failure is observed, a decision maker does not severely revise her expectation downwards. As a consequence, for the same history including at least one failure, the posterior is always higher for a risky project than for a safer project. The magnitude of $d\theta(S,F)/d\overline{p}$ is largest when the experimental project is either bad or good equally likely. When only successes have been observed, the relative likelihood of $p = \overline{p}$ against $p = \underline{p}$ is the same for both risky and safer projects, due to the assumption that $\underline{p} = \alpha\overline{p}$. And therefore the posterior does not depend on the riskiness of project.

This lemma also implies that the range of the posterior is smaller for a risky project than a safer project. The upper bound of the posterior generated by observing only successful outcomes is the same for both types of projects. Nevertheless, the lower bound is bigger for a risky project.

Lemma 2 *If* $1-\overline{p} > \underline{p} \Leftrightarrow \overline{p} < (1+\alpha)^{-1}$*, then*

$$\theta(S+1,F+1) - \theta(S,F) > 0.$$

Otherwise,

$$\theta(S+1,F+1) - \theta(S,F) \leq 0.$$

Proof. By algebra. ■

That is, if a failure of a high quality project is more likely than a success of a low quality project, a decrease in the posterior due to one failure is smaller than an increase in the posterior due to one success. Note that this assumption is more likely to be satisfied if the project is riskier. Therefore, this lemma implies that a success in a risky project has a bigger impact on the posterior than a failure, in absolute terms.

We now study the problem of a single decision maker, which can undertake a project for $T+1$ times. Once the decision maker stops experimenting, no new information will be generated and therefore without loss of generality, we assume that once she stops, she does not resume experimenting.

Let $a(S,F) \in \{1,0\}$ for $S,F=\{0,1,2,...,T\}$ be the decision rule such that $a=1$ implies stopping and $a=0$ implies continuing.

If we define Ω_t as the discounted sum of expected revenues if she stops after period $t \in \{0,1,2,...,T-1\}$, her expected future payoff is

$$\Omega_t = \sum_{\tau=t+1}^{T} \delta^{\tau-t} R.$$

Let $V(S,F)$ be the discounted sum of future revenues after S successes and F failures if the optimal action is taken in the future. Then, we can define this value function as follows:

$$\begin{aligned} & V(S,F) \\ = \ & \max\left[\delta\pi(S,F)\left[Y+(V(S+1,F))\right]+\delta(1-\pi(S,F))V(S,F+1),\Omega_{S+F}\right], \end{aligned}$$

where

$$\begin{aligned} \pi(S,F) &= \theta(S,F)\overline{p}+(1-\theta(S,F))\alpha\overline{p} \\ &= \left[\alpha+\theta(S,F)(1-\alpha)\right]\overline{p}. \end{aligned}$$

We can solve this problem backwards. If $\theta(S',T-S') \geq \widehat{\theta}$ for $S'=\{0,...,T\}$, then the optimal solution is to undertake the experimental project in all periods. If $\theta(S',T-S') \leq \widehat{\theta}$ for $S'=\{0,...,T\}$, then the optimal solution is to undertake the traditional project in all periods. Suppose there exists $S' \in \{1,...,T\}$ such that

$$\theta\left(S',T-S'\right) > \widehat{\theta} > \theta\left(S'-1,T-S'+1\right).$$

Note that in the last period $T+1$, it is optimal to undertake the experimental action if and only if $\theta(S,T-S) > \widehat{\theta}$. Then, S' is the minimum number of success after T periods to make the experimental project optimal for the last period. Therefore,

$$V(S,T-S) = \begin{cases} \delta\pi(S,T-S)Y & \text{if } S \geq S' \\ \delta R & \text{otherwise} \end{cases}.$$

Once we find S', then we determine the optimal action in period T. The optimal action in earlier periods can be determined backwards.

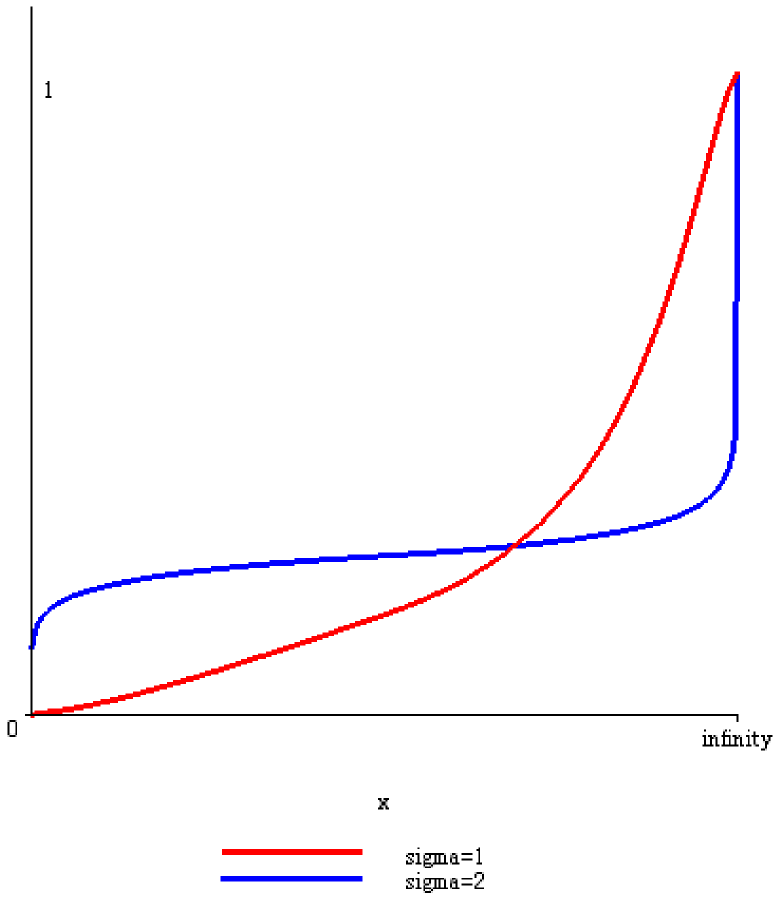

Figure 3: Posterior – $\theta_1(x)$

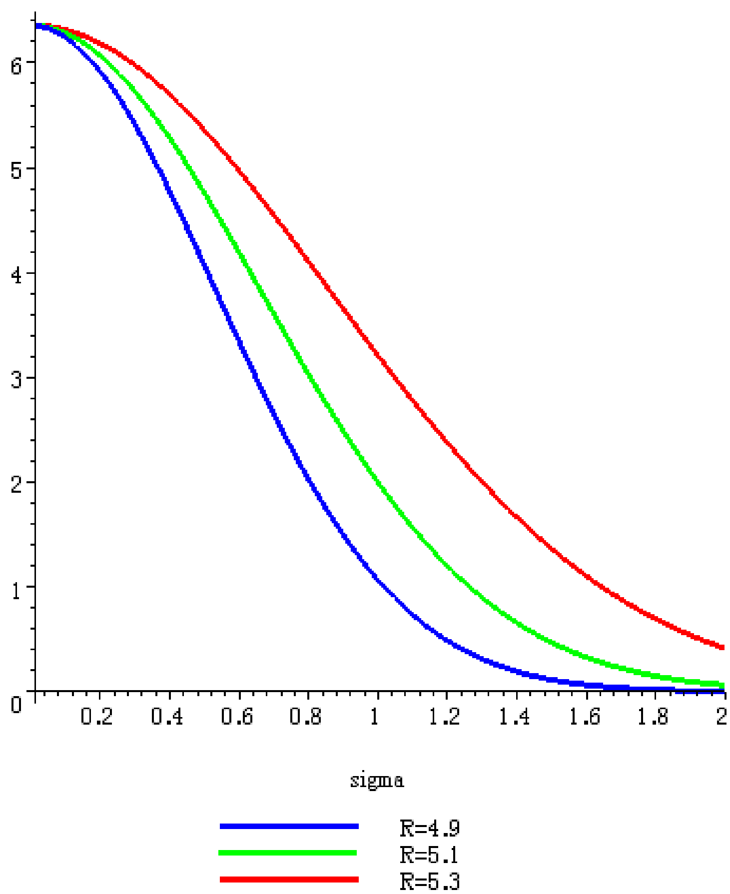

Figure 4: Threshold – $\widehat{(}x)$

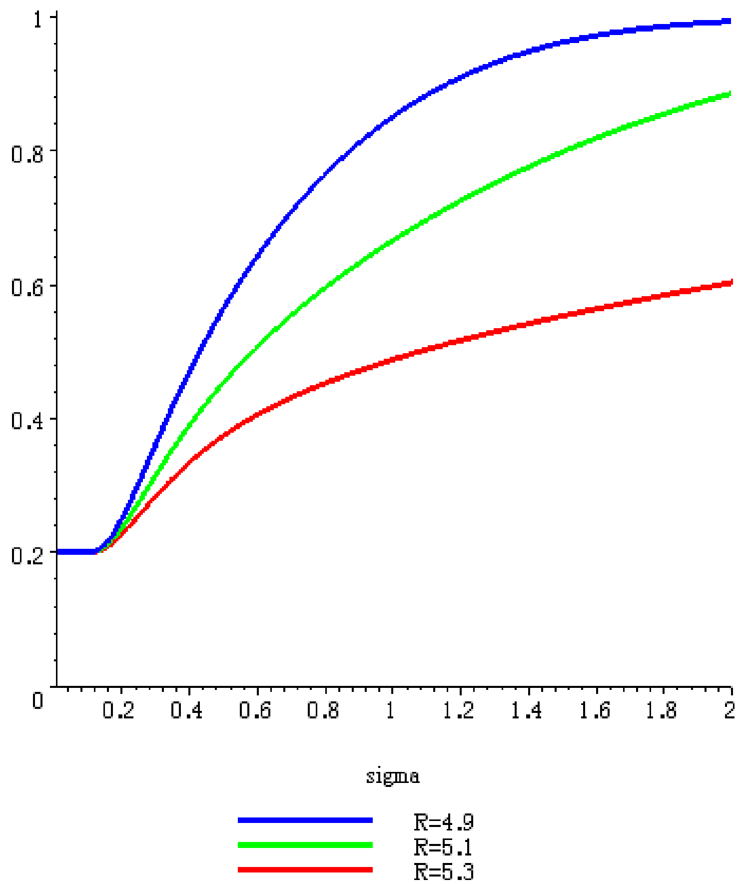

Figure 5: Probability of Continuation – $(1 - G(\widehat{x}))$

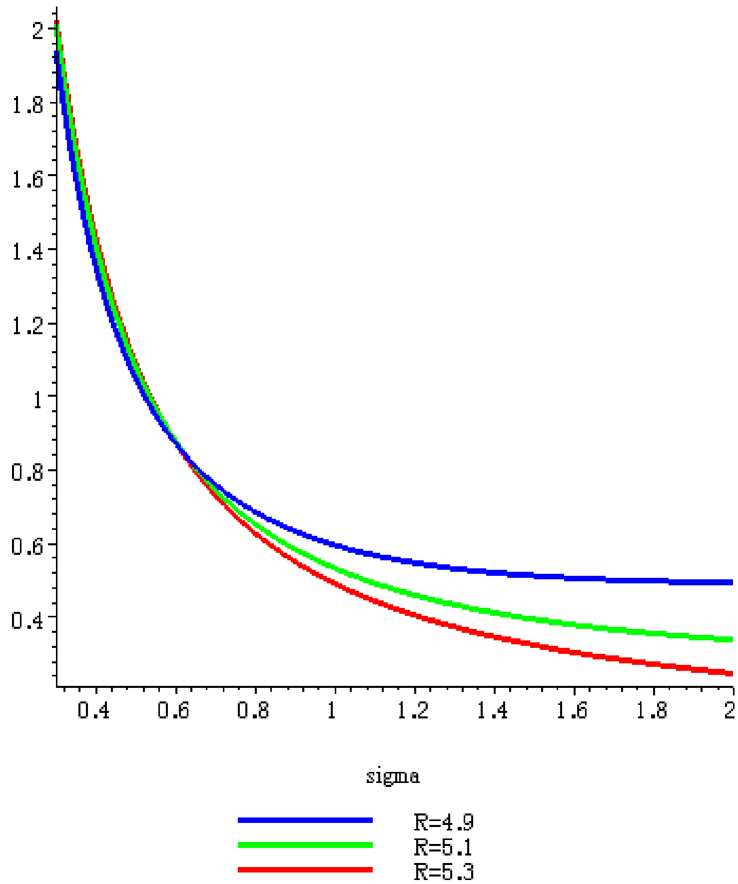

Figure 6: Conditional Option Value – $(1 - G(\widehat{x}))$

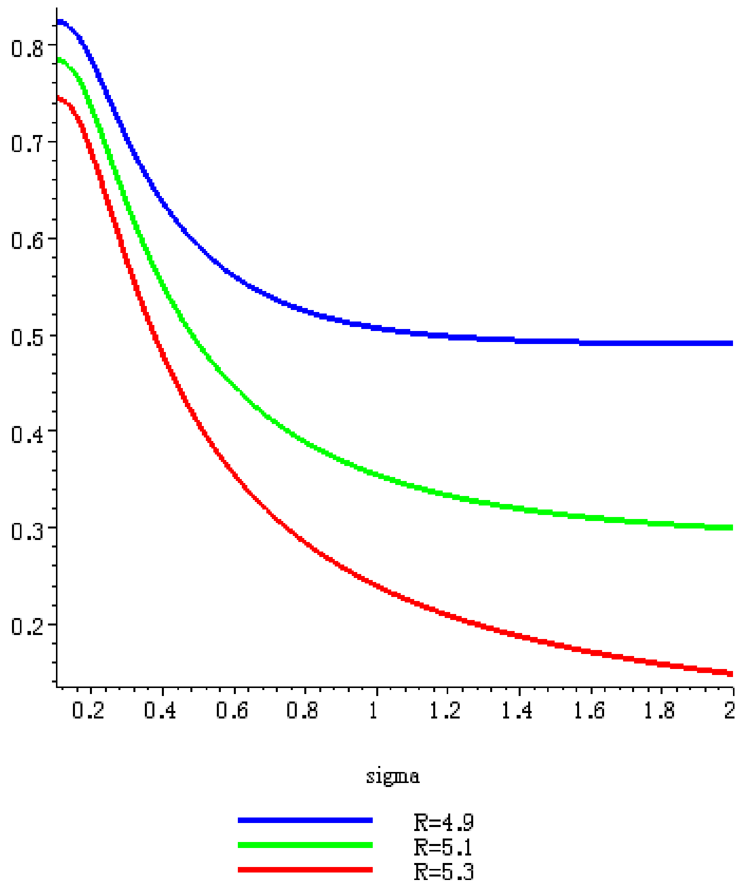

Figure 7: Option Value – π