

Consumer Credit Regulation and Lender Market Power*

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Abstract

We investigate the welfare consequences of consumer credit regulation in a dynamic, heterogeneous-agent model with endogenous lender market power. We incorporate a decentralized credit market with search and incomplete information frictions into an off-the-shelf Eaton-Gersowitz model of consumer credit and default. Lenders post credit offers and borrowers apply for credit. Some borrowers are informed and direct their application toward the lowest offers while others are uninformed and apply randomly. Equilibrium features price dispersion — controlling for a borrower’s default risk, there exists both high- and low-cost lending. Importantly, the distribution of loan prices and the extent of lenders’ market power are disciplined by borrowers’ outside options. We calibrate the model to match characteristics of the unsecured consumer credit market, including high-cost options such as payday loans. We use the calibrated model to evaluate interest rate ceilings. In a model with a competitive financial market, ceilings can only harm borrower welfare. In contrast, with lender market power interest rate ceilings can raise borrower welfare by reducing markups, but that requires households have some degree of financial illiteracy (lack of information about interest rates).

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1 Introduction

Alternative financial institutions (AFIs) provide high-cost loans to households in the US. The most notable are payday lenders that offer short-term loans at extremely high interest rates (with an average of around 350 percent annualized). These rates have been cited frequently as justification for regulatory actions, such as interest rate ceilings or loan-size limits, aimed at curbing the activities of AFIs that are often deemed as harmful to borrowers. These regulations are ubiquitous; a quarter of states ban payday lending or other AFIs completely, while others impose tight regulations. In this paper, we explore the aggregate and distributional welfare implications of these credit regulations.

In a competitive model of unsecured credit and default-based pricing (such as those used in [Chatterjee, Corbae, and Rios-Rull, 2008](#), or [Athreya, Tam, and Young, 2012](#)), restricting loan contracts cannot help consumers because they simply shrink budget sets; either the household is unaffected by the ceiling because their optimal borrowing choice leads to small enough default risk, or they are forced to reduce borrowing, and therefore current consumption. As a result, no welfare gains are available.¹ If we want to understand the argument for interest rate ceilings, we must therefore extend the model to allow for the possibility that loan terms are set inefficiently.

In this paper, we do so by departing from the assumption of perfect competition in order to allow lenders to possess market power. We incorporate search and information frictions following [Lester \(2011\)](#) and [Bethune, Choi, and Wright \(2020\)](#) into the workhorse model of unsecured credit and bankruptcy. Lenders post credit offers with commitment, and borrowers search for offers. Some households observe all posted terms and direct their search (we call these households informed), while others search randomly (uninformed). In equilibrium, two types of intermediaries emerge: low-cost and high-cost. Low-cost intermediaries compete for informed borrowers while high-cost intermediaries post terms to extract surplus from the uninformed. As a result, only uninformed households use the high-cost market, while both types use the low-cost market. Further, since wealth and income are publicly observable, the equilibrium loan terms and prevalence of low- and high-cost lenders depends on a household's state. Hence, the model

¹If the risk-free rate is endogenous, reduced borrowing leads to higher capital and therefore can potentially deliver welfare gains through higher wages. However, [Chatterjee et al. \(2008\)](#) show that the effect of bankruptcy regulations on the risk-free rate is small, and therefore this channel is unlikely to deliver significant gains.

features an endogenous market composition channel in which, e.g., low-wealth households are targeted more by high-cost lenders relative to high-wealth households, a pattern that holds in the calibrated economy. The equilibrium is constrained inefficient since there is over-entry of high-cost lenders that exploit their market power.² A planner who only has the instrument of a blunt interest rate ceiling may use it to indirectly tax high-cost lenders, but potentially at the risk of distorting other low-cost markets.³

Our model captures two features of public discourse about the need to regulate unsecured debt markets. First, interest rates are set using some degree of monopoly power, which is a common thread in political discussions of lending markets (and notably applies not only to AFIs but also to large credit card lenders like Bank of America, CitiBank, and Chase). Second, households are “financially unsophisticated” in a stylized sense; the uninformed households lack knowledge about the options that are available in the credit market, and, as will become clear, even the low-cost market exploits these households to some degree.

By combining these features, our environment produces an equilibrium pattern of trade that is consistent with a broad set of findings from the empirical literature on consumer credit: (i) information disclosure policies lead to a decrease in borrowing from payday lenders as in [Bertrand and Morse \(2011\)](#) and [Wang and Burke \(2022\)](#); (ii) low financial literacy or decision-making ability is strongly correlated with the use of high-cost loans as in [Lusardi and de Bassa Scheresberg \(2013\)](#) and [Carvalho, Olafsson, and Silverman \(2024\)](#); and (iii) a significant share of the observed heterogeneity in interest rates paid on household credit can be explained by measures of shopping intensity as in [Stango and Zinman \(2015\)](#) for credit cards or [Coen, Kashyap, and Rostom \(2023\)](#) for mortgages. Furthermore, our model is consistent with evidence documenting a “pecking order” of consumer credit products in which borrowers tend to take payday loans when they face restricted access to credit cards, either as a result of utilization rates or application denials, as in [Bhutta, Skiba, and Tobacman \(2015\)](#).

A feature of our equilibrium is that high-cost borrowing is more prevalent among lower wealth and income households since high-cost lenders target more offers to this part of the

²The case in which all borrowers are informed leads to the competitive search outcome that guarantees efficient entry of intermediaries, as in [Hosios \(1990\)](#).

³[Saldain \(2023\)](#) explores an alternative option in which households borrow excessively due to self-control problems and finds that borrowing constraints are already tight enough, due to high default rates, that households do not benefit from further regulation.

distribution because the potential surplus to extract is highest. Hence, as households borrow more over time, the model predicts that there is a switch from low-cost to high-cost borrowing.⁴ Finally, our model is consistent with the idea that operational and default costs justify a portion of the high interest rates observed in payday lending, as shown in [Flannery and Samolyk \(2005\)](#). In our model, lenders are *ex ante* identical, including in fixed costs. However, the probability that high-cost terms of trade are accepted is lower relative to the low-cost market. Hence, costs per loan are larger in the high-cost market, which in part explains the higher interest rates.

We calibrate the model to match the features of both the traditional unsecured credit market (credit card lenders) and the market for AFIs (payday lenders). Introducing lender market power helps overcome a challenge in calibrating standard competitive models: to generate interest rates high enough to be affected by regulatory ceilings, the default rate must be counterfactually high. In turn, the level of debt and borrowing must also be counterfactually low, as agents are unwilling to pay very high rates to borrow (or, if they are willing to pay those rates, wealth accumulation will be too small). The only way for these models to generate the empirical regularities in credit markets is to somehow exogenously impose the need to use the high-cost lender. We show that our model can generate interest rate spreads between high- and low-cost lenders in line with the data while remaining consistent with default rates, levels of borrowing, and participation in the high-cost AFI market. To do so, our model only requires that 4 percent of households are uninformed.

Our primary experiment is to study the aggregate and distributional effects of interest rate ceilings. We find that interest rate ceilings are welfare improving, with a welfare-maximizing ceiling of 27 percent annual percentage rate (APR). Despite the fact that there are only 4 percent of uninformed agents in the calibration, restricting high-cost lenders' rent-seeking behavior implies ceilings as low as a 20 percent APR increase aggregate welfare, with gains on average around 0.03 percent of annual consumption. Furthermore, the optimal ceiling is Pareto-improving over the calibrated equilibrium, improving the welfare of the poorest and least wealthy households by 0.32 percent of annual consumption. The reason blunt interest rate ceilings are effective tools in the model relies on two aspects of the calibrated economy. The first is that there is little overlap

⁴This pattern of lower wealth/income and higher high-cost borrowing, despite the availability of cheaper alternatives, has also been documented for the mortgage market in the UK by [Coen et al. \(2023\)](#).

in the equilibrium distribution of interest rates charged between the low- and high-cost lenders, consistent with empirical evidence.⁵ The second is that while there are potentially both positive and negative spillover effects, we find quantitatively large positive spillovers resulting from improved market composition – access to low-cost credit increases for the uninformed. Notably, even *informed* borrowers gain from restricting the high-cost market because they anticipate using it in the future. The largest gains are accumulated by the lowest wealth and income borrowers, who face the largest probability of borrowing of a high-cost lender and the most extractive terms of trade.

In sum, if all lenders *ex ante* compete in price to attract borrowers, then interest rate ceilings only harm welfare. However, if even a few borrowers are uninformed and, as a result, some lenders can post terms to extract their rent, then blunt interest ceilings can lead to meaningful welfare gains and Pareto improvements. Indeed, in Section 7 we show that the gains in welfare from implementing the optimal interest rate ceiling nearly attain the gains from making all borrowers informed.

Related literature Our work contributes to several strands of the literature on unsecured credit. First, there is a set of papers that departs from competitive pricing by introducing search frictions in the credit market.⁶ For instance, [Nosal and Drozd \(2008\)](#) and [Raveendranathan \(2020\)](#) introduce random search, while [Herkenhoff \(2019\)](#) and [Braxton, Herkenhoff, and Phillips \(2020\)](#) introduce competitive search.⁷ A second set of papers studies information asymmetries in the unsecured credit market, as in [Athreya et al. \(2012\)](#), [Livshits, MacGee, and Tertilt \(2016\)](#), [Sanchez \(2017\)](#), [Exler, Livshits, MacGee, and Tertilt \(2021\)](#), and [Chatterjee, Corbae, Dempsey, and Rios-Rull \(2023\)](#). We contribute to this literature by combining search and private information frictions that gives rise to an endogenous distribution of lender market power and interest rates.

Our paper is closest to those that study the effect of credit regulation in environments with frictional credit markets. [Cuesta and Sepúlveda \(2021\)](#) and [Galenianos and Gavazza \(2022\)](#) study

⁵For further discussion, see Section 4. The largest interest rate in the low-cost market is 34 percent while the smallest in the high-cost market is 100 percent, roughly consistent with empirical evidence on credit card and payday lender APRs.

⁶For competitive models, see, e.g., [Livshits, MacGee, and Tertilt \(2007\)](#) or [Chatterjee, Corbae, Nakajima, and Rios-Rull \(2007\)](#).

⁷Related, [Herkenhoff and Raveendranathan \(2021\)](#) study the effects of lender entry in an oligopoly model with a finite number of lenders.

static models while ours is dynamic, which allows the distribution of interest rates and market power to depend on the distribution of wealth, and vice versa. [Hatchondo and Martinez \(2017\)](#), [Raveendranathan and Stefanidis \(2024\)](#), and [Galenianos, Law, and Nosal \(2023\)](#) study dynamic models. We depart from these papers by (i) combining both random and competitive search within a dynamic model of default that produces an endogenous market composition of low- and high-cost lenders, (ii) jointly matching the distribution of interest rates for both traditional and high-cost unsecured credit, credit usage and default rates, and debt-to-income ratios, and (iii) considering asymmetric information over the information state of the consumers, which generates additional price dispersion even in the competitive segment of the credit market. These ingredients create a new rationale for interest rate ceilings in relation to the literature: an interest rate ceiling can limit over-entry of high-cost lenders, improving the access of uninformed households to low-cost credit along the extensive and intensive margins.

This paper is also related to the literature on payday lending in two ways. First, our model is consistent with a broad set of findings from the empirical literature on payday lending, such as [Bertrand and Morse \(2011\)](#), [Wang and Burke \(2022\)](#), [Agarwal, Skiba, and Tobacman \(2009\)](#), [Carvalho et al. \(2024\)](#) and [Bhutta et al. \(2015\)](#). Second, it relates to structural papers studying high-cost credit, such as [Skiba and Tobacman \(2008\)](#), [Li and Sun \(2021\)](#), [Allcott, Kim, Taubinsky, and Zinman \(2022\)](#), and [Saldain \(2023\)](#). Our contribution to this literature is to (i) consider market power together with default as a driver of interest rates in high-cost credit markets, and (ii) study regulations when high-cost options are not an assumption but arise endogenously; low- and high-cost options coexist and are jointly affected by regulations.

We also contribute to the search literature that studies intermediate cases between purely random and purely directed search, what is often referred to as “partially directed search.” Our modeling strategy is closest to [Lester \(2011\)](#) and [Bethune et al. \(2020\)](#) in the context of a goods market and to [Rabinovich and Wolthoff \(2022\)](#) in the context of a labor market.⁸ Our paper is the first to study partially directed search in the context of a consumer credit market. Further, we allow searchers (here, the borrowers) to have private information about their information type that, combined with persistent information types, generates a screening problem for low-cost

⁸There are alternative modeling assumptions that nest random and competitive search; see, e.g., [Delacroix and Shi \(2013\)](#), [Menzio \(2017\)](#), [Lentz and Moen \(2017\)](#), [Cheremukhin, Restrepo-Echavarria, and Tutino \(2020\)](#), [Wu \(2024\)](#), or [Shi \(2023\)](#).

lenders.

Finally, we contribute to the literature on financial literacy. Several papers have studied how financial literacy shapes wealth. [Lusardi and Mitchell \(2007\)](#) and [Behrman, Mitchell, Soo, and Bravo \(2012\)](#) study how financial literacy affects wealth accumulation; [van Rooij, Lusardi, and Alessie \(2011\)](#) how it affects participation in the stock market; and [Lusardi, Michaud, and Mitchell \(2017\)](#) how it affects wealth inequality. In addition, other papers have studied how financial literacy correlates with the credit market behavior of households, such as [Gorbachev and Luengo-Prado \(2019\)](#) and [Disney and Gathergood \(2013\)](#). Our paper connects both topics within the financial literacy literature. In this paper, the lack of financial literacy affects the terms of the credit market faced by households and, as a consequence, their capacity to smooth consumption, their precautionary savings, and wealth.

2 Environment

Time is discrete and infinite. There is a large measure of *ex ante* identical lenders and a unit measure of *ex ante* identical households that interact in a frictional market for defaultable, unsecured debt. Lenders are risk-neutral and can borrow at an exogenous rate of return $r > 0$. To lend in any period, they must enter the credit market at a fixed cost $\kappa > 0$. Upon entry, they post terms of trade, with commitment, which consist of the price of a loan q and an amount borrowed $a' < 0$. These terms can be made contingent on the observed state of the borrower, to be specified below.

Households discount the future at rate $\beta \in (0, 1)$ and have preferences over consumption within a period, $u(c)$, with $u'(c) > 0$, $u''(c) < 0$, and $\lim_{c \rightarrow 0} u'(c) = \infty$. They receive a stochastic endowment, y , whose process is given by a Markov transition matrix $\Pi(y'|y)$ and is i.i.d across households. Households can save and borrow using one-period, non-contingent debt. At the beginning of the period, a borrower can default on any outstanding debt. Upon default, debt is cleared, the defaulter spends a period in financial autarky and incurs a one-time utility penalty, $\lambda^i > 0$, that is drawn randomly, i.i.d. through time and across households, from a distribution $G(\lambda)$. A household's financial choice is denoted $k \in \{S, D\}$, to represent solvency and default, respectively.

Saving is frictionless and earns a risk-free return r . In order to borrow, a household must search for a lender in the credit market. When searching, borrowers can be either informed (I) or uninformed (U) about the set of credit offers posted by lenders. Being informed means they observe $h \in [2, \dots, \infty)$ draws from the distribution of offers before choosing where to direct their search. We focus on the limiting case $h \rightarrow \infty$, in which informed borrowers observe every offer and direct their search to the one yielding the highest expected utility.⁹ Uninformed households draw only one offer and so are effectively random searchers. All households have rational expectations about the equilibrium distribution of posted offers.

A household's financial information state is stochastic and follows an exogenous Markov process given by π_{ij} for $i, j \in \{I, U\}$. Households learn their information type simultaneously with their period endowment. Notice that a household's information state is independent of other idiosyncratic or aggregate state variables, so any equilibrium relationship between, e.g., a household's wealth and their information status will be endogenous. In an extension, we relax the exogeneity assumption and allow households to choose their financial information. The complete state of a household is given by $\mathbf{s} \equiv (a, y, k, j)$, which implies that lenders post credit offers $(q(\mathbf{s}), a'(\mathbf{s}))$.

We introduce two assumptions that serve to limit the ability of lenders to extract rents from uninformed borrowers. The first is that a household's information state is private information, while beginning of period wealth, current income, and financial status are observable and common knowledge to all agents. This assumption will generate informational rents to borrowers and induce a screening problem that potentially limits lenders' ability to compete *ex ante* for informed borrowers. How severe the information problem is depends on how many uninformed borrowers arrive in a given low-cost market, which is endogenous as it depends on the entry decisions of lenders and the past borrowing decisions of households. Second, we assume that all borrowers have the ability to initiate bargaining conditional on matching with a lender at a given posted terms of trade. In equilibrium, a lender would never post terms that generate a lower borrower surplus than the bargaining protocol. Hence, the ability to bargain directly limits the

⁹The case with $h = 2$ coincides with the noisy search equilibrium of [Burdett and Judd \(1983\)](#) (see, e.g., the analysis in [Acemoglu and Shimer, 2000](#), or [Bethune et al., 2020](#)). The primary inefficiency has a similar nature to the case with $h \rightarrow \infty$, however with the added complexity that there is now a continuous distribution of offers, with potential mass points, for a given type. We chose instead to keep the analysis simpler, and more intuitive, along this dimension.

amount of rents lenders can extract from the uninformed. How much the lender can extract is then a function of the bargaining weight, and we discipline this value in the calibration.¹⁰

As is standard in the competitive search literature (Wright, Kircher, Julien, and Guerrieri, 2021), a submarket consists of all lenders and borrowers posting and directing their search to the same terms of trade. Further, with heterogeneity, terms of trade and submarkets will differ across households. Within a submarket, borrowers and lenders are paired bilaterally according to a matching function. Let n represent the ratio of the measure of lenders to the measure of borrowers within a submarket (potentially varying across submarkets). Then, $\alpha(n)$ is the probability that a borrower gets matched to a lender and $\alpha(n)/n$ is the probability that a lender gets matched to a borrower. We assume that $\alpha(n)$ is continuous and $\alpha(0) = 0$, $\alpha'(n) > 0$, and $\lim_{n \rightarrow \infty} \alpha(n) = 1$. Unmatched borrowers are only allowed to save during the current period (although in general the optimal choice is to set $a' = 0$ so that they are simply paying off their existing balance). We focus on symmetric strategies for borrowers. Additionally, for simplicity we assume that uninformed borrowers only draw offers from submarkets indexed to their type.

In terms of timing within a period, we assume that agents first learn their information status, $j \in \{I, U\}$, then face their default/repayment decision, then make their consumption and savings decisions, including when informed where to direct their search if borrowing.

3 Equilibrium

We will show the equilibrium set of active submarkets consists of at most two submarkets for every solvent household with observable state (a, y) .¹¹ In one submarket, lenders will post terms of trade to cater to uninformed searchers and, therefore, effectively post the bargaining solution. We label this submarket “high cost.” In the other submarket, lenders will cater to the informed agents and post terms of trade competitively (to maximize informed borrowers’ expected sur-

¹⁰This last assumption also serves a technical purpose. If lenders can extract the entire surplus of uninformed borrowers then there is an indeterminacy in their choice of borrowing. Intuitively, in this case borrowers are indifferent across a menu of debt contracts that all deliver them zero surplus. Allowing for borrowers to have some positive bargaining power breaks this indeterminacy.

¹¹Hence, submarkets, and thereby prices, will differ both across households of different types (a, y) , and, within households of a given type. Notice that lenders can perfectly discriminate borrowers based on their observable state, (a, y) and solvency status, which implies that, out of equilibrium, if a borrower of type $(\hat{a}, \hat{y}, \hat{k})$ searches in submarket with terms posted for (a, y, k) , then lenders can commit to refuse trade.

plus). However, their ability to compete for informed agents will be limited by the presence of uninformed – but lucky – agents that randomly draw the competitive terms of trade. The presence of these uninformed borrowers induces a screening problem. We label this submarket “low cost.”¹² Finally, in equilibrium lenders will be indifferent to posting terms across all active submarkets, guaranteed by a free-entry condition.

We proceed by first defining the household’s value functions and the trade surpluses of lenders and borrowers. We then use these to characterize the terms of trade in the high- and low-cost submarkets. Finally, we characterize the equilibrium measures of high- and low-cost lenders.

Value functions and trade surpluses At the beginning of the period, a household with wealth a , income y , and information state j faces a default decision. Their lifetime utility is given by

$$v(a, y, j) = \max \left\{ v^d(a, y, j), v^s(a, y, j) \right\}, \quad (1)$$

where $v^d(a, y, j)$ represents the lifetime value of defaulting, given by

$$v^d(a, y, j) = u(y) - \lambda + \beta \mathbb{E}_{y', j' | y, j} [v(0, y', j')]. \quad (2)$$

, and $v^s(a, y, j)$ represents the lifetime value of remaining solvent, given by

$$v^s(a, y, j) = \alpha_h v^{s,h}(a, y, j) + \alpha_\ell v^{s,\ell}(a, y, j) + (1 - \alpha_h - \alpha_\ell) v^{s,n}(a, y, j). \quad (3)$$

In (2), defaulters spend a period in financial autarky, consume their endowment y , incur the utility cost of defaulting of λ , and start the next period with zero net assets.¹³ In (3), a solvent household can be in one of three states: borrowing from a high-cost lender with lifetime expected value of $v^{s,h}(a, y, j)$, borrowing from a low-cost lender with value $v^{s,\ell}(a, y, j)$, or saving with value

¹²Given constant returns to scale matching, a lender posting the low-cost terms would never deviate to terms other than posted by high-cost lenders as they still only attract uninformed random searchers but receive a lower surplus from trade.

¹³Default is therefore to be interpreted as Chapter 7 bankruptcy, in which all unsecured debt is eliminated and lenders are prohibited from garnishing any future labor income.

$v^{s,m}(a, y, j)$, defined by

$$v^{s,m}(a, y) = \max_{c, a' \geq 0} \left\{ u(c) + \beta \mathbb{E}_{y', j' | y, j} [v(a', y', j')] \right\} \quad (4)$$

$$s.t. \quad c = a + y - \frac{1}{1+r} a'.$$

The weights (α_h, α_ℓ) in (3) represent the probability a household of type (a, y, j) borrows in the high- and low-cost market, respectively. They are endogenous and the result of a household's state, their decisions (e.g. whether to save or borrow, or where to search), and the equilibrium distribution of lenders. For now we take these as given and characterize them explicitly later.

The value of borrowing in either submarket, $v^{s,m}(a, y, j)$ for $m \in \{h, \ell\}$, depends on the equilibrium terms of trade in that market, (q_m, a'_m) . For a given terms of trade, the value of borrowing in either submarket $m \in \{h, \ell\}$ is

$$v^{s,m}(a, y, j) \equiv u(c) + \beta \mathbb{E}_{y', j' | y, j} [v(a'_m, y', j')] \quad (5)$$

$$s.t. \quad c = a + y - q_m a'_m.$$

Using the value functions above, we can define a household's surplus from borrowing at terms (q_m, a'_m) for $m \in \{h, \ell\}$, as

$$\mathcal{S}^B(q_m, a'_m; a, y, j) \equiv v^{s,m}(a, y, j) - v^{s,n}(a, y, j) \quad (6)$$

In (6), the borrower's surplus is the difference between their lifetime utility of borrowing in market m at terms (q_m, a'_m) and the value of saving.

If a borrower opts to bargain, we assume the terms of trade are determined by the [Kalai \(1977\)](#) proportional bargaining solution.¹⁴ Let $\theta \in [0, 1]$ represent the borrower's share of the surplus

¹⁴Relative to the more well-known [Nash \(1950\)](#) bargaining solution, the [Kalai \(1977\)](#) solution imposes monotonicity. This monotonicity will be a useful feature when we consider the effects of interest rate ceilings since it implies that a binding constraint on the total surplus will weakly reduce the surplus of borrowers and lenders. The strength of the effect on each party is determined by θ .

(or bargaining power). The terms are given as the solution to

$$\bar{\mathcal{S}}^{\mathcal{B}}(a, y, j) = \max_{q, a'} \mathcal{S}^b(q, a'; a, y, j) \quad (7)$$

$$s.t. (1 - \theta)\mathcal{S}^{\mathcal{B}}(q, a'; a, y, j) = \theta u'(a + y - qa') \mathcal{S}^{\mathcal{L}}(q, a'; a, y, j). \quad (8)$$

The borrower's surplus from bargaining, $\bar{\mathcal{S}}^{\mathcal{B}}(a, y, j)$, represents their outside option in any credit meeting. It is determined by the share θ of the maximized total surplus of the match. We now turn to determining the terms of trade in the high- and low-cost submarkets.

A lender's expected surplus (or, equivalently, profit) from trade with a solvent household of type $(a, y, j \in \{I, U\})$ at credit terms (q, a') is

$$\mathcal{S}^{\mathcal{L}}(q, a'; a, y, j) = - \left[\frac{1 - \mathbb{E}_{y', j' | y, j} [d(a', y', j')]}{1 + r} - q \right] a'. \quad (9)$$

For each unit of debt, $-a' > 0$, lenders expect to be repaid $1 - \mathbb{E}_{y', j' | y, j} [d(a', y', j')]$ in the following period, discounted to the present by $1/(1 + r)$, where $d(a', y', j') \in \{0, 1\}$ represents the default decision of the borrower contingent on their state at the beginning of the following period. The term inside the square brackets, then, represents expected profits per unit lent. In competitive models of trade, this term equals zero and q exactly represents default risk. However, in our model, consistent with the evidence on unsecured debt pricing (e.g. in [Dempsey and Ionescu, 2021](#)), there exists a positive spread between the default premium and the loan price.

High-cost submarket terms of trade Let $q_{Uh}(a, y)$ and $a'_{Uh}(a, y)$ represent the posted terms of trade in the high-cost submarket for uninformed, solvent borrowers of type (a, y) and let $n_h(a, y)$ represent the market tightness.¹⁵ Suppressing the dependence on (a, y) , $\{q_{Uh}, a'_{Uh}, n_h\}$ are given

¹⁵We assume that high-cost lenders post the same terms for informed consumers, $q_{Ih} = q_{Uh}$ and $a'_{Ih} = a'_{Uh}$ that might, off equilibrium, direct their search to the high-cost submarket. We verify that, given this, the expected borrower surplus of informed agents searching in the high-cost submarket at these terms of trade is always lower compared to their expected surplus in the low-cost submarket. Hence, any high-cost submarket will feature no informed agents in equilibrium.

as the solution to

$$\max_{q, a'} \mathcal{S}^{\mathcal{L}}(q, a'; a, y, U) \quad (10)$$

$$s.t. \mathcal{S}^{\mathcal{B}}(q, a'; a, y, U) \geq \bar{\mathcal{S}}^{\mathcal{B}}(a, y, U), \quad (11)$$

plus the free-entry condition

$$\frac{\alpha(n_h)}{n_h} \mathcal{S}^{\mathcal{L}}(q_{Uh}, a'_{Uh}; a, y, U) \leq \kappa \quad (=" if $n_h > 0$)). \quad (12)$$

The terms of trade (q, a') are determined in (10)-(11) by maximizing the lender's surplus, subject to the participation constraint of borrowers that they achieve at least their surplus from bargaining. The participation constraint is always binding since, fixing an a' the lender's surplus is always increasing (borrower's decreasing) in the loan price q — the high-cost lender extracts as much as possible from uninformed borrowers, giving them their outside option from bargaining.

Then, given (q_{Uh}, a'_{Uh}) , the free-entry condition (12) pins down the tightness of the high-cost market. The left side is equal to the lender's expected surplus — the probability of matching with a borrower times their surplus — while the right side is the cost of entry. We define an uninformed borrower's expected surplus in the high-cost submarket as

$$v^{s,h}(a, y, U) = \alpha(n_h) S^{\mathcal{B}}(q_{Uh}, a'_{Uh}, U). \quad (13)$$

Low-cost submarket terms of trade The terms of trade and tightness in the low-cost submarket are given by $(\{q_{j\ell}(a, y), a'_{j\ell}(a, y)\}_{j \in \{I, U\}}, n_{\ell}(a, y))$ for household of type (a, y) . Compared to the high-cost market, lenders in the low-cost market face a screening problem since in equilibrium they attract both informed borrowers and (lucky) uninformed borrowers. We now index the posted terms of trade by information status of the borrower, $j \in \{I, U\}$, allowing lenders to post direct revelation mechanisms. The contract solves the following problem, suppressing the

dependence on (a, y) :

$$\max_{\{q_{j\ell}, a'_{j\ell}\}_{j \in \{I, U\}, n_\ell}} \alpha(n_\ell) \mathcal{S}^B(q_{I\ell}, a'_{I\ell}; a, y, I), \quad (14)$$

$$s.t. \mathcal{S}^B(q_{j\ell}, a'_{j\ell}; a, y, j) \geq \bar{\mathcal{S}}^B(a, y, j) \quad \text{for } j \in \{I, U\} \quad (15)$$

$$\mathcal{S}^B(q_{j\ell}, a'_{j\ell}; a, y, j) \geq \mathcal{S}^B(q_{-j\ell}, a'_{-j\ell}; a, y, j) \quad \text{for } j \in \{I, U\} \quad (16)$$

$$\frac{\alpha(n_\ell)}{n_\ell} \sum_{j \in \{I, U\}} \frac{\Gamma(a, y, j)}{\Gamma(a, y, I) + \Gamma(a, y, U)} \mathcal{S}^L(q_{j\ell}, a'_{j\ell}; a, y, j) = \kappa. \quad (17)$$

The solution maximizes informed borrowers' expected trade surplus subject to borrowers' participation constraints (15), incentive-compatibility constraints (16), and the free-entry condition for lenders (17), where $\Gamma(a, y, j)$ is the equilibrium measure of solvent households in state (a, y) for $j \in \{I, U\}$. Low-cost lenders post terms to compete for informed borrowers, but that competition is limited by the presence of some uninformed agents that can potentially misreport their type and trade at the terms for informed agents. Lenders therefore attempt to screen uninformed borrowers, but binding incentive compatibility constraints limit competition for informed agents. Finally, the free-entry condition (17) weights the expected lender surplus of trading with informed and uninformed borrowers of type (a, y) according to the equilibrium distribution.

It is helpful to understand why the lender cares about the information state of the borrower. Unlike some models of unsecured lending with asymmetric information where the private information involves the utility cost of default (e.g., [Athreya et al., 2012](#)), our lenders do not observe borrowers' current information state, which has no direct effect on the relative values of solvency and default since the terms are set after borrowers have already exploited their information in the current period. However, since a borrower's information state is persistent, their current state is informative about their future information, which does determine the relative value of default. Informed agents – that are more likely to be informed tomorrow – are generally less likely to default because the terms at which they borrow in the future are more favorable (they get better qs so they can roll over debt more easily). The lender today would benefit from identifying the uninformed and exploiting their relatively poor continuation value. This screening, in turn, limits the ability of the lender to extract the surplus of the uninformed who turn up in the low-cost market because they must be willing to select the appropriate contract.

Deriving closed-form expressions for the screening contract is infeasible since it depends on borrowers' continuation values, which themselves depend on the screening contracts they face in the future. However, as a preview of our quantitative results, we show that lenders can in fact separate informed and uninformed agents by offering the uninformed agents loan contracts with a low principal but high interest rate, compared to the informed. Uninformed agents have a stronger incentive to maintain low levels of debt since they face a higher probability of being uninformed in the future and, as a result, higher future expected interest rate costs rolling over debt balances. They are willing to pay higher interest rates in the current period to avoid higher debt.

Probability of trade and credit market composition Given (n_ℓ, n_h) we can define the *ex ante* probability that household (a, y) trades in submarket $i = \{\ell, h\}$, as $\alpha_i \equiv \alpha_i(a, y) \in [0, 1]$. Let $N_h \equiv N_h(a, y)$ and $N_\ell \equiv N_\ell(a, y)$ denote the equilibrium measures of lenders posting the high- and low-cost terms of trade, respectively, for households in state (a, y) . Then,

$$\alpha_\ell = \begin{cases} \alpha(n_\ell) & \text{if } j = I \\ \frac{N_\ell}{N_\ell + N_h} \alpha(n_\ell) & \text{if } j = U \end{cases} \quad \alpha_h = \begin{cases} 0 & \text{if } j = I \\ \frac{N_h}{N_\ell + N_h} \alpha(n_h) & \text{if } j = U \end{cases}. \quad (18)$$

In (18), the *ex ante* probability of trading in the low-cost submarket for an informed household is simply the probability of matching $\alpha(n_\ell)$. The *ex ante* probability for an uninformed household, however, depends on the endogenous market composition of low- and high-cost lenders, N_ℓ/N , where $N = N_h + N_\ell$, for that particular household type. Likewise, the probability of entering the high-cost submarket for an informed household is zero, while for an uninformed household it is the probability of matching $\alpha(n_h)$ times the probability of drawing a high-cost offer N_h/N . Hence, a household's expected borrowing cost depends not only on their information status but also their wealth and income through the endogenous market composition of high- and low cost lenders, $N_h = N_h(a, y)$ and $N_\ell(a, y)$.

To characterize N_h and N_ℓ , we use the definition of the tightness in submarket $i \in \{\ell, h\}$ as the ratio of lenders to borrowers in the market, along with the equilibrium search probabilities

in (18). This gives

$$n_\ell(a, y) = \frac{N_\ell(a, y)}{\Gamma(a, y, I) + \frac{N_\ell(a, y)}{N_\ell(a, y) + N_h(a, y)} \Gamma(a, y, U)} \text{ and} \quad (19)$$

$$n_h(a, y) = \frac{N_h(a, y)}{\frac{N_h(a, y)}{N_\ell(a, y) + N_h(a, y)} \Gamma(a, y, U)}. \quad (20)$$

The measures of borrowers and the low- and high-cost markets in (19) and (20) follow from the intuition above. Borrowers in the low-cost market are composed of all the informed agents and a fraction, N_ℓ/N of the uninformed agents. The remaining uninformed enter the high-cost market. Given (n_ℓ, n_h) as the solution to the contracting problems and the ergodic distribution of households, $\Gamma(a, y, j)$, we can solve (19)-(20) for (N_h, N_ℓ) for each (a, y) .¹⁶

4 Calibration

We set a time period in the model to be one month. We adopt a matching function of the form $\alpha(n) = \frac{n}{(1+n^v)^{\frac{1}{v}}}$. A first group of parameters is calibrated to values commonly used in the bankruptcy literature. The monthly risk-free rate is $r = \frac{0.01}{12}$. We choose the persistence and standard deviation of the endowment process to be $\rho_y = 0.95$ and $\sigma_y = 0.10$, respectively. The relative risk aversion parameter σ is set to 2. We assume the default cost distribution $G(\lambda)$ is Gumbel with a scale parameter μ and shape parameter ω . We set a large value for $\omega = 500$ so that the randomness in λ plays a negligible role in determining default, but the household problem remains convex, making computation easier.¹⁷

The remaining parameters are jointly calibrated within the model. These include the discount factor β , the entry cost of lenders κ , the scale parameter μ , the elasticity of the matching function v , the share of the surplus that the borrower receives when bargaining θ , and the probabilities that govern the information process π_{II} and π_{UU} . We calibrate these parameters to match the features of the unsecured credit market broadly defined to include both conventional lending (e.g., credit cards) and high-cost lending.

We target two sets of moments. First, a set of moments that are commonly used in the

¹⁶We omit the Komolgorov forward equation that determines the ergodic distribution of households $\Gamma(a, y, j)$ as it is standard.

¹⁷As $\omega \rightarrow \infty$ the optimal choice function approaches the maximum.

unsecured credit literature taken from [Athreya et al. \(2012\)](#) and [Sanchez \(2017\)](#): (i) the fraction of households with negative net worth, 12.5 percent, (ii) the average annual interest rate on credit cards, 12 percent, which we interpret as the average interest rate for low-cost lenders in the model, (iii) the ratio of aggregate unsecured debt over aggregate income, 1.23, and (iv) the annual fraction of households that file for bankruptcy, 1.2 percent. These moments primarily determine the discount factor, the stigma cost of default, the elasticity of the matching function, and the entry cost of lenders.

Second, we target moments related to high-cost borrowing. We approximate high-cost borrowing using data on the payday lending industry, which represents the largest share of high-cost borrowing. We obtain statistics from the 2016 Survey of Consumer Finances (SCF) and a study from the Consumer Financial Protection Bureau (CFPB), [CFPB \(2013\)](#). Several waves of the SCF ask survey respondents if they have taken a payday loan in the previous 12 months and [CFPB \(2013\)](#) provide aggregate data on interest rates and borrowing sequences (consecutive loans) from a nationwide but undisclosed payday lender. From these sources, we target (i) the average effective annualized interest rate paid in payday borrowing, 339 percent, (ii) the fraction of households in a given year that took out a payday loan, 3.4 percent, and (iii) the fraction of borrowing sequences in payday lending that last longer than a month, 45 percent, where borrowing sequences are defined as loans in consecutive periods. Targeting the effective high-cost interest rate disciplines the share of the surplus that high-cost lenders can extract from borrowers. Targeting the extensive margin of high-cost borrowing disciplines the steady-state fraction of uninformed households, while targeting the fraction of long high-cost borrowing sequences disciplines the persistence of the uninformed state.

We choose to calibrate the economy with no regulations. While 43 states plus the District of Columbia have explicit APR caps and four more have “unconscionability” restrictions, these regulations have numerous loopholes. For example, the Texas Fair Lending Alliance (<https://www.texasfairlending.org/the-basics>) notes that payday lenders may register as Credit Access Businesses under the Credit Services Organizations Act and broker loans between borrowers and third-party lenders; officially, the lender charges the capped interest rate (10 percent currently) but the broker charges a large and unregulated fee (over 500 percent on average). In other states, “rent-a-bank” schemes circumvent caps, in which payday lenders use a chartered

bank (which is regulated at the *federal* level) to issue a loan which is sold to the payday lender at a small markup.¹⁸ Obviously, in theory these payments are just interest (the ratio of the present value of repayments divided by the initial delivery of funds). We believe that it therefore is reasonable to impose no ceilings in the benchmark.

Table 1: Parameter calibration

Parameter	Value	Description
External		
ρ_y	0.95	Persistence income shock
σ_y	0.1	Standard deviation income shock
r	$\frac{0.01}{12}$	Risk-free interest rate
ω	500	Scale parameter Gumbel dist.
Jointly determined		
β	0.991	Discount factor
κ	0.00007	Entry cost lenders
λ	0.012	Stigma cost of default
θ	0.010	Borrower's share of surplus
v	28.39	Slope matching function
π_{II}	0.97	Prob. informed when informed
π_{UU}	0.41	Prob. uninformed when uninformed

We report the jointly calibrated parameters in Table 1 and the empirical moments and model counterparts in Table 2. The model is not only able to match standard moments used in the unsecured credit literature but also captures features of the high-cost lending sector that are a challenge for competitive, default-based pricing models, but of first-order importance to analyze interest rate ceilings. The challenge of the standard model is that to generate the interest rates observed in high-cost lending, the model must impose counterfactually high default rates. Introducing lender market power allows us to break the tight connection between default and interest rates. For example, the average interest rate across high-cost submarkets in the model is 304 percent, compared to the average payday interest rate in the data of 339 percent, while the default rate in the model of 1.3 percent remains in line with the data value of 1.20 percent. The calibration achieves this combination by setting the bargaining power of borrowers to a very low value ($\theta = 0.01$).¹⁹

¹⁸See <https://www.responsiblelending.org> for discussion of rent-a-bank schemes and the loopholes in federal law that permit them.

¹⁹The borrower's share of the surplus θ has to be positive, as the choice of new debt a'_i is indeterminate at the lower

Our model is consistent with evidence that operating costs are higher for payday lenders, as noted in [Flannery and Samolyk \(2005\)](#). To see why, note that lenders have the same fixed entry cost across submarkets κ , but the model endogenously generates a different expected fixed cost per loan. High-cost lenders face a lower probability of trade compared to low-cost lenders and therefore frequently pay costs but receive no revenue.²⁰

Our model is also consistent with the fact that a relatively small fraction of total unsecured credit is borrowed from high-cost lenders. In the data, the fraction of households that borrowed from AFI lenders in the previous year ranges from 3.4 percent to 7.8 percent, depending on how expansively AFI borrowing is defined.²¹ The fraction of households borrowing from a payday lender is 3.4 percent, while in the model it is 3.8 percent. To generate this number, the calibration imposes a low probability that an informed agent becomes uninformed $\pi_{IU} = 0.03$, while having a moderate persistence of the uninformed state $\pi_{UU} = 0.41$.

Table 2: Targeted moments: data and model

Moment	Data	Model	Source
Fraction NW<0 (%)	12.50	14.44	Athreya et al. (2012)
Default rate (annualized, %)	1.20	1.30	Athreya et al. (2012)
Debt-to-income ratio (%)	1.23	1.31	Sanchez (2018)
Mean Interest Rate Low-cost (annualized, %)	12.07	10.98	Athreya et al. (2012)
Mean Interest Rate High-cost (annualized, %)	339	304	CFPB (2012)
Fraction High-cost (prev. 12m, %)	3.4	3.8	SCF (2016)
Fraction of Long Borrowing Sequences – High-cost	0.45	0.33	CFPB (2012)

Finally, our model is also consistent with (untargeted) cross-sectional features of high-cost borrowing across the wealth distribution. In [Figure 2](#), we show the probability of searching in a submarket $\iota \in \{\ell, h\}, \frac{N_\iota}{N}$, for the median income borrower, plotted by wealth. Informed borrowers always search for credit in the low-cost submarket regardless of wealth, as the blue line shows. For uninformed households, the probability of searching in the high-cost market (illustrated as the gray line) tends to decrease in wealth, a . At higher levels of wealth, high-cost lenders find it too costly to enter, $N_h(a, y) = 0$, since the borrower’s surplus is too low to justify the

bound of $\theta = 0$. At the lower bound, any a'_h yields the same surplus to the borrower.

²⁰In the data, one reason AFIs are expensive is that they operate brick-and-mortar storefronts that have long hours and need to be located near public transit.

²¹The upper bound includes AFIs such as pawn shops, which are technically secured debts. Around 85 percent of pawned items are reclaimed by the borrower.

high expected fixed cost. As a result, wealthy uninformed borrowers only trade in the low-cost submarket. However, at lower wealth levels, high-cost lenders become more prevalent and crowd out low-cost options. The borrower surplus increases as wealth falls, since their outside option forces them to deleverage to zero net worth. Knowing this fact, high-cost lenders are induced to enter at higher rates, leading to a higher probability of uninformed, low-wealth borrowers trading with a high-cost lender. This endogenous result in the model is consistent with evidence from the Survey of Consumer Finances, as shown in [Figure 1](#). Payday borrowing is more common at lower levels of wealth, and the model generates this feature of the data endogenously.²²

Matching these facts is important for our policy exercises. Welfare gains from regulating interest rates are generally larger if lenders have more market power, so a model that assigns all the spread to market power will overstate those gains (admittedly, our market power parameter hits the lower bound, so it is not clear we could actually overstate these gains, but the general point holds). Similarly, a model that generates an excessively large high-cost market will also overstate the gains, since more uninformed agents will be stuck borrowing at very high rates, and one that gets the distribution of borrowers by wealth wrong will skew the welfare gains since it will miss on the distribution of marginal utility.

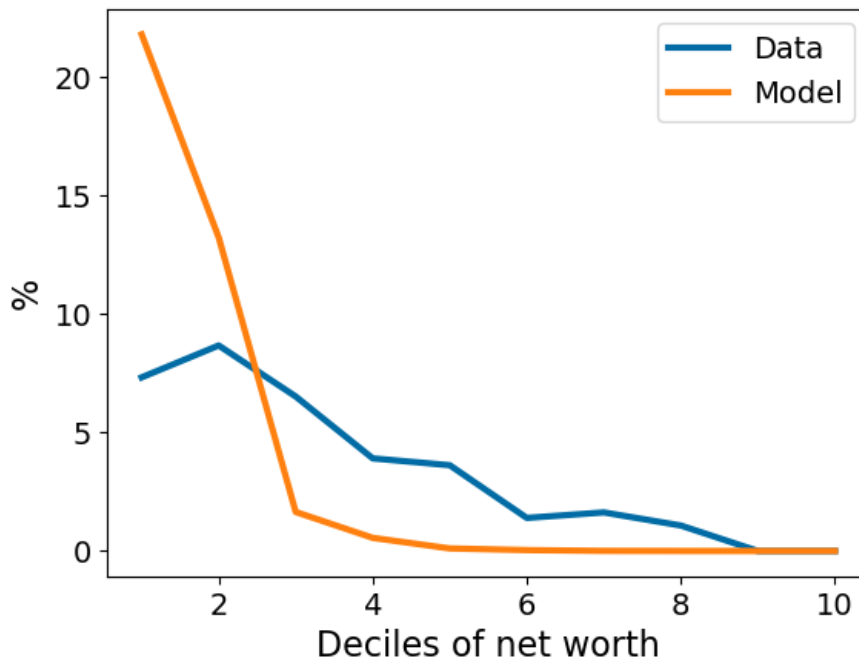
5 Interest Rate Dispersion

In this section, we characterize the rich dispersion in the interest rates generated by our model. The distribution of interest rates from the model is shown in [Figure 3](#) and summarized in [Table 3](#). Annualized interest rates for unsecured credit range from 6 to 923 percent. Low-cost lenders post rates on the lower end – between 6 and 66 percent – while high-cost lenders post rates on the higher end – between 90 and 923 percent. In the calibrated steady-state there is little overlap between the distribution of low- and high-cost interest rates. We now describe how this feature, which gives a first check that a blunt interest rate ceiling might be an effective tool to target the high-cost market, is consistent with the data.

Within the low-cost submarket, the rates paid by *informed* consumers shown in blue – up to 36 percent – match reasonably well the observed terms on credit cards. According to the

²²The correlation between wealth and payday use is also documented empirically in [Li \(2022\)](#).

Figure 1: Use of payday loans (data) and high-cost borrowing (model) in the previous 12 months



Survey of Consumer Finances (2016), the maximum interest rate on credit cards reported by households was 36 percent. The additional spread in low-cost submarkets borne by uninformed consumers shown in orange – rates between 36 and 66 percent – can be interpreted as additional financial costs, such as late fees, which are documented in [Grodzicki and Koulayev \(2021\)](#), or price dispersion between shoppers and non-shoppers, as documented in [Stango and Zinman \(2015\)](#).

Table 3: Interest rates across submarkets and information state

	Average	Std. dev.	Min.	Max.
Low-cost submarket	11.0	6.0	6.0	66.0
– Informed	10.8	5.6	6.0	38.0
– Uninformed	26.1	12.7	6.6	66.0
High-cost submarket	303.9	154.2	90.9	923.6

In [Table 3](#) we can also see that, in the low-cost submarket, uninformed consumers pay higher interest rates compared to informed consumers for a given (a, y) . Low-cost lenders screen uninformed borrowers by offering them lower debt levels at higher interest rates. Uninformed

Figure 2: Probability of searching in a submarket conditional on information state

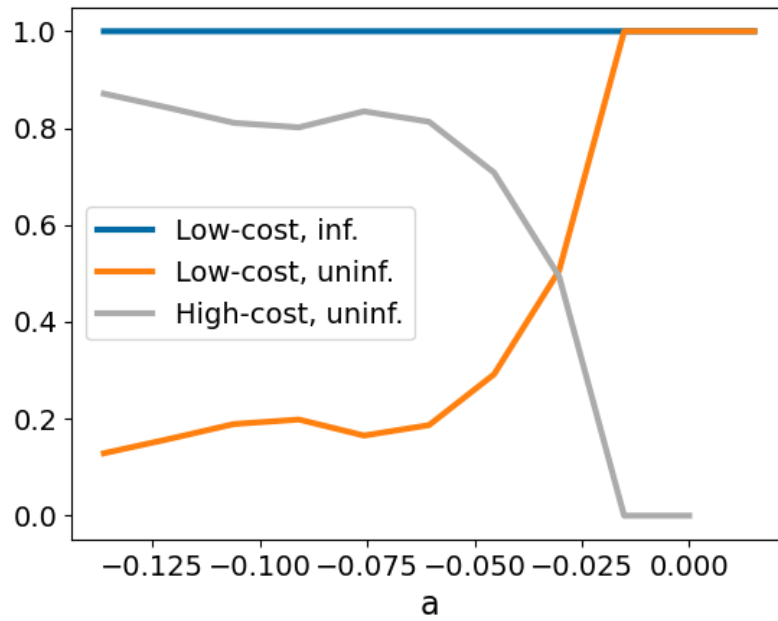
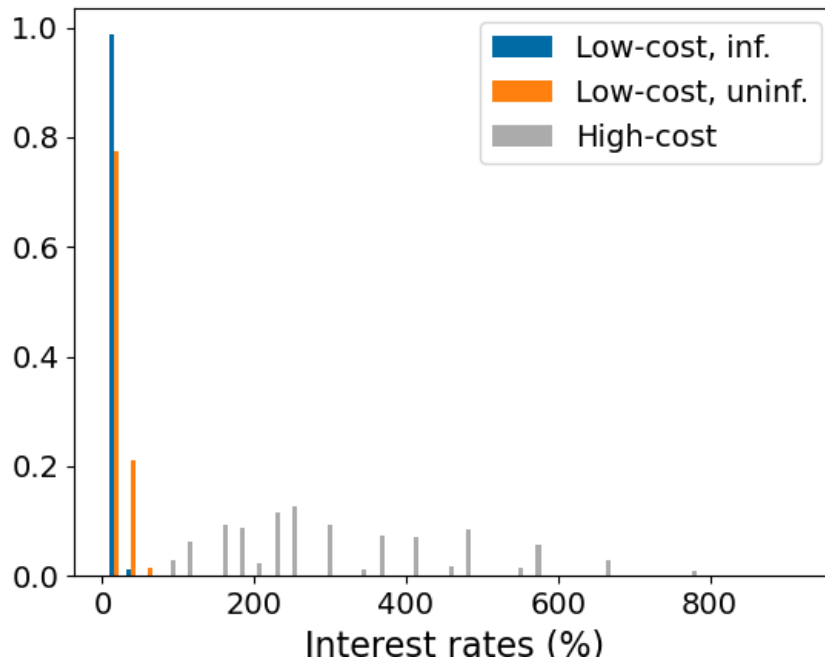


Figure 3: Relative frequency of interest rates, by submarket



borrowers have a stronger incentive to borrow less compared to the informed, as they are more likely to remain uninformed than an informed borrower is to become one, and carrying less debt

reduces their likelihood of meeting a payday lender next period and having to roll over their debt at a high interest rate. Although smaller loans carry less risk of default, the interest rate paid by an uninformed consumer is still higher. Through higher interest rates, lenders are able to extract surplus from uninformed consumers even in the low-cost submarket. However, low-cost lenders cannot extract as much surplus as high-cost lenders, since contracts have to satisfy incentive compatibility constraints and these are binding for the uninformed.

In [Table 4](#), we decompose the average interest rates paid into three components due to: i) default risk, ii) the fixed cost of entry, and iii) market power. For i) we compute an implied interest rate that only accounts for default risk, as it would in a competitive model:

$$q^D(a', y, j) = \frac{E[1 - d(a', y', j')]}{1 + r},$$

where d is the default rate in the next period. Second, we compute the interest rate that accounts for the fixed cost of entry κ :

$$q^{FC}(a', y, j) = -\frac{\kappa}{a'}.$$

We measure the contribution of market power as the difference between the equilibrium interest rate in our model and the interest rates that correspond to default risk plus fixed costs. The decomposition is shown in [Table 4](#).

Table 4: Decomposition of average interest rates by submarket and information state

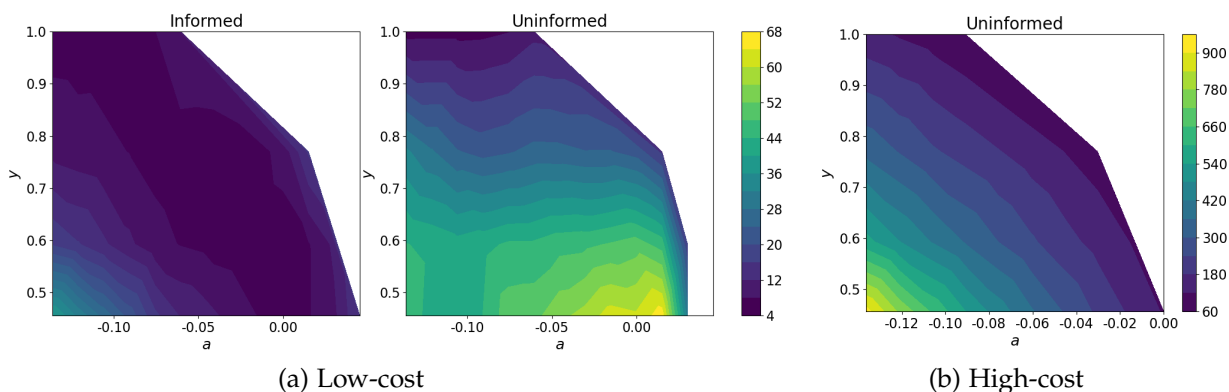
	Low-cost, inf.	Low-cost, uninf.	High-cost, uninf.
Equilibrium rate	10.8	26.0	303.9
Default risk only	10.1	5.5	8.3
Fixed cost only	1.3	6.6	1.3
Market power effect	-0.6	13.9	294.3

We find that most of the interest rates in the high-cost market are due to market power. Lenders have virtually all of the bargaining power and default rates are relatively low. In a frictionless model, the risk of default and the fixed entry cost would justify an average interest rate of 9.6 percent, compared to the effective interest rate of 303.9 percent. The large spread is therefore an indicator of substantial distortions relative to the competitive benchmark.

The composition of interest rates in the low-cost submarket is very different depending on

the information state of the consumer. Uninformed consumers subsidize informed consumers. Informed consumers pay an average interest rate of 10.8 percent, which is not enough to cover the risk of default and the fixed cost (the break-even rate is 11.4 percent). Uninformed consumers pay on average 26 percent. More than half of their rates are driven by market power, as an interest rate of 12.1 percent would be enough to cover the risk of default and fixed costs.

Figure 4: Interest rates by wealth and income in low-cost (left and middle panels) and high-cost markets (right panel)



Interest rates also vary throughout the wealth and income distribution, as shown in Figure 4. In high-cost markets, interest rates are higher for lower income and lower wealth (more debt) households since borrower surplus is high in these states and lenders can increase interest rates to extract more surplus. This variation holds in the low-cost submarkets as well, but in this case it is due to higher default risk. For the uninformed, their rates are higher for lower income.

6 The effects of interest rate ceilings

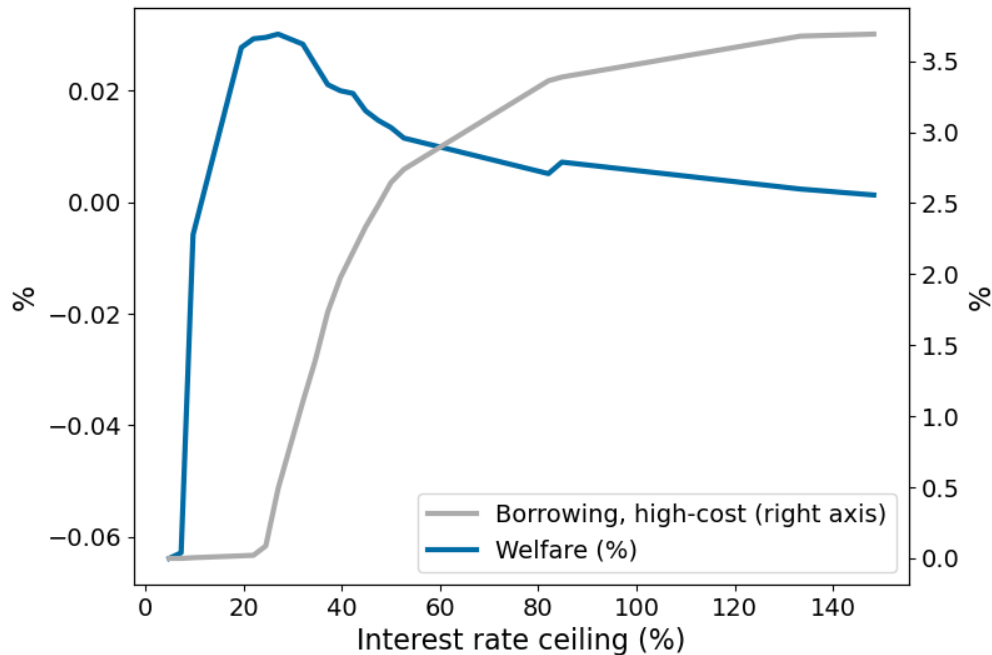
In this section, we investigate the effects of interest rate ceilings. In particular, we focus on interest rate ceilings that are noncontingent – they apply to all submarkets uniformly. These ceilings are one of the most common policies used by states to regulate high-cost lending, and they do not require any particular knowledge on the part of the regulator. In terms of the model, an interest rate ceiling \bar{r} corresponds to a price floor $\bar{q} = \frac{1}{1+\bar{r}}$ for $q_i(a'; a, y, j)$.

In our model, an interest rate ceiling restricts the division of surplus within a match. Directly affected submarkets that would have equilibrium interest rates above the ceiling may continue to

operate at lower rates after the ceiling is imposed or may shut down, depending on whether the total surplus remains positive at the ceiling or not. Furthermore, general equilibrium spillover effects imply that interest ceilings can alter the terms of trade and credit availability across other, not directly affected submarkets. Our goal is to characterize how interest rate ceilings alter the entire composition of the unsecured credit market and consumer welfare.

Normative effects Figure 5 illustrates our primary results. We show the consumption-equivalent aggregate welfare gain (or loss, if negative) from introducing progressively tighter interest rate ceilings into the model. The welfare gain is computed between the steady state of the model with a given interest rate ceiling and the steady state of the unregulated model.²³

Figure 5: Aggregate welfare gain of interest rate ceilings

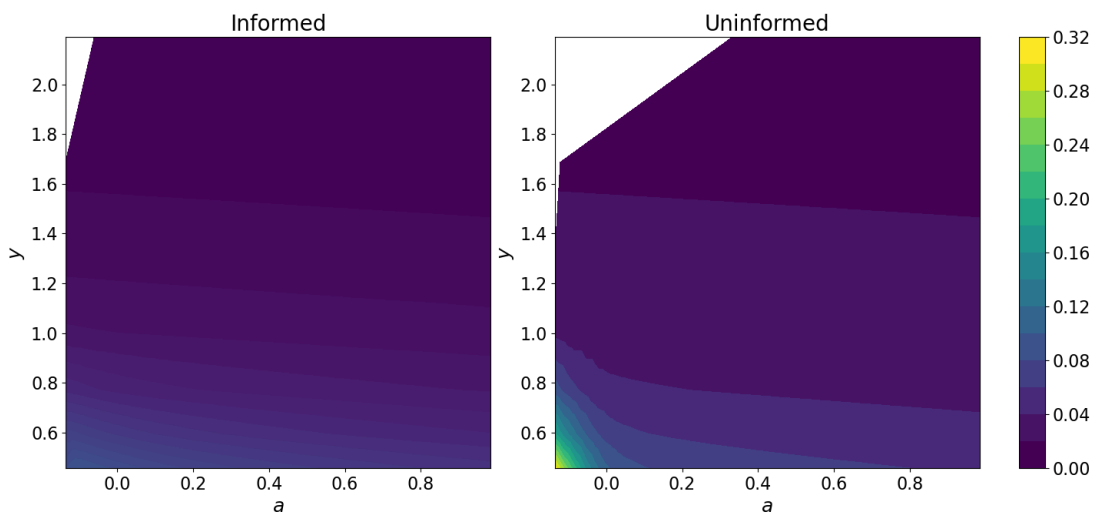


We find an optimal interest rate ceiling of 26 percent (annualized), which yields an aggregate welfare gain of 0.03 percent. The gains are heterogeneous across the wealth and income distributions, as shown in Figure 6. For example, low-income, low-wealth households will gain up to

²³We are abstracting from transitional dynamics, which would be prohibitively difficult to compute in our model. Given the absence of capital, it is likely that these transitions are very short and therefore are unlikely to yield substantially different welfare numbers.

0.32 percent from imposing the optimal ceiling. Importantly, the policy is *Pareto-improving*; no households lose from the optimal interest rate ceiling. Despite the blunt nature of the instrument, no one would oppose introducing it.

Figure 6: Welfare gain of a 26% interest rate ceiling

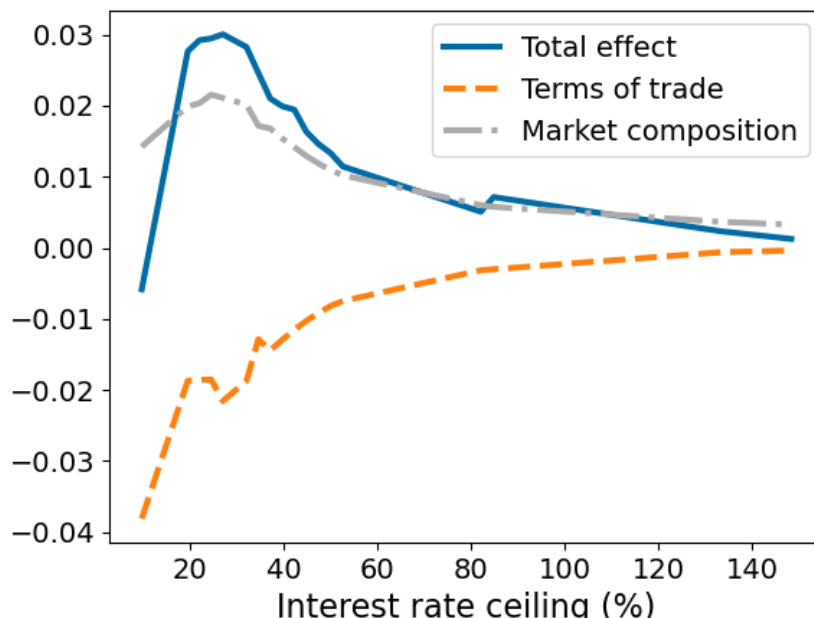


To grasp the source of welfare gains from the interest rate ceiling policy, we decompose the welfare gains using two partial equilibrium experiments. First, we compute the welfare gains of an economy with the terms of trade (interest rates and borrowing levels) of an economy with an interest rate ceiling but keeping fixed the tightness of each submarket from the unregulated economy. This allows us to measure to what extent the gains from the optimal ceiling are driven by changes in terms of trade alone and not by changes in the probability of matching with high- and low-cost lenders. We call this the “terms of trade” channel. In our second experiment, we do the opposite and compute the welfare gains with the tightness of the economy with a ceiling but fixing the interest rates and borrowing levels to the ones from the unregulated economy. We call this the “market composition” channel. The results are presented in [Figure 7](#).

Our results support the idea that the welfare gains from tighter interest rate ceilings are driven by general equilibrium effects through the market composition of high- and low-cost lenders, and not through changes in the terms of trade. As the high-cost submarket faces tighter interest rate ceilings, lenders exit, and uninformed consumers are then more likely to match with low-cost lenders. The terms of trade of informed consumers, the majority of the population in

the model, deteriorate with a tighter interest rate ceiling. Interest rate schedules for the informed shift upward toward higher interest rates as the interest rate ceiling is tightened.

Figure 7: Welfare decomposition: “market composition” versus “terms of trade” channels

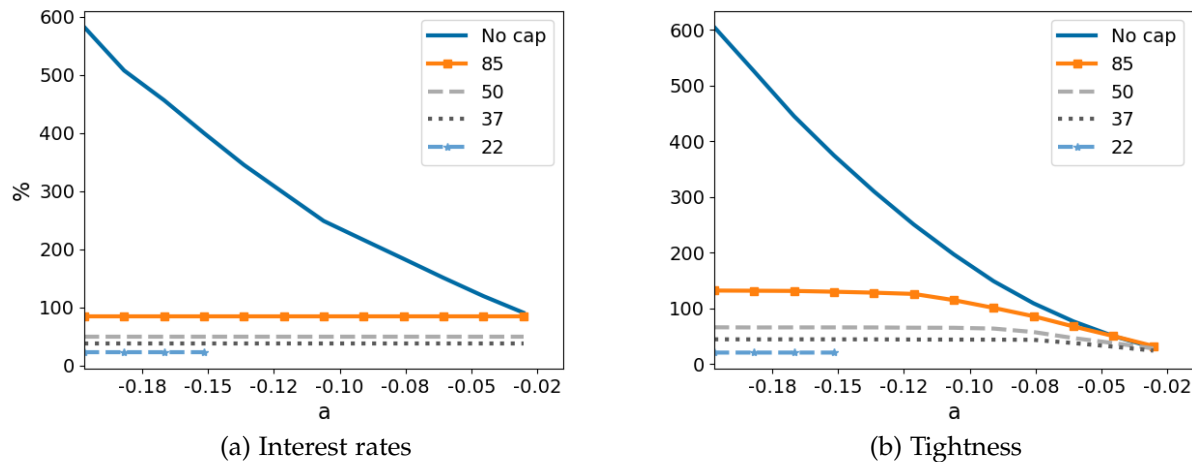


Positive effects The interest rate ceilings we investigate are binding for high-cost lenders.²⁴ Figure 8 shows the equilibrium interest rate and the tightness in the high-cost submarket under various interest rate ceilings. In each panel, the lines illustrate the interest rate and tightness as a function of the incoming wealth of the borrower, keeping income fixed at the median income of the borrowers. If the ceiling is not restrictive enough to close the market, the high-cost lender simply sets the price at the ceiling. As a result, borrowers that match with high-cost lenders face better prices (left panel). Among high-cost submarkets that are still active, profits conditional on trade are decreasing with a tighter interest rate ceiling, so lenders exit, and the tightness of the submarket goes down (right panel).

The reduced entry of high-cost lenders spills over to the *ex ante* probability of matching with a low-cost lender. To illustrate how ceilings alter the composition of the credit market, in Figure 9

²⁴In principle, even ceilings that do not bind could affect welfare by changing the outside option of borrowers and lenders in the bargain. Such ceilings would be very high, and the effects are likely to be very small because lenders are extracting essentially all the surplus.

Figure 8: Interest rates and tightness in the high-cost submarket, per interest rate ceiling



we plot the relative measure of lenders in submarket $\iota \in \{\ell, h\}$ over the total lenders across high- and low-cost submarkets (we denote this ratio as ω_ι), across wealth for a median income borrower. As the interest ceiling is lowered, there tends to be a higher chance of randomly meeting a low-cost lender when uninformed. This increase benefits borrowers, as they will be matched more frequently with low-cost markets. Notice that since information states can change, this increase also benefits informed borrowers. Consumption smoothing improves due to increased access to lower interest rates.²⁵

In terms of prices and borrowing in the low-cost submarket, we find that equilibrium interest rates and borrowing can increase or decrease in the low-cost submarket depending on an agent's information state, wealth, and income. Figure 10 shows average interest rates and the fraction of households borrowing for high and low submarkets, and the information state; in Figure 11 we show debt-to-income ratios and default rates.

First, consider Figure 10. Moving from right to left along the x-axis represents lower interest rate ceilings or, equivalently, higher price floors. Lower ceilings reduce the average interest rate and participation in high-cost submarkets by reducing the incentive of lenders to enter and post extractive terms of trade. Some high-cost markets still exist and lend at the ceiling, but others are destroyed. These effects spill over to the low-cost submarket. In the low-cost

²⁵In the appendix, we report a similar plot but with the probability of searching and meeting a high- or low-cost lender α_ι (see Figure 17). The changes are practically identical, which confirms that spillovers to the low-cost submarket from regulation occur through changes in ω_ι .

Figure 9: Market composition between high- and low-cost lenders, per interest rate ceiling

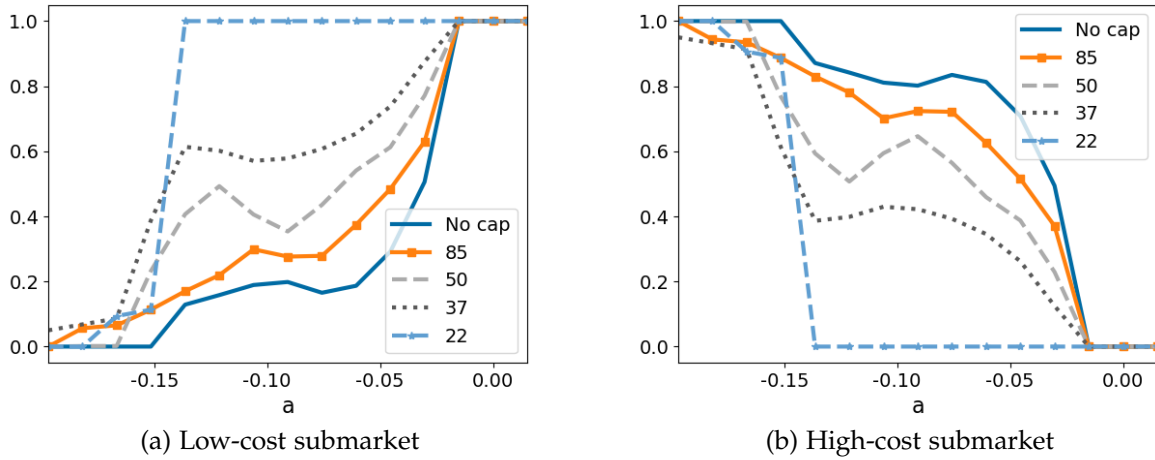
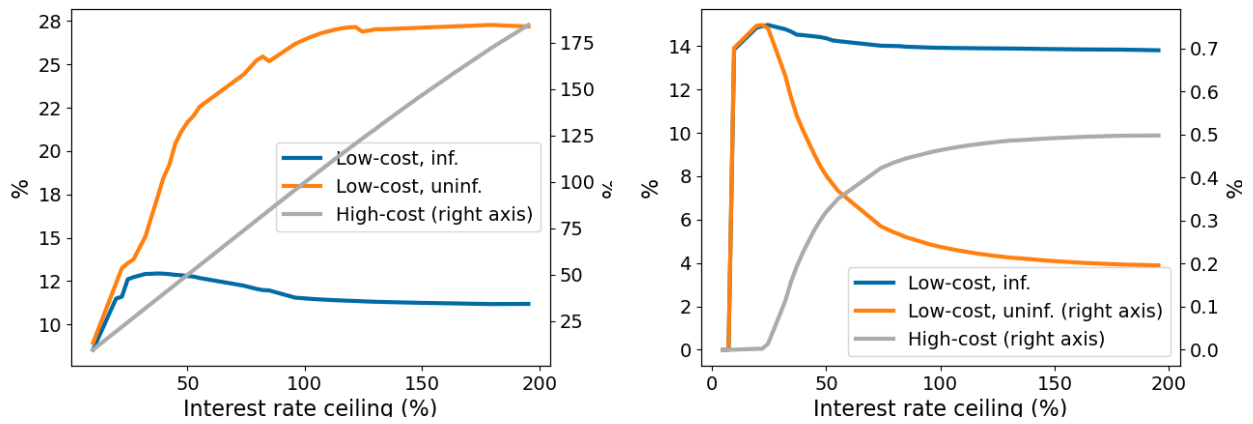
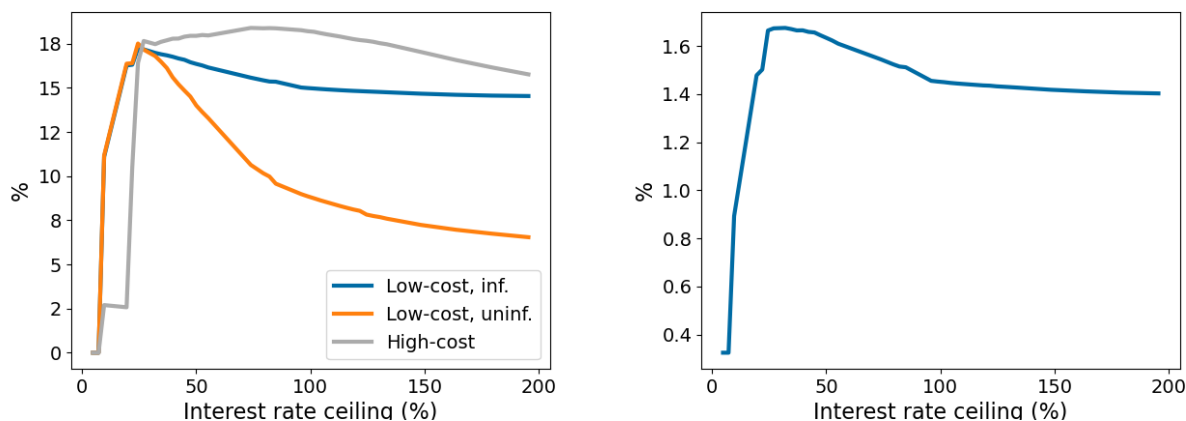


Figure 10: Average interest rates (left) and fraction of household borrowing in a submarket (right)



market, the extensive margin of borrowing increases as interest rates ceilings get tighter, for informed and uninformed households. Average interest rates increase for informed agents, while uninformed agents pay on average lower rates; the higher interest rates for the informed are driven by the congestion effect of more uninformed reducing the probability of matching, which is small because there are not many uninformed borrowers.

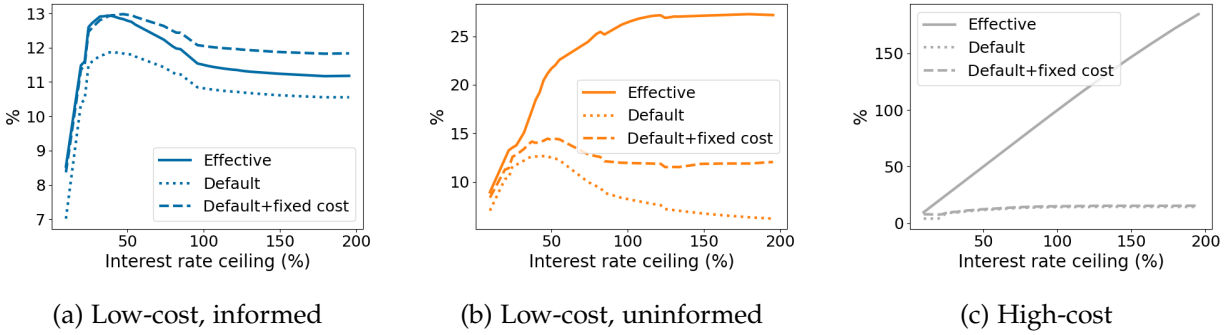
Figure 11: Debt-to-income and default rate



Along the intensive margin of borrowing, illustrated in the left panel of [Figure 11](#), informed and uninformed consumers are borrowing more in low-cost submarkets as the interest rate ceilings become tighter. High-cost borrowing increases as the interest rate ceiling is lowered to 75 percent and decreases at tighter ceilings. Overall, more borrowing increases the default rate of the economy, as shown in the right panel. However, when the interest rate ceiling is too tight – below 22 percent – tighter ceilings collapse the credit market as borrowing and average interest rates decrease to the point that there is no borrowing at all. Default rates are too high and entry too costly to justify any lending. Note that this rate is above some rates charged in the unregulated equilibrium, unlike what would happen in a competitive lending environment.

In [Figure 12](#), we decompose the path of interest rates for each interest rate ceiling as in [Table 3](#). We show the average interest rates (solid lines) together with the interest rate implied by the default risk (dotted lines) and default risk plus fixed costs (dash-dotted lines). We interpret the distance between effective interest rates and the interest rate that includes default risk and fixed cost as the effect of market power. The increase in rates for informed agents as ceilings are lowered is driven by larger loans, which carry a higher default risk but also a lower cross-

Figure 12: Average interest rates and components by interest rate ceiling and submarket



subsidization that they get from the presence of uninformed consumers. The lower rates for uninformed consumers are driven by lower market power despite the fact that the risk of default is increasing due to larger loans.

Sensitivity to $\pi_{I,I}$ We recompute the model for different values of the probability of remaining informed $\pi_{I,I}$. We choose to focus on this parameter because it is identified by the extensive margin of borrowing at high cost and plays an important role in the welfare gains from interest rate ceilings; regulations that only directly help the currently uninformed may not be Pareto-improving if information states are too persistent, as the benefits are expected to arrive too far in the future for the currently informed to gain. In particular, if information states are essentially permanent, then only indirect effects (which are negative) will affect the informed.²⁶

With this in mind, we choose a range of $\pi_{I,I}$ to target the range of the fraction of households borrowing from payday lenders across US states. As shown along the horizontal axis of Figure 13, payday borrowing varies substantially across states, ranging from 0.20 percent of households in Vermont to 6 percent in Mississippi.²⁷ If we include alternative financial services (AFS), the numbers are even larger, ranging between 5 and 21 percent, but the variation remains. Since high-cost borrowing in our model includes payday and some components of other AFS, we choose a range for $\pi_{I,I}$ such that the fraction of households borrowing in the high-cost submarket ranges between 3.4 percent (baseline) and 12 percent.

²⁶Here, we keep other parameters and regulations fixed and only consider variation in $\pi_{I,I}$. Fully recalibrating the model to different states poses some challenges since we do not observe some of our target moments at the state level (e.g., sequences of payday borrowing).

²⁷These statistics are calculated using the Unbanked/Underbanked supplement of the Current Population Survey. We exclude states like New York, where payday lending is banned outright.

Figure 13: Use of payday loans and AFS across US states

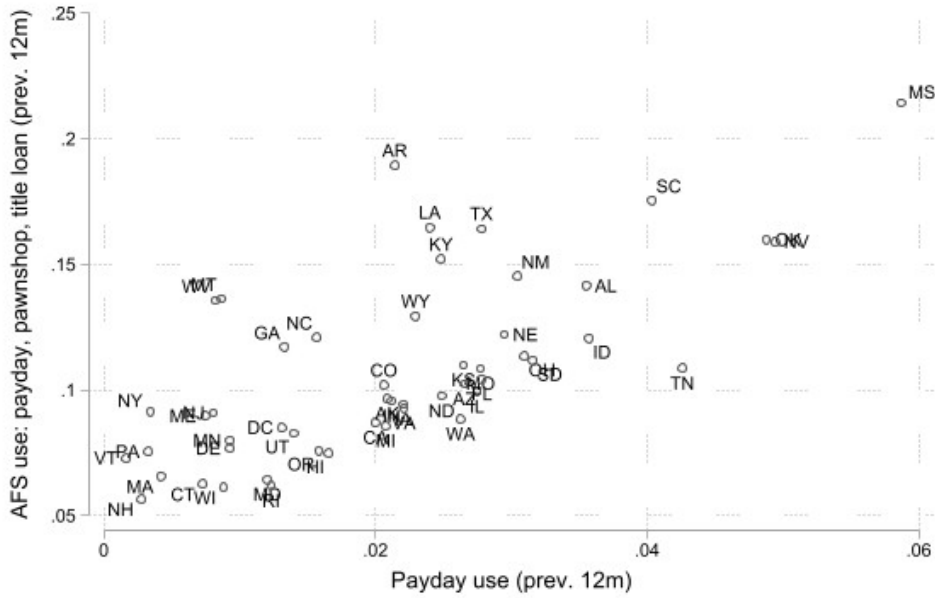
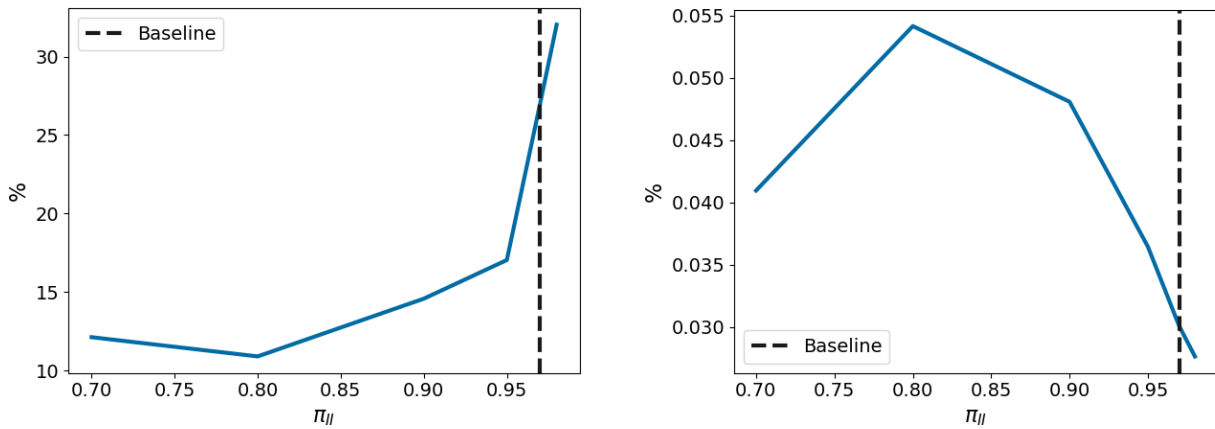


Figure 14: Optimal interest rate ceiling and welfare gains across $\pi_{I,I}$



(a) Optimal interest rate ceiling

(b) Welfare gain from optimal ceiling (%)

For every $\pi_{I,I}$, we recompute the optimal interest rate ceiling and calculate the welfare gains from optimal regulation, relative to an unregulated equilibrium. These curves are shown in the left and right panels of [Figure 14](#), respectively. As we decrease $\pi_{I,I}$ from the baseline value of 0.97, the optimal interest rate ceiling falls and welfare gains from optimal regulation increase. However, as the persistence of being informed falls low enough, optimal interest rate ceilings begin to increase.

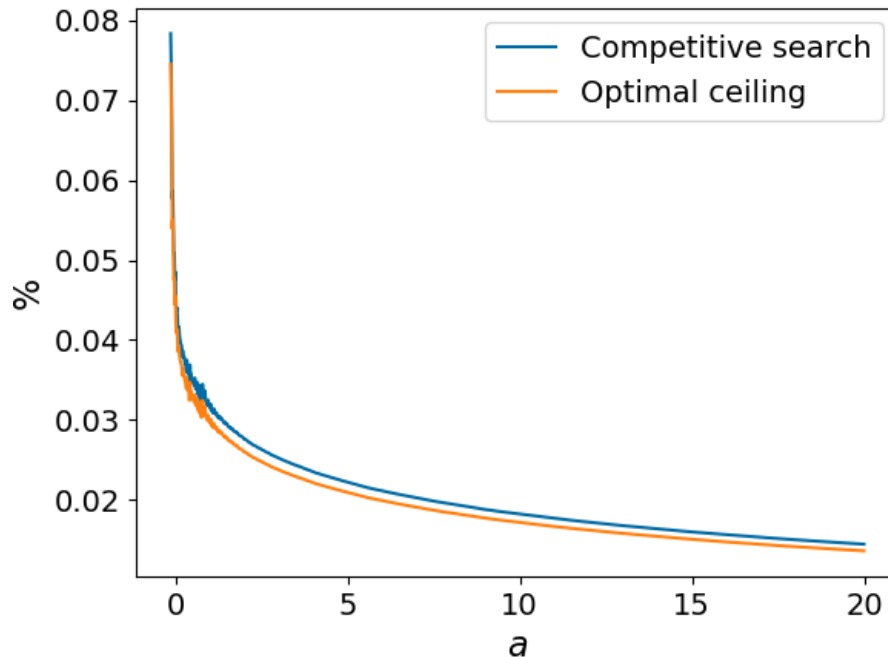
7 The Value of Information

In this section, we perform two additional experiments. First, we consider an economy where all households are informed. Second, we ask what a single uninformed agent would pay to become informed, holding the equilibrium fixed. The goal of these experiments is to shed light on the value of information in the model, and therefore the role it plays in our welfare results above.

All households are informed In this experiment, we assume that all households are informed, which turns the credit market into one with standard competitive search. We calculate the welfare gains relative to our benchmark economy; we interpret this change as the gains associated with either improved financial literacy or improved dissemination of facts about credit markets. This particular allocation is of interest because it is generally the best that a planner could implement without being able to affect search technologies.

We illustrate the welfare gains as a function of a household's wealth, a , in [Figure 15](#). The blue line shows the welfare gains in the competitive search model for households with a given wealth (varying income and information states), while the orange line shows the welfare gains in a model with a optimal interest rate ceiling. Aggregate welfare gains are 0.032 percent of consumption in an economy where all households are informed compared to our baseline model, only a small amount larger than the 0.03 percent welfare gain from imposing an optimal interest rate ceiling. The welfare gains in each model are close across the wealth distribution as well. This result indicates that the interest rate ceiling is close to implementing the welfare gains from the competitive search model, suggesting not only that interest rate ceilings are welfare-improving but that they are quite powerful. In addition, since implementing the fully optimal policy would

Figure 15: Welfare gains in an economy with competitive search and one with the optimal interest rate ceiling with respect to the baseline model

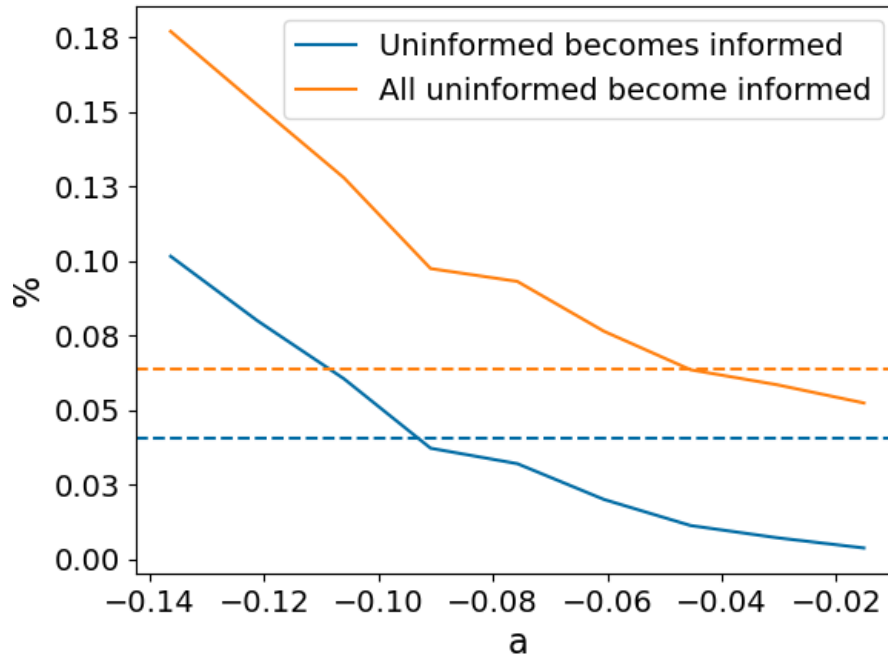


be complicated and involve a substantial amount of individual information about borrowers, it is helpful to understand the gains from simple, easy-to-administrate policies; that a blunt interest rate ceiling can capture nearly all the gains available is a result that is of practical value to policymakers.

One uninformed borrower becomes informed In this experiment, we ask households how much they value becoming informed while keeping the equilibrium constant. This experiment can be thought of as the value of a financial literacy program that is not big enough to change the equilibrium. We show the results in Figure 16. Note that this change is ultimately transitory, since the agent may become uninformed again in the future.

The value of being immediately informed for the *average* uninformed consumer is 0.04 percent of lifetime consumption. This gain is represented by the blue dashed line in Figure 16. The welfare gain decreases with wealth, as shown by the full blue line in the same plot. The welfare gain is due to better terms in the low-cost submarket; by directing their search, they can find substantially lower interest rates. Since households with higher wealth are less likely to borrow,

Figure 16: Welfare gains of uninformed households



they gain less from this exercise. Despite the fact that the intervention is transitory, there are still significant welfare gains compared to the more substantial intervention of making all consumers informed, illustrated in orange in Figure 16.

8 Conclusion

We study the positive and normative effects of interest rate ceilings in environments in which lenders possess market power. We find that annual interest rate ceilings as low as 20 percent can increase borrower welfare in the presence of uninformed agents whose surplus can be extracted more completely by lenders. Even if only a small fraction of agents are uninformed (4 percent in the calibrated model), the gains from imposing interest rate ceilings outweigh the costs for all agents.

Our model complements the findings in Saldain (2023) regarding the welfare effects of credit market regulations. In that paper, households with self-control problems could also benefit from interest rate ceilings but do not because their default incentives already lead to very tight bor-

rowing constraints. It could be fruitful to combine the two models, especially if the preferences of the borrowers are not observable.²⁸

We have explored alternative models. In particular, we considered a market structure in which all households could obtain two draws from the distribution of lenders; the resulting equilibrium again has two types of lenders, low and high cost, where high-cost lenders are used exclusively by those who failed to match in the low-cost market. These lenders are labeled “high cost” because they exploit the limited outside options of the borrowers who failed to obtain a low-cost match. While this model delivered interesting results on the spillovers of market power due to regulation in the high-cost market, the spreads were too small an order of magnitude and interest rate ceilings ended up reducing welfare as in the competitive model.²⁹

Our model does not feature dynamic information updating on the part of lenders. Credit scoring is a mechanism through which markets provide information on relevant and unobserved state variables, such as propensities to default. [Chatterjee et al. \(2023\)](#) show that dynamic scoring has important effects on the provision of competitive credit. Given that payday lenders typically do not report to the main three credit bureaus (Equifax, TransUnion, and Experian), but rather rely on their own information agency (Teletrack), it would be an interesting extension to consider the role of scores in our model and the welfare gains/losses associated with better tracking of past activity and the sharing of information.³⁰

Our model also does not feature delinquency (informal default; see [Athreya, Sanchez, Tam, and Young \(2018\)](#)). While the default rate on payday loans is high, it is more likely due to failure to repay than formal bankruptcy. In [Athreya et al. \(2018\)](#), after a borrower skips a payment, 60 percent of lenders choose a fixed “penalty” interest rate on delinquent debt and 40 percent make a take-it-or-leave-it offer of the new debt level to the borrower, taking into account the probability of repayment in the future. High penalty rates for delinquent debt could easily run afoul of the ceilings, which could lead to welfare gains through reduced interest rates but could also encourage delinquencies and lender exit. Whether this extension changes our answers

²⁸See also [Raveendranathan and Stefanidis \(2024\)](#), who study the role of restricting the ability of lenders to increase debt limits to households with self-control problems.

²⁹Details of these experiments are available upon request. We explored also an extension of our model to allow for random searchers to get multiple offers, but solving for the equilibrium structure was difficult; we intend to revisit that model in future research.

³⁰Accounts that get sold to debt collectors or result in court judgments do appear on credit reports, as do bankruptcies that discharge payday lender debt.

regarding the welfare gains from simple ceilings is left for future work.

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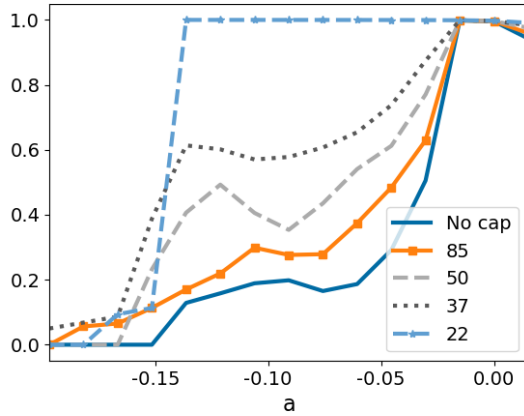
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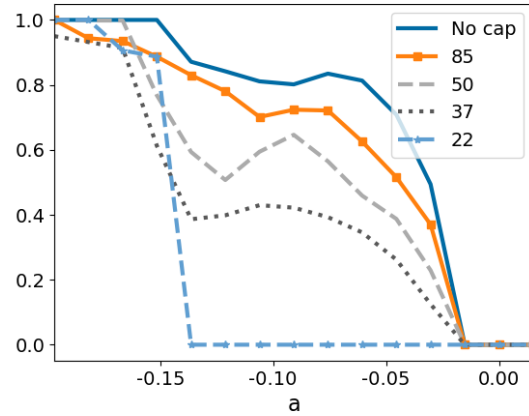
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A Supplementary Figures

Figure 17: Probability of searching and meeting a lender for uninformed consumers, per interest rate ceiling and submarket



(a) Low-cost submarket



(b) High-cost submarket

B Outline of Numerical Algorithm

We assume a matching function of the form $\alpha(n) = \frac{n}{(1+n^v)^{\frac{1}{v}}}$ and proceed as follows:

0. Guess $v^0(a, y, j), d^0(a, y, j), \Gamma^0(a, y, j)$. We use as a guess the solution from the model without private information.
1. Solve $v^{s,n}(a, y)$.
2. Solve $v^{s,l}(a, y)$ using local optimizer FFSQP.
3. Solve $v^{s,h}(a, y)$ using local optimizer FFSQP. Compute n_h using (12).
4. Compute α_l, α_h solving equations (18), (19) and (20).

$$\alpha_l = \alpha(n_l) \frac{\Gamma(a, y, I)n_l}{\Gamma(a, y, U)(n_h - n_l)} \quad (21)$$

$$\alpha_h = \alpha(n_h) \frac{n_h \Gamma(a, y, U) - n_l [\Gamma(a, y, I) + \Gamma(a, y, U)]}{\Gamma(a, y, U)(n_h - n_l)} \quad (22)$$

Need $\frac{n_h}{n_l} > \frac{\Gamma(a, y, I) + \Gamma(a, y, U)}{\Gamma(a, y, U)}$ for positive α_l, α_h .

5. Compute $v^s(a, y), v^d(a, y)$. Update $v^1(a, y), d^1(a, y), \Gamma^1(a, y, j)$.
6. If $v^0(a, y), d^0(a, y), \Gamma^0(a, y, j)$ close enough to updated values, finish; otherwise, update guess in step 0.