TFPR: Dispersion and Cyclicality*

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Abstract

This paper studies the determinants of TFPR, a revenue based measure of total factor productivity. Recent evidence points to the rapid increase in the dispersion of TFPR during the Great Recession period and more generally in other recessions. Since the distribution of TFPR is endogenous, it is important to understand its determination and behavior over the business cycle. The framework used to determine the distribution of TFPR is an overlapping generations model with monopolistic competition and state dependent pricing. Changes in the mean and the dispersion of a quantity based measure of total factor productivity, TFPQ, and monetary shocks are analyzed as exogenous variations that influence the distribution of TFPR. None of these shocks alone can generate countercyclical dispersion in TFPR **and** match observed countercyclical dispersion in price changes and countercyclical movements in the frequency of price changes. Large enough shocks to the dispersion in TFPQ along with a responsive monetary policy can match these facts. But the required monetary feedback does not reproduce the positive correlation between money innovations and the dispersion in TFPR seen in the data. In this framework, uncertainty *per se* plays a very limited role.

1 Motivation

The dispersion of productivity has been shown to be counter-cyclical.[1] This finding plays a major role in recent quantitative studies of aggregate fluctuations. A prominent example is Bloom (2009) which studies the effects of uncertainty over the dispersion of productivity on investment activity. Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) go further to document the business cycle implications of countercyclical dispersion in, *inter alia*, firm level productivity.[2] Bachmann and Bayer (2014b) provide complementary evidence from German data.[3] As another leading example, Vavra (2013) provides evidence that price changes are more dispersed in recessions and the frequency of price adjustment is higher. He argues that these patterns can be

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[1] See the evidence and discussion in, for example, Kehrig (2011), Bachmann and Bayer (2014a), and Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). The evidence is presented as changes in the distribution of total factor productivity and/or the correlation in the dispersion of total factor productivity with a measure of economic activity.

[2] Here there is an important but distinction between uncertainty and dispersion. Uncertainty refers to an *ex ante* situation of not knowing, say, some moment of the distribution of a random variable, such as not knowing the future variance. Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) contains both uncertainty and dispersion effects.

[3] They also add to the set of observations the procylicality in the dispersion of investment rates.

reproduced in a model with variations in the volatility of firm level productivity as these fluctuations induce some sellers to adjust prices upwards and others to adjust downwards.[4]

But, there is a fundamental inconsistency in the above description of evidence and models. It has to do with the term "productivity". The measured productivity that underlies the evidence is revenue based total factor productivity, hereafter TFPR. But, the models are built upon dispersion (and uncertainty) effects from changes in the distribution of a quantity based measure of total factor productivity, hereafter TFPQ.

The problem is that these are different measures of productivity, both in the data and in theory. The distinction between these measures of productivity is central to the empirical analysis in Foster, Haltiwanger, and Syverson (2008).[5] The facts presented in Foster, Haltiwanger, and Syverson (2008) make clear that: (i) the distributions of TFPQ and TFPR differ and (ii) the distribution of TFPR is not degenerate. The first point implies that any model attempting to study both of these distributions needs to rationalize the difference between TFPR and TFPQ. Further, that model, following the discussion in Hsieh and Klenow (2009), must explain why the distribution of TFPR is not degenerate.

The key difference between the distribution of TFPR and the distribution of TFPQ is the distribution of prices. Thus, understanding the cyclicality of TFPR dispersion requires a model of price determination. With that in mind, the central question of this paper is: what factors, both in the process of price determination and shocks, generate the observed countercyclical dispersion in TFPR?

This question is addressed through a model of state dependent pricing to obtain a mapping from the distribution of TFPQ to the distribution of TFPR. In contrast to the flexible price case, state dependent pricing due to menu costs introduces both extensive and intensive margins of pricing decisions and thus allows for a variety of factors, both monetary and real, to influence the distribution of TFPR.

In our environment, following Hsieh and Klenow (2009), in the absence of frictions, the distribution of TFPR would be degenerate. Interestingly, price stickiness is sufficient to create a non-degenerate distribution of TFPR, other types of frictions or wedges are not needed.[6] Thus state dependent pricing, in our setting, is a key input into the economic mechanism determining the distribution of TFPR.

This paper does not contest the cyclicality of TFPR dispersion. Rather, it studies the determinants of this cyclicality through the effects of three types of shocks: (i) variations in the distribution of TFPQ, (ii) aggregate money shocks and (iii) productivity shocks.[7] The first source of fluctuations seems natural: the dispersion in TFPR is driven by the dispersion in TFPQ. The second and third types of shocks can also generate changes in the dispersion in TFPR since, in our framework, the distribution of prices is endogenous.

The framework for the analysis is an overlapping generations model with monopolistic competition and sticky prices specified in section 2. Young agents have market power, set prices *ex ante* and can, at a cost,

[4] His calibration relies upon the same measures of dispersion as Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). The connection between firm specific shocks and the distribution of price changes is highlighted in Golosov and Lucas (2007) as well.

[5] As in Foster, Haltiwanger, and Syverson (2008), suppose the technology at the plant-level is $y = Af(l)$, where y is output, A is a technology shock, and l is the labor input into a technology $f(\cdot)$. TFPQ is given by $A = \frac{y}{f(l)}$ while TFPR is $pA = \frac{pY}{f(l)}$ where p is the relative output price.

[6] We are grateful to John Haltiwanger for emphasizing this point to us.

[7] These are leading sources of fluctuations but there are other candidates worth considering. Kehrig (2011) stresses the distributional implications of entry and exit as well as the distinction between durable and nondurable goods producers. Cooper and Schott (2013) emphasize the importance of variations in the cost of reallocation as generating aggregate fluctuations as well as variance in the dispersion of productivity.

change them *ex post*, that is once the monetary and productivity shocks are realized. Old agents take money earnings from youth as well as monetary policy induced transfers and spend them on a variety of goods. The analysis is conducted through a stationary rational expectations equilibrium for this environment.

One benefit of this model framework is the transparency of the equilibrium characterization as the *ex post* pricing decisions of sellers have no dynamic component. That is, the pricing problem is associated with young agents who set a price *ex ante* and have an option to pay a cost to adjust their price *ex post*. In old age, they are buyers not sellers. [8]

A second benefit is that money demand and monetary shocks are an integral part of the environment. The response of prices and quantities at both the individual and aggregate levels are fully determined in a stationary rational expectations equilibrium. There is no need to restrict the analysis to one-time unanticipated shocks.

Some of the results are quantitative and rely on a particular form of preferences and adjustment costs.[9] These are calibrated from the existing literature. Section 3 presents the quantitative model.

It is natural to think that variations in the dispersion of TFPR, hereafter denoted $disp_R$, is driven by changes in the dispersion of TFPQ, hereafter denoted $disp_Q$. This is indeed the case, despite the endogenous component of TFPR due to price setting. That is, variations in the distribution of TFPQ impacts TFPR dispersion directly and also indirectly through the frequency and magnitude of price adjustment. This includes the responsive of prices on both the extensive and intensive margins.

But this does not generate countercyclical $disp_R$. In the model, output is itself positively correlated with $disp_Q$. In a flexible price model, the increased dispersion of productivity allows for the reallocation of inputs towards more productive uses and output increases.[10] Here that effect exists also when prices are sticky. Thus one main finding is that a model economy driven by dispersion shocks to TFPQ fails to capture the counter-cyclicality in $disp_R$. To clear, this result is not immediate because of the endogeneity of price determination.

As suggested by Vavra (2013), shocks to the dispersion of TFPQ do succeed in matching two prominent features of pricing: (i) countercyclical dispersion in price changes and (ii) the countercyclical frequency of price adjustment. This qualitatively matches the data patterns Vavra (2013) uncovers but the model delivers the counterfactual prediction of procyclical $disp_R$.[11]

Note that this seems to contradict the findings of Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). That model relies on a "wait and see" aspect of investment with nonconvex adjustment costs that is not present in our setting. Thus we focus largely (though see below) on the effects of dispersion rather than volatility. The findings in Bachmann and Bayer (2013), Berger, Dew-Becker, and Giglio (2020) and Cooper and Schott (2013) and create considerable doubt that uncertainty dominates volatility effects. This further

[8] Other papers in the literature make assumptions that limit the power of state dependent pricing. For example, Christiano, Eichenbaum, and Evans (2005) invoke a Calvo pricing framework so that adjustment probabilities are exogenous and prices are indexed to the rate of lagged inflation. A similar approach is taken by Smets and Wouters (2007).

[9] Thus the overlapping generations framework is used here to provide a framework for conducting experiments in a stochastic equilibrium setting. The results are intended to be qualitative and suggestive for more details empirical analyses.

[10] This is sometimes called the Abel-Hartman effect and is discussed by Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) alongside the effects of uncertainty (see their section 5.2.4). These reallocation effects are emphasized in Cooper and Schott (2013).

[11] The distinction between the dispersion in TFPQ and TFPR is not discussed in Vavra (2013). The analysis does not evaluate the cyclicality of $disp_R$ and considers variation in $disp_Q$ along with the mean of TFPQ as sources of fluctuations

motivates our emphasis on pricing as the source of countercyclical $disp_R$.

Due to price setting behavior, $disp_R$ responds to other shocks. Holding fixed the distribution of TFPQ, money shocks alone can impact the distribution of TFPR through pricing decisions of sellers. In this case, the money shocks also generate countercyclical dispersion in price changes and in the frequency of price changes, but the dispersion of TFPR is again procyclical.

Finally, if fluctuations are driven by (aggregate) shocks to the mean of TFPQ, denoted μ_Q, then extreme shocks to the mean of TFPQ will reduce the dispersion in TFPR while increasing output. Interestingly, the same goes for the employment response to the shock: for extreme shocks aggregate employment increases with aggregate productivity, else it falls.[12] But in this setting the frequency of price adjustment is procyclical and the correlation of output and the dispersion of price changes is very close to zero.

Throughout these exercises, one theme emerges: there are non-linearities in the response of $disp_R$ to various shocks. Regardless of the source of aggregate fluctuations, $disp_R$ is generally lowest for extremely low and high realizations of the money shock and highest for the average state.

This suggests that allowing the monetary authority to respond to shocks to either $disp_Q$ or μ_Q shocks might alter the cyclical patterns. To study this, Section 5.1 adds a monetary feedback rule to the model and resolve the model to characterize the new equilibrium. Depending on the feedback rule and the source of the shock, we are able to match all of these moments. In particular, if fluctuations are driven by $disp_Q$ shocks **and** the monetary authority tightens money growth when $disp_Q$ is high, then the equilibrium displays countercyclical dispersion in TFPR (despite having procyclical dispersion in TFPQ), countercyclical frequency and dispersion of price changes as well as countercyclical employment dispersion. A similar finding about the role of monetary policy applies when both $disp_Q$ and μ_Q shocks are present and are perfectly negatively correlated. However, the monetary feedback, making money shocks dependent on $disp_Q$, produces a negative correlation between monetary innovations and $disp_R$, contrary to the data.

Section 6 looks at two additional properties of the model economy. First, as discussed by Vavra (2013) as well, there is another link between dispersion and monetary policy. An increase in the dispersion of real productivity reduces the effectiveness of monetary policy.[13] This reflects the fact that following an increase in $disp_Q$, both the frequency of adjustment and the dispersion of price adjustment increase.

Second, the model also provides insights into the pricing and output effects of uncertainty, as distinguished from dispersion shocks. In general, dispersion shocks refer to variations in the *ex post* distribution of idiosyncratic or aggregate shocks. The uncertainty effect arises *ex ante* as agents are uncertain of the future distribution from which the shocks are drawn.[14] In our model, an increase in *ex ante* uncertainty over the future distribution of idiosyncratic productivity influences the *ex ante* price set by sellers. Quantitatively we find that these effects are tiny. [15]

[12]This is clearly related to the findings in Galí (1999) and the literature that followed. Here we link these patterns to price setting at the plant-level.

[13]There is some additional empirical support for this proposition. Tenreyro and Thwaites (2016) argue that the response of the economy to monetary (federal funds rate) innovations is considerably stronger in expansions compared to recessions. The findings reported in Bachmann, Born, Elstner, and Grimme (2019) provide additional evidence linking price setting behavior to uncertainty.

[14]Under some assumptions, increases in uncertainty reduce economic activity. Bloom (2009) and related papers focus on the effects of uncertainty on spending on durables, such as firm capital.

[15]This is consistent with Vavra (2013) and Bachmann, Born, Elstner, and Grimme (2019).

Returning to our motivating question, in the end it does not appear that the endogenous evolution of the distribution of prices is enough to match the basic facts on pricing and to generate a countercyclical dispersion in TFPR. While we do succeed in finding setting in which all these moments are matched, this requires, as argued below, monetary interventions that are quite different from those observed in the data.

2 Model

We study these issues in an infinite horizon overlapping generations model with differentiated products and market power. Agents live for two periods, youth and old age. Generation t young agents produce and, when old, these agents consume a basket of goods produced by the next generation of young producers.

Saving occurs through the holding of fiat money. The quantity of fiat money is stochastic, representing monetary shocks.

Young producers are distinct in three dimensions. First, they produce a differentiated product. Second, their output is a stochastic function of their labor input. Finally, they have an idiosyncratic cost of price adjustment.

The focus of the analysis is on price setting by sellers. Each young agent freely sets a price *ex ante*. After the realization of their idiosyncratic and aggregate shocks, they decide to adjust their price or not.[16] Importantly, this *ex post* decision on readjustment depends on the realization of all shocks. In this way, the dispersion of the distribution of productivity shocks impacts the frequency of adjustment and thus the real effects of money shocks.

As in Lucas (1972), in the absence of price stickiness, there would be a stationary rational expectations equilibrium in which money was neutral. This is because money transfers are made to the old in proportion to money holding earned in youth. And, as in that paper, the analysis rests on the coexistence of real and nominal shocks. But, in our setting the friction of costly price adjustment replaces his assumption of imperfect information.[17]

2.1 Choice of Old Agents

Lifetime utility is represented by $u(c) - g(n) = \frac{c^{1-\sigma}}{1-\sigma} - g(n)$. Here c is a CES aggregator given by $c = \left(\sum_i c^{i\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$, with $\varepsilon > 1$.[18] The function $g(\cdot)$ is increasing and convex in hours worked, with $0 \leq n \leq 1$.

When old, agents take their money holdings from income earned in youth and allocate it across goods to maximize $u\left(\left[\sum_i (c^i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right)$, subject to a budget constraint of $\sum_i c^i p^i = M$ where M is their nominal income and p^i is the money price of good i.[19]

For these preferences, the demand for good i is given by

[16] These include idiosyncratic shocks to profitability and menu costs and aggregate shocks to money and the distribution of productivity.

[17] Of course, in his model the real shock was to the fraction of sellers in a particular market while we focus on productivity shocks.

[18] We normalize the number of young agents and thus products to 1. With the CES assumption, markups are constant.

[19] To simplify the notation, the time subscript is repressed. The money holdings come from income earned in youth as money is the store of value in this economy. Many other general equilibrium models, such as Dotsey, King, and Wolman (1999), impose money demand. In Golosov and Lucas (2007), money is in the utility function.

$$c^i = d(p^i, P, M) = (\frac{p^i}{P})^{-\varepsilon}\frac{M}{P}. \tag{1}$$

Here P is an aggregate price index defined as $P = \left(\sum_i (p^i)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$. Note that the only shock to demand is from variations in the stock of money, M: there are no product specific taste shocks common to all agents.

Let $V(\frac{M}{P})$ be the value of the solution to the optimization problem of an old agent with nominal income of M with prices given by P. Given the definition of c,

$$V(\frac{M}{P}) = u\left(\left[\sum_i \left((\frac{p^i}{P})^{-\varepsilon}\frac{M}{P}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\right) = u\left(\left[\sum_i \left((\frac{p^i}{P})^{-\varepsilon}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}\frac{M}{P}\right) \tag{2}$$

with P given above. From this, the marginal value of nominal income is given by $V_M = \frac{u'(c)}{P}$.

At this point, these the generic demands and values for an old age given nominal income and prices. We will take this structure and use it to study the choices of young agents in the OG framework, summarizing the utility they obtain when old through $V(\frac{M}{P})$.

2.2 Choice of Young Agents

We start with the pricing decisions of generation t young agents. When young agents choose the price of their product *ex ante*, they take into account the option, at a fixed cost, of adjusting their price *ex post*. Since this is a model of a menu rather than a quadratic cost at the micro-level, the *ex ante* price will influence the frequency of adjustment but not the *ex post* price conditional on adjustment.

As is common in the literature, see for example Galí (2015), agents are assumed to meet the demand forthcoming at their price. Thus the prices they set will determine their nominal income in youth.

This nominal income is held over time in the form of money to purchase consumption goods when old. Holdings of money are altered through monetary policy. Thus in our framework, money holdings and monetary policy interventions are made explicit.

To study the pricing choice, consider the *ex post* decision of generation t sellers. If they choose to adjust, these sellers choose a price $\tilde{p}$ to solve

$$W^a(s_t) = max_{\tilde{p}} E_{x_{t+1},P_{t+1}} V((R(\tilde{p}, P_t, M_t))x_{t+1}/P_{t+1}) - g(\frac{d(\tilde{p}, P_t, M_t)}{z_t}). \tag{3}$$

Here the demand, denoted $d(\tilde{p}, P_t, M_t)$ and specified in 1, is the spending of the old agents on the product of this seller.

Since this decision is made *ex post*, the value and the price depend on the current state $s_t \equiv (z_t, M_{t-1}, x_t, P_t)$. Here z_t is the current idiosyncratic productivity shock, M_{t-1} is the inherited money supply, x_t is the money shock and P_t is the aggregate price level, determined in equilibrium as described below.

In this model, the difference between TFPQ and TFPR is transparent. Here, z corresponds to the TFPQ measure of productivity.[20] It is exogenous to the seller. Variations in the distribution of z are studied and

[20] Throughout the mean of z is 1. The standard deviation of z, denoted σ_z, is referred to in the text as $disp_Q$.

termed dispersion shocks. The variable $\frac{\tilde{p}z}{P}$ is TFPR.[21] It is endogenous as prices are set by sellers. The distribution of TFPR responds to shocks insofar as sellers adjust prices in response to those shocks.[22]

Notice that the price set by these sellers is independent of any price they may have set *ex ante* so that the *ex ante* choice does not appear in the state. Importantly, once the cost of adjustment is incurred, the price reflects both the monetary shock and seller specific productivity. In this sense, there is an underlying complementarity at work. If a seller pays an adjustment cost to respond to one type of shock, then the marginal cost of responding to another type of shock is zero. This is important for the analysis that follows as it explains why price dispersion and thus TFPR dispersion is influenced by monetary policy.

With the production function of $y = zn$, the labor input of the seller is given by $\frac{d(\tilde{p}, P_t, M_t)}{z_t}$.[23] As the seller meets all demand, the labor input varies inversely with productivity, **given** demand.

There is a first-order condition of

$$E_{x_{t+1}, P_{t+1}} \left(u'(c_{t+1}) x_{t+1} \frac{d(p_t^i, P_t, M_t)(1-\varepsilon)}{P_{t+1}} \right) = g'(\frac{d(p_t^i, P_t, M_t)}{z_t}) \left(-\varepsilon \frac{d(p_t^i, P_t, M_t)}{p_t^i z_t} \right). \tag{4}$$

This is the standard condition for optimal price setting, equating marginal revenue with marginal cost.[24] But in this overlapping generations model, the value marginal revenue is determined by the marginal utility of the future consumption that can be acquired with the additional money income. And that income is itself impacted by future monetary policy, through the stochastic transfer x_{t+1}.

Denote this *ex post* optimal price by $p_t^i = \tilde{p}(z_t, M_{t-1}, x_t, P_t)$ for all sellers i. As specified below, the adjustment cost is written as a utility loss. This specification has a convenient property that the optimal price is independent of the adjustment cost. So, the extensive margin of adjustment will depend on the realized menu cost and idiosyncratic productivity but the intensive margin does not so that the price dispersion of adjusters reflects only heterogeneity in z_t.

Alternatively, if the seller does not adjust, then expected lifetime utility is given by:

$$W^n(s_t, \bar{p}) = E_{x_{t+1}, P_{t+1}} V((R(\bar{p}, P_t, M_t)) x_{t+1}/P_{t+1}) - g(\frac{d(\tilde{p}, P_t, M_t)}{z_t}). \tag{5}$$

Here, expected utility depends on the preset price, $\bar{p}$.

Given this, consider the *ex ante* choice. When this price is set, the young agent just knows the money supply from the past. Let $W^{xa}(M_{t-1})$ be the value to a young agent of setting the price *ex ante*. The value

[21] Since TFPQ is measured directly in simulated data, there is no need to infer TFPR from revenue and thus no discussion of output or revenue factor shares. See the discussion of these measurement issues in Decker, Haltiwanger, Jarmin, and Miranda (2019).

[22] Using a static, flexible price version of the model and returning to a point made earlier, $TFPR = pz = zq^{-\eta} = z^{1-\eta} n^{-\alpha\eta}$. From the first order condition with respect to n, if marginal cost of labor is ω, we have

$$(1-\eta)\alpha n^{(-\alpha\eta+\alpha-1)} z^{1-\eta} = \omega$$

At $\alpha = 1$, the FOC becomes $(1-\eta)n^{-\eta} z^{(1-\eta)} = \omega$ for this to hold for all z, implies $TFPR$ is independent of z. In our model, both price stickiness and non-linear production costs will contribute to the non-degenerate distribution of TFPR.

[23] When we discuss below the case of an aggregate TFP shock, then $y = Azn$, with A stochastic.

[24] To understand this condition in a static setting, let $d = (\frac{p}{P})^{-\varepsilon} y$ be the level of produce demand if the seller sets the price p and the aggregate price is P and the level of real spending is y. So $d_p = -\varepsilon \frac{d}{p}$. Further, revenue is given by $R = pd = p^{1-\varepsilon}(\frac{1}{P})^{-\varepsilon} y$. Hence $R_p = (1-\varepsilon)d$. The left side of 4 is the product of R_p and $\frac{u'(c_{t+1})x_{t+1}}{P_{t+1}}$. The right side is the product of d_p and the marginal disutility of work, $g'(\frac{d(p_t^i, P_t, M_t)}{z_t})\frac{1}{z_t}$.

is given by:

$$W^{xa}(M_{t-1}) = max_{\bar{p}} E_{(z_t, x_t, x_{t+1} P_t, P_{t+1})} [(1 - \Omega(F^*(s_t))) W^n(s_t, \bar{p}) + \int_0^{F^*(s_t)} W^a(M_{t-1}, x_t, P_t) - F] d\Omega(F) \quad (6)$$

where $F^*(s_t)$ is the critical menu cost in state s_t such that price adjustment occurs iff $F \leq F^*(s_t)$. Here the menu cost F has a cdf of $\Omega(\cdot)$. Let $\bar{p}^i(M_{t-1})$ denote the optimal *ex ante* choice.

2.3 SREE

The analysis is based on a stationary rational expectations equilibrium (SREE) with valued fiat money. The current aggregate state is represented as (M, x) where M is the inherited money supply and x is the current shock, so that the current money supply is Mx. At the individual supplier level, productivity and the cost of price adjustment are the two elements in the idiosyncratic state: (z, F).

There are four state dependent functions to be determined. The *ex ante* price set knowing only M is denoted $\bar{p}(M)$. The *ex post* price set by sellers who choose to adjust their price is given by $\tilde{p}(M, z, x)$, indicating the price depends on both the realized money shock and productivity. There is a critical level of the adjustment cost, $F^*(M, x, z)$, such that adjustment occurs iff $F \leq F^*(M, x, z)$. Finally, the *ex post* money price of goods, $P(M, x)$, clears the goods market.

At this point of the analysis, the distribution of the idiosyncratic productivity shocks is not in the state vector. An equilibrium is defined and characterized given that distribution. The effects of changing that distribution and adding further aggregate shocks on the characterization of a SREE are introduced and analyzed below.[25]

A SREE is a set of functions $(\bar{p}(M), \tilde{p}(M, z, x), F^*(M, x, z), P(M, x), W^n(M), W^a(M, x, z)$ such that:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P(M, x)$;

$$\hat{\varepsilon} \bar{p}(M) E_{x,x'} \left[\frac{x'}{P(Mx, x')} d(\bar{p}(M), M, x)) \right] = \frac{(E_{z,x} d(\bar{p}(M), M, x))^2}{z^2} \quad (7)$$

- $\tilde{p}(M, z, x)$ solves the *ex post* pricing problem given the state dependent price index $P(M, x)$

$$\hat{\varepsilon} \tilde{p}(M, z, x) E_{x'} (\frac{x'}{P(Mx, x')}) = \frac{d(\tilde{p}(M, z, x), M, x)}{z^2} \quad (8)$$

- at the critical adjustment cost $F^*(M, x, z)$, the seller is just indifferent between adjusting and not:

  $W^n(M) = W^a(M, z, x) - F$

  with $W^n(M)$ and $W^a(M, x, z)$ given by (3) and (5)

- $P(M, x)$ is the aggregate price index in state (M, x) given by:

[25] Essentially, if S is the aggregate state and s is the idiosyncratic state, then a SREE is a set of price functions $(\bar{p}(M), \tilde{p}(M, S, s), P(M, S, s)$, value functions $W^n(M), W^a(M, S, s)$, and a critical value of the price adjustment cost, $F^*(M, S, s)$ satisfying: (i) individual optimization by young price setters and old consumers, (ii) market clearing and (iii) consistency of beliefs and expectations for all states.

$$P(M,x) = [E_z(1-\Omega(F^*(M,x,z)))\bar{p}(M)^{1-\varepsilon} + E_z(\Omega(F^*(M,x,z))\tilde{p}(M,z,x)^{1-\varepsilon})]^{\frac{1}{1-\varepsilon}} \tag{9}$$

Here $d(\bar{p}(M), M, x) = (\frac{\bar{p}(M)}{P(M,x})^{-\varepsilon} I$ and $d(\tilde{p}(M,z,x), M, x) = (\frac{\tilde{p}(M,z,x)}{P(M,x})^{-\varepsilon} I$. Here $I = \frac{Mx}{P(M,x)}$ is the equilibrium determined real value of money holdings.

There are two main properties of a SREE that are verified in the analysis that follows.

Proposition 1. *There exists a SREE in which: (i) all prices are proportional to M and real quantities are independent of M and (ii) all prices are not proportional to x and so real quantities are not independent of x.*

First, the inherited money supply is neutral: i.e. prices are proportional to M and all real quantities are independent of M. Formally, this amounts to guessing and verifying that there is a SREE in which $\bar{p}(M) = QM$ where Q is an unknown constant and $\tilde{p}(M,x,z) = M\tilde{\phi}(x,z)$. From this all relative prices and thus quantities demanded (and thus supplied) are independent of M.

The second property is money non-neutrality. If prices were not costly to adjust, i.e. the distribution of F was degenerate at $F = 0$, then there would exist a SREE with prices proportional to Mx. In this case, real quantities would be independent of the current money supply, Mx. But, in the presence of non-degenerate menu costs, as long as some sellers choose not to adjustment their prices *ex post*, a SREE with prices proportional to Mx cannot exist simply because the preset price, $\bar{p}$, must be independent of x.[26]

In equilibrium, aggregate real GDP is given by: $Y(x) = \frac{Mx}{P(M,x)} = \frac{x}{\phi(x)}$. As σ_z changes, there will be both level effects, the gains to dispersion in productivity, as well as responsiveness changes, so that $\frac{d\phi(x)}{dx}$ or the elasticity should increase as σ_z increases.

3 Quantitative Analysis

The quantitative analysis rests upon a linear-quadratic economy where $u(c) = c, g(n) = \frac{n^2}{2}$. This section characterizes a SREE for this specification of preferences and then presents the functional form for the price adjustment cost used in the numerical exercises.

To be clear, the goal of the quantitative analysis is not to match data moments. This seems off the domain of the model as it is cast as an OG structure, with pricing decisions made once and thus apparently far from the high frequency decision problem that underlies plant and firm-level data.

But there is a benefit to this abstraction through an opportunity to clarify various channels of influence through the stationary rational expectations equilibrium of a monetary economy. In this setting, the monetary shocks as well as those to productivity are part of the basic economic environment and thus the responses are part of the equilibrium outcome. The goal is to provide a framework upon which empirics can be generated. These points are brought out here and in the following section on additional implications.

[26] Formally, this requires that the support of menu costs be large enough so that even if all other sellers adjust their prices *ex post*, the remaining seller, for any x, will have a high enough adjustment cost so that adjustment will not occur. See Ball and Romer (1991) for a discussion of this related to multiplicity of equilibria.

Further, the OG model is not that far from the more standard models of Calvo price adjustment. In those models, as in the OG structure, the probability of price adjustment and the price set conditional on adjustment are both independent of the previously set price. Further, in some specifications, such as Christiano, Eichenbaum, and Evans (2005), price setters who do not adjust get to freely reset prices based upon inflation. This added feature further reduces the role of history for price setting. Or, put differently, the fact that *ex post* price setters do not look beyond the current period is also present in these other formulations. This point is reinforced by the quantitative analysis which generates familiar patterns of price adjustment.

3.1 Linear-Quadratic Economy

For this case, the SREE defined above becomes a set of functions $(\bar{p}(M), \tilde{p}(M,z,x), F^*(M,x,z), P(M,x)$ such that:

- $\bar{p}(M)$ solves the *ex ante* pricing problem given the state dependent price index $P(M,x)$;

$$\hat{\varepsilon}\bar{p}(M)E_{x,x'}\left[\frac{x'}{P(Mx,x')}d(\bar{p}(M),M,x))\right] = \frac{(E_{z,x}d(\bar{p}(M),M,x))^2}{z^2} \tag{10}$$

- $\tilde{p}(M,z,x)$ solves the *ex post* pricing problem given the state dependent price index $P(M,x)$

$$\hat{\varepsilon}\tilde{p}(M,z,x)E_{x'}(\frac{x'}{P(Mx,x')}) = \frac{d(\tilde{p}(M,z,x),M,x)}{z^2} \tag{11}$$

- at the critical adjustment cost $F^*(M,x,z)$, the seller is just indifferent between adjusting and not: $W^n(M) = W^a(M,z,x) - F$

- $P(M,x)$ is the aggregate price function in state (M,x) given by:

$$P(M,x) = E_z(1-\Omega(F^*(M,x,z))\bar{p}(M) + E_z\Omega(F^*(M,x,z))\tilde{p}(M,z,x). \tag{12}$$

Throughout $d(\bar{p}(M),M,x) = (\frac{\bar{p}(M)}{P(M,x})^{-\varepsilon}Y$ and $d(\tilde{p}(M,z,x),M,x) = (\frac{\tilde{p}(M,z,x)}{P(M,x})^{-\varepsilon}Y$ and $Y = \frac{Mx}{P(M,x)}$.

3.2 Quantitative Approach

We first discuss how a SREE is computed and then present the various functional forms and parameterization used in the analysis. For the price setting problems of the SREE, except for the aggregate price level, $P(M,x)$, all of the other state variables are exogenous. In contrast, the aggregate price level is an equilibrium object, and is therefore calculated from the choices of the sellers, as in (12). Thus the focus of the solution approach is to find the equilibrium price function, $P(M,x)$.

3.2.1 Finding a SREE

The numerical solution algorithm has two loops. In the outer loop, the aggregate price level for each point in the aggregate state space is set. The inner loop solves the *ex ante* and *ex post* choice problems for all points in the state space (x, z, F). To do so, sellers take the aggregate price function from the outer loop as given. Based on these values, the extensive margin decisions are determined. Finally, the probability of adjustment for each point on the state space is calculated. After taking the expectation over the idiosyncratic shocks, the algorithm computes the aggregate price level from the choices of all sellers and checks the convergence with the initial guess. If there is no convergence, then the probability of adjustment is reset in the outer loop until convergence is achieved.

Note that there is no approximation involved here. The approach simply solves a system of equations to find a SREE. So unlike an approach based upon Krusell and Smith (1998), there are no moments *per se* used to characterize an equilibrium.[27]

3.2.2 Distributions

Idiosyncratic productivity shocks have a mean of 1 and a standard deviation denoted by σ_z.[28] The menu cost distribution follows Dotsey and Wolman (2019), using a tangent function given by:

$$G(F) = \frac{1}{\omega}\left\{\tan(\frac{F-\kappa_2}{\kappa_1}) + \nu \cdot \pi\right\} \tag{13}$$

with

$$\kappa_1 = \frac{\bar{F}}{[\tan^{-1}(\omega - \nu\cdot\pi) + \tan^{-1}(\nu\cdot\pi)]}; \qquad \kappa_2 = \arctan(\nu\cdot\pi)\cdot\kappa_1. \tag{14}$$

The upper bound on the fixed cost, $\bar{F}$, controls the extent of price stickiness. As $\bar{F}$ increases, higher values for menu cost is now available, making the adjustment harder. The curvature parameters (ω, ν), are chosen so that $G(F)$ is monotonically increasing.

By changing $(\omega, \nu, \bar{F})$, this functional form can approximate the Calvo setup, a single fixed cost of adjustment or, as in our case, a distribution with adjustment probabilities that are flatter for a wide range of F. In order to capture small price changes, agents can draw a zero menu cost, with probability ψ.

3.3 SREE: Quantitative Properties

This section lays out the properties of the SREE for this linear-quadratic economy. Among other things, this discussion verifies the properties of the SREE stated earlier. Further it makes clear that the overlapping generations model with state dependent price adjustment retains the essential features of the more standard infinitely lived agent specifications.

For this analysis, the distribution of idiosyncratic productivity shocks is taken as given, so that σ_z is not in the state vector. The effects of changes in this distribution are discussed in the next section.

[27] This in part reflects the simplification that price setters do not have inherited prices set in the past. Hence, there is no joint distribution of past prices and productivity in the state space.

[28] Given the one period nature of price setting, there is no gain to specifying the AR(1) for these shocks.

Table 1: Parameterization

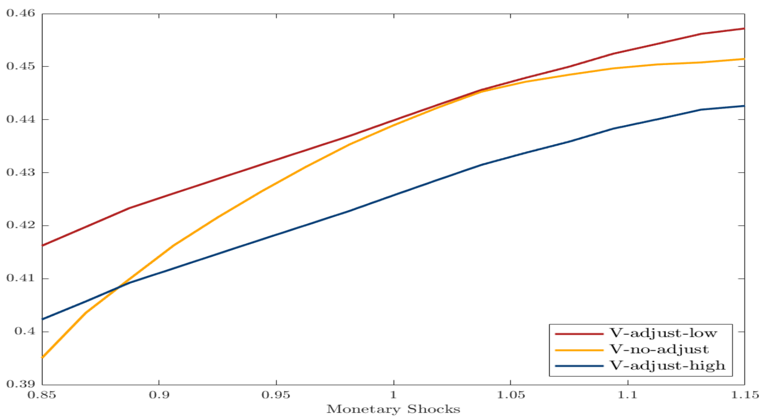
Parameter	Value	Description	Source
ϵ	3	Elasticity of substitution between products	Common in the literature
ρ	0.95	Persistence parameter of the AR(1) process	Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018)
σ_z	0.08	Dispersion of TFPQ	Foster, Haltiwanger, and Syverson (2008)
Menu Cost Distribution			
ψ	0.053	Probability of zero menu cost	Dotsey and Wolman (2019)
$\bar{F}$	0.033	Upper bound on menu cost	Dotsey and Wolman (2019)
ω	41.9	Curvature parameter	Dotsey and Wolman (2019)
ν	2.8	Curvature parameter	Dotsey and Wolman (2019)

3.3.1 Seller Choices

This section illustrates the quantitative properties of the seller's choices for the linear-quadratic economy. The parameterization is shown in Table 1.

As in the traditional state dependent pricing model, prices are adjusted only for sufficiently large monetary shocks and the region of adjustment depends on the adjustment costs. This property is illustrated for a given productivity shock to make clear that our model also contains the usual result. A second important property is the interaction of the real productivity shock and the monetary shock: adjustment can occur even for monetary shocks near their expected value if productivity shocks are sufficiently large.

Figure 1: Adjustment Values



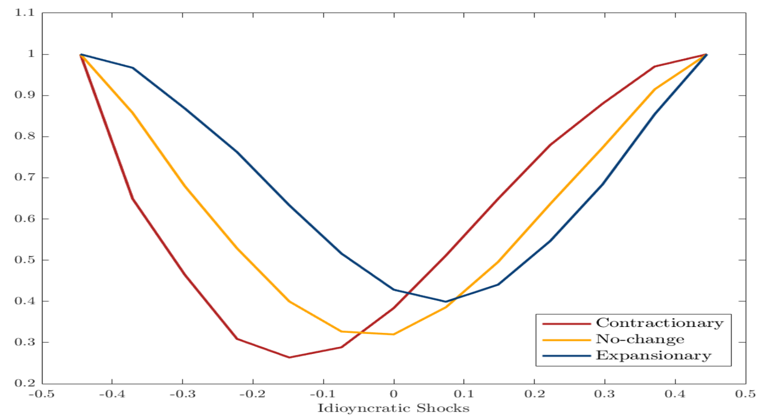
This figure shows the choice to adjust or not for different realizations of the money shock, x, for two values of the menu cost.

Figure 1 illustrates the *ex post* choices of a seller.[29] The figure shows the value of no-adjustment along with the values of adjustment for two levels of the menu cost. The blue value is associated with a higher menu cost than the red value. As is clear, for sufficiently low (high) values of the money shock x, adjustment is preferred to no adjustment. The region of adjustment is larger for the low menu cost. As seen, if the menu

[29]This is from a case in which there are no idiosyncratic productivity shocks.

cost draw is high enough, the seller will not adjust its price for any x in the range shown.

Figure 2: Adjustment Choice as a function of z



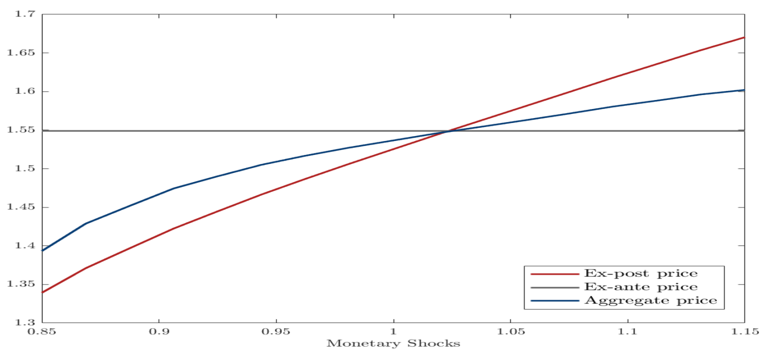
This figure shows the choice to adjust or not for different realizations of the idiosyncratic profitability shock, for 3 values of the monetary shock

Figure 2 illustrates the dependence of price adjustment on the real productivity shock, for three values of x. For each value of x, there is a realization of the productivity shock such that all firms choose not to adjustment their price. The higher the money shock, the higher is this critical productivity to offset the incentive to adjust.

The frequency of adjustment is then U-shaped around this minimum. The fact that the model economy produces this shape for the adjustment rate is important for two reasons. First, it confirms that state dependent pricing in the overlapping generations model produces patterns that are similar to other models. Second, as the analysis develops, the aggregate economy will display non-monotonic responses to various types of shocks. Those patterns can be traced back to the U-shaped adjustment rate.

3.3.2 Aggregate Effects

Figure 3: Prices



This figure shows patterns of prices for different realizations of the money shock, x.

Given these responses at the firm level, we now turn to briefly describe the aggregate implications of the model in terms of the overall price level, the frequency of adjustment and output. We return to these effects when we look at other implications of the model.

Figure 3 shows the aggregate price as well as the *ex ante* and *ex post* prices as a function of the monetary shock. The aggregate price is clearly an increasing function of the money shock. The aggregate price, which is a CES index, is a combination of the state independent *ex ante* price and the state dependent *ex post* price. It is increasing in the money shock, reflecting responses on both the extensive and intensive margins. The frequency of adjustment, as established earlier, is U-shaped, while the *ex post* price is monotonically increasing in the shock.

Figure 4 illustrates the dependence of the adjustment frequency on the monetary shock and the real effects of the shock. Focusing on the blue line in the left panel, for low dispersion, the frequency of price adjustment is a U-shaped function of the money shock.[30] Note that the adjustment frequency does not have a minimum at 0, reflecting the presence of the real shocks which create an independent value of adjustment.

The response of the aggregate price (index) to the money shock is shown in the top right panel. Here, reflecting both the extensive and intensive margins, the price level increases when the money supply expands. But, as indicated in the bottom left panel, the level of real economic activity output is clearly increasing in the monetary shock as well. This reflects both the stickiness of some prices and the choice of adjusted prices that are not proportional to the money supply.

This is summarized by the first row of Table 2. For the benchmark parameterization, about 25% of the sellers adjust their prices *ex post.* The average dispersion in the price change is about 0.10, reflecting the underlying distribution in productivity. The correlation of output and the money shock is nearly 1.

Table 2: Pricing Moments

Case	Mean ($Freq_{\Delta P}$)	Mean ($disp_{\Delta P}$)	$corr(y, x)$
Benchmark	0.2556	0.0975	0.9959
Higher Product Substitutability	0.4799	0.0737	0.7035
High Labor Supply Elasticity	0.2166	0.1081	0.9956

This table shows basic moments in response to money shocks, for the baseline model and other parameterizations discussed in sub-section 5.2.

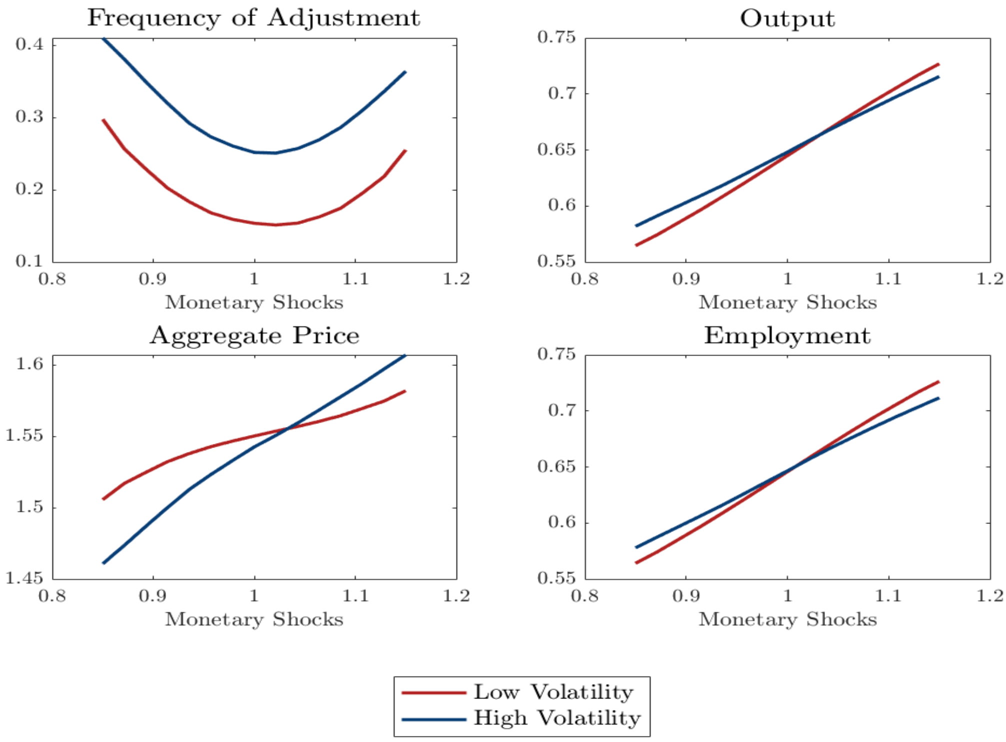
4 Cyclicality of TFPR Dispersion

The model of state dependent prices provides a basis to study the cyclicality of TFPR dispersion. As noted earlier, many theories are about the dispersion in TFPQ while, as emphasized by Foster, Haltiwanger, and Syverson (2008), the measurement commonly taken from plant-level studies is TFPR not TFPQ. Output and revenue measures of productivity are not same and their distributions may covary in different ways over the business cycle.

The question is whether the model of price setting can reproduce the counter-cyclical dispersion in TFPR seen in the data. This depends both on price setting behavior and exogenous variations. Here the exogenous

[30]We return to a comparison of the low and high dispersion cases below.

Figure 4:



This figure shows the effects of money shocks on the price adjustment rate, output, employment and the aggregate price.

variations include changes in $disp_Q$, money shocks and changes in the mean of TFPQ.[31]

Table 3 summarizes our findings. It displays for the three sources of variation, the cyclical patterns of dispersion in TFPR, the dispersion of price changes, employment and the frequency of price adjustment.[32] The table is constructed using our baseline parameters. Variations on these parameters are reported in sub-section 5.2.

The table is discussed in detail in this section, first by looking at each shock independently. This allows us to focus on the cyclical effects of each shock independently. We then allow the monetary authority to respond to variations in the mean and dispersion of TFPQ and study the implications for the dispersion of

[31]The SREE characterized above is for the case of money shocks alone. Exogenous variations in the dispersion of the idiosyncratic productivity shocks, $disp_Q$, is introduced as a mean preserving spread of z. For the model with changes in the mean of TFPQ, as noted earlier, the production function at the individual level becomes $y = Azn$ where A is stochastic. In both cases the SREE is redefined and computed with the additional state variable.

[32]Here we report correlations with the level of output as there is nothing to detrend. The same properties emerge from splitting the sample into "expansions" and "contractions" and computing conditional moments as displayed in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and Ilut, Kehrig, and Schneider (2018).

TFPR.

The results are best evaluated relative to moments from the data, which are provided as well.[33] As discussed, the dispersion of TFPR is countercyclical, the dispersion of price changes and frequency of price changes are countercyclical as is the dispersion of employment growth.

Table 3: Cyclical Variations

Case	$corr(y, disp_R)$	$corr(y, disp_{\Delta P})$	$corr(y, disp_n)$	$corr(y, freq)$
Data	-0.45	-0.41	-0.50	-0.27
		$disp_Q$		
Model	0.4292	-0.1348	-0.3413	-0.1695
Flex. P	0.9977	0.8626	0.9974	na
		More TFPQ Dispersion		
Model	-0.0238	-0.0069	0.0191	0.0855
Flex. P	0.9911	0.9715	0.9910	na
		x		
Model	0.4574	-0.2397	-0.4863	-0.1311
		μ_Q		
Model	-0.0183	-0.0010	-0.3863	0.1878
Flex. P	0.9402	-0.5259	-0.3264	na

This table shows the correlation between output and the dispersion of TFPR, the dispersion and frequency of price changes and the dispersion of employment for three different types of shocks. For the row labeled "Data", the correlations of prices changes and frequency with output come from Vavra (2013). The correlation of output and the dispersion in TFPR comes from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) though the correlation reported there is output growth not its level. The correlation of employment dispersion and growth is from Ilut, Kehrig, and Schneider (2018).

4.1 Effects of Variation in TFPQ Dispersion

The analysis of counter-cyclical variation in TFPR dispersion starts with an obvious hypothesis: variations in $disp_Q$ drive the cyclicality of $disp_R$. To order for this explanation to be consistent with data patterns, it must be that: (i) increased dispersion in TFPQ creates increased dispersion in TFPR and (ii) increased dispersion in TFPQ causes economic downturns. We demonstrate that the model does not produce these patterns: **variations in the dispersion of TFPQ do not generate countercyclical fluctuations in the dispersion of TFPR.**

Specifically, here we study the effects on $disp_R$ of an increase in $disp_Q$, modeled as a mean preserving spread in the distribution of z.[34] To be clear, the effects highlighted here come from realized changes in the distribution of TFPQ. Another channel, studied below, is on the uncertainty caused by future changes in this distribution. Throughout, $disp_R$ is always less than $disp_Q$, as it is in the data summarized by Foster,

[33] The correlation of output (growth) and $disp_R$ comes from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). Kehrig (2011) finds that the correlation of (detrended) output and the dispersion of productivity is -0.293 for non-durables and -0.502 for durables, Table 2. His Table 4 makes clear that the countercyclicality is robust to various output measures. The moments on the dispersion and frequency of price changes comes from Vavra (2013), Table 4, and calculated at the business cycle frequency. The correlation of employment (growth) and dispersion is from Table I.3 in Ilut, Kehrig, and Schneider (2018).

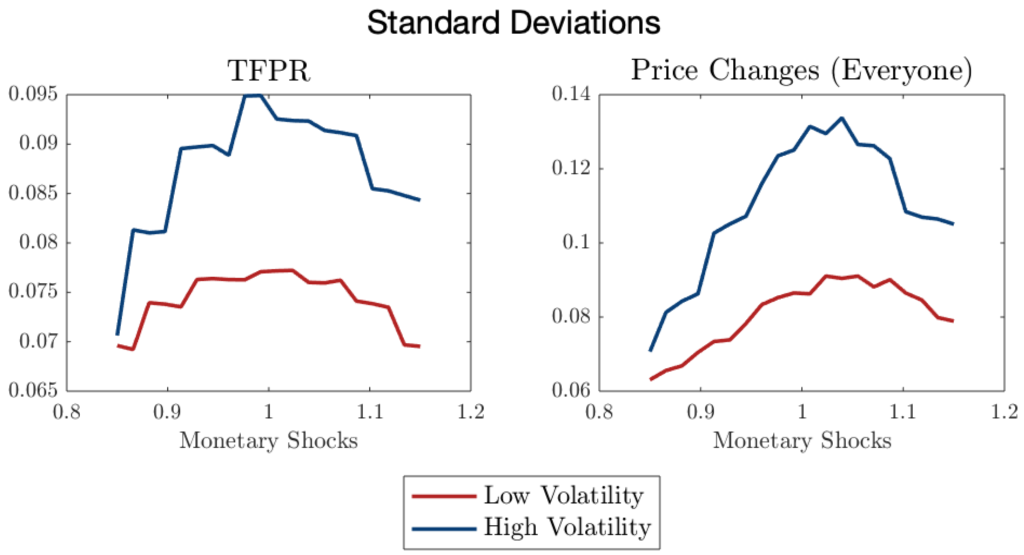
[34] This is parameterized to match the cyclical change in TFPR dispersion as reported in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018). This determines the magnitude not the cyclicality of the change in the dispersion of TFPR. This parameterization applies when all the shocks, $(disp_Q, x, \mu_Q)$, are present to match the unconditional movements in the dispersion of TFPR.

Haltiwanger, and Syverson (2008). Again, the magnitude of this difference depends on the state of the economy.

Variations in $disp_Q$ will impact $disp_R$ in two ways. First, of course, there is the direct effect: given prices, an increase in $disp_Q$ will translate into an increase in TFPR dispersion. Second, pricing behavior will adjust, potentially magnifying (reducing) the effects of the increase in $disp_Q$. The sign and size of this latter effect will depend on the properties of the revenue function and, as emphasized by our model, the pattern of price adjustment.

Figure 5 illustrates the effects of variations in $disp_Q$ on the $disp_R$, for different values of the money shock. Clearly an increase in $disp_Q$ leads to an increase in $disp_R$. The magnitude of this effect though does depend on the money shock, as discussed further below.

Figure 5: Response of Dispersion in TFPR and Price Changes



This figure shows the effects of variations in $disp_Q$ on $disp_R$ and the dispersion of price changes.

What are the effects of an increase in $disp_Q$ on output? If the increased dispersion causes a reduction in output, then shocks to $disp_Q$ will create counter-cyclical dispersion in TFPR, as in the data.

In an economy with flexible prices, a mean preserving spread in plant-level productivity will typically increase output as factors are reallocated to take advantage of high productivity plants. This property holds in our overlapping generations model with monopolistic competition as well if there are no costs of price adjustment.

In a setting with price rigidities this reallocation will be weaker and may not even occur. To see why, consider a seller with a high productivity realization and a high adjustment cost. Without price adjustment, the high productivity means that the seller will supply a relatively low level of labor to meet demand.[35] But if the realized adjustment cost was low, the seller would choose a lower price and expand production and

[35]This reflects the assumption that the seller meets demand at the posted price.

employment. Thus the output response to increased dispersion is hampered by price inflexibility.[36]

Returning to Figure 4, at $x = 1$, the response of the economy to an increase in $disp_Q$ is a slightly lower aggregate price level and thus slightly higher aggregate output. In this case, without a response by the monetary authority, variations in $disp_Q$ are procyclical and lead to procyclical variations in $disp_R$ as indicated in Table 3.

Also from Table 3, the model predicts that the dispersion in price changes is countercyclical, consistent the evidence provided in Vavra (2013). The dispersion in employment is positively correlated with $disp_Q$. This makes sense, as the dispersion in productivity increases, so will the dispersion in employment. This effect is particularly strong for sellers who choose not to adjustment their price. Finally, the frequency of price adjustment is countercyclical in the model driven by $disp_Q$ shocks.

The findings about the cyclicality of the dispersion in price changes and frequency are consistent with the findings of Vavra (2013). But the model is inconsistent with the data in terms of the motivating observation of countercyclical dispersion in TFPR.

4.2 Money Shocks

A second shock comes from monetary innovations, x. Due to price rigidities, monetary shocks impact real output. From Figure 4, output and aggregate prices both respond positively to monetary shocks. Here we focus on the effects of money shocks on the dispersion of TFPR holding fixed the distribution of TFPQ.

Can they produce counter-cyclical TFPR dispersion? Figure 5 illustrates the effects of money shocks on the standard deviation of TFPR. As indicated in the top left panel, for either extremely low or high monetary shocks (around $\pm$ 10%), the dispersion in TFPR is actually lower than it is at the mean value of the money shock.

As indicated by the right panel, the price changes for the adjusters are less dispersed in the tails of the money shock distribution since sellers with less dispersed values of the productivity shock are induced to adjust their price when x is either very high or low. In doing so, they respond largely to the common shock to the money stock, thus reducing the dispersion in TFPR.[37]

Since real output increases with the money shock, the model implies that the standard deviation of TFPR is not a monotone function of economic activity when fluctuations are induced by money shocks. It can be lower in recessions and also lower in expansions when the money shocks take relatively extreme values. **Thus, the model can produce counter-cyclical dispersion in TFPR, for a given distribution of TFPQ, but only when money shocks are surprisingly large**.

This suggests an empirical exercise that goes beyond the traditional focus on correlations between output and the dispersion of TFPR. From this model, the effects of the money shock on output and $disp_R$ depend on whether the money shock is above or below its average value, here $x = 1$. To the extent fluctuations are driven by money shocks correlation of output and $disp_R$ ought to be positive conditioning on below average innovations and negative for above average.

[36] As discussed in sub-section 5.2, the magnitude of the effect also depends on the marginal cost of employment.

[37] So, in contrast to Vavra (2013), it seems that dispersion and frequency can move in the same direction for some monetary shocks. Though clearly a reduction in x, increases the frequency but reduces the dispersion of change.

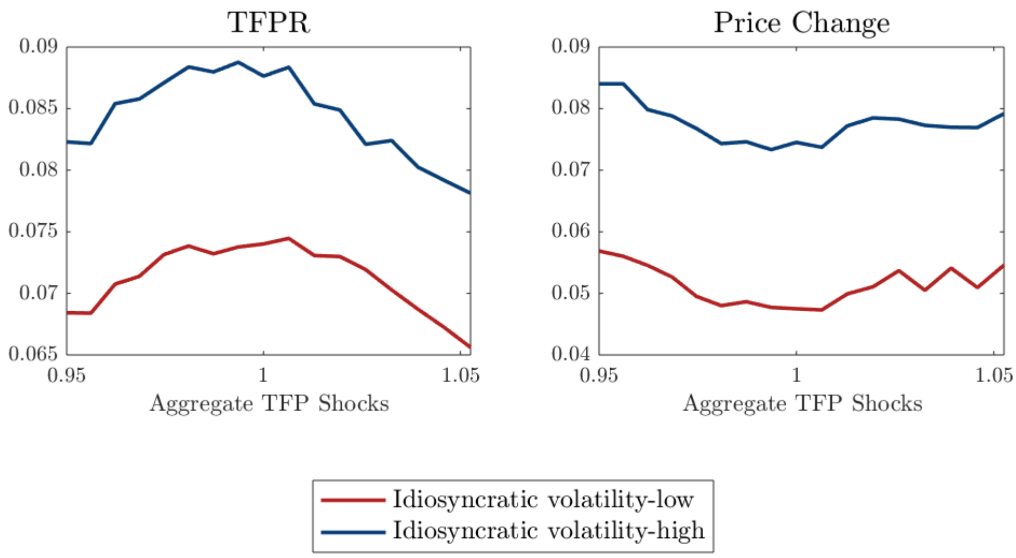
Leaving aside the nonlinearities, the basic correlations are indicated in Table 3. Here the overall correlation of output and $disp_R$ is positive, contrary to data. The model does reproduce a reduction in the dispersion of price changes during expansions. The dispersion in employment is lower in the expansion. This is because the higher frequency of price changes reduces the response of employment to productivity. Finally, the frequency of adjustment is countercyclical, but again this is a nonlinear relationship, partly masked by looked at this correlation.

4.3 Shocks to the Mean of TFPQ

The final source of variation is the more standard shock to the average productivity, i.e. the mean of TFPQ, denoted μ_Q. As before, the interest is in the cyclicality of the dispersion in TFPR, as reported in Table 3 for this case.

Figure 6 summarizes the findings. The horizontal axis measures the *ex post* realization of μ_Q. The left panel shows the dispersion in TFPR and the right indicates the dispersion in price changes. These are shown for two different levels of $disp_Q$.

Figure 6: Effects of TFPQ shocks on Dispersion of TFPR and Price Changes



This figure shows the effects of shocks to the mean of TFPQ on the standard deviations of TFPR and price changes.

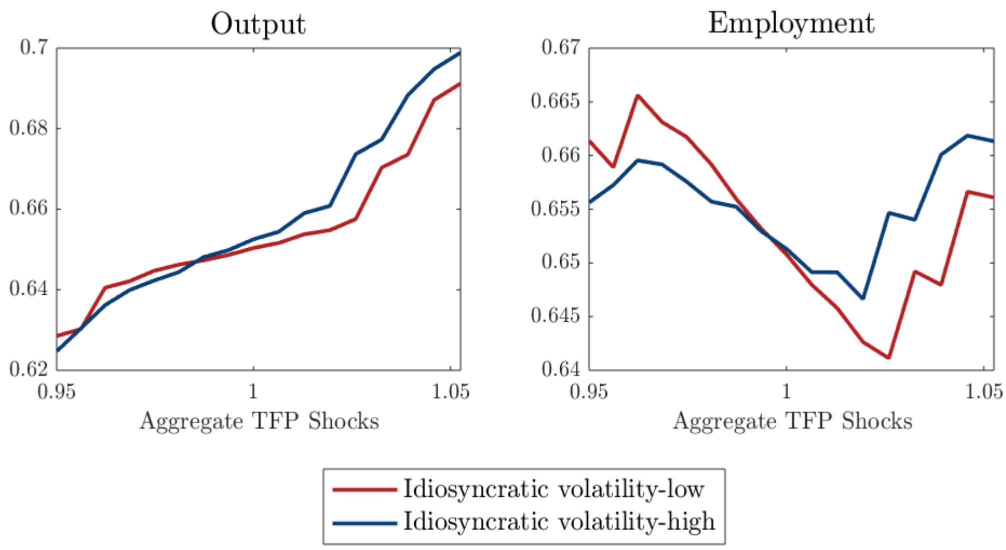
From the left panel, the relationship between μ_Q and $disp_R$ is not monotone. When the mean of TFPQ is either very large or very small, then $disp_R$ is lower. This again reflects the response of price setters. In the tails, the mean of TFPQ becomes a dominant force so that the dispersion of price changes falls. Note the asymmetry: the response of $disp_R$ is much larger for large realizations of μ_Q than for small ones. These patterns hold for both values of $disp_Q$.

As for the cyclical properties of a μ_Q shock, Figure 7 illustrates the effects of TFPQ shocks on output and employment, for two levels of TFPQ dispersion. Shocks to μ_Q are always procyclical. This is the case for both low and high dispersion in TFPQ.

Combining this with Figure 6, the relationship between output and the dispersion in TFPR is not monotone. If an economy with a relatively high aggregate TFPQ shock is compared with one with an average shock, then $disp_R$ appears to be pro-cyclical. But if the comparison is between an average economy and one in a low μ_Q state, both output and dispersion are below average.

The right side of Figure 6 returns to the point about employment and productivity in modelprice models made in Galí (1999). Except for the lowest and highest productivity states, employment is falling productivity. This arises from the assumption that producers meet demand at the posted price. Hence sellers that do not adjust their price will decrease employment in the face of a productivity shock, be it aggregate or idiosyncratic. From Figure 6, this effect is dominating for most of the aggregate productivity states. But, in the highest set of μ_Q realizations, there are enough sellers adjusting their prices in response to the aggregate shock, that employment increases with productivity.

Figure 7: μ_Q shock effects on Output and Employment



This figure shows the effects of shocks to the mean of TFPQ on output and employment.

This interpretation is supported by behavior at the producer level. Table 4 reports regression results estimated from simulated data for three experiments characterized by the type of shocks: (i) idiosyncratic productivity shocks alone, (ii) idiosyncratic and aggregate productivity shocks and (iii) idiosyncratic and aggregate productivity and monetary shocks. The dependent variable is either the (log of) producer employment or output. The independent variation is the product of the idiosyncratic and the aggregate productivity shock, as in the revenue function.

For the employment column, the negative coefficients for those sellers choosing not to adjust their price indicate the role of these rigidities on the employment response. The negative effective is present, though weaker, even when monetary shocks are in the model. For the adjusters, the effect of productivity on employment is always positive.

The same is true for the output of adjusters: output expands with either productivity or money shocks. For non-adjusters, idiosyncratic productivity shocks have no output effects since demand is given. But, if there is a positive aggregate productivity shock then the sales of non-adjusters decline. Though nominal spending is held fixed, aggregate prices are lower so that a seller not adjusting its price has a high relative price and thus lower sales. When there is a money shock as well, the overall impact is to create a positive correlation of output and productivity, even for the non-adjusters.

Returning to the evidence, there are two issues. First, the aggregate response shown in Figure 6 clearly is dependent on the fraction of each type of producer, i.e. on the adjustment rate. This will determine the size of the two regions and the slope within each. At the aggregate level, the analysis points to a very non-linear response of employment to productivity, where the magnitude of the response depends on the underlying adjustment rate which itself is state dependent.

Second, at the plant level, there is ample evidence, for example in Decker, Haltiwanger, Jarmin, and Miranda (2019) that plant-level employment is increasing in profitability. To what extent this response interacts with price setting at the plant-level remains an open question, made difficult to address by the lack of evidence on prices and employment at the micro-level.

Table 4: Dependence of Employment and Output on Productivity

Shock	Employment		Output	
	Adj.	No Adj.	Adj.	No Adj.
z	0.331	-0.656	1.007	0
(z, A)	0.472	-0.847	1.164	-0.172
(z, A, x)	0.482	-0.444	1.192	0.180

This table shows the effects of idiosyncratic,(z), aggregate productivity, (A), and monetary shocks, x, on producer-level employment and output conditioning on price adjustment status.

Overall, our findings for the case of shocks to the mean of TFPQ are summarized in Table 3. The dispersion in TFPR is indeed countercyclical, as in the data, though the magnitude is very small. This is driven by the asymmetric response of the $disp_R$ to aggregate TFPQ shocks. The dispersion in price changes in also countercyclical, but again only slightly. Finally, the frequency of price adjustment is procyclical, counter to the data. **Thus shocks to the mean of TFPQ cannot match the data patterns.**

5 Extensions and Robustness

Here we undertake some extensions of the analysis. Given that the shocks, taken alone, are not able to match data patterns, a key extension looks at active monetary policy. A discussion of robustness is included as well.

5.1 Monetary Feedback Rules

From Figures 4 and 5, the effects of changes in the dispersion of profitability shocks, $disp_Q$, on the dispersion of TFPR, $disp_R$, were clearly dependent on the money shock. For example, from Figure 4, output actually falls slightly when there is an increase in $disp_Q$ along with a large positive money shock. And, from Figure 5, the effects of an increase in $disp_Q$ on $disp_R$ were quite small for extreme values of the money shock, particularly when money growth is low.

To study these interactions further, this sub-section allows some response by the monetary authority to the shocks to the mean and dispersion of TFPQ.[38] As we see, allowing the monetary authority to link the distribution of x to the aggregate state can alter the cyclicality of $disp_R$. In this way, the implications of the model can be brought closer to some features of the data.

In particular, we highlight two cases which produce counter-cyclical $disp_R$ as well as counter-cyclical price changes. In the first, the economy is driven by fluctuations in $disp_Q$. When this dispersion is above average, the monetary authority intervenes so that the mean value of x is below average. In the second, the economy is driven by fluctuations in the mean value of TFPQ. When aggregate productivity is above average, the monetary authority intervenes and selects a mean value of x that is above average as well. In addition, following the evidence in Vavra (2013), only when there are $disp_Q$ shocks do we also produce countercyclical frequency of adjustment.

Table 5: Cyclical Variations: Monetary Feedback Rules

Case	$corr(y, disp_R)$	$corr(y, disp_{\Delta P})$	$corr(y, disp_n)$	$corr(y, freq)$
Data	-0.45	-0.41	-0.50	-0.27
		$disp_Q$		
$\zeta = 2.0$	0.2066	0.0089	-0.3909	0.1470
$\zeta = 1.3$	0.1296	-0.0334	-0.4602	0.1522
$\zeta = 0$	0.4292	-0.1348	-0.3413	-0.1695
$\zeta = -1.3$	-0.0093	-0.0146	-0.2517	0.0099
$\zeta = -2.0$	-0.0254	0.0065	-0.1257	-0.0812
		μ_Q		
$\zeta = 2.0$	-0.3044	0.0682	-0.2829	0.4294
$\zeta = 1.3$	-0.0596	-0.0868	-0.3630	0.3130
$\zeta = 0.0$	-0.0183	-0.0010	-0.3863	0.1878
$\zeta = -1.3$	0.2459	-0.0865	-0.3665	0.0320
$\zeta = -2.0$	0.2340	-0.0621	-0.3505	0.1814

This table shows the correlation between output and the dispersion of TFPR, the dispersion and frequency of price changes and the dispersion of employment for different monetary feedback rules. For the row labeled "Data", the correlations of prices changes and frequency with output come from Vavra (2013). The correlation of output and the dispersion in TFPR comes from Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) though the correlation reported there is output growth not its level. The correlation of employment dispersion and growth is from Ilut, Kehrig, and Schneider (2018).

Specifically, suppose that the evolution of the money supply is given by:

$$M_{t+1} = M_t x_{t+1} = M_t[\phi(s_{t+1}) + \tilde{x}_{t+1}]. \tag{15}$$

In this specification, the money stock follows the same stochastic process as above, with x_{t+1} representing the period $t+1$ money shock that is not predictable given period t information. But here, the growth of the money supply, $[\phi(s_{t+1}) + \tilde{x}_{t+1}]$ has two components. The first is the feedback rule where $\phi(s_{t+1})$ allows

[38] In doing so, it makes clear the advantage of using an economy with valued money to study monetary policy.

money growth to depend on the period $t+1$ state of the economy. The second is the money shock, as above denoted $\tilde{x}_{t+1}$.

Following Table 3, we focus on two specific cases, distinguished by the source of fluctuations in the aggregate economy. In the first, the monetary authority responds to changes in the dispersion of TFPQ. Let μ_{disp_Q} be the average value of $disp_Q$ and consider

$$\phi(disp_Q) = \zeta(disp_Q - \mu_{disp_Q}). \tag{16}$$

In a similar fashion, let μ_{μ_Q} be the average value of the mean of TFPQ and consider

$$\phi(\mu_Q) = \zeta(\mu_{\mu_Q} - \mu_Q). \tag{17}$$

In both formulations, the feedback is characterized by ζ.

Given a monetary feedback rule, it is straightforward to extend the analysis of a SREE from sub-section 3.1 to include (15).[39]

Note that the monetary feedback rule impacts agents both as young price setters and as old agents, both in terms of the distribution of the stochastic transfer and the equilibrium prices they face as buyers. As in the previous analysis, all of the newly created money is distributed as a proportional transfer. But in this specification, it is feasible for the monetary authority to link these transfers to the current state of the economy. If prices were perfectly flexible, there would be no real effects of this monetary policy. Further, since private agents share the information of the monetary authority, there is no information transmitted to the private sector by this policy.

The SREE was characterized for both shocks to μ_Q and $disp_Q$, allowing both negative and positive responses by the monetary authority. The results are reported in Table 7 for a couple of values of ζ. In addition, the moments calculated under the assumption of flexible prices is included for comparison. There are two cases that generate countercyclical dispersion in both TFPR and prices changes and thus match data patterns. Only one of these also creates countercyclical adjustment frequency.

Consider first the results when the economy is driven by variations in $dispQ$, along with money shocks. In this case, the only feedback rules that generate countercyclical dispersion in $dispR$ arise when the monetary authority sets $\zeta = -1.3$ and $\zeta = -2.0$. With this policy, the monetary authority responds to higher than average dispersion in idiosyncratic profitability shocks by reducing the average growth of the money supply. As output is positively correlated with $disp_Q$, the monetary authority appears to be leaning against the wind.

The mechanics are, in part, made clear by the panel in Figure 5 relating $disp_R$ to x. Since $disp_R$ is asymmetrically related to x, if the mean of the money shock (through the feedback rule) falls when the dispersion in TFPQ rises, then the dispersion in TFPR can fall. And, from Figure 4, the lower than average

[39]For example, in the case of shocks to $disp_Q$, the *ex post* pricing equation becomes

$$\hat{\varepsilon}\tilde{p}(M,z,disp_Q,x)E_{x',disp'_Q}\left(\frac{x'}{P(Mx,disp'_Q,x')}\right) = \frac{d(\tilde{p}(M,z,disp_Q,x),M,x)}{z^2} \tag{18}$$

where $x = \phi(disp_Q) + \tilde{x}$. A similar modification is needed for the *ex ante* problem.

stock of money will imply higher output when $disp_Q$ is higher. Putting the pieces together, a high value of $disp_Q$ triggers, through the feedback rule, a lower mean for the money shock so that: (i) output on average is higher, (ii) $disp_R$ is lower and (iii) the dispersion in price changes in lower as well.[40] That is, while $disp_Q$ is procyclical, $\zeta < 0$ induces, through optimal pricing behavior a countercyclical $disp_R$.

This case matches other features of the data. For $\zeta = -1.3$ the model generates countercyclical dispersion in price changes. And at $\zeta = -2.0$, the frequency of price adjustment is countercyclical as well. But none of these correlations are as large as in the data moments taken from Vavra (2013).

Note that this result does not occur without monetary feedback. As noted earlier, with $\zeta = 0$ the model does match both the countercyclical dispersion in price changes and generates countercyclical frequency of price changes but it does not create procyclical dispersion in TFPR. Further, the result clearly requires state dependent pricing. If prices are flexible, then again the dispersion in TFPR is procyclical.

The lower block studies variations in aggregate TFPQ, denoted μ_Q. If aggregate fluctuations are driven by a combination of shocks to the mean of TFPQ and monetary injections, the model is able to generate countercyclical $disp_R$ for $\zeta \geq 0$. And in some cases, the model also generates countercyclical variations in $disp_{\Delta p}$. But, in no cases is the frequency of adjustment countercyclical.

5.2 Alternative Parameters

This sub-section looks at the robustness of our findings with respect to the parameterization of the problem. There are two key parameters: (i) the elasticity of substitution between products, ε and (ii) the convexity in the disutility of work, $g(n) = \frac{n^{1+\phi}}{1+\phi}$.

The point is to understand how the shapes of these functions impact the results in Table 7. In particular, there we see that only in the presence of $disp_Q$ shocks and $\zeta < 0$ can the model reproduce the basic features of: (i) countercyclical dispersion in TFPR, (ii) countercyclical dispersion in price changes and (iii) countercyclical frequency of adjustment.

The baseline model has a quadratic disutility of work so that $\phi = 1.0$ and $\varepsilon = 3$. In the high elasticity of substitution case, $\varepsilon = 4$. With products more substitutable, a sellers whose price is, for example, very high compared to competitors will lose a lot of sales and this will create an incentive for price adjustment. So all else the same, monetary shocks will have smaller effects on output. Further, in the case of an increase in the dispersion of productivity, there will be a larger output gain since demand and therefore production is more easily reallocated to high productivity production sites.

The high labor supply elasticity sets $\phi = 0.4$. The reduction in the curvature of the disutility, towards being more linear, makes the marginal cost of production less variable. This reduces the probability of adjustment as the cost of not adjusting the price is lower. It also impacts the price chosen by adjusters.

The last two rows of Table 2 present the pricing moments for these two alternative parameterizations. Indeed, with higher product substitutability, price adjustment is more frequent and the correlation of output and the money shock is lower. The dispersion of price changes is also lower since sellers respond more to common than idiosyncratic shocks.

[40] Here the statements are on average since the feedback influences the mean of the stochastic transfer, leaving some randomness.

From Table 2 , equilibrium outcome with a lower ϕ looks more like the baseline. It is important to keep in mind that ϕ has no direct impact on the output (employment) response of sellers who do not adjust their price. These sellers, by assumption, meet demand. The lower ϕ reduces the adjustment rate since the marginal cost of meeting fluctuations demand is lower, as shown in Table 2.

A final robustness check looks at an alternative parameterization in the $disp_Q$ case. The baseline specification set the process for $disp_Q$ to match the magnitude of changes in the dispersion of TFPR reported in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) in an economy with multiple shocks, including those to the mean of TFPQ. Here we allow only shocks to the standard deviation in z as well as monetary shocks and set the distribution of the standard deviation of the idiosyncratic shocks to match the changes in the dispersion in TFPR. The result leads to more dispersion in the standard deviation of z relative to the baseline.[41]

Table 6: Cyclical Variations: Robustness

Case	$corr(y, disp_R)$	$corr(y, disp_{\Delta p})$	$corr(y, disp_n)$	$corr(y, freq)$	$E(U)$
Data	-0.45	-0.41	-0.50	-0.27	na
		Baseline			
$disp_Q, \zeta = 0$	0.4292	-0.1348	-0.3413	-0.1695	0.4347
$disp_Q, \zeta = -1.3$	-0.0093	-0.0146	-0.2517	0.0099	0.4385
$mu_Q, \zeta = 0.0$	-0.0183	-0.0010	-0.3863	0.1878	0.4301
$mu_Q, \zeta = 2.0$	-0.3044	0.0682	-0.2829	0.4294	0.4284
		High Product Substitutability			
$disp_Q, \zeta = 0$	0.2020	0.0412	-0.2066	0.0822	0.4619
$disp_Q, \zeta = -1.3$	0.0842	0.0812	-0.1555	0.1272	0.4579
$mu_Q, \zeta = 0$	0.1656	-0.2432	-0.4949	0.2213	0.4570
$mu_Q, \zeta = 2.0$	-0.1669	-0.3750	-0.5912	0.5640	0.4542
		High Labor Supply Elasticity			
$disp_Q, \zeta = 0$	0.0348	0.0889	-0.2521	0.1785	0.3719
$disp_Q, \zeta = -1.3$	-0.1156	0.0258	-0.2493	0.0161	0.3728
$mu_Q, \zeta = 0$	0.0770	-0.0124	-0.2388	0.0667	0.3669
$mu_Q, \zeta = 2.0$	-0.2111	-0.1221	-0.4239	0.3641	0.3631
$mu_Q, \zeta = 2.0$	-0.5503	-0.2227	-0.5148	0.6877	0.4268
		Larger $disp_Q$ Variation			
$disp_Q, \zeta = 1.3$	0.3871	0.3855	0.4300	0.4417	0.4337
$disp_Q, \zeta = 0$	-0.0065	0.0274	0.0521	0.1142	0.4376
$disp_Q, \zeta = -1.3$	-0.4568	-0.4300	-0.4232	-0.3693	1.5218
		$disp_Q, \mu_Q$			
Baseline					
$disp_Q, \zeta = 0$	-0.1945	-0.2621	-0.4789	0.1378	0.4302
$disp_Q, \zeta = -1.3$	0.0181	-0.4690	-0.5616	-0.1323	0.4318
Larger $disp_Q$ Variation					
$disp_Q, \zeta = 0$	-0.6740	-0.6255	-0.8448	0.8450	0.4279
$disp_Q, \zeta = -1.3$	-0.7807	-0.6856	-0.6101	-0.5894	0.4492

This table shows moments for different monetary policy rules at the baseline and alternative parameters parameter values. Here E(U) is expected utility.

Table 6 shows the resulting patterns of correlations for these cases. The top panel reproduces the patterns

[41]Specifically, the calibrated values of TFPQ dispersion are $(0.1126, 0.1283, 0.1455)$ in the baseline. For the robustness check, the values are $(0.0571, 0.1284, 0.2111)$.

Table 7: Correlation between x and $disp_R$

Case	$corr(x, disp_R)$
Data	0.045
$disp_Q$	
Baseline	
$\zeta = 1.3$	0.8534
$\zeta = 0$	0.1083
$\zeta = -1.3$	-0.7367
Larger $disp_Q$ Variation	
$\zeta = 1.3$	0.3857
$\zeta = 0$	-0.0257
$\zeta = -1.3$	-0.4790
$disp_Q, \mu_Q$	
Baseline	
$\zeta = 1.3$	0.8361
$\zeta = 0$	0.0765
$\zeta = -1.3$	-0.6732
Larger $disp_Q$ Variation	
$\zeta = 1.3$	0.0796
$\zeta = 0$	-0.3834
$\zeta = -1.3$	-0.5993

from the baseline parameters and the others are the three experiments.

In terms of fitting the moments, the increased competitiveness through higher product substitutability never produces countercyclical adjustment frequencies. Only when there are shocks to the mean of TFPQ and $\zeta = 2$ is the dispersion of TFPR countercyclical. In this case, so is the dispersion of price changes.

With a high labor supply elasticity, the model produces countercyclical dispersion in TFPR for both $disp_Q$. In some of these cases, the dispersion of price changes is also countercyclical but the frequency remains procyclical throughout.

For the final specification with higher dispersion in the standard deviation of z, the model seems to match the moments. For $\zeta = -1.3$, all of the key variables are countercyclical, as is the dispersion of employment. Given this "success" it is worth exploring the welfare associated and the conduct of monetary policy associated with this case.

The column labeled "E(U)" calculates expected utility of a representative agent in a SREE with the parameterization given in the table. Expected utility varies with the specification on utility parameters, particularly the labor supply elasticity, as well as the monetary feedback rule.

Looking specifically at the large $disp_Q$ variation case, E(U) slightly lower when $\zeta < 0$ compared to the case of no intervention. In this sense there is no welfare gain from this intervention.

Table 7 displays the correlation between the money shock, x, and the dispersion of TFPR, $disp_R$. Here the focus in on $disp_R$ rather than $disp_Q$ as the former is observed.

For Table 7, the data row is calculated as follows. First, x refers to the monetary policy shocks proxied with the high frequency/narrative shock series of Miranda-Agrippino and Ricco (2018). This series proxies for the changes in monetary policy which are not captured by the endogenous component reacting to the output or inflation gap. Second, following Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), Decker, Haltiwanger, Jarmin, and Miranda (2019) we identify TFPR shocks using a cost-share approach at the firm level. We calculate $disp_R$ at the sectoral level by year and obtain annual $disp_R$ by averaging over the sectoral dispersion each year. Finally, the correlation between x and $disp_R$ is calculated.

For the model, when the feedback rule entails $\zeta < 0$ to that money innovations are negatively correlated with $disp_Q$. As indicated in the two bottom panels of the table, this pattern is sustained in looking at the correlation of x, and $disp_R$. Notably, in the case of the larger variation in $disp_Q$, the correlation is strongly negative and far from the data. In this sense, the monetary policy that generates moments qualitatively matching the data in Table 6, are far from the monetary innovations seen in the data.

5.3 Correlated Shocks

In many studies, such as Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018) and Vavra (2013) the shock to dispersion and to the mean of TFPQ are studied jointly. Here we follow the baseline model in Vavra (2013) and assume the shocks are perfectly negatively correlated: $corr(disp_Q, \mu_Q) = -1$.

The moments from this exercise are shown in the bottom panel of Table 6. There are two cases. One in which both $disp_Q$ and μ_Q took on the same three values as in the baseline model and using the values for $disp_Q$ from the "Larger $disp_Q$ variation" parameterization. For each, we allow two values of ζ.

Of the cases explored, the one that most closely matches the four data moments occur when the variation in $disp_Q$ is large and, once again, $\zeta < 0$. In the case the correlations are more strongly negative compared to the model with large $disp_Q$ variations alone. When $\zeta = 0$, the models do not generate countercyclical variation in the frequency of price adjustment.

But again, looking at Table 7, the monetary policy needed to match these moments is again counter to the data. That is, with $\zeta < 0$, the correlation of the monetary innovation and $disp_R$ is negative.

6 Other Implications

This section looks at other implications of the model. The first is the interaction between the dispersion of productivity shocks and the impact of monetary policy. The second is the effects of uncertainty rather than dispersion on pricing.

6.1 Real Dispersion and the Effects of Monetary Policy

This section continues to study the interaction between money shocks and TFPR dispersion. But instead of asking whether money shocks can create TFPR dispersion, here we study how the real effects of money depends on TFPQ dispersion.

There are two important empirical findings that guide this discussion. First, Vavra (2013) argues that the dispersion of price changes is counter-cyclical as is the frequency of price adjustment. Second, Tenreyro and Thwaites (2016), output is less responsive to monetary policy during recessions.

Putting these pieces together, if TFPR dispersion is counter-cyclical than recessions are associated with more frequent price adjustment and thus a smaller impact of monetary policy. This leads Vavra (2013) to argue that shocks to nominal spending will have a smaller effect on output when the dispersion of firm level productivity is higher.

We use our model, with its explicit distinction between TFPQ and TFPR, to study the effects of monetary shocks. The question is whether the real impact of these shocks is lower when $disp_R$ is higher, given that this dispersion is endogenous.[42]

As we have already seen, changes in the distribution of z will influence price setting and thus will the impact of monetary policy in a SREE. Intuitively, more variability in the distribution of z implies that price adjustment, given a monetary shock, is more likely and thus the real effects of the monetary shock will be reduced. This is a variant of the point made by Vavra (2013).

This is illustrated in Figure 5 which compares low and high dispersion cases. Here the focus is on the response of real output and prices to x, rather than on the shifts in these curves due to variations in TFPQ dispersion. Clearly the frequency of adjustment is higher when the dispersion of z is higher. From the right diagram, prices are more responsive to monetary shocks in the high uncertainty case so that output, left bottom, is less responsive.

Table 8 quantifies the effects of changes in dispersion on the response of output to a monetary innovation. It does so by regressing the log of real GDP on the (log of the) monetary shock. From that table, the response of output to a monetary innovation is 9% points higher in the low idiosyncratic uncertainty case. The bottom part of Table 8 shows the complementary effects of money shocks on prices.

Table 8: Regression of output and prices on log(x)

	Constant	Slope
Output Response		
Low $disp_Q$	-0.4298	0.8866
High $disp_Q$	-0.4381	0.7993
Price Response		
Low $disp_Q$	0.4303	0.1167
High $disp_Q$	0.4392	0.2087

Does this analysis support the finding of Tenreyro and Thwaites (2016) on the cyclical effectiveness of monetary policy? It would iff recessions were associated with large dispersion in TFPR. But, as discussed above, the sources of aggregate fluctuations studied here, particularly variations in $disp_Q$, do not generate counter-cyclical dispersion in TPPR.

6.2 Effects of Uncertainty

The distinction between uncertainty and dispersion is often blurred. The main effect of uncertainty, again expressed in Bloom, Floetotto, Jaimovich, Eksten, and Terry (2018), is to create an incentive to wait and

[42] In contrast to Vavra (2013), we do so in a model with exogeneous variations in money rather than nominal spending "shocks".

allow the uncertainty to be resolved. To the extent this leads to a decrease in spending, largely on durables, the uncertainty can be recessionary. This is often quite different from the positive effects of dispersion which can lead to an expansion in output, as discussed above.

The previous discussion highlighted the effects of dispersion on the frequency of price adjustment and thus the real effects of monetary shocks. Here we focus on how *ex ante* price and *ex post* respond to uncertainty over a distribution, not the realization of that change.

Our analysis includes distributions over three dimensions: (i) idiosyncratic productivity, (ii) money transfers and (iii) aggregate productivity. Thus in principle one can study the effects of uncertainty with respect to each of these three distributions.

To do so, it is natural to create a Markov switching process for the dispersion of, say, idiosyncratic productivity. Price setters in period t would know the distribution of these shocks last period but in setting their *ex ante* price, the period t distribution, as well as that for period $t+1$ would not be known. Further, for those who adjust *ex post*, the uncertainty would remain over the distribution in the following period when they are consumers.[43] This is the nature of the uncertainty.

One extreme version of this Markow switching process is for the dispersion to be permanently high (low). It turns out that for the price setting problem of young agents, the *ex ante* price is essentially the same with high dispersion of the idiosyncratic productivity shock as it is for the low dispersion case. In fact, this is true when the uncertainty is over the money transfer or the aggregate productivity distributions.

Given this, it is unlikely that *ex ante* uncertainty matters for the price setting problem. This is verified explicitly for the case of uncertainty over idiosyncratic productivity. Even if there is a positive probability of a regime shift in the distribution of z, the *ex ante* price is essentially unchanged.

This is an important finding. It makes clear that the effects come from dispersion not uncertainty. This is consistent with Berger, Dew-Becker, and Giglio (2020) who argue, at least for aggregate shocks, that uncertainty *per se*, had a negligible effect on real activity.

7 Conclusion

This paper studies the factors that determine the distribution of TFPR in an economy with sticky prices and firm specific productivity shocks. It characterizes and studies the properties of TFPR in a stationary rational expectations equilibrium.

Using this framework, the model is used to determine the cyclicality of the dispersion in TFPR as well as other key pricing moments, the cyclicality of both the frequency of price changes and their dispersion. This is studied by determining pricing decisions and thus the distribution of TFPR in the face of aggregate shocks to: (i) the dispersion of TFPQ, (ii) the money supply, (iii) the mean of TFPQ. These are very conventional shocks for an aggregate economy, with recent attention given to variations in the dispersion of TFPQ. The quantitative analysis is based upon the stationary rational expectations equilibrium of the model economy for given variations in exogenous variables.

[43] Thus the expectation on the left side of (11) is extended to include the conditional expectation over the future dispersion.

The findings are not supportive of the view that variations in the dispersion of TFPQ drive countercyclical variations in TFPR. As indicated in the analysis, this can only arise from a particular form of monetary intervention: the money supply innovation must be negatively correlated with variations in the dispersion of TFPQ. This is the case even when the dispersion shock is accompanied with an offsetting change in the mean of TFPQ. Absent such monetary interventions, dispersion in TFPQ and TFPR are procyclical, reflecting the gains to reallocation associated with increased productivity dispersion.

Focusing on the case in which the model can reproduce both countercyclical TFPR and match pricing patterns, the paper provides evidence that the resulting monetary policy is not consistent with the data. In particular, the correlation between monetary innovations and the dispersion in TFPR is slightly positive. But, in order to match moments, the model requires this correlation to be quite negative.

Admittedly these results are suggestive rather than definitive. The OG model, with only one period of price setting, misses some of the forward looking aspect of price adjustment. But, as argued in the text, the pricing behavior in the model is similar to that produced by other state dependent pricing models. On the data side, it would be desirable to have higher frequency observations on both prices and quantities upon which to base a structural estimation exercise.

Throughout these exercises, one theme emerges: non-linearities in the response of $disp_R$ to shocks. Regardless of the source of aggregate fluctuations, $disp_R$ is generally lowest for extremely low and high realizations and highest for the average state. This property of the model, driven by the U-shaped response of the frequency of price changes to money surprises, makes it useful to study the impact of monetary and productivity shocks using non-linear statistical models.

Finally, the model is used to study the effects of uncertainty on pricing. It seems clear that the effects highlighted in our analysis stem from dispersion not uncertainty. One interesting extension of our model would be to include some of the adjustment cost structure that creates a real options effect, as in Bloom (2009), **coupled with** state dependent pricing.

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