Heterogeneous Agents Dynamic Spatial General Equilibrium*

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Abstract

I develop a dynamic model of migration and labor market choice with incomplete markets and uninsurable income risk to quantify the effects of international trade on workers' earnings, savings and reallocation decisions. Macroeconomic conditions in different labor markets and idiosyncratic shocks shape agents' labor market choices, consumption, earnings, and asset accumulation over time. Despite the rich heterogeneity, the model is highly tractable as the optimal consumption, labor supply, capital accumulation, migration and reallocation decisions of individual workers across different markets have closed-form expressions and can be aggregated. I study the asymmetric impact of international trade on the evolution of employment, earnings, wealth, and inequality, and decompose the frictions workers face to reallocate across U.S. sectors and regions into those with a transitory effect and those with long-lasting consequences.

Keywords: International trade, migration, spatial equilibrium, dynamic Roy models, human capital, wealth, inequality.

JEL Classification: F16, F66, R23, E21, E24, J24, J61.

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1 Introduction

In theory, globalization and international trade are good, as they promote a more efficient allocation of resources across countries and across different factors of production, expanding production possibilities and income. In practice, however, considerable debate exists on whether globalization has adverse effects, particularly on the labor market. Workers with different characteristics and skills may be asymmetrically exposed to the large swings in labor demand caused by these forces, and while on aggregate there could be important benefits, these may not be evenly distributed across individuals.

In recent years, an important body of literature analyzed the effects of import competition, highlighting the displacement of workers from the manufacturing sector and from production occupations. For individual workers, depressed labor market conditions in some industries and regions translated into job displacement, protracted periods of unemployment, earnings losses and sectoral and occupational change.[1]

The very influential works of Autor, Dorn, and Hanson (2013) and Autor, Dorn, Hanson, and Song (2014) on the effects of the surge of Chinese imports into the U.S. economy show that workers more exposed to trade with China endure long-lasting earnings loses. In this paper I complement these facts and show that the wealth of more exposed workers is also negatively affected by the increased trade with China. Moreover, I show how residual wage inequality, that is, differences in pay that cannot be accounted for by workers' observed characteristics, increases in local labor markets more exposed to trade competition relative to less exposed ones.

A few economic mechanisms may explain these facts. On the one hand, trade exposure may lead to job displacement, which destroys industry and/or occupation specific human capital. Thus, compared to non-displaced workers, those affected by trade will find reemployment in positions for which they are poorly matched and with a low labor productivity. On the other hand, if workers seek reemployment in similar industries and occupations to avoid human capital loses, labor demand and wages will remain depressed, affecting earnings. In terms of wealth, exposed workers can use their savings or reduce their investments in human capital or assets as a way to smooth consumption over time. In addition, regions with more exposed industries will see a relative decline in the value of real estate and revenues of local businesses as economic conditions remain weak, affecting individuals' wealth. As argued by Xu, Ma, and Feenstra (2019), the "China shock" operated in part through the housing market and

[1] For a discussion of labor market effects of international trade, see Artuç, Chaudhuri, and McLaren (2010); Autor, Dorn, and Hanson (2013); Caliendo, Dvorkin, and Parro (2019); Dix-Carneiro (2014); Helpman, Itskhoki, and Redding (2010); Pierce and Schott (2016); Traiberman (2019).

find evidence that commuting zones that experienced larger increases in import exposure also had smaller increases in housing prices. Thus, the margins of adjustment to a trade shock extend beyond labor reallocation across industries or regions, and include also changes in consumption, savings, and investments in human capital and wealth, and these heterogeneous effects may increase inequality.

To rationalize the empirical evidence, I develop a heterogeneous agents dynamic spatial general equilibrium model of labor reallocation and regional migration where workers invest in their human capital and save in assets. The model extends the setup developed in Caliendo, Dvorkin, and Parro (2019) and Dvorkin and Monge-Naranjo (2019) to an economy with heterogeneous agents facing idiosyncratic shocks affecting their human capital, or efficient units of labor, and the returns on their assets.

Every period workers make a set of optimal discrete and continuous choices to maximize lifetime utility. The model combines elements of the dynamic discrete choice literature, such as labor reallocation over industries or occupations and migration over regions, with optimal decisions on continuous variables, such as human capital investments and assets accumulation. In my setting, the evolution of a worker's human capital and assets is driven by his labor market choices, idiosyncratic labor-market-specific shocks and the costs of switching industries/occupations and migrating.

In this economy, agents are heterogeneous in the level of human capital and the assets they bring from the previous period and are subject to idiosyncratic shocks affecting their labor supply and the effective return of their assets. The effect of these shocks on the evolution of individual human capital and assets depend on investments and on the reallocation and migration decision of workers. I characterize the worker's human capital and asset investment choices and the equilibrium assignment of workers to labor markets. I show how individual consumption/investment decisions can be written as an optimal portfolio problem conditional on labor market choices, leading to decision rules that are homogeneous in wealth and a human capital and asset allocation that is similar across individuals but differs by labor market. Exploiting properties of extreme value distributions and the optimal consumption and savings policies, I show how the ensuing discrete-continuous problem leads to a tractable characterization.

Solving for general equilibrium across time and space in dynamic models with heterogeneous forward-looking agents is, in general, a difficult task. The reason is that this requires to keep track of the evolution of the whole distribution of local characteristics across all labor markets.[2] In each labor market, the total supply of human capital and assets depends on

[2] In spatial models with heterogeneous agents, this distribution typically cannot be summarized or approx-

the sum of individual supplies across heterogeneous workers arriving from all locations and sectors. This requires aggregating across different sources of heterogeneity, namely, individual idiosyncratic shocks, individual asset holdings and human capital. Moreover, individuals do not chose a labor market at random, but their labor market reallocation and migration decision depends on individual characteristics, past labor market history and the realization of idiosyncratic shocks which shape workers' comparative advantage in different labor markets. A key result of the paper is that individual investment decisions and the stocks of human capital and assets can be aggregated, and the equilibrium evolution of the aggregate supplies of human capital and assets across labor markets can be represented in closed-form.

Workers are heterogeneous and their earnings and wealth evolve over time as a result of idiosyncratic shocks and optimal decisions generating rich patterns of income and wealth inequality over the life-cycle and over time. The model delivers tractable expressions that characterize the evolution of earnings and wealth inequality in each labor market in closed-form. As these expressions depend on model parameters and endogenous variables, it is possible to analyze the different forces that shape income and wealth inequality across time and space.

In the model workers face costs in terms of human capital and assets to reallocate across sectors and move across regions. In addition, there are switching and mobility costs in terms of utility and also parameters related to the distribution of idiosyncratic shocks. These frictions and parameters can vary arbitrarily by origin and destination, and by workers' characteristics, resulting in a very large set of parameters or fundamentals in the model. I extend dynamic exact hat algebra techniques to this class of heterogeneous agent models and show that the model can be solved in changes over time without needing to pin-down the values for these parameters. The intuition is that some key moments in the data are sufficient statistics of the frictions workers face when moving or reallocating. Different from other applications of dynamic exact hat algebra, the initial values of the variables needed do not directly map with an object in the data. In this work I show which moments in the data identify the initial values of the endogenous variables required for these techniques. Moreover, I extend these methods to characterize the evolution of inequality across time and space.

I then use the model to study how worker's individual choices change with the exposure to import competition from China, and in turn how this choices shape the equilibrium allocation of workers to different labor markets, the dynamics of aggregate human capital and assets, the behavior of earnings inequality, and the welfare of the different workers in the economy in general equilibrium.

imated by a few aggregate moments, or by a few paths of prices.

An important recent literature on dynamic labor reallocation and migration following Artuç, Chaudhuri, and McLaren (2010) explores some of these frictions, but for the most part they are modeled as switching costs in terms of utility, disconnected from human capital and earnings. The recent works by Caliendo, Dvorkin, and Parro (2019); Dix-Carneiro (2014); Traiberman (2019) are some important examples, but workers are hand-to-mouth, with no savings and limited human capital accumulation due to experience, greatly constraining the amount of heterogeneity. A recent exception, although applied to a different context, is the work by Coen-Pirani (2021), who develops a model of migration with human capital (or labor productivity) accumulation due to learning-by-doing, that interacts with worker's reallocation/migration decision in an economy with perfectly symmetric regions.

It is important to highlight that individuals in my model face idiosyncratic earnings risk and capital-income risk. These sources of risk are not new to the literature but a much larger body of work in heterogeneous agents models with incomplete markets models focuses on earnings risks. Some notable exceptions are the works by Angeletos (2007), Moll (2014) and Guvenen, Kambourov, Kuruscu, and Ocampo (2022), who develop incomplete markets models with idiosyncratic capital-income risk. Empirically, fluctuations in capital-income over time for the same individual and large differences in the level of capital income across individuals are a prevalent feature of the data. Moreover, while in reality some risks on assets' return can be diversified or hedged, a very large share of the capital stock in the United States is in the form of private businesses and private equity, and real estate, and individuals tend to have a poorly diversified portfolio. Benhabib, Bisin, and Zhu (2011) develops an incomplete markets model with both stochastic labor and capital income processes. My paper contributes to this literature by adding geographic and sectoral components, with workers optimally reallocating across industries and occupations and migrating across regions, where the reallocation decision influences the future evolution of capital, earnings and reallocation.

By and large, the recent literature on dynamic worker reallocation and migration has abstracted from capital accumulation decisions. However, in a recent influential paper Kleinman, Liu, and Redding (2023) incorporate investment and capital dynamics in different sectors and regions by immobile rentiers. In their setup workers cannot save, thus frictions affecting the evolution of financial assets are not taken into account by workers. Relative to Kleinman, Liu, and Redding (2023), this paper studies how the ownership of capital is distributed in the economy and how it is affected by individuals' migration and reallocation decisions and by international trade. Ferriere, Navarro, and Reyes-Heroles (2021), Giannone, Li, Paixao, and Pang (2020), and Greaney (2020) develop models with asset accumulation and labor reallocation/migration decisions, but the setting rapidly becomes intractable as the number of labor markets increase. My model remains tractable and can be easily extended to savings and investment in several types of assets and different kinds of human capital. Carroll and Hur (2020) develop a heterogeneous agent model with earnings risk and incomplete markets but abstracts from labor reallocation and mobility. Lyon and Waugh (2019) allow for labor reallocation across industries, but a labor market is defined at the level of each individual good variety in a trade model.

Bilal and Rossi-Hansberg (2021) argued that workers sort over industries and regions to exploit their comparative advantage, but also trade-off static gains in terms of amenities and wages for future earnings potential and capital accumulation. The evolution of workers' human and assets in my model also depend on workers comparative advantage and their reallocation decisions, taking into account the dynamic gains and losses on both forms of capital.

Helpman (2018) has recently conducted an exhaustive review of the literature studying globalization and inequality and argues that (p. 159) "residual wage inequality is not only important in size, it is also considerably responsive to foreign trade conditions. For this reason an analysis of the effects of globalization on inequality is most likely incomplete if it disregards the impact of the trade environment on residual wage inequality." Here I study the effects of a trade shock on earnings inequality. I show how these effects depend on differences in the rate at which workers accumulate human capital in different labor markets and on endogenous choices affecting the volatility of their earnings.

This paper is organized as follows. Section 2 presents evidence on the long-lasting effects of international trade on earnings and wealth. Section 3 develops a dynamic model of migration and labor market choice with incomplete markets and uninsurable income risk. Section 4, presents the quantitative exercise, comparing the effects of trade at the micro and macro levels between the calibrated version of model and the data. This section also decomposes the effects of trade and discusses the role of main frictions. Section 5 concludes.

2 Imports, earnings and wealth: empirical evidence

In this section I follow closely Autor, Dorn, and Hanson (2013) and Autor, Dorn, Hanson, and Song (2014) and study how exposure to increased competition from China affects (1) the evolution of earnings and wealth of U.S. workers and (2) average income from wages and capital across commuting zones, and (3) inequality across commuting zones.

2.1 Individual level effects

First, I study the effects of import exposure on individual workers. For this, I use data from the National Longitudinal Survey of Youth 1979 (NLSY79), which is a nationally representative survey of a cohort of over 12,500 young men and women living in the United States in 1979. Individuals in this cohort were ages 14 to 22 when first interviewed in 1979 and the U.S. Bureau of Labor Statistics interviewed these individuals yearly up to 1994 and every two years after that. This survey contains information on demographic characteristics, education and employment choices, earnings and wealth, among other things.[3] In the Appendix I show that moments and percentiles from the distribution of wealth in the NLSY79 are similar to those computed using the Survey of Consumer Finance, except at the very top (percentile 95 and above) of the distribution.

As argued in Autor et al. (2013) and Autor et al. (2014), the expansion in U.S.-China international trade since the reestablishment of diplomatic relations in the early 1970's was gradual and did not really take off until the early 1990's. For example, in 1987 U.S. imports from and exports to China represented around 1.5% of all U.S. imports or exports, while trade with Canada or Europe was over ten times larger. However, by the year 2000, U.S. imports from China represented over 8% of all imports, and by 2017 this figure peaked at around 22%. Thus, following these works, I take 1991 to be the year in which the "China shock" starts and use workers' industry of employment in the year 1991 to isolate the dynamic effects of workers' exposure to import competition.[4] Workers reallocation to other industries after 1991, is part of the dynamic response to the shock, thus focusing on the industry in 1991 abstracts from selection into other industries after the shock.

In the empirical analysis, the trade exposure of a U.S. worker to Chinese imports is measured as,

$$\Delta IP_j = \frac{\Delta M_j^{U-C}}{Y_j + M_j - X_j} \tag{1}$$

where ΔM_j^{U-C} is the change in U.S. imports from China between 1991 and 2007 for industry j, Y_j is total U.S. production of industry j in 1991, M_j and X_j are total U.S. imports and exports, of industry j in 1991. The subindex j denotes the industry of employment of the worker in 1991. In this way, ΔIP_j represents the change in import penetration of industry j between 1991 and 2007 as a share of initial absorption.

[3]Information on wealth after the year 2000 is every four years and there is no information on wealth in 1991.

[4]Disaggregated measures of U.S. imports from China by industry or goods are not available prior to 1991, which conditions the choice of the starting period.

I study the evolution of real earnings and wealth post 1991 and how it changes due to the exposure of Chines imports. In particular, let $\tilde{E}_{ij\tau}$ and $\tilde{W}_{ij\tau}$, be defined as,

$$\begin{aligned} \tilde{E}_{ij\tau} &= \sum_{t=1992}^{\tau} \frac{E_{ijt}}{\bar{E}_{ij0}} \times 100, \\ \tilde{W}_{ij\tau} &= \frac{W_{ij\tau} - W_{ij0}}{|\bar{W}_{ij0}|} \times 100, \end{aligned}$$

for $1992 \leq \tau \leq 2015$, where E_{ijt} and W_{ijt} are real earnings and wealth, respectively, in period t for individual i that was employed in industry j in 1991, and $\bar{W}_{ij0}$ is the average wealth of that individual between 1988 and 1990, and $\bar{E}_{ij0}$ is average yearly earnings of that individual between 1988 and 1991. Since information on earnings is every two years after 1994, I multiply by two this variable for $t \geq 1996$. In this way, $\tilde{E}_{ij\tau}$ is the cumulative earnings since 1992 relative to earnings before the China shock and $\tilde{W}_{ij\tau}$ is the change in wealth relative to wealth before the shock. Since initial wealth is negative for an important fraction of individuals, around 15% of the sample, I use the absolute value in the denominator.

I run the following regression for different periods,

$$Y_{ij\tau} = \beta_0 + \beta_1 \, \Delta IP_{ij} + \beta_2 \, IP_{ij} + Z'_{ij} \, \beta_4 + e_{ij\tau}, \tag{2}$$

where $Y_{ij\tau}$ is either E_{ijt} or W_{ijt}, and Z'_{ij} is a set of controls that include dummies for workers' gender, race, education, and foreign born status, tenure at the firm in 1991, dummies for firm size in 1991, and average earnings between 1988 and 1991. As before, subindex j denotes worker i industry of employment in 1991. Variable IP_{ij} denotes the level of import penetration of industry j in 1991, which is computed similar to equation (3) but using the level of imports in 1991 in the numerator rather than the changes. The sample consists of all individuals with average earnings between 1988 and 1991 larger than $1,000 constant dollars of the year 2007. Keep in mind that the NLSY79 follows a cohort of workers and this cohort was between 26 and 34 years old in 1991, that is, individuals in the early part of their prime work years.

Similar to Autor et al. (2014), I instrument variable ΔIP_{ij} in the regressions with ΔIPO_{ij}, which is constructed in a similar way but replacing ΔM_j^{U-C} in equation (3) with the change in imports from China between 1991 to 2007 of non-U.S. high-income countries.

Figure 1 shows the estimated values of coefficient β_1 in the regression for different periods of time, together with 90% confidence intervals. Panel (a) in the Figure shows the effect of the "China shock" as measured by import penetration on workers' cumulative earnings as a percent of average earnings of the worker before the shock, while panel (b) shows the effect

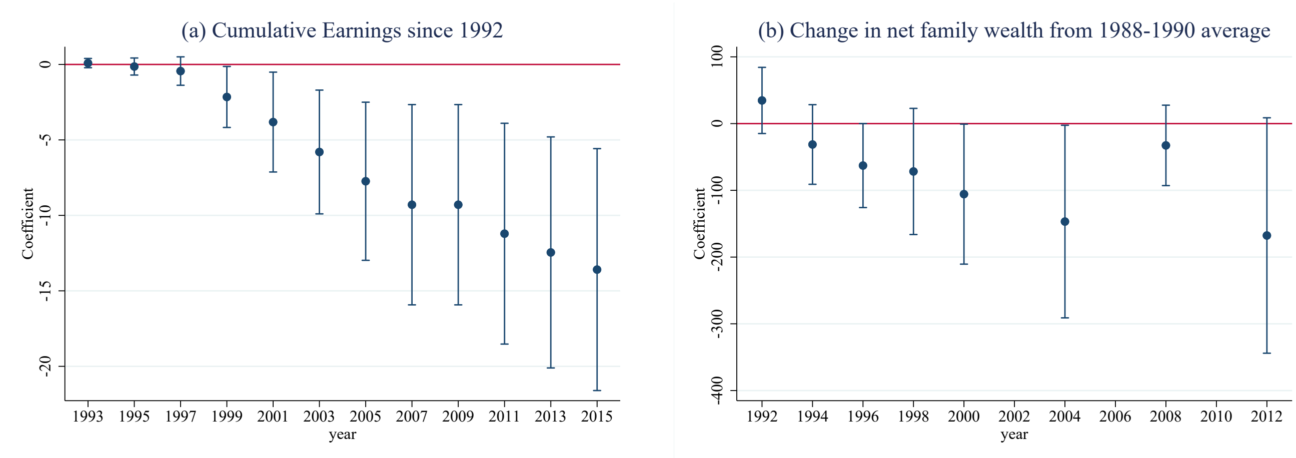

Figure 1: Estimated coefficients on Import Penetration from China

on the change in wealth relative to average wealth before the shock. Panel (a) is similar to the left panel of Figure III in Autor et al. (2014) and the point estimates on the coefficient for cumulative earnings are similar to those estimated in that work up to 2007, which is the last year they study. Point estimates for the effects on wealth are negative and large, but estimates have wide confidence bands and are statistically significative only in 1996 and 2004.

The economic impact can be quantified in the following way. The median exposure to Chinese imports of a manufacturing worker in 1991 (median value of ΔIP_j) is 3.8. By definition, a non-manufacturing worker in 1991 has zero exposure to trade with China. Since the estimated coefficient in 2007 is -9.3, the median exposure translates to a decrease in cumulative earnings up to 2007 of -35% of initial average earnings. In the sample, initial average earnings between 1988 and 1991 are around 34 thousand dollars, thus the decrease in earnings due to trade exposure is around 12 thousand dollars. Over time the effect is larger, with a point estimate of -14 in 2015, which translates into a drop of 53% of cumulative earnings for the median exposure, or a drop in 18 thousand dollars for the median exposed worker in manufacturing.[5]

The decrease in wealth is larger, with a point estimate in 2004 of -140. Given the median exposure of 3.8, this translates into a decrease of 530% in wealth relative to the average level of wealth . Median average wealth in 1988-1990 in the sample is 19 thousand dollars, which translates into a drop in wealth for a manufacturing worker with the median exposure of 100 thousand dollars. In 2004, median wealth is 120 thousand dollars and mean wealth is close to 300 thousand, thus trade can have sizable effects on worker's wealth.

[5]The dollar amounts discussed in this section are all expressed in constant dollars of the year 2007.

The empirical evidence in this section shows that workers more exposed to trade endure long-lasting earnings and wealth loses and that workers. A few economic mechanisms may explain these facts. On the one hand, trade exposure may lead to job displacement, which destroys industry and/or occupation specific human capital. Thus, compared to non-displaced workers, those affected by trade will find reemployment in positions for which they are poorly matched and with a low labor productivity. On the other hand, if workers seek reemployment in industries and occupations similar to those they had in the past in order to avoid human capital loses, labor demand and wages in exposed industries and occupations will remain depressed, affecting earnings. In terms of wealth, if the periods of job loss are perceived as transitory, workers will use their savings as a way to smooth consumption over time. Moreover, an important part of worker's wealth is in the form of real estate. Regions with more exposed industries will see a relative decline in the value of housing as local economic conditions remain weak, affecting workers' wealth.

2.2 Effects across commuting zones

In the previous subsection I provided empirical evidence on the effects of import competition on the accumulation of earnings and the stock of financial wealth of individuals. I now use data by commuting zones to provide evidence on the effects of import competition on *income from financial wealth*. For this, I follow Autor et al. (2013) and show how the surge in Chinese imports affected earnings and capital income. For this I use county income data from the Internal Revenue Service (IRS). The IRS publishes yearly data on total wages and salaries, dividends before exclusion, and interest received for all U.S. counties. This information comes from the individual's income tax returns filed with the IRS and aggregates by county using the address reported by individuals in their tax return.[6]

As in Autor et al. (2013), the exposure of a U.S. commuting zone to Chinese imports is measured as,

$$\Delta IPW_{uit} = \sum_j \frac{L_{ijt}}{L_{ujt}} \frac{\Delta M_{j\tau}^{U-C}}{L_{it}} \tag{3}$$

where, similar to before, ΔM_{jt}^{U-C} is the change in U.S. imports from China in between year t and $t+\tau$, L_{it} is total employment in commuting zone i in year t, and the ratio L_{ijt}/L_{ujt} is the share of employment in industry j, commuting zone i in year t relative to total employment in that industry an year in the United States. In the analysis I use two different periods: the years 1991 to 2000 and 2000 to 2007 and use the stacked first difference specification across

[6] See https://www.irs.gov/statistics/soi-tax-stats-county-data.

two periods as in Autor et al. (2013).

To compute the outcome variables I aggregate the county-level IRS data to commuting zones and compute income from dividends and interest per capita by commuting zone and year.[7] I then take the change in log per-capita income between year t and $t+\tau$. In addition I also compute the change in log per-capita wage and salary income, all in real terms.

I run the following regression for different periods,

$$\Delta Y_{it} = \beta_0 + \beta_1\, \Delta IPW_{uit} + Z'_{it}\, \beta_2 + e_{it}, \tag{4}$$

where Y_{it} is the change in log per capita income of commuting zone i, by type of income, from year t to $t+\tau$, and Z'_{it} is a set of controls that include Census division dummies, lagged shares of manufacturing employment, college educated, foreign born, women employment, routine occupations, and lagged values of the average offshorability index of occupations. The sample consists of 722 commuting zones and two time periods, leading to 1444 observations. I re-scale all changes to 10-year equivalent differences.

Table 1: Effect of Import exposure on per capita income

	Ten-year equivalent log-point changes (1991-2007)		
	Dividends	Interest received	Wages and salaries
Δ IPW	-0.87*	-0.48	-2.50***
	(0.45)	(0.55)	(0.30)

Note: estimated coefficient β_1 from two-stage least-squares regression in equation (4). $\Delta\, IPW_{uit}$ instrumented using the change in imports from China of other advanced countries. Regression include the full set of controls used by Autor et al. (2013). Stacked differences: 1991-2000 & 2000-2007. Per capita variables constructed using total number of exceptions claimed on income returns. $N = 1444$ observations. Robust standard errors in parenthesis. Stars indicate statistical significance of point estimates: * 10%, ** 5%, *** 1%.

Table 1 shows that a thousand dollars increase in import exposure per worker over a decade lowers per-capita income from dividends by 0.9 log points. The point estimate for per-capita income from interest is negative, but not statistically different from zero. As a point of comparison, the last column of the table shows the effect on per-capita income from wages, with a coefficient of -2.5. It is worth noting that the change in income from wages in the IRS data is the combination of lower wages by those employed and lower levels of employment in

[7] Per capita measures are simply the total income divided by the total number of personal exemptions, which approximates the population. I aggregate county-level to commuting zones using the cross-walk by David Dorn (`https://www.ddorn.net/data.htm`).

the commuting zones. Table 6 in in Autor et al. (2013) shows a smaller estimated coefficient (-0.8), but this is the change in average wages among those employed.

Since the 10-year equivalent interquartile range in import exposure growth across commuting zones was approximately 1.6 thousand dollars per worker, the point estimate of the regression implies that the per-capita divined income of a commuting zone at the 75th percentile of import exposure decreased by 1.4 percent relative to a commuting zone at the 25th percentile of exposure over ten years, while for per-capita wage and salaries income the decrease is close to 4 percent.

Helpman (2018) has recently conducted an exhaustive review of the literature studying the impacts of globalization on inequality and argues that (p. 159) "residual wage inequality is not only important in size, it is also considerably responsive to foreign trade conditions. For this reason an analysis of the effects of globalization on inequality is most likely incomplete if it disregards the impact of the trade environment on residual wage inequality."

Here I offer complementary evidence using a similar empirical strategy as the one I presented before to quantify the effects of import competition on residual wage inequality. For this, I use information on wages from the U.S. Census of Population and the American Community Survey to compute measures of hourly wages and weekly wages.[8]

The computation of wages excludes self-employed workers, those working in the public sector, and individuals with missing wages, weeks or hours. Moreover, I restrict the sample to individuals with age between 23 and 63.[9]

An important debate has developed on whether the rise in residual wage inequality in the United States was mostly an episodic event of the 1980s, with limited persistence and that stopped by mid-1990s (Card & DiNardo, 2002; Lemieux, 2006) or, rather, a long-lived process generating persistent increases in wage inequality since 1970 and still ongoing (Autor, Katz, & Kearney, 2008). A central component of this debate are the changes in the composition of the labor force due to changes in the number of workers with higher levels of education. To control for this, I compute measures of residual wage inequality for all workers and for males with a level of education of high-school or less.[10]

[8]See Ruggles et al. (2023).

[9]Hourly wages are computed as yearly wage and salary income divided by the product of weeks worked and usual weekly hours. Weekly wages are yearly wage and salary income divided by weeks worked. Top-coded yearly wages are multiplied by a factor of 1.5 and hourly wages are set not to exceed this value divided by 50 weeks times 35 hours. Hourly wages below the first percentile of the national hourly wage distribution are set to the value of the first percentile. Wages are inflated to the year 2007 using the Personal Consumption Expenditure Index.

[10]I compute residual wage inequality as the difference between observed wages and wages predicted by a Mincer-regression that includes gender fixed effects, race fixed effects, a polynomial of order 4 in age, industry fixed effects and state fixed effects. Regressions are run separately for years 1990, 2000, and 2007.

Table 2 shows the results for both samples and both measures of inequality. Across all specifications, increased import competition from China leads to an increase in the variance of residual log-wages in the more exposed commuting zones relative to less exposed ones. Connecting these results to the worker-level evidence of the previous subsection, earnings of workers more exposed to import competition evolve differently, growing at a lower rate relative to less exposed workers. Then, it is likely that geographic areas with with a larger concentration of employment in more exposed industries will see an increase in inequality.

The point estimate for the sample of all workers implies that the variance of residual log-wages of a commuting zone at the 75th percentile of import exposure increased by 0.72 points more than a commuting zone at the 25th percentile over ten years. For the group of male workers with up to high school education, the increased in this measure of inequality is of 0.48 points across these commuting zones.[11]

Table 2: Effect of Import exposure on log-wage inequality

	Ten-year equivalent changes in the variance of log-wages x 100 (1991-2007)			
	weekly wages		hourly wages	
	all	males, HS	all	males, HS
Δ IPW	0.45**	0.31**	0.44*	0.30*
	(0.21)	(0.15)	(0.23)	(0.18)

Note: estimated coefficient β_1 from two-stage least-squares regression in equation (4). ΔIPW_{uit} instrumented using the change in imports from China of other advanced countries. Regression include the full set of controls used by Autor, Dorn, and Hanson (2015) and instruments also for routine employment. Stacked differences: 1991-2000 & 2000-2007. N = 1444 observations. Robust standard errors in parenthesis. Stars indicate statistical significance of point estimates: * 10%, ** 5%, *** 1%.

Taking together, the evidence in this section suggests that individuals and labor markets more exposed to a surge in import competition display lower levels of income from labor and from financial assets and lower levels of financial wealth relative to less exposed individuals or less exposed labor markets. Moreover, commuting zones more exposed to international trade see an increase in residual wage inequality relative to less exposed areas.

[11] As a point of reference, the increase in the variance of residual log-weekly wages and log-hourly wages between 1990 and 2000 is of 5 and 4 points, respectively.

3 Labor market choice, human capital accumulation and wealth

To understand the empirical evidence of the previous section, I propose a model of labor reallocation and regional migration where workers invest in their human capital and save in financial assets. The model extends the setup developed in Caliendo, Dvorkin, and Parro (2019) and Dvorkin and Monge-Naranjo (2019) to an economy with heterogeneous agents facing idiosyncratic shocks affecting the individual returns to human capital and their assets.

I consider an overlapping generations model in which workers with characteristics e enter the economy at an initial age, $a = 0$, and work for a fixed number of periods before retiring at age $a = A$. Workers have standard log-preferences.[12] At time t, the utility of a worker of type e and age a is given by,

$$U_t^{a,e} = \log\left(c_t^{a,e}\right) + E\left[\sum_{s=1}^{A-a-1} \beta^s \log\left(c_{t+s}^{a+s,e}\right)\right] + \beta^{A-a} U_{R,t+A-a}^{A,e},$$

where $0 < \beta < 1$ is the worker's discount factor and $U_{R,t+A-a}^{A,e}$ is a level of utility in retirement.[13]

The economy has N geographic regions and I industries or sectors. For age $1 \leq a \leq A-1$, the worker starts each period attached to one of $j = 1, ..., J$ labor markets, where a labor market is an industry-region pair such that $J = N \times I$. The worker carries over from the previous period a level of human capital and assets which describe, respectively, the total efficiency units of labor of the worker in her labor market (industry/region) and her stock of savings in units of physical capital up to that date. At the beginning of each period, the worker may switch to a different labor market, ℓ. As in Dvorkin and Monge-Naranjo (2019), the decision to switch to a different labor market is shaped by costs affecting workers' human capital and assets. In particular, I assume there is a matrix $\tau^{a,e}$ of size $J \times J$ that captures the average transferability of human capital across labor markets. That is, each element of this matrix, $\tau_{j\ell}^{a,e} > 0$, determines the fraction of human capital h that can be transferred from the current labor market j to a new one ℓ. Some labor markets may use a similar set of skills, and the loss of human capital for those transitions is low, which implies a value of $\tau_{j,\ell}^{a,e}$ close to one. For other transitions, the skills or experience in one labor market may not be useful in other activities, and $\tau_{j\ell}^{a,e}$ will be low.[14] Let $\tilde{\tau}_{j\ell}^{a,e} = -\log\left(\tau_{j\ell}^{a,e}\right)$.

[12]The analysis extends to CRRA preferences, but some expressions are more involved.

[13]The discount factor may incorporate a constant death probability or labor market exit probability. For simplicity I abstract from heterogeneity in the duration of workers' lifetimes.

[14]Note that $\tau_{j\ell}^{a,e}$ may be larger than one in the model which would imply that human capital upgrades, on

Migrating across regions or switching to a different labor market also affect the return on workers' assets. There are several possible reasons for this. On the one hand, workers may incur in important monetary costs when moving across regions. In other cases, a labor market switch may reflect a change in employer, enduring a short spell with no labor earnings or may involve a signing bonus that increases available resources. Alternatively, the worker may spend resources in conducting interviews or obtaining certifications required for a new job. Finally, workers can spend resources in the sale and purchase of real estate or stakes in privately owned businesses with an important geographic or sectoral component. The matrix $\psi^{a,e}$, of size $J \times J$ and with strictly positive elements, reflects a common component to asset changes for switchers. Let $\tilde{\psi}^{a,e}_{j\ell} = -\log\left(\psi^{a,e}_{j\ell}\right)$. Note these costs, and those related to the transferability of human capital, can vary in arbitrary ways by origin and destination, age and worker's type.

In addition, each period workers face idiosyncratic shocks to their individual human capital and assets evolution or depreciation, captured by the random vectors $\tilde{\epsilon}^{h,a,e} \in \mathbb{R}^J_+$ and $\tilde{\epsilon}^{k,a,e} \in \mathbb{R}^J_+$, respectively. In this way, the matrices $\tau^{a,e}_{j\ell}$ and $\psi^{a,e}_{j\ell}$ capture a common component of transferability or depreciation of human capital and assets due to reallocation and migration, and the shocks $\tilde{\epsilon}^{h,a,e}$ and $\tilde{\epsilon}^{k,a,e}$ capture idiosyncratic forces.

In each market there is a common real rental rate per unit of the asset, $\tilde{r}_{\ell,t}$, $\tilde{r}_{\ell,t}$ and a common return per unit of human capital, $\tilde{w}_{\ell,t}$. When deciding which labor market to reallocate to, workers weight the effect of current and future economic conditions in the different labor markets, the effects of frictions and individual shocks. Thus, given a level of human capital and assets at the beginning of the period, h_t and k_t, last period's labor market j, and the realization of the vectors vectors $\tilde{\epsilon}^{h,a,e}_t$ and $\tilde{\epsilon}^{k,a,e}_t$, after chosing labor market ℓ, the worker's real income from earnings and assets is $\tilde{w}_{\ell,t}\left(1 - \tilde{\tau}^{a,e}_{j\ell} + \tilde{\epsilon}^{h,a,e}_{j\ell}\right) h_t + \tilde{r}_{\ell,t}\left(1 - \tilde{\psi}^{a,e}_{j\ell} + \tilde{\epsilon}^{k,a,e}_{j\ell}\right) k_t$. Note that, by affecting the depreciation rate and effective units of human capital and assets available, idiosyncratic shocks $\tilde{\epsilon}^{h,a,e}_{j\ell}$ and $\tilde{\epsilon}^{k,a,e}_{j\ell}$ affect the net return from human capital and assets. In this way, these idiosyncratic shocks shape the comparative advantage that workers have in supplying human capital and assets in different markets, and all else equal, workers will tend to migrate/reallocate to markets with more favorable realizations of these shocks.[15]

It is important to highlight that individuals in the model face both idiosyncratic earnings risk and capital-income risk. In this way my model connects with an important number of

average, with that transition.

[15]But note that, different from models where idiosyncratic shocks and self-selection due to comparative advantage in static contexts, here the effects of these shocks and the selection decision has lasting consequences in the accumulation of human capital and assets.

works that develop incomplete markets models with idiosyncratic capital-income risk.[16] Empirically, fluctuations in capital-income over time for the same individual and large differences in the level of capital income across individuals are a prevalent feature of the data. Moreover, while some of the risks related to the return on assets in the real world can be diversified or hedged, individuals tend to have a poorly diversified portfolio, and a very large share of the capital stock in the United States is in the form of private businesses and private equity, and housing. In this way, my assumptions on idiosyncratic stochastic earnings and capital-income connect to a recent literature and find support in the data. As I discuss later, it is possible to extend the model to include different types of assets or different types of human capital/skills, with different levels of risk, volatility and reallocation/transferability frictions .

For tractability, I now make the following assumption on the distribution of random variables $\tilde{\epsilon}_{j\ell}^{h,a,e}$ and $\tilde{\epsilon}_{j\ell}^{k,a,e}$.

Assumption 1: I assume that $\tilde{\epsilon}_{j\ell}^{h,a,e} = \tilde{\epsilon}_{j\ell}^{k,a,e} + \tilde{\nu}_j$, where $\tilde{\nu}$ is a *scalar* random variable independently distributed normal across time and across individuals with mean zero and variance $\sigma_{\nu,j}^2$. Moreover, let $\epsilon_{j\ell}^{k,a,e} = \exp\left(\tilde{\epsilon}_{j\ell}^{k,a,e}\right)$. I assume that random variables $\epsilon_{j\ell}^{k,a,e}$ are distributed i.i.d. Frechet with scale parameter $\lambda_\ell^{a,e}$ and shape parameter α. For simplicity, I drop the superindex k and subindex j and write $\epsilon_\ell^{a,e}$.[17]

In this way, shocks affecting the evolution of human capital and assets are correlated, but not perfectly due to the effects of variable $\tilde{\nu}_j$. Note that the variance of the shock $\tilde{\nu}_j$ depends on the worker's last period's labor market.

After making their labor market choice and obtaining their earnings and income from assets, workers chose how much to consume and how much to invest in human capital and assets for the next period. Thus, the budget constraint, in real terms, for a worker that chose labor market ℓ is,

$$c_{j\ell,t}^{a,e} + i_{j\ell,t}^{h,a+1,e} + i_{j\ell,t}^{k,a+1,e} = \tilde{w}_{\ell,t}\left(1 - \tilde{\tau}_{j\ell}^{a,e} + \tilde{\nu}_j + \tilde{\epsilon}_\ell^{a,e}\right) h_t + \tilde{r}_{\ell,t}\left(1 - \tilde{\psi}_{j\ell}^{a,e} + \tilde{\epsilon}_\ell^{a,e}\right) k_t.$$

And the evolution of human capital and assets follow,

$$\begin{aligned} h_{j\ell,t+1}^{a+1,e} &= \left(1 - \tilde{\tau}_{j\ell}^{a,e} + \tilde{\nu}_j + \tilde{\epsilon}_\ell^{a,e}\right) h_t + i_{j\ell,t}^{h,a+1,e}, \\ k_{j\ell,t+1}^{a+1,e} &= \left(1 - \tilde{\psi}_{j\ell}^{a,e} + \tilde{\epsilon}_\ell^{a,e}\right) k_t + i_{j\ell,t}^{k,a+1,e}. \end{aligned}$$

Note that I am treating the investment in human capital similar to the investment in assets.

[16]See, for example, Angeletos (2007), Moll (2014) and Guvenen et al. (2022). In addition, Benhabib et al. (2011) develops a tractable incomplete markets model with both stochastic labor and capital income processes.

[17]Since shocks $\tilde{\epsilon}_\ell^{a,e}$ and $\tilde{\nu}_j$ are i.i.d., I omit the time subscript.

In this way, I am following a large literature on human capital models in the tradition of Ben-Porath (1967), in which human capital is an asset that workers invest in, contributing to an expanding stock of knowledge and skills useful in the labor market. However, the human capital investment function I assume here is a restricted version of that in Ben-Porath (1967), as I am assuming that workers invest only resources to accumulate human capital, while in the seminal paper workers also invest time to increase human capital.[18] Finally, note that I assume that the technology to transform one unit of final goods in human capital and assets is linear, at the rate of one-to-one. This assumption can be relaxed, and the rate of transformation of final goods to human capital or assets need not be one-to-one, but to preserve tractability it has to be linear and identical across different labor markets.

After proper substitution and rearrangement, I can express the budget constraint as,

$$c_{j\ell,t}^{a,e} + h_{j\ell,t+1}^{a+1,e} + k_{j\ell,t+1}^{a+1,e} = w_{\ell,t}\, \nu_j\, \tau_{j\ell}^{a,e}\, \epsilon_{\ell}^{a,e}\, h_t + r_{\ell,t}\, \psi_{j\ell}^{a,e}\, \epsilon_{\ell}^{a,e}\, k_t, \tag{5}$$

where the variables without a tilde are the exponential of the respective variable.[19]

Entering cohort. At the beginning of each period, a new generation of individuals enter the economy with age $a = 0$. They arrive with some level of human capital and assets, h_t and k_t, and face a labor market choice similar to that of older workers in the economy, except that new individuals are not attached to any past labor market. In particular, new individuals observe a vector of labor market opportunities $\epsilon^{0,e}$ and ν^0 and decide in which labor market to participate, and make their consumption/savings decision. I assume that $\epsilon^{0,e}$ and ν^0 are distributed i.i.d. Frechet and log-normal, respectively, but the parameters of the distribution for the entering cohort may differ from older workers.

For the entering cohort the budget constraint is,

$$c_{\ell,t}^{0,e} + h_{\ell,t+1}^{1,e} + k_{\ell,t+1}^{1,e} = w_{\ell,t}\, \nu^0\, \epsilon_{\ell}^{0,e}\, h_t + r_{\ell,t}\, \epsilon_{\ell}^{0,e}\, k_t. \tag{6}$$

3.1 Recursive problem of the worker

I set up the problem of the worker recursively in individual state variables and characterize the optimal choices under some additional assumptions.[20]

Denote by $V_{j,t}^{a,e}(h_t, k_t, \epsilon, \nu_j)$ the expected life-time discounted utility of the worker of type

[18] In particular, I follow King and Rebelo (1990) and Krebs (2003) which formulate models of human and physical capital accumulation with investment (and disinvestment) using final goods.

[19] Moreover, I am using the approximation that $\tilde{x} \approx \log(1+\tilde{x})$ for $\tilde{x}$ small.

[20] Thus, I keep using the subindex t with some variables as this captures changes in aggregate economic conditions.

e with age a current labor market j, a level of human capital h_t, savings k_t and with a vector of labor market opportunities ϵ. The Bellman Equation (BE) that defines this value function can be written as,

$$V_t^{a,e}\left(h_t, k_t, \epsilon^{a,e}, \nu\right) = \begin{cases} \max\limits_{\ell, c_{\ell,t}^{0,e}, h_{\ell,t+1}^{1,e}, k_{\ell,t+1}^{1,e}} \left\{ \log(\chi_{\ell,t}^{0,e}) + \log\left(c_{\ell,t}^{0,e}\right) + \beta\, E\left[V_{\ell,t+1}^{1,e}\left(h_{\ell,t}^{1,e}, k_{\ell,t}^{1,e}, \epsilon^{1,e}, \nu_\ell'\right)\right]\right\}, & \text{if } a = 0; \\ \max\limits_{\ell, c_{j\ell,t}^{a,e}, h_{j\ell,t+1}^{a+1,e}, k_{j\ell,t+1}^{a+1,e}} \left\{ \log(\chi_{j\ell}^{a,e}) + \log\left(c_{j\ell,t}^{a,e}\right) + \beta\, E\left[V_{\ell,t+1}^{a+1,e}\left(h_{j\ell,t+1}^{a+1,e}, k_{j\ell,t+1}^{a+1,e}, \epsilon^{a+1,e}, \nu_\ell'\right)\right]\right\}, & \text{if } 1 \leq a < A; \\ \frac{1}{1-\beta} \log\left(\varsigma_R\left(w_{j,t}\, \nu_j\, h_t + r_{j,t}\, k_t\right)\right) & \text{if } a = A. \end{cases} \tag{7}$$

Subject to,

$$c_{j\ell,t}^{a,e} + h_{j\ell,t+1}^{a+1,e} + k_{j\ell,t+1}^{a+1,e} = w_{\ell,t}\, \tau_{j\ell}^{a,e}\, \nu_j\, \epsilon_\ell^{a,e}\, h_t + r_{\ell,t}\, \psi_{j\ell}^{a,e}\, \epsilon_\ell^{a,e}\, k_t, \qquad \text{for } 1 \leq a < A, \tag{8}$$

$$c_{\ell,t}^{0,e} + h_{\ell,t+1}^{1,e} + k_{\ell,t+1}^{1,e} = w_{\ell,t}\, \nu^0\, \epsilon_\ell^{0,e}\, h_t + r_{\ell,t}\, \epsilon_\ell^{0,e}\, k_t, \qquad \text{for } a = 0, \tag{9}$$

where $E[\cdot]$ is the expectation over the next period's idiosyncratic shocks, $\epsilon^{a+1,e}$, ν_ℓ'. $\chi_{j\ell}^{a,e}$ are non-pecuniary costs of switching labor markets in terms of utility, as usually assumed in the literature. In retirement, workers make no choices and simply receive a final utility that depends on their level of human capital, assets and the shock ν at the time of retirement. This particular functional form can be derived from a model where workers optimally consume a constant fraction of their wealth every period under log-preferences. For tractability, I am assuming that utility in retirement is multiplied by the inverse of $(1-\beta)$, as this simplifies the expressions.[21]

Equation (8) is the budget constraint of the worker, with consumption and gross investment on the left hand side and sources of income on the right. As discussed before, human capital accumulation is affected by labor market reallocation decisions and the idiosyncratic shocks. Similarly for assets.

Note the timing of the model. The worker starts the period and observes the realization of idiosyncratic shocks and the aggregate state and decides where to relocate. The human capital and assets the worker is able to rent in markets are net of any reallocation costs or depreciation. At the end of the period, the worker decides the amount of consumption and the investment in human capital and assets.

To characterize this Bellman Equation and find a tractable solution to the problem I take a few steps. I follow Krebs (2003, 2006) and write the problem in terms of effective wealth,

[21] This can be interpreted not only as consumption in retirement but also as some utility from bequest. Parameter ς_R can be used to adjust the importance of this assumption, as upon retirement worker's can consume only a fraction of their human capital and assets. In the quantitative exercise, I assume the the initial human capital and assets of the newborn generation depends on the human capital and assets not consumed by the workers that retire.

that is, the sum of human and financial, as in Angeletos (2007). This way of setting up the problem relates closely to the optimal portfolio choice problem in Merton (1969); Samuelson (1969).[22] Define effective wealth as $W_t = h_t + k_t$, and let $\theta_t = h_t/W_t$ be the share of effective wealth that is human capital, then we can write the optimization problem as,

$$V^{a,e}_{j,t}\left(W,\theta,\epsilon^{a,e},\nu_j\right) = \begin{cases} \max\limits_{\ell, c^{0,e}_{\ell,t}, W^{1,e}_{\ell,t+1}, \theta^{1,e}_{\ell,t+1}} \left\{ \log(\chi^{0,e}_\ell) + \log(c^{0,e}_{\ell,t}) + \beta\, E\left[V^{1,e}_{\ell,t+1}\left(W^{1,e}_{\ell,t+1}, \theta^{1,e}_{\ell,t+1}, \epsilon^{1,e}, \nu'_\ell \right)\right]\right\}, & \text{if } a = 0; \\ \max\limits_{\ell, c^{a,e}_{j\ell,t}, W^{a+1,e}_{j\ell,t+1}, \theta^{a+1,e}_{j\ell,t+1}} \left\{ \log(\chi^{a,e}_{j\ell}) + \log(c^{a,e}_{j\ell,t}) + \beta\, E\left[V^{a+1,e}_{\ell,t+1}\left(W^{a+1,e}_{j\ell,t+1}, \theta^{a+1,e}_{j\ell,t+1}, \epsilon^{a+1,e}, \nu'_\ell \right)\right]\right\}, & \text{if } 1 \le a < A; \\ \frac{1}{1-\beta} \log\left(\varsigma_R \left(\omega^{A,e}_j(\theta,\nu_j)\right) W \right) & \text{if } a = A. \end{cases} \tag{10}$$

Subject to,

$$\begin{aligned} c^{a,e}_{j\ell,t} + W^{a+1,e}_{j\ell,t+1} &= \omega^{a,e}_{j\ell,t}(\theta,\nu_j)\,\epsilon^{a,e}_\ell\, W, && \text{for } 1 \le a < A, \\ c^{0,e}_{\ell,t} + W^{1,e}_{\ell,t+1} &= \omega^{0,e}_{\ell,t}(\theta,\nu^0)\,\epsilon^{0,e}_\ell\, W, && \text{for } a = 0, \end{aligned}$$

where $\omega^{a,e}_{j\ell,t}(\theta,\nu_j) = \left[w_{\ell,t}\,\tau^{a,e}_{j\ell}\,\nu_j\;\theta + r_{\ell,t}\,\psi^{a,e}_{j\ell}\,(1-\theta)\right]$, for $1 \le a < A$, and a similar expression for retirement and entering cohorts, with the exception of the frictions, $\omega^{A,e}_{j,t}(\theta,\nu_j) = w_{j,t}\,\nu_j\,\theta + r_{j,t}\,(1-\theta)$, and $\omega^{0,e}_{\ell,t}(\theta,\nu^0) = w_{\ell,t}\,\nu^0\,\theta + r_{\ell,t}\,(1-\theta)$.

In this way, the problem boils-down to an optimal portfolio decision problem with CRRA utility, where the portfolio decision is how much to invest in human capital and how much in assets. Different from other settings in the spirit of Merton (1969) and Krebs (2003), there is a discrete choice decision each period and the return on human capital and assets and the optimal portfolio allocation will be affected by this.

The necessary conditions that characterize an optimal recursive decision on future effective wealth and portfolio shares, conditional on the choice of labor market ℓ, are,

$$c^{a,e}_{j\ell,t}(W,\theta,\epsilon,\nu_j)^{-1} = \beta \frac{\partial E[V^{a+1,e}_{\ell,t+1}\left(W^{a+1,e}_{j\ell,t+1},\theta^{a+1,e}_{j\ell,t+1},\epsilon',\nu'_\ell\right)]}{\partial W^{a+1,e}_{j\ell,t+1}} \quad \text{if } 1 \le a < A, \forall\, j\ell; \tag{11}$$

$$\frac{\partial E[V^{a+1,e}_{\ell,t+1}\left(W^{a+1,e}_{j\ell,t+1},\theta^{a+1,e}_{j\ell,t+1},\epsilon',\nu'_\ell\right)]}{\partial \theta^{a+1,e}_{j\ell,t+1}} = 0 \quad \text{if } 1 \le a < A, \forall\, j\ell; \tag{12}$$

and similar conditions for the entering cohort. Condition (11) is the intertemporal Euler equation that optimally characterizes the consumption-savings decision, and (12) characterizes the optimal investment share in the two types of capital, human and financial. Note that in both cases, the choices for next period initial wealth and human capital share are defined in period t and conditional on the chosen labor market ℓ.

[22] See also Toda (2014) for a recent generalization of the results.

3.2 Workers' optimal decisions

The following proposition characterizes the optimal policies of the individual problem.

Proposition 1 -*Optimal consumption, portfolio and reallocation choices:* *Given values for the state variables θ and W, and realizations of idiosyncratic shocks ν_j and ϵ, at the beginning of the period, the following policies on consumption, total next period effective wealth and human capital share, satisfy the necessary conditions for the problem of the worker of type e and age a with past labor market j, for $a < A$,*

$$\begin{aligned} c^{a,e}_{j\ell,t}\left(W,\theta,\epsilon,\nu_j\right) &= (1-\beta)\,\omega^{a,e}_{j\ell,t}(\theta,\nu_j)\,\epsilon^{a,e}_{\ell}W, \\ W^{a+1,e}_{j\ell,t+1}\left(W,\theta,\epsilon,\nu_j\right) &= \beta\,\omega^{a,e}_{j\ell,t}(\theta,\nu_j)\,\epsilon^{a,e}_{\ell}W, \\ \theta^{a+1,e}_{j\ell,t+1}\left(W,\theta,\epsilon,\nu_j\right) &= \theta^{a+1,e}_{\ell,t+1}, \end{aligned}$$

where $\theta^{a+1,e}_{\ell,t+1}$ depends only on worker's characteristics (age, type), the chosen labor market ℓ, and aggregate economic conditions.

Moreover, the value function is log-additive in initial effective wealth, W, and the ex-ante value function, $E_\epsilon\left[V^{a,e}_{j,t}\left(\theta,\nu_j,\epsilon,W\right)\right] = \frac{v^{a,e}_{j,t}(\theta,\nu_j)}{1-\beta} + \frac{1}{1-\beta}\log(W)$, for $1 \leq a \leq A-1$ can be characterized recursively as,

$$v^{a,e}_{j,t}\left(\theta,\nu_j\right) = \mathbb{C} + \frac{1}{\alpha}\log\left[\sum_{\ell=1}^{J}\exp\left(\alpha\left[(1-\beta)\log(\chi^{a,e}_{j\ell}) + \log\left(\omega^{a,e}_{j\ell,t}(\theta,\nu_j)\right) + \beta\,E\left[v^{a+1,e}_{\ell,t+1}(\theta^{a+1,e}_{\ell,t+1},\nu'_\ell)\right]\right] + \alpha\log(\lambda^{a,e}_{\ell})\right)\right],$$

and for $a = 0$,

$$v^{0,e}_{t}\left(\theta_t,\nu^0,\right) = \mathbb{C} + \frac{1}{\alpha^0}\log\left[\sum_{\ell=1}^{J}\exp\left(\alpha^0\left[(1-\beta)\log(\chi^{0,e}_{\ell}) + \log\left(\omega^{0,e}_{\ell,t}(\theta,\nu^0)\right) + \beta\,E\left[v^{1,e}_{\ell,t+1}(\theta^{1,e}_{\ell,t+1},\nu'_\ell)\right]\right] + \alpha^0\log(\lambda^{0,e}_{\ell})\right)\right],$$

where $\mathbb{C} = \frac{\bar{\gamma}}{\alpha} + (1-\beta)\log\left(1-\beta\right) + \beta\log(\beta)$, $\bar{\gamma}$ is Euler's constant, and $v^{a,e}_{j,t}$ is the unitary ex-ante value function with $v^{A,e}_{j,t}\left(\theta,\nu_j\right) = \log\left(\varsigma_R\,\omega^{A,e}_{j\ell,t}(\theta,\nu_j)\right)$.

In addition, the share of individuals that choose labor market ℓ for age $a = 0$,

$$\mu^{0,e}_{\ell,t}\left(\theta,\nu^0\right) = \frac{\exp\left(\alpha^0\left[(1-\beta)\log(\chi^{0,e}_{\ell}) + \log\left(\omega^{0,e}_{\ell}(\theta,\nu^0)\right) + \beta\,E\left[v^{1,e}_{\ell,t}(\theta^{1,e}_{\ell,1},\nu'_\ell)\right]\right] + \alpha^0\log(\lambda^{0,e}_{\ell})\right)}{\sum_{m=1}^{J}\exp\left(\alpha^0\left[(1-\beta)\log(\chi^{0,e}_{m}) + \log\left(\omega^{0,e}_{m}(\theta,\nu^0)\right) + \beta\,E\left[v^{1,e}_{m,t}(\theta^{1,e}_{\ell,1},\nu'_\ell)\right]\right] + \alpha^0\log(\lambda^{0,e}_{m})\right)}$$

and for age $1 \leq a \leq A-1$ *are,*

$$\mu^{a,e}_{j\ell,t}(\theta,\nu_j) = \frac{\exp\left(\alpha\left[(1-\beta)\log(\chi^{a,e}_{j\ell}) + \log\left(\omega^{a,e}_{j\ell}(\theta,\nu_j)\right) + \beta\, E\left[v^{a+1,e}_{\ell,t}(\theta^{a+1,e}_{\ell,t+1},\nu'_\ell)\right]\right] + \alpha\log(\lambda^{a,e}_\ell)\right)}{\sum_{m=1}^J \exp\left(\alpha\left[\log((1-\beta)\chi^{a,e}_{jm}) + \log\left(\omega^{a,e}_{jm}(\theta,\nu_j)\right) + \beta\, E\left[v^{a+1,e}_{m,t}(\theta^{a+1,e}_{\ell,t+1},\nu'_\ell)\right]\right] + \alpha\log(\lambda^{a,e}_m)\right)}.$$

And the optimal human capital share satisfies the following conditions,

$$E_{\nu'_\ell}\left[\frac{w_{\ell,t+1}\,\nu'_\ell - r_{\ell,t+1}}{w_{\ell,t+1}\,\nu'_\ell\,\theta^{A,e}_{\ell,t+1} + r_{\ell,t+1}\,(1-\theta^{A,e}_{\ell,t+1})}\right] = 0, \text{ for } a = A, \tag{13}$$

$$E_{\nu'_\ell}\left[\sum_{m=1}^J \mu^{a,e}_{\ell m,t+1}\left(\theta^{a,e}_{\ell,t+1},\nu'_\ell\right)\frac{w_{m,t+1}\,\tau^{a,e}_{\ell m}\,\nu'_\ell - r_{m,t+1}\,\psi^{a,e}_{\ell m}}{w_{m,t+1}\,\tau^{a,e}_{\ell m}\,\nu'_\ell\,\theta^{a,e}_{\ell,t+1} + r_{m,t+1}\,\psi^{a,e}_{\ell m}\,(1-\theta^{a,e}_{\ell,t+1})}\right] = 0, \text{ for } 1 \leq a \leq A-1. \tag{14}$$

The first step in the proof, fully derived in the Appendix, is to compute the expression for the ex-ante value function and its derivatives. I show that policy functions for consumption and effective wealth accumulation are homogeneous of degree one in the initial wealth of the period and on shocks ϵ, and that the Bellman equation is log-additive in effective wealth. Intuitively, this says that the lifetime utility and consumption/investment decision of individuals with the same state variables, except effective wealth, scale proportionally with W. In addition, since this is a finite horizon dynamic programing problem, the ex-ante value function (expectation with respect to the ϵ shocks) always exists, is finite and unique, and I show that it is differentiable with respect to W and θ. The homogeneity of the decision rules in terms of W and ϵ makes the discrete choice problem tractable, leading to usual closed-form expressions for the ex-ante value function and conditional choice probabilities.

A few comments are in order. First, markets are incomplete and workers self-insure against idiosyncratic shocks by adjusting their consumption, savings, *and mobility choices*. Then, as in standard incomplete markets models, workers adjust their consumption and savings due to idiosyncratic shocks, but in my model, mobility and reallocation provides a way to "escape" unfavorable labor market conditions (i.e. low w and r) and also unfavorable realizations of idiosyncratic (comparative advantage) shocks, introducing an additional margin of adjustment. Standard dynamic discrete choice models of reallocation an mobility typically feature hand-to-mouth workers or assume complete markets, and individuals do not use savings as a margin of adjustment to idiosyncratic shocks.[23]

[23]Idiosyncratic shocks in standard dynamic discrete choice models of reallocation an mobility are typically shocks to preferences. In the appendix I show that if those same shocks are assumed to affect earnings and comparative advantage, then a dynamic discrete choice model with hand-to-mouth workers with log-utility would lead to expressions that are identical to a model with preference shocks.

Second, note that risk, or the volatility of idiosyncratic shocks, affects the share of effective wealth invested in human capital and thus, the expected lifetime utility of a labor market, the amount of consumption and savings, and the mobility choices of individuals. In this way, different locations or labor markets are an asset (Bilal & Rossi-Hansberg, 2021) that deliver different returns to different individuals, but also have associated different levels of risk. The reallocation and mobility choices of workers take into account their current and future expected returns, net of frictions, of the different labor markets according to their individual comparative advantage *and* their risk.[24]

Equation (14) shows that the policy function for the share of human capital does not depend on last period labor market j, the effective wealth W at the beginning of the period (due to the homogeneity of the problem in effective wealth) or the labor market opportunity shocks ϵ. Since the shocks ν'_ℓ are independently distributed, we have that the optimal share of human capital depends only on the aggregate state and the characteristics of the chosen labor market ℓ, but not on individual state variables, i.e. $\theta^{a+1,e}_{j\ell,t+1}(W,\theta,\epsilon,\nu) = \theta^{a+1,e}_{\ell,t+1}$.

Note that the condition that characterizes the optimal share of human capital for workers in this economy is different from that in Krebs (2003). While equation (13) is similar to that in Krebs (2003), here there are many potential portfolio choices, one for each labor market the worker chooses at age $A-1$. The main reason for the similarity is that I assume that the worker cannot select a different labor market at age A. On the other hand, equation (14) differs from Krebs (2003) since the optimal choice for the share of human capital for period one takes into account that the worker may subsequently move to a different labor market and future reallocation affects the returns to human capital and assets. Clearly, if $J = 1$, then $\mu = 1$ and there will be only one element in the sum, thus the expression will boil down to those in Krebs (2003).

Finally, note that while mobility rates do not vary with the level of *effective wealth* due to the homogeneity of the problem, mobility rates will depend not only on worker's characteristics (type and age), but also on the share of human capital relative to assets. Thus, workers with a higher share of assets may move less if the frictions to migrate and reallocate assets are high relative to staying.[25]

Simplification of equilibrium conditions.- I now obtain simpler expressions for equilibrium conditions using $\log(1+x) \approx x$, or $1+x \approx e^x$ for x small. Let a variable with a tilde be related to the original variable in the following way, $x = e^{\tilde{x}}$, or $\tilde{x} = \log(x)$, then we can

[24] In this work I abstracted from aggregate risk, that is volatility in, for example, wages and rental rates. It is possible to extend the analysis in that direction, in which case, consumption, savings and mobility decisions would also be influenced by the different levels of risk of different labor markets.

[25] In this way, the mobility rate in the model may vary with the level of *financial wealth*.

write,

$$\omega_{j\ell,t}^{a,e}(\theta,\nu) \;\approx\; (1+\tilde{w}_{\ell,t}+\tilde{\nu}_j-\tilde{\tau}_{j\ell}^{a,e})\theta+(1+\tilde{r}_{\ell,t}-\tilde{\psi}_{j\ell}^{a,e})(1-\theta),$$

and

$$w_{m,t+1}\,\tau_{\ell m}^{a,e}\,\nu_\ell'-r_{m,t+1}\,\psi_{\ell m}^{a,e} \;\approx\; \tilde{w}_{m,t+1}-\tilde{\tau}_{\ell m}^{a,e}+\tilde{\nu}_\ell'-(\tilde{r}_{m,t+1}-\tilde{\psi}_{\ell m}^{a,e}).$$

It is important to highlight that these approximations do not imply a linearization or a Taylor expansion of equilibrium conditions around some particular point. In fact, if the model was written in continuous time, the previous expressions would be exact. I prefer to use discrete time as most works in the literature proceed in this way.

Then, the following Lemma characterizes simpler equilibrium conditions,

Lemma 1 *Using* $\log(1+x)\approx x$*, optimal conditions for the individual problem can be expressed as,*

$$v_{j,t}^{a,e}(\theta,\tilde{\nu}_j) \;=\; \mathbb{C}+\frac{1}{\alpha}\log\left(\sum_{\ell=1}^{J}\exp\left(\alpha\left[(1-\beta)\tilde{\chi}_{j\ell}^{a,e}+(\tilde{w}_{\ell,t}-\tilde{\tau}_{j\ell}^{a,e})\theta+(\tilde{r}_{\ell,t}-\tilde{\psi}_{j\ell}^{a,e})(1-\theta)\right]+\right.\right.$$
$$\left.\left.+\alpha\tilde{\lambda}_\ell^{a,e}+\alpha\beta E_{\nu'}\left[v_{\ell,t+1}^{a+1,e}(\theta_{\ell,t+1}^{a+1,e},\tilde{\nu}_\ell')\right]\right)\right)+\tilde{\nu}_j\,\theta,$$

$$v_t^{0,e}(\theta,\tilde{\nu}^0) \;=\; \mathbb{C}+\frac{1}{\alpha^0}\log\left(\sum_{\ell=1}^{J}\exp\left(\alpha^0\left[(1-\beta)\tilde{\chi}_\ell^{0,e}+\tilde{w}_{\ell,t}\theta+\tilde{r}_{\ell,t}(1-\theta)\right]+\right.\right.$$
$$\left.\left.+\alpha^0\tilde{\lambda}_\ell^{0,e}+\alpha^0\beta E_{\nu'}\left[v_{\ell,t+1}^{1,e}(\theta_{\ell,t+1}^{1,e},\tilde{\nu}_\ell')\right]\right)\right)+\tilde{\nu}^0\,\theta,$$

$$\mu_{j\ell,t}^{a,e}(\theta)=\frac{\exp\left(\alpha\left[(1-\beta)\tilde{\chi}_{j\ell}^{a,e}+(\tilde{w}_{\ell,t}-\tilde{\tau}_{j\ell}^{a,e})\theta+(\tilde{r}_{\ell,t}-\tilde{\psi}_{j\ell}^{a,e})(1-\theta)\right]+\alpha\tilde{\lambda}_\ell^{a,e}+\alpha\beta E_{\tilde{\nu}_\ell'}\left[v_{\ell,t+1}^{a+1,e}(\theta_{\ell,t+1}^{a+1,e},\tilde{\nu}_\ell')\right]\right)}{\sum_{n=1}^{J}\exp\left(\alpha\left[(1-\beta)\tilde{\chi}_{jn}^{a,e}+(\tilde{w}_{n,t}-\tilde{\tau}_{jn}^{a,e})\theta+(\tilde{r}_{n,t}-\tilde{\psi}_{jn}^{a,e})(1-\theta)\right]+\alpha\tilde{\lambda}_n^{a,e}+\alpha\beta E_{\nu'}\left[v_{n,t+1}^{a+1,e}(\theta_{n,t+1}^{a+1,e},\tilde{\nu}_\ell')\right]\right)},\tag{15}$$

$$\mu_{\ell,t}^{0,e}(\theta)=\frac{\exp\left(\alpha^0\left[(1-\beta)\tilde{\chi}_\ell^{0,e}+\tilde{w}_{\ell,t}\theta+\tilde{r}_{\ell,t}(1-\theta)\right]+\alpha^0\tilde{\lambda}_\ell^{0,e}+\alpha^0\beta E_{\tilde{\nu}_\ell'}\left[v_{\ell,t+1}^{1,e}(\theta_{\ell,t+1}^{1,e},\tilde{\nu}_\ell')\right]\right)}{\sum_{n=1}^{J}\exp\left(\alpha^0\left[(1-\beta)\tilde{\chi}_n^{0,e}+\tilde{w}_{n,t}\theta+\tilde{r}_{n,t}(1-\theta)\right]+\alpha^0\tilde{\lambda}_n^{0,e}+\alpha^0\beta E_{\nu'}\left[v_{n,t+1}^{1,e}(\theta_{n,t+1}^{1,e},\tilde{\nu}_\ell')\right]\right)},$$

$$\theta_{m,t+1}^{A,e}=\frac{\tilde{w}_{m,t+1}-\tilde{r}_{m,t+1}}{\sigma_{\nu,m}^2},\tag{16}$$

$$\sum_{m=1}^{J}\mu_{\ell m,t+1}^{a+1,e}(\theta_{\ell,t+1}^{a+1,e})\frac{\left[\tilde{w}_{m,t+1}-\tilde{\tau}_{\ell m}^{a+1,e}-\sigma_{\nu,\ell}^2\,\theta_{\ell,t+1}^{a+1,e}-\tilde{r}_{m,t+1}+\tilde{\psi}_{\ell m}^{a+1,e}\right]}{1+\tilde{r}_{m,t+1}-\tilde{\psi}_{\ell m}^{a+1,e}+(\tilde{w}_{m,t+1}-\tilde{\tau}_{\ell m}^{a+1,e}-\tilde{r}_{m,t+1}+\tilde{\psi}_{\ell m}^{a+1,e})\theta_{\ell,t+1}^{a+1,e}-\frac{\sigma_{\nu,\ell}^2}{2}(\theta_{\ell,t+1}^{a+1,e})^2}=0,\tag{17}$$

where, as assumed earlier, $\tilde{\nu}_j = \log(\nu_j)$ is i.i.d normal with zero mean and variance $\sigma^2_{\nu,j}$.

It is interesting to highlight additional features of this economy. First, equation (15) shows that the mobility matrix μ does not depend on the shock ν_j. The intuition is that the earnings shock ν_j is a common component of returns across all choices, and thus does not influence the reallocation decision. As is usual in dynamic models of labor reallocation, worker's mobility depends positively on labor market conditions, as reflected by returns $\tilde{w}_\ell$, $\tilde{r}_{\ell,t}$, and continuation values, $v^{a+1,e}_{\ell,t+1}$. Moreover, if frictions to reallocate from j to ℓ are large, such that these transitions are very costly for workers, mobility will between j and ℓ will be low. If parameter $\tilde{\lambda}^{a,e}_\ell$ is high, the realization of the shock $\epsilon^{a,e}_\ell$ for labor market ℓ will tend to be larger, thus favoring more mobility of workers into that market. An important point of departure relative to standard models in the literature is the endogenous dependence of mobility on the share of human capital, θ, a predetermined variable. Workers with higher levels of human capital relative to assets will favor transitions into labor markets in which the returns to human capital are larger. As discussed later, since the optimal share of human capital depends on returns and risk, mobility rates will also be influenced by the amount of income risk, i.e. the volatility of idiosyncratic shocks, in the different markets.

In addition, conditions (16) and (17) show that the state variable θ will be identical across all individuals with the same past labor market j and same age and type. This implies that the probability of making a transition from j to ℓ is the same for all individuals in j with the same age and type. This property is useful to connect the model with the data, as the model-implied mobility rates do not vary with the realization of (potentially unobserved) idiosyncratic shocks, only by origin-destination and demographic characteristics.

Under perfect foresight of wages and rental rates, we obtain a closed-form solution for the optimal human capital share in the last period of the worklife $a = A$, (16), as individuals do not reallocate in this period. This expression is the result by Merton (1969) for the case of log-utility. For other ages, we can characterize the optimal portfolio choices by computing the expectation over ν'_ℓ. Equation (17) shows that finding the optimal human capital share for each labor market, age $a < A$ and type, boils-down to finding the zero of a (well-behaved) non-linear function of a single variable. Moreover, the optimal human capital share depends on $\sigma^2_{\nu,\ell}$. All else equal, workers will optimally chose to lower their investments in human capital if expected earnings shocks are more volatile.

In the Appendix I show how to extend the model for multiple types of assets and different types of human capital. These extensions can be useful, for example, when assets have different levels of exposure and returns/risk to local (spatial) conditions. For example the return and risk of some assets, like housing and local businesses, may be strongly linked to a geography, but those of federal government bonds less so. In addition, workers may invest and accumulate different types of human capital, each with different characteristics, like cognitive, manual, and interpersonal skills, with returns and volatility that may be heterogeneous across industries and occupations, and with different frictions or cost of reallocation.

Solving for general equilibrium across space and time in dynamic models with heterogeneous forward-looking agents is, in general, a difficult task. The reason is that this requires to keep track of the evolution of the whole distribution of local labor market characteristics across space, which in this model includes the distribution of assets and human capital and the realization of the idiosyncratic shocks in each market. In spatial models with heterogeneous agents, this multidimensional distribution typically cannot be summarized by a few economy-wide moments or by a few paths of prices. The results in Proposition 1 and Lemma 1 are important not only to characterize the solution to the individual problem, but also to characterize the aggregate consumption and investment decisions across heterogeneous individuals, and the aggregate supply of human capital and assets *in each labor market* ℓ.

As shown in the Proposition 1, the ex-ante indirect utility of an individual is homothetic in effective wealth and, conditional on a labor market choice, the consumption and savings/investment decisions are homothetic in effective wealth and in the ϵ shock for the chosen market. In addition, conditional choosing labor market ℓ, all workers (of the same age and type) invest the same fraction of their effective wealth in human capital and assets, as $\theta^{a+1,e}_{\ell,t+1}$ only depends on the chosen labor market. The following proposition shows how to characterize the *aggregate* consumption and investment, the aggregate supply of labor and assets and the dynamics of effective wealth in each labor market.[26]

Dynamics of aggregate labor and assets supply. Let $\Lambda^{a,e}_j$ be the share of workers with characteristics a, e and past labor market j at the beginning a period, with $\sum_{j=1}^{J} \Lambda^{a,e}_j = 1$. In each period of time, all workers with identical e and a and with past labor market j have the same share of effective wealth in the form of human capital, θ_j. Since $\mu_{j\ell}(\theta_j)$ denotes the fraction of workers with past labor market j that chose market ℓ in period t, we have that $\Lambda^{a+1,e}_{\ell} = \sum_{j=1}^{J} \mu^{a,e}_{j\ell}(\theta_j)\, \Lambda^{a,e}_j$.

Proposition 2 -*Aggregate capital, labor supply and effective wealth dynamics:* *Assume that $\theta^{0,e}_t$ is the same for all newborn workers with type e in each period t. Then, the law of motion of effective wealth across labor markets for $1 \leq a < A$, can be characterized*

[26] The results in Proposition 2 connect to Gorman (1961) aggregation, but extended to a dynamic context, with idiosyncratic shocks and selection.

recursively as,

$$E[W^{a+1,e}_{\ell,t+1}] = \sum_{j=1}^{J} \frac{\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t})\,\Lambda^{a,e}_{j,t}}{\Lambda^{a+1,e}_{\ell,t+1}}\,\beta\,\left[\left(w_{\ell,t}\,e^{\sigma^2_{\nu,j}/2}\,\tau^{a,e}_{j\ell}\right)\,\theta^{a,e}_{j,t} + \left(r_{\ell,t}\,\psi^{a,e}_{j\ell}\right)(1-\theta^{a,e}_{j,t})\right] \times$$
$$\Gamma(1-1/\alpha)\,\lambda^{a,e}_{\ell}\,(\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))^{-1/\alpha}E[W^{a,e}_{j,t}]. \qquad (18)$$

The total supply of labor (human capital) and assets by workers of type e and $1 \le a < A$ are,

$$\bar{h}^{a,e}_{\ell,t} = \sum_{j=1}^{J} \mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t})\,\Lambda^{a,e}_{j,t}\,e^{\sigma^2_{\nu,j}/2}\,\Gamma(1-1/\alpha)\,\lambda^{a,e}_{\ell}\,(\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))^{-1/\alpha}\tau^{a,e}_{j\ell}\,\theta^{a,e}_{j,t}\,E[W^{a,e}_{j,t}],$$
$$\bar{k}^{a,e}_{\ell,t} = \sum_{j=1}^{J} \mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t})\,\Lambda^{a,e}_{j,t}\,\Gamma(1-1/\alpha)\,\lambda^{a,e}_{\ell}\,(\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))^{-1/\alpha}\psi^{a,e}_{j\ell}\,(1-\theta^{a,e}_{j,t})\,E[W^{a,e}_{j,t}].$$

Finally, the total expenditures of of final goods for consumption and investment in effective wealth by workers of type e and $1 \le a < A$ are,

$$\bar{c}^{a,e}_{\ell,t} = \sum_{j=1}^{J} \mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t})\,\Lambda^{a,e}_{j,t}\,(1-\beta)\,\left[\left(w_{\ell,t}\,e^{\sigma^2_{\nu,j}/2}\,\tau^{a,e}_{j\ell}\right)\,\theta^{a,e}_{j,t} + \left(r_{\ell,t}\,\psi^{a,e}_{j\ell}\right)(1-\theta^{a,e}_{j,t})\right] \times$$
$$\Gamma(1-1/\alpha)\,\lambda^{a,e}_{\ell}\,(\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))^{-1/\alpha}E[W^{a,e}_{j,t}],$$

$$\bar{i}^{W,a,e}_{\ell,t} = \sum_{j=1}^{J} \mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t})\,\Lambda^{a,e}_{j,t}\left[\beta\,\left(w_{\ell,t}\,e^{\sigma^2_{\nu,j}/2}\,\tau^{a,e}_{j\ell}\,\theta^{a,e}_{j,t} + r_{\ell,t}\,\psi^{a,e}_{j\ell}(1-\theta^{a,e}_{j,t})\right) - \left(e^{\sigma^2_{\nu,j}/2}\,\tau^{a,e}_{j\ell}\theta^{a,e}_{j,t} + \psi^{a,e}_{j\ell}(1-\theta^{a,e}_{j,t})\right)\right] \times$$
$$\Gamma(1-1/\alpha)\,\lambda^{a,e}_{\ell}\,(\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))^{-1/\alpha}E[W^{a,e}_{j,t}].$$

The Appendix also shows the expressions for $a = 0$ and $a = A$ and contains the proof.

The importance of Proposition 2 can be grasped by highlighting the heterogeneity in the problem. In this economy, agents are heterogeneous in the level of human capital and assets they bring from the previous period. In addition, are subject to idiosyncratic shocks affecting their labor supply and assets (or their returns). The impact of these shocks on individual's evolution of human capital and assets depend not only on the realization of the shocks, but on workers' reallocation and migration decisions. Thus, the total supply of human capital and assets in labor market ℓ depends on the aggregation across workers arriving from all previous labor markets j, across all these sources of heterogeneity (past labor market, different levels of human capital and assets, individual realization of idiosyncratic shocks) and selection patterns. Proposition 2 tells us that in this model, it is possible to characterize the aggregate supply of labor and assets and their dynamics over time.

Total factor supply and consumption for the whole market ℓ can easily be obtained by aggregating across age and type. For instance, let $\Psi^{a,e}$ be the mass of individuals with age a and type e in the population, such that $\sum_{a,e} \Psi^{a,e} = 1$, then total labor and capital supply in market ℓ are, $L_{\ell,t} = \sum_{a,e} \Psi^{a,e} \bar{h}^{a,e}_{\ell,t}$ and $K_{\ell,t} = \sum_{a,e} \Psi^{a,e} \bar{k}^{a,e}_{\ell,t}$, respectively, and total expenditures on final goods by individuals, for consumption and investment, is $F_{\ell,t} = \sum_{a,e} \Psi^{a,e} \left(\bar{c}^{a,e}_{\ell,t} + \bar{i}^{W,a,e}_{\ell,t} \right) = C_{\ell,t} + I^{W}_{\ell,t}$.

3.3 Long-run growth

In this economy, individuals use resources to expand their human capital and assets over the life-cycle. Equation (18) shows the evolution of over time and over the life-cycle of the average effective wealth in the cross-section of individuals in each market. Imagine a situation (for example, in the long-run) in which wages and rental rates are constant period after period. It is easy to show using the expressions in Proposition 1 that the ex-ante unitary value function, $v^{a,e}_{j,t}$, mobility rates, $\mu^{a,e}_{j\ell,t}$, and human capital shares, $\theta^{a,e}_{j,t}$, will be constant. However, note that even if these elements are constant over time, it is possible for average effective wealth, and thus average consumption, to display sustained growth if the stock of human capital and assets of the entering cohort depend on the stock of human capital and assets of the exiting cohorts.[27]

To see this more clearly, define $\mathcal{G}^{a,e}_{t}$ to be the transition matrix of average wealth, with j, ℓ element defined as:[28]

$$\mathcal{G}^{a,e}_{j\ell} = \frac{\mu^{a,e}_{j\ell}(\theta^{a,e}_{j})\, \Lambda^{a,e}_{j}}{\Lambda^{a+1,e}_{\ell}}\, \beta\, \left[\left(w_\ell\, e^{\sigma^2_{\nu,j}/2}\, \tau^{a,e}_{j\ell} \right)\, \theta^{a,e}_{j} + \left(r_\ell\, \psi^{a,e}_{j\ell} \right) (1 - \theta^{a,e}_{j}) \right] \Gamma(1{-}1/\alpha)\, \lambda^{a,e}_{\ell}\, (\mu^{a,e}_{j\ell}(\theta^{a,e}_{j}))^{-1/\alpha}, \tag{19}$$

which, for constant wages and rental rates, the matrix $\mathcal{G}$ will be time invariant. Then, we can

[27]Note that, sustained growth is a situation where the average wealth and average consumption of newer cohorts is higher than the consumption of previous cohorts. That is, comparing individuals of the same age at different points in time.

[28]The following discussion extends the insights of Dvorkin and Monge-Naranjo (2019) to this economy with a consumption/savings decision and endogenous human capital and asset allocation.

write equation (18) in matrix form as,

$$E[W^{a+1,e}] = (\mathcal{G}^{a,e})'\, E[W^{a,e}], \qquad \text{for } 0 \leq a \leq A-1.$$

Now, assume there is a matrix $\mathcal{G}^{0,e}$ with strictly positive entries such that $E[W^{0,e}] = (\mathcal{G}^{0,e})'\, E[W^{A,e}]$. This assumption simply states in which way the effective wealth of the exiting cohort connects with the effective wealth of the entering cohort. In the context of the model, this matrix can be a function of $(1-\varsigma_R)$, which is the fraction of effective wealth not consumed by the individuals in retirement. Under this assumption, we can stack the vectors of average effective wealth by age and type and write,

$$E[W] = (\mathcal{G})'\, E[W]$$

The matrix $\mathcal{G}$ is non-negative and, under adequate assumptions on $\mathcal{G}^{0,e}$, also irreducible. Then, by the Perron-Frobenius theorem for irreducible non-negative matrices, the largest eigenvalue of $\mathcal{G}$, denoted by g^W, is always simple (multiplicity one), real, and positive. Moreover, the associated eigenvector to this eigenvalue, denoted by $E[W^*]$, has all its coordinates strictly positive and in the long-run, the vector of average effective wealth from one period to the next behaves as,

$$E[W_{t+1}] = g^W\, E[W_t] \propto E[W^*].$$

Therefore, the model can generate sustained growth of average effective wealth, and thus human capital and assets, over time if $g^W > 1$. Changes in wages and rental rates have both a direct and indirect impact on the *growth rate* of effective wealth and consumption. On the one hand, higher returns will lead (directly) to higher savings and more accumulation over time. On the other hand, changes in wages and rental rates may induce individuals (indirectly) to change their allocation of effective wealth over human capital and assets, and their *mobility and reallocation* decisions, leading to changes in the evolution of effective wealth over time.

In this way, the model generates growth that changes endogenously with economic conditions since all factors of production are reproducible and the resources/endowments of the entering cohort are related to the resources of the exiting cohort. By incorporating migration and reallocation decisions, the model extends several works in the literature that develop models of endogenous growth due to investments in human capital, such as Jones, Manuelli, and Rossi (1993); King and Rebelo (1990) in the context of an infinitely lived representative agent, or Krebs (2003) in a model with heterogeneous agents in a single sector/single region economy.[29]

The following proposition extends the results in Dvorkin and Monge-Naranjo (2019) to an economy with an endogenous consumption/savings decision, human capital accumulation, and life-cycle structure with ex-ante heterogeneity.

Proposition 3 *Assume that the vectors of returns of human capital and assets are strictly positive and that $\mathcal{G}^{0,e}$ has strictly positive entries. Then, there exists a unique invariant distribution of workers over labor markets, i.e., $\Lambda_\infty^{a+1,e} = (\mu^{a,e})' \Lambda_\infty^{a,e}$, with $\Lambda_{j,\infty}^{a,e} > 0$ for all a, e, and j, with $\sum_{j=1}^{J} \Lambda_{j,\infty}^{a,e} = 1$. Moreover, the sequence $\{\Lambda_t^{a,e}\}_{t=0}^{\infty}$ converges to Λ_∞ from any initial distribution $\Lambda_0^{a,e}$. In addition, there is a unique (up to scale) balanced growth path of average effective wealth across labor markets, where average effective wealth grows at gross rate $g^W > 0$, and g^W is the Perron root of $\mathcal{G}$. Moreover, the economy converges to $E[W_{t+1}] = g^W E[W_t]$ from any initial vector $E[W_0] > 0$.*

Finally note that, if the effective wealth of the entering cohort is exogenous and identical period after period, then there will be no long-run growth of average effective wealth in this economy, and the long-run distribution of average effective wealth will be unique and invariant over time given constant returns of human capital and assets.

3.4 Inequality

I now show that it is possible to characterize higher order moments of the distribution of effective wealth or functions of it. In particular, I will focus here on characterizing the evolution of the cross-sectional of "residual" earnings, as these are widely analyzed moments in the literature that study inequality.[30]

Denote by $\tilde{z}_{j\ell,t}^{i,a,e}$ the log labor earnings of worker i with age a, type e, past labor market j working in labor market ℓ. Then,

$$\tilde{z}_{j\ell,t}^{i,a,e} = \tilde{w}_{\ell,t} + \tilde{\nu}_j^i - \tilde{\tau}_{j\ell}^{a,e} + \tilde{\epsilon}_\ell^{i,a,e} + \log(\theta_{j,t}^{a,e}) + \log(W_{j,t}^{i,a,e}) \tag{20}$$

Where the superscript i denotes variables that are individual-specific, such as the realization of the idiosyncratic shocks and the worker's effective wealth. It is clear from the equation that the variance of idiosyncratic shocks will influence the variance of log-earnings directly.

[29]See also Uzawa (1965) and Lucas (1988) for models of endogenous growth with human capital accumulation using a different production function for human capital, where investment in human capital are functions of time.

[30]It is straightforward to construct overall inequality measures using these moments of "residual" earnings.

In addition, since past realizations of the shocks affect today's value of log-effective wealth, the variance of idiosyncratic shocks will also affect the the variance of log-earnings and its evolution over time. It is important to highlight two additional forces not directly seen in the equation. On the one hand, individuals will select into labor markets influenced by favorable realizations of idiosincratic shocks, thus the variance of log-earnings will be influenced by selection. On the other hand, even if wages and human capital share are identical across individuals with the same characteristics (state variables), the returns on human capital and assets and human capital share of past labor markets will influence the evolution of log-effective wealth, contributing indirectly to earnings inequality. Then, it is important to highlight that even when the variance of idiosyncratic shocks remains constant, changes in economic conditions due to, for example, international trade, will change wages and asset returns, worker's choices and selection, affecting inequality.[31]

Denote with $E[\log(W_{j,t}^{a,e})]$ and $Var[\log(W_{j,t}^{a,e})]$ the mean and variance of log-effective wealth of workers with age a and type e in labor market j at time t. The following proposition characterizes the evolution of the cross-sectional variance of log-earnings over the life-cycle and over time in this economy.

Proposition 4 ***Moments of log-effective wealth and log-earnings.*** *Given the mean and variance of log-effective wealth for the entering cohort of workers for all t, and let the human capital share of the entering cohort for workers of type e, $\theta_t^{0,e}$, be identical across these workers in each t, then the evolution of the first two moments of the distribution of log-effective wealth and log-earnings across labor markets can be characterized by the following expressions,*

$$
\begin{aligned}
E[\log(W_{j\ell,t+1}^{a+1,e})] &= \log(\beta) + \tilde{r}_{\ell,t} - \tilde{\psi}_{j\ell}^{a,e} + \left(\tilde{w}_{\ell,t} - \tilde{\tau}_{j\ell}^{a,e} - r_{\ell,t} + \tilde{\psi}_{j\ell}^{a,e}\right) \theta_{j,t}^{a,e} + \frac{\bar{\gamma}}{\alpha} + \log(\lambda_{\ell}^{a,e}) - \log(\mu_{j\ell,t}^{a,e})/\alpha + E[\log(W_{j,t}^{a,e})], \\
Var[\log(W_{j\ell,t+1}^{a+1,e})] &= \frac{\pi^2}{6\,\alpha^2} + \sigma_{\nu,j}^2\,(\theta_{j,t}^{a,e})^2 + Var[\log(W_{j,t}^{a,e})], \\
E[\log(W_{\ell,t+1}^{a+1,e})] &= \sum_{j=1}^{J} \frac{\mu_{j\ell,t}^{a,e}(\theta_{j,t}^{a,e})\Lambda_{j,t}^{a,e}}{\Lambda_{\ell,t+1}^{a+1,e}} E[\log(W_{j\ell,t+1}^{a+1,e})], \\
Var[\log(W_{\ell,t+1}^{a+1,e})] &= \sum_{j=1}^{J} \frac{\mu_{j\ell,t}^{a,e}(\theta_{j,t}^{a,e})\,\Lambda_{j,t}^{a,e}}{\Lambda_{\ell,t+1}^{a+1,e}} \Big[Var[\log(W_{j\ell,t+1}^{a+1,e})] + \left(E[\log(W_{j\ell,t+1}^{a+1,e})]\right)^2 - \left(E[\log(W_{\ell,t+1}^{a+1,e})]\right)^2\Big],
\end{aligned}
$$

[31] These extend the results in Dvorkin and Monge-Naranjo (2019) which only had selection as a force shaping inequality. The model adds additional endogenous choices of workers which also influence inequality.

$$
\begin{aligned}
E[\tilde{z}^{a,e}_{j\ell,t}] &= \tilde{w}_{\ell,t} - \tilde{\tau}^{a,e}_{j\ell} + \frac{\bar{\gamma}}{\alpha} + \log(\lambda^{a,e}_{\ell}) - \log(\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t}))/\alpha + \log(\theta^{a,e}_{j,t}) + E[\log(W^{a,e}_{j,t})], \\
Var[\tilde{z}^{a,e}_{j\ell,t}] &= \sigma^2_{\nu,j} + \frac{\pi^2}{6\,\alpha^2} + Var[\log(W^{a,e}_{j,t})], \\
E[\tilde{z}^{a,e}_{\ell,t}] &= \sum_{j=1}^{J} \frac{\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t})\,\Lambda^{a,e}_{j,t}}{\Lambda^{a+1,e}_{\ell,t+1}}\, E[\tilde{z}^{a,e}_{j\ell,t}], \\
Var[\tilde{z}^{a,e}_{j\ell,t}] &= \sum_{j=1}^{J} \frac{\mu^{a,e}_{j\ell,t}(\theta^{a,e}_{j,t})\,\Lambda^{a,e}_{j,t}}{\Lambda^{a+1,e}_{\ell,t+1}} \left[Var[\tilde{z}^{a,e}_{j\ell,t}] + \left(E[\tilde{z}^{a,e}_{j\ell,t}]\right)^2 - \left(E[\tilde{z}^{a,e}_{\ell,t}]\right)^2 \right],
\end{aligned}
$$

where $\bar{\gamma}$ is Euler's constant ≈ 0.5772, and π is the mathematical constant ≈ 3.1416.

Proposition 4 can be extended to derive expressions for the dynamics of the variance of log-income from assets and financial wealth.

Then, an important feature of the model, highlighted in Propositions 2 and 4, is that one can obtain the aggregate supplies of efficient units of labor and assets, and moments of income and wealth inequality in closed form, with no need to simulate individual realization of shocks and individual employment histories. In addition to being a very convenient feature for quantitative work, these closed-form expressions permit a very transparent analysis of how the different forces at play shape workers' individual and aggregate consumption and savings decisions, mobility and inequality.

4 Production and the demand for labor and capital

Production follows closely the open economy model of Eaton and Kortum (2002), extended to multiple sectors and regions within a country as in Caliendo, Parro, Rossi-Hansberg, and Sarte (2018) and Caliendo, Dvorkin, and Parro (2019), and capital in production (equipment) as in Kleinman, Liu, and Redding (2023). Firms in industry i and region n are able to produce many varieties of intermediate goods. The technology to produce these intermediate goods requires efficient units of labor and capital, which are the primary factors of production, and materials, which consist of goods from all sectors. Total factor productivity (TFP) of an intermediate good is composed of two terms, a time-varying sectoral-regional component, A^{ni}_t, which is common to all varieties in ni, and a variety-specific component, $\mathbb{z}^{ni}$.

Intermediate Goods Producers: The output for a producer of an intermediate variety with efficiency $\mathbb{z}^{ni}$ is given by,

$$
q^{ni}_t = \mathbb{z}^{ni} \left(A^{ni}_t \, (k^{ni}_t)^{\xi^n} (l^{ni}_t)^{1-\xi^n} \right)^{\gamma^{ni}} \prod_{k=1}^{J} (M^{ni,nk}_t)^{\gamma^{ni,nk}},
$$

where l_t^{ni}, k_t^{ni} are labor and capital inputs, respectively, and $M_t^{ni,nk}$are material inputs from industry k and region n demanded by a firm in industry i to produce q units of an intermediate variety with efficiency $\mathbb{z}^{ni}$. Material inputs are final goods from region n and industry k. Parameter $\gamma^{ni} \geq 0$ is the share of value added in the production of sector i and region n, and $\gamma^{ni,nk} \geq 0$ is the share of materials from sector k in the production of sector i and region n. The production function exhibits constant returns to scale such that $\sum_{k=1}^{J} \gamma^{ni,nk} = 1 - \gamma^{ni}$. The parameter ξ^n is the share of capital in value added. The nominal unit price of an input bundle is

$$x_t^{ni} = B^{ni} \left(P_t^n (\tilde{r}_{ni,t})^{\xi^n} \; (\tilde{w}_{ni,t})^{1-\xi^n} \right)^{\gamma^{ni}} \prod\nolimits_{k=1}^{J} (P_t^{nk})^{\gamma^{ni,nk}}, \tag{21}$$

where B^{ni} is a constant and P_t^{ni} also applies to goods used as materials in production, as described below.[32] Then, the unit cost of an intermediate good $\mathbb{z}^{ni}$ at time t is $\frac{x_t^{ni}}{\mathbb{z}^{ni} \, (A_t^{ni})^{\gamma^{nj}}}$.

Trade costs are represented by $\kappa_t^{ni,mi}$ and are of the iceberg type. One unit of any variety of intermediate good i shipped from region m to n requires producing $\kappa_t^{ni,mi} \geq 1$ units in region m. If a good is nontradable, then $\kappa = \infty$. Competition implies that the price paid for a particular variety of good i in region n is given by the minimum unit cost across regions, taking into account trade costs, and where the vector of productivity draws received by the different regions is $\mathbb{z}^i = (\mathbb{z}^{1i}, \mathbb{z}^{2i}, \ldots, \mathbb{z}^{Ni})$. That is, using $\mathbb{z}^i$ to index varieties,

$$p_t^{ni}(\mathbb{z}^i) = \min_m \left\{ \frac{\kappa_t^{ni,mi} \; x_t^{mi}}{\mathbb{z}^{mi} (A_t^{mi})^{\gamma^{mi}}} \right\}.$$

Local Sectoral Aggregate Goods. Intermediate goods demanded from sector i and from all regions are aggregated into a local sectoral (final) good denoted by Q and that can be thought as a bundle of goods purchased from different regions. In particular, let Q_t^{ni} be the quantity produced of aggregate sectoral goods i in region n and $\tilde{q}_t^{ni}(\mathbb{z}^i)$ be the quantity demanded of an intermediate good of a given variety from the lowest-cost supplier. The production of local sectoral goods is given by

$$Q_t^{ni} = \left(\int (\tilde{q}_t^{ni}(\mathbb{z}^i))^{1-1/\eta^{ni}} \; d\phi^i(\mathbb{z}^i) \right)^{\eta^{ni}/(\eta^{ni}-1)},$$

[32] Since I defined factor prices, $\tilde{r}_{ni,t}$ and $\tilde{w}_{ni,t}$, in real terms, I multiply by the cost of the final good basket, P_t^n, to express them in nominal terms.

where $\phi^j(\mathbb{z}^i) = \exp\left\{-\sum_{n=1}^{N}(\mathbb{z}^{ni})^{-\vartheta^i}\right\}$ is the joint distribution over the vector $\mathbb{z}^i$, with marginal distribution given by $\phi^{ni}(\mathbb{z}^{ni}) = \exp\left\{-(\mathbb{z}^{ni})^{-\vartheta^i}\right\}$, and the integral is over R_+^N. For nontradable sectors, the only relevant distribution is $\phi^{ni}(\mathbb{z}^{ni})$ since sectoral good producers use only local intermediate goods. There are no fixed costs or barriers to entry and exit in the production of intermediate and sectoral goods. Competitive behavior implies zero profits at all times.

Local sectoral aggregate goods are used as materials for the production of intermediate varieties as well as for final consumption. Note that the fact that local sectoral aggregate goods are not traded does not imply that consumers are not purchasing traded goods. On the contrary, both intermediate goods producers and households, via the direct purchase of the local sectoral aggregate good, purchase tradable varieties.

Given the properties of the Fréchet distribution, the price of the sectoral aggregate good i in region n at time t is

$$P_t^{ni} = \Gamma^{ni}\left(\sum_{m=1}^{N}(x_t^{mi}\kappa_t^{ni,mi})^{-\vartheta^i}(A_t^{mi})^{\vartheta^i\gamma^{mi}}\right)^{-1/\vartheta^j}, \tag{22}$$

where Γ^{ni} is a constant.[33] To obtain (22), we assume that $1+\vartheta^i > \eta^{ni}$. Following similar steps as earlier, we can solve for the share of total expenditure in market (n, i) on goods i from market m. In particular,

$$\pi_t^{ni,mi} = \frac{(x_t^{mi}\kappa_t^{ni,mi})^{-\vartheta^i}(A_t^{mi})^{\vartheta^i\gamma^{mi}}}{\sum_{m=1}^{N}(x_t^{mj}\kappa_t^{ni,mj})^{-\vartheta^i}(A_t^{mj})^{\vartheta^i\gamma^{mj}}}. \tag{23}$$

This equilibrium condition reflects that the more productive market mi is, given factor costs, the cheaper the cost of production is in market mi and, therefore, the more region n purchases sector j goods from region i. In addition, the easier it is to ship sector j goods from region i to n (lower $\kappa^{ni,mi}$), the more region n purchases sector j goods from region i. This equilibrium condition resembles a gravity equation.

The price of the final good basket in region n is, $P_t^n = \prod_{i=1}^{I}\left(\frac{P_t^{ni}}{\zeta^i}\right)^{\zeta^i}$.

Finally, I assume that the final good used for consumption and investment is a Cobb-Douglas aggregator using the local sectoral aggregate good from all industries in each region with parameters ζ_i, where $\sum_{i=1}^{I}\zeta_i = 1$.

[33] In particular, the constant Γ^{ni} is the Gamma function evaluated at $1 + \left(1 - \eta^{ni}/\vartheta^i\right)$.

5 General equilibrium and balanced growth path

Let X_t^{ni} be the total expenditure on sector i good in region n. Then, goods market clearing implies

$$X_t^{ni} = \sum\nolimits_{k=1}^{J} \gamma^{nk,ni} \sum\nolimits_{m=1}^{N} \pi_t^{mk,nk} \, X_t^{mk} + \zeta^i \, P_t^n \, F_{n,t}, \tag{24}$$

where the first term on the right-hand side is the value of the total demand for sector i goods produced in n used as materials in all sectors and regions in the economy, and $P_t^n \, F_{n,t}$ is the value of the final demand of good i in region n, which workers use to consume and invest in human capital and assets, with $F_{n,t} = \sum_{i=1}^{I} F_{ni,t}$.

Labor market clearing in region n and sector i is,

$$L_t^{ni} = \frac{\gamma^{ni} \, (1-\xi^n)}{P_t^n \, \tilde{w}_t^{ni}} \sum\nolimits_{m=1}^{N} \pi_t^{mi,ni} \, X_t^{mi}, \tag{25}$$

while the market clearing for capital in region n and sector i must satisfy,

$$K_t^{ni} = \frac{\gamma^{ni} \, \xi^n}{P_t^n \, \tilde{r}_t^{ni}} \sum\nolimits_{m=1}^{N} \pi_t^{mi,ni} \, X_t^{mi}. \tag{26}$$

Definition 1 *General equilibrium.- Equilibrium in this economy is defined as a sequence of prices for goods, P_t^n, P_t^{ni}, and $p_t^{ni}(a^i)$, and rental rates of factors of production, $\tilde{r}_{ni,t}$ and $\tilde{w}_{ni,t}$, sequences of quantities of goods produced, $q_t^{ni}(a^{ni})$ and Q_t^{ni}, sequences of aggregate consumption, investments, labor and assets supplies, $C_{\ell,t}$, $I_{\ell,t}^W$, $L_{\ell,t}$, and $K_{\ell,t}$, sequences of optimal individual decision rules and value functions that depend on time and individual state variables, $c_{j\ell,t}^{a,e}(W,\theta,\epsilon,\nu_j)$, $W_{j\ell,t+1}^{a+1,e}(W,\theta,\epsilon,\nu_j)$, $\mu_{j\ell,t}^{a,e}(\theta)$, $\theta_{\ell,t+1}^{a+1,e}$, $v_{j,t}^{a,e}(\theta,\tilde{\nu}_j)$, for all ages a and types e, and a sequence of distributions of effective wealth across labor markets ℓ, $\Upsilon_{W,t}^{a,e}$, such that,*

- *Given prices, individuals optimally chose consumption, investment in human capital and assets, as described in Proposition 1 and Lemma 1.*

- *Given prices, the ex-ante unitary value function and migration/reallocation probability satisfy the conditions in Proposition 1 and Lemma 1.*

- *Aggregate consumption, investments, supply of human capital and assets is the result of aggregating individual supply according to $\Upsilon_{W,t}^{a,e}$ together with the measure of individuals by age and type.*

- *Given prices, firms optimally chose labor, capital and materials demand to produce individual goods varieties and aggregate varieties by industry and region.*
- *Goods and factor markets clear.*
- *The sequence of distributions of effective wealth over time and across industries and regions is consistent with individual decisions.*

As discussed before, it is possible for this economy to to exhibit sustained growth if the effective wealth of the newborn cohort is connected to the effective wealth of exiting cohorts. The following defines a balanced growth path equilibrium.

Definition 2 *Balanced Growth Path.- If $\mathcal{G}^{0,e}$ has strictly positive entries, a balanced growth path equilibrium, is a general equilibrium were rental rates of human capital and assets, and the prices of goods are constant over time. Mobility matrices, the share of employment, ex-ante unitary value functions, and human capital shares in each labor market are constant over time. The distribution of effective wealth across labor markets is constant over time. The average effective wealth, aggregate consumption, investment, final demand, and supplies of human capital and assets grow at the constant gross rate g^W each period. The quantities produced of intermediate varieties and final goods grow at the constant gross rate g^W each period.*

In the model workers face costs in terms of human capital and assets to reallocate across sectors and move across regions. In addition, there are reallocation costs in terms of utility and parameters related to the distribution of idiosyncratic shocks. These frictions and parameters can vary arbitrarily by origin and destination, and also by worker's characteristics. In this way, quantitative work requires to pin-down values for a very large set of parameters or fundamentals. I extend dynamic exact hat algebra techniques to this heterogeneous agent model and show how the model can be expressed in changes, and given initial values of endogenous variables which are identified directly by moments in the data. Thus, the model can be solved without having to pin-down the value of this very large set of parameters. In addition, I extend dynamic exact hat algebra to characterize the evolution of inequality.

As in Caliendo, Dvorkin, and Parro (2019), the following proposition shows a convenient way to conduct quantitative exercises with this model by extending dynamic hat algebra techniques.

Proposition 5 ***Dynamic exact hat algebra:*** *If $\tau^{ae}_{j\ell} = \Xi^{ae}_{j\ell}\,\psi^{ae}_{j\ell}$, where $\Xi^{ae}_{j\ell}$ is known, and initial mobility rates $\mu^{ae}_{j\ell,0}$, initial transferability matrix $\mathcal{M}^{ae}_0$, initial share of human capital,*

$\theta_{j,0}^{a;e}$ and mean effective wealth by age, $E[W_{j,0}^{a,e}]$, type and labor market, initial trade shares and expenditures, $\pi_0^{ni,mi}$and X_0^{ni}, and initial returns on human capital and assets are observed, then we can express the equilibrium conditions that characterize the solution to this economy over time, in changes. Moreover, equilibrium conditions in changes do not require information on the level of parameters, or fundamentals, $\tau_{j\ell}^{ae}$, $\psi_{j\ell}^{ae}$, and $\chi_{j\ell}^{ae}$, $\lambda_\ell^{a,e}$, for the worker's problem, or $\kappa^{ni,mi}$ and A^{ni} on production and trade. Moreover, given a change in fundamentals, we can compute the transition to the new equilibrium without requiring information on the level of fundamentals.

The Appendix contains the details of the proof and lists the expressions used to perform the quantitative analysis. The main intuition behind Proposition 5 is that the matrices $\mu_0^{a,e}$ and $\mathcal{M}_0^{a,e}$ contain information about the frictions faced by workers when migrating and reallocating (sufficient statistics), as discussed by Caliendo, Dvorkin, and Parro (2019) which I extend to the dynamics of earnings and wealth with heterogeneous agents. Note that, assuming $\Xi_{j\ell}^{ae}$ known is somewhat restrictive, but is only needed to apply dynamic exact hat algebra in a simple way. In applications one simplifying assumption can be $\Xi_{j\ell}^{ae} = 1$, which means that frictions on human capital and assets are identical. Alternatively, assuming $\Xi_{j\ell}^{ae} = \xi$, would imply that frictions differ by a known constant. Without this assumption solving the model and performing counterfactuals using dynamic exact hat-algebra would require knowledge of the level of frictions $\tau_{j\ell}^{ae}$ and $\psi_{j\ell}^{ae}$, but not for $\chi_{j\ell}^{ae}$, $\lambda_\ell^{a,e}$, $\kappa^{ni,mi}$, or A^{ni}.

I now extend dynamic exact hat algebra techniques to characterize the dynamics of inequality without having to recover the level of fundamentals. For this, define the matrix of log-transferability $\tilde{\mathcal{M}}^{ae}$, with typical element,

$$\tilde{\mathcal{M}}_{j\ell,t}^{ae} = \log(\lambda_\ell) - \tilde{\tau}_{j\ell}^{a,e} + \frac{\bar{\gamma}}{\alpha} - \frac{1}{\alpha}\log\left(\mu_{j\ell,t}^{a,e}\right),$$

where $\bar{\gamma}$ is Euler's constant.

Lemma 2 ***Dynamic exact hat algebra for inequality:*** *If the conditions in Proposition 5 are satisfied, and the values at $t = 0$ of the first two moments of log-earnings and log-effective wealth are observed, the first two moments of log-earnings and log-effective wealth for the entering cohort are known for all t, and the matrix of log-transferability $\tilde{\mathcal{M}}_0^{ae}$ for period $t = 0$ is observed, then we can characterize the evolution of the first two moments of log-earnings and log-effective wealth given initial conditions and can characterize how they change under counterfactual experiments without needing information on the level of fundamentals $\tau_{j\ell}^{ae}$, $\psi_{j\ell}^{ae}$, and $\chi_{j\ell}^{ae}$, $\lambda_\ell^{a,e}$.*

In this way, dynamic hat algebra techniques are useful not only to solve the model and obtain the equilibrium path of endogenous variables, but also to compute the dynamics of inequality. In the following section I use these techniques to quantify the effects of a trade shock on the economy.

6 Quantitative Analysis

In the first part of this section, I conduct a simple exercises to understand the mechanism driving mobility, earnings and wealth changes in the model after a shock. The next subsection abstracts from general equilibrium and studies the effects of an exogenous change. After this I turn to the full quantitative exercise.

6.1 Simple example

Take an economy with three industries such that $J = 3$. These industries are fully symmetric before the shock hits. I calibrate a period to be three years. Individuals are identical in all respects except their age. A=10 such that individuals have a worklife of 30 years. I make $\lambda = 0.95$ and $\alpha = 20$ such that earnings and assets have an upward trajectory over the life of individuals and the variance of (log) earnings and assets resembles that of the U.S. economy. Frictions are such that individuals that the elements outside of the diagonal of matrices τ and ψ are 0.86 and 0.87 respectively. In this way industry switchers see their stock of human capital and assets fall by approximately 15%, all else equal (slightly higher for assets). This implies that industry mobility is around 15% over three years. The earnings shock ν has a variance of 0.1. Finally, the initial returns on human capital and assets, $\tilde{w}$ and $\tilde{r}$ are 10% and 7%, respectively, in all industries.[34] In this way, initial human capital share of effective wealth θ is 0.3. Due to symmetry, employment is evenly distributed across all industries.

In period 1 the economy is hit by a shock that lowers the return on human capital and assets in industry one. In particular, the shock is a decrease of 100 basis points in the return of human capital and assets in that industry. I solve the worker's optimization problem given these fixed set of returns and simulate employment histories for a large number of individuals. For each of these simulated agents, I compute accumulated earnings since the shock (returns to human capital) and percent change in assets since initial period to connect with the analysis

[34] Thus, on average the wage is 1.1.

of Section 2. Then, I run the following regression for different periods since the shock,

$$Y_{ij} = \beta_0 + \beta_1\, dummy_{i1} + e_{ij},$$

where $Y_{ij\tau}$ is either the accumulated earnings of individual i since period zero and relative to that initial period, or the percent change in the stock of assets since period zero. The variable dummy is equal to one if the individual was employed in industry one at the time of the shock.

Figure 2 shows the estimates of coefficient β_1 for different periods since the shock. The estimates show the differential effects on earnings and assets of workers exposed to the shock relative to those that were not exposed. As the figure shows, the exposure to the shock has a (relative) negative effect on the evolution of earnings and assets.

Figure 2: Estimated coefficients on cumulative earnings and assets regressions

The effects on earnings and assets on exposed workers are the result of different forces. On the one hand, for workers that stay in industry one, the effects of the shock are small initially since returns on human capital and assets change modestly. However, over time, the differentially lower rates of return in industry one imply a lower growth rate of effective wealth for workers that stay in that industry, and the effects on the stocks increases over time, leading to lower levels of human capital and assets. On the other hand, workers that switch to a different industry, can benefit from the higher returns, but they must endure a drop in their human capital and assets of a considerable magnitude, in this example of around to 15%. Total employment in industry 1 gradually contracts, partly due to an increased outflow and partly due to a decrease in inflows. The higher outflow implies a higher decline in effective wealth across workers exposed and a lower-than-normal decline in effective wealth among non-exposed workers as some of them do not switch.

Towards the end of their work life workers load more of their effective wealth into human capital due to the higher frictions in this example, and this is the reason why the effect on earnings moderates in period 9, but increases for assets.

6.2 Quantitative model

I now work with a large-scale quantitative version of the model. For this I use the dynamic exact hat algebra techniques in Proposition 5 and Lemma 2. While these techniques are convenient for the quantitative analysis, as a large set of parameters do not need to be estimated/calibrated, they require information on the initial value of many endogenous variables and also the value of some key parameters.[35]

Different from much of the recent literature using dynamic exact hat algebra methods, some of the initial values of the endogenous variables I need to apply these methods do not have a direct counterpart in the data.[36] However, it is possible to manipulate the model's equilibrium conditions and obtain expressions that allows the estimation of these initial conditions. The next subsections discuss how to estimate these objects. Moreover, as is usually the case, parameters related to the variance of shocks need to be estimated. But the estimation strategy differs notably from past works. The reason is that the idiosyncratic shocks faced by individuals affect their earnings. Thus, we can use moments about earnings dynamics for individuals to estimate these parameters, in particular, the variance of the shocks ϵ that heavily influence workers' mobility and reallocation decisions.[37]

For this I will use data from the Survey of Income and Program Participation (SIPP), which is a series of panels of U.S. households and contains information on industry of employment, earnings, wealth, and state of residence.[38]

[35] To solve the model in the computer, I proceed in a similar way as Caliendo, Dvorkin, and Parro (2019), iterating on paths of wages and rental rates until all equilibrium conditions are satisfied. It would also be possible to find a linear approximation to the equilibrium conditions and adapt the tools developed in Kleinman, Liu, and Redding (2023) to solve the model.

[36] For example, Caliendo, Dvorkin, and Parro (2019) used data on mobility rates, average wages and employment shares, which are readily available in the data. While I also use mobility rates and employment shares, I also need values for average effective wealth, among others.

[37] I connect with the empirical literature on earnings dynamics (Lillard & Willis, 1978; MaCurdy, 1982), but note that this large literature on random income processes abstracts from selection of workers over industries and regions due to shocks affecting workers' comparative advantage. The model delivers expressions that allow me to correct for selection in the estimation.

[38] The SIPP is a widely used survey in applied work in economics studying earnings and employment dynamics. However, the information about wealth in the survey has been used only scarcely. In the Appendix I show that moments of the wealth distribution in the SIPP are similar to the same moments computed using the Survey of Consumer Finances, except at the very top of the distribution, which gives confidence on the use of this data.

In this section I assume $\tau^{a,e}_{j\ell} = \psi^{a,e}_{j\ell}$, to use the results from Proposition 5 and Lemma 2. As discussed before, this constraints somewhat the value of some frictions or fundamentals, but many others can still vary in an arbitrary way. I assume that the discount factor, β, the trade elasticity, ϑ, and the share of consumption of the retired ς_R, are known or pin-down beforehand.

6.2.1 Identification of α and $\sigma^2_{\nu,j}$, $\theta^{a,e}_{j,0}$, and $\tilde{w}_{j,t} - \tilde{r}_{j,t}$.

I now discuss which moments in the data identify the different parameters and variables needed to conduct the quantitative exercises. My identification strategy uses mostly information on workers' log-earnings and, in some cases, also information log-income from assets. Since the data has a limited number of observations and the moments are conditional on workers' age, type, and labor market of origin and destination, in the Appendix I discuss additional assumptions I use to estimate these moments using the SIPP data.[39]

Define log-income from assets as,

$$\tilde{y}^{i,a,e}_{j\ell,t} = \tilde{r}_{\ell,t} - \tilde{\psi}^{a,e}_{j\ell} + \tilde{\epsilon}^{i,a,e}_{\ell} + \log(1 - \theta^{a,e}_{j,t}) + \log(W^{i,a,e}_{j,t}). \tag{27}$$

We can compute the variance of the difference between log-earnings and log-income from assets,

$$Var\left[\tilde{z}^{i,a,e}_{j\ell,t} - \tilde{y}^{i,a,e}_{j\ell,t}\right] = \sigma^2_{\nu,j}.$$

Thus, we can obtain estimates of $\sigma^2_{\nu,\ell}$ using this moment. The following moments identify $\tilde{w}_{\ell,t} - \tilde{r}_{\ell,t}$ and $\theta^{a,e}_{j,t}$ given information on mobility rates and $\sigma^2_{\nu,j}$,

$$E\left[\tilde{z}^{i,a+1,e}_{\ell m,t+1} - \tilde{y}^{i,a,e}_{j\ell,t}\right] = \tilde{w}_{j,t} - \tilde{r}_{j,t} + \log\left(\frac{\theta^{a,e}_{j,t}}{1 - \theta^{a,e}_{j,t}}\right),$$

$$\theta^{A,e}_{m,t} \approx \frac{\tilde{w}_{m,t} - \tilde{r}_{m,t}}{\sigma^2_{\nu,m}},$$

$$\sum_{m=1}^{J} \mu^{a,e}_{\ell m,t} \frac{\left[\tilde{w}_{m,t} - \sigma^2_{\nu,\ell}\,\theta^{a,e}_{\ell,t} - \tilde{r}_{m,t}\right]}{1 + \tilde{r}_{m,t} + (\tilde{w}_{m,t} - \tilde{r}_{m,t})\theta^{a,e}_{\ell,t} - \frac{\sigma^2_{\nu,\ell}}{2}(\theta^{a,e}_{\ell,t})^2} = 0,$$

Intuitively, moments of the differences in income from labor and assets are informative of the

[39] With a sufficiently large number of observations across these cells, these additional assumptions would not be needed. These assumptions simply impose a parametric structure on the dependence of some moments on age and limit how they change with workers characteristics and labor markets.

return premium and volatility. If the difference is positive and large, on average, the model demands either a large difference in returns, or a large human capital share.

We can compute the change in log earnings for the same worker over two consecutive periods using equation (20) and optimal effective wealth, in logs. Then, the variance in the cross-section of the change in log-earnings and log income from assets is,

$$Var\left[\tilde{z}^{i,a+1,e}_{\ell m,t+1} - \tilde{z}^{i,a,e}_{j\ell,t}\right] \;=\; \frac{\pi^2}{6\,\alpha^2} + (1-\theta^{a,e}_{j,t})^2\,\sigma^2_{\nu,j} + \sigma^2_{\nu,\ell},$$

which allows to identify α.

Thus, identification of these parameters rests largely on the assumption that α and $\sigma^2_{\nu,j}$ do not vary with workers' characteristics and are constant over time.

6.2.2 Identification of initial values for $E[W^{a,e}_{j,0}]$, $\mathcal{M}^{a,e}_{j\ell,0}$, $\tilde{\mathcal{M}}^{a,e}_{j\ell,0}$, $E[\log(W^{a,e}_{j,0})]$, and $Var[\log(W^{a,e}_{j,0})]$.

Conditional on the values obtained previously, we can identify the transferability matrices and moments of effective wealth using moments of workers' earnings using the following conditions,

$$\begin{aligned}
E\left[\tilde{z}^{i,a,e}_{j\ell,t}\right] &= \tilde{w}_{\ell,t} + \log(\theta^{a,e}_{j,t}) + \tilde{\mathcal{M}}^{a,e}_{j\ell,t} + E[\log(W^{a,e}_{j,t})] \\
E\left[\tilde{z}^{i,a+1,e}_{\ell m,t+1} - \tilde{z}^{i,a,e}_{j\ell,t}\right] &= \tilde{w}_{m,t+1} - (1-\theta^{a,e}_{j,t})(\tilde{w}_{\ell,t} - \tilde{r}_{\ell,t}) + \log(\theta^{a+1,e}_{\ell,t+1}) - \log(\theta^{a,e}_{j,t}) + \log(\beta) + \tilde{\mathcal{M}}^{a,e}_{j\ell,t} \\
E\left[z^{i,a,e}_{j\ell,t}\right] &= w_{\ell,t}\, e^{\sigma^2_{\nu,j}/2}\, \theta^{a,e}_{j,t}\, \mathcal{M}^{a+1,e}_{\ell m,t}\, E[W^{a,e}_{j,t}] \\
E\left[z^{i,a+1,e}_{\ell m,t+1}/z^{i,a,e}_{j\ell,t}\right] &= \beta\; e^{\sigma^2_{\nu,j}/2}\, e^{\sigma^2_{\nu,\ell}/2} \left(\frac{w_{m,t+1}}{w_{\ell,t}}\right)\left(\frac{\theta^{a+1,e}_{\ell,t+1}}{\theta^{a,e}_{j,t}}\right)\left[w_{\ell,t}\, e^{\sigma^2_{\nu,j}/2}\theta^{a,e}_{j,t} + r_{\ell,t}(1-\theta^{a,e}_{j,t})\right]\mathcal{M}^{a+1,e}_{\ell m,t+1} \\
Var\left[\tilde{z}^{i,a,e}_{j\ell,t}\right] &= \frac{\pi^2}{6\,\alpha^2} + \sigma^2_{\nu,j} + Var[\log(W^{a,e}_{j,t})]
\end{aligned}$$

In the Appendix I derive similar expressions for the entering cohort. As is clear from the expressions, given previously found factor prices and human capital shares, the matrices $\tilde{M}^{a,e}_{j\ell,t}$ and $\tilde{\mathcal{M}}^{a,e}_{j\ell,t}$ are identified by moments of the changes in earnings for individuals that stay or move across labor markets. Intuitively, since frictions $\tau^{a,e}$ and parameters $\lambda^{a,e}$ are the main components of these matrices, the way in which earnings change over the life-cycle and with mobility across labor markets is informative about these parameters.[40]

Given these values, it is straightforward to compute the total supply of labor and assets,

[40]Note that, given the expressions that define these matrices and the identification strategy laid out here, it is possible to recover the values of frictions $\tau^{a,e}$ and parameters $\lambda^{a,e}$. I use this to recover these values needed for the simulation of individual earnings and mobility histories of workers, as discussed later.

consumption, investment and the final demand of goods. With information on consumption share parameters, the trade elasticity, factor returns, the final demand of goods, the factor supplies and trade shares, we can compute all initial conditions needed for the supply side of the model.[41]

6.2.3 XXX

7 Conclusion

I develop an heterogeneous agent dynamic spatial general equilibrium model of labor market choice (industry/occupations) and migration (regions) with human capital and assets accumulation and use it to quantify the effects of increased import competition on earnings and wealth. In my setting, the evolution of workers' human capital and assets is driven by their labor market choices, idiosyncratic labor-market-specific shocks, and the costs of switching/migrating. I characterize the worker's human capital and asset investment choices and equilibrium assignment of workers to labor markets, which involves a set of discrete-continuous decisions over time. I show how individual consumption/investment decisions can be written as a optimal portfolio problem given labor market choices, leading to decision rules that are homogeneous in wealth and portfolio choices that are similar across individuals. Exploiting properties of extreme value distributions I show how the ensuing discrete problem leads to a tractable characterization. A key result is that individual decisions can be aggregated and the resulting evolution of the aggregate supply of human capital and assets across labor markets can be represented in closed-form. I extend dynamic exact hat algebra techniques to this heterogeneous agent model and show how the model can be computed in changes given initial values of endogenous variables which are identified directly by moments in the data. I also extend dynamic exact hat algebra to characterize the dynamics of inequality.

I then use the model to quantitatively study how worker's individual choices change with the exposure to import competition from China, and in turn how this choices shape the equilibrium allocation of workers to different labor markets, the dynamics of aggregate human capital and assets, the behavior of earnings inequality over time and space, and the welfare of the different workers in the economy.

The model developed here provides a tractable quantitative framework to study other interesting questions, such as the effects of climate change on the dynamics of consumption,

[41] This simply is the solution to the static equilibrium at period $t = 0$. Some minor adjustments may be needed for all equilibrium conditions to be satisfied simultaneously.

wealth, inequality and growth, for finely detailed geographic areas.

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