Doing without Nominal Rigidities

Real Effects of Monetary Policy in a Monetary World

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Abstract

I develop a quantitative model of money as a medium of exchange, built on search frictions in the product market, which provides an alternative theory for the real effects of monetary policy. Due to matching uncertainty, firms operate below full capacity, and households carry money that ends up unused. A reduction in the nominal interest rate decreases the opportunity cost of holding money, pushing up households' money demand. This results in a decrease in money velocity and an increase in capacity utilization, as it becomes easier for firms to match households with money to purchase their goods. I estimate the model to match the impulse response functions to a stimulative monetary policy shock in a vector autoregression and compare it to a model with nominal rigidities. The search-based model's response to the shock displays positive and persistent effects on consumption, investment, and employment. Moreover, it better matches the procyclical response of labor productivity and the countercyclical response of the labor share than the model with nominal rigidities considered.

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1. Introduction

A notable development in macroeconomics over the last 25 years has been the disappearance of money from most models of monetary policy. One reason is the perceived success of the New Keynesian model in matching the effects of monetary policy shocks estimated through vector autoregressions (VARs).[1] In that model, money's unit of account role is the key to understanding the real effects of monetary policy rather than the role of medium of exchange. This renders modeling money unnecessary. In contrast, models that emphasize money's role as the medium of exchange, such as cash-in-advance or shopping-time models, have not been shown to match the data to the same extent.

In this paper, I develop a model of product market search frictions where money is the medium of exchange. It provides an alternative theory for the real effects of monetary policy. In this model, all trade is bilateral between a household and a trading post of the firm, and money is required to settle the transaction. The matching uncertainty for firms and households implies that firms cannot operate at full capacity and households carry unused money balances. A reduction in the nominal interest rate reduces the opportunity cost involved in holding money. Consequently, households are willing to carry more liquidity. In equilibrium, it becomes harder for households to use all their money, resulting in a decrease in money velocity, but it becomes easier for firms to find buyers for their products, increasing capacity utilization. As a consequence, output and household income go up. Furthermore, the increase in capacity utilization is equivalent to an increase in total factor productivity. Hence, as a second-order effect, firms respond by demanding more inputs of production to increase capacity. Overall, when combined with labor market search frictions, this theory predicts that a persistent reduction in the nominal interest rate leads to an increase in output and employment, an increase in labor productivity, a decrease in labor share, an increase in real money supply, and a decrease in money velocity. All of these features are present in the data. In contrast, a basic New Keynesian model with sticky prices and wages produces the same effect on output and employment, but gets the impact on labor productivity and labor share incorrectly. In addition, it does not speak to the behavior of monetary variables.

It is well known in the New Keynesian literature that real frictions are key for the model's quantitative performance. To evaluate the performance of the monetary search

[1] See Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), Christiano, Eichenbaum, and Trabandt (2016), among others.

model I propose relative to a New Keynesian framework, I include variable capital utilization and investment adjustment costs in both models, as well as preferences with internal habit formation in the New Keynesian model and a portfolio adjustment cost in the monetary model. Variable capital utilization has been proposed as a solution to the NK model's inability to increase labor productivity following a decrease in the nominal interest rate. Also, the New Keynesian model commonly includes preferences with internal habit formation, which are used to generate hump-shaped responses to monetary policy shocks. I show that adding a portfolio adjustment cost achieves the same aim in the monetary search setting. I also add a cash constraint to the New Keynesian model to have a prediction for the behavior of money in that setting.

I estimate the two models using an impulse response matching procedure, i.e., I find the parameters that minimize a weighted distance between each of the respective model's impulse response functions and the impulse response functions from a vector autoregression. The response of nine variables are matched: the nominal interest rate, output, consumption, investment, inflation, employment, labor productivity, labor share, and real money supply (using the money zero maturity aggregate). The fit of the two models is close. For the full set of variables, the search-based model delivers a 2% lower weighted root mean squared error than the New Keynesian model. This result is driven by a better fit for labor productivity and labor share.

Two objections are commonly made to models of money that emphasize its medium of exchange role and liquidity-based channels for monetary policy. The first is that they ignore financial development. In a recent paper, Lagos and Zhang (2022) address this criticism from a theoretical standpoint. They show that financial intermediary market power makes money relevant even when no agents hold it in equilibrium.[2] The second objection is that under the current monetary policy operational framework with interest on excess reserves, the economy is satiated in liquidity, and one must consider monetary policy without monetary frictions (see, for example, Cochrane 2014). To address these concerns, I propose a simple extension of my model in which banks issue deposits and households choose to hold all their liquid assets in this form. It clarifies that the statistic determining the importance of monetary policy's liquidity-based effects is the pass-through of the risk-free interest rate into the nominal interest rate on liquid assets.

[2] Their paper is a response to Woodford (1998) who shows that in a cash-in-advance economy with cash and credit goods, there is no discontinuity at the limit as the proportion of cash goods goes to zero. Woodford's result has provided justification for the idea that a cashless economy can well approximate an economy with high money velocity. The work of Lagos and Zhang (2022) underlines that this result is specific to the cash-in-advance economy analyzed by Woodford.

If the interest rate paid on the liquid asset moves one-to-one with the other interest rates, Central Bank actions cannot change the desirability of liquidity and affect the economy through this channel. Until 2011, under Regulation Q, demand deposits could not be remunerated with interest in the United States, so this pass-through was equal to 0, just as with Central Bank-issued money. Cirelli (2022) shows that even the interest rates on savings accounts move less than one-to-one with market rates, suggesting that the relevant measure of liquidity to think about the liquidity effects of monetary policy is broader than M1. I also use this extension to underscore that the economy behaves no differently in a regime of interest on excess reserves as long as banks do not transmit this rate into their deposits.

The model I develop builds on the monetary search literature. Kiyotaki and Wright (1989, 1993) developed the search-theoretic approach to money. In those papers, goods and money are indivisible. Shi (1997) and Lagos and Wright (2005) refine this, allowing for divisibility of money and goods.[3] My paper's contribution is to develop a tractable model focused on the short-term effects of monetary policy whose quantitative performance can be evaluated relative to the data.[4] Three key ingredients are needed. The first is risk insurance in the product market, a feature of Shi (1997), which allows the model to retain tractability without using a centralized market. This is important to have an intertemporal substitution motive, often absent due to the use of quasilinear preferences. The second is the use of directed search as in Menzio, Shi, and Sun (2013) rather than random search and Nash bargaining.[5] The third is preinstalled production capacity – firms hire their factors of production in advance and know that a part of it will go unused due to matches with households that do not occur. This leads to a notion of unutilized capacity.[6] These same ingredients are put together in Mennuni (2022) to explain why money can be demanded in excess of spending needs even in the presence of credit. However, he does not analyze the dynamic implications for the short-term effects of monetary policy. My paper is also one of the first to evaluate the performance of a monetary search model in the data. Aruoba and Schorfheide (2011) put together and estimate a model that combines monetary search frictions with the New Keynesian

[3]See Williamson and Wright (2010) for a review of the so-called "New Monetarist" literature.

[4]Much of the monetary search literature is directed towards the long-run effects of monetary policy. Some papers do analyze the effects of a monetary injection, for example, Menzio, Shi, and Sun (2013), Rocheteau, Weill, and Wong (2021), and Chiu and Molico (2021).

[5]Random search is the norm in monetary search models. However, several papers have used directed search. Rocheteau and Wright (2005) explore the different market structures that can be used.

[6]Monetary search models that include labor market search frictions also have a similar notion. However, unsold output is often taken to the centralized market where it is sold. See, for example, Berentsen, Menzio, and Wright (2011).

framework. However, their focus is different, as they are interested in evaluating the welfare costs of the two frictions. In that sense, monetary search frictions in their model are relevant for the welfare costs of inflation, not for the short-term effects of monetary policy.

Several papers have looked at the effects of monetary policy when money is the medium of exchange. For example, Cooley and Hansen (1997) study an economy with a cash-in-advance constraint and information frictions in the style of Lucas (1972). They find that monetary shocks can induce significant volatility in real variables, but consumption and productivity are countercyclical in response to these shocks. Cooley and Quadrini (1999) and Li (2011) combine limited participation in asset markets with labor market search frictions. These models can produce large liquidity effects and positive real effects of monetary policy. However, they face the same challenge as the New Keynesian model to produce procyclical productivity and countercyclical labor share because they do not include the capacity utilization channel present in my model.

This paper also relates to the search literature emphasizing the importance of product market frictions for the business cycle, such as Bai, Ríos-Rull, and Storesletten (2019), Petrosky-Nadeau and Wasmer (2015), and Huo and Ríos-Rull (2020). In these models, consumers need to exert effort to find firms and purchase goods. Changes in consumer effort can generate or amplify business cycle fluctuations. My model abstracts from search effort and focuses instead on the role played by money in making matches result in exchanges. Qiu and Ríos-Rull (2021) focus on some of the same weaknesses of the New Keynesian framework as does this paper, particularly the response of labor productivity and labor share to monetary policy shocks. They suggest introducing product market search frictions and consumer search effort into a New Keynesian environment as a solution. This provides an amplification channel for nominal rigidities. In contrast, I propose an alternative theory for the real effects of monetary policy.

The rest of the paper is structured as follows. In section 2, I present a two-period endowment economy with the directed monetary search friction. This model can be analytically solved and illustrates how monetary policy has real effects in this environment. Section 3 builds on this by setting up a quantitative model with endogenous production capacity and additional real frictions. Section 4 calibrates and estimates this model and compares its performance to that of the New Keynesian model. Section 5 introduces banks into the two-period environment. Section 6 concludes.

2. A Two-Period Endowment Economy

To illustrate the critical mechanism by which monetary policy produces real effects in this paper, I start by building the simplest model in which it can play a part. While the model is close to much of the work developed in the money search literature, some idiosyncrasies, such as directed search and preinstalled capacity, deserve a careful exposition. As will be seen, many of the interesting features of the framework already surface in this simple setting.

Environment Consider an economy with two periods and aggregate endowments given by y_1 in period 1 and y_2 in period 2. There is a representative household, a representative firm, a Government, and a Central Bank. In period one there is a monetary search friction: the endowment of goods is held by the firm, and to purchase it the household must match with a trading post set up by the firm. Because of the anonymous bilateral nature of trade, money is necessary for exchange. For simplicity, period 2 is a pure exchange economy – the household holds the endowment, and the price of the good in terms of money clears the market. All the action of the model is in period 1. However, the two periods are necessary to produce a monetary friction. Money is the numeraire.

In period 1, there are search frictions in the product market, and search is directed. The household decides the submarket in which to search for goods, while the firm decides where to set up trading posts to sell its endowment of goods. Submarkets are indexed by the triplet (θ, p, q), where θ is the tightness of the submarket (how many trading posts there are relative to buyers) and (p, q), are the terms of trade of the transaction in the case of a match (q units for p units of money each). In a submarket (θ, p, q), the probability of a match for the household is given by $\rho^h(\theta)$, while the probability of a match for a trading post is $\rho^f(\theta)$. These matching probabilities are a function of the tightness of the submarket because the matching function $M_P(b, s)$ has constant returns to scale, so that $\rho^h(\theta)$ is given by $M_P(b, s)/b$ and $\rho^f(\theta)$ is given by $M_P(b, s)/s$, where s is the number of buyers and s is the number of sellers (trading posts) in the submarket.[7]

There is perfect risk-sharing in the product market in the style of Shi (1997): some individuals get a match and spend their money holdings and others do not, but goods and unused money are then pooled together, such that there is a representative household

[7]A necessary assumption on the matching function is that $\rho^h(\theta)\rho^f(\theta)$ is increasing on θ.

which consumes $\rho^h(\theta)q$ and spends $\rho^h(\theta)\, pq$ units of money with certainty.[8] Hence, in this environment, the household trades off the percentage of money holdings spent with the price. Consumption of c units of the good yields a utility given by the utility function $u(c)$.

Government and Central Bank The household starts the period with nominal government debt holdings D. The government will roll over this debt by issuing bonds at a nominal interest rate i, controlled by the Central Bank through open market operations: the Central Bank will issue money to buy some of the bonds the Government wants to sell, matching the interest rate with its target. Hence, of its total initial government debt holdings D, households will end up with a part in government bonds B with a promised nominal interest payment of i, and a part in money M with a nominal interest rate equal to 0. The household income, in the form of profits from the firm, is only paid at the end of period 1, after the product market. Then, the household gets taxed lump-sum the amount the government needs to pay its debt. This is given by:

$$T = D(1+i) - iM. \tag{1}$$

The government pays interest on its debt, making total payments $(1+i)D$; however, a part of this, iM, is interest paid to the Central Bank, which is rebated back to the government.

Firm The firm has y_1 units of the good and wants to set up trading posts in submarkets to sell these goods. A trading post in submarket (θ, p, q) must hold the quantity of goods q for sale in case of a match with a buyer. The units allocated to this trading post will be unsold if this specific trading post does not match a household. Thus, the firm's problem consists of choosing the mass of trading posts to set up in each submarket, denoted $s(\theta, p, q)$, in order to maximize revenue, subject to the constraint that the quantity allocated to the trading posts equals the endowment of goods available. Notice that because trading posts are a continuum, there is no uncertainty from the perspective of the firm. By setting a unit mass of trading posts in a submarket with tightness θ, the firm knows that a mass $\rho^f(\theta)$ of these will have a match and the remaining will not. However, because the firm has to preallocate the goods, it cannot just have them in

[8] One can conceptualize this as a family made of a continuum of household members, each visiting the product market at the same time searching for goods, or simply as different households engaging in perfectly enforced state-contingent contracts amongst each other.

those trading posts that will have a match. This results in the following firm problem:

$$\max_{s(\theta,p,q)} \Pi = \int \rho^f(\theta) \times p \times q \times s(\theta,\ p,\ q).d(\theta,\ p,\ q),$$
$$s.t.: \int q \times s(\theta,\ p,\ q).d(\theta,\ p,\ q) \leq y_1.$$

In this problem, $s(\theta,\ p,\ q)$ appears linearly both in the objective function and the constraint. For any submarket with a positive number of buyers, the solution to the problem requires that the revenue per unit in a trading post, given by $\rho^f(\theta)\,p$, is the same across all submarkets in which the firm picks $s(\theta,\ p,\ q) > 0$. Moreover, amongst these submarkets, the firm is indifferent about the mass of trading posts to set up in each, so $s(\theta,\ p,\ q)$ can be the exact mass that makes θ the tightness of that submarket. There are different equilibria where some submarkets have no trading posts because there are no buyers, and there are no buyers because the firm does not choose to set up trading posts. Such equilibria are often ruled out by "trembling hand" arguments in the directed search literature.[9] Hence, the firm problem yields the following constraint that describes the locus of active submarkets in which the household can search:

(2) $$\rho^f(\theta)\,p = \mu,$$

where μ is the revenue per unit of good allocated to each trading post and is an equilibrium object. In equilibrium, firm profits will be given by:

(3) $$\Pi = \mu\, y_1.$$

Household At the beginning of period 1, the household has D nominal government debt and decides how much to hold in government bonds a and how much to hold in money m that can be used to purchase goods in the product market. At the beginning of period 2, the household has any remaining money holdings that it did not spend, it receives the payment of the government bonds with interest, the firm profits Π, and it pays the lump-sum taxes T. Its period 2 endowment y_2, with price p_2, is worth $p_2 y_2$. This yields a period 2 budget constraint determining how much the household can consume in this period, c_2.

Then, the problem of the household consists of choosing money holdings and government bond holdings, as well as the submarket in which to search for goods, subject

[9] See Menzio and Shi (2010).

to the budget constraints and two restrictions on the choice of submarket – that its money holdings are sufficient to pay for the transaction in the submarket chosen, and that the firm is willing to set up trading posts in that submarket. The latter is given by condition (2) above. The household takes as given the firm revenue per unit in the active submarkets μ, profits Π, taxes T, the nominal interest rate i, and period 2 price p_2. This results in the following problem:

$$\max_{m,a,(\theta,p,q),c_1,c_2} u(c_1) + \beta u(c_2)$$
$$s.t.: m + \frac{a}{1+i} = D,$$
$$c_1 = \rho^h(\theta)q,$$
$$pq \leq m,$$
$$\rho^f(\theta)\, p = \mu,$$
$$p_2 c_2 = m - \rho^h(\theta)\, pq + a + \Pi - T + p_2 y_2.$$

In the last condition, $\rho^h(\theta)\, pq$ is the amount of money the household actually ends up spending. This is because each household member spends pq units of money in the event of a match, and the matching probability $\rho^h(\theta)$ will equal the mass of members who find a trading post and spend the money.

Equilibrium To describe equilibrium in this model, it is convenient to write the period 2 price in terms of the aggregate price in period 1, P, and the inflation rate π_2:

$$p_2 = P(1+\pi_2).$$

P is an equilibrium object, taken as given by the household, which will equal the price of the submarket visited by the representative household, p. This is an innocuous variable change, which will be helpful to explain the price level indeterminacy result that arises in the model.

Using the aggregate conditions (1) and (3), the household problem can be written in terms of the following 5 variables: i, μ, M, P and π_2. Let $m(i,\mu,M,P,\pi_2)$, $a(i,\mu,M,P,\pi_2)$, $\theta(i,\mu,M,P,\pi_2)$, $p(i,\mu,M,P,\pi_2)$, $q(i,\mu,M,P,\pi_2)$, $c_1(i,\mu,M,P,\pi_2)$, $c_2(i,\mu,M,P,\pi_2)$ be the solution to the household problem. The following four conditions characterize the equilibrium in this model:

$$m(i,\mu,M,P,\pi_2) = M,$$

$$c_1(i, \mu, M, P, \pi_2) = \rho^f\left(\theta(i, \mu, M, P, \pi_2)\right) y_1 \Leftrightarrow q(i, \mu, M, P, \pi_2).\theta(i, \mu, M, P, \pi_2) = y_1,$$

$$c_2(i, \mu, M, P, \pi_2) = y_2,$$

$$p(i, \mu, M, P, \pi_2) = P.$$

The first three conditions are the money market clearing, period 1 product market equilibrium, and period 2 market clearing. Product market equilibrium does not entail that the household consumes all the endowment but rather that household consumption equals the goods sold by the firm. Because of matching frictions, a part of the endowment will not find a buyer and will be unsold. The last condition is just the definition of aggregate price; it is a byproduct of rewriting the price p_2 in terms of P and π_2, which resulted in one additional aggregate variable but also one additional condition.

The household decision To provide further intuition into the household decision, I rewrite the household problem, plugging in the constraints into the objective function. Notice that if $i > 0$, the inequality constraint $pq \leq m$ should bind – because carrying money is costly, the household only wants to carry money that it would use in the event of a match.

$$\max_{\theta, c_1} u\left(c_1\right) + \beta u\left(\frac{-\frac{\mu}{\rho^f(\theta)} c_1\left(1 + \frac{i}{\rho^h(\theta)}\right) + \mu y_1 + iM}{P(1+\pi_2)} + y_2\right).$$

This problem can be broken down into two parts: the decision of what submarket to visit and the intertemporal decision. That is, given a consumption c_1, what is the tightness θ of the submarket that the household wants to visit, and given this submarket choice, how much the household wants to consume in period 1. In the choice of submarket, the household faces a trade-off. Consider the decision of visiting a market with low tightness. In such a submarket, the matching probability for the household is lower. Due to perfect risk-sharing, the household can still consume the same amount because it can have fewer of its members matching a trading post, but each of them purchasing more units (lower θ, higher q). However, to do so, the household must carry more money. The foregone interest rate lost by carrying money will be more significant. For the firm, this lower tightness means a higher matching probability, which implies fewer goods unsold. As a consequence, the firm can give the household a lower price. Hence, putting it all together, the household trades off a lower price with higher lost interest. The level of the nominal interest rate can affect this trade-off.

Solution Using a CRRA utility function and a Cobb-Douglas matching function $M_P(b,s) = Ab^{1-\nu}s^{\nu}$ it is possible to derive a closed-form solution to this model. Detailed derivations are presented in the Appendix. Given a policy choice, $i = i^*$, one can find the solution to all the real variables of the model as well as period 2 inflation. However, the price level P is left indeterminate:

$$\theta^* = A^{-\frac{1}{\nu}}(i^*)^{\frac{1}{\nu}}\left(\frac{2\nu-1}{1-\nu}\right)^{\frac{1}{\nu}},$$

$$q^* = A^{\frac{1}{\nu}}(i^*)^{-\frac{1}{\nu}}\left(\frac{2\nu-1}{1-\nu}\right)^{-\frac{1}{\nu}} y_1,$$

$$\left(\frac{\mu}{P}\right)^* = A^{\frac{1}{\nu}}(i^*)^{1-\frac{1}{\nu}}\left(\frac{2\nu-1}{1-\nu}\right)^{1-\frac{1}{\nu}},$$

$$\left(\frac{M}{P}\right)^* = A^{\frac{1}{\nu}}(i^*)^{-\frac{1}{\nu}}\left(\frac{2\nu-1}{1-\nu}\right)^{-\frac{1}{\nu}} y_1,$$

$$(1+\pi_2^*) = \beta A^{\frac{1}{\nu}\sigma}(i^*)^{\left(1-\frac{1}{\nu}\right)\sigma}\left(\frac{2\nu-1}{1-\nu}\right)^{\left(1-\frac{1}{\nu}\right)\sigma}\left(\frac{\nu}{2\nu-1}\right)\left(\frac{y_1}{y_2}\right)^{\sigma}.$$

Price level indeterminacy The price level indeterminacy result above is not particular to this setting. At least since Sargent and Wallace (1975), it is known that a policy specification in which the monetary authority picks a nominal interest rate peg and the fiscal authority adjusts taxes passively, taking the equilibrium as given, does not determine the price level. This result is also familiar in the New Keynesian literature – it is why a Taylor rule that satisfies the Taylor principle of increasing the nominal interest rate by more than one-to-one with contemporaneous inflation is necessary. Here, I bypass this issue by focusing on the effects of monetary policy on the variables for which there is no indeterminacy (real variables and future inflation) and abstracting from the behavior of contemporaneous inflation, which depends on how one solves the indeterminacy.[10]

[10] The debate on achieving uniqueness in monetary models is chiefly orthogonal to the frictions one uses. The three most common solutions are the Fiscal Theory of the Price Level (FTPL), by which the fiscal authority would pick a real amount of taxes; the Quantity Theory of Money (QTM), by which the Central Bank chooses the nominal money supply rather than the nominal interest rate; and the Wicksellian, which involves having a rule for the nominal interest rate, instead of a peg (the Taylor rule being an example). See Leeper (1991) and Woodford (1995) for the FTPL, Lucas (1990) for an example of the QTM, and Woodford (1998) for an explanation of Wicksellian price determinacy.

Effects of a change in the nominal interest rate Using the expressions above, I can write the elasticities of the model variables with respect to the gross nominal interest rate, $R := 1 + i$, for an interior solution (in which $\rho^h(\theta) < 1$ and $\rho^f(\theta) < 1$). These roughly express the percentage change in a variable caused by a one percentage point increase in the nominal interest rate. The relevant range for ν is $[0.5, 1]$.[11]

$$\epsilon_R^V = 1\left(\frac{1+i}{i}\right) > 0,$$

$$\epsilon_R^{M/P} = -\left(\frac{1}{\nu}\right)\left(\frac{1+i}{i}\right) < 0,$$

$$\epsilon_R^C = -\left(\frac{1}{\nu} - 1\right)\left(\frac{1+i}{i}\right) < 0,$$

$$\epsilon_R^{\Pi/P} = -\left(\frac{1}{\nu} - 1\right)\left(\frac{1+i}{i}\right) < 0,$$

$$\epsilon_R^{1+\pi_2} = -\left(\frac{1}{\nu} - 1\right)\sigma\left(\frac{1+i}{i}\right) < 0.$$

The elasticity of velocity is positive, while the elasticity of real money holdings is negative. Essentially, real money demand goes down when the nominal interest rate goes up. Velocity increases, but by less than real money holdings decrease. Consumption falls as market tightness changes and more goods go unsold; the same is true of real profits. Interestingly, the effects on future inflation are negative – changes in the nominal interest rate affect the real interest rate.

2.1. Discussion

How does the model above deliver real effects from changes in the nominal interest rate? Due to product market search frictions, households cannot just choose to carry around the exact amount of money that they will need for consumption. Instead, a part of the money holdings carried around ends up unused. This means that when the nominal interest rate is high, the foregone interest lost by carrying this money is larger. Given the directed search setting, households choose the submarket to go to. Submarkets with higher tightness have a higher matching probability for the household and lower for the firm. When visiting these submarkets, the household ends up using

[11] With a Cobb-Douglas matching function this is the necessary and sufficient condition for $\rho^h(\theta)\rho^f(\theta)$ to be increasing in tightness.

more of the money it carries. However, the firm will have more of its goods unsold and will charge higher prices to be compensated. When choosing a submarket, the household weighs this trade-off: the foregone interest against the price differential across submarkets. When the nominal interest rate changes, it affects this trade-off, and the resulting change in tightness moves money velocity and capacity utilization.

How should one interpret the model's mechanism? What features of the real world is the model trying to describe that account for the real effects of monetary policy? The feature of reality that the model intends to capture is the uncertainty faced by consumers about how many desirable consumption opportunities they will be faced with in a period of time and the uncertainty that firms face about when customers will arrive, which makes it hard for them to operate at full capacity.[12] Such uncertainty means that households face a trade-off in their choice of how much liquidity to hold. By holding a lot of liquidity, a household's consumption is not restricted by it, but in periods where the household has fewer consumption opportunities, it loses the interest on that liquidity unnecessarily. Holding less liquidity implies less foregone interest but prevents a household from taking some desired consumption opportunities when many of these materialize. Changes in the opportunity cost of liquidity, given by the nominal interest rate, affect households' decisions of how much money to hold, determining the amount of consumption opportunities taken. In turn, this affects how easy it is for firms to find buyers with money willing to buy their products.

An important distinction between the model above and frameworks with nominal rigidities is that in the former, money is neutral: an unexpected increase in the nominal money supply without a corresponding change in the nominal interest rate would lead to a price jump with no change in real variables. The effects of monetary policy come through the cost of liquidity imposed by the nominal interest rate. Money is neutral but not superneutral, allowing a Central Bank to steer real economic allocations.

3. A Quantitative Representative Agent Model

I now build on the abovementioned model to construct an infinitely-lived production economy that can be estimated and evaluated in comparison to a vector autoregression.

[12]The European Commission surveys businesses in the manufacturing and services sectors about the percentage rate at which they use available resources. For manufacturing, this indicator goes back to 1985 and has fluctuated between a minimum of 66.5% in the COVID-19 recession and a maximum of 84.6%, with a long-run average of 80.9%. The indicator for services is more recent, having started only in 2011, and its highest historical value is 90.9%.

In this model, markets are still complete, and there is a representative household. However, there is now a continuum of good varieties indexed by $j \in [0, 1]$, each produced by a single firm with monopoly power. These firms must use labor and capital to produce goods according to a production function. I model a labor market with search frictions in the style of Mortensen and Pissarides. I also include variable capital utilization, investment adjustment costs, and a convex portfolio adjustment cost households must pay to adjust their real money holdings.

Labor market search frictions are important for the behavior of employment and the labor share. Variable capital utilization and investment adjustment costs are standard ingredients used in the New Keynesian literature, which I also include here to make the comparison smoother. The convex portfolio adjustment cost generates hump-shaped responses to monetary policy shocks, a feature of the impulse response functions estimated through vector autoregressions.

3.1. Setup

A continuum of size 1 of individuals is aggregated in a representative household that pools idiosyncratic risks. Time is discrete, and each period can be thought of as being composed of four subperiods. The first subperiod is a labor market with search frictions and random search. The second subperiod is a centralized asset market, in which the household chooses the composition of its portfolio in terms of money and bonds. The third subperiod is the consumption goods market. This is a decentralized market for goods varieties where search is directed, and money is the medium of exchange. In this subperiod, households buy goods from firms for consumption. Due to the presence of adjustment costs, vacancy costs, and investment, which involve the purchase of goods for purposes other than consumption, there is one more subperiod in which non-consumption goods are traded. There is a representative firm that purchases varieties from the monopolistic producers and aggregates them to be sold as investment goods, vacancy posting services, or adjustment cost services. The household and monopolistic firms can buy these from this representative firm in a frictionless market. This ensures that only the purchase of consumption goods is affected by the product market frictions. Income in wages, unemployment benefits, and dividends are received at the end of the period. Taxes are also paid at the end of the period.

I define the price level as the expenditure necessary to buy one unit of each variety, and I make it the numeraire.

Household Because of search frictions in the labor market, individuals in the economy can be employed or unemployed. At the end of a period, a portion δ^n of employed individuals suffer a separation shock and become unemployed. At the beginning of the following subperiod, the labor market opens. All unemployed individuals search for a job and find one with probability $\psi^h(\xi_t)$, where ξ_t is the tightness of the labor market, given by the ratio of vacancies to unemployment, and $\psi^h(\xi)$ is given by $M_L(u, v)/u$ where M_L is the labor market matching function, u is the number of unemployed individuals searching for a job and v is the number of job vacancies posted by firms. An individual who matches with a firm will become employed, and those who do not remain unemployed. Hence, employment, n_t, evolves according to:

$$n_t = (1 - \delta^n)n_{t-1} + \psi^h(\xi_t)(1 - (1 - \delta^n)n_{t-1}). \tag{4}$$

An employed individual is paid a wage, w_t, and an unemployed individual is paid unemployment benefits, $\bar{u}$. Because income is pooled, when n individuals are employed, labor income for the representative household, paid at the end of the period, is given by:

$$n_t.w_t + (1 - n_t)\bar{u}.$$

In the second subperiod of the model, the household chooses its asset portfolio: how much to hold in money and in government bonds. There is a convex adjustment cost that the household must pay to change its money balances. If in period $t - 1$ the household chose real money balances $\hat{m}_{t-1}$, and in period t it wants to choose a portfolio with real money balances $\hat{m}_t$, it must pay at the end of the period a real cost given by the function

$$\tau^m(\hat{m}_t, \hat{m}_{t-1}) = \frac{\iota^m}{2}\left(\frac{\hat{m}_t}{\hat{m}_{t-1}} - 1\right)^2 \hat{m}_{t-1},$$ [13]

where ι^m is a parameter determining the size of these costs.

The household derives utility from consumption according to utility function $u(c)$.

[13] Here I impose this convex adjustment cost in an ad-hoc way: it depends on period t beginning-of-period real money holdings relative to period $t - 1$ beginning-of-period real money holdings. It would be more natural to have a cost that depends instead on period t beginning-of-period nominal money holdings relative to period $t - 1$ end-of-period nominal money holdings. In the Appendix, I provide and discuss the necessary conditions on the behavior of fiscal policy that make the two formulations equivalent.

Varieties are aggregated according to a Dixit-Stiglitz aggregator:

$$c = \left(\int_0^1 c_j^{1/\lambda_g} . dj \right)^{\lambda_g},$$

where c_j denotes quantity of variety $j \in [0,1]$ and λ_g determines the elasticity of substitution of goods varieties.

To purchase varieties for consumption, the household must search for trading posts selling them. Each of the individuals in the household (a continuum of size 1) searches for a specific variety. The household takes as given the submarkets available and decides how much to purchase from the trading post in case of a match. To fix ideas, suppose that a specific variety $j \in [0,1]$ is sold in a submarket with price p and tightness θ. The household chooses $q(p,\theta)$ – the quantity it wants to purchase from a trading post selling this variety in this submarket in the case of a match. This means the household has to provide the individual searching for this variety with money equal to $pq(p,\theta)$ so that the individual has enough money to settle the transaction in case of a match. Consumption of this variety would equal $q(p,\theta)$ with probability $\rho^h(\theta)$ and 0 with probability $1 - \rho^h(\theta)$. Letting $\Phi(p,\theta)$ denote the measure of varieties being offered in submarkets with price p and tightness θ, it is possible to rewrite the Dixit-Stiglitz aggregation integrating over submarkets (p,θ) instead of varieties:

$$c = \left(\int_\Phi \rho^h(\theta)[q(p,\theta)]^{1/\lambda_g} . dp . d\theta \right)^{\lambda_g}.$$

Similarly, the condition that the household has enough money $\hat{m}$ for how much it wants to purchase of each variety in the event of a match can be written as:

$$\int_\Phi pq(p,\theta) . dp . d\theta \leq \hat{m}.$$

At the end of the period, the household is paid the income from labor, unemployment benefits, dividends as the firm's shareholder, and the interest on its government bond holdings. It must buy goods in a frictionless market to pay the real portfolio adjustment costs defined above. It also pays lump-sum taxes T_t. Separation shocks occur with probability δ^n.

Recursively, one can write the household problem as (omitting dependence on the aggregate state S to ease notation):

$$\text{(5)} \qquad V(z,m) = \max_{\hat{m}\geq 0, \hat{a}, q(p,\theta)} \left\{ u(c) + \beta \mathbb{E} V\left(z', \hat{m}\right) \right\}$$

$$s.t.: \hat{m} + \frac{\hat{a}}{1+i} = z,$$

$$c = \left(\int_{\Phi} \rho^h(\theta)[q(p,\theta)]^{1/\lambda_g}.dp.d\theta \right)^{\lambda_g},$$

$$\int_{\Phi} pq(p,\theta).dp.d\theta \leq \hat{m},$$

$$z' = \frac{\hat{m} - \int_{\Phi} \rho^h(\theta)\, pq(p,\theta).dp.d\theta + \hat{a} - \tau^m(\hat{m},m) + nw + (1-n)\bar{u} + \Pi - T}{1+\pi'}.$$

The first constraint is the portfolio decision. Given initial real wealth z, the household chooses how much to hold as money $\hat{m}$ and government bonds $\hat{a}$. Taking as given the availability of submarkets $\Phi(p,\theta)$, the household also decides how much to try to purchase of each variety. As discussed before, this can be reformulated as a choice over how much to purchase from each submarket. This is denoted by $q(p,\theta)$. At the end of the period, the household is paid back the government bonds with interest, labor income, unemployment benefits, and the dividends from the firms Π. It pays lump-sum taxes of T. It also must pay the real portfolio adjustment cost $\tau^m(\hat{m},m)$. Notice that $\hat{m}$ and $\hat{a}$ are measured in real terms; that is, $\hat{m}$ would be the choice of nominal money divided by the aggregate price level. Going into the next period, the real value of money and nominal government bond holdings is deflated by inflation, as shown in the last constraint describing z'.

Monopolistic producer of a variety Each monopolistic producer of a variety posts vacancies to hire labor and purchases capital to produce goods according to a production function $F(\gamma k, n)$, where γ is the capital utilization rate, k is the capital stock, and n is labor.

Search frictions in the labor market imply that each vacancy finds a worker only with probability $\psi^f(\xi)$, where this equals $M_L(u,v)/v = \psi^h(\xi)/\xi$. Thus, the evolution of the firm's labor force is given by:

$$n_t = (1-\delta^n)n_{t-1} + \psi^f(\xi_t)v_t,$$

where n_{t-1} is the number of workers the firm hired in the previous period and $\delta^n n_{t-1}$

of them will separate from the firm through the exogenous separation shocks. The firm chooses to post v_t vacancies and will hire $\psi^f(\xi_t)v_t$ labor units. Each vacancy the firm posts has a real cost of κ. Additionally, after a match has occurred, the firm needs to pay a fixed hiring cost of H as in Pissarides (2009).[14]

The firm's production function is Cobb-Douglas with constant returns to scale: $F(k, n) = Zk^{\alpha}n^{1-\alpha}$, where Z is a parameter that represents potential total factor productivity. Actual total factor productivity, however, will be below Z and is an endogenous object because of product market search frictions, which imply that not all potential output is traded. The firm holds the capital and chooses how much to invest to determine the next period's capital. Capital depreciates at rate δ^k.

The firm chooses the submarket (p, θ) to sell its variety and takes demand as given. Suppose the firm decides to sell its goods in a submarket with tightness θ and price p. Firstly, it must set up a mass of trading posts equal to θ in this submarket so that it delivers on the chosen tightness (the number of buyers searching for each variety equals one; hence, the tightness equals the mass of trading posts). Moreover, each of these trading posts must be able to accommodate the demand from the buyers, $q(p, \theta)$. Finally, the firm must also accommodate the demand from the representative firm, which aggregates goods for investment and adjustment cost purposes, which I denote $\tilde{q}(p)$. The latter does not depend on tightness because these exchanges occur in a frictionless market. Putting all this together, a monopolistic firm deciding to sell in a submarket (p, θ) needs to have a production capacity of at least $\theta q(p, \theta) + \tilde{q}(p)$.

As before, some of the firm's production capacity, allocated into trading posts that do not match a buyer, will end up unused.

The firm starts the period with capital predetermined. It can choose how much to invest – which will determine the next period's capital stock – and the capital utilization rate for the current period. There are convex investment adjustment costs given by

$$\tau^I(I_t, I_{t-1}) = \frac{\iota^I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_{t-1},$$

where ι^I is a parameter determining the scale of these costs. Operating capital at rate γ, involves a utilization cost given by $r(\gamma)k$.

[14]Pissarides proposes this hiring cost as a solution to the problem raised by Shimer (2005) – that a standard Mortensen-Pissarides model of the labor market does not produce sufficient volatility in employment. I discuss this in more detail in the description of the wage determination of the model.

The firm's recursive problem can be written as:

$$\text{(6)} \qquad \Omega(n^{-}, k, I^{-}) = \max_{(p,\theta), n, v \geq 0, \gamma, k', I} \mathbb{E}\left\{ \frac{\beta\chi'}{\chi} \left[\frac{\Pi}{1+\pi'} + \Omega(n, k', I) \right] \right\}$$

$$\begin{aligned} s.t.: \; & \theta q(p, \theta) + \tilde{q}(p) \leq F(\gamma k, n), \\ & \Pi = \theta\rho^{f}(\theta)\, pq(p, \theta) + p\tilde{q}(p) \\ & \quad - \kappa v - H(n - (1-\delta^{n})n^{-}) - I - \tau^{I}(I, I^{-}) - wn - a(\gamma)k, \\ & I = k' - (1-\delta^{k})k, \\ & n = (1-\delta^{n})n^{-} + \psi^{f}(\xi)v. \end{aligned}$$

Π is the firm's profits. The firm sets up θ trading posts, each with a probability of a match equal to $\rho^{f}(\theta)$, so the product of the two will denote the mass of trading posts matching a buyer. The revenue of each is given by $pq(p, \theta)$. The firm also sells goods in the frictionless market, where it makes $p\tilde{q}(p)$. The firm must pay for the vacancy costs, hiring costs, investment and corresponding adjustment costs, as well as capital utilization costs.

The firm discounts future profits according to the stochastic discount factor of the representative household, given by $\beta\chi'/\chi$, where χ is the representative household's marginal value of wealth.

I define an additional variable, which will be important in the wage determination: μ will denote the Lagrange multiplier on the first constraint. This is the shadow value of one additional unit of production capacity for the firm. It is a generalized notion of the per-unit revenue of the two-period model, hence why I denote it by the same letter.

Due to symmetry, all producers of varieties will make the exact same decisions. Hence, there will be a single active submarket (p, θ) in equilibrium, and each firm will have the same labor force n and the same capital k.

Demand for non-consumption goods If the representative firm purchasing varieties and aggregating them to sell investment goods, vacancy posting services, or adjustment cost services wants to produce Q units, it solves the following cost minimization problem:

$$\min_{\tilde{q}(p)} \int_{\Phi} p.\tilde{q}(p).dp$$

$$\left(\int_{\Phi} \tilde{q}(p)^{\frac{1}{\lambda_g}}.dp\right)^{\lambda_g} = Q.$$

Then, the demand for the variety of a monopolist selling at price p is:

$$\tilde{q}(p) = p^{-\frac{\lambda_g}{\lambda_g - 1}} Q.$$

Notice that in equilibrium,

$$Q_t = \left[\frac{\kappa}{\psi^f(\xi_t)} + H\right](n_t - (1-\delta^n)n_{t-1}) + I_t + \tau^I(I_t, I_{t-1}) + a(\gamma_t)k_t + \tau^m(\hat{m}_t, \hat{m}_{t-1}).$$

Wage Determination I let the steady-state real wage be determined by the solution to the Nash Bargaining problem between the representative household and the monopolistic firm (all firms are the same). However, as in Huo and Ríos-Rull (2020), I assume that both parties see themselves as wage takers, that is, they do not take into consideration the impact of their other decisions in the equilibrium wage.

To match employment dynamics in the data, one must deal with the Shimer puzzle. Shimer (2005) shows that in a calibrated Mortensen-Pissarides model of the labor market, total factor productivity shocks do not generate sufficient volatility in employment to match the business cycle time series data. In the model developed in this paper, monetary policy can affect employment because it affects capacity utilization. From the firm's perspective, the effects of monetary policy are equivalent to shocks in TFP. Consequently, the problem that productivity shocks do not generate enough volatility in employment is inherited from using the Mortensen-Pissarides labor market structure in the form of monetary policy shocks not generating enough volatility in employment. Different solutions to this puzzle have been proposed by Hall (2005), Hagedorn and Manovskii (2008), and Pissarides (2009), which could be adapted to the present setting. I take the simple route of considering a rigid real wage as in Hall (2005), combined with a fixed hiring cost as in Pissarides (2009), to achieve the right level of employment volatility, given that I use lower unemployment benefits than Hall (2005). Hence, Nash Bargaining will determine the equilibrium steady-state real wage; when shocks hit, the real wage is fixed at this level. It is worth emphasizing that the role of this assumption is very different from the role played by nominal rigidities in the New Keynesian model. In that model, nominal rigidities are the critical friction by which monetary policy can have real effects. Here, this is not the case. The imposition of real rigidities on the wage determines how monetary policy affects the different margins of dividends, wages, and employment.

To write the Nash Bargaining problem, one must compute the value of having an additional worker employed for the household and the same for the firm for a given wage $\tilde{w}$. Denoting the marginal value of wealth at the beginning of a period, V_z, as χ, and the marginal value of an additional employed worker at the beginning of a period, V_n, as υ, the value of negotiating a wage of $\tilde{w}$ for the household is given by:

$$\mathbb{E}\left[\beta\chi'\frac{\tilde{w}-\bar{u}}{1+\pi'}+\beta(1-\delta^n)\upsilon'\right]\equiv V^W_{Emp}(\tilde{w}),$$

$$\upsilon=\mathbb{E}\left[\beta(1-\psi^h(\xi))\chi'\frac{(w-\bar{u})}{1+\pi'}+\beta(1-\delta^n)(1-\psi^h(\xi))\upsilon'\right].$$

That is, from the employment of the marginal worker, the household will get an additional income at the end of the period of $\tilde{w}-\bar{u}$ because this worker is paid the negotiated wage rather than the unemployment benefits. Hence, the household will start the next period with an additional $(\tilde{w}-\bar{u})/(1+\pi')$ in wealth. To get the value of this wealth measured in today's utility terms, it must be discounted by β and multiplied by the marginal value of wealth, χ'. On top of this, if the worker is not subject to a separation shock, the household starts the next period with one more employed worker, and hence, its value is also increased by the discount factor β, multiplied by the probability of not being subject to the separation shock $(1-\delta^n)$, multiplied by the derivative of the value function with respect to employment υ'.

In turn, the value of an additional worker for the firm at negotiated wage $\tilde{w}$ is given by:

$$\mathbb{E}\left\{\mu F_n(\gamma k,n)+\frac{\beta\chi'}{\chi}\left[-\frac{\tilde{w}}{1+\pi'}+\frac{\beta\chi''}{\chi'}\frac{\left(\frac{\kappa}{\psi^f(\xi')}+H\right)(1-\delta^n)}{1+\pi''}\right]\right\}\equiv V^F_{Emp}(\tilde{w}).$$

This is the derivative of the firm's value function with respect to n, ignoring the vacancy and hiring costs already incurred by the negotiation stage.

The Nash Bargaining problem determining the steady-state wage is:

$$w^*=\arg\max_{\tilde{w}}\left(V^W_{Emp}(\tilde{w})\right)^{\zeta}\left(V^F_{Emp}(\tilde{w})\right)^{1-\zeta}, \tag{7}$$

where ζ is the bargaining power of the household. Out of steady-state, the wage is fixed at this real level:

$$w_t = w^*. \tag{8}$$

Fiscal and Monetary Authorities In this model, policy constitutes choosing a sequence for the money supply M_t, the bond supply B_t, the nominal interest rate i_t, and taxes T_t, subject to household and firm optimality conditions, equilibrium conditions, and debt boundedness. I divide instruments between a fiscal and a monetary authority. The fiscal authority is responsible for taxes and total public debt D_t, which is not the level of outstanding bond holdings B_t because the monetary authority undertakes open market operations in which it buys some of this debt with printed money and keeps the debt in its balance sheet to set the nominal interest rate i_t. Hence, the monetary authority picks i_t and determines how D_t is divided into bond holdings B_t and money holdings M_t (given the household's demand for these assets, which depends on the nominal interest rate), such that

$$D_t = \frac{B_t}{1 + i_t} + M_t. \tag{9}$$

There is a law of motion for the consolidated Government given by:

$$\frac{B_t}{1 + i_t} + M_t + \frac{T_{t-1} - (1 - n_{t-1})\bar{u}}{1 + \pi_t} = \frac{B_{t-1}}{1 + \pi_t} + \frac{M_{t-1}}{1 + \pi_t}. \tag{10}$$

At the end of period $t - 1$, the Government collects the lump-sum taxes and pays the unemployment benefits. Following that, at the beginning of period t, it must pay its liabilities from the previous period, which in real terms are given by the right-hand side of the above equation. To fund the payment of these liabilities, the government issues new total public debt $D_t = B_t/(1 + i_t) + M_t$. This means that the fiscal authority can pick taxes, and public debt is determined by equation (10).

To close the model, one needs to specify the policy rules for the fiscal and monetary authorities and the shocks.

Policy and Determinacy Two policy rules are needed in this model. One for the fiscal authority that determines how the choice of taxes responds to the evolution of public debt, and one for the monetary authority that determines how the nominal interest rate responds to aggregate variables such as inflation and unemployment. As was the case in the two-period model, not all policy rules will achieve determinacy of the price level. Because I am only interested in computing the model's response to a monetary policy shock, I do not specify policy rules here. I define a recursive monetary equilibrium for general policy rules and then discuss how I solve the model following a monetary policy shock.

Equilibrium In equilibrium, all monopolistic firms make the same decisions. Equilibrium in the goods market entails:

$$\theta_t q(p_t, \theta_t) = F(\gamma_t k_t, n_t) - \left[\frac{\kappa}{\psi^f(\xi_t)} + H\right]\left(n_t - (1-\delta^n)n_{t-1}\right) - I_t - \tau^I(I_t, I_{t-1}) - \tau^m(\hat{m}_t, \hat{m}_{t-1}) - r(\gamma_t)k_t. \tag{11}$$

Notice that this is not the typical market clearing condition. As in the two-period model, a part of the production capacity allocated to the consumption market will end up unused because the trading post does not match a buyer.

Equilibrium in the money market is given by:

$$\hat{m}_t = M_t, \tag{12}$$

while equilibrium in the bond market entails:

$$\hat{a}_t = B_t, \tag{13}$$

I have made the expenditure necessary to purchase one unit of each variety the numeraire. This means:

$$\int_0^1 p_j.dj = 1.$$

Given that in equilibrium, all firms set the same price, this simplifies the condition to:

$$p_t = 1. \tag{14}$$

By Walras' law, only 3 conditions out of (11)-(14) are necessary.

3.2. Recursive Monetary Equilibrium

I define a Recursive Monetary Equilibrium of this model that can accommodate the addition of different shocks. The aggregate state variables of the model are the previous period employment level n^{-}, the capital stock K, previous period investment I^{-}, previous period real money balances M^{-}, total public debt inherited from the previous period $\tilde{D}^{-} \equiv (B^{-} + M^{-} - T^{-} + (1 - n^{-})\bar{u})$, and shocks ϵ. Thus, we have $S = (n^{-}, K, I^{-}, M^{-}, \tilde{D}^{-}, \epsilon) \in \mathcal{M}$ where $\mathcal{M}$ is the appropriate domain for the aggregate state space. Shocks evolve according to a Markov process described by a matrix Π_{ϵ}.

Definition A Recursive Monetary Equilibrium is given by: i) the household's value function $V(z, m, S)$ and policy functions for money holdings $\hat{m}(z, m, S)$, bond holdings $\hat{a}(z, m, S)$, and goods demand by submarket $q(p, \theta; z, m, S)$; ii) the demand for goods from the representative firm that sells non-consumption goods $\tilde{q}(p; S)$; iii) the firm's value function $\Omega(n^{-}, k, I^{-}, S)$, goods shadow-value $\mu(n^{-}, k, I^{-}, S)$, and policy functions for labor demand $n(n^{-}, k, I^{-}, S)$, capital $k'(n^{-}, k, I^{-}, S)$, capital utilization $\gamma(n^{-}, k, I^{-}, S)$, investment $I(n^{-}, k, I^{-}, S)$, price $p(n^{-}, k, I^{-}, S)$, and tightness $\theta(n^{-}, k, I^{-}, S)$; iv) functions with domain $\mathcal{M}$ determining all aggregate variables – labor market tightness $\xi(S)$, real wage $w(S)$, inflation $\pi(S)$, dividends $\Pi(S)$, real money supply $M(S)$, real bond supply $B(S)$, total public debt $D(S)$, taxes $T(S)$, nominal interest rate $i(S)$, and marginal value of wealth $\chi(S)$ – and; v) an aggregate law of motion for the aggregate state $S' = J(S)$; such that:

a. The household's value function and policy functions satisfy the Bellman Equation for the household's problem as described in (5).
b. The firm's value function, goods shadow-value, and policy functions satisfy the Bellman Equation for the firm's problem as described in (6).
c. The equations determining the policy rules of the fiscal and monetary authorities are satisfied and guarantee price level determinacy. These could be a fiscal policy in which taxes $T(S)$ respond strongly enough to increases in debt $D(S)$ (passive fiscal policy) and a policy rule for the nominal interest rate that constitutes active policy.
d. The law of motion for the Government, given by equation (10), is satisfied, and so is the definition of total public debt, given by equation (9).
e. Dividends $\Pi(S)$ are determined as defined in the firm problem in (6).
f. The marginal value of wealth satisfies:

$$\chi(S) = \frac{\partial V\left(\frac{\tilde{D}^{-}}{1+\pi(S)}, M^{-}, S\right)}{\partial z}$$

To understand the expression above, note that the representative household's total wealth is given by: $\frac{\tilde{D}}{1+\pi}$. To see why this is the case, notice that before the previous period's product market, the household held the aggregate money and bond supply. Because the household gets all wages and dividends, all the money spent ends up being received back by the representative household in these income payments. Hence, at the end of the period, the household holds $M^{-} + B^{-}$ and pays taxes but gets unemployment benefits. Therefore, this amounts precisely to $\tilde{D}^{-}$.

g. The real wage satisfies equation (8).

h. The money market clears, meaning that supply equals demand:

$$\hat{m}\left(\frac{\tilde{D}^{-}}{1+\pi(S)}, M^{-}, S\right) = M(S),$$

i. The bond market clears

$$\hat{a}\left(\frac{\tilde{D}^{-}}{1+\pi(S)}, M^{-}, S\right) = B(S).$$

j. The aggregate price level is the numeraire:

$$p\left(n^{-}, K, I^{-}, S\right) = 1.$$

k. The law of motion for the aggregate state satisfies five different conditions. Employment evolves according to (4). The capital stock evolves according to the firm's policy function for capital, and so does investment. The aggregate state for real money balances in the next period equals the money supply $M(S)$. Public debt evolves according to:

$$\tilde{D}(S) = B(S) + M(S) - T(S) + (1 - n\left(n^{-}, K, I^{-}, S\right))\bar{u}.$$

3.3. Model computation

To estimate the model and evaluate its impulse response functions to a monetary policy shock, I solve it through a first-order perturbation (linearization) around the steady state. Details on the first-order conditions of the model and steady state are provided in the Appendix.

Feeding a nominal interest rate path to the model with passive fiscal policy (taxes adjust to ensure debt boundedness for any equilibrium) results in indeterminacy. Consider a sequence for the nominal interest rate: $\{i_t\}_{t=0}^{\infty}$. There is a continuum of equilibria possible. All equilibria have the same path for all variables with the exception of initial inflation, π_0, and the discounted value of future government surpluses. I focus on the equilibrium in which initial period inflation is unaffected by the shock, meaning that it stays at the steady state level, $\pi_0 = \pi^*$. This is the identification restriction commonly imposed in VARs to identify monetary policy shocks (Alvarez, Atkeson, and Edmond 2009 use the same strategy to compute the impulse response function to a nominal interest rate shock in a different monetary model). The monetary authority can implement this equilibrium as unique by announcing both the sequence for the nominal interest rate $\{i_t\}_{t=0}^{\infty}$ and an initial nominal money supply $\tilde{M}_0$. Because the interest rate sequence determines the path for real money demand $\{M_t\}_{t=0}^{\infty}$, the monetary authority needs to set $\tilde{M}_0$ according to

$$\frac{\tilde{M}_0}{\tilde{M}_{-1}} = \frac{M_0}{M_{-1}}(1+\pi^*).$$

4. The Quantitative Effects of a Monetary Policy Shock

This section presents the details of the calibration and estimation of the model. It also describes briefly a New Keynesian model that I compare the search-based model to. Finally, it presents the results: I display the effects of monetary policy in my model and evaluate the fit to the vector autoregression relative to the New Keynesian model.

4.1. Calibration and Estimation

4.1.1. Functional forms

I consider a model period to equal one month, which seems natural as the frequency of labor income payment. I use a CRRA utility function,

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

a Cobb-Douglas matching function for the labor market,

$$M_L(u, v) = A_L v^{\gamma_L} u^{1-\gamma_L},$$

and a CES matching function for the product market,

$$M_P(b, s) = A_P \left[\omega_P s^{\rho_P} + (1-\omega_P) b^{\rho_P}\right]^{\frac{1}{\rho_P}}.$$

4.1.2. Parameter values

The inverse elasticity of intertemporal substitution σ is set to 2, and the discount factor β is chosen to get a steady-state annual real interest rate of 2% ($\beta = 0.998351$). For the labor market, I set the elasticity of the matches to vacancies, given by ν_L, to 0.28, which is in the interval of estimates reviewed by Petrongolo and Pissarides (2001). The worker bargaining power is set to 0.72 to satisfy Hosio's condition, and the monthly separation rate is set to 0.034, which is also standard in the literature (see Shimer 2005 or Hall 2005). Unemployment benefits are set to a 45% replacement rate. The matching productivity in the labor market, A_L, is set to target a steady-state unemployment rate of 6%. The remaining labor market parameters, κ and H, are part of the set of parameters to be estimated. I set $\alpha = 0.36$ and the capital depreciation rate, δ^k, to equal 10% annually. I follow Christiano, Eichenbaum, and Evans (2005) in assuming that the capital utilization cost function $r(\gamma)$ is such that $\gamma = 1$ in steady state and $r(1) = 0$. There is, then, a single parameter relevant to the dynamics of the linearized model, which is $\sigma_r = r''(1)/r'(1)$. I set this equal to 0.01 as in that paper.

I let the matching productivity of the product market A_P target a 95% steady state capacity utilization rate. This choice is somewhat arbitrary but should be sufficiently below 100 to ensure there is room for monetary policy to push it up. The other two parameters of the product market matching function ω_P and ρ_P, the elasticity of substitution of good varieties λ_g, the size of the portfolio adjustment costs ι^m, and investment adjustment costs ι^I are estimated.

Table 1 summarizes the information on each parameter of the model and how they are determined.

4.2. Data and VAR estimation

Here, I follow closely Christiano, Trabandt, and Walentin (2011). I collect seasonally adjusted quarterly data for the period 1960Q1-2008Q4, constructing a VAR with two lags and the following variables:

Search-based model

Parameter	Description		Target
Preferences			
β	Discount factor	Calibrated	Annual real rate of 2 %
σ	Inverse IES	Calibrated	2.0
Labor market			
A_L	Matching productivity	Calibrated	Steady-state unemployment of 6%
ν_L	Elasticity of matching function	Calibrated	Petrongolo and Pissarides (2001): 0.28
ζ	Worker bargaining power	Calibrated	Hosios' condition: 0.72
δ^n	Separation rate	Calibrated	Shimer (2005): 0.034
κ	Vacancy cost	Estimated	—
H	Hiring cost	Estimated	—
Production			
Z	Potential TFP	Calibrated	Normalization
α	Capital share	Calibrated	Labor share of 64%
δ^k	Capital depreciation rate	Calibrated	10% annually
σ_r	Costs of capital utilization	Calibrated	Christiano et al. (2005): 0.01
ι^I	Investment adjustment costs	Estimated	—
Goods market			
λ_g	Elasticity of substitution good varieties	Estimated	—
A_P	Matching productivity	Calibrated	95% steady state capacity utilization
ω_P	Elasticity of matching function	Estimated	—
ρ_P	Substitution in matching function	Estimated	—
Asset market			
ι^m	Portfolio adjustment costs	Estimated	—
Government policy			
$\bar{u}$	Unemployment benefits	Calibrated	Steady state replacement ratio of 45%
$\bar{\pi}$	Inflation target	Calibrated	Steady state inflation target of 2%

TABLE 1. Details of calibration and estimation strategy by parameter

$$Y = \begin{bmatrix} \triangle \ln(relPriceInvestment) \\ \triangle \ln(realGDP/hours) \\ \triangle ln(GDPDeflator) \\ unemploymentRate \\ capacityUtilization \\ \ln(hours) \\ \ln(realGDP/hours) - \ln(realWage) \\ \ln(nominalC/nominalY) \\ \ln(nominalI/nominalY) \\ \ln(vacancies) \\ jobSeparationRate \\ jobFindingRate \\ \ln(hours/laborForce) \\ FedFundsRate \\ \triangle \ln(MZM) \end{bmatrix}$$

Except for the growth rate of the monetary aggregate money zero maturity (MZM), the set of variables in this VAR is the same as the one in Christiano, Trabandt, and Walentin (2011) and Christiano, Eichenbaum, and Trabandt (2016). I use MZM to emphasize that the concept of money that is relevant for the liquidity effects of monetary policy is broader than M1 and should include other liquid assets whose return is dominated by the nominal interest rate on government bonds due to liquidity premia. In the Appendix, I show the results for the vector autoregression using M1 as the monetary aggregate.

I use the same identification scheme as Christiano, Trabandt, and Walentin (2011). The identifying restriction for the monetary policy shock is that the only variables it affects contemporaneously are the Federal Funds rate and the monetary aggregate. Using the estimated VAR, I compute the impulse response functions to a monetary policy shock for the variables described in Table 2. These are the impulse response functions used for model estimation. I use bootstrap methods to compute the error covariance matrix of the impulse response functions.

Variable	Unit of measure
Federal Funds Rate	Annual percentage rate deviation from steady-state
Output	% deviation from steady-state
Consumption	% deviation from steady-state
Investment	% deviation from steady-state
Unemployment rate	p.p. deviation from steady-state
Inflation rate	Annual percentage rate deviation from steady-state
Labor productivity	% deviation from steady-state
Labor share	% deviation from steady-state
Real MZM supply	% deviation from steady-state

TABLE 2. Variables whose impulse response functions are used in the model estimation

4.3. New Keynesian model with cash constraint

To compare the performance of the search-based model developed in this paper with the New Keynesian model, I also calibrate and estimate a medium-scale New Keynesian model. The search-based model has product and labor market search frictions, while the New Keynesian model has nominal rigidities in these markets (price and wage stickiness). Both models have variable capital utilization and investment adjustment costs. Instead of portfolio adjustment costs, the New Keynesian model has preferences with internal habit formation, which also results in hump-shaped responses. Finally, the model has a cash constraint, which means that real money holdings equal consumption. Monetary policy follows a Taylor rule of the form:

$$1 + i_t = (1 + i_{t-1})^{\phi_i} \left[\left(\frac{1+\pi_t}{1+\pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y^*} \right)^{\phi_y} \right]^{1-\phi_i} \epsilon_t^m.$$

Further details are provided in the Appendix.

4.4. Impulse Response Matching

Impulse response matching is a type of Classical Minimum Distance estimator. Assume the structural model is correct and $\Psi(\theta)$ is the mapping from model parameters θ to the quarterly impulse response functions. Let $\hat{\Psi}$ represent a vector that collects the impulse response functions estimated by the VAR for the different variables and horizons. This is a stacked vector of size $N \times T$, where N is the number of variables one wants to match, and T is the number of periods in each impulse response function. $\hat{\Sigma}$ is, in turn, the error covariance matrix estimated by bootstrap. Impulse response matching finds the model parameters, $\hat{\theta}$, that minimize a weighted average distance between the model impulse response functions and the VAR impulse response functions:

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \left(\hat{\Psi} - \Psi(\theta) \right)' W \left(\hat{\Psi} - \Psi(\theta) \right).$$

W is a positive definite $(N \times T)$-by-$(N \times T)$ weighting matrix.

In my setting, I use 9 variables and a horizon of 15 quarters. Following common practice, I use a diagonal weighting matrix with elements equal to the inverse of the diagonal elements of $\hat{\Sigma}$. Because I estimate the VAR using a Cholesky decomposition that imposes a zero contemporaneous effect of the monetary shock on many of the variables, I set the weight corresponding to these equal to zero. Having chosen a monthly period for the model, I aggregate the monthly impulse response functions to get $\Psi(\theta)$.

I first estimate the New Keynesian model with the Taylor Rule. Then, I use the interest rate path produced in the estimated New Keynesian model impulse response function to set the path for the nominal interest rate in the search-based model. Then, I estimate the search-based model.

4.5. Estimation Results

Figure 1 displays the impulse response functions in the vector autoregression (with 95% confidence interval bands) and in the two estimated models. The search-based model is able to generate a large and sustained response of output and consumption that aligns well with the vector autoregression. Investment and employment also increase, though their responses are weaker than the ones found in the data. The difference between the two models is most noticeable for inflation, labor share, and monetary variables.

To understand why the two models have a very different inflation response, notice that in the New Keynesian model, the economy is initially operating at capacity. A decrease in the nominal interest rate raises demand, but nominal rigidities imply that prices do not adjust fully immediately. The effect on inflation is slow and spreads over several quarters, allowing for an effect on quantities. In the search-based model, the mechanism is different. A decrease in the nominal interest rate increases real money demand, as households are more willing to hold liquidity, making it easier for firms to find buyers. This force is not inflationary; the nominal money supply is not increased by more than the increase in real money supply. Instead, the dynamics of inflation are driven by the Euler Equation. Given a certain path for consumption and a path for the nominal interest rate, inflation will adjust to deliver a certain path for the real interest rate. The path of aggregate consumption implies an initial decrease in the real interest rate, which is lower than the decrease in the nominal interest rate; hence, expected inflation falls. Limited participation in asset markets could potentially be an avenue to bring the response of the model closer to the data, as it would drive a disconnect between the real rate in the economy and the aggregate consumption path (see Khan and Thomas 2015).

The labor share in the New Keynesian model displays a counterfactual increase following the monetary policy shock. In contrast, the search-based model is able to deliver a response in line with the data. The labor share is equal to the real wage divided by labor productivity. Hence, the reason why the New Keynesian model fails to deliver a countercyclical labor share response relates to its weak labor productivity response.

Variable	Search model (1)	New Keynesian model (2)	(1)/(2)
Nominal interest rate	0.1163	0.1163	1.0000
Output	0.0979	0.0986	0.9926
Consumption	0.0579	0.0941	0.6146
Investment	0.4199	0.3575	1.1746
Unemployment rate	0.0620	0.0406	1.5248
Inflation rate	0.1249	0.1097	1.1388
Labor productivity	0.0302	0.0404	0.7490
Labor share	0.0152	0.0994	0.1532
Real money supply	0.6675	0.6171	1.0817
All variables	0.0906	0.0923	0.9819

TABLE 3. Weighted Root Mean Squared Errors by variable in the two models

The search-based model's capacity utilization channel provides a mechanism by which the economy can get more output from the same input factors.

The New Keynesian model's real money supply response is muted because the model does not have any mechanism to change money velocity. Clearly, in the data, following the shock, money velocity decreases, meaning that the real money supply goes up by more than output. After five quarters, money velocity seems to increase. The search-based model is able to match the initial fall in money velocity, but it cannot generate the subsequent increase. The model's tight inverse relationship between money velocity and capacity utilization is at odds with the data. Adding search effort may be a way of helping the model better match the data in this regard.

Table 3 summarizes some of the information contained in the impulse response functions. It displays the weighted root mean squared errors (RMSE) for the two models. These measure how well the impulse response functions of the model, following estimation, match the impulse response functions of the VAR. The weights are the same ones used in estimation. The last row contains the weighted root mean squared error for all the variables included in the estimation; this is the value that the estimation procedure minimized. Column 3 shows the ratio between the RMSE of the two models. The search-based model has a slightly lower RMSE. Perhaps surprisingly, using this quantitative measure, the performance of the New Keynesian model seems to be better for real money than the search-based model. This is because while the search model is able to better match the initial behavior of real money, its slow dynamics back to steady-state penalize it for quarters 5 to 15.

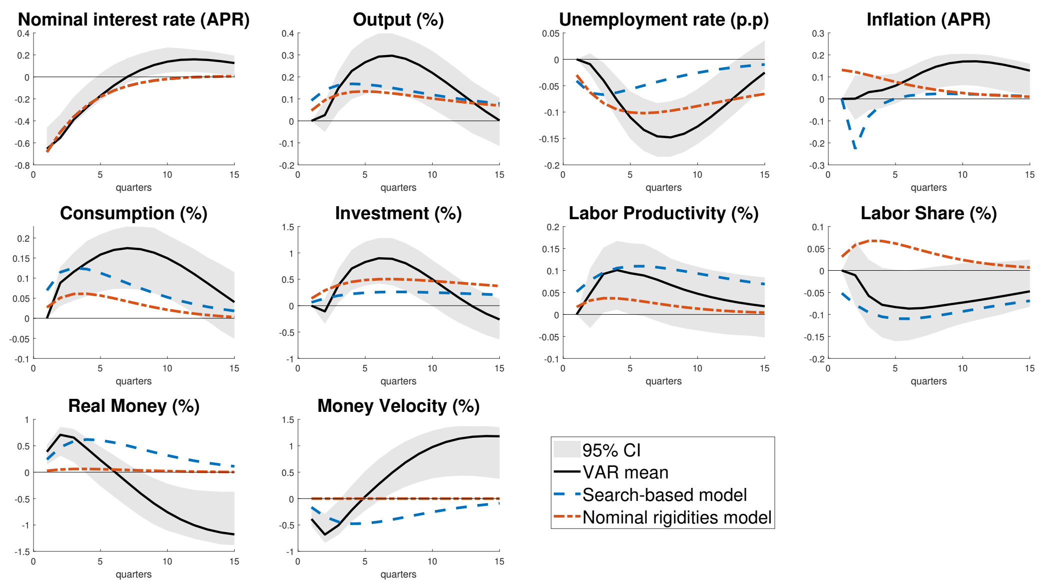

FIGURE 1. Impulse response functions following a monetary policy shock in vector autoregression, search-based model, and New Keynesian model.

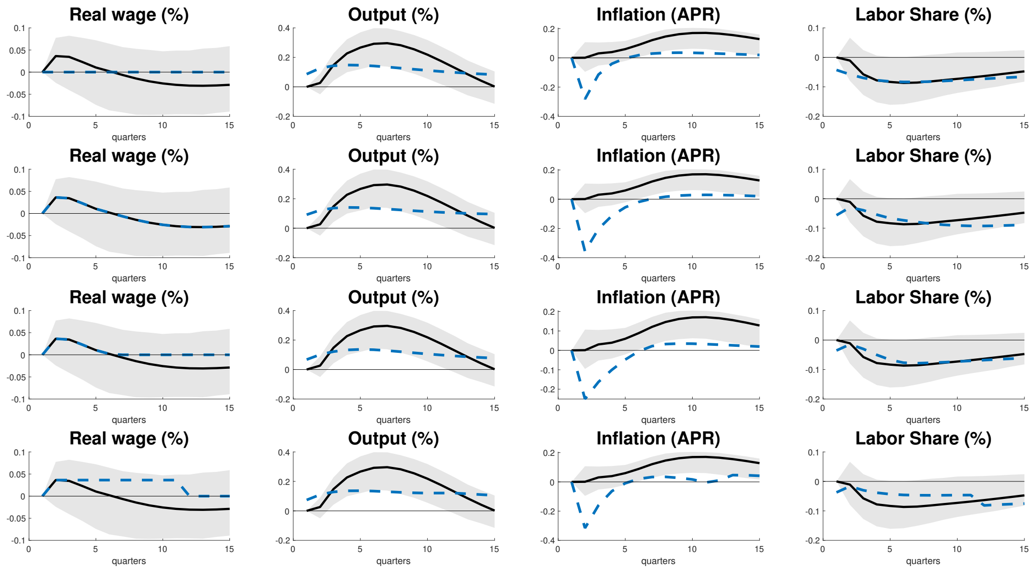

FIGURE 2. Impulse response functions following a monetary policy shock in vector autoregression and the search model, under different model paths for the real wage

4.6. Robustness to real wage behavior

In the search model estimated above, the real wage was fixed at the steady state level. Under such a rigid real wage, the labor share is the inverse of labor productivity. As a robustness exercise, I reestimate the model under alternative specifications for the behavior of the real wage following the monetary policy shock.

Due to the random search and bargaining environment, one can choose a path for the real wage exogenously as long as it satisfies individual rationality for both parties involved. Figure 2 displays the results. Each of the four rows contains the impulse response function following the monetary policy shock for the real wage, output, inflation, and the labor share under different specifications for the behavior of the real wage. The first row presents the results under the real wage specification estimated before. The second row displays the responses under a real wage path that exactly mimics the one in the vector autoregression. The third and fourth row consider other real wage responses in which the real wage remains elevated relative to the data for longer. Results clearly show that the countercyclical response of the labor share does not depend on any strong assumptions about the behavior of the real wage. Because the response of labor productivity to the monetary shock is strongly procyclical, the model can deliver a countercyclical labor share even with significant increases in real wages.

5. Banks and Monetary Policy Implementation

The model presented above did not include a financial system. The Central Bank issued all money, and the economy had a single nominal interest rate. While this is common practice in many models of monetary policy, in this particular setting, it raises the question of how the model's mechanism changes in a modern financial system. In this section, I show a simple way of incorporating banks in the two-period model presented in section 2 and discuss the conditions under which liquidity-based effects of monetary policy are relevant in this setting. Essentially, the gap between the nominal interest rate on government bonds and the nominal interest rate on liquid assets becomes the variable by which monetary policy can affect the economy. In the model above, the liquid asset was money and paid a 0 interest rate. The liquid asset may pay a positive nominal interest rate with bank deposits. Monetary policy's effectiveness is larger when there is not a substantial pass-through from the nominal interest rate on government bonds to the deposit rate. That is when the Central Bank can affect the gap between the interest rate on government bonds and liquid assets significantly.

Environment Relative to the original setting, the model now includes a representative bank owned by the representative household. This bank issues deposits and makes loans to households. It can also hold government bonds and reserves at the Central Bank. Lagos and Zhang (2022) show that the market power of financial intermediaries is key to understanding why money may remain important even in an economy where it does not circulate. I model market power in the deposit market through search frictions: households match a bank in each period and then need to bargain the terms of trade on their deposits. The outside option for the household is to hold cash. For simplicity, I assume the market for loans, government bonds, and reserves is perfectly competitive. Only the market for deposits deviates from Walrasian market clearing. I assume the timing is as follows: at the beginning of a period, the household makes its decision about how much to take in bank loans, how much to invest in government bonds, and how much to hold in liquid assets. The household then matches with a bank, having made this decision, and bargains how much of its liquid assets to hold as deposits and at what nominal interest rate. Thus, the household's disagreement point consists of holding all its liquidity as cash, while the bank's involves not getting the profits it would make from using the deposits to increase its loans and government bond holdings.[15]

Government and Central Bank The Government has an initial debt of D^g and needs to issue debt to finance its payment. Because the household owns the banks, it is without loss of generality to assume that the household holds this initial debt. The Central Bank can issue reserves R by buying government debt and picks the nominal interest rate it pays on these reserves, i^R. The Central Bank can also issue cash and takes demand for this asset as given to ensure parity with reserves (one unit of reserves trades for one unit of cash). However, in equilibrium, no cash will circulate as long as banks' nominal interest rate on deposits is above 0. The Central Bank's behavior aims to control the nominal interest rate on government bonds i. Banks and households will hold part of the initial government debt, D^g, in the form of government bonds B. The remaining will end up in the Central Bank's balance sheet as an asset, backed by reserves held by banks:

$$D^g = \frac{B}{1+i} + \frac{R}{1+i^R}.$$

At the end of period 1, the government taxes the household lump sum to pay its debt.

$$T = D^g(1+i) - \left[\left(\frac{1+i}{1+i^R}\right) - 1\right] R. \tag{15}$$

The government pays interest on its debt, making total payments $(1+i)D^g$; however, a part of this is interest paid to the Central Bank. I assume that Central Bank profits are rebated back to the Government. This is given by the second term on the right-hand side of equation (15). Notice how the Central Bank has positive profits when $i^R < i$, that is, when it buys government debt with reserves, and the interest it gets on the government debt is larger than the interest it pays on reserves.

Bank The bank can issue deposits and loans; it purchases government bonds and reserves. It has a reserve requirement constraint, determined by ρ:

$$\rho D \leq R.$$

[15] One may see a timing inconsistency here. If the household has decided on loans at the beginning of the period, would a bank not have decided it in advance too? This can be made coherent by assuming that there is a Walrasian interbank market where, by a no-arbitrage condition, banks lend to each other at the interest rate on government bonds. When bargaining with a household, then, a bank sees the disagreement point as involving zero profits because it can just borrow at the market rate whatever it already committed to lending.

Reserves are paid the nominal interest rate i^R, deposits pay a nominal interest rate i^D, bonds pay i, and loans pay i^L. I let the market for loans be perfectly competitive, with loans having an issuance cost of κ_L. For the deposit market, I assume that each household matches a bank and must negotiate the conditions for the deposit. The household's outside option is to hold money instead of deposits, which pay a zero nominal interest rate. The nominal interest rate on deposits will be determined by Nash Bargaining, and no household will hold cash in equilibrium.

Suppose a bank gets D in deposits and bargained an interest rate on deposits of i^D, then its maximization problem solves:

$$\begin{aligned}\Pi^B(D, i^D) = \max_{R,B,L}\ & R + B + L - D \\ s.t.:\ & \frac{D}{1+i^D} = \frac{R}{1+i^R} + \frac{B}{1+i} + \frac{L}{1+i^L}(1+\kappa_L), \\ & \rho D \leq R.\end{aligned}$$

This yields the following no-arbitrage condition:

$$1 + i^L = (1+i)(1+\kappa_L).$$

Firm There is no change to the firm problem.

Household The household owns the firm and the bank and will get the profits at the end of period 1, after the product market trade occurs and profits are materialized. However, at the beginning of period 1, the household's wealth is the initial government debt. The household has to decide the composition of its portfolio.

The problem of the household consists of choosing deposits, government bonds, loans, and the submarket in which to search for goods, subject to the budget constraint and two restrictions on the choice of submarket – that its liquid asset holdings are sufficient to pay for the transaction in the submarket chosen, and that the firm is willing to set up trading posts in that submarket. The household takes as given the firm revenue per unit in the active submarkets μ, profits Π (which sum firm and bank profits), taxes T, the nominal interest rate on government bonds i, the nominal interest rate of loans i^L, and period two price p_2. The nominal interest rate on deposits will depend on the household's asset portfolio decision because this will affect the Nash Bargaining solution.

It is helpful to divide the asset composition decision into two stages. In stage 1, the household chooses its portfolio composition in terms of loans, government bonds, and liquid assets. In stage 2, the household matches a bank and bargains how much liquid assets to deposit in the bank and at which nominal interest rate. The household then goes to the product market with asset holdings of a in government bonds, l in bank loans, $\hat{d}$ in deposits, and $\hat{m}$ in cash. The value function before going into the product market, having negotiated an interest rate i^D on deposits, is given by:

$$
\begin{aligned}
W(a, l, \hat{d}, \hat{m}, i^D) = \max_{(\theta, p, q), c_1, c_2} \quad & u(c_1) + \beta u(c_2) \\
s.t.: \quad & c_1 = \rho^h(\theta).q, \\
& pq \leq \hat{m} + \frac{\hat{d}}{1 + i^D}, \\
& \rho^f(\theta)\, p = \mu, \\
& p_2 c_2 = \hat{m} + \hat{d} + a - \rho^h(\theta)\, pq - l + \Pi - T + p_2 y_2.
\end{aligned}
$$

In stage 2 of the asset composition problem, the household holds government bonds, loans, and cash. It negotiates with the bank how much of this cash to turn into deposits and the interest it receives. The following problem characterizes the Nash Bargaining solution:

$$
\begin{aligned}
\max_{\hat{d} \geq 0, i^D, \hat{m}} \quad & [W(a, l, \hat{d}, \hat{m}, i^D) - W(a, l, 0, m, i^D)]^{\chi} [\Pi^B(\hat{d}, i^D)]^{1-\chi}, \\
s.t.: \quad & \frac{\hat{d}}{1 + i^D} + \hat{m} \leq m.
\end{aligned}
$$

where χ is the bargaining power of the household.

The value function of the household in stage 1 is given by:

$$
\begin{aligned}
V = \max_{a \geq 0, l \geq 0, m \geq 0} \quad & W(a, l, \hat{d}^*(a, l, m), \hat{m}^*(a, l, m), i^D(a, l, m)) \\
s.t.: \quad & \frac{a}{1 + i} + m = D^g + \frac{l}{1 + i^L}.
\end{aligned}
$$

where $\hat{d}^*(a, l, m)$, $\hat{m}^*(a, l, m)$, and $i^D(a, l, m)$ denote the terms of trade in the negotiation in stage 2, given the choices in stage 1.

Characterizing the household problem The solution to the bargaining problem will involve a nominal interest rate on deposits above zero, and the household will choose to hold all its liquid assets in the form of deposits.[16] Using this result, one can write the household problem as:

$$\begin{aligned}
\max_{d\geq 0, a\geq 0, l\geq 0, (\theta, p, q), c_1, c_2} \quad & u(c_1) + \beta u(c_2) \\
s.t.: \; & \frac{a}{1+i} + \frac{d}{1+i^D(d,a,l)} = D^g + \frac{l}{1+i^L}, \\
& c_1 = \rho^h(\theta) q, \\
& pq \leq \frac{d}{1+i^D(d,a,l)}, \\
& \rho^f(\theta). \, p = \mu, \\
& p_2 c_2 = d + a - \rho^h(\theta)\, pq - l + \Pi - T + p_2 y_2.
\end{aligned}$$

The nominal interest rate on deposits depends on the asset choice of the household, and the household internalizes this in its decision.

Government bonds pay an interest rate lower than the rate on loans, hence the household will not hold these two assets simultaneously. Whether the household needs to borrow depends on the size of its initial wealth D^g. Focusing on the case in which $l \geq 0$ and $a = 0$, the problem simplifies to:

$$\max_{c_1, \theta} u(c_1) + \beta u\left(\frac{-\frac{\mu}{\rho^f(\theta)} c_1 \left(1 + \frac{i^L - i^D(d,a,l)}{\rho^h(\theta)}\right) + \Pi + \left[\left(\frac{1+i}{1+i^R}\right) - 1\right] R}{P(1+\pi_2)} + y_2 \right). \tag{16}$$

Where I omit the conditions connecting d, a and l to c_1 and θ. The optimization problem in (16) is very similar to the one the household faced in the model with money, given in (2). Now i has been replaced with $i^L - i^D(d,a,l)$. This means that the effects of monetary policy depend now on how this gap changes with a change in the Central Bank's target for i. If the bank's bargaining power is 1, $i^D = 0$, and the elasticities of this model are the same as in the model without banks. As the bank's bargaining power deviates from this, the interest rate on deposits moves more with i, and the effects of monetary policy become slightly weaker. This suggests that within this model, the pass-through of the interest rate to the deposit rate is the crucial statistic to understand

[16] If the bank's bargaining power equals 1 then the deposit rate is exactly 0. One can assume the household still holds its liquid assets as deposits, as it is indifferent.

the relevance of liquidity-based effects of monetary policy.

Monetary policy implementation In the model without banks, the Central Bank had control over the composition of outstanding government assets and targeted the nominal interest rate i. Now, the Central Bank has one more instrument. Not only does it still have control over the composition of outstanding government assets, but it can also set the interest rate on reserves. This means it has one additional degree of freedom and can implement monetary policy in two different ways.

Scarce reserves regime:

a. The target for the interest rate on government bonds is higher than the interest on reserves: $i > i^R$.
b. Banks hold only required reserves $R = \rho D$.
c. The Central Bank must passively accommodate the demand for reserves and implement open market operations to change the composition of public debt between reserves and outstanding government bonds.

Excess reserves regime:

a. The target for the interest rate on government bonds is equal to the interest on reserves: $i = i^R$.
b. Banks' demand for reserves is perfectly elastic.
c. Open market operations are neutral.

Nothing fundamentally changes in the effects of monetary policy between the two regimes. They constitute two ways the Central Bank can control i.

6. Conclusion

Readers familiar with Woodford (1998) might have noticed that the title of this paper contains a reference to that paper's title "Doing Without Money: Controlling Inflation in a Post-Monetary World". In that paper, Woodford criticized the macro monetary literature for its insistence on conceiving monetary policy in terms of a choice of the money supply. He shows that interest rate rules such as the Taylor Rule may achieve price level determinacy even in the cashless limit of the economy. In this paper, I do not weigh in on the price level determinacy questions that Woodford tried to resolve. However, over the last 25 years, monetary macroeconomics has evolved not only towards an agreement that monetary policy should be thought in terms of a nominal interest rate but also towards a belief that the short-term real effects of monetary policy are unrelated to the liquidity services that money provides and are instead fully explained by nominal rigidities. This might be because models of nominal rigidities have been shown to match several of the features associated with the effects of monetary policy. In contrast, models of money as a medium of exchange have not been as successful in this endeavor. In this paper, I model monetary search frictions in a way that can easily be embedded into a quantitative business cycle framework and investigate the quantitative ability of such a model to match the effects of monetary policy. The model is successful in many ways and can generate a large procyclical response of labor productivity and a countercyclical response of the labor share, which the New Keynesian model cannot. It also matches the initial response of money velocity to the monetary policy shock.

The monetary search frictions used here can be easily applied to different settings. For example, they can be used to study monetary policy in environments with household heterogeneity. Directed search results in a block recursive structure, which makes a heterogeneous agent model computationally tractable. Using these frictions instead of nominal rigidities in these contexts may be particularly advantageous because getting the behavior of the labor share correctly is necessary to get the distributional effects of monetary policy right. The extension I present with financial intermediaries can also be useful in such a setting where the distinction between borrowers and lenders can play an important role.

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