International Diversification, Reallocation, and the Labor Share*

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April 13, 2023

Abstract

How does growing international financial diversification affect firm-level and aggregate labor shares? We study this question using a novel framework of firm labor choice in the face of aggregate risk. The theory implies a cross-section of labor risk premia and labor shares that appear as markups in firm-level data. International risk sharing leads to a reallocation of labor towards riskier/low labor share firms alongside a rise in within-firm labor shares, matching key micro-level facts. We use cross-country firm-level data to document a number of empirical patterns consistent with the theory, namely: (i) riskier firms have lower labor shares and (ii) international financial diversification is associated with a reallocation towards risky/low labor share firms. Our estimates suggest the reallocation effect has dominated the within effect in recent decades; on net, increased financial integration has reduced the corporate labor share in the US by about 2.5 percentage points, roughly one-third of the total decline since the 1970s.

*We thank Andy Atkeson, Gadi Barlevy, Adrien Bilal, Harold Cole, Nicolas Crouzet, Jan Eeckhout, Francois Gourio, Kyle Herkenhoff, Bart Hobijn, Loukas Karabarbounis, Matthias Kehrig, Simon Mongey, Sebnem Kalemli-Ozcan, Miguel Palacios, David Thesmar and various seminar and conference participants for comments and suggestions. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

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1 Introduction

The last 40 years have witnessed a global decline in labor's share of income (Karabarbounis and Neiman, 2014) concurrent with a rapid deepening in international financial integration (Lane and Milesi-Ferretti, 2018). This paper explores the links between the international diversification of risk, risk-sharing between firms and workers, and the labor share. The paper stresses the dual (and competing) effects of deeper international diversification on the aggregate labor share: a *within-firm* effect that increases firm-level and aggregate labor shares and a *reallocation* effect that shifts production towards lower labor share firms, which decreases the aggregate labor share. We derive conditions to empirically quantify these two effects and find that the latter has dominated, explaining the negative relationship between international financial integration and the aggregate labor share observed in the data.

Figures 1 and 2 display the two main empirical patterns motivating our study. First, Figure 1 shows that at the macro-level, the aggregate labor has fallen substantially over recent decades, while simultaneously, foreign equity liabilities – the value of foreign holdings of domestic risky, equity-type assets – have increased dramatically. These patterns hold both for the US and globally.[1]

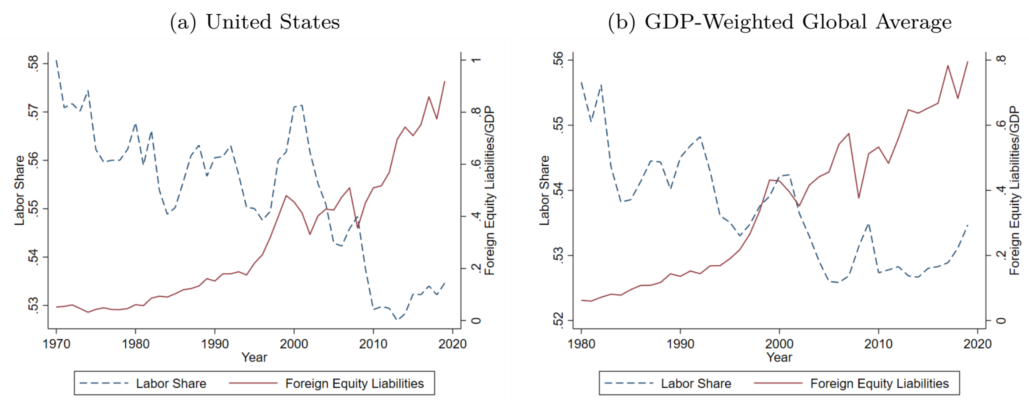

Figure 1: Trends in Labor Share and Foreign Equity Holdings

Notes: Figure displays the aggregate labor share (left-axis) and foreign equity liabilities (the value of foreign holdings of domestic equity), normalized by GDP (right-axis). Panel (a) displays statistics for the United States; panel (b) displays GDP-weighted averages across 27 countries classified as advanced economies by Lane and Milesi-Ferretti (2018). Data are from the OECD (labor share) and the External Wealth of Nations database (Lane and Milesi-Ferretti, 2018), and are described in detail in Section 4.1.

Second, at the micro-level, Figure 2 decomposes changes in the aggregate labor share into a reallocation component, i.e., changes in production shares across firms, holding firm-level

[1] Foreign equity liabilities is the sum of portfolio equity investment and FDI. Details of the data are in Section 4.1. In Appendix D.2, we show that similar patterns hold using the corporate sector labor share and using the foreign equity share, i.e., foreign equity liabilities as a fraction of the total value of the corporate sector, rather than GDP, which controls for the important valuation effects documented in Atkeson et al. (2022).

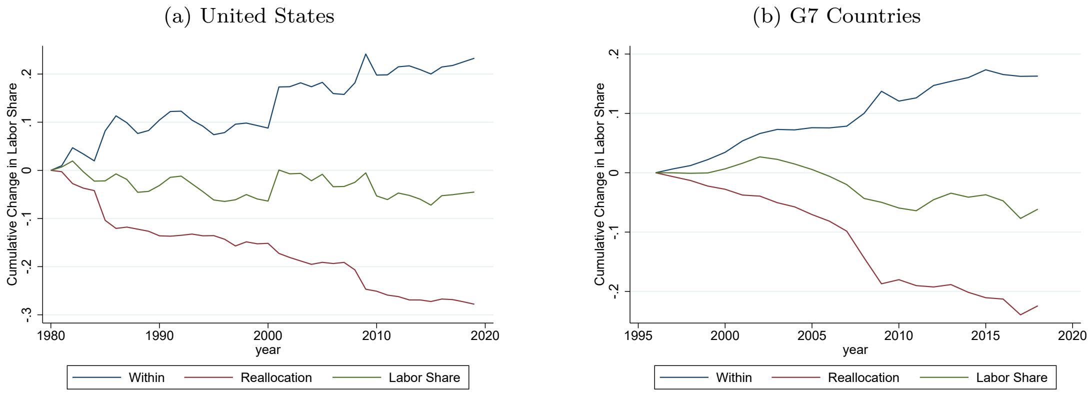

Figure 2: Labor Share Decomposition into Reallocation and Within Components

Notes: Figure displays the decomposition of changes in the aggregate labor share into reallocation and within components. Panel (a) displays statistics for the US; panel (b) displays arithmetic averages across five G7 countries. Data for the US are from Compustat. Data for the G7 countries are from Orbis and include Germany, France, the UK, Italy and Japan.

labor shares fixed, and a within component, i.e., changes in firm-level labor shares, holding production shares fixed. We calculate these components using the following decomposition (derived formally in Section 2.2):[2]

$$\Delta LS_{t+1} = \underbrace{\sum_i \Delta\hat{Y}_{it+1} \times LS_{it+1}}_{\text{reallocation effect}} + \underbrace{\sum_i \hat{Y}_{it} \times \Delta LS_{it+1}}_{\text{within effect}}, \tag{1}$$

where LS_{it} and LS_t denote firm-level and aggregate labor shares, respectively, $\hat{Y}_{it} = \frac{Y_{it}}{Y_t}$ firm shares of aggregate value-added, and Δ the difference operator. The figure clearly illustrates the key role of reallocation across firms in driving declines in the aggregate labor share: the micro-data point to the reallocation component as the main force behind the fall in the labor share, while, if anything, the within effect has been increasing the labor share. These patterns are consistent with recent evidence on micro and macro labor shares as forcefully documented in Hartman-Glaser et al. (2019), Autor et al. (2020) and Kehrig and Vincent (2021), who also decompose the changes in the US labor share into within and reallocation components and find that the decline in the aggregate results from the latter dominating the former.

In light of these patterns, we propose a simple yet novel framework linking aggregate risk – and opportunities for international risk-sharing – to the allocation of resources across firms and both micro-level and aggregate labor shares. Heterogeneous firms choose inputs under both idiosyncratic and aggregate uncertainty and hence factor payments are determined before the

[2] In Appendix D.3, we extend the decomposition to separate within vs. cross-industry reallocation and find that the former accounts for the majority of the total reallocation component.

realization of shocks.[3] While idiosyncratic risk can be diversified away, aggregate risk cannot, and heterogeneity in firms' risk exposures leads to cross-sectional dispersion in labor risk premia and labor shares. Intuitively, firms insure workers against aggregate risk, but the price of such insurance depends on firms' exposure to that risk and their ability to diversify it away. The firm-specific labor share depends both on the share of labor in the production function and on the covariance between firm-specific risk, i.e., the firm-level marginal production of labor, and market risk, encompassed by the stochastic discount factor (SDF) used to price cash flows. If firm productivity is procyclical and the SDF countercyclical, as standard theory and empirics suggest, then this covariance is negative, reducing labor's share of expected income. Firms that are more exposed to aggregate risk, i.e., for which the covariance between firm and market risk is more negative, display a lower labor share.

This key novel result has important implications in the presence of opportunities for international diversification and risk-sharing. Domestic investors diversify country-specific risk by exchanging equity shares with investors in other countries. Limits to diversification are captured by a cost of international trade in these assets. As the cost of international trade in financial assets falls, international diversification and risk-sharing grow, changing the nature of aggregate risk. In particular, increasing diversification reduces the price of risk and hence, the implicit cost to firms of providing wage insurance to workers and labor risk premia. Such a change induces dual effects on the domestic labor share: (i) the decrease in the risk premium faced by firms leads to a decline in the cost of wage insurance and hence to an increase in the labor share within each individual firm. This is the within-firm effect. On the other hand, (ii) diversification opportunities disproportionately benefit firms with higher risk exposure and lower labor share, leading to a reallocation of production towards these firms, which expand. This reallocation effect can generate a decline in the aggregate labor share even as within-firm labor shares increase, consistent with the macro- and micro-level patterns in Figures 1 and 2.

The aggregate consequences of financial diversification and risk-sharing depend on whether the within-firm insurance effect or between firm reallocation effect dominates. We study a series of parametric examples showing that the impact of such diversification on the aggregate labor share takes a non-monotone U-shape: there is a unique threshold in the risk premium above (below) which a decrease in risk leads to a decline (increase) in the labor share. It is only in the extreme case in which market risk can be fully diversified that the labor share is fully determined by the relative importance of labor inputs in the production function. One striking implication of the theory is that increasing diversification does not lead to a perpetual decline

[3] The environment extends recent work studying the impact of risk adjustments in the capital allocation (David et al., 2022; David and Zeke, 2022) to the allocation of labor. Donangelo et al. (2018) also study firm-level labor shares and risk premia.

in the labor share – there exists a threshold level of diversification such that increases beyond this point will lead to a reversal and the labor share begins to rise. At the limit, with full risk-sharing, labor share is completely determined by labor's share in the production function. Notably, though the aggregate labor share takes a non-monotone U-shape with respect to the extent of diversification, we show that worker welfare is monotonically increasing in financial integration. The result highlights that in environments with firm heterogeneity and aggregate risk, the labor share may not be an accurate gauge of movements in worker welfare and indeed, there may be configurations in which the two measures move inversely.

Our empirical analysis consists of two parts. First, we use cross-country, firm-level micro-data to establish two novel predictions of the theory: (i) firms that are more exposed to aggregate risk have lower labor shares, and (ii) growing international diversification has led to a reallocation of inputs and production to risky/low labor share firms. We document strong evidence for these patterns using data on US firms from Compustat/CRSP and for a sample of foreign firms from Compustat Global. The results hold under a number of different specifications including various controls and sets of fixed-effects to capture additional sources of unobserved heterogeneity, speaking to the robust nature of these relationships.

Second, with these results in hand, we show how a first-order approximation to the model's equilibrium conditions yields a quantitative mapping between the strength of these connections, namely, the regression coefficients estimated under predictions (i) and (ii) and the reallocative and within effects of international diversification, and hence the consequences of the recent growth in such diversification for the aggregate labor share. The estimates suggest that the increase in diversification experienced in the US, i.e., the growth in the share of US domestic equity assets held by foreigners, has reduced the US labor share by about two and a half percentage points over recent decades, which accounts for about one-third of the total decline experienced in the US corporate sector.

Applying similar calculations to our sample of foreign firms/countries suggests that the growth in financial diversification for those countries reduced labor share by a smaller amount, about 0.3 percentage points, since the late 1980s (the beginning of our foreign sample), which represents almost 15% of the total decline in the corporate sector in those countries. Decomposing the difference between the US and the foreign countries shows that the main difference stems from the response of the resource allocation to growing international diversification and risk sharing: whereas increasing diversification has led to a substantial reallocation towards risky/low labor firms in the US, these reallocation effects have been more muted in other countries, limiting the extent of the labor share decline.

The paper is organized as follows. Section 2 outlines the model and demonstrates how micro and macro labor shares are linked to the price of risk in a standard production environment with uncertainty, and further, to opportunities for international risk-sharing. Section 3 derives sharp estimating equations for two of the key predictions of the model and shows how the estimation results can be used along with the model's equilibrium conditions and other observable moments of the data to quantify the impact of international diversification on both micro and aggregate labor shares via the reallocation and within-firm effects. Section 4 describes the data, empirical implementation and results. We conclude in Section 5.

Related literature. Our paper builds a bridge between two literatures: the literature on international financial integration and the literature on the global dynamics of the labor share. That international integration favors risk-taking and growth has been demonstrated theoretically by Obstfeld (1994). Empirically, Thesmar and Thoenig (2011) show using French firm-level data that diversification in ownership leads to more risk-taking at the firm level. Levchenko et al. (2009) find sector-level volatility increases permanently following international financial liberalization, suggesting an underlying risk-taking channel. Levchenko (2005) shows that the risk-sharing benefits of international financial integration may not be passed on to workers when access to the international insurance market is unevenly distributed and domestic risk-sharing is limited to self-enforcing contracts. Although the mechanism is quite different, our results share a similar theme – in our framework, in which only global "capitalists" trade internationally in financial assets, their endogenous risk-taking and reallocation of production towards riskier firms can lead to a reduction in workers' share of national income.

Karabarbounis and Neiman (2014) document a global decline in the labor share and a vast ensuing literature has examined the causes of this decline, as recently summarized in Grossman and Oberfield (2021). Proposed candidates range from technical change and the relative price of capital goods (Karabarbounis and Neiman, 2014) to the rise of superstar firms (Autor et al., 2020; Lashkari et al., 2018), automation (Acemoglu and Restrepo, 2018, 2020; Autor and Salomons, 2018; Hubmer and Restrepo, 2021), increased trade globalization (Elsby et al., 2013) and changing market power of firms and workers (Barkai, 2020; Benmelech et al., 2020; Stansbury and Summers, 2020). Our paper provides a novel theory along with strong empirical support, namely, the role of financial globalization and its implications for the labor share due to international risk-sharing and its effects on domestic risk-taking and labor market outcomes.

Our theoretical mechanism and empirical findings stress the dual within and reallocation effects observed at the micro-level, relating them both to changes in the nature of aggregate risk induced by increasing financial globalization. Perhaps closest to our work, Hartman-Glaser et al. (2019) emphasize the role of within-firm risk sharing between capitalists and workers in conjunction with increasing idiosyncratic volatility, whereas we study the effects of heterogeneous exposure to aggregate risk on the ex-ante distribution of expected labor shares and the implications of international diversification on the nature of this risk.[4] With these differences in mind, we view our empirical results and theoretical explanation as complementary to theirs.

In our framework, labor risk premia show up as markups in firm-level data, which connects our paper to a recent and growing literature studying the distribution of firm-level markups and its evolution over recent decades, important examples of which include De Loecker et al. (2020) and Edmond et al. (2018). Our theory provides a note of caution in interpreting measured markups as pure rents to the firm – rather, in our setting, the measured markup captures the risk premia required by the firm to bear labor market risk, and thus may not be indicative of firm market power. Eeckhout and Veldkamp (2021) present a related theory, relating firms' increasing access to data to risk premia and markups.

2 The Model

This section lays out our theory linking the labor share to the nature of aggregate risk and heterogeneous exposure to that risk across firms. For simplicity, we focus on a static input choice problem, but the results extend to dynamic versions as well. We focus first on a closed-economy setting and derive sharp comparative statics with respect to changes in the nature of aggregate risk. We then embed this setup into a multi-country setting and link these changes to the degree of international financial diversification and risk-sharing.

2.1 Risk, Input Allocation and the Labor Share

A set of heterogeneous technologies, i.e., firms, produce a homogeneous good according to[5]

$$Y_i = A_i K_i^{\alpha_1} L_i^{\alpha_2}, \quad \alpha_1 + \alpha_2 < 1 \ .$$

Firms differ in their productivity, A_i, which is composed of an anticipated component, $\mathbb{E}[A_i]$, and an unanticipated shock, $\mathcal{E}_i = \frac{A_i}{\mathbb{E}[A_i]}$. To ease notation later, we denote returns to scale with $\nu \equiv \alpha_1 + \alpha_2$.

Input choices are made prior to the realization of shocks and payments to factors of production cannot be state-contingent. In this sense, firms insure workers against the realization of shocks: wage payments are independent of these shocks, which are then fully reflected in

[4] There is significant evidence that workers are insured within the firm, and especially so against temporary shocks (Guiso et al., 2005), and further that labor choices are made under considerable uncertainty (David, Hopenhayn, and Venkateswaran, 2016).

[5] For our main analysis, we assume each firm operates a single technology. In Appendix A.1, we show that the same theoretical results hold if firms operate multiple technologies.

fluctuations in firm profits. Firms choose inputs to maximize the expected discounted value of cash flows, i.e., to solve

$$\max_{L_i,K_i} \mathbb{E}\left[\Lambda\left(A_i K_i^{\alpha_1} L_i^{\alpha_2} - WL_i - RK_i\right)\right] , \tag{2}$$

where Λ is a stochastic discount factor (SDF) used to price all cash flows.[6]

Firm-level labor shares. The optimality condition with respect to labor yields the share of expected sales paid to labor, which we refer to as *expected labor share*:[7]

$$\frac{WL_i}{\mathbb{E}\left[Y_i\right]} = \alpha_2\left(1-\kappa_i\right), \quad \text{where} \quad \kappa_i = -\text{cov}\left(\frac{\Lambda}{\mathbb{E}\left[\Lambda\right]}, \frac{A_i}{\mathbb{E}\left[A_i\right]}\right) . \tag{3}$$

Expression (3) is a key building block in our analysis and shows that the expected labor share of income is not simply equal to labor's share in production, α_2; rather, labor's share of income additionally depends on a firm-specific risk premium, κ_i, given by (the negative of) the covariance of firm-level productivity with the SDF. Firms that co-move more negatively with the SDF, i.e., have higher productivity and profits when the discount factor is low, are riskier and have lower expected labor shares. Notably, the result holds even in the Cobb-Douglas case with common parameters across firms, showing that the standard equivalence between labor's share of income and production fails to hold when firms make labor choices in the face of uncertainty and aggregate risk. Further, the risk premium shows up as what would otherwise be measured as a wedge in firm-level labor shares and hence helps rationalize recent findings of significant and heterogeneous price markups (e.g., De Loecker et al., 2020; Edmond et al., 2018) and/or wage markdowns (e.g., Berger et al., 2022; Yeh et al., 2022), despite the fact that the economy is in fact perfectly competitive.[8]

The intuition for the result is as follows: firms fully insure workers and thus bear the entirety of cash flow risk. The risk premium captures the cost of this insurance, which leads risky firms to hire fewer workers and reduce their payments to labor below what their expected productivity and the going wage rate would dictate. To see this clearly, we can derive the relative allocation

[6] The exact timing of when payments to inputs are made is not crucial, only that these payments are not contingent on the realization of shocks.

[7] The realized labor share equals expected labor share adjusted for the realization of the unanticipated shock, specifically, $LS_i \equiv \frac{WL_i}{Y_i} = \frac{WL_i}{\mathbb{E}[Y_i]}\frac{1}{\mathcal{E}_i}$. Because the shock is strictly exogenous and always multiplicatively separable, we work primarily with expected labor share throughout.

[8] To see this clearly, we can rewrite (3) as $\mathbb{E}\left[MRPL_i\right] = \frac{1}{1-\kappa_i}W$, which highlights that the risk premium shows up as a firm-specific wedge that causes the expected marginal product of labor to exceed the wage.

of labor as:

$$\frac{L_i}{L} = \frac{\left(\mathbb{E}\left[A_i\right]\left(1-\kappa_i\right)\right)^{\frac{1}{1-\nu}}}{\sum_i \left(\mathbb{E}\left[A_i\right]\left(1-\kappa_i\right)\right)^{\frac{1}{1-\nu}}} , \tag{4}$$

where $L = \sum_i L_i$. The expression shows that – conditional on expected productivity – risky firms (high κ_i) have lower expected labor shares and relative allocations of labor compared to safer firms (small κ_i). The capital allocation, i.e., $\frac{K_i}{K}$, is also described by (4).

The aggregate labor share. The aggregate expected labor share can be written as an output-weighted average of firm-level expected labor shares:[9]

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \sum_i \frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} \frac{WL_i}{\mathbb{E}\left[Y_i\right]} , \tag{5}$$

where firm-level output shares satisfy

$$\frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]} = \frac{\mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(1-\kappa_i\right)^{\frac{\nu}{1-\nu}}}{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(1-\kappa_i\right)^{\frac{\nu}{1-\nu}}} . \tag{6}$$

Expression (5) reveals a key insight of our study – the aggregate labor share is shaped by the cross-section of firm-level labor shares and output shares both, and thus by the joint distribution of these two micro-level objects. Further, expressions (3) and (6) highlight that both labor shares and output shares are functions of the cross-section of risk premia and hence the effects of risk premia on the aggregate labor share manifest themselves through both of these margins.

The aggregate expected labor share is given by

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \alpha_2 \left(1 - \sum_i \frac{\mathbb{E}\left[Y_i\right]}{\mathbb{E}\left[Y\right]}\kappa_i\right) = \alpha_2 \frac{\sum_i \left(\mathbb{E}\left[A_i\right]\left(1-\kappa_i\right)\right)^{\frac{1}{1-\nu}}}{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(1-\kappa_i\right)^{\frac{\nu}{1-\nu}}} . \tag{7}$$

In the absence of risk adjustments in the allocation, i.e., $\kappa_i = 0 \;\forall\; i$, the aggregate expected labor share is always equal to α_2. More generally, however, it depends on the full set of risk premia through their effects on both firm-level labor shares and firm-level shares of aggregate output.

Generalizations. Expression (3) is derived under three simplifying assumptions: (i) Cobb-Douglas production; (ii) labor and capital chosen under the same information set; (iii) firms fully

[9]Analogous to firm-level labor shares, the realized aggregate labor share depends on the expected aggregate labor share and additionally the realization of shocks. Specifically, $LS \equiv \frac{WL}{Y} = \frac{WL}{\mathbb{E}[Y]}\frac{1}{\mathcal{E}}$, where $\mathcal{E} = \frac{\sum_i A_i (\mathbb{E}[A_i](1-\kappa_i))^{\frac{\nu}{1-\nu}}}{\sum_i \mathbb{E}[A_i]^{\frac{1}{1-\nu}}(1-\kappa_i)^{\frac{\nu}{1-\nu}}}$ is identical to an aggregate TFP shock, i.e., $\mathcal{E} = \frac{TFP}{\mathbb{E}[TFP]}$, where $TFP = \frac{Y}{K^{\alpha_1}L^{\alpha_2}}$.

insuring workers. Each of these assumptions can be relaxed. For example: (i) Appendix A.1 shows that the result extends to a broader class of production functions, including a more general CES function of capital and labor (notably, in the CES case, the result holds independently of assumptions on the elasticity of substitution between capital and labor, i.e., whether it is smaller or larger than one); (ii) it is straightforward to verify that (3) goes through under a more complicated information structure in which capital is chosen prior to/under less information than labor, which has been found to be the empirically relevant case;[10] (iii) Appendix A.1 shows that a modified version of (3) holds when firms only partially insure workers, augmented to reflect the effects of risk over the wage. Appendix A.1 also shows that analogous expressions hold under different assumptions on the boundaries of the firm, specifically, if firms operate multiple technologies rather than one. In that case, the relevant object at the firm level is the average risk premium across the firm's technologies, weighted by each technology's share of the firm's total output.

2.2 Changes in the Price of Risk

We can decompose the risk premium into the product of two terms, capturing the quantity and price of risk:

$$\kappa_i = \underbrace{-\frac{\text{cov}\left(\mathcal{E}_i, \Lambda\right)}{\text{std}\left(\Lambda\right)}}_{\text{quantity of risk } = \ \mathcal{Q}_i} \times \underbrace{\frac{\text{std}\left(\Lambda\right)}{\mathbb{E}\left[\Lambda\right]}}_{\text{price of risk } = \ \mathcal{P}} .$$

The quantity of risk, $\mathcal{Q}_i$, is firm-specific and captures the sensitivity, or exposure, of firm productivity/profitability to the SDF. The price of risk is common across firms and is a function of the volatility, i.e., standard deviation, of the SDF. If agents are risk-neutral and hence the SDF constant, the risk premium is everywhere zero. If agents are risk averse and shocks to the SDF are correlated with shocks to productivity, this is no longer the case and the risk premium drives a wedge between labor's share of income and production. More concretely, if firm productivity is procyclical and the SDF countercyclical, as standard theory and empirics suggest, then $\text{cov}\left(\mathcal{E}_i, \Lambda\right) < 0$, which implies a positive risk premium, reducing the expected labor share. The risk premium is larger (depressing expected labor share more) for firms with a higher quantity of risk, i.e., for more procyclical firms that covary more negatively with the SDF. The magnitude of this effect is increasing in the price of risk.

As a salient example, consider a log-linear SDF (alternatively, a log-linear approximation to any SDF) that is function of a single aggregate shock, so that $\log \Lambda - \mathbb{E}\left[\log \Lambda\right] = -\lambda \log A$ where A without subscript denotes the aggregate shock and λ the loading of the SDF on the

[10] See, e.g., David, Hopenhayn, and Venkateswaran (2016), who find that firm-level capital and labor choices are both made under uncertainty, but the former under more uncertainty than the latter.

shock. The price of risk is approximately $\mathcal{P} = \lambda\sigma_a$ and the quantity of risk for firm i is approximately $\mathcal{Q}_i = \beta_i\sigma_a$, where $\beta_i \equiv \frac{\text{cov}(\log A_i, \log A)}{\sigma_a^2}$ and σ_a denotes the standard deviation of the shock. The quantity of risk is exogenous and a function of shocks alone, whereas the price of risk is potentially endogenous in general equilibrium since the sensitivity of the SDF to the shock, λ, may depend on agents' choices (for example, their diversification opportunities).[11] We explore this setting further in Example 2 below.

Now consider a change in the price of risk, i.e., in $\mathcal{P}$.[12] Such a change induces the following effects on the aggregate expected labor share:

$$\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \mathcal{P}} = \underbrace{\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \mathcal{P}} \frac{WL_i}{\mathbb{E}[Y_i]}}_{\text{reallocation effect}} + \underbrace{\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \mathcal{P}}}_{\text{within effect}}. \tag{8}$$

The first term in (8) is a *reallocation effect*: it captures the contribution of changes in the allocation of resources – and hence output – across firms, conditional on the distribution of firm-level labor shares. The second term is a *within effect*: it captures the contribution of changes in firm-level labor shares, conditional on the distribution of resources.[13]

Examples. For more insight into how the pricing of risk affects the labor share, we need to put more structure on the environment. We consider two examples: first, one in which there are two production technologies, a risky and a safe. Second, we consider a version where Λ and A_i are log-linear functions of an aggregate shock and the loading of A_i's on this shock are normally distributed across firms. In both cases, we show that following a fall in the price of risk, the reallocation effect decreases the aggregate labor share while the within effect increases it. Further, we derive conditions on the price of risk such that the reallocation effect dominates and the aggregate labor share falls as the price of risk does.

Example 1: one risky, one safe technology. There are two types of technologies, i.e., $i \in \{s, r\}$. The safe technology has productivity always equal to its expectation and the risky technology has productivity that is stochastic and negatively correlated with the SDF. The safe technology

[11] For example, in the case of CRRA utility, λ is the product of the coefficient of relative risk aversion and a term capturing the sensitivity of consumption to the realization of the shock. The former is exogenous and the latter endogenous.

[12] More generally, we can represent this as a change in the dynamics of the SDF. Formally, define a function χ that maps the set of exogenous shocks, $\{A_i\}$, to a value of the SDF, i.e., $\chi : \{A_i\} \to \Lambda$. A change in the function χ leads to a change in the price of risk.

[13] A discrete analog to (8) can be written as $\Delta LS_{t+1} = \underbrace{\sum_i \left(\frac{Y_{it+1}}{Y_{t+1}} - \frac{Y_{it}}{Y_t}\right) LS_{it+1}}_{\text{reallocation effect}} + \underbrace{\sum_i \frac{Y_{it}}{Y_t}\left(LS_{it+1} - LS_{it}\right)}_{\text{within effect}}$, which is expression (1).

bears no risk premium and hence $\frac{WL_s}{\mathbb{E}[Y_s]} = \alpha_2$, i.e., the labor share of this type is pinned down by the production technology. The risky technology features a positive risk premium, κ_r, which is increasing in the price of risk.

Proposition 1 formalizes the effects of a change in the price of risk on the aggregate expected labor share:

Proposition 1. *A decrease in the price of risk, $\mathcal{P}$, and thus κ_r implies:*

(i) The within effect increases labor share: $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \kappa_r} = -\alpha_2 \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} < 0.$

(ii) The reallocation effect reduces labor share: $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \kappa_r} \frac{WL_i}{\mathbb{E}[Y_i]} = \frac{\alpha_2 \nu}{1-\nu} \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\kappa_r}{1-\kappa_r} > 0.$

(iii) There exists a threshold $\overline{\kappa_r} > 0$ such that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r} > 0$ iff $\kappa_r > \overline{\kappa_r}$.

Proof. See appendix A.2. □

Part (i) of the proposition shows that as the price of risk falls, so does the implicit cost to firms of insuring workers' wages and hence the labor share within the risky technology increases (the labor share within the safe technology is of course constant). Part (ii) shows that the falling price of risk leads to a reallocation of inputs and output towards the risky sector, increasing its share of economic activity. These two forces act in opposing directions on the aggregate expected labor share: the within effect raises it while the reallocation effect lowers it. Part (iii) of the proposition shows that there exists a threshold, $\overline{\kappa_r}$, such that when the price of risk and thus κ_r are high enough, the reallocation effect dominates and the aggregate expected labor share falls as the price of risk does.

We illustrate these patterns in panel A of Figure 3. At the micro-level, the risky firm's labor share increases as the price of risk falls while that of the safe firm is constant. The risky firm's share of inputs and output also increase in response to the decline in the price of risk. When the price of risk is above the threshold, i.e., $\kappa_r > \overline{\kappa_r}$, the reallocation effect is larger than the within effect and a fall in the price of risk leads to a decline in the aggregate labor share; if $\kappa < \overline{\kappa_r}$, then the opposite holds. The figure also underscores a novel implication of the theory: due to the two competing forces at work, the aggregate labor share is not monotonic in the price of risk. If the price of risk falls sufficiently, the labor share reverses its decline at the threshold, $\overline{\kappa_r}$, and then begins to rise, eventually stabilizing at its share in the production function, α_2. Thus, to extent the observed decline in the labor share is related to a reduction in the price of risk, the theory in fact predicts an eventual recovery.

Example 2: Gaussian shocks and risk exposures. The same intuition applies in environments with richer firm heterogeneity. Assume now that there are a continuum of technologies with heterogeneous exposures to a single aggregate shock, A, and that the SDF is an affine function of the same shock, i.e., we have the following system (where lower-case denotes natural logs):

$$\begin{aligned} a_i &= \mathbb{E}[a_i] + \beta_i a, \qquad \beta_i \sim \mathcal{N}\left(1, \sigma_\beta^2\right), \quad a \sim \mathcal{N}\left(0, \sigma_a^2\right) \\ \lambda &= \mathbb{E}[\lambda] - \lambda a, \qquad \lambda > 0 \,. \end{aligned}$$

The first expression shows that firms differ in their exposure to the aggregate shock, with the degree of this heterogeneity captured by the dispersion in β_i, σ_β^2. By definition, the average exposure is unity. The second expression shows that the SDF is decreasing in the aggregate shock – capturing the usual intuition that marginal utility is countercyclical – with a loading on the shock given by λ. To keep the expressions as transparent as possible, we assume $\log \mathbb{E}[e^{a_i}]$ and β_i are independent. The risk premium for firm i is approximately equal to $\kappa_i = \beta_i \lambda \sigma_a^2$ and the price of risk to $\mathcal{P} = \lambda \sigma_a$.

Proposition 2 proves an analog of Proposition 1:

Proposition 2. *A decrease in λ and thus the price of risk, $\mathcal{P}$, implies:*

(i) The within effect increases labor share: $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \lambda} < 0$ iff $\sum_i \beta_i \frac{L_i}{L} > 0$, i.e., iff the employment-weighted aggregate risk exposure is positive.

(ii) The reallocation effect reduces labor share: $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \lambda} \frac{WL_i}{\mathbb{E}[Y_i]} > 0$.

(iii) There exists a threshold $\overline{\lambda} = \frac{1-\nu}{1+\nu} \frac{1}{\sigma_\beta^2 \sigma_a^2}$ such that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \lambda} > 0$ iff $\lambda > \overline{\lambda}$.

Proof. See appendix A.2. □

The intuition is the same as for Proposition 1. We illustrate the results for this case in Panel B of Figure 3. The left-hand plot shows that the within-firm labor share of risky firms increases as the price of risk falls and the slope is steeper the riskier (i.e., higher beta) is the firm. The middle plot shows as an example the relative share of inputs and output for a firm with $\beta = 1$ compared to a riskless firm with $\beta = 0$.[14] The riskier firm's share of inputs and output increase in response to the decline in the price of risk. Finally, as before, the right-hand plot shows that the aggregate expected labor share first falls then increases with the price of risk.

The results from these examples can be further generalized. For example, in Appendix A.3 we derive an analog of Propositions 1 and 2 assuming only a linear factor structure of risk and no further structure on the distributions of shocks or firm risk exposures. Similar results hold under a log-linear factor structure as well.

[14] The relative share of firms with $\beta_i \neq 1$ is equal to the relative shares of the $\beta = 1$ firm to the power of β_i.

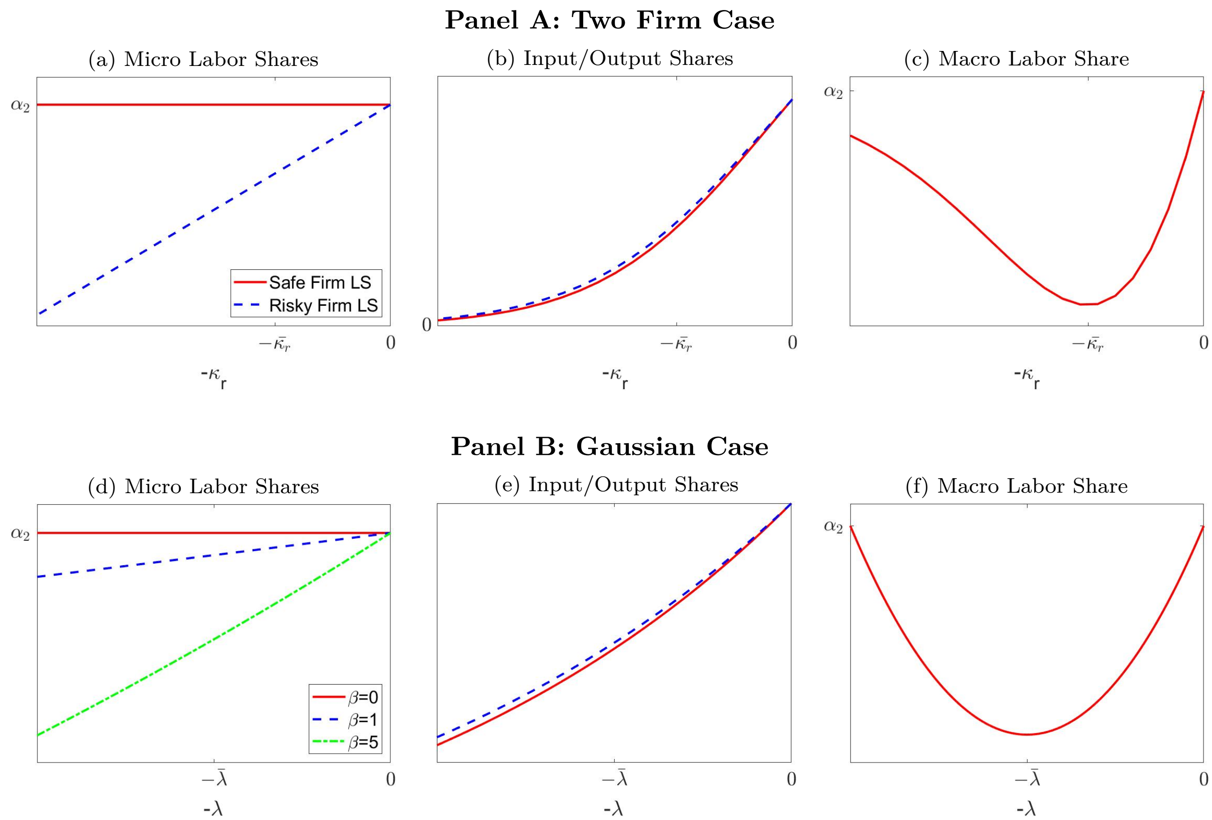

Figure 3: The Price of Risk and the Labor Share

2.3 International Diversification

The previous results illustrated the connection between the price of risk and labor's share of income both at the micro and macro levels. In this section we close the model in general equilibrium and link changes in the price of risk to increasing opportunities for international diversification.

There are a continuum of islands, i.e., "countries," indexed by j. Consumption goods are homogeneous and fully mobile across countries (i.e., there are no frictions on trade in goods). Labor is immobile across countries while financial assets are imperfectly mobile, described in more detail below.

In each country a continuum of firms operate one of two production technologies, a risky and a safe, indexed by $i \in \{s, r\}$, as outlined in Example 1 above. The safe technology has productivity always equal to its expectation, while the productivity of the risky technology depends on the realization of a country-specific shock. For simplicity, we assume these shocks are uncorrelated across countries.

There are two types of agents in each country: workers and capitalists. Workers provide labor for production and consume a portion of the final good; they cannot trade financial assets. Capitalists consume their portion of the final good and can trade a limited set of financial instruments: equity shares in firms, both domestic and foreign, and a risk-free bond. Timing works as follows: in an initial period 0, firms decide the quantity of capital and labor to employ and capitalists receive an endowment that can be either consumed or used for capital investment and/or to purchase financial assets. In the ensuing period (period 1), shocks are realized, production occurs, workers are paid their wages and capitalists their profits and both workers and capitalists consume their shares of production of the final good.

Appendix B.1 lays out in detail the problem of each agent in the economy and formally defines the equilibrium. Here, we give a more brief description and put our focus on the key optimality conditions, namely, the portfolio allocation decisions of capitalists and how they provide a sharp link between the degree of international diversification and the price of risk and hence the labor share.

Worker and firm problems. Workers have utility over consumption and leisure, which they maximize subject to their budget constraint. Because workers cannot trade financial assets they are strictly hand-to-mouth and simply consume their labor income.[15]

The firm's problem is identical to expression (2). In equilibrium, the domestic capitalist will always be the marginal investor for the domestic firms and therefore we can use $\Lambda_j = \rho \frac{U'(C_{j1})}{U'(C_{j0})}$ as the relevant SDF pricing the payouts – and hence determining the input choices – of firms in country j, where ρ denotes the time discount factor, C_{jt} consumption of country j capitalists at time t and $U'(\cdot)$ their marginal utility of consumption.[16]

Capitalist problem. Capitalists in country j have initial shares in domestic and foreign firms, $S_{ij0}\ \forall\ i$ and $S_{jih0}\ \forall\ i, h \neq j$, respectively, and receive an endowment of goods E_j in the first period, which they can consume, sell to firms who transform it into productive capital, or exchange for financial assets, i.e., equity shares in domestic and/or foreign firms and a risk-free bond. The risk-free bond can be traded internationally without frictions and hence has a common rate of return R_f. Crucially, purchasing an equity share of a foreign firm, say in country h, incurs an additional cost equal to a fraction τ_h of the amount invested. Thus, there are limits to international diversification. This cost can be interpreted as an explicit/implicit "tax" or as a reduced-form representation of informational or administrative costs of foreign investment. In the second period, capitalists receive the operating profits firms pay out to their shareholders and use those funds to purchase consumption goods.

[15]Because they are hand-to-mouth, the exact form of the worker utility function plays no role for our results.

[16]Appendix B.1 provides conditions such that in equilibrium the domestic capitalist will always be the marginal investor for domestic firms.

Capitalists have CRRA preferences over consumption in each period.[17] They act to maximize the expected discount sum of utility subject to each period's budget constraint and non-negativity constraints on consumption and firm equity shares (i.e., there is no shorting).

Diversification and the price of risk. It is straightforward to verify that there are two distinct types of equilibria depending on the level of τ_j. In particular, there exists a threshold, τ_j^{aut}, such that if $\tau_j \geq \tau_j^{aut}$ the shares of all country j firms are held by country j capitalists. We call this "financial quasi-autarky" (the risk-free bond may still be traded by that country and/or domestic capitalists may still hold foreign assets). In the quasi-autarky equilibrium, the domestic allocation depends only on domestic preferences, technology and the (common) risk-free rate. In contrast, if $\tau_j < \tau_j^{aut}$, then in equilibrium the shares of the risky firm in country j are held in positive quantities by both domestic and foreign capitalists; shares of the safe firm in country j are never held by foreign capitalists when $\tau_j > 0$. Note that if $\tau_j = 0$, there are no limits to trade in international equity assets and capitalists achieve complete diversification. In this case, the economy features risk neutral pricing, i.e., the price of risk and all risk premia are zero.

In an interior equilibrium where $\tau_j < \tau_j^{aut}$, the optimality conditions for domestic and foreign capitalists' (from some country h) holdings of domestic firm shares yield the following expressions for the date 0 share price of the firm, which must both hold jointly in equilibrium:

$$P_{rj} = \mathbb{E}\left[\Lambda_j V_{rj}\right], \qquad P_{rj}\left(1+\tau_j\right) = \mathbb{E}\left[\Lambda_h V_{rj}\right] \ , \tag{9}$$

where $V_{rj} = A_{rj} K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} - W_j L_{rj} - R_j K_{rj}$ denotes the ex-post value of the firm, which is simply equal to its cash flows. Intuitively, if both foreign and domestic investors own shares of the risky firm then their cost-adjusted valuations of the firm must be equal. The valuation of the domestic capitalist can be expressed as the riskless discounted value of the firm (by the common risk-free interest rate) less a risk premium due to the covariance of the domestic SDF with the firm's productivity and so its cash flows. The foreign investor's valuation, on the other hand, is equal to the risk-free discounted value adjusted for the cost of foreign investment: the technology bears no risk premium for the foreign investor from country h because in equilibrium their SDF is independent of country j productivity.[18]

[17]We make this assumption largely for simplicity. We can prove many of our results under the weaker restrictions that the capitalist utility function is a continuous, increasing and concave function of consumption.

[18]Specifically, we can rewrite the first condition in (9) as $P_{rj} = \frac{\mathbb{E}[V_{rj}]}{R^f} + \text{cov}\left(\Lambda_j, V_{rj}\right)$ and the second condition as $P_{rj} = \frac{1}{1+\tau_j}\frac{\mathbb{E}[V_{rj}]}{R_f}$.

Combining these expressions yields the following condition:

$$\kappa_{rj} = -\text{cov}\left(\frac{\Lambda_j}{\mathbb{E}\left[\Lambda_j\right]}, \frac{A_{rj}}{\mathbb{E}\left[A_{rj}\right]}\right) = \frac{\tau_j\left(1-\nu\right)}{1+\tau_j\left(1-\nu\right)}\ , \tag{10}$$

which shows that the risk premium in country j is pinned down by the cost of investment to foreign investors. It is easily verified that $\frac{\partial \kappa_{rj}}{\partial \tau_j} > 0$. As the cost to foreign investors falls, they increase their demand for domestic risky firm shares. In equilibrium, market clearing necessitates that domestic capitalists reduce their holdings of these shares as a fraction of their portfolio and thus their consumption and SDF become less sensitive to the realization of productivity of the domestic risky firm. It follows that the covariance of the capitalist SDF with the domestic productivity shock falls and hence so does the price of risk.[19]

Diversification and the labor share. We apply (10) to the results from section 2.1 to derive expressions for expected micro and macro labor shares and the labor allocation as functions of τ_j for $\tau_j \in [0, \tau_j^{aut})$. At the micro-level, the risky firm expected labor share is given by

$$\frac{L_{rj}W_j}{E\left[Y_{rj}\right]} = \alpha_2 \frac{1}{1+\tau_j\left(1-\nu\right)}\ ,$$

and, as before, that of the safe firm is simply equal to α_2. As τ_j falls, so does the risk premium, increasing the expected labor share for the risky technology.

The relative allocation of labor to the risky firm satisfies

$$\frac{L_{rj}}{L_j} = \frac{\left(\mathbb{E}\left[A_{rj}\right]\frac{1}{1+\tau_j(1-\nu)}\right)^{\frac{1}{1-\nu}}}{\mathbb{E}\left[A_{sj}\right]^{\frac{1}{1-\nu}} + \left(\mathbb{E}\left[A_{rj}\right]\frac{1}{1+\tau_j(1-\nu)}\right)^{\frac{1}{1-\nu}}}\ .$$

A fall in τ_j reduces the risk premium and leads to a reallocation of resources towards the risky technology. Thus, increasing diversification induces both the within and reallocation effects on the aggregate labor share.

Last, the aggregate expected labor share is given by:

$$\frac{W_jL_j}{\mathbb{E}\left[Y_j\right]} = \alpha_2 \frac{1+\left(\frac{\mathbb{E}[A_{rj}]}{\mathbb{E}[A_{sj}]}\frac{1}{1+\tau_j(1-\nu)}\right)^{\frac{1}{1-\nu}}}{1+\left(\frac{\mathbb{E}[A_{rj}]}{\mathbb{E}[A_{sj}]}\right)^{\frac{1}{1-\nu}}\left(\frac{1}{1+\tau_j(1-\nu)}\right)^{\frac{\nu}{1-\nu}}}\ .$$

[19] Although the price of risk unambiguously falls with growing diversification, due to the reallocation of inputs to the risky sector, the aggregate equity premium may not. Thus, the theory does not necessarily predict that observed measures of aggregate risk premia fall as diversification rises, but rather, can be consistent with increasing aggregate risk premia, which has been found in several recent papers, e.g., Farhi and Gourio (2018).

Proposition 3 formalizes the effects of changes in diversification opportunities on the aggregate expected labor share:

Proposition 3. *For $\tau_j < \tau_j^{aut}$, a decrease in the cost of foreign investment, τ_j, and thus the risk premium, κ_{rj}, implies:*

(i) Domestic (foreign) holdings of the risky firm fall (rise) if $\frac{S_{rj1}}{S_{rj0}} > 1-\nu$, i.e., if the domestic capitalist share of the domestic risky firm is not too small.[20]

(ii) The within effect increases labor share: $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \tau_j} < 0$.

(iii) The reallocation effect decreases labor share: $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \tau_j} \frac{WL_i}{\mathbb{E}[Y_i]} > 0$.

(iv) There exists a threshold $\overline{\tau_j}$ such that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \tau_j} > 0$ iff $\tau_j > \overline{\tau_j}$.[21]

Proof. See Appendix B.2. □

Parts (ii)-(iv) of the proposition are the analogs of Propositions 1 and 2, but now link changes in the labor share directly to changes in the primitive parameter determining diversification opportunities. Part (i) of the proposition is important for our empirical implementation below and shows that foreign holdings of domestic risky firms monotonically increase as the cost of diversification falls. As the cost for foreign investors falls, their demand for (and thus holdings of) domestic risky firms rises; this reduces the shares of these firms held by domestic capitalists and hence the sensitivity of their consumption to domestic shocks, lowering risk premia. The key implication for our purposes is that we can use (observable) foreign holdings of domestic risky assets as a proxy for the (unobservable) cost of trade in international financial securities.

Diversification, the labor share and worker welfare. Proposition 3 shows that the labor share is non-monotonic in the extent of international diversification, at first falling as diversification increases but eventually reaching a threshold where it reverses its decline and rises with diversification. A natural question is: what are the implications for worker welfare? Or put another way, is the labor share an accurate gauge of worker welfare in this environment? Corollary 3.1 shows that the answer to the latter question is: not necessarily.

[20] Note that the condition is sufficient, but not necessary. More specifically, the condition states that the domestic capitalist not sell too large a share of her initial endowment of domestic risky firm shares. Given the degree of home bias in the data and the fact that typical estimates are ν are large and close to one, the condition is likely to hold.

[21] In some parameterizations (i.e., if there is relatively little risk or capitalists are close to risk neutral) it is possible that $\overline{\tau_j} > \tau_j^{aut}$.

Corollary 3.1. *If the inverse Frisch elasticity of labor supply is positive (non-negative), then worker welfare is strictly (weakly) monotonically increasing in diversification.*

Proof. See Appendix B.3. □

In contrast to the labor share, worker welfare is everywhere increasing with the extent of diversification. The result implies that there is a region where the labor share and worker welfare move in opposite directions and hence are negatively correlated: the former declines as the latter rises. Importantly, however, this does not mean that a declining labor share is necessarily indicative of rising welfare: the negative relationship we would observe in the data is observational, not structural, and there is another region where the two are positively correlated. The key takeaway from the result is that studying movements in the labor share alone may not be sufficient to assess changes in worker welfare since the two may not move in tandem; instead, understanding the underlying causes of changes in the labor share is crucial to reach accurate normative conclusions.

3 Empirical Predictions and Quantification

In this section we lay out two key micro-level empirical implications of the theory. We then show how we can use these empirical estimates along with the other equations of the model to quantify the implications of growing financial integration on firm-level labor shares (the within effect), the allocation of labor across firms (the reallocation effect) and the aggregate labor share.

3.1 Empirical Predictions

The theory suggests two key empirical relationships at the micro-level:

Prediction 1: risky firms have lower labor shares. In an important building block of our theory, expression (3) implies that riskier firms should have lower labor shares. In Appendix C, we show that, assuming a linear factor structure of risk, a linear approximation to the firm's optimality condition yields a negative relationship between firm-level labor shares and observed equity market betas, which proxy for the firm's exposure to aggregate risk:

$$\log \frac{WL_{ij}}{Y_{ij}} \approx \log \alpha_2 - \beta^e_{ij}\overline{\kappa_j}c_j - \log \mathcal{E}_{ij} \ , \tag{11}$$

where β^e_{ij} denotes the equity market beta, i.e., the exposure of firm i in country j stock market returns to the country j aggregate market return, which is proportional to the firm's fundamental beta with a factor of proportionality equal to c_j, specifically, $\beta_{ij} = c_j \beta^e_{ij}$. The term $\overline{\kappa_j}$ is proportional to the price of risk and $\mathcal{E}_{ij} = \frac{A_{ij}}{\mathbb{E}[A_{ij}]}$ captures the realization of unanticipated shocks. The expression suggests the following regression specification:

$$\log LS_{ij} = \gamma_0 + \gamma_\beta \beta^e_{ij} + \varepsilon_{ij} \ , \tag{12}$$

where $\gamma_\beta = -\overline{\kappa} c$ and $\varepsilon_{ij} = -\log \mathcal{E}_{ij}$.[22] The theory predicts that the coefficient $\gamma_\beta < 0$, i.e., firms with high stock market betas have low labor shares, and the magnitude of γ_β is informative about the price of labor market risk.

Prediction 2: international diversification is associated with a reallocation towards risky/low labor share firms. Expressions (4) and (6) imply that firm risk premia affect the allocation of resources across firms and firm shares of expected and realized output. Proposition 3 further shows that increasing international diversification, and hence risk-sharing, reduces the price of country-specific risk and leads to a reallocation of labor and output towards risky/low labor share firms. In Appendix C, we show that, again assuming a linear factor structure of risk, we can derive the following linear approximation to a firm's share of total industry inputs/output as a function of the firm's expected productivity and equity market beta:

$$\begin{aligned} \log \frac{L_{ij}}{L_j} &\propto \frac{1}{1-\nu} \log \mathbb{E}\left[A_{ij}\right] - \frac{1}{1-\nu} \beta^e_{ij} \overline{\kappa_j} c_j \\ \log \frac{\mathbb{E}\left[Y_{ij}\right]}{\mathbb{E}\left[Y_j\right]} &\propto \frac{1}{1-\nu} \log \mathbb{E}\left[A_{ij}\right] - \frac{\nu}{1-\nu} \beta^e_{ij} \overline{\kappa_j} c_j \ . \end{aligned} \tag{13}$$

Part (i) of Proposition 3 proves that the price of risk, $\overline{\kappa_j}$, is monotonically decreasing in the foreign equity share, i.e., the value of foreign investors' holdings of domestic equity as a fraction of the total value of domestic equity, denoted FES_j, and making this substitution yields regression specifications of the form

$$\log \frac{Z_{ij}}{\overline{Z}_j} = \gamma_0 + \gamma_{\beta,FES} \beta^e_{ij} \times \log FES_j + \varepsilon_{ij} \ , \tag{14}$$

where $Z = \{\mathbb{E}\left[Y\right], L\}$ denotes labor or expected output and $\overline{Z}$ their respective industry totals.[23] The expression shows that firm shares of aggregate labor and output are in part determined by the interaction of FES, which captures international risk-sharing and is inversely related to

[22] The theory implies that the coefficient γ_β may differ across countries. In practice, Section 4 estimates γ_β separately for the US and a pooled set of non-US developed countries.

[23] The equation also holds for realized output, with the addition of an error term capturing the realization of unanticipated shocks, $\log \mathcal{E}_{ij} - \log \mathcal{E}_j$.

the price of risk, with stock market betas. The theory predicts that the coefficient $\gamma_{\beta,FES} > 0$, i.e., growing international diversification, measured by the foreign equity share, by reducing the price of risk, leads to a reallocation of inputs and output to risky/high beta firms.

Robustness. Expressions (12) and (14) continue to hold under various assumptions in the theory, such as the shape of the production function or the boundary of the firm. For example, Appendix A.1 develops an analog of (3), (4) and (6) (and hence (12) and (14)) under more general production functions, including a more flexible CES between capital and labor; that appendix also shows that versions of these expressions go through in an environment where firms operate multiple technologies (with heterogeneous levels of risk), rather than a single one. Thus, expressions (12) and (14) are robust predictions of the theory.

Importantly, a similar robustness does not hold for all the predictions of even our simple model, suggesting care must be taken when investigating its empirical implications. For example, in the case where firms operate multiple technologies, Appendix A.1 shows that part (ii) of Proposition 3 holds only at the level of a single technology; the within effect of international diversification and changes in the price of risk on labor shares at the firm-level, which is then made up of a bundle of technologies, is ambiguous, since there is an accompanying reallocation of resources across these technologies within the firm. This within-firm reallocation effect is not measurable with firm-level data alone. These robustness properties in large part motivate our focus on Predictions 1 and 2. Further, as we show next, we can use the empirical estimates from equations (12) and (14) along with the structural equations of the model to quantify the impact of increasing diversification on the aggregate labor share.

3.2 Quantification

The firm-level empirical specifications in expressions (12) and (14) can be combined to yield estimates of the impact of international diversification on the aggregate labor share, and further, to separate its effects through reallocation vs. changing labor shares for each technology.

Reallocation effect. Guided by expression (8) we can write the reallocation effect as the change in the aggregate labor share due to changes in the output shares of firms, i.e.,

$$Realloc_t = \sum_i \left(\Delta \frac{Y_{it}}{Y_t} \right) LS_{it} \ ,$$

where Δ denotes the first-difference operator, i.e., $\Delta X_t = X_t - X_{t-1}$ for any variable X. Appendix C shows that this can be approximated as

$$Realloc_t \approx \overline{LS}_t \text{cov}\left(\Delta \log \frac{Y_{it}}{Y_t}, \log LS_{it}\right) , \tag{15}$$

where $\overline{LS}_t$ is the unweighted mean labor share. The expression relates the magnitude of the reallocation effect to the mean labor share and the covariance of the change in firm output shares with firm labor shares.

Finally, using the expression for $\log LS_i$ in (12) and for $\log \frac{Y_i}{Y}$ in (14) implies

$$\text{cov}\left(\Delta \log \frac{Y_{it}}{Y_t}, \log LS_{it}\right) = \gamma_\beta \gamma_{\beta,FES} \text{var}\left(\beta_i^e\right) \Delta \log FES_t ,$$

where $\text{var}\left(\beta_i^e\right)$ is the cross-sectional variance of firm stock market betas, $\Delta \log FES_t$ is the percentage change in the foreign equity share, and $\gamma_\beta, \gamma_{\beta,FES}$ are the regression coefficients estimated in (12) and (14), respectively. Thus, we have a simple expression to quantify the reallocation effect as a function of observable statistics – the mean labor share, the dispersion in stock market betas and the observed changes in foreign holdings of domestic assets – as well as the estimated regression coefficients from predictions 1 and 2:

$$Realloc_t \approx \overline{LS}_t \gamma_\beta \gamma_{\beta,FES} \text{var}\left(\beta_i^e\right) \Delta \log FES_t . \tag{16}$$

The expression shows that the magnitude of the reallocation effect induced by an increase in the foreign equity share – and hence, a decrease in the price of risk – depends on (i) the strength of the relationship between risk exposure and labor share, γ_β, (ii) how responsive the allocation is to changes in the foreign equity share and the price of risk, $\gamma_{\beta,FES}$, and (iii) the extent of such reallocation opportunities, captured by the cross-sectional dispersion in betas, $\text{var}\left(\beta_i^e\right)$.

Within effect. We can similarly use the estimated coefficients from (12) and (14) to quantify the magnitude of the within effect. Again guided by (8) and assuming a linear factor structure of risk, we can write the within effect as the change in the aggregate labor share due to changes in firm-level labor shares, i.e.,

$$Within_t = \sum_i \frac{Y_{it}}{Y_t} \Delta LS_{it} = -\left(\Delta \overline{\kappa}_t\right) \alpha_2 \overline{\beta}_t ,$$

where $\overline{\beta}_t = \sum_i \frac{Y_{i,t}}{Y_t} \beta_i$ is the output-share weighted average beta.

Appendix C derives the following two approximations linking changes in the price of risk,

$\Delta\overline{\kappa}_t$, and the output-share weighted average beta, $\overline{\beta}_t$, to a single parameter and four observable moments that we estimate or calibrate, namely, the regression coefficients from predictions 1 and 2, γ_β and $\gamma_{\beta,FES}$, the change in the foreign equity share, the level of the market equity premium, and the parameter ν, which governs returns to scale:

$$\frac{\Delta\overline{\kappa}}{\overline{\kappa}} \approx \frac{1-\nu}{\nu}\frac{\gamma_{\beta,FES}}{\gamma_\beta}\Delta \log FES \tag{17}$$

$$\overline{\kappa}\overline{\beta} \approx (1-\nu)\left(\mathbb{E}\left[r^e\right] - r_f\right) . \tag{18}$$

Expression (17) shows that the stronger is the reallocation effect (bigger $\gamma_{\beta,FES}\Delta FES$), the larger is the change in the price of risk, while the stronger is the relation between firm risk exposure and labor share (bigger γ_β), the larger is the level of the price of risk. Expression (18) links the price of risk and an appropriate average of the quantity of risk to the market equity premium. Combining (17) and (18) yields an expression for the within effect as a function of the two regression coefficients γ_β and $\gamma_{\beta,FES}$, the change in the foreign equity share, the level of the equity premium, and the production parameters, α_2 and ν:

$$within_t \approx -\alpha_2 \frac{(1-\nu)^2}{\nu}\frac{\gamma_{\beta,FES}}{\gamma_\beta}\left(\mathbb{E}\left[r^e_t\right] - r_{ft}\right)\Delta \log FES_t . \tag{19}$$

4 Empirical Analysis

In this section, we assess the empirical predictions of the theory described in Section 3 and use the estimates to quantify the implications of increasing diversification on the aggregate labor share through both within-firm and reallocation effects.

4.1 Data and Measurement

Our analysis combines a number of datasets. At the country level, we measure international diversification by calculating the share of domestic equity held by foreign investors. For each country, the total value of foreign investors' equity holdings – i.e., foreign equity liabilities – is obtained from the External Wealth of Nations database (Lane and Milesi-Ferretti, 2018) as the sum of foreign equity holdings via portfolio investment and FDI. This measure includes foreign holdings of equity in both publicly traded and non-publicly traded firms. We compute the total value of domestic equity for both public and private firms as the product of the total earnings of the business sector, measured as total operating surplus of businesses from OECD national accounts, and the price/earnings ratio, calculated using Compustat stock market data for each country. This calculation yields an estimate of the total value of domestic equity for both publicly traded and private firms. Dividing the value of foreign equity liabilities by the total value of domestic equity gives the share of domestic equity held by foreign investors, which corresponds to variable FES.[24] We obtain cross-country panel data on aggregate labor shares from the OECD.[25]

We use three firm-level datasets. The first is annual data on firm accounting statements and stock returns for US publicly traded firms from the merged Compustat/CRSP database. We use data from the period 1973-2019. The Compustat/CRSP data enable us to compute measures of firm risk exposures from high quality financial market data with relatively good coverage (at least among large, publicly traded firms). Specifically, we proxy for these exposures using firms' stock market (CAPM) betas, as suggested by the empirical specifications derived in Section 3. We estimate firm betas using regressions of firm-level daily stock market returns on the daily aggregate market return for all trading days within a calendar year.[26] By definition, a market capitalization weighted average of firm-level betas is always equal to one, which implies that a reallocation towards riskier firms, by making the market portfolio itself riskier, would reduce measured betas, despite the fact that firm-level technologies are unchanged. This poses a challenge for our predictions relating to reallocation. To this end, we construct a measure of *relative* beta for each firm by residualizing the estimated stock market betas on industry-year fixed effects and computing the average of these residuals by firm. From hereon, we use these relative stock market betas as the main measure of firm risk exposure in our empirical analyses.

Measuring firm-level labor shares for US firms from the Compustat data presents challenges – while the number of employees is reported for most firms in Compustat, only a small share of US firms report labor compensation. Thus, we can directly compute the labor share for only about 8% of US firms. For this set of firms, we calculate "reported" labor share as

$$LS_{it} = \frac{\text{LABEX}_{it}}{\text{OIBDP}_{it} + \Delta\text{INVFG}_{it} + \text{LABEX}_{it}}\,, \tag{20}$$

i.e., as the share of labor expense in value-added, where LABEX denotes labor expense (Compustat variable XLR), OIBDP operating income before depreciation and ΔINVFG change in inventories of finished goods.

[24] Our calculations implicitly assume that privately held firms have similar valuation ratios (i.e., price/earnings ratios) as publicly traded firms; Lane and Milesi-Ferretti (2018) similarly use market prices of publicly traded firms to calculate changes in the value of foreign equity liabilities of both public and private firms. Thus, our measure of FES relies on the valuation of publicly traded firms to calculate both the numerator and denominator. Full details are in appendix D.1. Appendix D.5 shows that our results are not sensitive to this assumption, for example, normalizing foreign equity liabilities by stock market capitalization or GDP yields similar results.

[25] Data on labor share through 2020 were obtained from Haver Analytics.

[26] We obtain the daily market return from Ken French's data library, available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Due to the small set of firms that report labor compensation, we compute two additional measures of labor share for US firms: the first is labor intensity, calculated as total employees divided by sales, denoted $\frac{L}{Y}$. Because all of our analyses include industry fixed-effects, labor intensity is proportional to labor share if all firms within an industry have the same average compensation per employee.[27] Our second additional measure follows the approach laid out in Donangelo et al. (2018), who infer labor compensation per employee for firms with missing data from the industry-level average of firms that do report compensation. Following Donangelo et al. (2018), we call this measure "extended" labor share, denoted ELS.[28]

We obtain analogous data for foreign firms from Compustat Global. The data cover the period 1987-2019. We include developed countries for which at least 500 firms report data in at least one year. The resulting set of countries are Australia, Germany, France, the UK, Japan, Korea, Singapore, Sweden and Taiwan.[29] We estimate stock market betas for these firms following a similar approach as for the US, using regressions of firm-level daily stock market returns on the daily market return for each respective country for all trading days within a calendar year.[30] We construct firm relative betas by residualizing the stock market betas on country-industry-year fixed effects and computing the average of the residuals for each firm. Labor compensation is much more widely reported in Compustat Global (as compared to Compustat US), particularly for European firms. Thus, we can directly calculate the labor share for each firm as the ratio of labor compensation to value-added (calculated the same way as in the US). Table 1 displays summary statistics of our sample, including the number of observations which report sufficient data to directly calculate the labor share. While only a small fraction of US firms report the necessary variables to compute the labor share, a majority in the European countries – France, Germany, and the UK – do.[31]

[27] We focus primarily on differences in labor shares across firms within an industry since labor shares can vary across industries for a variety of reasons, e.g., differences in production technologies. Moreover, previous work has documented that the labor share decline is foremost a within – rather than across – industry phenomenon (e.g., Kehrig and Vincent, 2021).

[28] Specifically, for the firms that do not report labor expense, labor expense is imputed as $\text{EMP}_{it} \times \text{avg.}\left(\frac{\text{LABEX}_{it}}{\text{EMP}_{it}}\right)$, where the latter term is the average compensation per employee for firms that report the necessary data. The imputed labor expense is then used in (20) to calculate labor share for firms that do not report labor expense. Donangelo et al. (2018) show that both reported and extended labor shares in the Compustat data are highly correlated with labor shares calculated from Census data, noting that the result "...validates our Compustat-based labor share measure and suggests that future researchers who are unable to access Census data can use the proxies we construct from Compustat."

[29] Since many of our specifications account for country-industry-year fixed effects, countries with few observations would likely be absorbed in those specifications. Very few firms in Japan and Korea report labor compensation (so we cannot compute firm-level labor shares), but we include these countries in specifications not involving labor share.

[30] We obtain country-level market returns from Wharton Research Data Services (WRDS), which computes market return indices for each country in the Compustat Global database.

[31] Japan and Korea have very few firms reporting labor share, but have a large number of firms that enter specifications without labor share, such as the reallocation regressions.

Table 1: Summary Statistics

Country	AUS	DEU	FRA	GBR	JPN	KOR	SGP	SWE	TWN	USA
Observations	28,452	14,881	15,384	32,050	68,554	17,582	10,640	10,811	28,440	202,675
— With LS_{it}	5,520	9,898	10,508	18,704	12	38	4,340	4,224	2,142	19,622
— With β_{it}	19,926	9,748	9,649	21,845	61,067	16,211	7,404	7,169	23,574	189,714
Unique Firms	2,563	1,118	1,190	2,978	4,389	2,057	748	1,001	2,117	17,565
Mean β_{it}	0.727	0.481	0.466	0.434	0.648	1.049	0.739	0.629	0.839	0.807
S.D. β_{it}	0.535	0.489	0.456	0.449	0.437	0.413	0.486	0.432	0.396	0.575
S.D. Relative β_i	0.253	0.272	0.260	0.245	0.252	0.220	0.230	0.259	0.257	0.354
Mean LS_{it}	0.500	0.618	0.625	0.494	0.455	0.270	0.495	0.614	0.478	0.596
S.D. LS_{it}	0.282	0.242	0.234	0.279	0.073	0.176	0.232	0.262	0.241	0.200

Notes: Table reports summary statistics of firm-level variables for the countries in our sample. An observation is a firm-year. We report the unweighted mean and standard deviation of firm market betas (on the respective country market excess return), β_{it}, as well as labor shares, LS_{it}. We report the standard deviation of the firm's relative beta, the firm-level fixed effect after controlling for industry-year fixed effects; by construction the mean relative beta is zero. We omit observations with labor shares in excess of 1 or below 0, and β_{it} is trimmed at the 2% level.

The third firm-level dataset we use is cross-country panel data from the Orbis database. Orbis includes both public and private firms. The data cover the period 1980-2019. We calculate firm-level labor shares as the ratio of labor expenses (cost of employees) to value-added, which we measure as the sum of operating profits (EBIT) and labor expenses. We include countries for which at least 10,000 firms report data in at least one year. We include advanced economies not classified as tax havens or financial centers by Lane and Milesi-Ferretti (2018). The final Orbis sample contains 27 countries.

4.2 Firm-level Risk and Labor Share

Prediction 1 implies that firms that are more exposed to aggregate risk have, on average, lower labor shares. Figure 4 presents bin-scatter plots of firm relative betas – our measure of risk exposure – against labor shares. Panel (a) displays the results for US firms and panel (b) displays results for foreign firms. For US firms, we plot extended labor share, though a similar figure obtains using the other measures of labor share described above. For the US, we residualize both firm betas and labor shares on industry-time fixed-effects, and for foreign firms we residualize on country-industry-time fixed-effects.[32] The figure illustrates that both in the US and abroad, higher beta firms within a given country/industry/year have lower labor shares.

Tables 2 and 3 investigate this relationship more formally by estimating regressions of firm

[32]Bin-scatter plots have been widely used in applied microeconomics to visualize relationships between variables in large datasets since Chetty and Szeidl (2005) and Chetty et al. (2009). With controls, this procedure first residualizes both the x and y variables on the fixed-effects and then adds back the unconditional means. Each point in the figure then represents the average of the residualized x and y variable for percentile bins of the x variable.

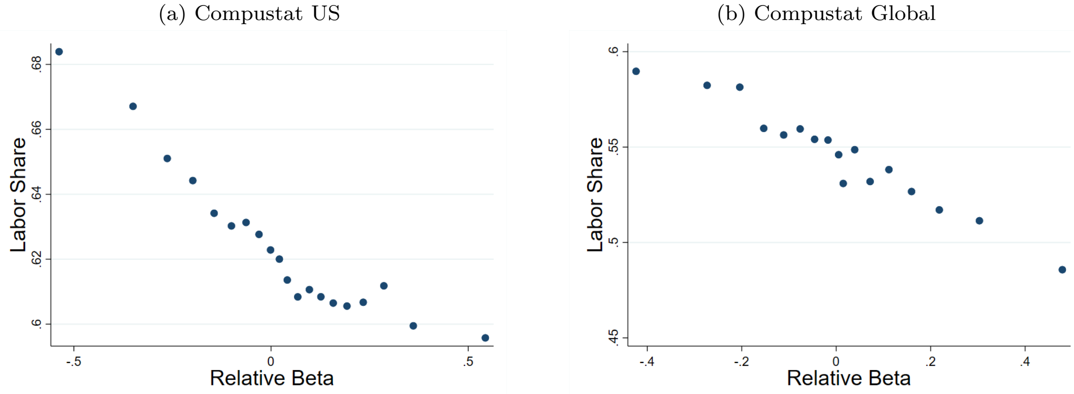

Figure 4: Firm-Level Risk Exposures and Labor Shares

Notes: Figure displays bin-scatter plots of firm-level labor shares and relative betas, for US firms in panel (a) and foreign firms in panel (b). Panel (a) controls for industry-year fixed-effects and panel (b) for country-industry-year fixed-effects. Each point in the figure is a percentile bin of firm betas.

Table 2: Firm-Level Risk Exposure and Labor Share – US Firms

	(1) $\log \frac{L}{Y}$	(2) $\log ELS$	(3) $\log LS$	(4) $\log \frac{L}{Y}$	(5) $\log ELS$	(6) $\log LS$
Relative Beta	-0.227***	-0.169***	-0.0804**	-0.277***	-0.252***	-0.210***
	(-12.94)	(-11.65)	(-2.56)	(-15.68)	(-16.03)	(-6.28)
Industry-Year F.E.	yes	yes	yes	yes	yes	yes
Controls				yes	yes	yes
R^2	0.674	0.551	0.718	0.682	0.570	0.758
Observations	168999	122866	12237	163720	118291	10864

Notes: Table reports regressions of (log) firm labor shares on firm relative betas and controls. We present results for three measures of labor share in Compustat US: labor intensity ($\frac{L}{Y}$), extended labor share (ELS), and reported labor share (LS). We trim all measures at the 2% level. Controls include firm size and age. Data are from Compustat/CRSP. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

labor shares on relative betas as laid out in expression (12) for US and foreign firms, respectively. We include controls for firm age and size and industry-year fixed effects. Table 2 reports results for US firms for the three measures of labor share introduced above, namely, labor intensity, extended labor share and reported labor share for the subset of firms that report labor compensation. The association between firm beta and labor share is both statistically and economically significant: for example, the estimated coefficients imply that a one standard deviation increase in firm beta is associated with a reduction in firm labor share of between about 7% and 10%, or between 4 and 6 percentage points.[33]

Table 3 reports analogous regressions for foreign firms with and without controls and with various sets of fixed-effects. Across these specifications, the results are qualitatively similar to

[33]From Table 1, the cross-sectional standard deviation of relative betas in the US is 0.354. Multiplying this by the coefficients in columns (4)-(6) of Table 2 yields a 7-10% lower labor share. To translate into percentage points, we multiply the percentage change by 0.6, which is approximately the median labor share of US firms in the sample.

Table 3: Firm-Level Risk Exposure and Labor Share – Foreign Firms

	(1)	(2)	(3)	(4)	(5)	(6)
Relative Beta	-0.630***	-0.612***	-0.552***	-0.497***	-0.479***	-0.447***
	(-5.59)	(-5.72)	(-5.49)	(-5.31)	(-5.14)	(-4.79)
F.E.	*yr*	*ctry* × *yr*	*ind* × *ctry* × *yr*	*yr*	*ctry* × *yr*	*ind* × *ctry* × *yr*
Controls				yes	yes	yes
R^2	0.0996	0.169	0.485	0.148	0.209	0.529
Observations	47050	47038	37582	33000	32988	24880

Notes: Table reports regressions of (log) firm labor shares on firm relative betas and controls. We trim all measures at the 2% level. Controls include firm size and age. Data are from Compustat Global. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

the US and imply a significant relationship between firm beta and labor share, both statistically and economically: for example, following an analogous calculation as in the US, the estimated coefficients imply that a one standard deviation increase in firm beta is associated with a reduction in labor share of about 11%.

Additional specifications. Appendix D.4 reports analogous results to Tables 2 and 3 under various assumptions and slices of the data. In particular, Tables 6 and 7 in that appendix show that similar results hold using time-varying measures of beta along with firm fixed-effects, which allow us to control for unobserved persistent firm characteristics that may be correlated with both labor share and beta; Table 8 reports regressions of firm labor share on both country and global risk exposure for firms in Compustat Global and shows that, even controlling for exposure to global risk, firms with greater exposure to country risk have lower labor shares;[34] Tables 9 and 10 show that the link between risk and labor share is not driven by so-called "superstar" firms.[35]

4.3 International Diversification and Reallocation

Prediction 2 implies that increasing international diversification leads to a reallocation of resources towards risky/low labor share firms. Figure 5 presents bin-scatter plots of annual firm-level employment growth against either firm beta or labor share. Panels (a) and (c) display the results for US firms and panels (b) and (d) displays results for foreign firms. The figure illustrates that riskier/lower labor share firms have grown substantially more than safer/higher labor share ones, implying that there has been a reallocation of employment towards the former set of firms.

[34] Consistent with a multi-factor extension of our model, exposures to both country and global risk are associated with lower labor shares.

[35] We follow Kroen, Liu, Mian, and Sufi (2021) and define superstar firms as the top 5% of firms within each Fama-French industry (in a given country-year).

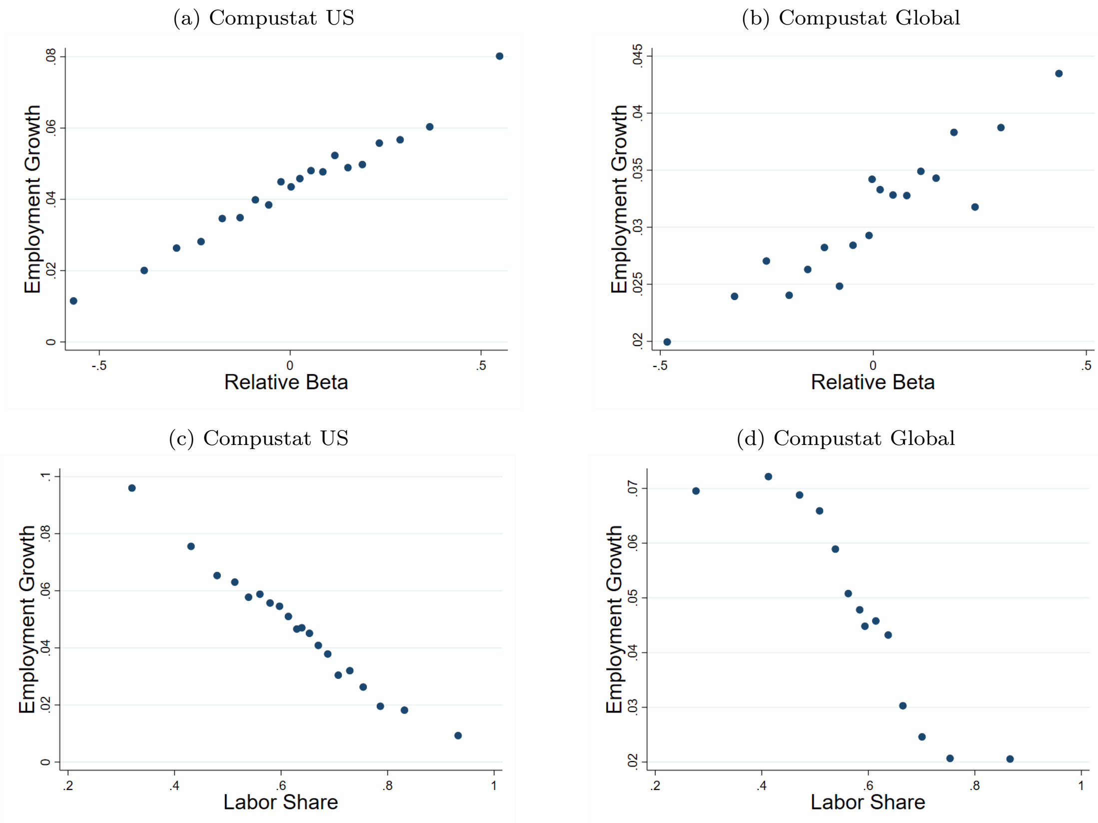

Figure 5: Risk Exposure, Labor Share and Employment Reallocation

Notes: Figure displays bin-scatter plots of annual firm-level employment growth rates and relative betas/labor shares. Labor shares are time-varying and are lagged by one year relative to employment growth. Each point in the figure is a percentile bin of firm betas or labor shares.

Table 4 formally investigates whether increases in international diversification, measured as the share of domestic firm equity held by foreign investors, leads to a reallocation of input and output shares to riskier firms by estimating regressions of firm shares of industry employment/sales on the interaction of the (log) foreign equity share with firm relative beta as implied by (14). The key implication of the theory is that when the foreign equity share increases, riskier firms should become larger, measured both by input utilization (e.g., employment) and sales. We include both firm fixed-effects and country-industry-year fixed effects (industry-year for the US) and control for firm age to account for possible life-cycle effects. The table shows positive and significant coefficients across specifications, implying that, in line with the theory, a larger foreign equity share (fraction of domestic equity held abroad) is associated with a reallocation of both inputs and output to riskier firms, which then make up a larger share of industry employment and sales.

The coefficient estimates in Table 4 are not only statistically significant, but also economically meaningful. For example, the estimates in the US imply that the increase in the foreign

Table 4: International Diversification and Reallocation

	US Firms		Foreign Firms	
	(1)	(2)	(3)	(4)
	L	Y	L	Y
Relative Beta × log FES	0.938***	1.147***	0.279***	0.256***
	(12.95)	(14.80)	(5.23)	(4.34)
Ind × Ctry × Yr F.E.	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes
Controls	yes	yes	yes	yes
R^2	0.940	0.942	0.968	0.981
Observations	170515	171231	96395	142731

Notes: Table reports regressions of firm log share of industry employment (L) and sales (Y) on the interaction of firm relative beta with the log foreign equity share. Columns (1) and (2) report results using CRSP/Compustat data for US firms and columns (3) and (4) report results using Compustat Global for foreign firms. We trim all measures at the 2% level. Standard errors are clustered two ways by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

equity share of about 2.0 log points from 6.6% in 1973 to 48.7% in 2019 leads a firm with a beta one standard deviation above the median to increase its share of industry employment (sales) by about 1.4% (1.7%) per year as compared to the median firm. In the global sample, the increase in the foreign equity share of about 0.75 log points from 17.4% in 1987 (the start of the Compustat global sample and thus our measure of total equity value and foreign equity share) to 36.9% in 2019 implies that a firm with a beta one standard deviation above the median increased its share of both industry employment and sales by about 0.3% per year as compared to the median firm.

Additional specifications. Appendix D.5 reports analogous results to Table 4 under a number of alternative specifications. First, Table 11 shows that similar results hold using various measures of the intensity of foreign equity ownership including foreign equity liabilities normalized by domestic market capitalization or GDP, and using only foreign portfolio equity holdings (and thus excluding FDI) in the measure of foreign equity ownership. Second, Table 12 reports the results of a specification in growth rates, in which case the inclusion of firm fixed-effects enables us to control for firm-specific growth trends. Third, because some metrics of financial positions are related to trade flows (though our measure is gross foreign equity ownership as a share of total domestic equity, not net financial flows), Table 13 investigates whether the results are driven by tradable sectors and finds, if anything, that the reallocation trends are slightly weaker in tradable sectors and hence are unlikely to be driven by trade dynamics. Table 14 additionally controls for the interaction of labor share with foreign equity share; the interaction of firm risk exposure with foreign equity share remains significant, suggesting that increased diversification is not merely linked with reallocation towards lower labor share firms which happen to be on average riskier, but rather depends crucially on risk characteristics per se. Table

15 shows that the results are not driven by superstar firms.

4.4 International Diversification and the Aggregate Labor Share

The empirical estimates in Sections 4.2 and 4.3, together with expressions (16) and (19) imply quantitative magnitudes for the strength of the reallocation and within effects stemming from growing international diversification, and the net effect on the aggregate labor share.

We first quantify the implications of the increase in the foreign equity share in the US for US firms and the US labor share. Columns (4)-(6) of Table 2 yields estimates of the coefficient γ_β for the three measures of labor share we use in the US data ranging from from -0.210 to -0.277. Column (1) of Table 4 yields an estimate of $\gamma_{\beta,FES}$ of 1.147. The cross-sectional variance of firm (relative) betas in the US is about 0.12 and the unweighted mean labor share of US firms in Compustat is about 0.60. Finally, the change in the US foreign equity share from 1973 to 2019 was 2.0 log points. Working with the smallest estimate of γ_β and substituting these values into (16) implies

$$Realloc_t^{US} \approx \underbrace{\overline{LS}_t}_{0.60} \underbrace{\gamma_\beta}_{-0.21} \underbrace{\gamma_{\beta,FES}}_{1.147} \underbrace{\text{var}\left(\beta_i^e\right)}_{0.12} \underbrace{\Delta \log FES_t}_{2.0} = -0.036 \; .$$

The calculation shows that the increase in foreign diversification since 1970 induced a reallocation effect that reduced US labor share by about 3.6 percentage points (using the other estimates of γ_β yields slightly larger values).

To obtain an estimate for the within effect, we need to parameterize the equity premium and the parameters governing returns to scale in production, i.e., α_2 and ν. We set the overall returns to scale to $\nu = 0.85$, a standard value in the literature on firm dynamics, and the production coefficient on labor to $\alpha_2 = 0.60$. We assume an equity premium of 5%. Substituting these values into (19) implies

$$Within_t^{US} = -\underbrace{\alpha_2}_{0.60} \underbrace{\frac{(1-\nu)^2}{\nu}}_{0.026} \underbrace{\frac{\gamma_{\beta,FES}}{\gamma_\beta}}_{-5.47} \underbrace{\left(\mathbb{E}\left[r^e\right] - r_f\right)}_{0.05} \underbrace{\Delta \log FES_t}_{2.0} = 0.0087 \; .$$

The calculation shows that the increase in foreign diversification since 1970 induced a within effect that increased the US labor share by about 0.87 percentage points. Combined, the estimates suggest that the within and reallocation effects attributable to greater international diversification have reduced the aggregate labor share in the US by roughly 2.5 percentage points. By comparison, according to the OECD data, the aggregate labor share in the US corporate sector fell by about 6.5 percentage points over the period 1973-2019, suggesting that international diversification and risk sharing can account for roughly one-third of the overall decline (the total US labor share including the non-corporate sector fell by about 3.6 percentage points over the same period).

We can perform analogous calculations for foreign firms, though we face the limitation that our measure of the total value of domestic equity begins only in 1987 due to data availability in Compustat Global. Column (6) of Table 3 yields an estimate of $\gamma_\beta = -0.447$, while column (4) of Table 4 yields an estimate of $\gamma_{\beta,FES} = 0.256$. The average cross-sectional variance of firm relative betas is 0.062, while the unweighted mean labor share is 0.55. Finally, for the countries corresponding to the firms in our sample, the mean change in foreign equity share from 1987 to 2019 was 0.98 log points. Substituting these values into (16) implies that the increase in foreign diversification since 1987 induced a reallocation effect that reduced the labor share by 0.38 percentage points for this sample of non-US (developed) countries. Using these estimates and the same production function parameters and equity premium as for the US, expression (19) implies that the increase in foreign diversification induced a within effect that raised labor share for the non-US countries by 0.04 percentage points. Combined, the estimates suggest that the growth in foreign diversification reduced the aggregate labor share in these countries by 0.34 percentage points. By comparison, the decline in the GDP-weighted average corporate sector labor share in these countries over the same period was 2.6 percentage points, suggesting that international diversification and risk sharing can account for almost 15% of the overall decline (the GDP-weighted decline in the total labor share including the non-corporate sector in these countries was about 1 percentage point over the same period).

Reallocation and changes in the US vs. foreign labor share. Our findings suggest that international diversification has had a larger impact on the aggregate labor share in the US than in our sample of foreign countries. The result stems from two features of the data: first, our sample of US firms begins in 1973 whereas our sample of foreign firms begins notably later, in 1987. Because the increase in the foreign equity share from 1973 to 2019 in the US is roughly double that in the foreign countries from 1987 to 2019, the impact on the aggregate labor share would mechanically be about double the magnitude in the US compared to the foreign countries, holding all other pieces of the calculation fixed.

A second, more subtle difference is due to the reallocation response to diversification, captured by the coefficient $\gamma_{\beta,FES}$. In the US, this coefficient is large, about 1.15, which implies that growing diversification induces a significant degree of labor reallocation across firms. In contrast, this coefficient is markedly smaller in the sample of foreign firms (though statistically significant), about 0.26, which suggests a much more muted response of the labor allocation to diversification. Indeed, holding all else fixed, if the allocative response to increasing diversification in the foreign countries was the same as in the US, the labor share response would have been more than four times larger. To the extent that the more modest response of the allocation in the foreign countries is due to larger frictions or distortions that inhibit the reallocation process, the smaller impact of diversification on the labor share in those countries may be a symptom of more sluggish overall business dynamism and lack of adjustment flexibility in the labor market as compared to the US.

4.5 Industry-Level Labor Shares

Although we have focused on firm-level and aggregate labor shares, the theory also has implications for industry-level labor shares. With heterogeneity in risk exposures and thus labor shares across firms within a sector, we can link the sectoral-level labor share to the price of risk and the mean and dispersion of labor shares across firms. Consider the case of Gaussian risk exposures from Example 2 in Section 2.2 for a given industry s in country j. In this case, Appendix C derives the elasticity of the change in the industry-level labor share to a change in the price of risk as:

$$\frac{\partial \log \frac{W_j L_{sj}}{\mathbb{E}[Y_{sj}]}}{\partial \lambda_j} = \frac{1}{\lambda_j}\left(-\log \alpha_{2,sj} + \overline{\log \frac{W_j L_{sj}}{\mathbb{E}\left[Y_{sj}\right]}} + \frac{1+\nu}{1-\nu}\text{var}\left(\log \frac{W_j L_{sj}}{\mathbb{E}\left[Y_{sj}\right]}\right)\right) ,$$

where $\overline{\log \frac{W_j L_{sj}}{\mathbb{E}[Y_{sj}]}}$ and $\text{var}\left(\log \frac{W_j L_{sj}}{\mathbb{E}[Y_{sj}]}\right)$ denote the mean and cross-sectional variance of (log) expected labor shares in industry-country sj, and $\alpha_{2,sj}$ an industry-country specific production function parameter. The expression shows that industries with larger dispersion in risk exposures, and hence labor shares, have greater scope for reallocation in response to changes in the price of risk. Thus, the reallocation effect should be larger in those industries. In industries with higher mean risk exposure and hence, lower average labor share, the within effect should be smaller. The expression suggests a regression of the following form:

$$\log LS_{sj} = \gamma_0 + \gamma_{sj} + \gamma_\mu \overline{\log LS_{sj}} \times \log FES_j + \gamma_\sigma \text{var}\left(LS_{sj}\right) \times \log FES_j + \varepsilon_{sj} \ , \tag{21}$$

which tests whether the mean and dispersion of firm-level labor shares interacted with the foreign equity share are associated with changes in industry-level labor shares. The theory predicts $\gamma_\mu < 0$ and $\gamma_\sigma < 0$ (the term γ_{sj} denotes an industry-country fixed-effect).

We use cross-country firm-level data from Orbis to investigate this prediction. Recall that Orbis contains data on both publicly traded and private firms, and because we cannot compute our measure of risk exposure (which relies on stock market data) for private firms, we were precluded from using these data in our analyses above. Expression (21), however, does not rely

(a) Variance of Labor Shares (b) Mean Labor Share

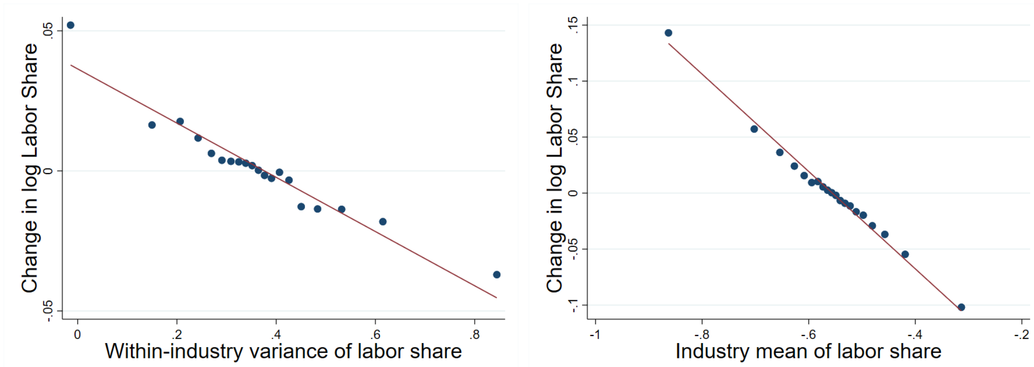

Figure 6: Industry Labor Share Growth vs. Mean/Variance of Firm Labor Shares

Notes: Figure displays bin-scatter plots of the one year ahead log change in industry-level labor shares vs. the variance/mean of firm-level log labor shares within an industry, after controlling for country-industry fixed-effects. Each point in the figure is a bin of industry-years, sorted by the variance/mean of labor shares within the industry-year.

on measures of risk exposure, and thus we are able to use the Orbis sample, which includes smaller, non-publicly traded firms.

Figure 6 displays bin-scatter plots of the growth rate of industry-level labor shares against the (lagged) mean and variance of firm-level log labor shares within the industry. The figure illustrates that, consistent with the theory, industries with greater heterogeneity in firm-level labor shares have experienced larger declines in the industry-level labor shares, as have industries with higher mean firm-level labor shares.

To more formally test this relationship, Table 5 reports results of panel regressions of industry-country-year labor shares on the interaction of the foreign equity share with the mean and standard deviation of lagged firm (log) labor shares within that industry as suggested by expression (21). We include a rich set of fixed-effects (indeed, this is one of the main benefits of turning to the Orbis data), namely, industry-year, country-year and industry-country effects (the unit of observation is industry-country-year), and include the interacted variables separately as controls. Columns (1) and (2) of the table show that, in line with the theory, country-industries with greater dispersion in firm-level labor shares experience larger declines in their industry-level labor share in response to increases in the foreign equity share, as do country-industries with higher average labor shares. Columns (3) and (4) show that the results continue to hold when controlling for the interaction of the mean and standard deviation of firm employment with the foreign equity share, confirming that the regressions are not simply picking up the effects of dispersion in firm size.

Table 5: International Diversification, Heterogeneity and Industry-Level Labor Shares

	(1)	(2)	(3)	(4)
$\overline{\log LS} \times \log FES$	-0.229***	-0.0848**	-0.196***	-0.0713**
	(-5.15)	(-2.61)	(-4.58)	(-2.22)
$\text{var}\,(\log LS) \times \log FES$	-0.128***	-0.0331**	-0.0986***	-0.0293*
	(-4.49)	(-2.10)	(-3.41)	(-1.85)
$\overline{\log L} \times \log FES$			0.0219***	0.00309
			(5.13)	(0.46)
$\text{var}\,(\log L) \times \log FES$			-0.00514**	0.00679
			(-2.45)	(1.55)
Fixed effects		yes		yes
R^2	0.467	0.800	0.480	0.806
Observations	39255	37828	37170	35721

Notes: Table reports regressions of industry-level labor shares on the mean and standard deviation of firm (log) labor shares within that industry interacted with the foreign equity share. All specifications include the interacted variables individually as controls. Fixed effects, when included, consist of industry-year, country-year and industry-country effects. Standard errors are two-way clustered by industry-country and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

5 Conclusion

In this paper, we have developed a theory of the allocation of inputs and labor's share of income in an environment with heterogeneous firms and aggregate risk. In equilibrium, riskier firms that are more exposed to aggregate shocks exhibit lower labor shares and are allocated a smaller share of inputs/outputs relative to their productivity. The magnitude of the risk adjustments in the allocation depends critically on the price of risk. In the presence of opportunities for international financial diversification, the price of risk is endogenous to the degree of international risk sharing.

Growth in such diversification, as experienced by the US and other advanced economies over the past decades, reduces the price of risk and leads to two competing effects on the aggregate labor share: first, the fall in the price of risk leads to an increase in the labor share for a given firm; second, the fall in the price of risk leads to a reallocation of inputs and production to riskier/low labor share firms. If the second effect dominates the first, growing diversification can lead to a decline in the aggregate labor share, even while within-firm labor shares rise.

The key empirical predictions of the theory are supported in the US data, as well as in a sample of advanced countries. Riskier firms have, all else equal, a lower labor share. International diversification leads to a reallocation of labor and output towards riskier/low labor share firms, contributing to the decline in aggregate labor share. A model-based decomposition of the within and reallocation effects shows that the latter effect dominates. Our estimates suggest that about a third of the decline in the corporate sector labor share in the US can be attributed to international diversification.

Finally, we find that the reallocation effect is not homogeneous across countries, but in fact is considerably stronger for the US than for other advanced economies. Understanding the potential role of labor market frictions in explaining this differential response to financial globalization is left for further research.

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Appendix

A Derivations and Proofs for Sections 2.1 and 2.2

This appendix provides derivations and proofs for the results and propositions in Sections 2.1 and 2.2.

A.1 Generalizations

Alternative production functions. Consider a generic production function in labor, capital, and possibly other inputs, $F(K_i, L_i, X_i)$. Assuming that labor is chosen under uncertainty, the optimality condition yields

$$\mathbb{E}\left[\Lambda\left(MRPL_i\right)\right] = \mathbb{E}\left[\Lambda\right] W$$

where the marginal revenue product of labor is given by $MRPL_i = \frac{\partial F(\cdot)}{\partial L_i}$. Rearranging,

$$\mathbb{E}\left[MRPL_i\right]\left(1 + \text{cov}\left(\frac{\Lambda}{\mathbb{E}\left[\Lambda\right]}, \frac{MRPL_i}{\mathbb{E}\left[MRPL_i\right]}\right)\right) = W$$

The expression shows that firms do not set the expected MRPL equal to the wage, but rather, the optimality condition features an adjustment for risk that depends on the covariance of the MRPL with the SDF. We can write the expected labor share of income as:

$$\frac{WL_i}{\mathbb{E}\left[Y_i\right]} = \frac{\mathbb{E}\left[MRPL_i\right] L_i}{\mathbb{E}\left[Y_i\right]}\left(1 + \text{cov}\left(\frac{\Lambda}{\mathbb{E}\left[\Lambda\right]}, \frac{MRPL_i}{\mathbb{E}\left[MRPL_i\right]}\right)\right)$$

The Cobb-Douglas case in expression (3) is a special case where $\frac{\mathbb{E}[MRPL_i]L_i}{\mathbb{E}[Y_i]}$ is equal to α_2.

As a second salient example, consider the case of a CES production function in capital and labor:

$$Y_i = A_i\left(K_i^{\rho}\left(1-\theta\right) + \theta L_i^{\rho}\right)^{\frac{\nu}{\rho}}$$

Following similar steps again yields an analog of (3):

$$\frac{WL_i}{\mathbb{E}\left[Y_i\right]} = \frac{\nu\theta}{\left(\frac{K}{L}\right)^{\rho}\left(1-\theta\right) + \theta}\left(1 - \kappa_i\right) \ ,$$

where $\frac{K}{L} = \frac{\sum_i K_i}{\sum_i L_i}$ is the aggregate capital/labor ratio and κ_i is as defined in the main text. It is straightforward to verify that equations (4) and (6) continue to hold exactly. The aggregate expected labor share is given by

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \frac{\nu\theta}{\left(\frac{K}{L}\right)^{\rho}\left(1-\theta\right) + \theta}\frac{\sum_i\left(\mathbb{E}\left[A_i\right]\left(1-\kappa_i\right)\right)^{\frac{1}{1-\nu}}}{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(1-\kappa_i\right)^{\frac{\nu}{1-\nu}}}$$

which is the analog of expression (7). Note that in this CES case, the results do not hinge on the elasticity of substitution between capital and labor (equal to $\frac{1}{1-\rho}$), specifically, whether it is greater or less than one.

Partial insurance. In the case that firms only partially insure workers, the wage will be state-contingent, i.e., dependent on the realization of shocks. Firms choose inputs to satisfy

$$\max_{L_i,K_i} \mathbb{E}\left[\Lambda\left(A_i K_i^{\alpha_1} L_i^{\alpha_2} - WL_i - RK_i\right)\right],$$

where the wage (and potentially, the cost of capital) are now uncertain. The first order condition for labor gives

$$\alpha_2 \mathbb{E}\left[A_i\right]\left(1+\operatorname{cov}\left(\frac{\Lambda}{\mathbb{E}\left[\Lambda\right]}, \frac{A_i}{\mathbb{E}\left[A_i\right]}\right)\right) K_i^{\alpha_1} L_i^{\alpha_2 - 1} = \mathbb{E}\left[W\right]\left(1+\operatorname{cov}\left(\frac{\Lambda}{\mathbb{E}\left[\Lambda\right]}, \frac{W}{\mathbb{E}\left[W\right]}\right)\right)$$

and rearranging yields expected labor's share of income:

$$\frac{\mathbb{E}\left[WL_i\right]}{\mathbb{E}\left[Y_i\right]} = \alpha_2 \frac{1+\operatorname{cov}\left(\frac{\Lambda}{\mathbb{E}[\Lambda]}, \frac{A_i}{\mathbb{E}[A_i]}\right)}{1+\operatorname{cov}\left(\frac{\Lambda}{\mathbb{E}[\Lambda]}, \frac{W}{\mathbb{E}[W]}\right)}$$

which is a modified version of (3), augmented to reflect the effects of risk over the wage. It is straightforward to verify that equations (4) and (6) continue to hold exactly. The aggregate expected labor share is given by

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \frac{\alpha_2}{1+\operatorname{cov}\left(\frac{\Lambda}{\mathbb{E}[\Lambda]}, \frac{W}{\mathbb{E}[W]}\right)} \frac{\sum_i \left(\mathbb{E}\left[A_i\right]\left(1-\kappa_i\right)\right)^{\frac{1}{1-\nu}}}{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(1-\kappa_i\right)^{\frac{\nu}{1-\nu}}}$$

which is the analog of (7), where κ_i is as defined in the main text.

Boundaries of the firm. Assume each firm i operates a collection of technologies, indexed by k. Each technology produces a homogeneous good with technology $Y_{ik} = A_{ik} K_{ik}^{\alpha_1} L_{ik}^{\alpha_2}$. Thus, the firm's aggregate output, labor and capital are the sum over the technologies they operate: $Y_i = \sum_{k\in i} Y_{ik}$, $L_i = \sum_{k\in i} L_{ik}$, $K_i = \sum_{k\in i} K_{ik}$.

Following the same steps as in the main text, we can derive the firm-level expected labor share as

$$\frac{WL_i}{E\left[Y_i\right]} = \alpha_2\left(1-\kappa_i\right), \quad \text{where} \quad \kappa_i = \sum_{k\in i} \frac{\mathbb{E}\left[Y_{ik}\right]}{\mathbb{E}\left[Y_i\right]} \kappa_{ik} \tag{22}$$

and κ_{ik} is the risk premium of technology k inside firm i, given by $\kappa_{ik} = -\operatorname{cov}\left(\frac{\Lambda}{\mathbb{E}[\Lambda]}, \frac{A_{ik}}{\mathbb{E}[A_{ik}]}\right)$. Expression (22) is the clear analog of (3) and shows that firm expected labor share are determined by the average of the risk premium of each of its technologies, weighted by the expected output of each technology as a share of total firm output. Technology-level expected output shares satisfy:

$$\frac{\mathbb{E}\left[Y_{ik}\right]}{\mathbb{E}\left[Y_i\right]} = \frac{\mathbb{E}\left[A_{ik}\right]^{\frac{1}{1-\nu}}\left(1-\kappa_{ik}\right)^{\frac{\nu}{1-\nu}}}{\sum_{k\in i} \mathbb{E}\left[A_{ik}\right]^{\frac{1}{1-\nu}}\left(1-\kappa_{ik}\right)^{\frac{\nu}{1-\nu}}}$$

and the firm's share of aggregate output and labor

$$\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} = \frac{\sum_{k\in i}\mathbb{E}[A_{ik}]^{\frac{1}{1-\nu}}(1-\kappa_{ik})^{\frac{\nu}{1-\nu}}}{\sum_i\sum_{k\in i}\mathbb{E}[A_{ik}]^{\frac{1}{1-\nu}}(1-\kappa_{ik})^{\frac{\nu}{1-\nu}}}$$
$$\frac{L_i}{L} = \frac{\sum_{k\in i}\mathbb{E}[A_{ik}]^{\frac{1}{1-\nu}}(1-\kappa_{ik})^{\frac{1}{1-\nu}}}{\sum_i\sum_{k\in i}\mathbb{E}[A_{ik}]^{\frac{1}{1-\nu}}(1-\kappa_{ik})^{\frac{1}{1-\nu}}}$$

which are the clear analogs of (4) and (6).

The aggregate expected labor is again the output-weighted average of firm-level labor shares, and can be expressed as

$$\frac{WL}{E[Y]} = \alpha_2\frac{\sum_i\sum_{k\in i}\mathbb{E}[A_{ik}]^{\frac{1}{1-\nu}}(1-\kappa_{ik})^{\frac{1}{1-\nu}}}{\sum_i\sum_{k\in i}\mathbb{E}[A_{ik}]^{\frac{1}{1-\nu}}(1-\kappa_{ik})^{\frac{\nu}{1-\nu}}}$$

or as a weighted average of technology or firm risk premia:

$$\frac{WL}{E[Y]} = \alpha_2\left(1-\sum_i\sum_{k\in i}\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}\frac{\mathbb{E}[Y_{ik}]}{\mathbb{E}[Y_i]}\kappa_{ik}\right) = \alpha_2\left(1-\sum_i\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}\kappa_i\right)$$

where κ_i is as defined in (22).

Assuming a linear factor structure of risk and following similar steps as in Appendix C, we can derive the firm's equity market beta as

$$\beta_{ij}^e \equiv \frac{\partial\left(r_{ij}^e - r_f\right)}{\partial r_j^M} = \frac{1}{1-\nu}\frac{\beta_i}{1-\beta_i\overline{\kappa}}\left(\frac{\partial r_j^M}{\partial A}\right)^{-1}$$

where $\beta_i^e = \sum_{k\in i}\frac{\mathbb{E}[Y_{ik}]}{\mathbb{E}[Y_i]}\beta_{ik}$, i.e., the firm's stock market beta is determined by an output-weighted average of the betas of each of its technologies. To a first order approximation,

$$\beta_{ij}^e \approx \frac{1}{c}\beta_{ij}, \quad c = \frac{1}{1-\nu}\left(\frac{\partial r_j^M}{\partial A}\right)^{-1} \tag{23}$$

Using expressions (22) to (23) and following the same steps as in Appendix C, we can derive versions of (12) and (14), showing the same empirical specifications continue to hold.

Finally, while expression (8) continues to hold, we can separate the components of reallocation into three parts: across-firm reallocation, within-firm reallocation, and changes in the labor shares of technologies:

$$\frac{\partial\frac{WL}{\mathbb{E}[Y]}}{\partial\mathcal{P}} = \underbrace{\sum_i\frac{\partial\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial\mathcal{P}}\frac{WL_i}{\mathbb{E}[Y_i]}}_{\text{reallocation across firms}} + \underbrace{\sum_i\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}\sum_{k\in i}\frac{\partial\frac{\mathbb{E}[Y_{ik}]}{\mathbb{E}[Y_i]}}{\partial\mathcal{P}}\frac{WL_{ik}}{\mathbb{E}[Y_{ik}]}}_{\text{reallocation within firms}} + \underbrace{\sum_i\sum_{k\in i}\frac{\mathbb{E}[Y_{ik}]}{\mathbb{E}[Y]}\frac{\partial\frac{WL_{ik}}{\mathbb{E}[Y_{ik}]}}{\partial\mathcal{P}}}_{\text{within technology}}$$

In one important difference between this setup in which firms operate multiple technologies and the baseline case where they operate only a single one, the impact of changes in the price of risk – and hence, international diversification – on firm-level labor share now depends on both reallocation within the firm and the change in the labor share for each technology it operates:

$$\frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \mathcal{P}} = \sum_{k\in i} \frac{\partial \frac{\mathbb{E}[Y_{ik}]}{\mathbb{E}[Y_i]}}{\partial \mathcal{P}} \frac{WL_{ik}}{\mathbb{E}\left[Y_{ik}\right]} + \sum_{k\in i} \frac{\mathbb{E}\left[Y_{ik}\right]}{\mathbb{E}\left[Y_i\right]} \frac{\partial \frac{WL_{ik}}{\mathbb{E}[Y_{ik}]}}{\partial \mathcal{P}}$$

Because the within-firm reallocation effect (first term in the expression) and within-technology effect (second term in the expression) have opposite signs, there are not clear predictions for the relationship between the price of risk, international risk-sharing, and changes in within-firm labor shares.

A.2 Proofs of Propositions 1 and 2

Proof of Proposition 1.

(i) Taking the derivative of the labor share of each technology directly yields the expression.

(ii) Using (6), the risky sector share of expected output is

$$\frac{\mathbb{E}\left[Y_r\right]}{\mathbb{E}\left[Y\right]} = \frac{\mathbb{E}\left[A_r\right]^{\frac{1}{1-\nu}} \left(1-\kappa_r\right)^{\frac{\nu}{1-\nu}}}{\mathbb{E}\left[A_s\right]^{\frac{1}{1-\nu}} + \mathbb{E}\left[A_r\right]^{\frac{1}{1-\nu}} \left(1-\kappa_r\right)^{\frac{\nu}{1-\nu}}}.$$

Taking the derivative w.r.t κ_r yields

$$\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \kappa_r} = -\frac{\nu}{1-\nu} \frac{\left(1-\kappa_r\right)^{\frac{\nu}{1-\nu}-1} \mathbb{E}\left[A_r\right]^{\frac{1}{1-\nu}} \mathbb{E}\left[A_s\right]^{\frac{1}{1-\nu}}}{\left(\mathbb{E}\left[A_s\right]^{\frac{1}{1-\nu}} + \mathbb{E}\left[A_r\right]^{\frac{1}{1-\nu}} \left(1-\kappa_r\right)^{\frac{\nu}{1-\nu}}\right)^2}$$

which can be simplified as

$$\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \kappa_r} = -\frac{\nu}{1-\nu} \frac{\mathbb{E}\left[Y_r\right]}{\mathbb{E}\left[Y\right]} \frac{\mathbb{E}\left[Y_s\right]}{\mathbb{E}\left[Y\right]} \frac{1}{1-\kappa_r}$$

Similarly,

$$\frac{\partial \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]}}{\partial \kappa_r} = -\frac{\nu}{1-\nu} \frac{\mathbb{E}\left[Y_r\right]}{\mathbb{E}\left[Y\right]} \frac{\mathbb{E}\left[Y_s\right]}{\mathbb{E}\left[Y\right]} \frac{1}{1-\kappa_r}$$

and substituting for sectoral labor shares

$$\begin{aligned} \sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \kappa_r} \frac{WL_i}{\mathbb{E}\left[Y_i\right]} &= -\frac{\nu}{1-\nu} \frac{\mathbb{E}\left[Y_r\right]}{\mathbb{E}\left[Y\right]} \frac{\mathbb{E}\left[Y_s\right]}{\mathbb{E}\left[Y\right]} \frac{1}{1-\kappa_r} \left(\alpha_2\left(1-\kappa_r\right)-\alpha_2\right) \\ &= \frac{\alpha_2 \nu}{1-\nu} \frac{\mathbb{E}\left[Y_r\right]}{\mathbb{E}\left[Y\right]} \frac{\mathbb{E}\left[Y_s\right]}{\mathbb{E}\left[Y\right]} \frac{\kappa_r}{1-\kappa_r} \end{aligned}$$

Since $\kappa_r \in (0,1)$, then $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \kappa_r} \frac{WL_i}{\mathbb{E}[Y_i]} > 0$.[36]

[36] Note that $\kappa > 1$ would imply negative expected labor's share of income for the risky firm, which is impossible. In other words, risk premia are so large that the risky firm chooses no labor in equilibrium and therefore there is not an interior solution. $\kappa > 0$ follows from the fact that A_i is negatively correlated with the SDF.

(iii) Combining the results in parts (i) and (ii) yields:

$$\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r} = -\alpha_2 \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} + \alpha_2 \frac{\nu}{1-\nu} \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\kappa_r}{1-\kappa_r}$$

which is equal to zero when $\kappa_r = \overline{\kappa_r}$ where $\overline{\kappa_r}$ satisfies

$$1 + \frac{\mathbb{E}[A_r]^{\frac{1}{1-\nu}}}{\mathbb{E}[A_s]^{\frac{1}{1-\nu}}} (1-\overline{\kappa_r})^{\frac{1}{1-\nu}} - \frac{1-\nu(1-\alpha_2)}{1-\nu} \overline{\kappa_r} = 0$$

Next, note that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r}$ has the same sign as

$$-1 + \frac{\nu}{1-\nu} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\kappa_r}{1-\kappa_r}$$

and the derivative of this w.r.t. κ_r is

$$\frac{\partial \left(-1 + \frac{\nu}{1-\nu} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{\kappa_r}{1-\kappa_r}\right)}{\partial \kappa_r} = \frac{\nu}{1-\nu} \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]} \frac{1}{(1-\kappa_r)^2} + \frac{\nu}{1-\nu} \frac{\kappa_r}{1-\kappa_r} \frac{\partial \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]}}{\partial \kappa_r}$$

Both of these terms are positive if $\kappa_r \in (0,1)$ since $\frac{\partial \frac{\mathbb{E}[Y_s]}{\mathbb{E}[Y]}}{\partial \kappa_r} = -\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \kappa_r}$ and the proof of part (ii) above shows $\frac{\partial \frac{\mathbb{E}[Y_r]}{\mathbb{E}[Y]}}{\partial \kappa_r} < 0$.

Because $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r}$ is a strictly increasing function of κ_r that is equal to zero at $\kappa_r = \overline{\kappa_r}$, it immediately follows that $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r} > 0$ if $\kappa_r > \overline{\kappa_r}$ and $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \kappa_r} < 0$ if $\kappa_r < \overline{\kappa_r}$.

□

Proof of Proposition 2.

(i) Using the properties of the log-normal distribution,

$$\begin{aligned}
\kappa_i &= -\mathrm{cov}\left(\frac{\Lambda}{\mathbb{E}[\Lambda]}, \frac{A_i}{\mathbb{E}[A_i]}\right) = -\frac{e^{\mathbb{E}[a_i]+\mathbb{E}[\lambda]+\frac{1}{2}\left(\sigma_\lambda^2+\sigma_{a_i}^2\right)}\left(e^{\mathrm{cov}(\lambda,a_i)}-1\right)}{\mathbb{E}[\Lambda]\,\mathbb{E}[A_i]} \\
&= -e^{\mathrm{cov}(\lambda,a_i)} + 1 \\
&= -e^{-\beta_i \lambda \sigma_a^2} + 1
\end{aligned}$$

and substituting into (3) and (6) gives firm-level expected labor shares

$$\frac{WL_i}{\mathbb{E}[Y_i]} = \alpha_2 e^{-\beta_i \lambda \sigma_a^2}$$

and output shares

$$\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} = \frac{\mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}}{\sum_i \mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}}$$

Using the fact that

$$\frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \lambda} = -\beta_i \sigma_a^2 \frac{WL_i}{\mathbb{E}[Y_i]}$$

we can derive

$$\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \lambda} = -\left(\sum_i \beta_i \frac{L_i}{L}\right) \frac{WL}{E[Y]} \sigma_a^2$$

which is negative if $\sum_i \beta_i \frac{L_i}{L} > 0$.

(ii) First, taking derivatives and rearranging, we can derive

$$\frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \lambda} = \frac{\nu}{1-\nu} \sigma_a^2 \frac{\mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}}{\left(\sum_i \mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}\right)^2} \left(\sum_h (\beta_h - \beta_i) \mathbb{E}[A_h]^{\frac{1}{1-\nu}} \left(e^{-\beta_h \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}\right)$$

so that

$$\frac{WL_i}{\mathbb{E}[Y_i]} \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \lambda} = \frac{\alpha_2 \nu}{1-\nu} \sigma_a^2 \frac{\mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{1}{1-\nu}}}{\left(\sum_i \mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}\right)^2} \left(\sum_h (\beta_h - \beta_i) \mathbb{E}[A_h]^{\frac{1}{1-\nu}} \left(e^{-\beta_h \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}\right)$$

Summing over i and rearranging,

$$\begin{aligned}
\sum_i \frac{WL_i}{\mathbb{E}[Y_i]} \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \lambda} &= \frac{\alpha_2 \nu}{1-\nu} \sigma_a^2 \frac{1}{\left(\sum_i \mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}\right)^2} \\
\times & \left(\sum_i \mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{1}{1-\nu}} \sum_i \beta_i \mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}\right. \\
- & \left. \sum_i \beta_i \mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{1}{1-\nu}} \sum_i \mathbb{E}[A_i]^{\frac{1}{1-\nu}} \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}\right)
\end{aligned}$$

and using the assumption that $\mathbb{E}[A_i]^{\frac{1}{1-\nu}}$ is independent of β_i, we have

$$\sum_i \frac{WL_i}{\mathbb{E}[Y_i]} \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \lambda} = \frac{\alpha_2 \nu}{1-\nu} \sigma_a^2 \frac{\sum_i \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{1}{1-\nu}} \sum_i \beta_i \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}} - \sum_i \beta_i \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{1}{1-\nu}} \sum_i \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}}{\left(\sum_i \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}}\right)^2}$$

which is positive if

$$\sum_i \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{1}{1-\nu}} \sum_i \beta_i \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}} - \sum_i \beta_i \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{1}{1-\nu}} \sum_i \left(e^{-\beta_i \lambda \sigma_a^2}\right)^{\frac{\nu}{1-\nu}} > 0$$

To simplify the expression, we use the following property of normal-log-normal mixtures: consider two jointly normal random variables, x_1 and x_2, i.e.,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}\right)$$

Then

$$\mathbb{E}\left[x_1 e^{x_2}\right] = e^{\mu_2}\left(\sigma_{12} + \mu_1\right) e^{\frac{1}{2}\sigma_2^2}$$

When $x_2 = a x_1$ for some constant a, the result implies

$$\mathbb{E}\left[x_1 e^{a x_1}\right] = e^{a\mu_1 + \frac{1}{2}a^2\sigma_1^2}\left(a\sigma_1^2 + \mu_1\right)$$

Using this result, we have

$$\begin{aligned}
& \sum_i \left(e^{-\beta_i\lambda\sigma_a^2}\right)^{\frac{1}{1-\nu}} \sum_i \beta_i \left(e^{-\beta_i\lambda\sigma_a^2}\right)^{\frac{\nu}{1-\nu}} - \sum_i \beta_i \left(e^{-\beta_i\lambda\sigma_a^2}\right)^{\frac{1}{1-\nu}} \sum_i \left(e^{-\beta_i\lambda\sigma_a^2}\right)^{\frac{\nu}{1-\nu}} \\
= \quad & -e^{-\frac{\nu}{1-\nu}\lambda\sigma_a^2 + \frac{1}{2}\left(\frac{\nu}{1-\nu}\lambda\sigma_a^2\right)^2\sigma_\beta^2}\left(\frac{\nu}{1-\nu}\lambda\sigma_a^2\sigma_\beta^2 - 1\right) e^{-\frac{1}{1-\nu}\lambda\sigma_a^2 + \frac{1}{2}\left(\frac{1}{1-\nu}\lambda\sigma_a^2\right)^2\sigma_\beta^2} \\
+ \quad & e^{-\frac{1}{1-\nu}\lambda\sigma_a^2 + \frac{1}{2}\left(\frac{1}{1-\nu}\lambda\sigma_a^2\right)^2\sigma_\beta^2}\left(\frac{1}{1-\nu}\lambda\sigma_a^2\sigma_\beta^2 - 1\right) e^{-\frac{\nu}{1-\nu}\lambda\sigma_a^2 + \frac{1}{2}\left(\frac{\nu}{1-\nu}\lambda\sigma_a^2\right)^2\sigma_\beta^2} \\
= \quad & \lambda\sigma_a^2\sigma_\beta^2 e^{-\lambda\sigma_a^2 + \frac{1}{2}\frac{1+\nu}{1-\nu}\left(\lambda\sigma_a^2\right)^2\sigma_\beta^2}
\end{aligned}$$

which is strictly positive for $\lambda > 0$, proving the result.

(iii) Write the aggregate expected labor share as

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \alpha_2 \frac{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(e^{-\beta_i\lambda\sigma_a^2}\right)^{\frac{1}{1-\nu}}}{\sum_i \mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}\left(e^{-\beta_i\lambda\sigma_a^2}\right)^{\frac{\nu}{1-\nu}}}$$

and using the assumption that $\mathbb{E}\left[A_i\right]^{\frac{1}{1-\nu}}$ is independent of β_i,

$$\frac{WL}{\mathbb{E}\left[Y\right]} = \alpha_2 \frac{\sum_i \left(e^{-\beta_i\lambda\sigma_a^2}\right)^{\frac{1}{1-\nu}}}{\sum_i \left(e^{-\beta_i\lambda\sigma_a^2}\right)^{\frac{\nu}{1-\nu}}}$$

Evaluating the summations and taking logs,

$$\log\frac{WL}{\mathbb{E}\left[Y\right]} = \log\alpha_2 - \lambda\sigma_a^2 + \frac{1}{2}\left(\lambda\sigma_a^2\right)^2\sigma_\beta^2\frac{1+\nu}{1-\nu}$$

which has derivative w.r.t. λ:

$$\frac{\partial\log\frac{WL}{\mathbb{E}[Y]}}{\partial\lambda} = \sigma_a^2\left(\frac{1+\nu}{1-\nu}\lambda\sigma_a^2\sigma_\beta^2 - 1\right) \tag{24}$$

from which the result immediately follows.

□

A.3 Linear Factor Structure

We can prove an analog of Propositions 1 and 2 in the case of a linear factor structure of risk, but no other assumptions. There are a continuum of technologies with heterogeneous exposure to a single aggregate shock, A, where $\mathbb{E}[A] = 0$. Firm productivity takes the form

$$A_i = \mathbb{E}[A_i](1 + \beta_i A) + \varepsilon_i$$

where ε_i captures purely idiosyncratic, and thus diversifiable, risk. Firm-level risk premia are then given by

$$\kappa_i = -\mathrm{cov}\left(\frac{A_i}{\mathbb{E}[A_i]}, \frac{\Lambda}{\mathbb{E}[\Lambda]}\right) = \beta_i \overline{\kappa}$$

where $\overline{\kappa}$ is common among firms and is proportional to the price of risk:[37]

$$\overline{\kappa} = -\mathrm{cov}\left(A, \frac{\Lambda}{\mathbb{E}[\Lambda]}\right)$$

Proposition 4 proves an analog of Propositions 1 and 2:

Proposition 4. *A decrease in the price of risk, $\mathcal{P}$, and thus $\overline{\kappa}$ implies:*

(i) The within effect increases labor share: $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \overline{\kappa}} < 0$ iff $\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \beta_i > 0$, i.e., if the output-weighted aggregate risk exposure is positive.

(ii) The reallocation effect reduces labor share: $\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \overline{\kappa}} \frac{WL_i}{\mathbb{E}[Y_i]} > 0$.

(iii) If the price of risk is sufficiently small, the within effect dominates: $\frac{\partial \frac{WL}{\mathbb{E}[Y]}}{\partial \overline{\kappa}} \leq 0$.

Proof of Proposition 4.

(i) Substituting for the derivative of firm-level labor share with respect to $\overline{\kappa}$ into the within effect immediately yields the result:

$$\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \frac{\partial \frac{WL_i}{\mathbb{E}[Y_i]}}{\partial \overline{\kappa}} = -\alpha_2 \sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]} \beta_i$$

(ii) Substituting for relative output shares from expression (6) and differentiating yields

$$\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \overline{\kappa}} \frac{WL_i}{\mathbb{E}[Y_i]} = -\frac{\alpha_2 \nu}{1-\nu}\left(\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}\beta_i - \left(\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}(1-\overline{\kappa}\beta_i)\right)\left(\sum_i \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}\beta_i (1-\overline{\kappa}\beta_i)^{-1}\right)\right)$$

We can consider $\frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}$ as the cross-sectional output-share weighted probability distribution, with covariance and expectation of any variable with respect this distribution denoted $\mathrm{cov}_Y(\cdot)$ and $\mathbb{E}_Y[\cdot]$, respectively, and thus:

$$\begin{aligned}\sum_i \frac{\partial \frac{\mathbb{E}[Y_i]}{\mathbb{E}[Y]}}{\partial \overline{\kappa}} \frac{WL_i}{\mathbb{E}[Y_i]} &= -\frac{\alpha_2 \nu}{1-\nu}\Big(\mathbb{E}_Y[\beta_i]\,\mathrm{cov}_Y\left[(1-\overline{\kappa}\beta_i), (1-\overline{\kappa}\beta_i)^{-1}\right] \\ &- \mathbb{E}_Y[1-\overline{\kappa}\beta_i]\,\mathrm{cov}_Y\left[\beta_i, (1-\overline{\kappa}\beta_i)^{-1}\right]\Big) \\ &= \frac{\alpha_2 \nu}{1-\nu}\mathrm{cov}_Y\left(\beta_i, (1-\overline{\kappa}\beta_i)^{-1}\right)\end{aligned}$$

[37] To be precise, $\overline{\kappa} = -\mathcal{P} corr(A, \Lambda)\, \sigma(A)$ where $\mathcal{P}$ is the price of risk as defined in the text. To a first-order approximation, $corr(A, \Lambda) = -1$ since A is the only source of aggregate consumption risk.

Note that for firms to use non-zero quantities of inputs, it must be the case that $\overline{\kappa}\beta_i < 1$. Therefore the first covariance is negative and the second is positive, implying that the entire expression is positive.

(iii) The derivative of the aggregate labor share with respect to κ is simply the sum of the within and reallocation effects:

$$\frac{\partial \frac{WL}{Y}}{\partial \overline{\kappa}} = \alpha_2 \left(\frac{\nu}{1-\nu} \left(\text{cov}_Y \left[\beta_i, (1-\overline{\kappa}\beta_i)^{-1} \right] \right) - \mathbb{E}_Y \left[\beta_i \right] \right)$$

As the price of risk converges to zero ($\overline{\kappa} \to 0$), the expression approaches $-\alpha_2 \mathbb{E}_Y [\beta_i] < 0$.

□

B Derivations and proofs for Section 2.3

This appendix provides derivations and proofs for the results and proposition in Section 2.3.

B.1 Individual Problems and Equilibrium Definition

Firm problem and the marginal investor. Firms act to maximize the market value of their shares. This optimization is complicated by the fact that there are multiple possible investors. To address this issue, we introduce an additional penalty to the firm of being disproportionately foreign-owned relative to other firms of the same risk type in the same country. We then place appropriate assumptions on the penalty to ensure that firms always choose inputs under the domestic SDF.

Formally, the problem of firm k of type i in country j takes the form

$$\max_{L_{kij}, K_{kij}} \max \left\{ \mathbb{E}\left[\Lambda_j V_{kij}\right], \ \max_{h \neq j} \frac{\mathbb{E}\left[\Lambda_h V_{kij}\right]}{1 + \tau_j + \tau^*_{ij}\left(S^*_{kij}\right)} \right\}$$

where $V_{kij} = A_{kij} K^{\alpha_1}_{kij} L^{\alpha_2}_{kij} - W_j L_{kij} - R_j K_{kij}$ and $\tau^*_{ij}\left(S^*_{kij}\right)$ denotes the penalty for being overly foreign-owned. Define $S^*_{kij} = \frac{1-S_{kij}}{1-S_{ij}}$ where $S_{ij} = \int_{k \in ij} S_{kij} dk$ is the average fraction of firm shares that are held domestically for all firms of type i and S_{kij} is the fraction of firm k shares that are held domestically.[38] The definition shows that the penalty is a function of the fraction of firm k shares that are held by foreign capitalists relative to the average fraction that are held by foreigners across all firms of the same type i.

In the absence of the penalty $\tau^*_{ij}(S^*_{kij})$, consider an equilibrium in which (1) all firms of a certain risk profile in country j make the same choices and (2) are jointly owned by domestic and (some) foreign investors. If input decisions are made using any single SDF, firms have incentives to deviate: if input choices are made using the domestic SDF, the firm can increase its value to foreign investors by tilting its choices towards risk-neutrality. If input choices are made using the foreign (risk-neutral) SDF, the firm can increase its value to domestic investors by changing tilting its choices to be more risk-averse. If they are not following either, then deviations towards the direction suggested by either SDF can increase their value. It is straightforward to verify that there is no such equilibrium.[39]

[38] For simplicity, we assume there is a measure 1 continuum of firms of each type.

[39] More precisely, the only possible equilibrium may be one in which firms of the same type are either wholly owned by domestic or foreign investors with each firm making input choices using the corresponding SDF and

However, if we add the penalty $\tau_{ij}^*(S_{kij}^*)$ and assume that this penalty is large if the firm is wholly foreign-owned (when other firms of the same type are not), the incentive to deviate is eliminated and firms always make input choices using the domestic SDF. In this case, we can work with a representative firm of each type i. The firm's input choice problem takes exactly the form in (2). The first order conditions yield

$$\begin{aligned}\alpha_2 \mathbb{E}\left[\Lambda_j A_{ij} K_{ij}^{\alpha_1} L_{ij}^{\alpha_2 - 1}\right] &= \mathbb{E}\left[\Lambda_j\right] W_j \\ \alpha_1 \mathbb{E}\left[\Lambda_j A_{ij} K_{ij}^{\alpha_1 - 1} L_{ij}^{\alpha_2}\right] &= \mathbb{E}\left[\Lambda_j\right] R_j\end{aligned}$$

Worker problem. Workers choose consumption and labor to maximize utility $U_{wj}\left(C_{wj}, L_j\right)$ subject to their budget constraint

$$C_{wj} = W_j L_j$$

where C_{wj} denotes consumption of workers in country j and $L_j = \sum_i L_{ij}$ total labor supply of those workers to all firms i in country j. The first order conditions yield:

$$\frac{\partial U_{wj}}{\partial L_j} + W_j \frac{\partial U_{wj}}{\partial C_{wj}} = 0$$

i.e., workers equate the wage to the marginal rate of substitution between consumption and leisure. Note that there is no uncertainty in the worker's problem: because the wage and labor supply are known, consumption is effectively pinned down. This is the sense in which firms are insuring workers.

Capitalist problem. Capitalists receive an endowment of goods, E_j, and initial shares in domestic firms, $S_{ij0}\ \forall\ i$, and foreign firms, $S_{jih0}\ \forall i, h \neq j$. In period 0 they choose consumption, sale of capital to firms and period 1 holdings of financial assets – domestic and foreign equity shares and the risk-free bond – to maximize expected discounted utility. They must also pay the cost of foreign equity holdings. In period 1, they receive the payouts from firms, the return on the risk-free bond and payments on capital. The capitalist's problem takes the form:

$$\max_{C_{j0}, K_j, B_j, \left(S_{ij1} \forall i\right), \left(S_{jih1} \forall i, h\right)} U\left(C_{j0}\right) + \rho \mathbb{E}\left[U\left(C_{j1}\right)\right]$$

$$\text{s.t.}$$

$$\begin{aligned}C_{j0} &= E_j - K_j - \sum_i \left(S_{ij1} - S_{ij0}\right) P_{ij} - \sum_{h \neq j} \sum_i \left(S_{jih1} - S_{jih0}\right) P_{ih} - Q B_j - \sum_{h \neq j} \sum_i \tau_h S_{jih1} P_{ih} \\ C_{j1} &= \sum_i S_{ij1} V_{ij} + \sum_{h \neq j} \sum_i S_{jih1} V_{ih} + B_j + R_j K_j \\ S_{ij1} &\geq 0\ \forall\ i, \quad S_{jih1} \geq 0\ \forall\ i, h\end{aligned}$$

where

$$V_{ij} = A_{ij} K_{ij}^{\alpha_1} L_{ij}^{\alpha_2} - W_j L_{ij} - R_j K_{ij} \tag{25}$$

thus there is no equilibrium with a representative firm of each type with no incentive to deviate in terms of its input choices.

and P_{ij} denotes the share price of firm type i in country j, $Q = \frac{1}{R_f}$ is the price of the risk-free bond and R_j the price of capital. The first order conditions yield:

$$\begin{aligned} Q &= \mathbb{E}\left[\Lambda_j\right] \\ 1 &= R_j \mathbb{E}\left[\Lambda_j\right] \\ 0 &= \left(P_{ij} - \mathbb{E}\left[\Lambda_j V_{ij}\right]\right) S_{ij1}, \quad \forall\ i \\ 0 &= \left(P_{ij}\left(1+\tau_j\right) - \mathbb{E}\left[\Lambda_h V_{ij}\right]\right) S_{hij1}, \quad \forall\ i, h \neq j \end{aligned} \tag{26}$$

where

$$\Lambda_j = \rho \frac{U'\left(C_{j1}\right)}{U'\left(C_{j0}\right)}$$

Definition of equilibrium. An equilibrium consists of a set of (1) physical allocations $\{L_{ij}, K_{ij}, Y_{ij}, Y_j, C_{wj}, C_{j0}, C_j\}$, (2) asset holdings $\{S_{ij1}, S_{jih1}, B_j, K_j\}$ and (3) prices $\{W_j, R_j, P_{ij}, \Lambda_j, R_f, V_{ij}\}$ such that workers and capitalists maximize utility, firms maximize market valuations and goods, shares, bond and capital and labor markets clear, i.e.,

$$\begin{aligned} \sum_j C_{j0} + \sum_j \tau_j \sum_i \left(1 - S_{ij1}\right) P_{ij} + \sum_j K_j = \sum_j E_j, &\qquad \sum_j \left(C_{j1} + C_{wj}\right) = \sum_j Y_j \\ S_{ij1} + \sum_{h\neq j} S_{hij1} = 1, &\qquad \sum_j B_j = 0 \\ \sum_j K_j = \sum_j \sum_i K_{ij}, &\qquad L_j = \sum_i L_{ij} \end{aligned}$$

B.2 Proof of Proposition 3

Proof. Results (ii)-(iv) follow directly from proposition 1, since $\frac{\partial \kappa_{rj}}{\partial \tau_j} > 0$. The proof of result (i) follows:

Dividing the period 0 budget constraint by $\mathbb{E}\left[\Lambda_j\right]$ and combining with the period 1 budget constraint and rearranging yields

$$\begin{aligned} C_{j1} &= \sum_i S_{ij1}\left(V_{ij} - \frac{P_{ij}}{\mathbb{E}\left[\Lambda\right]}\right) + \sum_{h\neq j}\sum_i S_{jih1}\left(V_{ih} - \frac{\left(1+\tau_h\right) P_{ih}}{\mathbb{E}\left[\Lambda\right]}\right) + B_j\left(1 - \frac{Q}{\mathbb{E}\left[\Lambda\right]}\right) \\ &+ K_j\left(R_j - \frac{1}{\mathbb{E}\left[\Lambda\right]}\right) + T_j - \frac{C_{j0}}{\mathbb{E}\left[\Lambda\right]} \end{aligned}$$

where we have used the fact that in equilibrium, the expectation of the SDF is common across countries (and denote this common expectation without country subscript) and define the total discounted value of the goods endowment and initial shares as

$$T_j = \frac{E_j}{\mathbb{E}\left[\Lambda\right]} + \frac{\sum_i S_{ij0} P_{ij} + \sum_{h\neq j}\sum_i S_{jih0} P_{ih}}{\mathbb{E}\left[\Lambda\right]}$$

Substituting from the first order conditions in (26),

$$C_{j1} = \sum_i S_{ij1}\left(V_{ij} - \frac{\mathbb{E}\left[\Lambda_j V_{ij}\right]}{\mathbb{E}\left[\Lambda\right]}\right) + \sum_{h\neq j}\sum_i S_{jih1}\mathbb{E}\left[V_{ih}\right]\left(\frac{V_{ih}}{\mathbb{E}\left[V_{ih}\right]} - 1\right) + T_j - \frac{C_{j0}}{\mathbb{E}\left[\Lambda\right]}$$

Domestic capitalists will always fully diversify their holdings of foreign equity so that

$$\sum_{h\neq j}\sum_{i} S_{jih1}\mathbb{E}\left[V_{ih}\right]\left(\frac{V_{ih}}{\mathbb{E}\left[V_{ih}\right]}-1\right)=0$$

and thus

$$C_{j1}=\sum_{i} S_{ij1}\left(V_{ij}-\frac{\mathbb{E}\left[\Lambda_j V_{ij}\right]}{\mathbb{E}\left[\Lambda\right]}\right)+T_j-\frac{C_{j0}}{\mathbb{E}\left[\Lambda\right]} \tag{27}$$

Capital and labor choices satisfy

$$\begin{aligned} W_j L_{ij} &= \left(1-\kappa_{ij}\right)\alpha_2 \mathbb{E}\left[A_{ij}\right] K_{ij}^{\alpha_1} L_{ij}^{\alpha_2} \\ R_j K_{ij} &= \left(1-\kappa_{ij}\right)\alpha_1 \mathbb{E}\left[A_{ij}\right] K_{ij}^{\alpha_1} L_{ij}^{\alpha_2} \end{aligned}$$

and substituting into (25),

$$V_{ij}=K_{ij}^{\alpha_1}L_{ij}^{\alpha_2}\left(A_{ij}-\nu\left(1-\kappa_{ij}\right)\mathbb{E}\left[A_{ij}\right]\right)$$

while

$$\begin{aligned} \mathbb{E}\left[V_{ij}\right] &= K_{ij}^{\alpha_1}L_{ij}^{\alpha_2}\mathbb{E}\left[A_{ij}\right]\left(1-\nu\left(1-\kappa_{ij}\right)\right) \\ \mathbb{E}\left[\Lambda_j V_{ij}\right] &= \mathbb{E}\left[\Lambda_j\right]\mathbb{E}\left[A_{ij}\right]K_{ij}^{\alpha_1}L_{ij}^{\alpha_2}\left(1-\nu\left(1-\kappa_{ij}\right)-\kappa_{ij}\right) \end{aligned}$$

so that

$$V_{ij}-\frac{\mathbb{E}\left[\Lambda_j V_{ij}\right]}{\mathbb{E}\left[\Lambda\right]}=K_{ij}^{\alpha_1}L_{ij}^{\alpha_2}\left(A_{ij}-\left(1-\kappa_{ij}\right)\mathbb{E}\left[A_{ij}\right]\right)$$

For the safe firm, this term is clearly zero, and substituting for the risky firm in (27),

$$C_{j1}=S_{rj1}K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}\left(A_{rj}-\left(1-\kappa_{rj}\right)\mathbb{E}\left[A_{rj}\right]\right)+T_j-\frac{C_{j0}}{\mathbb{E}\left[\Lambda\right]}$$

Taking the derivative w.r.t. τ_j yields

$$\frac{\partial C_{j1}}{\partial \tau_j}=S_{rj1}K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}\left(\left(\frac{\partial S_{rj1}}{\partial \tau_j}\frac{1}{S_{rj1}}+\frac{\partial K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}}{\partial \tau_j}\frac{1}{K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}}\right)\left(A_{rj}-\left(1-\kappa_{rj}\right)\mathbb{E}\left[A_{rj}\right]\right)+\frac{\partial \kappa_{rj}}{\partial \tau_j}\mathbb{E}\left[A_{rj}\right]\right)+\frac{\partial T_j}{\partial \tau_j}-\frac{1}{\mathbb{E}\left[\Lambda\right]}\frac{\partial C_{j0}}{\partial \tau_j} \tag{28}$$

The remainder of the proof consists of substituting for the derivatives and manipulating (28) to derive conditions such that $\frac{\partial S_{rj1}}{\partial \tau}>0$.

Input choices. Combining the optimality conditions for capital and labor, we can derive

$$\begin{aligned} K_{rj} &= \left(\left(1-\kappa_{rj}\right)\mathbb{E}\left[A_{rj}\right]\left(\frac{\alpha_1}{R_j}\right)^{1-\alpha_2}\left(\frac{\alpha_2}{W_j}\right)^{\alpha_2}\right)^{\frac{1}{1-\nu}} \\ L_{rj} &= \left(\left(1-\kappa_{rj}\right)\mathbb{E}\left[A_{rj}\right]\left(\frac{\alpha_1}{R_j}\right)^{\alpha_1}\left(\frac{\alpha_2}{W_j}\right)^{1-\alpha_1}\right)^{\frac{1}{1-\nu}} \end{aligned}$$

and hence

$$K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} = (1-\kappa_{rj})^{\frac{\nu}{1-\nu}} \mathbb{E}\left[A_{rj}\right]^{\frac{\nu}{1-\nu}} \left(\frac{\alpha_1}{R_j}\right)^{\frac{\alpha_1}{1-\nu}} \left(\frac{\alpha_2}{W_j}\right)^{\frac{\alpha_2}{1-\nu}}$$

Taking the derivative yields

$$\frac{\partial K_{rj}^{\alpha_1} L_{rj}^{\alpha_2}}{\partial \tau_j} = -\frac{1}{1-\nu} K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} \left(\frac{\nu}{1-\kappa_{rj}} \frac{\partial \kappa_{rj}}{\partial \tau_j} + \frac{\alpha_2}{W_j} \frac{\partial W_j}{\partial \tau_j}\right) \tag{29}$$

where we have used the fact that $R_j = R = \frac{1}{\mathbb{E}[\Lambda]}$ is exogenous to country j variables, but W_j is endogenous.

Aggregate labor demand satisfies

$$L_j = \left(((1-\kappa_{rj})\,\mathbb{E}\left[A_{rj}\right])^{\frac{1}{1-\nu}} + \mathbb{E}\left[A_{sj}\right]^{\frac{1}{1-\nu}}\right) \left(\left(\frac{\alpha_1}{R}\right)^{\alpha_1} \left(\frac{\alpha_2}{W_j}\right)^{1-\alpha_1}\right)^{\frac{1}{1-\nu}}$$

and taking the derivative w.r.t. τ_j yields

$$\frac{\partial L_j}{\partial \tau_j} = -\frac{1-\alpha_1}{1-\nu} \frac{L_j}{W_j} \frac{\partial W_j}{\partial \tau_j} - \frac{1}{1-\nu} \frac{1}{1-\kappa_r} L_{rj} \frac{\partial \kappa_{rj}}{\partial \tau_j} \tag{30}$$

Denote the inverse Frisch elasticity with φ_j such that φ_j satisfies

$$\frac{1}{\varphi} = \frac{\partial L_j}{\partial W_j} \frac{W_j}{L_j}$$

or,

$$\frac{\partial L_j}{\partial \tau_j} = \frac{1}{\varphi} \frac{L_j}{W_j} \frac{\partial W_j}{\partial \tau_j} \tag{31}$$

We assume that $\varphi \geq 0$. Combining (30) and (31) and rearranging,

$$\frac{\partial W_j}{\partial \tau_j} = -\frac{\frac{1}{1-\nu}}{\frac{1}{\varphi} + \frac{1-\alpha_1}{1-\nu}} \frac{1}{1-\kappa_{rj}} \frac{W_j L_{rj}}{L_j} \frac{\partial \kappa_{rj}}{\partial \tau_j} \tag{32}$$

and substituting into (29),

$$\frac{\partial K_{rj}^{\alpha_1} L_{rj}^{\alpha_2}}{\partial \tau_j} = -\frac{\partial \kappa_{rj}}{\partial \tau_j} K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} c_{kl} \tag{33}$$

where

$$c_{kl} = \frac{1}{1-\nu} \frac{1}{1-\kappa_{rj}} \left(\nu - \frac{\frac{\alpha_2}{1-\nu}}{\frac{1}{\varphi} + \frac{1-\alpha_1}{1-\nu}} \frac{L_{rj}}{L_j}\right) > 0$$

Substituting into (28)

$$\begin{aligned} \frac{\partial C_{j1}}{\partial \tau_j} &= S_{rj1} K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} \left(\left(\frac{\partial S_{rj1}}{\partial \tau_j} \frac{1}{S_{rj1}} - \frac{\partial \kappa_{rj}}{\partial \tau_j} c_{kl}\right) (A_{rj} - (1-\kappa_{rj})\,\mathbb{E}\left[A_{rj}\right]) + \frac{\partial \kappa_{rj}}{\partial \tau_j} \mathbb{E}\left[A_{rj}\right]\right) \\ &+ \frac{\partial T_j}{\partial \tau_j} - \frac{1}{\mathbb{E}\left[\Lambda\right]} \frac{\partial C_{j0}}{\partial \tau_j} \end{aligned} \tag{34}$$

Consumption. By definition, we have

$$\mathbb{E}[\Lambda] = \frac{\mathbb{E}[U'(C_{j1})]}{U'(C_{j0})} \quad \Rightarrow \quad U'(C_{j0}) = \frac{\mathbb{E}[U'(C_{j1})]}{\mathbb{E}[\Lambda]}$$

where in some abuse of notation, we include the subjective discount factor ρ in $U'(C_{j1})$. Define $F(X) = U'^{-1}(X)$. Then,

$$C_{j0} = F\left(\frac{\mathbb{E}[U'(C_{j1})]}{\mathbb{E}[\Lambda]}\right), \quad \frac{\partial C_{j0}}{\partial \tau_j} = F'\left(\frac{\mathbb{E}[U'(C_{j1})]}{\mathbb{E}[\Lambda]}\right)\frac{1}{\mathbb{E}[\Lambda]}\frac{\partial \mathbb{E}[U'(C_{j1})]}{\partial \tau_j}$$

Substituting into (34), taking expectations and noting that $\frac{\partial \mathbb{E}[U'(C_{j1})]}{\partial \tau_j} = \mathbb{E}\left[U''(C_{j1})\frac{\partial C_{j1}}{\partial \tau_j}\right]$, we can derive

$$\frac{\partial \mathbb{E}[U'(C_{j1})]}{\partial \tau_j} = \frac{U''(C_{j1})\left(S_{rj1}K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}\left(\left(\frac{\partial S_{rj1}}{\partial \tau_j}\frac{1}{S_{rj1}} - \frac{\partial \kappa_{rj}}{\partial \tau_j}c_{kl}\right)(A_{rj} - (1-\kappa_{rj})\mathbb{E}[A_{rj}]) + \frac{\partial \kappa_{rj}}{\partial \tau_j}\mathbb{E}[A_{rj}]\right) + \frac{\partial T_j}{\partial \tau_j}\right)}{1 + \mathbb{E}\left[U''(C_{j1})\frac{1}{\mathbb{E}[\Lambda]^2}F'\left(\frac{\mathbb{E}[U'(C_{j1})]}{\mathbb{E}[\Lambda]}\right)\right]} \tag{35}$$

Next, note that

$$1 - \kappa_{rj} = \frac{\mathbb{E}[U'(C_{j1})A_{rj}]}{\mathbb{E}[U'(C_{j1})]\,\mathbb{E}[A_{rj}]}$$

and taking the derivative,

$$-\frac{\partial \kappa_{rj}}{\partial \tau_j} = \frac{\mathbb{E}\left[U''(C_{j1})A_{rj}\frac{\partial C_{j1}}{\partial \tau_j}\right]\mathbb{E}[U'(C_{j1})] - \mathbb{E}\left[U''(C_{j1})\frac{\partial C_{j1}}{\partial \tau_j}\right]\mathbb{E}[U'(C_{j1})A_{rj}]}{\mathbb{E}[U'(C_{j1})]^2\,\mathbb{E}[A_{rj}]} \tag{36}$$

It will prove convenient to split C_{j1} and its derivative into stochastic and non-stochastic components. Using

$$C_{j1} = S_{rj1}K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}(A_{rj} - (1-\kappa_{rj})\mathbb{E}[A_{rj}]) + T_j - \frac{1}{\mathbb{E}[\Lambda]}F\left(\frac{\mathbb{E}[U'(C_{j1})]}{\mathbb{E}[\Lambda]}\right)$$

we can write

$$A_{rj} = \frac{C_{j1} - \overline{C}_{j1}}{S_{rj1}K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}} \tag{37}$$

where

$$\overline{C}_{j1} = -S_{rj1}K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}(1 - \kappa_{rj}\mathbb{E}[A_{rj}]) + T_j - \frac{1}{\mathbb{E}[\Lambda]}F\left(\frac{\mathbb{E}[U'(C_{j1})]}{\mathbb{E}[\Lambda]}\right)$$

and using

$$\begin{aligned}\frac{\partial C_{j1}}{\partial \tau_j} &= S_{rj1}K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}\left(\left(\frac{\partial S_{rj1}}{\partial \tau_j}\frac{1}{S_{rj1}} - \frac{\partial \kappa_{rj}}{\partial \tau_j}c_{kl}\right)(A_{rj} - (1-\kappa_{rj})\mathbb{E}[A_{rj}]) + \frac{\partial \kappa_{rj}}{\partial \tau_j}\mathbb{E}[A_{rj}]\right) + \frac{\partial T_j}{\partial \tau_j}\\ &- \frac{1}{\mathbb{E}[\Lambda]^2}F'\left(\frac{\mathbb{E}[U'(C_{j1})]}{\mathbb{E}[\Lambda]}\right)\frac{\partial \mathbb{E}[U'(C_{j1})]}{\partial \tau_j}\end{aligned}$$

and the fact that

$$S_{rj1}K_{rj}^{\alpha_1}L_{rj}^{\alpha_2}(A_{rj} - (1-\kappa_{rj})\mathbb{E}[A_{rj}]) = C_{j1} - T_j + \frac{1}{\mathbb{E}[\Lambda]}F\left(\frac{\mathbb{E}[U'(C_{j1})]}{\mathbb{E}[\Lambda]}\right)$$

we can write

$$\frac{\partial C_{j1}}{\partial \tau_j} = \left(\frac{\partial S_{rj1}}{\partial \tau_j} \frac{1}{S_{rj1}} - \frac{\partial \kappa_{rj}}{\partial \tau_j} c_{kl} \right) C_{j1} + c_{\tau_j} \tag{38}$$

where

$$\begin{aligned} c_{\tau_j} &= \left(\frac{\partial S_{rj1}}{\partial \tau_j} \frac{1}{S_{rj1}} - \frac{\partial \kappa_{rj}}{\partial \tau_j} c_{kl} \right) \left(-T_j + \frac{1}{\mathbb{E}[\Lambda]} F \left(\frac{\mathbb{E}\left[U'\left(C_{j1}\right)\right]}{\mathbb{E}[\Lambda]} \right) \right) \\ &+ S_{rj1} K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} \frac{\partial \kappa_{rj}}{\partial \tau_j} \mathbb{E}\left[A_{rj}\right] + \frac{\partial T_j}{\partial \tau_j} - \frac{1}{\mathbb{E}[\Lambda]^2} F' \left(\frac{\mathbb{E}\left[U'\left(C_{j1}\right)\right]}{\mathbb{E}[\Lambda]} \right) \frac{\partial \mathbb{E}\left[U'\left(C_{j1}\right)\right]}{\partial \tau_j} \end{aligned}$$

Substituting (37) and (38) into (36) and using the assumption of CRRA utility $U\left(C_{j1}\right) = \frac{C_{j1}^{1-\gamma}}{1-\gamma}$, some lengthy algebra yields

$$\frac{\partial \kappa_{rj}}{\partial \tau_j} = \frac{\gamma \bar{c}_{\tau_j}}{S_{rj1} K_{rj}^{\alpha_1} L_{rj}^{\alpha_2}} \frac{\mathbb{E}\left[C_{j1}^{-\gamma}\right]^2 - \mathbb{E}\left[C_{j1}^{-\gamma+1}\right]\mathbb{E}\left[C_{j1}^{-\gamma-1}\right]}{\mathbb{E}\left[C_{j1}^{-\gamma}\right]^2 \mathbb{E}\left[A_{rj}\right]}$$

and substituting for the definition of c_{τ_j} and rearranging,

$$\begin{aligned} \frac{\partial \kappa_{rj}}{\partial \tau_j} \frac{1}{\gamma} \frac{\mathbb{E}\left[C_{j1}^{-\gamma}\right]^2 \mathbb{E}\left[A_{rj}\right] S_{rj1} K_{rj}^{\alpha_1} L_{rj}^{\alpha_2}}{\mathbb{E}\left[C_{j1}^{-\gamma}\right]^2 - \mathbb{E}\left[C_{j1}^{-\gamma+1}\right]\mathbb{E}\left[C_{j1}^{-\gamma-1}\right]} &= \left(\frac{\partial S_{rj1}}{\partial \tau_j} \frac{1}{S_{rj1}} - \frac{\partial \kappa_{rj}}{\partial \tau_j} c_{kl} \right) \left(-T_j + \frac{1}{\mathbb{E}[\Lambda]} F \left(\frac{\mathbb{E}\left[U'\left(C_{j1}\right)\right]}{\mathbb{E}[\Lambda]} \right) \right) \\ &+ S_{rj1} K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} \frac{\partial \kappa_{rj}}{\partial \tau_j} \mathbb{E}\left[A_{rj}\right] + \frac{\partial T_j}{\partial \tau_j} - \frac{1}{\mathbb{E}[\Lambda]^2} F' \left(\frac{\mathbb{E}\left[U'\left(C_{j1}\right)\right]}{\mathbb{E}[\Lambda]} \right) \frac{\partial \mathbb{E}\left[U'\left(C_{j1}\right)\right]}{\partial \tau_j} \end{aligned}$$

and substituting from (35) and using the form of $F(\cdot)$ and $F'(\cdot)$ implied by CRRA utility, we can rearrange the expression after some lengthy algebra to obtain

$$\begin{aligned} \frac{\partial S_{rj1}}{\partial \tau_j} \frac{1}{S_{rj1}} &= \frac{\partial \kappa_{rj}}{\partial \tau_j} \left(c_{kl} + \frac{S_{rj1} K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} \mathbb{E}\left[A_{rj}\right]}{T_j} \left(\frac{1}{\gamma} \frac{\mathbb{E}\left[C_{j1}^{-\gamma}\right]^2}{\mathbb{E}\left[C_{j1}^{-\gamma+1}\right]\mathbb{E}\left[C_{j1}^{-\gamma-1}\right] - \mathbb{E}\left[C_{j1}^{-\gamma}\right]^2} \right. \right. \\ &\times \left. \left. \left(1 + \mathbb{E}\left[C_{j1}^{-\gamma-1}\right] \frac{1}{\mathbb{E}[\Lambda]^2} \left(\frac{\mathbb{E}\left[C_{j1}^{-\gamma}\right]}{\mathbb{E}[\Lambda]} \right)^{-\frac{1}{\gamma}-1} \right) + 1 \right) \right) + \frac{\partial T_j}{\partial \tau_j} \frac{1}{T_j} \end{aligned} \tag{39}$$

It is straightforward to verify that the first term is positive since $\frac{\partial \kappa_{rj}}{\partial \tau_j} > 0$, $c_{kl} > 0$ and for any random variable $x > 0$, $\mathbb{E}\left[x^{-\gamma+1}\right]\mathbb{E}\left[x^{-\gamma-1}\right] - \mathbb{E}\left[x^{-\gamma}\right]^2 > 0$.

Endowment. Recall that

$$T_j = \frac{E_j}{\mathbb{E}[\Lambda]} + \frac{\sum_i S_{ij0} P_{ij}}{\mathbb{E}[\Lambda]} + \frac{\sum_{h \neq j} \sum_i S_{jih0} P_{ih}}{\mathbb{E}[\Lambda]}$$

The first and last terms are independent of τ_j. Turning to the middle term, substituting for input choices, we can derive

$$P_{ij} = \mathbb{E}[\Lambda]\,\mathbb{E}\left[A_{ij}\right]\left(1 - \kappa_{ij}\right)\left(1 - \nu\right) K_{ij}^{\alpha_1} L_{ij}^{\alpha_2}$$

so that for the risky sector,

$$\frac{\partial P_{rj}}{\partial \tau_j} = \left(\frac{\partial K_{rj}^{\alpha_1} L_{rj}^{\alpha_2}}{\partial \tau_j} \frac{1}{K_{rj}^{\alpha_1} L_{rj}^{\alpha_2}} - \frac{\partial \kappa_{rj}}{\partial \tau_j} \frac{1}{1-\kappa_{rj}} \right) \mathbb{E}\left[\Lambda\right] \mathbb{E}\left[A_{rj}\right] \left(1-\kappa_{rj}\right) \left(1-\nu\right) K_{rj}^{\alpha_1} L_{rj}^{\alpha_2}$$

and using (33),

$$\frac{\partial P_{rj}}{\partial \tau_j} = -\frac{\partial \kappa_{rj}}{\partial \tau_j} \left(c_{kl} P_{rj} + \mathbb{E}\left[\Lambda\right] \mathbb{E}\left[A_{rj}\right] \left(1-\nu\right) K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} \right) \tag{40}$$

For the safe sector,

$$\frac{\partial P_{sj}}{\partial \tau_j} = \frac{\partial K_{sj}^{\alpha_1} L_{sj}^{\alpha_2}}{\partial \tau_j} \mathbb{E}\left[\Lambda\right] \mathbb{E}\left[A_{sj}\right] \left(1-\nu\right)$$

Input choices satisfy

$$\begin{aligned} K_{sj} &= \left(\mathbb{E}\left[A_{sj}\right] \left(\frac{\alpha_1}{R_j}\right)^{1-\alpha_2} \left(\frac{\alpha_2}{W_j}\right)^{\alpha_2} \right)^{\frac{1}{1-\nu}} \\ L_{sj} &= \left(\mathbb{E}\left[A_{sj}\right] \left(\frac{\alpha_1}{R_j}\right)^{\alpha_1} \left(\frac{\alpha_2}{W_j}\right)^{1-\alpha_1} \right)^{\frac{1}{1-\nu}} \end{aligned}$$

and thus

$$\frac{\partial K_{sj}^{\alpha_1} L_{sj}^{\alpha_2}}{\partial \tau_j} = -\frac{\alpha_2}{1-\nu} K_{sj}^{\alpha_1} L_{sj}^{\alpha_2} \frac{1}{W_j} \frac{\partial W_j}{\partial \tau_j}$$

and using (32) and substituting,

$$\frac{\partial P_{sj}}{\partial \tau_j} = P_{sj} \frac{\alpha_2}{1-\nu} \frac{\frac{1}{1-\nu}}{\frac{1}{\varphi} + \frac{1-\alpha_1}{1-\nu}} \frac{1}{1-\kappa_{rj}} \frac{L_{rj}}{L_j} \frac{\partial \kappa_{rj}}{\partial \tau_j} \tag{41}$$

Combining (40) and (41),

$$\frac{\partial T_j}{\partial \tau_j} = \frac{1}{\mathbb{E}\left[\Lambda\right]} \left(S_{sj0} P_{sj} c_{kls} \frac{\partial \kappa_{rj}}{\partial \tau_j} - S_{rjo} \frac{\partial \kappa_{rj}}{\partial \tau_j} \left(c_{kl} P_{rj} + \mathbb{E}\left[\Lambda\right] \mathbb{E}\left[A_{rj}\right] \left(1-\nu\right) K_{rj}^{\alpha_1} L_{rj}^{\alpha_2} \right) \right)$$

where

$$c_{kls} = \frac{\alpha_2}{1-\nu} \frac{\frac{1}{1-\nu}}{\frac{1}{\varphi} + \frac{1-\alpha_1}{1-\nu}} \frac{1}{1-\kappa_{rj}} \frac{L_{rj}}{L_j} > 0$$

Finally, combining with (39), we have

$$\begin{aligned} \frac{\partial S_{rj1}}{\partial \tau_j} \frac{1}{S_{rj1}} &= \frac{\partial \kappa_{rj}}{\partial \tau_j} \left(c_{kl} \left(1 - \frac{\frac{S_{rj0} P_{rj}}{\mathbb{E}[\Lambda]}}{T_j} \right) + c_{kls} \frac{\frac{S_{sj0} P_{sj}}{\mathbb{E}[\Lambda]}}{T_j} + \frac{P_{rj}}{\mathbb{E}\left[\Lambda\right] T_j \left(1-\nu\right) \left(1-\kappa_{rj}\right)} \right. \\ &\times \left. \left(S_{rj1} \frac{1}{\gamma} \frac{\mathbb{E}\left[C_{j1}^{-\gamma}\right]^2 + \mathbb{E}\left[C_{j1}^{-\gamma-1}\right] \left(\frac{\mathbb{E}\left[C_{j1}^{-\gamma}\right]}{\mathbb{E}[\Lambda]} \right)^{-\frac{1}{\gamma}+1}}{\mathbb{E}\left[C_{j1}^{-\gamma+1}\right] \mathbb{E}\left[C_{j1}^{-\gamma-1}\right] - \mathbb{E}\left[C_{j1}^{-\gamma}\right]^2} + S_{rj1} - S_{rj0} \left(1-\nu\right) \right) \right) \end{aligned}$$

It is straightforward to verify that $\frac{\partial \kappa_{rj}}{\partial \tau_j} > 0$, $c_{kl} > 0$, $c_{kls} > 0$ and that for any random variable

$x > 0$, $\mathbb{E}\left[x^{-\gamma+1}\right]\mathbb{E}\left[x^{-\gamma-1}\right] - \mathbb{E}\left[x^{-\gamma}\right]^2 > 0$. Thus, $\frac{S_{rj1}}{S_{rj0}} > 1-\nu$ is a sufficient condition for the entire expression to be positive.

□

B.3 Labor Share and Worker Welfare

From (30), we have

$$\frac{\partial L_j}{\partial \tau_j} = -\frac{1-\alpha_1}{1-\nu}\frac{L_j}{W_j}\frac{\partial W_j}{\partial \tau_j} - \frac{1}{1-\nu}\frac{1}{1-\kappa_r}L_{rj}\frac{\partial \kappa_r}{\partial \tau_j}$$

Substituting for $\frac{\partial \kappa_r}{\partial \tau_r} = \frac{1-\nu}{(1+\tau_j(1-\nu))^2}$ and combining with (31), the elasticity of the wage w.r.t. τ_j satisfies

$$\frac{\partial W_j}{\partial \tau_j}\frac{\tau_j}{W_j} = -\frac{\kappa_r}{\frac{1}{\varphi}(1-\nu)+1-\alpha_1}\frac{L_{rj}}{L_j}$$

which is strictly negative so long as $\varphi > 0$. Thus, a decrease in τ_j unambiguously increases the wage. Holding labor supply fixed, workers are strictly better off at the lower τ_j and because labor supply is a control variable for workers, any chosen change in labor supply must further increase their utility. Thus, increasing diversification (decreasing τ_j) improves worker welfare.

C Empirical Predictions and Quantification

Fundamental vs. stock market betas. We first relate the firm's labor market risk premium, κ_{ij}, and fundamental beta, β_{ij}, to its equity market beta, β^e_{ij}, which is a standard measure of risk exposure and is readily observable in the data.

The gross return on firm equity is the value of firm payouts in the second period divided by its price in the first period (see proof of Proposition 3):

$$R^e_{ij} = \frac{V_{ij}}{\mathbb{E}\left[\Lambda_j V_{ij}\right]} = \frac{\left(\frac{A_{ij}}{\mathbb{E}[A_{ij}]} - (1-\kappa_{ij})\,\nu\right)}{\mathbb{E}\left[\Lambda_j\right](1-\kappa_{ij})(1-\nu)}$$

and subtracting and dividing by the risk-free rate, R_f, we can obtain the following approximation to the log excess return on equity:

$$r^e_{ij} - r_f \equiv \log R^e_{ij} - \log R_f \approx \frac{\frac{A_{ij}}{\mathbb{E}[A_{ij}]} - (1-\kappa_{ij})}{(1-\kappa_{ij})(1-\nu)} \tag{42}$$

Assuming the linear factor structure of risk laid out in Appendix A.3, we have

$$r^e_{ij} - r_f = \frac{\beta_{ij}A_j + \varepsilon_{ij} + \beta_{ij}\overline{\kappa_j}}{(1-\beta_{ij}\overline{\kappa_j})(1-\nu)}$$

and taking derivatives with respect to the market excess return, r^M_j, the firm's equity market beta is equal to

$$\beta^e_{ij} \equiv \frac{\partial\left(r^e_{ij} - r_f\right)}{\partial r^M_j} = \frac{1}{1-\nu}\frac{\beta_{ij}}{1-\beta_{ij}\overline{\kappa_j}}\left(\frac{\partial r^M_j}{\partial A_j}\right)^{-1}$$

which is positive so long as the aggregate market excess return responds positively to aggregate productivity shocks. To a first order approximation, we can write

$$\beta_{ij}^{e} \approx \frac{1}{c_j}\beta_{ij}, \quad c_j = \frac{1}{1-\nu}\left(\frac{\partial r_j^M}{\partial A_j}\right)^{-1} \tag{43}$$

which shows that the firm's observed equity market beta, β_{ij}^{e}, is monotonically increasing in its fundamental beta, β_{ij}, with a constant of proportionality equal to c_j.

Prediction 1. Using expression (3), along with the linear factor structure laid out in Appendix A.3 and expression (43), it is straightforward to derive the following approximation to the firm-level labor share:

$$\log \frac{W_j L_{ij}}{Y_{ij}} \approx \log \alpha_2 - \beta_{ij}^{e}\overline{\kappa_j}c_j - \log \mathcal{E}_{ij}$$

which is (11).

Prediction 2. Expressions (4) and (6) imply

$$\begin{aligned} \log \frac{L_{ij}}{L_j} &= \frac{1}{1-\nu}\log \mathbb{E}[A_{ij}] + \frac{1}{1-\nu}\log\left(1-\kappa_{ij}\right) + c_{Lj} \\ \log \frac{\mathbb{E}\left[Y_{ij}\right]}{\mathbb{E}\left[Y_j\right]} &= \frac{1}{1-\nu}\log \mathbb{E}[A_{ij}] + \frac{\nu}{1-\nu}\log\left(1-\kappa_{ij}\right) + c_{Yj} \end{aligned}$$

where c_{Lj}, c_{Yj} are common within a country and satisfy:

$$\begin{aligned} c_{Lj} &= -\log \sum_i \left(\mathbb{E}\left[A_{ij}\right]\left(1-\kappa_{ij}\right)\right)^{\frac{1}{1-\nu}} \\ c_{Yj} &= -\log \sum_i \mathbb{E}\left[A_{ij}\right]^{\frac{1}{1-\nu}}\left(1-\kappa_{ij}\right)^{\frac{\nu}{1-\nu}} \end{aligned}$$

Assuming a linear factor structure, substituting from (43) and taking a linear approximation yields equation (13).

Quantification. As in the text, the reallocation effect can be written

$$Realloc_t = \sum_i \left(\Delta \frac{Y_{it}}{Y_t}\right) LS_{it}$$

and taking a second-order approximation,

$$\begin{aligned} Realloc_t &\approx \overline{LS}_t \Delta e^{\text{cov}\left(\log \frac{Y_{it}}{Y_t}, \log LS_{it}\right)} \\ &\approx \overline{LS}_t \Delta \text{cov}\left(\log \frac{Y_{it}}{Y_t}, \log LS_{it}\right) \\ &= \overline{LS}_t \text{cov}\left(\Delta \log \frac{Y_{it}}{Y_t}, \log LS_{it}\right) \end{aligned}$$

which is (15).

As in the text, the within effect can be written

$$Within_t = \sum_i \frac{Y_{it}}{Y_t} \Delta LS_{it}$$

Assuming the linear factor structure of risk in Appendix A.3, we have

$$\kappa_{it} = \beta_i \overline{\kappa}_t$$

and combining with (3),

$$\Delta LS_{it} = -\alpha_2 \beta_i \Delta \overline{\kappa}_t$$

and we can write

$$Within_t = -\alpha_2 \overline{\beta}_t \Delta \overline{\kappa}_t, \quad \text{where} \quad \overline{\beta}_t = \sum_i \frac{Y_{it}}{Y_t} \beta_i$$

From (11) and (12), we have

$$\gamma_\beta = -\overline{\kappa} c$$

and from (13) and (14)

$$\gamma_{\beta,FES} \Delta \log FES = -\frac{\nu}{1-\nu} c \Delta \overline{\kappa}$$

Combining:

$$\frac{\Delta \overline{\kappa}}{\overline{\kappa}} = \frac{1-\nu}{\nu} \frac{\gamma_{\beta,FES}}{\gamma_\beta} \Delta \log FES$$

which is (17).

To derive (18), first note that (42) holds for each firm, but also in the aggregate, so that the excess market return is given by

$$r_j^e - r_f = \frac{\frac{A_j}{\mathbb{E}[A_j]} - (1-\kappa_j)}{(1-\kappa_j)(1-\nu)}$$

where $\kappa_j = \sum_i \frac{\mathbb{E}[Y_{ij}]}{\mathbb{E}[Y_j]} \kappa_{ij}$ is an output-weighted average for firm-level risk premia. Taking expectations and assuming a linear factor structure of risk

$$\mathbb{E}\left[r_j^e\right] - r_f = \frac{\overline{\beta_j} \overline{\kappa_j}}{1 - \overline{\beta_j} \overline{\kappa_j}} \frac{1}{1-\nu}$$

where $\overline{\beta_j} = \sum_i \frac{\mathbb{E}[Y_{ij}]}{\mathbb{E}[Y_j]} \beta_{ij}$. Taking a first-order approximation yields

$$\overline{\kappa_j} \overline{\beta_j} = (1-\nu) \left(\mathbb{E}\left[r_j^e\right] - r_f \right)$$

Industry-level labor shares. Assume there are multiple industries, indexed by s, and that shocks and risk exposures are both Gaussian as in Example 2 in Section 2.2. Within each industry-country, risk exposures are normally distributed with mean $\overline{\beta}_{sj}$ and variance $\sigma^2_{\beta,sj}$. Following the proof of Proposition 2, we can derive an analog of (24) and write the derivative of the industry-level labor share with respect to the pricing of risk as:

$$\frac{\partial \log \frac{W_j L_{sj}}{\mathbb{E}[Y_{sj}]}}{\partial \lambda_j} = \sigma^2_{aj} \left(\frac{1+\nu}{1-\nu} \lambda_j \sigma^2_{aj} \sigma^2_{\beta,sj} - \overline{\beta}_{sj} \right)$$

Using the fact that

$$\log \frac{W_j L_{sj}}{\mathbb{E}\left[Y_{sj}\right]} \approx \log \alpha_{2,sj} - \lambda_j \beta_{ij} \sigma_{aj}^2$$

and substituting, we have

$$\begin{aligned} \text{var}\left(\log \frac{W_j L_{sj}}{\mathbb{E}\left[Y_{sj}\right]}\right) &= \lambda_j^2 \sigma_{aj}^4 \sigma_{\beta,sj}^2 \\ \overline{\log \frac{W_j L_{sj}}{\mathbb{E}\left[Y_{sj}\right]}} &= \log \alpha_{2,sj} - \lambda_j \sigma_{aj}^2 \overline{\beta}_{sj} \end{aligned}$$

where $\text{var}\left(\log \frac{W_j L_{sj}}{\mathbb{E}[Y_{sj}]}\right)$ is the variance of log expected labor shares in industry-country sj and $\overline{\log \frac{W_j L_{sj}}{\mathbb{E}[Y_{sj}]}}$ is the mean. We can write

$$\frac{\partial \log \frac{W_j L_{sj}}{\mathbb{E}[Y_{sj}]}}{\partial \lambda_j} = \frac{1}{\lambda_j}\left(-\log \alpha_{2,sj} + \overline{\log \frac{W_j L_{sj}}{\mathbb{E}\left[Y_{sj}\right]}} + \frac{1+\nu}{1-\nu}\text{var}\left(\log \frac{W_j L_{sj}}{\mathbb{E}\left[Y_{sj}\right]}\right)\right)$$

Finally, using the negative monotonic relationship between FES and λ, the expression suggests a regression of the form

$$\log LS_{sj} = \gamma_0 + \gamma_{sj} + \gamma_\mu \overline{\log LS_{sj}} \times \log FES_j + \gamma_\sigma \text{var}\left(LS_{sj}\right) \times \log FES_j + \varepsilon_{sj}$$

i.e., the change in industry-labor share depends on the interactions of the mean labor share and the variance of labor shares with the change in the foreign equity share, and the term γ_{sj} denotes an industry-country fixed-effect. The theory implies $\gamma_\mu < 0$ and $\gamma_\sigma < 0$, that is an increase in the foreign equity share disproportionately reduces the industry-level labor share in industries with a low average beta and hence a high mean labor share. Intuitively, the strength of the within effect is smaller in these industries. Similarly, an increase in the foreign equity share disproportionately reduces the industry-level labor share in industries with high dispersion in betas and hence in labor shares. Intuitively, the strength of the reallocation effect is larger in these industries.

D Empirical Analysis

D.1 Data and Measurement

Section 4.1 describes the three micro-level datasets we use, as well as the measure of foreign equity liabilities, which is the sum of foreign portfolio equity liabilities and FDI liabilities. This includes both publicly traded and privately held firms. To calculate the foreign equity share, we first compute the total value of domestic equity as the product of total business earnings multiplied by the price/earnings ratio of publicly traded firms. We measure total business income as the total operating surplus of business establishments from the OECD (series NFB2GP). This measure does not include mixed income, is pre-tax, and does not include deductions for capital consumption or depreciation.

We calculate price/earnings ratios of publicly traded firms as market capitalization divided by EBITDA. For the US, we compute the aggregate price/earnings ratio as the sum of firm-level market capitalization divided by the sum of firm-level earnings (EBITDA). The aggregate price/earnings ratios in foreign countries with smaller publicly traded sectors turns out to be quite noisy due to composition effects: the addition/subtraction of a few large firms or changes in the industrial composition of these firms leads to large changes in the aggregate price/earnings ratio. We control for these compositional effects as follows: we estimate a panel regression of firm price/earnings ratios on industry-year and country-year fixed effects. We then extract the country-year component of the firm-level price/earnings ratios, which is then free of industry-year specific factors and firm-specific factors. Multiplying the price/earnings ratio in each country-year by the measure of total earnings from the OECD yields and estimate of the total value of domestic equity.

D.2 Aggregate Labor Share and Foreign Equity Holdings

Figure 7 plots two variants of Figure 1. The top row of the figure displays trends in the corporate sector labor share and foreign equity liabilities. The bottom row displays trends in the aggregate labor share and the foreign equity share, calculated as foreign equity liabilities divided by the total value of the domestic corporate sector. Across these measures, the figure illustrates the concurrent trends of a declining labor share alongside a dramatic increase in international financial diversification.

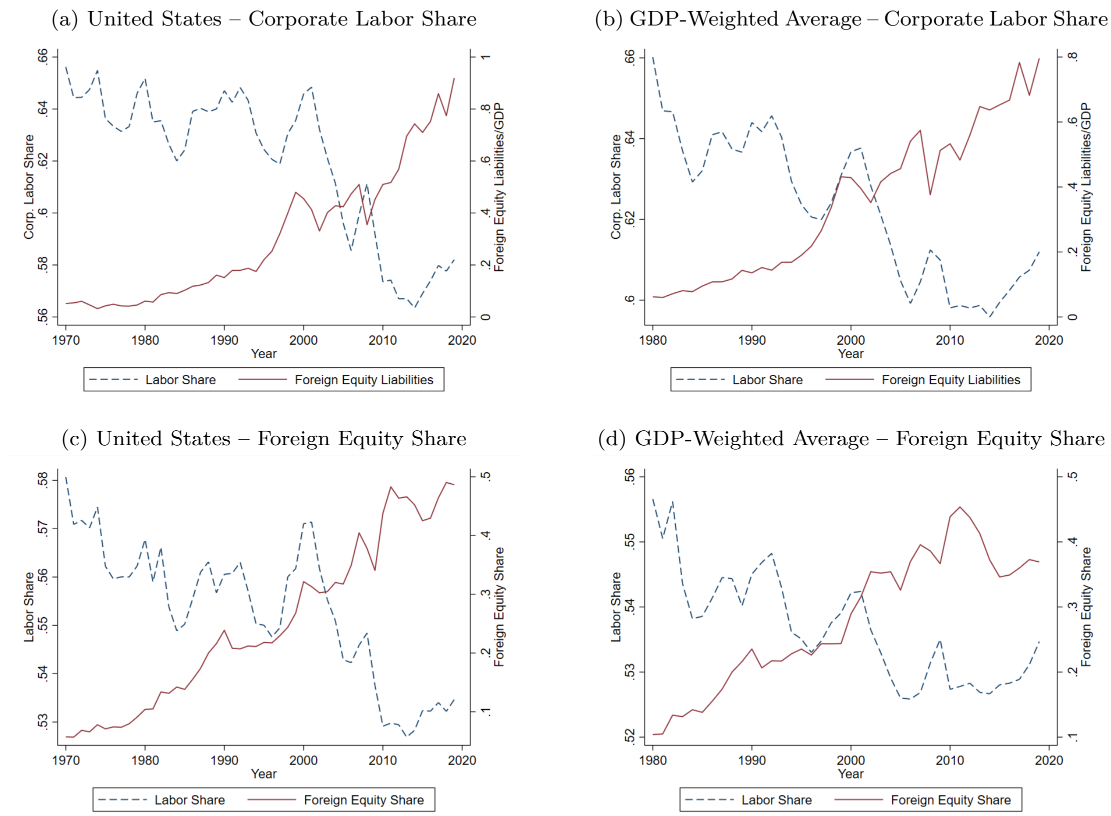

Figure 7: Trends in Labor Share and Foreign Equity Holdings – Alternative Measures

Notes: Figure displays measures of the labor share (left-axis) and foreign equity holdings (right-axis). Panels (a) and (c) display statistics for the United States; panels (b) and (d) displays GDP-weighted averages across 27 countries classified as advanced economies by Lane and Milesi-Ferretti (2018). Panels (a) and (b) display the corporate sector labor share and foreign equity liabilities normalized by GDP; panels (c) and (d) display the aggregate labor share and the foreign equity share, calculated as foreign equity liabilities divided by the total value of the domestic corporate sector. Data are from the OECD (labor share) and the External Wealth of Nations database (Lane and Milesi-Ferretti, 2018), and are described in detail in Section 4.1.

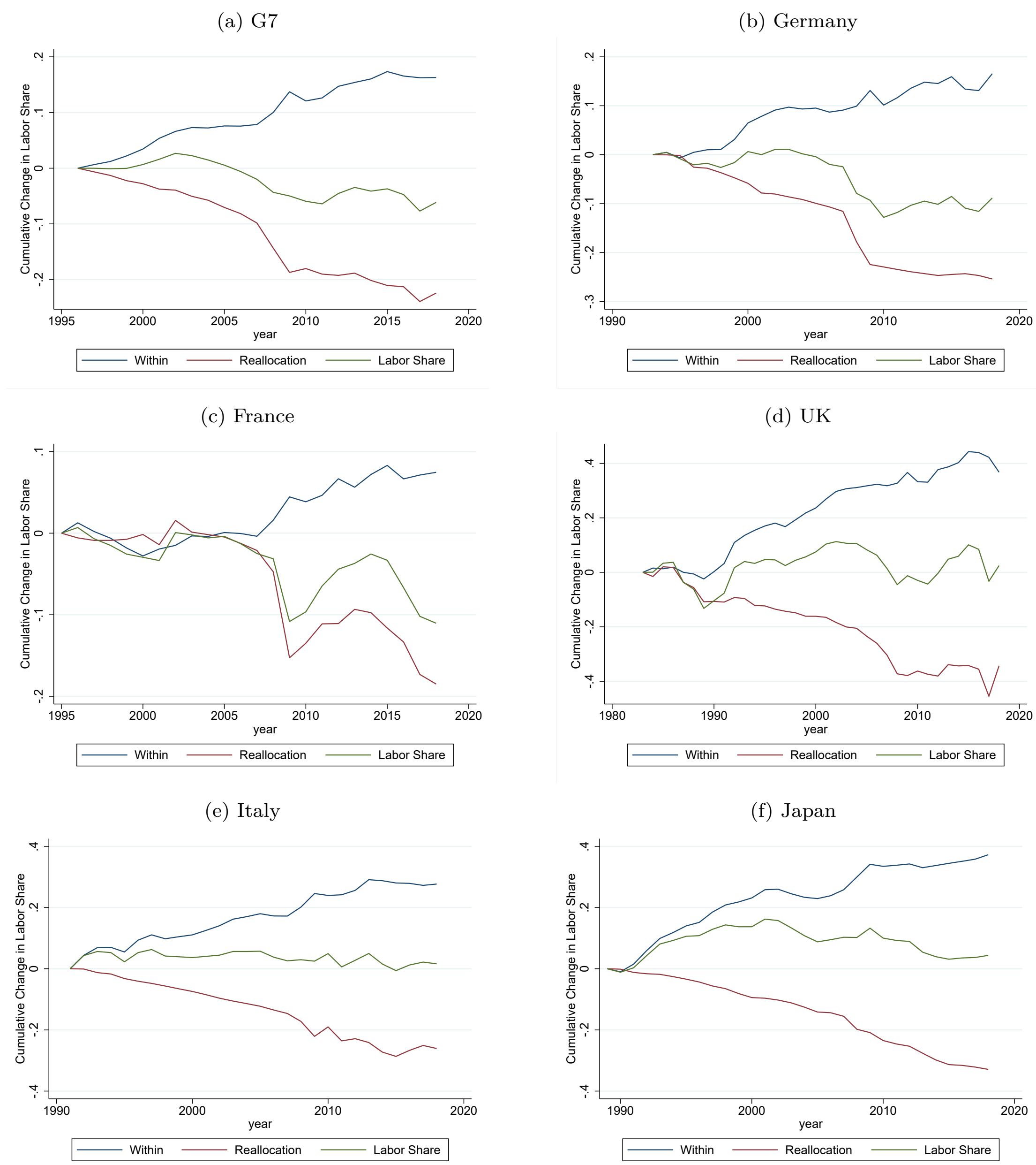

Figure 8: Labor Share Decomposition into Reallocation and Within Components

Notes: Figure displays the decomposition of changes in the aggregate labor share into reallocation and within components. Panel (a) displays arithmetic averages across five G7 countries, while panels (b)-(f) implement the decomposition individually for each country. Data are from Orbis.

D.3 Labor Share Decomposition

Figure 8 displays the within vs. reallocation decomposition individually for the five countries included in Figure 2. The figure shows that in all countries, the reallocation effect has been a force reducing the labor share in recent decades, while the within effect has been increasing it.

Figure 9 displays an analog of Figure 2, further separating the reallocation component into within vs. cross-industry reallocation using the following extension of expression (1):

$$\Delta LS_{t+1} = \underbrace{\sum_s \left(\frac{Y_{st+1}}{Y_{t+1}} - \frac{Y_{st}}{Y_t} \right) LS_{st+1}}_{\text{cross-industry reallocation}} + \underbrace{\sum_s \frac{Y_{st}}{Y_t} \sum_{i \in s} \left(\frac{Y_{it+1}}{Y_{st+1}} - \frac{Y_{it}}{Y_{st}} \right) LS_{it+1}}_{\text{within-industry reallocation}} + \underbrace{\sum_i \frac{Y_{it}}{Y_t} \left(LS_{it+1} - LS_{it} \right)}_{\text{within effect}}$$

where s indexes industries, i firms and t time. The term LS_{st} denotes the industry-level labor share, equal to $LS_{st} = \sum_{i \in s} \frac{Y_{it}}{Y_{st}} LS_{it}$. The figure shows that on average, and in the majority of countries, the within-industry reallocation component accounts for a larger share of the labor share decline than the cross-industry component.

D.4 Firm-level Risk and Labor Share

Tables 6 and 7 report regressions of firm labor shares on time-varying measures of firm risk exposure, controlling for firm fixed-effects for our US and international samples, respectively. We compute annual stock market betas by regressing, for each firm-year, the firm's daily stock returns on the aggregate market return in its country. The measure may not be ideal for quantification – measures of firm risk can be quite persistent, time-varying measures are more exposed to sampling error, and if there are any adjustment frictions in the labor choice then the labor share may react to changes in firm risk only with lags. Nonetheless, the link between firm risk and labor share remains strongly statistically significant using time varying market betas, even after controlling for firm fixed-effects.

Table 8 reports regressions of firm labor shares on both country and global risk exposure for firms in Compustat Global.[40] Since the measure of global market returns we use, Ken French's Global Developed Market Factor, is available at the monthly frequency, we regress firm-level monthly returns on both the monthly country market return and global market return using five-year rolling windows. We then compute firm relative risk exposures (betas) to these factors by residualizing both betas on country-industry-year fixed effects. The measure of firm relative risk is the average of the firm relative betas over time. The table shows that higher exposure to both country and global risk are associated with lower labor shares, which is what the theory would predict if both sources of risk are priced. The fact that country-specific risk has a lower risk premium associated with the labor share is consistent with the presence of some, but not complete, diversification across countries.

Tables 9 and 10 report regressions of firm labor share on firm relative betas, but we omit "superstar" firms, measured following Kroen, Liu, Mian, and Sufi (2021) as the top 5% of firms by market capitalization within each Fama-French industry each country-year.[41] The results are qualitatively and quantitative similar to those in main text, if anything the link between labor share and risk is slightly greater in magnitude for the global dataset when superstar firms are dropped. Thus it is unlikely that our results are driven by superstar firms.

D.5 International Diversification and Reallocation

Table 11 reports the results of the reallocation regressions (as in Table 4) using several alternative measures of foreign equity holding intensity: foreign equity liabilities divided by stock market capitalization, foreign equity liabilities divided by GDP, and foreign portfolio equity liabilities (i.e., excluding FDI) divided by market capitalization. Each row in the table represents a separate regression. The results are qualitatively and quantitatively similar to Table 4.

[40] We focus on non-US firms as foreign market returns are less correlated with global returns than is the US market return.

[41] For country-industry-years with less than 20 observations, we drop the firm with the largest market cap.

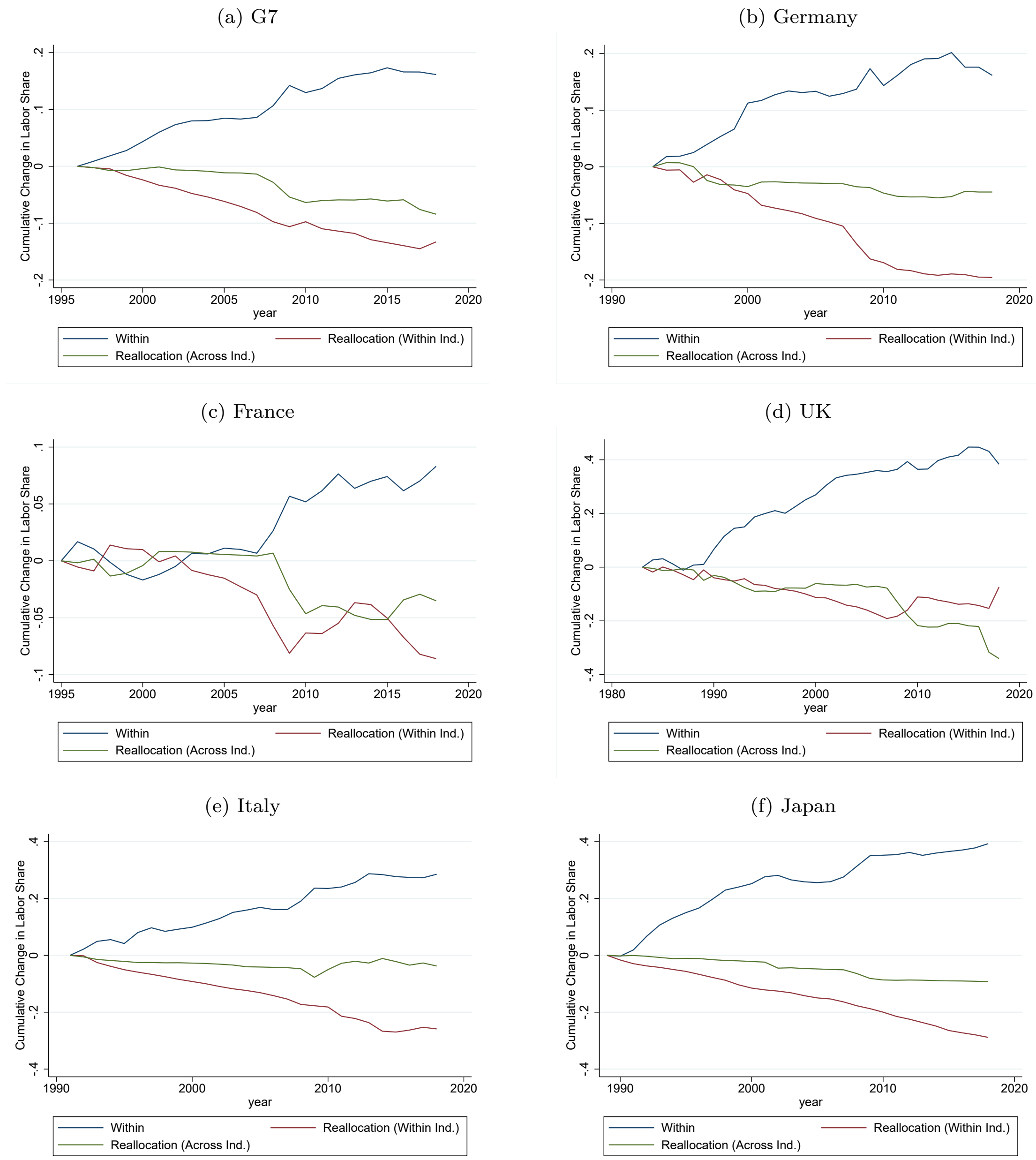

Figure 9: Labor Share Decomposition: Within and Across Industry Reallocation

Notes: Figure displays the decomposition of changes in the aggregate labor share into across-industry reallocation, within-industry reallocation, and within components. Panel (a) displays arithmetic averages across five G7 countries, while panels (b)-(f) implement the decomposition individually for each country. Data are from Orbis.

Table 12 reports the results of the reallocation regressions in five year changes, i.e.,

$$\Delta \log \frac{Z_{ij}}{\overline{Z}_j} = \gamma_0 + \gamma_{\beta,FES} \beta_{ij}^e \times \Delta \log FES_j + \varepsilon_{ij}$$

We again include firm and industry-country-year fixed-effects, as well as controls for firm age, and cluster standard errors two ways by firm and year. The firm fixed-effect controls for firm-specific

Table 6: Time-Varying Risk Exposure and Labor Share – US Firms

	(1) $\log \frac{L}{Y}$	(2) $\log \frac{L}{Y}$	(3) $\log ELS$	(4) $\log ELS$	(5) $\log LS$	(6) $\log LS$
Relative Beta	-0.105***	-0.0320***	-0.107***	-0.0433***	-0.0757***	-0.0283***
	(-13.23)	(-7.85)	(-12.42)	(-10.76)	(-6.12)	(-4.05)
Industry-Year F.E.	yes	yes	yes	yes	yes	yes
Firm F.E.		yes		yes		yes
Controls	yes	yes	yes	yes	yes	yes
R^2	0.682	0.906	0.570	0.851	0.758	0.936
Observations	155681	153973	113662	111855	10612	10337

Notes: Table reports regressions of (log) firm labor shares on annual firm betas and controls. We present results for three measures of labor share in Compustat US: labor intensity ($\frac{L}{Y}$), extended labor share (ELS), and reported labor share (LS). We trim all measures at the 2% level. Controls include firm size and age. Data are from Compustat/CRSP. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 7: Time-Varying Risk Exposure and Labor Share – Foreign Firms

	(1)	(2)	(3)	(4)
Relative Beta	-0.119***	-0.0396**	-0.155***	-0.0483**
	(-4.90)	(-2.43)	(-5.72)	(-2.18)
Ind × Ctry × Yr F.E.	yes	yes	yes	yes
Firm F.E.		yes		yes
Controls			yes	yes
R^2	0.537	0.811	0.570	0.818
Observations	21189	19471	14432	13277

Notes: Table reports regressions of (log) firm labor shares on annual firm betas and controls. We trim all measures at the 2% level. Controls include firm size and age. Data are from Compustat Global. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 8: Firm-Level Risk Exposure and Labor Share – Domestic and Global Risk

	(1)	(2)	(3)	(4)
Relative Beta	-0.0279	-0.0327*	-0.0366**	-0.0470**
	(-1.33)	(-1.72)	(-2.32)	(-2.26)
Relative Global Beta	-0.0389*	-0.0431**	-0.0490***	-0.0681***
	(-2.03)	(-2.46)	(-3.43)	(-3.44)
F.E.	yr	$ctry \times yr$	$ind \times ctry \times yr$	$ind \times ctry \times yr$
Controls				yes
R^2	0.0567	0.157	0.521	0.555
Observations	40905	40897	32076	21476

Notes: Table reports regressions of (log) firm labor shares on firm relative betas on both the domestic market factor and the global market factor and controls. We trim all measures at the 2% level. Controls include firm size and age. Data are from Compustat Global. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

trends in such a specification. The table shows that the results are qualitatively similar whether the equation is estimated in levels or growth rates.

Table 13 reports the results of the reallocation regressions allowing for different coefficients, $\gamma_{\beta,FES}$, for firms in tradable and non-tradable sectors.[42] There is no evidence that the result is driven by

[42] We follow Müller and Verner (2021) in classifying the manufacturing, agriculture and mining sectors as

Table 9: Firm-Level Risk Exposure and Labor Share Excluding Superstar Firms – US Firms

	(1)	(2)	(3)	(4)	(5)	(6)
	$\log\left(\frac{L}{Y}\right)$	logels	logls	$\log\left(\frac{L}{Y}\right)$	logels	logls
Relative Beta	-0.212***	-0.151***	-0.0715**	-0.294***	-0.261***	-0.216***
	(-12.55)	(-10.76)	(-2.25)	(-15.77)	(-16.82)	(-6.22)
Industry-year F.E.	yes	yes	yes	yes	yes	yes
Controls				yes	yes	yes
R^2	0.672	0.556	0.727	0.684	0.582	0.760
Observations	156647	112750	10389	153659	110378	9779

Notes: Table reports regressions of (log) firm labor shares on firm relative betas and controls excluding superstar firms, defined as the top 5% of firms by market cap within each industry-year (or the largest market cap firm if there are less than 20 firms in an industry-year). We present results for three measures of labor share in Compustat US: labor intensity ($\frac{L}{Y}$), extended labor share (ELS), and reported labor share (LS). We trim all measures at the 2% level. Controls include firm size and age. Data are from Compustat/CRSP. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 10: Firm Risk Exposure and Labor Share Excluding Superstar Firms – Foreign Firms

	(1)	(2)	(3)	(4)	(5)	(6)
Relative Beta	-0.775***	-0.774***	-0.564***	-0.568***	-0.576***	-0.511***
	(-5.67)	(-5.80)	(-5.24)	(-4.88)	(-4.97)	(-4.85)
F.E.	yr	$ctry \times yr$	$ind \times ctry \times yr$	yr	$ctry \times yr$	$ind \times ctry \times yr$
Controls				yes	yes	yes
R^2	0.109	0.166	0.512	0.153	0.202	0.540
Observations	28712	28695	21136	21112	21092	14769

Notes: Table reports regressions of (log) firm labor shares on firm relative betas and controls excluding superstar firms, defined as the top 5% of firms by market cap within each industry-country-year (or the largest market cap firm if there are less than 20 firms in an industry-country-year). We trim all measures at the 2% level. Controls include firm size and age. Data are from Compustat Global. Standard errors are two-way clustered by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

tradable sectors: the estimates are positive and significant in both tradable and non-tradable sectors, and indeed, slightly larger in magnitude in the non-tradable sectors.

Table 14 reports the results of the reallocation regressions controlling for the interaction of the foreign equity share with firm labor share (as well as for firm labor share by itself). The goal is to control for potential alternative explanations in which international diversification is associated with reallocation towards low labor share firms who happen to be on average riskier, but the relationship is not due to risk per se. The table shows that while a larger foreign equity share does seem to be associated with reallocation towards low labor share firms, the main coefficient of interest, on the interaction of firm risk with the foreign equity share, remains statistically significant and quantitatively similar to the baseline results in Table 4.

Table 15 reports the results of the reallocation regressions omitting "superstar" firms, defined as in Kroen, Liu, Mian, and Sufi (2021). The table shows that the relationship between the foreign equity share and reallocation towards riskier firms remains positive and significant, as shown by the coefficient estimate on the interaction of the foreign equity share with relative beta.

tradable and the remaining non-financial sectors as non-tradable.

Table 11: Reallocation and Alternative Measures of Diversification

	US Firms		Foreign Firms	
	(1)	(2)	(3)	(4)
	L	Y	L	Y
Specification 1:				
Relative Beta $\times \log \frac{\text{Foreign Equity}}{\text{Market Capitalization}}$	1.013***	1.224***	0.490***	0.511***
	(11.89)	(13.46)	(3.46)	(5.08)
Specification 2:				
Relative Beta $\times \log \frac{\text{Foreign Equity}}{\text{GDP}}$	0.574***	0.711***	0.368***	0.424***
	(12.54)	(14.52)	(8.25)	(8.38)
Specification 3:				
Relative Beta $\times \log \frac{\text{Foreign Portfolio Equity}}{\text{Market Capitalization}}$	0.994***	1.204***	0.516***	0.512***
	(11.96)	(13.73)	(3.56)	(5.23)
Ind × Ctry × Yr F.E.	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes
Controls	yes	yes	yes	yes

Notes: Table reports regressions of firm log share of industry employment (L) and sales (Y) on the interaction of firm relative beta with the log of various measures of foreign equity share. Columns (1) and (2) report results using CRSP/Compustat data for US firms and columns (3) and (4) report results using Compustat Global for foreign firms. We trim all measures at the 2% level. Standard errors are clustered two ways by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 12: International Diversification and Reallocation – Growth Rates

	US Firms		Foreign Firms	
	(1)	(2)	(3)	(4)
	L	Y	L	Y
Relative Beta $\times \Delta \log \text{FES}$	0.218**	0.238**	0.118***	0.224***
	(2.35)	(2.44)	(2.91)	(4.63)
Ind × Ctry × Yr F.E.	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes
Controls	yes	yes	yes	yes
R2	0.536	0.563	0.647	0.586
Observations	101612	103398	51965	83503

Notes: Table reports regressions of firm log share of industry employment (L) and sales (Y) on the interaction of firm relative beta with the five year change in log foreign equity share. Columns (1) and (2) report results using CRSP/Compustat data for US firms and columns (3) and (4) report results using Compustat Global for foreign firms. We trim all measures at the 2% level. Standard errors are clustered two ways by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 13: International Diversification and Reallocation – Tradable vs. Non-Tradable Sectors

	US Firms		Foreign Firms	
	(1)	(2)	(3)	(4)
	L	Y	L	Y
Relative Beta × log FES, Non-tradables	1.034***	1.248***	0.383***	0.311***
	(11.54)	(13.81)	(5.45)	(4.55)
Relative Beta × log FES, Tradables	0.888***	1.092***	0.185***	0.214***
	(11.52)	(12.77)	(3.45)	(3.50)
Ind × Ctry × Yr F.E.	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes
Controls	yes	yes	yes	yes
R2	0.940	0.942	0.968	0.981
Observations	170515	171231	96395	142731

Notes: Table reports regressions of firm log share of industry employment (L) and sales (Y) on the interaction of firm relative beta with the log foreign equity share, with separate coefficients for tradable and non-tradable sectors. Columns (1) and (2) report results using CRSP/Compustat data for US firms and columns (3) and (4) report results using Compustat Global for foreign firms. We trim all measures at the 2% level. Standard errors are clustered two ways by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 14: International Diversification and Reallocation – Controlling for Labor Share

	US Firms		Foreign Firms	
	(1)	(2)	(3)	(4)
	L	Y	L	Y
Relative Beta × log FES	0.869***	0.997***	0.202**	0.395***
	(12.10)	(14.18)	(2.62)	(3.17)
Labor Share × log FES	-0.00824*	-0.115***	-0.0282*	-0.00133
	(-2.01)	(-7.68)	(-1.83)	(-0.09)
Ind × Ctry × Yr F.E.	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes
Controls	yes	yes	yes	yes
R2	0.941	0.954	0.978	0.978
Observations	137990	138655	21117	27850

Notes: Table reports regressions of firm log share of industry employment (L) and sales (Y) on the interaction of firm relative beta with the log foreign equity share, as well as the interaction of firm labor share with the log foreign equity share. Columns (1) and (2) report results using CRSP/Compustat data for US firms and columns (3) and (4) report results using Compustat Global for foreign firms. We trim all measures at the 2% level. Standard errors are clustered two ways by firm and year. t-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 15: International Diversification and Reallocation Excluding Superstar Firms

	US Firms		Foreign Firms	
	(1)	(2)	(3)	(4)
	L	Y	L	Y
Relative Beta $\times \log \text{FES}$	0.898***	1.098***	0.198***	0.1000**
	(12.65)	(14.71)	(3.77)	(2.44)
Ind × Ctry × Yr F.E.	yes	yes	yes	yes
Firm F.E.	yes	yes	yes	yes
Controls	yes	yes	yes	yes
R2	0.939	0.942	0.973	0.985
Observations	159833	160593	72864	103698

Notes: Table reports regressions of firm log share of industry employment (L) and sales (Y) on the interaction of firm relative beta with the log foreign equity share. Columns (1) and (2) report results using CRSP/Compustat data for US firms and columns (3) and (4) report results using Compustat Global for foreign firms. We trim all measures at the 2% level. We omit superstar firms, defined as the top 5% of firms by market cap within each industry-country-year (if there are less than 20 firms, we omit the largest market cap firm). Standard errors are clustered two ways by firm and year. *t*-statistics in parentheses. Significance levels are denoted by: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.