

# Intangible Economies of Scope: Micro Evidence and Macro Implications\*

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## Abstract

Do economies of scope within firms transmit macro shocks across industries? I exploit exogenous variation in foreign demand faced by US multi-industry manufacturers to identify this mechanism. I find that a positive demand shock in one industry of a firm increases its sales in another only when both industries use the same *intangible* inputs. I develop a general equilibrium model of multi-industry firms and estimate that scope economies are driven by the scalability and non-rivalry of intangible inputs under joint production. Cross-industry spillovers due to scope economies account for 20 percent of the equilibrium response of productivity to market size. Applied to US trade, the model predicts large productivity spillovers from industry shocks, particularly across industries that use more intangible inputs.

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# 1 Introduction

Multi-industry firms account for three-quarters of US manufacturing output. Is a firm like General Electric simply a collection of independent industry segments, or do shared inputs generate economies of scope? The existing literature offers competing theories but no empirical consensus. When inputs (e.g. management) are rival and constrained within the firm, growth in one industry comes at the expense of others (Lucas, 1978). On the other hand, when inputs (e.g. scientists) are non-rival and scalable, growth in one industry might breed success in another (Penrose, 1959). In the absence of microeconomic evidence, quantitative models of multi-industry firms have remained ambivalent on how these intra-firm mechanisms shape an economy's response to aggregate shocks.

This paper provides empirical evidence of scope economies within the firm and develops a model to find that these internal effects generate quantitatively large aggregate spillovers across industries. I begin by examining how a firm's output in one industry is affected by demand shocks it receives in other industries. To do so, I assemble panel data from the US Economic Census on the sales and exports by industry of all US manufacturing firms. I leverage variation in a firm's exports across foreign destinations and products along with changes in the size of these markets to construct plausibly exogenous shifters of demand for each industry of the firm.<sup>1</sup>

I find that a positive demand shock in one industry of a firm increases its sales in another industry only when both industries use the same *intangible* inputs in production. I use the BEA's input-output (I/O) and capital flow tables to infer industry-level expenditure shares on intangible inputs—comprised of professional services (e.g. R&D, engineering, management), information (e.g. IT and software), and the leasing of intangible assets.<sup>2</sup> Unlike intangibles, other inputs do not generate spillovers across the industries that use them in common. These findings suggest that intangible inputs have distinct economic properties, consistent with evidence on their transfer across plants within the firm (Atalay et al., 2014) and their synergy and scalability (Haskel and Westlake, 2017).<sup>3</sup>

To rationalize these reduced-form spillovers, I develop a quantitative general equilibrium model of multi-industry firms. I introduce two elasticities that enrich properties of variable

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<sup>1</sup>Existing papers have used this shift-share identification strategy to construct demand shocks at the firm-level, using data from countries such as Denmark (Hummels et al., 2014), France (Mayer et al., 2016), and Portugal (Garín and Silverio, 2018). In my paper, the scale and scope of US exporters allows demand shocks to vary across industries *within* the firm.

<sup>2</sup>See Table 7 for a full list. The data includes both capitalized and current expenses. McGrattan (2017) uses a similar classification and the same data to estimate a multi-sector RBC model with intangible capital.

<sup>3</sup>Intangibles are also important in the aggregate economy. Almost 10 percent of gross manufacturing output is expensed on intangibles in 1997. Corrado and Hulten (2010) estimate that total investments in intangibles in the U.S. exceed 11% of GDP. Many intangibles are tradable services that magnify US international exposure (Gervais and Jensen, 2019) and regional wage inequality (Eckert, 2019; Eckert et al., 2019).

inputs under joint production: (i) how *scalable* are inputs within the firm, and (ii) how *rival* are their contributions across industries. These elasticities vary across inputs and determine the direction and strength of economies of scope (and scale).<sup>4</sup> My joint production framework generalizes benchmark models of heterogeneous firms (Melitz, 2003), nesting constant returns to scale and independence across industries (Bernard et al., 2010) as special cases.

I model input scalability and rivalry in a setting that reflects the uncertainty inherent in knowledge creation.<sup>5</sup> Consider an input such as ceramics scientists, who generate valuable research ideas within General Electric. GE deploys each idea in the industry where it serves the highest value. Gemstone scintillators are deployed in X-ray machines to enable high sensitivity scanning, whereas SiC-SiC ceramic matrix composites are deployed in aviation turbines to improve their hot gas path.<sup>6</sup> But *ex-ante*, the contribution of ceramics scientists is uncertain in terms of both the number of ideas that surface (a Poisson process) and the match-specific quality between an idea and an industry (a Fréchet random variable). Scalability is measured by the effectiveness of additional scientists towards increasing the arrival rate of ideas. Non-rivalry is captured by the variance in the match-specific qualities between each idea and industry. The higher is the variance, the larger is the *ex-ante* expected contribution of ceramics scientists, and the more likely are their research ideas to be deployed *ex-post* across different industries.<sup>7</sup>

Economies of scope arise whenever inputs in production are more scalable than rival. An increase in demand in one industry of the firm generates two competing effects on its sales in another. The more rival are inputs, the more the firm substitutes production towards the shocked industry, lowering output in others. The more scalable are inputs, the more other industries stand to benefit. These properties of inputs determine an empirically observable matrix of cross-industry elasticities of sales to demand shocks. Because industries differ in input requirements, scope economies (and thus cross-industry spillovers) might be positive for some pairs of industries while negative for others.

I leverage this mapping between the model and the data to structurally estimate scale and

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<sup>4</sup>Whereas other papers estimate properties of specific intangible inputs in isolation—ranging from R&D (Aw et al., 2011), marketing (Arkolakis, 2010), management (Bloom et al., 2019), to ICT (Fort, 2016; Lashkari et al., 2019)—my quantitative framework accounts jointly for the role of all production inputs in all industries and aggregates to match BEA industry-by-input expenditures. These other papers focus on scalability of the particular intangible input but do not consider their rivalry in use under joint production.

<sup>5</sup>This is inspired by the seminal papers on firm resources and diversification (Penrose, 1959; Gort, 1962) and a concurrent literature in management (Teece, 1982; Wernerfelt, 1984; Chandler, 1990). Griliches (1979) and Spence (1984) provide early models on knowledge capital and cost reduction that have been used to study diversification (Jovanovic, 1993) and R&D spillovers (Klette, 1996) within the firm.

<sup>6</sup>Source: <https://www.ge.com/research/technology-domains/materials/ceramics>

<sup>7</sup>There is *ex-ante* option value embedded in the firm's *ex-post* ability to deploy an idea to the best industry among competing uses. This option value increases with variance.

rivalry elasticities of inputs, using demand shocks as instruments.<sup>8</sup> While the identification insight is more general, I estimate a parsimonious set of elasticities for lack of statistical and computational power. I assume that *generic* inputs—including agriculture, manufacturing, and labor—are acquired at constant marginal costs and completely rival across industries.<sup>9</sup> I estimate one pair of scale and rivalry elasticities common to all intangible inputs, and another pair common to the set of residual inputs in the I/O tables. The elasticities on residual inputs reflect potential spillovers due to competing mechanisms, such as financial constraints and span-of-control.

I use simulated method of moments to estimate input elasticities that match the same within-firm, cross-industry covariances of sales growth and demand shocks in the data. Absent firm-industry level data on input use, a critical (but sufficient) source of variation comes from BEA data on industry-level input expenditures. Variation in these expenditures reflect differences in technology coefficients (common to all firms) in the model. I jointly estimate these coefficients and use their variation across input-industry pairs to differentiate elasticities of intangible inputs from residual inputs. For example, a more positive response of aviation turbine sales to X-ray machine demand suggests that inputs more intensively used in the two industries (e.g. intangibles) are more scalable and less rival compared to other inputs. I also use variation in own-industry and cross-industry responses to shocks to differentiate scale from rivalry elasticities.

Estimates suggest that intangible inputs are more scalable than rival and thus generate economies of scope. In contrast, residual inputs are mildly less scalable than rival. These elasticities are statistically significant, precisely estimated, and consistent with the reduced-form evidence. Despite its parsimony, the estimated model replicates cross-sectional moments not targeted in estimation, such as the firm scope distribution and joint production patterns across pairs of industries.

Finally, I use the estimated model to quantify the extent to which internal economies of scope affect the aggregate consequences of industry demand shocks. I exploit the model’s analytical tractability to express the elasticity of productivity and output to arbitrary industry shocks in terms of simple propagation matrices. These matrices permit a decomposition of the aggregate productivity response into changes that occur as a result of scale economies (own-industry effects) versus scope economies (cross-industry effects). Given the

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<sup>8</sup>Despite the simplicity and minimal data requirements, the use of demand shifters to identify economies of scope has not been implemented, at least in the context of US manufacturing. Most papers estimate economies of scope by appealing either to cost concepts developed in [Baumol et al. \(1982\)](#) or the input distance function in [Färe and Primont \(1995\)](#), and usually do so only for a handful of products at a time, e.g. agricultural cooperatives ([Pokharel and Featherstone, 2019](#)).

<sup>9</sup>This assumption corresponds to the knife-edge case of generic inputs being completely scalable and completely non-rival and is consistent with standard models and the reduced-form evidence.

unrestricted nature of production elasticities, the model nests as special cases the macro responses in quantitative multi-industry Ricardian models ([Costinot and Rodríguez-Clare, 2014](#)) and their extensions that feature variable (own-)industry returns to scale ([Kucheryavyy et al., 2019](#); [Bartelme et al., 2019](#)).

I find quantitatively large aggregate productivity spillovers from scope economies. Under a calibration of the model to the US data in 2017, I examine the impact of foreign demand shocks on output and productivity when internal economies of scale and scope are the only general equilibrium forces. I estimate that scope economies contribute to an aggregate elasticity of productivity to output of 0.04, more than 20% of the total productivity response. The model can be easily extended to settings where spillovers are also driven by external interactions among firms. Under an alternative production structure with input-output linkages ([Caliendo and Parro, 2014](#)), I find that introducing economies of scope more than doubles the response of productivity to foreign demand shocks.

These aggregate findings mask significant heterogeneity. Industries that use more intangible inputs are stronger transmitters and beneficiaries of productivity spillovers. For example, cross-industry spillovers account for 60% of the total productivity response when demand in the electromedical apparatus industry rises. This occurs because firms respond by scaling up intangible inputs. Non-rivalry leads to productivity improvements in other industries of these firms that use the same intangible inputs. But for an industry like flavoring syrup that relies less on intangible inputs and more on residual inputs (like tangible capital), I find that spillovers can even be reversed due to diseconomies of scope, accounting for negative 6% of the total productivity response.

As a proof of concept, I show that these microeconomic properties of joint production change our assessment of the price and productivity impacts of US trade policy. I analyze the impact on the US economy of unilateral tariffs of 20% on all Chinese imports. While import protection raises the prices of foreign goods, it also lowers prices of domestic goods through scale and scope economies induced by greater domestic market access. I estimate that the tariff policy would raise the US manufacturing CPI by 0.8% in my model, compared to 1.12% absent the response of domestic prices. I also use the model's propagation matrix to construct alternative tariffs that achieve the same reduction in US imports from China (41%) but more than halve the adverse CPI effects (to 0.38%) by biasing protection towards industries with stronger internal economies of scale and scope.

My paper contributes to a growing body of research on multi-product firms. In benchmark models that explain product switching and reallocation in the data ([Klette and Kortum, 2004](#); [Bernard et al., 2010](#); [Mayer et al., 2014](#)), firms emerge with a portfolio of independent

industries and products.<sup>10</sup> Other papers generate interdependencies across products using features that range from cannibalization (Eckel and Neary, 2009; Feenstra and Ma, 2007), innovation (Dhingra, 2013), span-of-control (Nocke and Yeaple, 2014), to carry-along trade (Bernard et al., 2018).<sup>11</sup> In contrast, I propose and quantify a different mechanism: the scalability and non-rivalry of intangible inputs generate economies of scope.<sup>12</sup> Whereas most models of multi-product firms are solved *within* an industry equilibrium and feature one-directional (zero or negative) spillovers, I develop a multi-industry general equilibrium framework where both within-firm and aggregate spillovers are unrestricted.

The concept of non-rivalry in my model takes inspiration from the theory of the multinational enterprise (Helpman, 1984; Markusen, 1984), in which headquarters provides non-rival, intangible services to other plants of the firm. An empirical strand of this literature quantifies knowledge spillovers between parent and affiliate plants of multinationals (Keller and Yeaple, 2009, 2013; Cravino and Levchenko, 2016), but does not study the cross-industry dimension.<sup>13</sup> Despite the difference in application, my estimates of spillovers within the multi-industry firm are close to the parent-affiliate spillovers (of roughly 20%) estimated in Bilir and Morales (2019), and also consistent with the positive relationship between domestic and foreign operations of US multinationals (Desai et al., 2009). Since most multinationals also operate in multiple industries, the results in my paper suggest that much of multinational knowledge spillovers could also be generating economies of scope.

Lastly, the quantitative results in my paper complement several literatures that focus on *external* interactions as the source of industry spillovers. This is true of papers in macroeconomics that study the propagation of shocks across production networks (Gabaix, 2011; Acemoglu et al., 2012, 2016; di Giovanni et al., 2018; Baqaee and Farhi, 2019; Liu, 2019; Lim, 2018), as well as research on agglomeration externalities (Ellison et al., 2010) and innovation spillovers (Bloom et al., 2013). My paper offers the requisite microeconomic evidence to distinguish internal effects from external effects, revealing that *intra*-firm economies also shape the macro propagation of shocks.

The rest of the paper is structured as follows. Section 2 describes the data and provides reduced-form evidence of scope economies. Section 3 develops a quantitative model of joint

<sup>10</sup>The assumption of independent production functions (i.e. separable across industries) also appears in multi-product productivity estimation such as De Loecker et al. (2016) and Orr (2019). Other papers in the IO literature do consider joint production functions but often do so with either a few industries at a time or in highly parametrized settings. For example, the Cobb-Douglas functional form (Grieco and McDevitt, 2016; Dhyne et al., 2017) presupposes diseconomies of scope.

<sup>11</sup>See also, among others, Arkolakis et al. (2019) and Macedoni and Xu (2019).

<sup>12</sup>In a contemporaneous paper, Boehm et al. (2019b) study the role of *tangible* intermediate input linkages for economies of scope. They focus instead on the entry margin using data from India.

<sup>13</sup>A recent paper by Boehm et al. (2019a) studies multinational spillovers arising from vertical relationships. Other papers relate intra-firm spillovers to internal capital markets (Lamont, 1997; Giroud and Mueller, 2019), plant-level shared resources (Borusyak and Okubo, 2016), and distance (Giroud, 2013; Gumpert et al., 2019).



production. Section 4 structurally estimates the model, and Section 5 uses these estimates to quantify aggregate industry spillovers. Section 6 concludes.

## 2 Empirical Evidence on Economies of Scope

This section provides empirical evidence of scope economies in US manufacturing firms. I construct an exhaustive dataset on their sales and exports by industry, and develop an identification strategy that exploits plausibly exogenous changes to foreign market size as shifters of demand specific to an industry within the firm. I find that a demand shock in one industry of a firm increases its sales in another industry only when both industries use the same intangible inputs.

### 2.1 Data and Descriptive Evidence

I construct a firm-industry-level panel dataset containing the universe of US manufacturing firms from 1997 to 2012. These data are assembled by matching, at the firm-industry level, production data from the quinquennial US Economic Census (EC) with customs data from the annual Longitudinal Foreign Trade Transaction Database (LFTTD). Product trailer files in the Census of Manufactures (part of the EC) contain data on sales across detailed product lines within each establishment of a firm. I construct firm sales in industry  $j$  in year  $t$ ,  $X_{fjt}$ , by summing up shipments of all products that fall within industry  $j$  over all plants owned by the firm.<sup>14</sup>

I define an industry,  $j$ , at the maximal level of disaggregation that permits concordability across census, customs, and BEA taxonomies over this time period.<sup>15</sup> This yields 206 industries. I refer to this classification (roughly at the level of 5-digit NAICS) as ‘BEAX’ and work with this taxonomy throughout the rest of the paper. This is the most disaggregated level at which input-output flows (across industries) are available in BEA data, which is crucial for constructing variation on input use by industry. This definition of industries also allows the quantitative analysis to connect directly to existing models featuring input-output linkages.

Table 1 summarizes the span of US manufacturing activity attributable to multi-industry firms. These firms are important across multiple dimensions, suggesting that reallocation within these firms may have first-order effects on industry-level changes in the economy.<sup>16</sup>

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<sup>14</sup>I abstract from the plant dimension and focus on the industry boundaries within the firm. Any single establishment firm that sells products spanning multiple industries is defined as a multi-industry firm. Instances of such firms are non-trivial and account for more than 5% of gross manufacturing output.

<sup>15</sup>This ends up generating a slight coarsening of the classification in the BEA Input-Output Use tables. More details on the match and construction of this concordance can be found in the Data Appendix.

<sup>16</sup>Bernard et al. (2010) provide a more detailed overview using the same data up to 1997.

Table 1: Statistics on Multi-Industry Manufacturing Firms

	1997	2002	2007	2012
<i>Share of aggregate outcome</i>				
Manuf. sales	.75	.75	.75	.76
by primary industry	.46	.49	.51	.52
by secondary industries	.29	.26	.24	.24
External manuf. sales	.74	.74	.74	.75
Misclassification-prone sales	.09	.08	.07	.07
Manuf. employment	.62	.63	.60	.60
Exports	.84	.80	.81	.76
Imports	.82	.79	.79	.77
Firms	.19	.20	.20	.20
<i>Mean and median scope</i>				
Mean number of industries	2.69	2.73	2.63	2.65
Median number of industries	2	2	2	2
Mean number of sectors	1.69	1.74	1.69	1.70
Median number of sectors	2	2	2	2

Source: US Economic Census. Multi-industry firms are firms producing manufacturing products that fall in at least two distinct industry classifications. The definition of an industry is at a hand-constructed BEAX level (of which there are 206)—roughly corresponding to 5-digit NAICS. External manuf. sales is equal to the firm's gross manufacturing sales less its total inter-plant shipments reported. Misclassification-prone refers to the instances where sales would be misattributed by an analyst who only has data at the plant level and attributes all sales of the plant to the plant's industry classification. Sectors refer to 3-digit NAICS codes (of which there are 21).



One-fifth of all US manufacturers operate in two or more industries. These firms account for more than three quarters of manufacturing sales, exports, imports, and a slightly lower share of employment. The second and third lines of the table show that sales within these firms are not too skewed towards their primary (single highest grossing) industry. The primary industry of each firm accounts for only roughly two-thirds of the firm's total sales. The remaining industries (just a single one for the median multi-industry firm) are classified under 'secondary' and account for one quarter of total gross output.

I find that the use of product-trailer information to apportion plant-level shipments across industries makes a quantitative difference for the aggregate accounting of industry sales. The fifth row of Table 1 reports that between 7 and 9 percent of total manufacturing sales would be misclassified (by industry) if a data analyst attributes the sales of each plant to the plant's main industry (which, in many datasets, is the only degree of disaggregation available).<sup>17</sup>

A classic explanation for the firm boundary is that firms integrate vertically related industries to avoid holdup and contracting inefficiencies. Consistent with [Atalay et al. \(2014\)](#), I find that within-firm shipments of goods are a small fraction of firms' overall sales and thus do not appear to explain firm size and scope. I define external manufacturing sales as the total sales less inter-plant shipments (among plants within the firm), and find that these external sales of multi-industry firms still account for 74 percent of gross output in manufacturing. Still, to mitigate concerns that intra-firm (inter-plant) shipments might respond at the margin and drive cross-industry spillovers, I use *external* sales as the main empirical variable in the remainder of the paper.

Under my industry classification, the mean scope among multi-industry firms is only 2.7 and stays stable over the years. This broad classification of what is an industry sets my paper apart from others that emphasize the multi-*product* margin of the firm. I abstract from the product dimension within an industry (other than to use as variation in the identification strategy). This alleviates concerns that demand-side substitution and cannibalization effects might be driving observational spillovers.<sup>18</sup>

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<sup>17</sup>This detail is important for my identification strategy and my findings of spillovers. I do not find statistically significant spillovers when I ignore the product dimension within the firm. This statistic is also an underestimate of the total degree of misclassification in the data, since many single-unit firms do not have their sales broken down in the product trailers. See the Data Appendix for more information.

<sup>18</sup>Interestingly, while the median multi-industry firm operates in two industries, those two industries also span multiple *sectors* (defined as 3-digit BEAX, of which there are 27). To the extent that demand complementarities and input-output relationships are stronger within a sector rather than across sectors, this fact suggests that neither explanation may be prominent for the median firm.

## 2.2 Identifying Economies of Scope

One explanation for the empirical size advantage of multi-industry firms is that they derive cost savings in a given industry's production from their scale of output in another industry. Denote a cost function dual to a firm's production technology as  $C(q_1, \dots, q_J; \vec{w})$ , where  $q_j$  describes output in industry  $j \in \{1, \dots, J\}$ , and  $\vec{w}$  stands for a vector of input prices the firm takes as given.<sup>19</sup> I define economies of scope as follows:

**Definition 1 (Economies of Scope)** *There are local economies of scope in the production of  $j$  and  $k$  if marginal costs of producing  $q_j$  is falling with  $q_k$  around the current output vector  $\vec{q}$ :*

$$C_{jk}(\vec{q}) \equiv \frac{\partial^2 C(\vec{q})}{\partial q_j \partial q_k} < 0.$$

This definition differs from the traditional one—sub-additivity of the total cost function—developed by Baumol et al. (1982).<sup>20</sup> In their setting, economies of scope refer to cost savings when production in a set of industries occurs jointly instead of separately over a given partition of that set. Instead, by focusing on local changes, Definition 1 circumvents the empirical challenge of identifying fixed costs over different sets of industries.<sup>21</sup>

Proposition 1 shows the conditions under which  $C_{jk}$  (and thus economies of scope) are identified from just panel data on sales,  $X_j$ , and demand shifters across industries of the firm,  $d \log S_k$ . The exclusion restriction is that the demand shifters are uncorrelated with unobserved demand and supply shocks. This result is useful for my empirical setting, which lacks data on input prices and expenditures.

**Proposition 1 (Identification Benchmark)** *Let  $S_k$  denote an exogenous and relevant shifter of demand in industry  $k$  of the firm. Let  $\psi_{jk}$  denote the observable elasticity of sales in industry  $j$  with respect to demand shifters in industry  $k$ :*

$$d \log X_j = \psi_{jk} d \log S_k, \quad \forall j, k \in J. \quad (1)$$

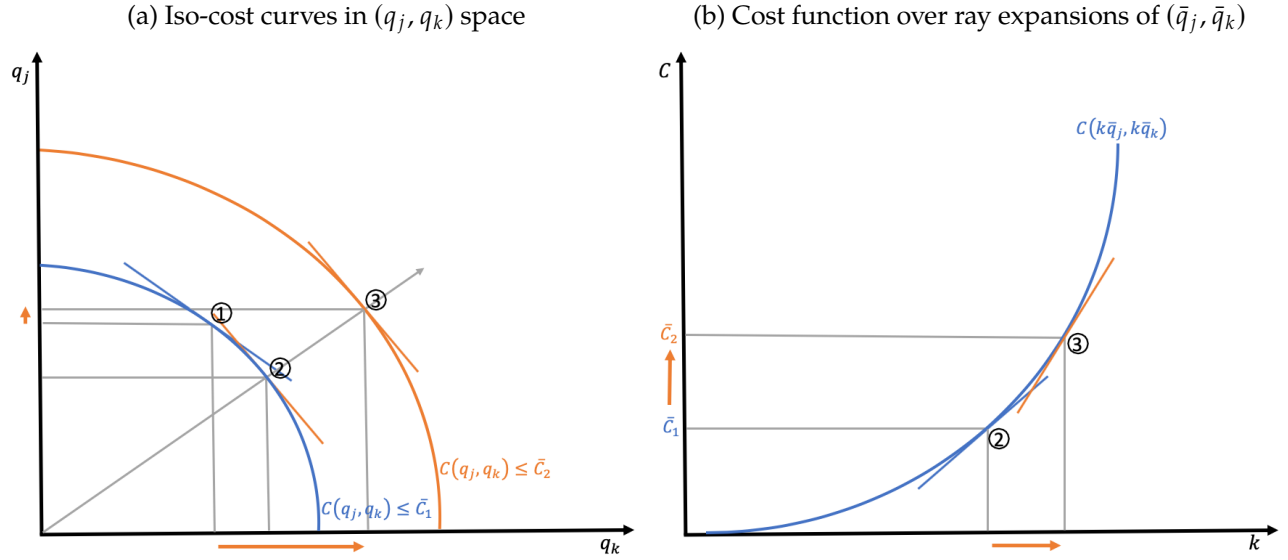
*The Hessian of the cost function,  $\{C_{jk}\}$ , is identified from observable sales-elasticities,  $\{\psi_{jk}\}$ , for a firm that maximizes profits under a known residual demand function and produces observable quantities  $q_j$  at an interior solution  $\vec{q} > 0$  satisfying second order conditions.*

<sup>19</sup>I interpret  $q_j$  as a one-dimensional index summarizing all the characteristics of goods the firm produces within that industry, including the number of varieties, product appeal, quality, and physical quantity.

<sup>20</sup>Formally, my definition is neither sufficient nor necessary for the cost function to be sub-additive. In the absence of fixed costs, local economies of scope à la Definition 1 at all points of production is equivalent to cost function sub-additivity (and sub-modularity).

<sup>21</sup>In settings with many industries, the dimensionality of potential partitions explodes. Their criteria yields 2<sup>J</sup> potentially differing measures of economies of scope at any output vector  $\vec{q}$  relative to just  $J \times J$  in my setting.

Figure 1: Using Demand Shocks to Identify the Cost Function



This figure displays the profit-maximization decisions of a firm under a convex cost function. Curves in panel (a) indicate feasible production bundles  $\{q_j, q_k\}$  conditional on costs  $\bar{C}$ . Panel (b) plots total costs  $C$  for different proportional expansions of a bundle of outputs, i.e. along the (2)-(3) ray depicted in panel (a).

Proposition 1 extends the textbook logic of using demand shifters to identify the marginal cost function of a single good. For graphical intuition, consider a firm producing in two industries,  $j$  and  $k$ .<sup>22</sup> The curved lines in Figure 1a indicate the firm's iso-cost curves—the set of feasible production bundles  $(q_j, q_k)$  holding total costs constant. The more concave is the curve, the more *non-rival* are inputs in production of  $j$  and  $k$ .<sup>23</sup> The convex curve in Figure 1b takes a different slice of the firm's cost surface—it traces out total costs along a particular  $(q_j, q_k)$  ray (such as that depicted by the grey arrow in panel (a)). The more convex is the curve, the more *scalable* are inputs in the production of  $(q_j, q_k)$ . The firm's optimal production choice is characterized by tangency points in (a) where the ratio of marginal revenues are equal to the ratio of marginal costs, and (b) where the ray marginal revenue is equal to the ray marginal cost.

A demand shock that increases marginal revenue in industry  $k$  generates substitution and scale effects within the firm. Holding total costs unchanged, the firm adjusts along its iso-cost curve from point (1) to a new tangency point (2), favoring production of  $q_k$ . The more rival are inputs, the stronger is the substitution effect. On the other hand, given the increase in profitability per bundle sold, the firm is incentivized to increase its scale—its

<sup>22</sup>While the cost function has to be convex in order for an interior solution to exist in the case of perfect competition, this is no longer true when the firm faces a downward sloping residual demand curve, which generates concavity in the marginal revenue function.

<sup>23</sup>In the limit where all inputs are perfectly non-rival, the iso-cost curve is Leontief.

input expenditures. This adjustment is depicted by the movement from point (2) to point (3) in panel (b), and the corresponding expansion of the iso-cost curve shown in panel (a). The more scalable and the less rival are inputs in production, the stronger is the scale effect relative to the substitution effect, and the stronger is the net increase in  $q_j$  (as depicted). Observations of the changes in quantities and knowledge of the movement of tangents (relative prices) in both planes thus allow marginal changes in total costs,  $C_{jk}$ , to be identified.

These responses traced out in Figure 1 nest the case where production is disjoint and the cost function is separable across industries. Under this knife-edge case, which is assumed implicitly in many canonical models of multi-product firms, scale and substitution effects exactly cancel out (so that there are no cross-industry spillovers, i.e.  $\psi_{jk} = 0$ ).

Before turning to a model that provides the necessary structure to identify economies of scope, the intuition above suggests that much can be gleaned from the off-diagonal, cross-industry responses of firm sales to demand shifters,  $\psi_{jk}$ . The remainder of this section presents reduced-form evidence on this cross-industry response margin within the firm.

## 2.3 Reduced-Form Implementation

### 2.3.1 Overview

I run the following empirical analog of equation (1) from Proposition 1 to estimate the impact of demand shocks ( $\Delta \log S_{fkt}$ ) a firm receives in various industries  $k$  on sales growth ( $\Delta \log X_{fjt}$ ) in a given industry  $j$ :

$$\Delta \log X_{fjt} = \psi^{OWN} \Delta \log S_{fjt} + h_{fj}(\{\Delta \log S_{fkt}\}_{k \neq j}; \psi^{CROSS}) + \text{Controls}_{fj} + FE_{jt} + \epsilon_{fjt}, \quad \forall f, j, t = \{2, 3\}, \quad (2)$$

where  $t = 1, 2, 3$  are labels for the years 1997, 2002, and 2007, and  $\Delta$  is a first-difference operator between  $t$  and  $t - 1$  (a five-year difference in the data).<sup>24</sup>

Equation (2) isolates own-industry shocks from cross-industry shocks, so that  $h_{fj} \neq 0$  (a non-zero cross-industry response) can be interpreted as a test of economies of scope. I use various functional forms,  $h_{fj}(\cdot)$ , to index the potential impact on industry  $j$  of demand shocks in *other* industries  $k$ ,  $\{\Delta \log S_{fkt}\}_{k \neq j}$ .<sup>25</sup> The inclusion of industry-year fixed effects sweeps away any supply and demand shocks common to all firms in an industry, while a variety

<sup>24</sup>I drop the year 2012 in all but summary statistics because (i) the global recession generated correlated shocks across countries, industries, as well as firms, jeopardizing variation in the instrument, and (ii) the relevance of industry characteristic information contained in BEA expenditure shares (which I hold fixed to 1997) is diminished. (Changes to BEA accounting rules on intangibles and the lack of fixed-asset tables prevent the direct use of BEA data after 1997.)

<sup>25</sup>This significantly reduces dimensionality. There are in principle  $J^2 = 42, 436$  cross-industry elasticities  $\psi_{jk}$ .

of  $Controls_{fj}$  account for non-parallel growth trends that depend on initial characteristics specific to a firm-industry  $fj$ .

I conduct my empirical analysis on as large a sample of data as possible. I include all US firms that are multi-industry in each base year ( $t - 1$ ) for which empirical measures of demand shocks can be constructed in at least one industry. Observations are continuing industries of these firms over two periods of time,  $t$  and  $t - 1$ .<sup>26</sup> I provide summary statistics on these firms and the regression variables in Appendix Table 8. My final regression sample of roughly 5000 firms per year accounts for over half of all US manufacturing gross output.

### 2.3.2 Demand Shocks

I construct firm-industry level demand shocks,  $\Delta \log S_{fkt}$ , as a shift-share instrument. I interact pre-existing variation in a firm's exports in an industry across foreign destinations ( $n$ ) and products ( $h$ ) with changes in the size of each of these foreign markets. I use the growth in imports (excluding imports from the US) of each foreign destination-product market,  $\Delta \log IMP_{nht}^{US}$  as a proxy for a shift in the size of that market, so that a demand shock relevant to an industry of the firm can be constructed as:

$$\Delta \log S_{fkt} = s_{fj,t-1}^* \sum_n \sum_{h \in H_j} s_{fnh|fj,t-1} \Delta \log IMP_{nht}^{US}, \quad (3)$$

where  $s_{fj,t-1}^*$  is the firm's export intensity in that industry in year  $t - 1$  and  $s_{fnh|fj,t-1}$  is the share of firm total exports in industry  $j$  that go to destination  $n$  and HS6 product  $h$ .<sup>27</sup> I use annual customs data on firm exports by destination and product to construct the export intensity and share variables, and the BACI Comtrade dataset on annual global trade flows between countries at the HS6 level to construct the import growth variable.<sup>28</sup>

My identification assumption is that these demand shocks are exogenous (unaffected by any of the firm's decisions) as well as conditionally uncorrelated with (i) unobservable supply-side shocks in other industries (that shift the firm's cost function) and (ii) unobservable

<sup>26</sup>For example, suppose firm  $f$  produces in industries A and B in 1997 but only produces in A in 2002. As long as the firm received a demand shock in either industry A or B in 1997, I include the firm in the sample (where it takes up a single observation,  $f = f$ ,  $j = A$ ). However, if in 2002 the firm switches to producing industries C and D, there is no intensive margin overlap and this firm does not appear in my sample.

<sup>27</sup>The use of both census and customs data sources on firm exports to construct the instrument is most similar to Aghion et al. (2019). There are roughly 5000 HS6 product codes  $h$ , and only 206 industries  $j$ , with very few instances of a HS6 code  $h$  concurring to more than one industry  $j$ . See the Data Appendix for how I deal with instances of carry-along trade and product-industry concordance splits.

<sup>28</sup>In the data, changes in import growth  $\Delta \log IMP_{nht}^{US}$  could be driven by both (i) changes in the level of demand in that market, and (ii) changes in the degree of foreign or home producer competition in that market that affect the price index. Both sources of variation are valid shifters of a US firm's residual demand in that market, though they would move residual demand in opposite directions. Empirically, I find that these demand shocks act as positive shifters of marginal revenue for US exporters, suggesting that the first force dominates.

demand-side shocks in other industries. A distinguishing feature of this research design is that the underlying source of variation behind  $\Delta \log S_{fkt}$  is not simply at the industry level ( $k$ ), but rather at the level of foreign destinations and HS6 products, of which there are over 1 million combinations.<sup>29</sup> The scale and scope of US multi-industry exporters also contributes to variation in the demand shock, even across firms within an industry. The median number of product-destination export markets within an industry of a firm in my sample is 6.2, and the mean is 24.1.

One threat to identification relates to exogeneity. Excluding US exports in the measures of foreign market import growth  $\Delta \log IMP_{nht}^{\sim US}$  ensures that demand shocks are not contaminated by any common supply-side shocks affecting US exporters. Nevertheless, if a US exporter has a large enough presence in a foreign market, supply-side changes within the US exporter could very well affect that market's import growth from other countries. For example, Indian imports of X-ray scanners from other countries could fall if GE became more productive at making them (and started exporting more to India). To avoid these potentially endogenous changes in foreign import growth, I construct the shock in equation (3) using only variation from the export markets of a firm in which it has a market share below 10%.<sup>30</sup>

### 2.3.3 Input Proximity

I construct  $h_{fj}(\{\Delta \log S_{fkt}\}_{k \neq j}; \psi^{CROSS})$  as a weighted average over shocks that the firm receives in other industries  $k \neq j$ , for two types of weights: (i) relative sales, and (ii) relative input-expenditures.

The sales weighting assumes that industries have symmetric spillover effects, so that a shock received in another industry  $k$  should be larger whenever that industry is larger (based on sales) relative to other industries  $k'$  within the firm:

$$h_{fj}^{SYM} \equiv \psi^{SYM} \sum_{k \neq j} \left( \frac{X_{fk,t-1}}{\sum_{k \neq j} X_{fk,t-1}} \right) \Delta \log S_{fkt}.$$

However, this functional form presupposes a common sign for spillovers (i.e.  $\psi_{jk} = \psi^{SYM} \leq 0$ ). In reality, whereas some pairs of industries may be complements in production, other pairs might be substitutes. I explore the alternative hypothesis that input proximity might drive spillovers. If inputs in production differ in their scalability and rivalry properties,

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<sup>29</sup> An alternative would have been to use government military expenditures as exogenous sources of spending, or the “China shock” popularized by Autor et al. (2013) as a measure of import competition. But both of these shocks vary only at the industry level. In addition, the China shock was a massive shock to both input prices and output competition. Changes in input prices are problematic in my setting because they could be firm-specific and act as a shifter of the firm's cost function, thus violating the exclusion restriction.

<sup>30</sup> The reduced-form results are robust with and without this re-classification.



industry pairs will tend to have positive cross-elasticities whenever they rely more commonly on inputs that are more scalable than rival. I test whether intangible inputs satisfy this description by constructing a bilateral  $j$ -to- $k$  measure of proximity based on expenditure shares on intangibles:

$$h_{fj}^{INT} \equiv \psi^{INT} \sum_{k \neq j} \sum_{m \in \mathcal{M}_f^{INT}} \beta_{jm} \left( \frac{\beta_{km} X_{fk,t-1}}{\sum_{k \neq j} \beta_{km} X_{fk,t-1}} \right) \Delta \log S_{fkt},$$

where the sales share weights from the *SYM* functional form are now replaced by a product of two terms. The first term,  $\beta_{jm}$ , is the expenditure share of industry  $j$  on input  $m$  according to input-output tables. The second term (in parentheses) measures the importance of industry  $k$  in the firm's overall spending on input  $m$  relative to the firm's other industries  $k' \neq j$ .

Data on input-by-industry expenditure shares  $\beta_{jm}$  come from the BEA input-output and capital flow tables in 1997:

$$\beta_{jm} \equiv \frac{E_{mj} + I_{mj}}{\sum_m E_{mj} + I_{mj}},$$

where  $E_{mj}$  are the expenses of industry  $j$  on input  $m$  and  $I_{mj}$  are the capitalized investments on input  $m$  made by industry  $j$  to form new capital.<sup>31</sup> Using these industry-level coefficients to impute resource utilization is potentially advantageous even if data on *firm-specific* input expenditures were available. For example, I avoid the bias that would occur if firms scaled up expenditures on relevant inputs in anticipation of pairs of industries that are about to grow.

I define intangible inputs as industries that belong to the following set of BEAX root families: all 'professional and technical services' industries (54), management of companies and enterprises (55), the leasing of intangible assets (533), and information (51).<sup>32</sup> Expenditures by the manufacturing sector on these inputs are non-trivial: they amount to over 9% of manufacturing gross output in 1997. More details on the exact input industries and variation in expenditures across inputs and industries can be found in the Data Appendix Section [A.1](#)

<sup>31</sup>Because the capital flow table ceases to exist (at least publicly) after 1997 and national accounting rules on intangibles changed over time, I hold the shares fixed at their 1997 values throughout the analysis. Accounting rules mean that some types of inputs are more likely to be capitalized than expensed. Different from traditional input-output analysis which focuses on inputs whose value depreciate fully within a year, I include information on both types of spending to construct the measure of proximity. Results are robust to using just expenditure flows from the I/O use table. See the Data Appendix for more details.

<sup>32</sup>Examples of industries under BEAX 54 include scientific R&D, engineering, consulting, architectural, advertising, and legal services. Although results are robust to including other industries (52, 531, and 532) under BEAX sector 5 (finance, insurance, real estate, and other rental leasing), I do not include them in baseline results because of the separate way that financial inputs affect businesses compared to real inputs. Moreover, placebo exercises show that proximity in these other BEAX-'5' inputs alone does not generate positive and significant spillovers.



and accompanying Table 7.

Using these functional forms, I estimate equation (2) and test whether  $\psi^{INT}$  and/or  $\psi^{SYM}$  are zero, which would be true under the null of no scope economies. I also test whether within-firm spillovers across industries vary with the intensity of use of intangible inputs (i.e. if  $\psi^{INT}$  is different from  $\psi^{SYM}$ ).

## 2.4 Intra-Firm Spillovers

Table 2 reports regression coefficients,  $\psi^{OWN}$  and  $\psi^{CROSS} = \{\psi^{SYM}, \psi^{INT}\}$ , from estimating variants of equation (2). I first show, in column (1), that a demand shock in one industry of the firm increases the firm’s sales in the same industry. The own-industry response,  $\psi^{OWN}$ , is independently positive, which confirms that demand shocks are a shifter of own-industry sales.<sup>33</sup>

The next two columns, (2) and (3), show that spillovers from demand shocks in other industries appear to be a wash when evaluated separately. The average cross-elasticity,  $\psi^{CROSS}$ , is statistically insignificant from zero. This could occur for one of two reasons: either (i) there are no scope economies, or (ii) some pairs of industries are complements in production while others are substitutes, so that the net effect comes out to zero (due to omitted variables bias).

Results in the remaining columns, starting with column (4), come in strongly in favor of the latter hypothesis. When allowing spillovers to be heterogeneous across industries, I find that spillovers are stronger and more positive among industry pairs that use more intangible inputs. Controlling for this positive effect concentrated around intangible input-intensive industries, the remaining spillover (based on sales weights) is negative. These coefficients are consistently estimated and statistically significant across a battery of controls in spite of substantial collinearity around the two measures of cross-industry shocks.<sup>34</sup> These results suggest that intangible inputs are more scalable and non-rival in use within the firm compared to *tangible* capital and other inputs.<sup>35</sup>

Column (5), the preferred specification, accounts jointly for own as well as cross-industry

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<sup>33</sup>I also check, in Appendix Table 9, that this ‘first-stage,’ own-shock relevance is not driven by the export intensity variable  $s_{fj,t-1}^*$  picking up pre-trends in growth rates (i.e. if more export-intensive industries of the firm grow faster). Estimates are robust to including as controls a full set of industry-year dummy variables interacted with the export intensity variable. This is recommended by Borusyak et al. (2018) for specifications that work with ‘incomplete’ shift-share research designs.

<sup>34</sup>This collinearity—at 0.9—is consistent with regression columns (2) and (3) being independently insignificant due to omitted variables bias.

<sup>35</sup>For example, consider the difficulties faced by a firm opening a new plant, or constructing a new assembly line, both of which are intuitively rival and hard-to-scale inputs. Columns (2)-(4) of Appendix Table 10 confirms this intuition by estimating a negative cross-industry impact of demand shocks around industries that share tangible inputs.

Table 2: Cross-Industry Spillovers within the Firm

Sales growth, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)	(5)	(6)
Own-industry shock, $\Delta \log S_{fjt}$ $\psi^{OWN}$	0.45*** (0.10)				0.46*** (0.09)	0.37* (0.19)
Cross-industry shocks, $\{\Delta \log S_{fkt}\}_{k \neq j}$						
(i) Intangible expenditure weighted $\psi^{INT}$			0.81 (1.01)	7.51*** (2.22)	8.00*** (2.25)	13.31*** (3.58)
(ii) Sales weighted $\psi^{SYM}$		-0.03 (0.11)		-0.74*** (0.24)	-0.83*** (0.24)	-1.67*** (0.52)
Industry-year-FE	✓	✓	✓	✓	✓	✓
Firm-year-FE						✓
Observations	21,500	21,500	21,500	21,500	21,500	17,500
$R^2$	0.06	0.06	0.06	0.06	0.06	0.39

This table displays responses of firm-industry sales to demand shocks across the firm's range of industries, in 5-year differences over the period 1997-2007. Standard errors are clustered at the firm level. Observations are at the firm-industry-year level, for continuing observations of a sample of multi-industry firms with at least one industry directly exporting. Results are unweighted but robust to weighting by the inverse within-firm share of sales of industry  $j$ . Results are robust to deflating outcomes and also shocks. The magnitude and significance of coefficients  $\psi$  are robust to inclusion of a host of control variables, including: initial period firm size, firm-industry size, export status, export intensity, as well as controls for the shares in the functional forms used to collapse shocks in other industries, and the interaction of these shares with other initial-period firm-industry variables.

demand shocks. I estimate a strong and positive own-industry response to demand shocks, and cross-industry responses that continue to be stronger and more positive the more that industries within the firm share intangible inputs. Overall, these results suggest that economies of scope exist within the firm (I reject the null that  $\psi_{jk}$  is zero for all industry pairs), and are increasing in industries' joint utilization of intangible inputs ( $\psi^{INT}$  is statistically different from  $\psi^{SYM}$ ).

While the magnitudes of these elasticities are best assessed through the lens of an economic model (in Section 3), I offer a back-of-the-envelope calculation to show that they are sizable in this reduced-form specification. These calculations also demonstrate that the net effect of cross-industry spillovers within the firm can be either positive or negative, depending on the intensity of use of intangible inputs among the firm's mix of industries.

Consider a firm that operates an industry  $j$  that has an intangible input proximity with the firm's other industries,  $k \neq j$ , of 0.09, the mean value in the sample. If this firm receives a common demand shock in all of its industries equal to one standard deviation in the sample (8.2%), the firm's sales in industry  $j$  would rise by 2.7%. This can be decomposed into an own-industry response (3.8%) and a negative cross-industry response (-1.1%). For another firm receiving the same shocks but whose industry  $j$  is more proximate (by one standard deviation above the mean) in its use of intangible inputs with its other industries  $k \neq j$ , the cross-industry response rises to a positive 1%. The total increase in sales in industry  $j$  is now 4.8% ( $= 3.8\% + 1\%$ ), with cross-industry spillovers accounting for more than one-fifth of this response.

All the displayed results in Table 2 are robust to the inclusion of an exhaustive set of firm-industry level controls (see column 1 of Appendix Table 10). They are robust to controlling for the pre-period size of the firm's industry  $j$ , which captures the possibility that large and small firm-industry segments might be on different growth trends. Results are robust to controlling for own-firm-industry export intensity and export status. Additional controls include firm-wide covariates such as firm total size (in a pre-period) and firm export intensity.

To placate concerns that the weights used to construct  $h_{fj}^{INT}$  and  $h_{fj}^{SYM}$  might be picking up correlated unobserved changes across industries, I construct additional controls that measure how pre-existing presence, sales, export intensity, and export status, of the firm in other industries  $k$  might affect sales growth in  $j$  when weighted using the same intangible input and sales weights. For any firm-industry level variable  $Y_{fk}$ , I construct:

$$Control_{fj}^{INT}(\{Y_{fk}\}_{k \neq j}) \equiv \psi_Y^{INT} \sum_{k \neq j} \sum_{m \in \mathcal{M}_f^{INT}} \beta_{jm} \left( \frac{\beta_{km} X_{fk,t-1}}{\sum_{k \neq j} \beta_{km} X_{fk,t-1}} \right) Y_{fkt},$$

and likewise for  $Control_{fj}^{SYM}(\{Y_{fk}\}_{k \neq j})$  using sales shares over  $k \neq j$  as weights. These vari-

ables control for the possibility that firm sales in  $j$  may have grown faster simply because the firm had a large amount of pre-existing sales in industry  $k$ , or because the firm was exporting in industry  $k$ , etc. I find that they affect neither the significance nor magnitude of the estimated spillover coefficients,  $\psi^{INT}$  and  $\psi^{SYM}$ . Reassuringly, these controls demonstrate that variation used to identify spillovers is not coming exclusively from the initial period distribution of export shares and sales intensities, but rather the interaction of these shares with changes in foreign market size.

Finally, column (6) of Table 2 shows that results remain robust to controlling for firm-year fixed effects, which soak up unobserved supply and demand shocks that jointly affect all industries of the firm (so that variation comes from both differences across industries within the firm and differences across firms within an industry).<sup>36</sup>

### 2.4.1 Other Input Linkages

Are there other relevant dimensions of industries that are correlated with spillovers? I construct alternative spillover functions based on correlation in input use for any blocks of inputs in the BEA I/O tables. I define these analogously to the measure of proximity over intangible inputs,  $h_{fj}^{INT}$ . For any block of inputs  $m \in BLK$ , I construct  $h_{fj}^{BLK}$  according to the same functional form, using relative input expenditures over industries within that block as weights:

$$h_{fj}^{BLK} \equiv \psi^{BLK} \sum_{k \neq j} \sum_{m \in \mathcal{M}_f^{BLK}} \beta_{jm} \left( \frac{\beta_{km} X_{fk,t-1}}{\sum_{k \neq j} \beta_{km} X_{fk,t-1}} \right) \Delta \log S_{fkt}.$$

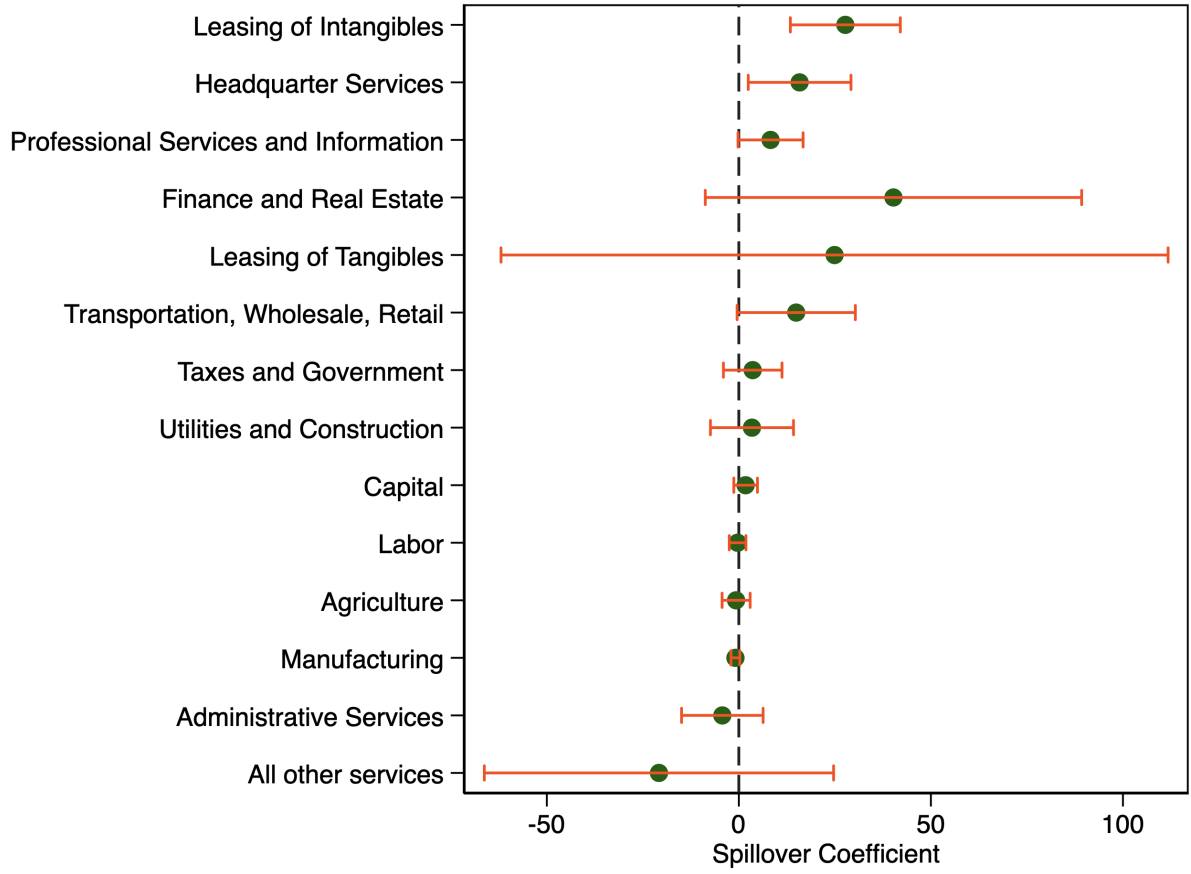
I re-produce the specification in column (5) of Table 2 but now replace  $h_{fj}^{INT}$  with  $h_{fj}^{BLK}$ .<sup>37</sup> Coefficients on input-based transmission are displayed in Figure 2 and the corresponding regression table can be found in the Data Appendix under Table 11. The first three rows of Figure 2 break down my classification of intangibles into three blocks of inputs. I find that each block is positive and statistically significant. With the exception of transportation, wholesale and retail service inputs, which is mildly positive, no remaining input block in the IO tables appear to drive spillovers in any significant way. In particular, the estimates on inputs such as capital, labor, agriculture, and manufacturing are all precisely estimated at zero. These results strongly suggest that intangibles have distinct characteristics from other inputs.<sup>38</sup>

<sup>36</sup>The magnitudes become more polarized because inclusion of firm-year fixed effects soak up any net firm-wide responses that are induced by economies of scope. For this reason the specification without firm-year fixed effects (column 5) is preferred.

<sup>37</sup>To enable a fair comparison, I include the same controls as the original column (5)—the own-industry shock and the sales-weighted spillover function,  $h_{fj}^{SYM}$ .

<sup>38</sup>Ultimately, the only factor preventing joint estimates of transmission elasticities for each input block in the

Figure 2: Cross-Industry Spillovers by Type of Input



This figure displays estimates of  $\psi^{BLK}$  (spillover coefficient) from replacing  $h_{fj}^{INT}$  with  $h_{fj}^{BLK}$  in alternative versions of regression specification (5) of Table 2, where  $BLK$  corresponds to the input category across the rows of the Figure. Point estimates are in green and 95% confidence intervals in orange. See Table 11 for the corresponding regression table and for more detail on the BEAX sector codes of these inputs.

### 2.4.2 Response of Intangible Inputs within the Firm

The finding that spillovers are positively correlated with intangible input use is suggestive of a particular *mechanism*: when firms scale up intangible inputs in response to a demand shock in one industry, the non-rivalry of these inputs also benefits other industries within the firm. I use the data on hand to test a necessary condition behind this mechanism.

I show that firm-wide expenditures on a particular category of intangibles I can measure—purchased professional services—rise in response to firm-wide demand shocks, in line with the hypothesis that these resources are scalable.<sup>39</sup> Table 3 reports the response of a set of firm-level outcome variables to firm-level demand shocks. I study the response of (i) purchased professional services and compare it to the response of (ii) sales, (iii) capital expenditures, and (iv) payroll. I estimate the following regression:

$$\Delta \log Y_{ft} = \tilde{v} \sum_k \eta_{fk,t-1} \Delta \log S_{fkt} + \epsilon_{ft}, \quad (4)$$

where  $\eta_{fk,t-1}$  are weights that measure the relative propensity of demand shocks in industry  $k$  to shift firm-wide outcomes, depending on the outcome variable.<sup>40</sup>

I find that purchased professional services rise in response to firm-wide demand shocks, with an elasticity of 0.65. This elasticity reflects the joint effect of input scalability and the responsiveness of marginal revenue to the empirically constructed demand shifter. I leverage and come back to this relationship when structurally estimating the model (in Section 4).<sup>41</sup> In comparison, capital expenditures and payroll all respond with a lower elasticity than purchased professional services, consistent with the hypothesis that intangibles are more scalable than these other inputs.

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same regression specification is the finiteness of the sample size. Whenever three or more blocks of inputs are included, collinearity kicks in and all estimated  $\psi$  coefficients—except  $\psi^{INT}$ —lose significance. For an example, refer to specification (4) of Table 10 in the Data Appendix, which controls jointly for tangible, symmetric, and intangible spillover variables.

<sup>39</sup>Data on these expenditures are only available for a subset of firms in the main regression specification. These expenditures are available at the level of the firm and comprise of expenditures on outsourced professional services that include purchased software and data processing, management, and advertising that are reasonably a subset of what would fall within the classification of intangibles in BEA data. For more details, see the Data Appendix.

<sup>40</sup>See the Data Appendix for the precise definitions. For the professional services outcome variable, I use the BEA I/O table implied expenditure shares by industry on intangible inputs. For sales as the outcome, I use firm sales weights. For capex and payroll, I use the relative expenditures on that input category by industry (implied by the BEA I/O table) as weights.

<sup>41</sup>This consistency with theory is the primary motivation for using input-expenditure weights  $\eta_{fk,t-1}$  instead of a simple relative sales (market size) as weights. Magnitudes do not change much and significance is retained when using relative sales (market size) as weights across all four outcome variables in Table 3.

Table 3: Firm-level Margins of Response to Demand Shocks

	(1) Purchased Prof. Services	(2) Sales	(3) Capex	(4) Payroll
Outcome-relevant demand shock $\sum_k \eta_{fk,t-1} \Delta \log S_{fkt}$	0.65*** (0.22)	0.37*** (0.15)	0.47 (0.37)	0.25** (0.11)
Year-FE	✓	✓	✓	✓
Observations	3,900	3,900	3,900	3,900
$R^2$	0.02	0.05	0.04	0.01

This table displays responses of firm outcomes to aggregated demand shocks across the firm's range of industries, in 5-year differences over the period 1997-2007. Standard errors are clustered at the firm level. Observations are at the firm-year level, for continuing firms with at least one industry directly exporting and reporting non-zero purchased professional services.

## 2.5 Discussion

These reduced-form results provide evidence that sales in one industry of the firm responds to demand shocks in other industries, and that these cross-industry spillovers are more positive among industries that share more intangible inputs.

For the spillovers in Table 2 to be interpreted as a test of scope economies, the only requirement is that demand shocks are conditionally uncorrelated with unobservables in *other* industries. In particular, a demand shock in industry  $k$  can be arbitrarily correlated with unobserved supply and demand shocks exclusive to that same industry—this would merely change the interpretation of the own-industry elasticity,  $\psi^{OWN}$ . Demand shocks also do not need to be unanticipated: the coefficients pick up precisely the endogenous supply side response in industry  $j$  to demand shifters in industry  $k$ . The fact that a particular shock is anticipatable  $t$  years ahead of time simply changes the interpretation of the time horizon without changing the fact that there has been a response.

My empirical strategy is resilient to two main identification threats. The first concern is that demand shocks in any industry are correlated with unobserved demand shocks in other industries, inducing omitted variables bias. But the lack of significance in either of the spillover functions in columns (2)-(3) of Table 2 rule out a simple correlation structure. Still, it could be the case that the import growth patterns in foreign markets ( $nh$  combinations) are more positively correlated across industries that co-use more intangibles. I directly test and reject this hypothesis in the data. A related concern is that proximity in the use of intangibles is actually correlated with demand-side complementarity. In this story, the firm only needs a demand shock in one of these industries to be observed to be selling more goods in both. However, results continue to be significant when I purge from the sales of industry  $j$  all



exports to destination countries where the demand shocks (for industries  $k$ ) originated.<sup>42</sup>

The second threat to identification is omitted variables bias induced by a correlation between demand shifters in one industry of the firm and unobserved supply-side shocks in other industries of the firm. Since all regression specifications include industry-year fixed effects, remaining supply-side shocks have to be firm-specific.<sup>43</sup> My preferred interpretation is that firm-specific supply-side innovations are the precise result of these cross-industry demand shocks, and therefore a part of the spillover response.

There is, however, a less organic interpretation: firms that anticipate positive supply-side shocks in a pair of industries  $j, k$  select into exporting in those industries (and, in particular, to fast-growing markets with higher demand shocks). For this interpretation to also explain the null coefficients in columns (2) and (3) of Table 2, suppose for the sake of argument that firms are systematically better at anticipating these effects in intangible input-intensive industries. Under this interpretation, growth is a function of pre-existing joint production and exporting patterns. I show that this hypothesis does not survive the following placebo exercise. I re-assign firm-industry exporters in each industry  $k$  different export demand shocks drawn from the empirical distribution of shocks in that industry.<sup>44</sup> I keep all remaining firm variables (such as firm-industry production weights, and other controls used) the same in the regression but find no false positive spillover coefficients.<sup>45</sup>

The Data Appendix summarizes findings from additional robustness exercises. I test and reject other mechanisms that could have generated the spillovers, such as common export destinations as a non-rival firm resource, and vertical (both upstream and downstream) relationships between  $k$  and  $j$ .

### 3 Quantitative Model

To interpret and quantify the reduced-form spillovers, I develop a tractable general equilibrium model of multi-industry firms with variable economies of scale and scope. My framework generalizes workhorse models of heterogeneous firms featuring monopolistic competition and CES demand (Melitz, 2003). It allows any variable input in production to have unrestricted degrees of scalability and rivalry within the firm. These characteristics

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<sup>42</sup>I also find that results are robust to controlling for latent demand shocks—a measure of demand for industry  $j$  of the firm not from where it is currently exporting the goods (which is  $\Delta \log S_{fjt}$ ) but from the other export destinations to which it currently sells goods in other industries.

<sup>43</sup>For example, suppose that the price of IT capital and software fell over 1997 to 2007. The industry-year fixed effects take care of this by absorbing any decline in input prices in industries that are heavily dependent on IT capital. Industry dependence on IT capital comes purely from aggregate (I/O) and not firm-level data, eliminating any concern about firm-specific endogeneity.

<sup>44</sup>This is  $\{\Delta \log S_{f'k}\}_f$  but still constructed using the firm's own (actual) export-intensity weighting.

<sup>45</sup>These results are undisclosed but available upon request.

of inputs generate economies of scope and rationalize cross-industry spillovers in the data. The first half of this section introduces the functional form assumptions and discusses the microeconomic predictions.

My model is also tractable in general equilibrium and aggregates to match industry-level expenditures on inputs (including intangibles). I derive analytical predictions for the elasticity of the price index in any industry with respect to market size shifters in another industry—what I call aggregate economies of scope. This matrix of aggregate cross-industry elasticities depends on the microeconomic scale and rivalry elasticities of variable inputs within the firm. The second half of this section derives these predictions and uses stylized examples to build intuition.

### 3.1 Micro Production Framework

A continuum of firms compete, in each industry  $j$ , under monopolistic competition facing CES demand with elasticity  $\sigma_j$ . This assumption shuts down demand-side and strategic complementarities and variable markups so that the only source of interdependencies in a firm's profit maximization problem comes from properties of joint production.

I begin, in Assumptions 1 and 2, with a description of the technology for producing quality-adjusted units of output,  $q_{fj}$ . The production function is agnostic about whether firms' input expenditures serve to improve product quality, appeal or physical efficiency.<sup>46</sup>

**Assumption 1 (Production Function)** *Production of quality-adjusted quantities  $q_{fj}$  is Cobb-Douglas in (i) a bundle of generic inputs,  $l_{fj}$ , (ii) an exogenous profitability shifter  $\tilde{\xi}_{fj}$ , and (iii) an acquired level of knowledge capital,  $\varphi_{fj}$ :*

$$q_{fj} = l_{fj}^{\gamma_j} \cdot \tilde{\xi}_{fj} \cdot \varphi_{fj}. \quad (5)$$

Generic inputs  $l_{fj}$  are purchased at constant unit prices  $c_j$ , and  $\gamma_j \in [0, \frac{\sigma_j}{\sigma_j-1})$  is the elasticity of outputs with respect to generic inputs.

This production function breaks down what would be a conventional measure of 'productivity' in the spirit of Melitz (2003) and Bernard et al. (2010) into two components: an exogenous component  $\tilde{\xi}_{fj}$ , which I call a profitability shifter, and an endogenous component  $\varphi_{fj}$ , which I call knowledge capital. By solving out for the conditional choice of generic inputs and pricing, I express gross profits  $\pi_{fj}$  and sales  $X_{fj}$  as a function of industry-wide

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<sup>46</sup>Given the broad definition of what is an industry (the manufacturing sector is divided into only 206 industries), firms competing in the same industry should not be expected to be making varieties differentiated only by their physical marginal costs of production. Indeed, Hottman et al. (2016) find, using Nielsen scanner data, that more than half of the variation in firm size in their sample can be attributed to product appeal alone.

aggregates (residual profits,  $B_j$ ) and the two firm-industry specific ‘productivity’ terms:

$$\pi_{fj} = (1 - \varsigma_j)X_{fj} = B_j \xi_{fj} \varphi_{fj}^{\frac{\sigma_j-1}{\sigma_j(1-\varsigma_j)}}, \quad (6)$$

where  $\varsigma_j \equiv \gamma_j \frac{\sigma_j-1}{\sigma_j}$  is a constant share of industry sales expensed on generic inputs observable from I/O tables,  $\xi_{fj} \equiv \tilde{\xi}_{fj}^{\frac{\sigma_j-1}{\sigma_j(1-\varsigma_j)}}$  is a convenient re-normalization (of a purely exogenous term), and industry-wide residual profits is given by

$$B_j = (1 - \varsigma_j) \left( \frac{c_j}{\varsigma_j} \right)^{\frac{\varsigma_j}{\varsigma_j-1}} \left( P_j^{\sigma_j-1} Y_j \right)^{\frac{1}{\sigma_j(1-\varsigma_j)}}, \quad \forall j. \quad (7)$$

where  $c_j$  denotes the unit cost of the generic input bundle,  $P_j$  describes the standard CES industry price index, and  $Y_j$  denotes aggregate expenditures on goods from  $j$ .<sup>47</sup>

Different from generic inputs, the contribution of *specialized* inputs in production is not deterministic ex-ante. Specialized inputs serve to increase the firm’s knowledge capital,  $\varphi_{fj}$ , an index that captures the combined profitability effect of firm-industry-specific attributes, such as customer lists, warehousing capabilities, product design, and brand capital. These attributes are customized to the firm and impossible to acquire directly on the market. To build up these attributes, the firm relies on the ideas generated (or the tasks performed) by different types of specialized inputs, which I denote by  $m = 1, \dots, M$ .<sup>48</sup> The firm plays the role of internally allocating ideas (or tasks) to industries where they generate the highest value.<sup>49</sup>

The contribution of specialized inputs towards advancing knowledge capital is uncertain ex-ante (when budgeting and input expenditure decisions are made). The first source of uncertainty is in the number of ideas generated. Even if the firm allocates a large budget to R&D (a type of input,  $m$ ), it may come up short on actual R&D ideas that improve knowledge capital. I model the arrival of (discrete) type- $m$  ideas within the firm,  $A_{fm}$ , as a Poisson process with rate  $I_{fm}$ . The arrival rate can be increased by the firm through higher

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<sup>47</sup>This section does not need to take a stance on whether the economy is open or closed;  $P_j$  and  $Y_j$  can reflect import competition and export market opportunities, as long as there is no selection margin.

<sup>48</sup>There are no ex-ante assumptions on what these inputs need to be: some of these specialized inputs (such as R&D workers) might fall under the ‘intangible’ classification adopted in the reduced-form, while some others (administrative assistants) might not. In the context of the model, ideas are also synonymous with specialized tasks (performed by labor, or machines).

<sup>49</sup>Though the deployment of ideas is fully rival by assumption, the value of specialized inputs that generate them is not, as will become clear.

expenditures on that specialized input, subject to a convex cost function:

$$C_m(I_{fm}) = \frac{\rho_m - 1}{\rho_m} w \left( \frac{I_{f m t}}{Z} \right)^{\frac{\rho_m}{\rho_m - 1}}, \quad \forall m,$$

where  $\rho_m \in (1, \infty)$  is an index of scalability,  $Z$  is a technological coefficient, and  $w$  is the price of labor (the single factor of production in the model).

The second source of uncertainty is the match-specific quality between an idea and an industry. For example, an engineer hired to improve battery longevity in vacuum cleaners may end up generating ideas that are more useful when deployed towards increasing battery efficiency in electric vehicles. I model the match-specific contribution of an idea (indexed  $i$ ) to knowledge capital in any industry (denoted  $\phi_{fjm,i}$ ) as an independent random draw from a Fréchet distribution:

$$Pr(\phi_{fjm,i} \leq x) = e^{-x^{-\theta_m}}, \quad \forall j, \forall i = 1, \dots, A_{fm},$$

where the shape parameter  $\theta_m \in (1, \infty)$  indexes predictability (the variance of the distribution decreases with  $\theta$ ).

With multiple ideas  $i$  arriving from multiple input types  $m = 1, \dots, M$  varying in quality, the firm's solution is intractable without further assumptions on how the ideas are combined. I proceed in the rest of the paper with an additive separability assumption.

**Assumption 2 (Knowledge Capital)** *Firms choose a single industry  $j$  in which to deploy each idea  $i$ ,  $\phi_{fjm,i}$ , knowing that the cumulative effect of deployed ideas on profit-relevant knowledge capital  $\varphi_{fj}$  is additive:*

$$\frac{\sigma_j^{-1}}{\sigma_j^{(1-\varsigma_j)}} = \sum_m \tilde{\alpha}_{mj} \sum_i^{A_{fm}} \phi_{fjm,i} \mathbf{1}_{fjm,i}, \quad \forall j$$

where  $\tilde{\alpha}_{mj}$  is a technology parameter governing the average quality of an idea from specialized input  $m$  applied to industry  $j$ , and  $\mathbf{1}_{fjm,i}$  is the firm's deployment decision variable: equal to 1 if idea  $i$  from input type  $m$  is deployed to industry  $j$ , and 0 otherwise.

With this assumption, the optimal industry in which to deploy any given idea is independent of past or future decisions.<sup>50</sup> Despite these parametric assumptions, the model of production described here generalizes that in standard models of firms, where knowledge

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<sup>50</sup>This is crucial for tractability in the model, though none of the qualitative predictions hinge on this parametrization. Taken literally, the parametrization makes directional sense. The value of an idea towards improving knowledge capital  $\varphi$  is increasing with the degree of product differentiation in an industry,  $1/\sigma_j$ , and decreasing with the output elasticity with respect to generic inputs,  $\gamma_j$ .

capital is implicitly exogenously determined at birth. Given that the scalability ( $\rho$ ), predictability ( $\theta$ ), and technology coefficients  $\tilde{\alpha}_{mj}$  are fully general in my setting, I nest these benchmark models whenever the cost function is restrictively convex ( $\rho_m \rightarrow \infty$ ).<sup>51</sup>

Two properties of the firm's profit maximization problem are noteworthy. First, discreteness in the number of ideas that arrive means that the firm can very well end up with zero accumulated knowledge capital in certain (or even, all) industries. Given the necessity of knowledge for production (through the Cobb-Douglas specification), the allocation of ideas to knowledge capital affects the firm's extensive margin (the set of industries in which the firm is active). This decision, however, is fully endogenous. The firm chooses the mean Poisson arrival rate,  $I_{fm}$ , and, among the ideas that do arrive, it chooses the industry in which that idea is deployed,  $\mathbf{1}_{fjm,i}$ . The model thus explains intensive and extensive margin reallocation in the data without the need to turn to literal fixed costs typical in this class of models.<sup>52</sup>

Second, the technology parameters  $\tilde{\alpha}_{mj}$  allow the quantitative model to exactly match heterogeneity in input-by-industry expenditures in the BEA Tables. For example, the fact that advertising innovations (a particular  $m$ ) are more useful on average for certain industries  $j$  (e.g. consumer-facing ones) is captured by a higher value of  $\tilde{\alpha}_{mj}$  in the model, and a higher expenditure share in the data. I exploit this mapping in Section 4 to invert technology parameters from expenditure shares.

Given these assumptions, the ex-ante value provided by specialized inputs within the firm is partially non-rival and determined by the Fréchet shape parameter,  $\theta_m$ .

**Lemma 1 (Microfoundation for Non-rivalry)** *Given Assumptions 1 and 2, the expected contribution to gross profits of any idea generated by specialized input  $m$  is given by  $\Delta_{fm}/Z$ , where  $\Delta_{fm}$  is a power sum of residual profitability of input  $m$  over industries  $j$ :*

$$\Delta_{fm} \equiv \left( \sum_j \delta_{fjm}^{\theta_m} \right)^{\frac{1}{\theta_m}}, \quad \delta_{fjm} \equiv \xi_{fj} \alpha_{mj} B_j Z,$$

where  $\alpha_{mj} \equiv \tilde{\alpha}_{mj} \Gamma(1 - 1/\theta_m)$  is a renormalization of the technology parameter and  $\delta_{fjm}$  is an exogenous index of residual profitability of input  $m$  in industry  $j$  in firm  $f$ .

Intuitively, non-rivalry comes from ex-ante 'option value' generated by unpredictability in the match-specific quality of ideas. When unpredictability is high ( $\theta_m$  low), the expected

<sup>51</sup>Another way to motivate the additive separability assumption is to think of each idea as the genesis of a distinct product line within the industry of the firm, and have the firm operate discrete product lines of measure zero within each industry.

<sup>52</sup>Expenditure outlays on specialized inputs  $C_m(I_{fm})$  can be interpreted as 'variable fixed costs' of the firm.

value (to gross profits) of the best industry application of an idea is high, and therefore so is the ex-ante value of the corresponding specialized input  $m$  within the firm. There are two illustrative limit cases. As  $\theta_m \rightarrow 1$ , optionality is so large that the expected value of the best industry application is equal to the sum of residual profitability across all industries  $j$ . Specialized inputs (which generate these ideas) are thus fully non-rival in their expected value-added to the firm. On the other hand, as  $\theta_m \rightarrow \infty$ , the quality of each idea is fully predictable, resulting in no option value. The ex-ante expected value-added of specialized inputs is thus lower—equal to the marginal profitability of the single highest industry in the firm.

Despite being an ex-ante concept, non-rivalry in the expected value-added of specialized inputs also rationalizes ex-post observations of ‘non-rivalry’. Due to higher unpredictability, all else equal, non-rival inputs will generate ideas that are more likely to be deployed across more industries.

### 3.2 Firm Outcomes

**Timing.** The production framework introduced in this section can be tractably embedded in either dynamic or static equilibrium settings. For exposition, I consider a static representation.<sup>53</sup> Each firm maximizes profits by first choosing expenditures on specialized inputs,  $C(I_{fm})$ , knowing the exogenous terms in  $\delta_{fmj}$ , but before the number of ideas and their match-specific qualities are realized. The firm thus internalizes the higher expected value of more non-rival specialized inputs. After spending on specialized inputs, the firm undergoes an incubation period of time before production to harvest ideas that improve knowledge capital. It decides the industry in which to deploy each idea that arrives. Finally, the firm purchases generic inputs  $l$  and decides quantities of production in equilibrium conditional on its accumulated level of knowledge capital  $\varphi_{fj}$ .

Lemma 2 characterizes optimal choices made by the firm and the expected profits and entry patterns that result from those choices, where the expectation operator,  $\mathbb{E}$ , represents uncertainty over the number and quality of ideas generated by specialized inputs.

**Lemma 2 (Firm’s Solution)** *Given Assumptions 1 and 2, the expected gross profits of any firm  $f$  in industry  $j$  is a constant fraction  $(1 - \varsigma_j)$  of expected sales and given by*

$$\begin{aligned}\mathbb{E}[\pi_{fj}] &= (1 - \varsigma_j) \mathbb{E}[X_{fj}] \\ &= \sum_m \mu_{fmj} \Delta_{fm}^{\rho_m} w^{1-\rho_m},\end{aligned}\tag{8}$$

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<sup>53</sup>The additive separability assumption makes it easy to have a firm produce and harvest ideas over multiple periods; one simply needs an assumption on the rate of depreciation of knowledge capital. Indeed, this slight extension is used in the next section to structurally estimate the model given the panel structure of the data.

where  $\Delta_{fm}$  is the expected contribution to profits of a single idea generated by input  $m$ , as in Lemma 1, and  $\mu_{fmj}$  is the probability that the firm deploys an idea generated by input  $m$  in industry  $j$ :

$$\mu_{fmj} \equiv \frac{\delta_{fmj}^{\theta_m}}{\Delta_{fm}^{\theta_m}}.$$

The probability that a firm enters industry  $j$ , denoted  $\chi_{fj} = 1$ , is given by

$$Pr(\chi_{fj} = 1) = 1 - \exp\left(-Z \sum_m \mu_{fmj} \Delta_{fm}^{\rho_m - 1}\right), \quad (9)$$

and is independent across industries. Expected net profits of the firm are given by expected gross profits over all industries less total expenditures on specialized inputs:

$$\mathbb{E}[\Pi_f] = \sum_j \mathbb{E}[\pi_{fj}] - \sum_m C_m(I_{fm}) = \sum_m \frac{1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m}. \quad (10)$$

Equation (8) encapsulates all the microeconomic insights behind scale and rivalry. A firm's sales in an industry  $j$  always increases with own-industry residual profit shifters:  $B_j$  and  $\xi_{fj}$ . But sales is also affected by profitability shifters in other industries  $\xi_{fk} B_k$ , which show up in the total profitability terms  $\{\Delta_{fm}\}_m$ . The direction and magnitude of cross-industry effects depend on whether the scalability of each input ( $\rho_m$ ) is larger than its rivalry in use ( $\theta_m$ ).<sup>54</sup>

Intuitively, two things happen when profitability in  $k$  (either  $B_k$  or  $\xi_{fk}$ ) rises. First, the firm increases its expenditures on specialized inputs, particularly those inputs  $m$  where ideas contribute the most to knowledge capital in  $k$ . Second, because profitability in  $k$  is now relatively higher, the firm will more often find  $k$  to be the most profitable industry in which to deploy an idea.<sup>55</sup> While the first effect raises the number of *potential* ideas of type- $m$  that improve knowledge capital in another industry  $j$ , the second effect decreases the likelihood that an actual idea will be deployed to industry  $j$ . The net effect on accumulated knowledge capital in industry  $j$  depends on a trade-off between these two forces, averaged across the sets of inputs  $m$  that are most technologically relevant for production in  $j$  and  $k$ .

These expressions for firm sales and profits combine both the extensive margin probability

<sup>54</sup>The use of this Fréchet shape parameter to model choice shares was introduced to trade by Eaton and Kortum (2002). However, rather than focusing on  $\theta$  as a choice-share elasticity, my paper exploits the role of  $\theta$  in the Fréchet functional form for expected value.

<sup>55</sup>Variance of match-specific shocks need not be a driver of differences in  $\theta_m$ . Even if  $\theta_m = \theta$  for all inputs  $m$ , the model is isomorphic to idea  $\phi_{fmj}$  having different returns to scale in the generation of knowledge capital  $\varphi_{fj}$  in Assumption 2. See the online supplementary theory appendix for more details.



that a firm is active (in that industry) and the intensive margin quality of accumulated knowledge capital conditional on ‘entry’. Industry entry in the model is generated by the firm deploying a first idea in that industry  $j$ . Granularity in the arrival process of ideas, combined with the necessity of knowledge capital in the production function, thus rationalizes endogenous entry and exit without any need to introduce fixed costs.<sup>56</sup> The probability of firm entry into industry  $j$  is analytically characterized by equation (9) and endogenous to the same scale and rivalry properties of specialized inputs. These features of the model render the firm’s ex-ante problem well-defined and convex, significantly reducing dimensionality.

Proposition 2 log differentiates equation (8) to illustrate how cross-industry elasticities of expected sales (or gross profits) with respect to demand shocks depend on scale and rivalry parameters behind specialized inputs,  $\{\rho_m, \theta_m\}_m$ . Within the firm, own-industry elasticities are positive, while cross-industry elasticities are completely unrestricted in both magnitude and direction. Cross-industry elasticities appear as simple weighted averages of  $\rho_m - \theta_m$  over the set of inputs  $m$ ; they are positive whenever, on average, inputs common in both industries are more scalable than rival. Cross-elasticities are asymmetric since transmission is one-directional. They are larger whenever the shocked industry  $k$  is an important user of specialized inputs  $m$  (relative to other industries  $k'$ ), and whenever input type  $m$  is a more important input for production in industry  $j$  (relative to other inputs  $m'$ ).<sup>57</sup>

**Proposition 2 (Spillovers within the Firm)** *The elasticity of firm profits or sales,  $\mathbb{E}[X_{fj}]$ , to residual demand shocks in any industry  $k$  is given by:*

$$\psi_{fjk} = \frac{d \log \mathbb{E}[X_{fj}]}{d \log \xi_{fk} B_k} = \sum_m \lambda_{fjm} (\mu_{fjm} (\rho_m - \theta_m) + \theta_m \mathbf{1}_{j=k}), \quad \forall j, k, \quad (11)$$

where  $\mu_{fjm}$  are choice shares given in Lemma 2, and  $\lambda_{fjm}$  denote utilization shares: the share of gross profits of industry  $j$  attributable to knowledge capital contributions by input  $m$ :

$$\lambda_{fjm} \equiv \frac{\mu_{fjm} \Delta_{fm}^{\rho_m} w^{1-\rho_m}}{\sum_{m'} \mu_{fm'} \Delta_{fm'}^{\rho_{m'}} w^{1-\rho_{m'}}}.$$

Proposition 2 explains the competing presence of both positive and negative spillovers (generated by the same demand shocks) in the reduced form. It interprets positive spillovers

<sup>56</sup>The use of discrete stochastic processes to explain ‘zeros’ is inspired by Klette and Kortum (2004), Eaton et al. (2013), and Armenter and Koren (2014). But instead of having an exogenous statistical process drive granularity, my framework grounds zeros (the absence of a firm operating in an industry) in the endogenous profit-maximizing decisions of firms.

<sup>57</sup>Note that the asymmetry is true even if  $\alpha_{mj} = \alpha_{mk}$ , since the asymmetry also depends on firm-specific market sizes in industries  $j$  and  $k$ .

in the reduced form as coming from a set of inputs that has  $\rho_m > \theta_m$ , and negative spillovers as coming from other inputs that have  $\rho_m < \theta_m$ . The case where there are no interdependencies across industries occurs on a knife's edge in parameter space: spillovers are zero for all industry pairs if and only if  $\theta_m = \rho_m$ ,  $\forall m$ .

In the limit as firms in the regression sample become infinitely large, equation (11) becomes a close approximation to the functional forms  $h_{fj}$  assumed in the reduced-form (Section 2). In this limit, the expectation term  $\mathbb{E}[X_{fj}]$ —which normally reflects both extensive and intensive margins—approaches just the intensive margin specification in the reduced-form. I provide more information in the Theory Appendix.

### 3.3 Industry-level Spillovers

This subsection characterizes aggregate equilibrium predictions in the model. I endogenize industry-level residual profits,  $\{B_j\}_j$ , by aggregating the decisions of all firms. General equilibrium is a setting whereby, given anticipated residual profits,  $\{B_j\}_j$ , firms' decisions over input expenditures, idea deployment, and production give rise to the same equilibrium  $\{B_j\}_j$  as anticipated, clearing the goods market in each industry.

The lack of any literal fixed costs in the model makes aggregation smooth and convex.<sup>58</sup> The problem of the firm described earlier applies to *all* firms, not just the firms that are *ex-post* multi-industry. In the model, both small (low  $\xi_{fj}$ ) and large (high  $\xi_{fj}$ ) firms face the same convex specialized input expenditure decision. Smaller firms simply spend less, and in equilibrium, are more likely to either have zero ideas that arrive (in which case it becomes a latent firm with no sales) or only one (in which case it becomes a single-industry firm).<sup>59</sup>

To economize on exposition, I develop here the minimal equilibrium structure required to understand how scope economies generate cross-industry propagation. I assume that the mass of firms  $N$  in the economy is fixed and that a distribution  $G(\{\xi_{fj}\})$  describes pre-existing variation in exogenous profitability shifters across firms and industries. I abstract from factor price changes and input price changes.<sup>60</sup>

<sup>58</sup>Fixed costs are problematic in that they introduce non-convexities not just from the point of view of firms, but also from the point of the aggregate economy. That is, even if a solution to the firm's combinatorial discrete choice problem under a partially supermodular and submodular profit function were computationally feasible (perhaps by building on innovations in Jia (2008), Antràs et al. (2017) and Arkolakis and Eckert (2017)), solving for equilibrium residual profits  $\{B_j\}_j$  under firm heterogeneity is itself non-trivial and likely to admit multiple equilibria. Instead, the stochasticity introduced in the model obviates the need to confront these issues, while still allowing for endogenous extensive margin responses within the firm.

<sup>59</sup>Note that firms that have two or more ideas that arrive do not automatically become multi-industry firms. Due to the independent draws, two ideas could both be optimally deployed in the same industry, particularly if the firm has a particularly high latent profitability shifter  $\xi_{fj}$  in that industry.

<sup>60</sup>This setup is sufficient for estimating the model (in Section 4) since  $X_j$  is observed. Estimates of the model's elasticities are invariant to other GE specifications. In the quantification section (5), I close the model in a GE setting calibrated to the US economy, and more general details are given in the Theory Appendix.

In such a setting, equilibrium boils down to solving a system of  $J$  goods market clearing conditions that solve for industry price indices,  $P_j$ :

$$N \int_f \mathbb{E}[X_{fj}; \vec{P}] dG(\xi_f) = X_j, \quad \forall j, \quad (12)$$

where firm sales  $\mathbb{E}[X_{fj}; \vec{P}]$  as denoted in equation (8) depends on  $\vec{P}$  through residual profits  $B_j$ , given by equation (7).

Holding all else equal, equations (12), (8), and (7) can be log-differentiated to yield a system of cross-industry local elasticities of the price index with respect to equilibrium changes in gross output  $X_j$ . I define these macro elasticities in Proposition 3 as aggregate economies of scale and scope.<sup>61</sup>

**Proposition 3 (Aggregate Economies of Scale and Scope)** *The elasticity of the price index  $P_j$  with respect to equilibrium changes in gross output in other industries  $\{X_k\}_{k \in J}$  is given by:*

$$d \log P_j = \frac{1}{\sigma_j - 1} \sum_{k \in J} \underbrace{(\sigma_j(1 - \varsigma_j)[\mathbf{\Upsilon}^{-1}]_{jk} - \mathbf{1}_{j=k})}_{\equiv [\Psi]_{jk}} d \log X_k, \quad \forall j, \quad (13)$$

where  $\Psi$  is a  $J \times J$  spillover matrix that depends on inverting weighted averages of individual firm-level cross-industry elasticities:

$$[\mathbf{\Upsilon}]_{jk} \equiv \sum_m \bar{\lambda}_{jm} \bar{\mu}_{jmk} (\rho_m - \theta_m) + \mathbf{1}_{j=k} \sum_m \bar{\lambda}_{jm} \theta_m, \quad \forall j, k,$$

where the bar over utilization and choice shares  $\bar{\lambda}$  and  $\bar{\mu}$  indicate that they are weighted averages over individual firm-level firm-level shares:

- (i) Choice shares  $\bar{\mu}_{jmk}$  indicate the economy-wide propensity for ideas generated by input  $m$  to be deployed towards expanding knowledge capital in industry  $k$  (relative to other industries  $k'$ ) among firms that produce in  $j$ :

$$\bar{\mu}_{jmk} \equiv \int_f \frac{\lambda_{fjm} \mathbb{E}[\pi_{fj}]}{\int_f \lambda_{fjm} \mathbb{E}[\pi_{fj}] dG(\xi_f)} \mu_{fmk} dG(\xi_f).$$

- (ii) Utilization shares  $\bar{\lambda}_{jm}$  indicate the aggregate contribution to industry  $j$  of ideas generated by

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<sup>61</sup>This is, of course, related to the traditional definition of scale economies as an elasticity of costs to quantities. See Appendix Section B.6 for a derivation of that expression. Concepts as defined here are more useful in monopolistic competition equilibria. The counterfactual shocks I consider (in Section 5) can be constructed as exogenous shifters of market size.

input  $m$  (relative to other specialized inputs  $m'$ ):

$$\bar{\lambda}_{jm} \equiv \frac{\int_f \lambda_{fjm} \mathbb{E}[X_{fj}] dG(\xi_f)}{\int_f \mathbb{E}[X_{fj}] dG(\xi_f)}.$$

Proposition 3 demonstrates that the scalability and rivalry of specialized inputs *within the firm* create interdependence across aggregate industry price indices. This is a new transmission mechanism independent of general equilibrium factor price and input price changes (such as those from Heckscher-Ohlin models or input-output linkages).<sup>62</sup> Cross-industry macro propagation occurs if and only if off-diagonal terms in  $\Upsilon$  are non-zero. This requires inputs to be both (i) co-utilized in multiple industries, and (ii) differ in their scalability and rivalry in production,  $\rho \neq \theta$ .

To understand the first force, suppose that each industry is technologically isolated. This could occur if specialized inputs only serve one industry (i.e.  $m$  is industry-specific capital and ideas generate value only in one industry), or if residual profitability shifters of firms are positive for only one industry within the firm. In this extreme case,  $\Upsilon_{jk} = 0$  for all elements  $j \neq k$  and the inverse is thus a diagonal matrix. Given that other elements of equation (13) operate only on the main diagonal, the spillover matrix  $\Psi$  is also limited to the main diagonal. The fact that spillovers  $\Psi$  relate to the inverse of  $\Upsilon$  captures propagation forces under general equilibrium. Even if a pair of industries  $j$  and  $k$  have no technological overlap<sup>63</sup>, price indices in  $j$  can respond to price indices in  $k$  due to changes occurring vis-a-vis other parts of the economy.

The second condition requires that scale and rivalry effects do not perfectly offset each other within the firm. This perfect offset occurs only in the knife-edge case of Proposition 2 where there are no economies of scope. Scope economies (positive or negative) within the firm are a necessary condition for general equilibrium propagation across industries.

Because within-firm economies of scope are nested within a CES-MC framework, demand elasticities  $\sigma_j$  invariably play an important role in regulating the *overall* magnitude of economies of scale and scope. The value of  $(1/(\sigma_j - 1))$  is an upper bound for the strength of economies of scale.<sup>64</sup> Because of this, I later explore the sensitivity of quantitative results to

<sup>62</sup>One goal behind quantification in Section 5 is to interact and compare spillovers arising from scale and scope to those in I/O models. I show, in Proposition 5, that it is straightforward to nest I/O linkages in this setting—by replacing wage-related unit costs with an appropriated weighted index of factor and input prices using I/O shares as Cobb-Douglas weights.

<sup>63</sup>That is, either  $\alpha_{mj} = 0$  or  $\alpha_{mk} = 0$  for each  $m$  over the set of specialized inputs  $m = 1, \dots, M$ , or firm that have positive profitability shifters in  $j$  have zero shifters in  $k$  and vice versa. In this case  $\bar{\lambda}_{jm} \bar{\mu}_{jmk} = 0$  and off-diagonal terms in  $\Upsilon$  are zero.

<sup>64</sup>This is achieved as  $\rho_m \rightarrow \infty$ , since  $\frac{\sigma_j(1-\zeta_j)}{\sum_m \bar{\lambda}_{jm} \rho_m}$  is always positive.

different values of  $\sigma$ .

### 3.3.1 Relation to Existing Models

Proposition 3 generalizes the macro predictions of existing multi-industry GE models by introducing more degrees of freedom in how specialized inputs adjust within the firm, thereby opening up off-diagonals in the spillover  $\Psi$  matrix. Whenever there are no scope economies within firms,  $\Psi$  reduces to a main-diagonal matrix of own-industry scale elasticities. This knife-edge condition nests a wide class of models with variable own-industry economies of scale. Equation (13) reduces to the following:

$$d \log P_j = \frac{1}{\sigma_j - 1} \left( \frac{\sigma_j(1 - \varsigma_j)}{\sum_m \bar{\lambda}_{jm} \rho_m} - 1 \right) d \log X_j, \quad \forall j,$$

which, for enough flexibility in  $\rho_m$  and  $\sigma_j$ , is isomorphic to the predictions in recent models with variable own-industry returns to scale (Kucheryavyi et al., 2019; Bartelme et al., 2019).

Economies of scale in this setting are increasing in the scalability of inputs,  $\rho_m$ . In the limit as  $\rho_m \rightarrow \infty$  for all  $m$ , the industry-level scale elasticity ( $1/(\sigma_j - 1)$ ) is equivalent to that in a multi-industry Krugman (1980) model with industry-specific free entry. In this limit, the costs of expanding the arrival rate of ideas is linear, so the endogenous response within existing (including latent) firms to shifters of market size equals that achieved by outside entrants in the Krugman model.

Conversely, as  $\rho_m$  approaches the other limit of 1, knowledge capital becomes fixed. As the costs of adjusting the arrival rate of ideas approach infinity, firms respond to market size shifters only by adjusting their use of generic inputs. The returns to scale in generic inputs thus shapes the response of the price index to market size shifters. The higher is the returns to scale  $\gamma_j$ , the lower is  $1 - \varsigma_j$  and the stronger is the productivity response. If there are constant returns to scale behind generic inputs,  $\sigma_j(1 - \varsigma_j) = 1$  and the price index (productivity) does not respond to shifters of market size given a fixed set of firms. Consequently, there are no industry-level economies of scale. Re-introducing aggregate free entry in this limit case reproduces the setup of Bernard et al. (2010).

## 4 Structural Estimation

In this section, I estimate the model's parameters using moment conditions analogous to the within-firm spillovers ( $\psi_{fjk}$ ) examined in Section 2. The model provides the requisite assumptions and structure to leverage the identification insight of Proposition 1, and attributes

economies of scope to the scalability ( $\rho_m$ ) and rivalry ( $\theta_m$ ) of different specialized inputs. I proceed parsimoniously and estimate a pair of scale and rivalry elasticities common to all intangible inputs, and another pair common to a set of residual specialized inputs. Despite this parsimony, the matrix of intra-firm cross-industry spillovers in the model,  $\psi_{fjk}$ , is still fully unrestricted, permitting a direct connection to that in the data.

Relative to the reduced-form, the structural estimation procedure accounts for selection on unobservables, entry and exit, and allows empirical coefficients (that are estimated on only a continuing panel of multi-industry firms) to be re-interpreted as structural elasticities that affect all firms in the economy. Crucially, the model's aggregation properties enable the use of BEA industry-level data in estimation. I invert these data through the model's equilibrium conditions to identify the technology parameters behind specialized inputs as well as levels of residual profits in each year.

## 4.1 Framework

**Input Taxonomy.** I take an ex-ante stance on which inputs are generic (acquired at constant marginal costs and perfectly rival in use) versus specialized (acquired at convex costs and partially non-rival). Consistent with the sector-specific tests for rivalry and scalability in Figure 2, I classify all inputs in agriculture, mining, construction, and utilities, manufactures, and wholesale, retail and transportation, and labor as generic.<sup>65</sup> I then estimate a common set of elasticities ( $\rho^{INT}, \theta^{INT}$ ) for the set of intangible inputs in the reduced form, and another set of elasticities ( $\rho^{RES}, \theta^{RES}$ ) for the remaining inputs.<sup>66</sup> I let  $\Theta$  denote the parameter set containing all four elasticities.<sup>67</sup>

Since the reduced-form results do not uncover any bilateral expenditure-driven industry proximity other than intangibles that generates spillovers, I collapse the set of residual inputs in the I/O tables to one category. The elasticities ( $\rho^{RES}, \theta^{RES}$ ) serve as a broad catch-all for any latent mechanism contributing to economies (and diseconomies) of scope, such as financial frictions, span-of-control, as well as demand-side cannibalization (or complementarity) effects. I also collapse the set of intangible inputs into just the three broad categories shown in Figure 2 corresponding to (i) headquarters services, (ii) the leasing of intangible assets, and (iii) professional services and information. This significantly reduces the number of technology parameters that I need to invert for.

<sup>65</sup>These correspond to NAICS sectors 1, 2, 3, 4, and labor value added. Leaving intermediate inputs such as manufactures as generic has the added benefit of allowing for a comparison with quantitative effects from standard I/O linkages (i.e. from manufacturing inputs to manufacturing downstream industries).

<sup>66</sup>These correspond to NAICS sectors 52, 531, 532, 56, 6, 7, 8, 9, and capital value added.

<sup>67</sup>In principle, with unlimited data and computing power, one can estimate separate scale and rivalry elasticities for each type of input  $m$  (among the roughly 300 input categories in the BEA I/O table). In practice, the burden of precisely estimating more than 600 elasticities in a highly co-linear setting is insurmountable.



**Timing.** To leverage time-variation within the firm for structural estimation, I interpret each period in the data ( $t = \{1, 2, 3\}$ ) as separate outcomes of the static equilibrium described in the model. Every five years, firms re-optimize expenditures on specialized inputs and re-accumulate knowledge capital conditional on their latent profitability shifters  $\xi_{fjt}$ .<sup>68</sup> The only firm characteristic that persists across periods is the unobserved firm-industry profitability shifters,  $\xi_{fjt}$ , which explain dynamic persistence of firm outcomes in the data even in the absence of any shocks.

The mass of firms is fixed over this period of time, at  $N$ . But there will still be observed entry and exit of firms into and out of ‘active’ status. Recall that an inactive firm is a firm that, despite its chosen level of investment, has not had any ideas arrive that build knowledge capital. In any year in which this happens, the firm will have zero sales, fall out of the observed sample, and thus ‘exit’. Likewise, in any subsequent year in which the same firm does find ideas to build knowledge capital, the firm reappears as an entrant.<sup>69</sup>

**Parametrization.** While a sufficiently rich covariance structure in the cross-sectional distribution of  $\xi_{fj}$  alone can explain any pattern of co-production, it cannot explain the dynamic responses of firms to shocks in the reduced-form. For this reason, I rely on the variation induced by dynamic spillovers (Proposition 1) to estimate elasticities. In order to highlight the ability for the elasticities to explain co-production in the model, I impose a zero covariance structure on the draws of firm profitability shifters across industries (even when, in practice, they might be correlated). These statements are formalized in Assumption 3:

**Assumption 3 (Parametrization of Profitability Shifters)** *Latent firm-industry profitability indices are distributed joint lognormal according to:*

$$\begin{aligned}\xi_{fj,t=1} &= \zeta_{fj} \zeta_f, & \forall f, j \\ \log \zeta_{fj} &\sim i.i.d. \mathcal{N}(0, \gamma_0), & \log \zeta_f \sim i.i.d. \mathcal{N}(0, \gamma_1), & \forall f, j.\end{aligned}$$

The means of the lognormal distributions are isomorphic to proportional shifters in industry residual demand,  $B_j$ , so the normalization to 0 is without loss of generality. The

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<sup>68</sup>Think of  $\xi_{fj}$  as a stand-in for accumulated capital in a full-fledged dynamic model that doesn’t depreciate over the time horizon I study. These assumptions are consistent with a dynamic framework if knowledge capital decays fully within 5 years, and latent profitability shifters do not decay. I show in the Online Supplementary Appendix that the model can also accommodate an arbitrary depreciation rate and also an arbitrary time frame, including an infinite horizon model. The memoryless and additive separability properties of the model enable tractability even in a full-fledged dynamic setting.

<sup>69</sup>I assume that firm longitudinal identifiers in the Census data break whenever a firm goes into a period of inactivity. It is also worth pointing out that—due to endogenous spending  $I_{fm}$ , entry is also ‘directionally’ endogenous - if residual profits  $B_j$  gets smaller i.e. due to increased foreign competition, the probability of ‘entry’ in an industry is also smaller. Interestingly, the same can happen if an industry becomes more competitive (smaller  $B_j$  due to spending on specialized inputs by multi-industry firms with good fundamentals in other booming industries  $k$  that co-utilize these specialized inputs).



variance parameters  $\gamma_0, \gamma_1$  control the degree of ex-ante dispersion in productivities (i) across industries within the firm, and (ii) across firms. The former,  $\gamma_0$ , explains persistence in firm outcomes over time despite full depreciation of accumulated specialized inputs. I estimate  $\gamma_0$  by matching the aggregate share of industries of multi-industry firms that survive over 5-year intervals in the model to that in the data, equal to 0.42. The latter,  $\gamma_1$ , explains why some firms are larger than others. I estimate  $\gamma_1$  by matching the aggregate share of sales by multi-industry firms in 1997 in the model to that in the data, equal to 0.75.<sup>70</sup>

In each year, a discrete and measure-zero number of firms receive export demand shocks  $\{\Delta SHK_{fkt}\}_k$  as constructed in the data. I model the impact of these shocks as reduced-form shifters of firm profitability,  $\xi_{fkt}$ , according to the following assumption:<sup>71</sup>

**Assumption 4 (Impact of Demand Shocks)** *Demand shocks as constructed in the data affect firm-industry latent profitabilities according to*

$$\log \xi_{fj,t} = \log \xi_{fj,t-1} + \nu \Delta \log S_{fjt} \quad \forall f, j, t = \{2, 3\}.$$

The parameter  $\nu$  is a ‘first-stage’ elasticity that captures the relevance of demand shocks  $\Delta \log S_{fkt}$ , as they are measured in the data, at shifting firm profitability in a given industry. The reduced-form, firm-level response of intangible input expenditures to demand shifters, in Table 3, identifies  $\nu$  conditional on  $\rho^{INT}$ . Log-differentiating expenditures  $M_{ft}^{INT}$  on all intangibles in the model yields

$$\Delta \log M_{ft}^{INT} = \rho^{INT} \cdot \nu \sum_k \eta_{fk}^{INT} \Delta \log S_{fkt},$$

where the theoretical expenditure shares  $\eta_{fk}^{INT}$  are approximated using aggregate I/O expenditure shares of an industry on intangibles:

$$\eta_{fk}^{INT} \equiv \frac{\sum_{m \in INT} \mu_{fmk} \Delta_{fm}^{\rho_m}}{\sum_{m \in INT} \Delta_{fm}^{\rho_m}} \approx \frac{\beta_{k,INT} X_{fk}}{\sum_k \beta_{k,INT} X_{fk}}.$$

Column (1) of Table 3 estimates the combined elasticity  $\rho^{INT} \nu$  at 0.65. This ‘offline’ relationship yields a value of  $\nu$  conditional on any estimate of  $\rho$ , and helps reduce dimensionality of the non-linear search below.<sup>72</sup>

<sup>70</sup>These are estimates instead of calibrated parameters because required values of  $\gamma_0, \gamma_1$  change conditional on estimates of the model’s scale and rivalry elasticities.

<sup>71</sup>This assumption can be properly micro-founded in a multi-destination exporting model by having firms draw different latent profitability shifters across destinations. See the Online Supplementary Appendix for details.

<sup>72</sup>Of course,  $\nu$  as parametrized here is an only approximation: it is assumed to be constant across industries,

Table 4: Overview of Model Primitives and Source of Identification

Variable	Description	Source of Identification
$\Theta$	Scale and rivalry elasticities of specialized inputs	Within-firm spillovers, $\psi_{fjk}$
$\gamma_0$	Within-firm heterogeneity in $\xi_{fjt}$	Share of industries that continue (0.42)
$\gamma_1$	Across-firm heterogeneity in $\xi_{ft}$	Share of sales by multi-industry firms (0.75)
$\nu$	Responsiveness of $\xi_{fjt}$ to export demand shocks	Assumption 4, Table 3, $\rho^{INT}\nu = 0.65$
$N$	Mass of latent firms	All active and inactive firms (318,000)
$\varsigma_j$	Expenditures on generic inputs as a share of sales	Corresponding share in I/O Table
$\alpha_{mj}$	Effectiveness of input $m$ for industry $j$	I/O Table Expenditures $M_{mj}$
$B_{jt}$	Equilibrium industry-wide residual profit shifter	Gross Output $X_{jt}$
$Z$	Efficiency of innovation on the extensive margin	Share of multi-industry firms (0.2)

Table 4 provides an overview of the remaining macro-level variables that can be inverted from the data conditional on knowledge of  $\Theta, \gamma, \nu$ . First, I fix the mass of firms at  $N = 318000$ , the total number of unique firms ever to appear in the 1997 census of manufacturing (including administrative and inactive records). Second, I read off  $1 - \varsigma_j$  from the industry-level share of gross profits in sales.<sup>73</sup> Third, the technology coefficients and residual profit shifters  $\alpha_{mj}, B_{jt}$  are positively related to aggregate expenditures (by input) and industry sales per firm. With one degree of normalization afforded per industry, I choose to normalize the technology coefficient on the residual category of specialized inputs to 1. This normalization pins down  $B_{jt}$  and  $\alpha_{mj}$  for  $m \in INT$  in the year 1997 based on I/O table data, and fixing  $\alpha_{mj}$  to their 1997 levels, I can recover  $B_{jt}$  based on gross output data in subsequent years.<sup>74</sup> Finally, I estimate the technology parameter in the cost function for the arrival rate of ideas,  $Z$ , by matching the model's share of multi-industry firms to that in the data (0.2).<sup>75</sup> I normalize the wage to equal 1.

The estimation routine developed here is agnostic to general equilibrium details (i.e. whether there is trade, or whether there are input-output linkages among generic inputs). Estimates of the model's elasticities (and macro parameters in Table 4) are also invariant

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when in reality scale elasticities and changes to price indices in foreign destinations could make the impact of demand shocks different across industries. A more serious attempt could be made to leverage variation in micro responses for estimating demand elasticities  $\sigma_j$  directly; I do not take up that exercise in this paper.

<sup>73</sup>Importantly, note that this does not translate to economic net profits. Net profits are gross profits less any expenses incurred towards specialized inputs. This can vary across firms both because different types of firms incur different mixes of expenditures depending on their inherent profitability shifters  $\xi_{fj}$ , and also because sales at the firm-level are stochastic: a firm could have gotten lucky with a lot of high-quality ideas (and thus high sales) with a small amount of specialized input expenditures.

<sup>74</sup>I principle I can re-estimating  $\alpha_{mj}$  in each year. In practice, I don't have good industry-by-expenditure data on intangibles after 1997 due to changes to BEA accounting rules.

<sup>75</sup>This parameter shifts the importance of the extensive margin number of arrivals relative to the intensive margin quality per arrival. While it has no aggregate implications (aggregation includes both intensive and extensive margins), it does affect the computation of the conditional moment condition which matches firms based on observed outcomes on the extensive margin.

to values of generic input returns to scale  $\gamma_j$  and demand elasticities  $\sigma_j$ . Observations of industry sales  $X_{jt}$  and gross profits are sufficient for identifying macro industry-wide residual profits,  $B_j$ . These residual profit terms act as a macro sufficient statistic for the firm (along with technology parameters  $\alpha$  and idiosyncratic profitability shifters  $\xi$ ).

## 4.2 Inference

I define firm-industry level structural residuals,  $\epsilon_{fjt}$ , as the linear deviation of observed sales in the data from expected sales in the model (conditional on  $\xi_{ft}, B_t$ ):

$$\epsilon_{fjt} \equiv X_{fjt} - \mathbb{E}[X_{fjt} | \xi_{ft}].$$

Variation in  $\epsilon_{fjt}$  comes from stochasticity in  $\varphi_{fj}$  induced by Poisson and Fréchet-distributed shocks. The structural residuals are mean-independent of any other observable firm-level variables, such as demand shocks  $\Delta \log S_{fs}$  and industry presence  $\chi_{f,s}$  for any year  $s$ , since  $\xi_{ft}$  contains all the relevant information for the firm's investment and allocation decisions in any given year  $t$ . By the law of iterated expectations (on  $\xi_{ft}$ ):

$$\mathbb{E}_f[\epsilon_{fjt} \mid \Delta \log S_{fs}, \chi_{f,t-1}] = 0, \quad \forall f, j, t, s.$$

Features of the data such as demand shocks and extensive margin outcomes (in the past) can serve as 'instruments'.<sup>76</sup> I exploit variation in the vector of demand shocks,  $\Delta \log S_{fs}$ , to identify  $\Theta$ . Conducting inference in this way requires taking a stance on the relationship between export demand shocks  $\Delta \log S_{fs}$  and unobserved profitability shifters,  $\xi_{f,s-1}$ . Assumption 5 below stipulates conditional independence: export demand shocks are randomly assigned to firms conditional on pre-existing industry presence (i.e. the extensive margin):

**Assumption 5 (Conditional Independence)** *Export demand shifters are randomly assigned to firms conditional on pre-existing industry presence:*

$$\Delta \log S_{fjt} \perp \{\xi_{fk,t-1}, X_{fk,t-1}\}_{k \in J} \mid \{\chi_{fk,t-1}\}_{k \in J} \quad \forall j.$$

This assumption is a significant weakening over a simple (unconditional) exogeneity assumption, because it allows shocks to be non-parametrically correlated with past industry presence. This is almost necessary in my setting, where, by construction, only firms with multi-industry presence receive export demand shocks, and where firm-industry shocks are

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<sup>76</sup>Note that the vector of 'ones' are implicitly being used as instruments to identify  $B_{jt}$  by relying on data we have on the average firm outcome over the entire population  $X_{jt}/N$ , and the model's prediction for that given the parametric distribution of  $\xi_{fj}$ .

mechanically zero whenever a firm is not active in an industry. Another advantage of this assumption is that it takes into account potential correlations between shocks and firm latent productivity shifters. If, systematically, firms with higher  $\xi_{ft}$  are in more industries, and these firms receive larger shocks, this correlation would be accounted for in the moment conditions used for inference, which I derive in Proposition 4.

**Proposition 4 (Inference)** *Let  $\widetilde{\Delta \log S_{fkt}}$  denote demeaned demand shocks by industry-year. Define the following analytical sample moment conditions for a pair of industries  $j, k$  and year  $t = \{2, 3\}$ :*

$$m_{jkt} \equiv \frac{1}{n_{jk,t-1}} \sum_{f \in n_{jk,t-1}} \left( (X_{fjt} - X_{fj,t-1}) - \sum_{s \in S} \omega_{sf} m_{fjk}(\xi_s, \Delta \log S_{ft}) \right) \widetilde{\Delta \log S_{fkt}},$$

where  $n_{jk,t-1}$  is the set of firms  $f$  in the data with positive sales in  $j$  and  $k$  in year  $t-1$ ,  $s \in S$  denotes a sample of simulated firms with profitability shifters  $\xi_s$  drawn from distribution  $G(\xi_s)$  according to Assumption 3,  $\omega_{fs}$  refers to the probability that a simulated firm ( $s$ ) (with fundamentals  $\xi_s$ ) matches that of a firm ( $f$ ) in the data (industry presence  $\chi_{f,t-1}$ ) relative to other  $s' \in S$ :

$$\omega_{fs} \equiv \frac{\prod_j \Pr(\chi_{j,t-1} = \chi_{fj,t-1} | \xi_s)}{\sum_{s'} \prod_j \Pr(\chi_{j,t-1} = \chi_{fj,t-1} | \xi_{s'})},$$

and  $m_{fjk}$  is model-implied expected sales growth given by

$$m_{fjk} = \mathbb{E}[X_{fjt} | \xi'_s] - \mathbb{E}[X_{fj,t-1} | \xi_s, \chi_{fj,t-1} > 0],$$

where next-period latent profitability  $\xi'_s$  evolves conditional on  $\xi_s$  and empirical demand shocks  $\Delta \log S_{ft}$  according to Assumption 4.

At true parameter values  $\Theta, \gamma$ , as the data and simulation samples get large,  $N, S \rightarrow \infty$ , the sample moment  $m_{jkt} = 0$  for any  $j, k$  and  $t \in \{2, 3\}$ .

These moment conditions offer several advantages for inference. First, all expressions are closed-form analytical objects, so that no simulation is required (other than a fixed initial set of draws of  $\zeta_s$  used to parametrize  $\xi_s$ ). Second, I use demeaned versions of demand shocks to avoid the mean picking up differences in variances in firm growth across moments. Third, the moment conditions allow and utilize the endogeneity of industry (and firm-wide) exit as a response to shocks. Lastly, the sampling weights,  $\omega_{sf}$ , take care of the correlation between observed initial-period industry presence and the shocks received in the data, so that inference is conducted not from the full parametric distribution of  $\xi_f$  but from the relevant parts of the distribution that are more likely to yield the kinds of multi-industry firms observed in the sample.

In practice, because there are very few observations in each non-diagonal  $jk$  cell, I collapse the set of  $J \times J$  moments in each year to four sets of moments based on observable bilateral characteristics of industries. I make these grouping choices based on ex-ante characteristics of industries in the I/O tables in 1997:

1. Main diagonals of the matrix,  $j = k$ , for industries  $j$  where expenditures on intangibles as a share of gross output is higher than the mean. This helps identify the residual scale elasticity  $\rho^{INT}$ .
2. Main diagonals of the matrix,  $j = k$ , for industries  $j$  where expenditures on intangibles as a share of gross output is lower than the mean. This helps identify the residual scale elasticity  $\rho^{RES}$ .
3. Off-diagonals of the matrix,  $j \neq k$ , for industry pairs  $jk$  where the proximity in the joint use of intangibles is higher than the mean over all industry pairs. This helps identify the residual rivalry elasticity  $\theta^{INT}$ .
4. Off-diagonals of the matrix,  $j \neq k$ , for industry pairs  $jk$  where the proximity in the joint use of intangibles is lower than the mean over all industry pairs. This helps identify the residual rivalry elasticity  $\theta^{RES}$ .

In total, I estimate six non-linear parameters  $(\rho^{INT}, \theta^{INT}, \rho^{RES}, \theta^{RES}, \gamma_0, \gamma_1)$  using ten moments: four dynamic moments each year (for  $t = 2, 3$ ) corresponding to the four groupings of cells in the  $J \times J$  matrix, and two remaining cross-sectional moments (from  $t = 1$ ) used to estimate the variances in the lognormal distribution:  $\gamma_0, \gamma_1$ . I use the identity weighting matrix to weigh moments.<sup>77</sup>

Estimation proceeds as follows:

1. Simulate a fixed set of 2000 firms, with baseline draws of  $\zeta_{sj}, \zeta_s$  from standard normal distributions. I use stratified sampling to over-weigh firms with higher  $\zeta_s$ .
2. Guess a starting  $\hat{\Theta}, \hat{\gamma}_0, \hat{\gamma}_1$ , then repeat Steps 3-5 until convergence criterion is met.
3. Compute  $\xi_{sj}$  given  $\hat{\gamma}_0, \hat{\gamma}_1$  from Assumption 3 and  $\zeta_{sj}, \zeta_s$ .
4. Given  $\xi_{sj}$  and  $\hat{\Theta}$ , invert for  $\alpha_{mj}, B_{jt}, Z$  using macro data.
5. Compute the sample moment conditions in Proposition 4, stack the moments as described above, and use a bounded Nelder-Mead simplex search algorithm to adjust the guess of  $\hat{\Theta}, \hat{\gamma}_0, \hat{\gamma}_1$  given the change in the objective value.

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<sup>77</sup>Estimates do not change by much when using the optimal weighting matrix under 2-step GMM, which I compute when conducting the test of over-identifying restrictions.

Table 5: Estimated Model Elasticities

Parameter	Description	Estimate	S.E.
$\rho^{INT}$	Intangible Input Scalability	12.64	(0.39)
$\theta^{INT}$	Intangible Input Rivalry	3.61	(0.07)
$\rho^{RES}$	Residual Input Scalability	2.63	(0.05)
$\theta^{RES}$	Residual Input Rivalry	4.06	(0.12)
$\gamma_0$	Variance in $\zeta_{fj}$ draws	0.85	(0.03)
$\gamma_1$	Variance in $\zeta_f$ draws	0.99	(0.05)
Test of Over-identifying Restrictions: $7.28 \sim \chi_4^2$		$p = 0.12$	

This table reports estimates of structural parameters in the model. The six parameters are estimated on the sample of all multi-industry firms and the industries in which they are active, in years 2002-2007 and 1997-2002, using 10 moments. There are 13,000 firm-year observations used in the sample. Standard errors of estimates are computed based on results from 21 bootstrap samples, where I re-draw over both the data and the simulated  $\xi$  samples.

### 4.3 Results

Results are given in Table 5. Consistent with the directional results in the reduced-form, I find that intangible inputs have much higher scale than rivalry elasticities, whereas the opposite is true for the set of residual specialized inputs, and that these claims are statistically significant. The test of over-identifying restrictions does not reject the null that the moments used in estimation are jointly valid, suggesting a good fit to the micro variation in the data.

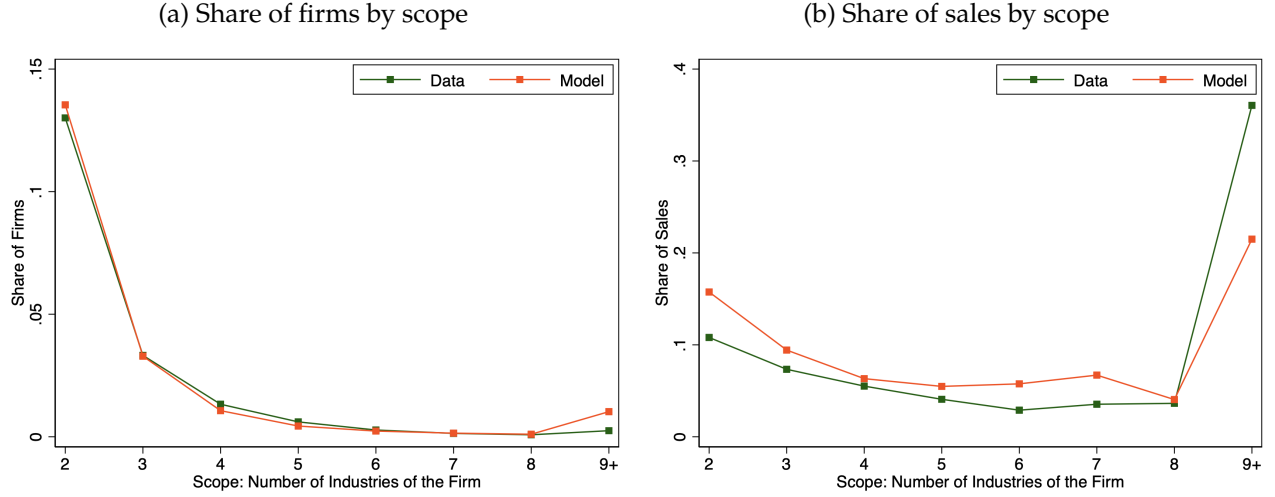
### 4.4 External Validation

Despite its parsimony, the model is capable of reproducing static features of the data that were not targeted during estimation (recall that variation used to identify  $\Theta$  comes from dynamic changes in firm sales over time).

First, the model matches the distribution of the number of firms and their sales over firm scope. Figure 3 shows that the data and the model form a close match—firms with higher scope are increasingly scarce but also increasing large in overall sales. Both the data and the model attribute a significant size premium to the extreme right tail of the firm scope distribution (firms with 9 or more industries), though the model undershoots the data by some amount. Appendix Table 12 reports the numbers behind these figures. Overall, the close fit between the model and data validate the function form assumptions (Poisson and Fréchet) used in the model.

Another feature in the data is a strong and increasing pattern of joint production associ-

Figure 3: External Validity: Distribution of Firms and Sales by Scope



Panel (a) plots the share of total firms accounted for by firms across the scope distribution, comparing the data (in 1997) to the model. Panel (b) plots the share of total sales accounted for by firms across the scope distribution, comparing the data to the model.

ated with industry co-utilization of intangible inputs. I measure joint production in a pair of industries  $jk$  as the share of industry  $j$  sales by firms with activities in  $k$ :

$$JointProd_{jk} \equiv \frac{\sum_f X_{fj} \mathbf{1}_{X_{fk} > 0}}{\sum_f X_{fj}},$$

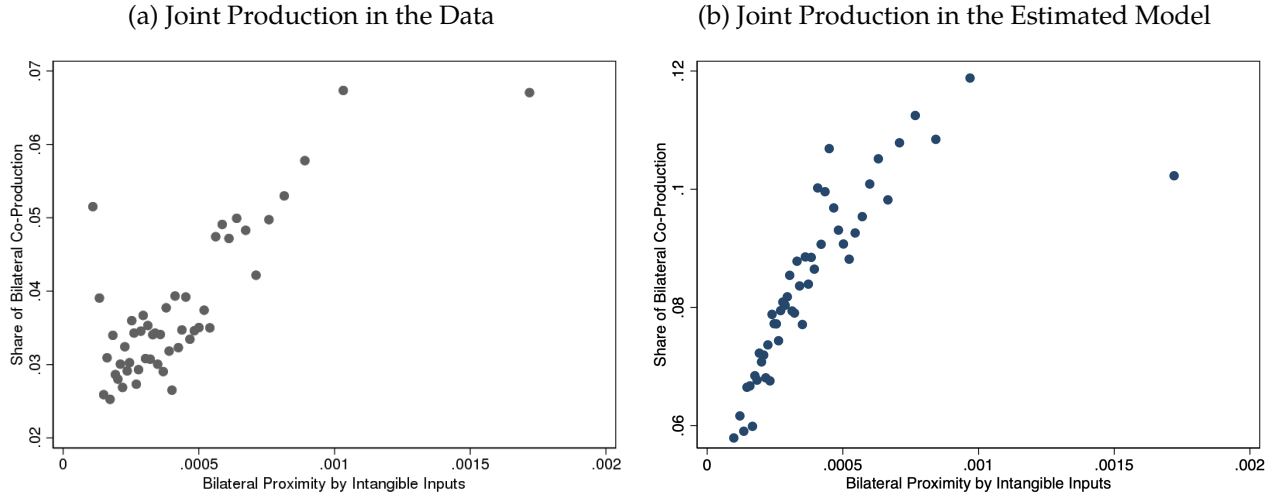
and industry co-utilization of intangible inputs as

$$Prox_{jk}^{INT} \equiv \sum_{m \in INT} \beta_{mj} \frac{\beta_{mk}}{\sum_{k'} \beta_{mk'}}.$$

Panel (a) of Figure 4 visualizes this relationship in the data in terms of a binscatter, for the full 42,240 pairs of industries  $j, k$ , where  $j \neq k$ , in 1997. Panel (b) in the same figure reproduces this relationship using the model's elasticities and parameters alone. The model predicts higher joint production among industry pairs that use similar intangible inputs. Joint production is high in these industries precisely because intangible inputs are scalable (resulting in higher investments by the firm) and non-rival (resulting in a greater fraction of times that ideas are deployed to both industries).



Figure 4: External Validity: Joint Production in the Data and the Model



These panels display the relationship between bilateral industry joint production (share of industry  $j$  sales by firms with activities in  $k$ ) and a measure of  $jk$  proximity in the utilization of intangible inputs, in (a) the data, and (b) the model.

## 5 Quantitative Implications

In this section, I embed the estimated model in open-economy general equilibrium to quantify US industry-level responses to exogenous foreign shocks and tariffs. I calibrate parameters of the open-economy model to data on the US economy in 2017.

Using analytical decompositions permitted by the spillover matrix,  $\Psi$ , I find that 20% of the aggregate productivity response to foreign demand shocks occurs as a result of scope economies and manifests in the form of cross-industry effects. Moreover, these productivity spillovers are unevenly distributed. Industries that utilize more intangible inputs are stronger transmitters as well as recipients of productivity spillovers.

Finally, I apply the model to analyze the price and productivity effects of a US trade policy. I estimate that a uniform 20% tariff on US imports from China raises the US manufacturing CPI by 0.8%, with domestic productivity responses mitigating the direct impact of higher foreign prices. I use the analytical spillover matrix  $\Psi$  to identify alternative tariff implementations that are biased towards industries with higher internal economies of scale and scope. I find alternative tariffs on Chinese imports that achieve the same (possibly political) objectives of reducing trade volumes with China—at a cost of only a 0.38% rise in the manufacturing CPI.

## 5.1 General Equilibrium Setting

I embed the model in an open economy setting where the only cross-industry propagation effects are due to either internal economies of scope or external input-output linkages (that I can turn on or off). This allows me to attribute spillovers precisely to economies of scope and illustrate how they interact with propagation effects driven by external linkages.

I assume that upper-tier demand is Cobb-Douglas across industries, which shuts down demand-side inter-industry substitutability. When transitioning to a setting with external input-output linkages, I open up the bundle of generic inputs,  $l_j$ , in equation (5). I assume that it is a Cobb-Douglas aggregator over labor and output from other manufacturing industries, and I calibrate these generic input expenditure shares to match those in the I/O table. I abstract from the free entry of new firms and focus on interactions among existing (including inactive) firms (under the estimated distribution of profitability shifters,  $\xi$ ). I calibrate the within-industry elasticity of substitution at  $\sigma_j = 5$  across the board and explore sensitivity of results to alternative values of  $\sigma_j$ .<sup>78</sup>

I model the US economy under trade with two foreign partners: China (singled out for the purposes of tariff counterfactuals), and a rest-of-the-world composite. While levels of foreign industry expenditures and foreign price indices are assumed to be exogenous, I endogenize the prices of goods sold by US firms. I abstract from wage effects by assuming that there is a large enough residual sector (e.g. intra and inter-temporal services) in which the US is a net exporter facing perfectly elastic foreign demand over the range of shocks considered. This pins down the wage under overall US trade balance (over combined trade in manufacturing and residual services). Instead of the wage adjusting, the manufacturing trade deficit becomes endogenous, allowing import protection to lead to export promotion through scale and scope economies. All firm profits (regardless of whether they are rebated or invested) are spent on the residual sector to shut out feedback effects through increased manufacturing demand.

Definition 2 in the Quantitative Appendix formalizes these assumptions in terms of equilibrium conditions in the economy. It also describes how I calibrate relevant model objects (i.e. exogenous foreign price indices) from aggregate industry-level production and trade data for the US in 2017.

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<sup>78</sup>The IO tables only allow me to read off values of  $\varsigma_j = \gamma_j \frac{\sigma_j - 1}{\sigma_j}$ . Setting  $\sigma_j$  to a common value across industries is a reasonable benchmark for the counterfactuals because it ensures that the asymmetric industry results are not driven by differences in demand substitutability.

## 5.2 Sizing up Economies of Scope

I log-differentiate the system of equations in Definition 2 to derive a unified analytical propagation matrix relating aggregate domestic industry productivity indices,  $PD_j$ , to any exogenous unit-elasticity shifter of market size,  $S_k$ .

**Proposition 5** *Under the open economy equilibrium characterized in Lemma 2, exogenous shocks to market size faced by US firms,  $d \log S_k$ , generate the following effects on domestic industry  $j$  manufacturing productivity  $d \log PD_j$ :*

$$d \log PD = - \left( \mathbb{I} - \Omega^P + \Psi (\mathbb{I} - \Omega^R)^{-1} \text{diag}(\lambda^{cpt}) \right)^{-1} \Psi (\mathbb{I} - \Omega^R)^{-1} d \log S, \quad (14)$$

where  $PD_j$  indexes productivity (inverse price) of domestic manufacturers:

$$PD_j \equiv N \int_f \mathbb{E} \left[ p_{fj}^{1-\sigma_j} \right] df,$$

$\Psi$  is the spillover matrix given in equation (13),  $\mathbb{I}$  is the identity matrix, and  $\Omega^P, \Omega^R$  are matrices containing external input-to-output coefficients:

$$[\Omega^P]_{jk} \equiv \frac{\gamma_{kj} \varsigma_j \sigma_j \lambda_{uuk}^c}{\sigma_k - 1}, \quad [\Omega^R]_{jk} \equiv \lambda_{uj}^s (1 - \lambda_{fin,j}^c) \frac{\gamma_{jk} \varsigma_k X_k}{\sum_{k'} \gamma_{jk'} \varsigma_{k'} X_{k'}},$$

$\lambda_{fin,j}^c$  is the share of final use in consumption in industry  $j$ , and  $\lambda_j^{cpt}$  is a measure of foreign competition given by

$$\lambda_j^{cpt} \equiv \sum_{d \in \{u, r, c\}} \lambda_{dj}^s (1 - \lambda_{udj}^c),$$

where  $\lambda_{udj}^c$  is the share of destination  $d$ 's consumption of industry  $j$  on goods sold by the US,  $u$ , and  $\lambda_{dj}^s$  is the share of US firms' total sales in industry  $j$  going to destination  $d$ .

Equation (14) presents a general formula for understanding the first order effects of exogenous unit-elasticity shifters of market size,  $d \log S$ , on the economy. Proposition 5 nests Proposition 3 as a special case when shifters of market size are domestic population  $L$ , the economy is in autarky (so  $\lambda_j^{cpt} = 0$ ), and there are no input-output linkages (so that  $\Omega^P = \Omega^R = 0$ ).

I use this analytical propagation matrix to decompose the effects of proportional increases in foreign market size<sup>79</sup> (across all industries) on the US manufacturing PPI. This is computed

<sup>79</sup>The term  $d \log S$  refers to a unit-elasticity demand shifter for US producers. To scale a foreign shock  $d \log \tilde{Y}$  into a unit-elasticity impact I use  $d \log S = (1 - \lambda_{uj}^s) d \log \tilde{Y}$ . With an alternative scaling, these increases in foreign market size can also be interpreted as reductions in US export trade costs.

as a weighted average of industry-level productivity responses, with weights equal to current output shares across industries,  $\lambda_j^{prod}$ :

$$d \log PPI = \sum_j \frac{\lambda_j^{prod}}{1 - \sigma_j} d \log PD_j.$$

The impact of foreign demand shocks on productivity can be decomposed by splitting up the matrix in equation (14) into (i) diagonal elements, representing the direct effect (of a shock in industry  $k$  on productivity in the same industry  $k$ ) versus (ii) off-diagonal elements, representing spillovers (a shock in  $k$  on productivity in  $j$ ).

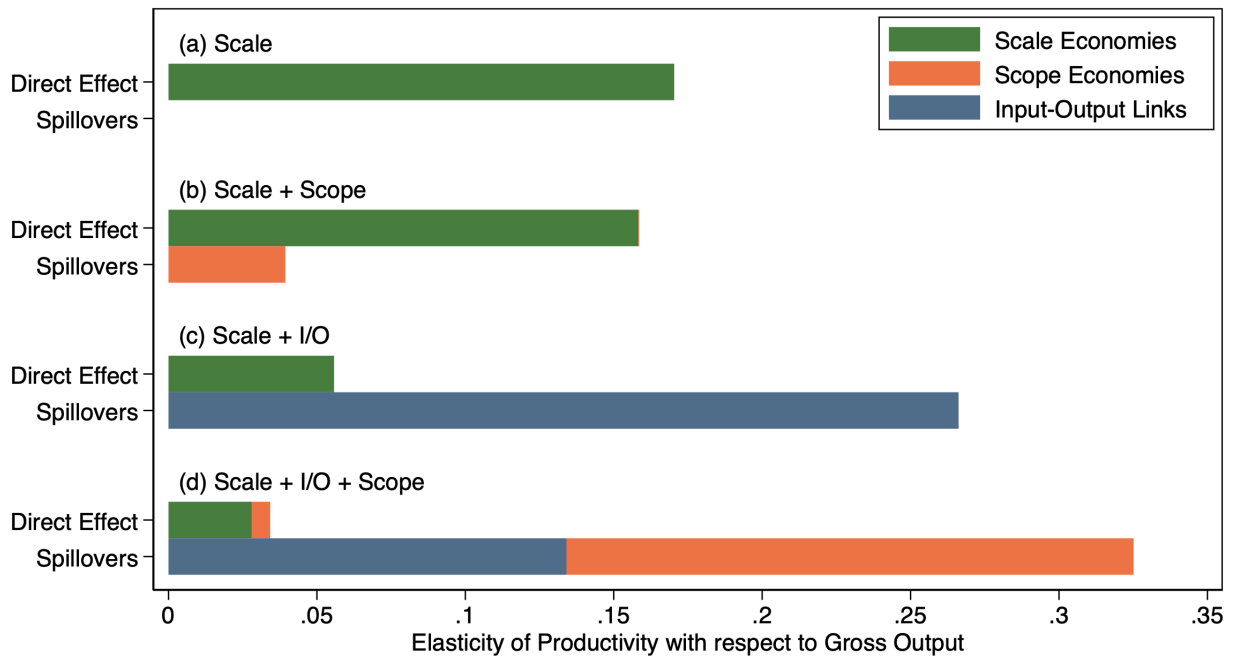
Figure 5 illustrates results from performing the decomposition above under four scenarios, labelled (a)-(d). To impose appropriate units on the exercise, in each scenario I scale the productivity change by the total response in gross output, so that the sum over direct effects and spillovers yields the macro elasticity of productivity to gross output. The accompanying numbers (along with a companion decomposition for the response of gross output to foreign shocks) are given in Appendix Table 13. The first two scenarios turn off external input-output linkages, while the latter two turn them on. In scenario (a), I pretend that an analyst has data only on own-industry responses in the micro spillover matrix of equation (13),  $\Upsilon$ , through, perhaps, estimating returns to scale using data on single-industry firms. In this scenario, the only impact of foreign shocks on the economy are direct, own-industry responses, depicted by the green bar in the figure.

Scenario (b) is my benchmark case and illustrates what happens when  $\Upsilon$  (and, by association,  $\Psi$ ) incorporates both scale and scope economies. The orange bar depicts productivity responses accruing due to cross-industry spillovers (economies of scope). On net, scope economies generate an elasticity of productivity to output of 0.04 and account for 20% of the total elasticity of productivity to gross output. The direct effect (green bar) is slightly diluted owing to a higher response of gross output (the scaling on the denominator).

Scenarios (c) and (d) of Figure 5 mirror the exercise in (a) and (b) but under a calibration featuring external input-output linkages across industries. Scenario (c) depicts these productivity responses in the presence of only own-industry returns to scale and external I/O linkages. Input-output linkages induce large propagation effects due to the resulting circular structure of production, depicted by the relative size of the blue bar. The total response of gross output is much higher under such a world, and because productivity is induced by scale, the total productivity response is now much higher, with a larger proportion explained by spillovers relative to the direct effects.

Finally, I bring together scale, scope and input-output linkages in scenario (d) by combin-

Figure 5: Elasticity of Productivity with respect to Gross Output



This figure decomposes the elasticity of the US manufacturing PPI (on the  $x$ -axis) with respect to gross manufacturing output (induced by a proportional increase in foreign demand) into (i) a direct, own-industry effect (green bar), (ii) spillovers that accrue due to scope economies (orange bar) and (iii) spillovers that accrue due to input-output linkages (blue bar). This decomposition is performed under four different scenarios, (a)-(d), corresponding to different underlying assumptions about the economy. For more details on the four scenarios, see the main text. Appendix Table 13 presents separately the responses of productivity and output used to compute these elasticities.

ing the full  $\Psi$  matrix with  $\Omega$ . I find that internal spillovers due to scope are quantitatively important not only in the setting without I/O links (comparing scenarios (a) and (b)) but also in the setting with external linkages (comparing scenarios (c) and (d)). In this latter case, allowing for internal scope economies more than doubles the total productivity response, almost all of which accrues due to cross-industry spillovers.

In addition to large cross-industry responses in the aggregate, I also find significant heterogeneity in the strength of spillovers across industries. I find that intangibles-intensive industries are stronger transmitters as well as recipients of productivity spillovers. Figure 6 plots, for each industry, the share of the total aggregate productivity change accruing as a result of net cross-industry spillovers, for a given industry-specific foreign demand shock under scenario (b) (where economies of scale and scope are the only forces at play). This share ranges from 0.71 for totalizing fluid meter and counting device manufacturing to -0.06 for flavoring syrup. Table 14 in the Quantitative Appendix presents a list of top and bottom industries. Interestingly, flavoring syrup is the only industry in which negative spillovers (owing to residual inputs being less scalable than rival) quantitatively overwhelm positive spillovers, causing a net negative productivity spillover in the event of a demand shock.

Finally, I explore the sensitivity of these results to alternative values of  $\sigma_j$  in the Quantitative Appendix. The finding that scope economies generate an equilibrium elasticity of productivity to gross output of 0.04 is insensitive across typical estimates of  $\sigma$  in the range of 3 to 10. I also allow  $\sigma_j$  to vary across industries by calibrating  $\sigma_j$  to match typical estimates of industry-level direct returns to scale or profit shares and find that scope economies account for roughly 20 percent of the total productivity response in both these instances.

### 5.3 Revisiting the Impact of Trade Protection

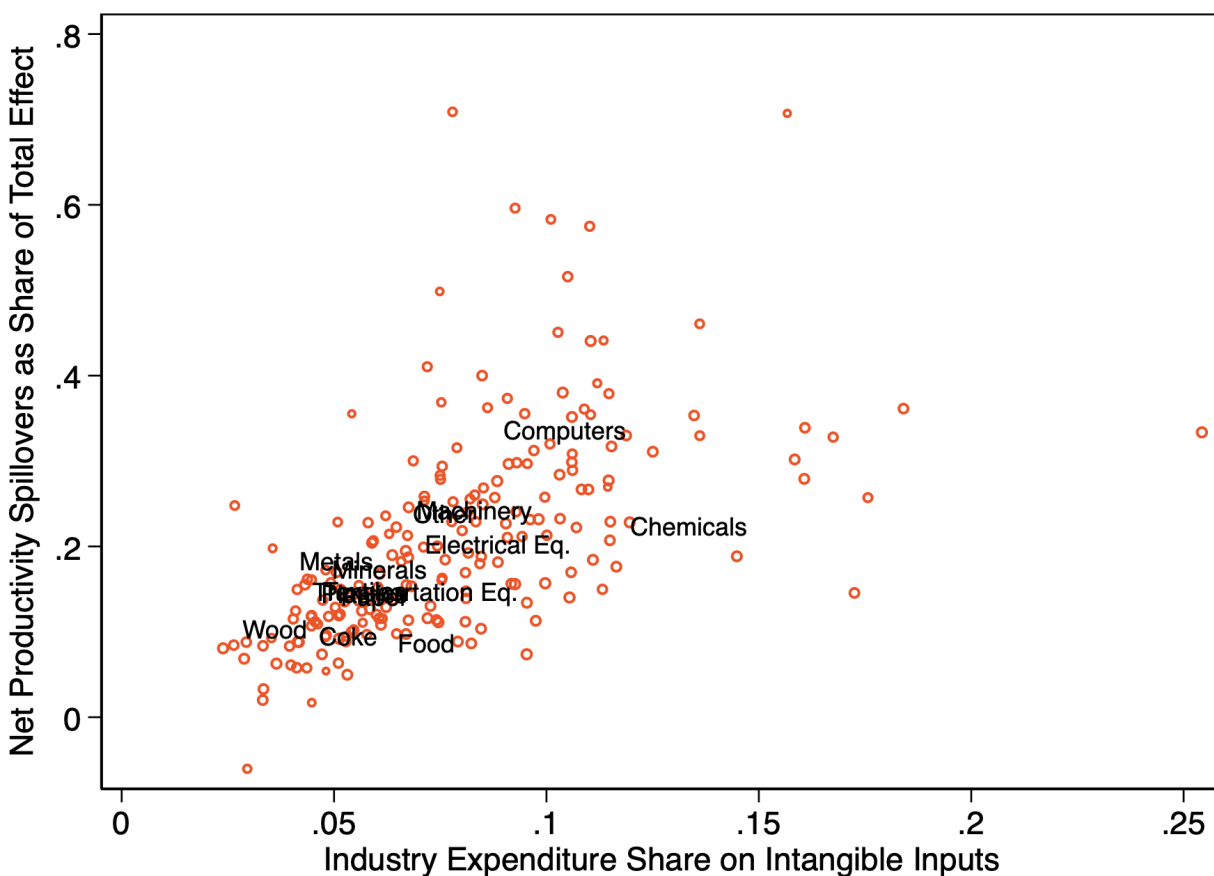
Given the findings of large and biased spillovers across industries, I apply the model to evaluate the aggregate consequences of US import tariffs on Chinese goods. I compare the impact of different tariff policies, holding all other foreign variables constant. I solve the system of equilibrium conditions in Definition 2 before and after the policy change using exact hat algebra. Table 6 displays the results and the Quantitative Appendix presents details on their definition and calculation.<sup>80</sup>

The first two columns of Table 6 display the aggregate impact of uniform 20% Chinese import tariffs on all industries. In column (1), I calibrate a neoclassical trade model with constant returns to scale and perfect competition to the same US data in 2017, and compute

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<sup>80</sup>To isolate the impact of my mechanism, I compute these results without taking into account external input-output linkages (with the underlying economy corresponding to benchmark scenario (b) in Figure 13. See the Appendix Table 16 for results under input-output linkages and a discussion.

Figure 6: Intangible-intensive Industries Drive Productivity Spillovers



This figure plots the net productivity spillover (as a share of the total productivity response) owing to an industry-level foreign demand shock, by industry expenditure share on intangible inputs. Each BEAX industry is represented by a circle on the diagram, and the size of each circle corresponds to the total elasticity of productivity with respect to gross output. The computations are performed under scenario (b) where economies of scale and scope are the only general equilibrium forces. Table 14 in the Quantitative Appendix presents a list of top and bottom industries. Overlaid in black text is the same statistic but aggregated to the level of broad 2-digit sectors. In order from the highest to lowest net spillovers, these sectors are: Computers, Machinery, Other, Chemicals, Electrical Equipment, Metals, Minerals, Textiles, Transportation Equipment, Plastic, Paper, Wood, Coke, and Food.



Table 6: Effects on the US Economy from US Tariffs on Chinese Imports

Model	(1)	(2)	(3)
Import tariffs on Chinese imports	Neoclassical CRS 20% Uniform	Scale + Scope 20% Uniform	Scale + Scope Alternative
<i>Change (%)</i>			
Consumer Price Index (CPI)	1.12	0.80	0.38
Producer Price Index (PPI)	0	-0.46	-0.67
Imports from China	-39.2	-40.5	-40.5
Imports from Rest of World	7.3	5.9	2.4
US Output	1.4	2.4	3.5
US Exports	0	2.1	2.8
Manufacturing Trade Deficit	-11.5	-19.3	-28.3
<i>As Share of Manufacturing Output (%)</i>			
New Manufacturing Profits	0	0.10	0.14
New Tariff Revenues	0.88	0.86	0.36

This table presents estimates of the impact of two different sets of tariffs on outcomes in the US economy, under two different model settings calibrated to match US national industry-level aggregates (industry output and trade patterns) in 2017. Column (1) presents results from a 20% increase in tariffs on Chinese imports across all industries, under a neoclassical model with constant returns to scale and perfect competition. Column (2) compares the impact of the same set of tariffs to my model with economies of scale and scope. Column (3) presents results from alternative tariffs identified by my model that minimize the CPI impact while achieving the same reduction in total imports from China. See Definition 2 for a characterization of the open economy equilibrium and Appendix Section C.4.6 for details on how I compute alternative tariffs.

the counterfactual price effects due to tariff policy. This corresponds to solving for the system of equations in Appendix Section C.4.5 with exogenous domestic prices. Column (1) serves as a useful benchmark to evaluate the mechanism of my model; here, the CPI rises by 1.12% due to Chinese imports becoming more expensive.

Column (2) applies my estimates of scale and scope economies and the full non-linear structure of the model to compute aggregate outcomes arising from the same tariff policy. Domestic US firms now benefit from reduced import competition, and prices of US goods fall not only in industries that are directly protected, but also in industries that are jointly produced. These endogenous responses of domestic prices (due to both scale and scope economies) substantially mitigate the price impact of import tariffs. The domestic PPI falls by 0.46%, lowering the overall rise in the CPI from 1.12% to 0.8%.<sup>81</sup> Other margins of the US economy also improve due to increased competitiveness of US goods: exports go up, the deficit shrinks, and import substitution towards other countries is less pronounced.

Are there other tariff implementations that lower imports from China by the same amount (41%) at the cost of an even smaller rise in the CPI? While the optimal tariff structure is difficult to solve for in this non-linear environment, I use a similar local propagation matrix to that in equation (14) to compute the contribution of import tariffs, by industry, towards reducing Chinese imports (a ‘benefit’) and increasing in the CPI (a ‘cost’). I start with a baseline vector of tariff values that are biased towards industries with higher benefits per unit cost, and scale up the tariffs proportionately so that they generate the same counterfactual decline in imports from China.

I find that the higher the bias, the smaller the reduction in the CPI needed to reduce Chinese imports by 41%.<sup>82</sup> When the degree of bias is such that tariffs on the most heavily affected industry reach 300%, the CPI impact reduces to 0.38%. Almost half of this reduction occurs as a result of a stronger domestic productivity response, indicated by a greater reduction in the PPI of 0.67%. Column (3) of Table 6 reports the impact on US aggregate economic outcomes from this policy. Due to the increased domestic productivity response, exports rise by 2.8%, the manufacturing trade deficit shrinks by 28.3%, and increased manufacturing net profits account for 0.14% of initial gross output.

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<sup>81</sup>Squaring these consumer price effects with new tariff revenues, the overall impact on welfare suggested by the results in column (1) is relatively low and consistent with estimates in [Amiti et al. \(2019\)](#) and [Fajgelbaum et al. \(2019\)](#). Adding an endogenous scale and scope response lowers the welfare costs of tariffs by even more.

<sup>82</sup>For details, see Section C.4.6 of the Quantitative Appendix.

## 6 Conclusion

This paper finds that economies of scope within firms drive the propagation of shocks across industries. I assemble panel data on industry-level sales of all US manufacturing firms and draw on plausibly exogenous shocks to their industry-specific market size to identify economies of scope. I provide reduced-form evidence that a demand shock in one industry of a firm increases its sales in another the more that the two industries share intangible inputs. I rationalize these results by developing a model of joint production under which inputs (including intangibles) differ in their degree of scalability and rivalry within the firm. I provide structural estimates of scale and rivalry elasticities and find that intangible inputs generate scope economies within firms.

Internal scope economies represent a new and quantitatively large mechanism for general equilibrium propagation. The aggregate response of productivity to market size would be roughly 20 percent lower if scope economies were not taken into account. Productivity spillovers are concentrated among industries that utilize more intangible inputs, suggesting that these industries should be more closely scrutinized in quantitative studies and policy evaluation. In light of these findings, I re-examine the consequences of US tariffs on imports from China. I identify alternative tariff policies that more than halve the adverse CPI effects by biasing protection towards industries with stronger internal economies of scale and scope.

There are at least two directions for further research. One is to apply the stochastic joint production framework to oligopolistic competition settings featuring strategic interactions among firms across and within industries. By incorporating firm-level data on research expenditures, patenting, levels of IT capital, and worker occupations, it might be possible to delve deeper into the knowledge production function of the firm, and provide evidence on how internal and external knowledge spillovers interact. Another direction is to exploit the model's tractability to study directed innovation and endogenous productivity across industries at the macro level. These results can improve our understanding of the origins, evolution, and correlation of comparative advantage across countries, as well as the welfare and productivity consequences of trade liberalization.

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# A Data Appendix

## A.1 Data Construction and Details

**Firms, Plants, and Products.** I assemble data from the Economic Censuses (EC), the Longitudinal Business Database (LBD), and the Longitudinal Firm Trade Transactions Database (LFTTD) to construct a portrait of firm activity over the years 1997 to 2012. The Censuses are conducted quinquennially in years ending with '2' and '7'. Data on product shipments made by establishments come from the product trailer (PT) files which are attached to the Census of Manufactures (CMF). These trailer files contain responses of establishments that are sent a CMF 'Long Form'. The long form is sent to all establishments belonging to multi-establishments firms as well as a sample of single-establishment firms. The long form elicits shipments made by the establishment at a disaggregated level (varying from 6 to 10 digit NAICS). There are two reasons why this process yields a strict underestimate of the significance of multi-product activity in the US economy. First, the long-form elicits questions about product sales over a pre-specified list of products (specific to the plant's classified industry). Although there is space for the firm to report shipments in products not covered by that pre-specified list, in practice firms rarely do. Second, the long-forms do not cover all single-establishment firms in the economy. A single-establishment firm could be selling in multiple industries but I will not see the breakdown of its sales over these industries if the firm was not sent a long-form.

Using firm identifiers in the LBD, I match establishments to their parent firms and aggregate industry-level shipments from the level of the plant to the level of the firm. The firm identifier in the LBD comes from information the census collects on the span of control of firms in the Company Organization Survey and from tax identifier and plant identifier information in the Business Register. An establishment is a physical location where business activity occurs. The firm is defined (by the census) as the highest level entity that controls more than 50% of each of the establishments we assign to the firm. I drop plants that are administrative records (for which sales data are imputed).

**External Sales.** The CMF contains data on the shipments of a plant that go towards other plants within the same firm (i.e. inter-plant, intra-firm). However, this data is not broken down at the product-line level. For plants that produce in multiple industries, I apportion this inter-plant shipment data into industry-level intra-firm shipments using shares taken from the plant's total sales across industries. I then define the external sales of a firm in each industry as its total sales in that industry minus its intra-firm shipments. I drop external sales computed in this way in any industries of the firm that (i) account for less than 0.5% of firm-wide external shipments and (ii) are never the main produced industry of any plant the firm owns. This is conservative and allows product shipments in very small industries of the firm to be entirely intra-firm. This also prevents the spurious adding / dropping of products simply because of changes to the PT forms over the years.

**Firm Trade Data.** The LFTTD contains the value of all import and export transactions, by trading country and by HS10 product, that each firm entity (a set of EIN tax codes) is a counter-party to. The CMF also contains data on plant-level shipments that are ultimately destined for export

markets (whether directly or indirectly through an intermediary). If the plant is a multi-industry plant, I apportion this plant-level shipment across the plant's industries using product trailer product shipment shares. I use both LFTTD and CMF sources of data on exports to construct the export demand shock, detailed below. Data on firm exports and imports from Table 1 come from the LFTTD.

**Country-level Trade Data.** I use data from BACI and Comtrade (bilateral country-level trade flows at the HS6 level) to generate the five-year growth rates in imports of a destination  $n$  in product  $h$  used in the analysis,  $\Delta \log IMP_{nht}$ .

**Intangible Inputs.** I use BEA input-output tables from 1997 for information on the use of inputs by industry. Table 7 lists the input industries from BEA input-output and capital flow tables that I classify as intangible inputs. These correspond to NAICS sectors 55, 54, 51, and 533. Although results are robust to including finance, insurance, real estate, and other rental leasing (NAICS 52, 531, and 532), I do not include them in my list of intangible inputs because of the separate way that financial inputs affect businesses compared to real inputs.<sup>83</sup>

The input-output tables record expenses on inputs that fully depreciate within one year. Because some intangible assets also have short depreciation rates and there are arbitrary rules around which inputs are expensed versus capitalized, I incorporate data from the capital flow tables on capitalized investments made by firms in manufacturing industries on intangible input industries (for example, a shoemaker investing in software capital). I count both capitalized investments and expensed investments as intangible input expenditures. The last three columns of Table 7 show data on aggregate expenditures on these input industries. I give statistics on the weighted mean (total manufacturing) expenditure share, as well as the 25th and 75th percentiles across industries.

In practice, it makes no difference to the results if I exclude data from the capital flow tables. Most intangible inputs circa 1997 were still expensed under the national accounts. Only four input industries had capitalized investments: software publishers (511200), architectural, engineering, and related services (541300), custom computer programming services (541511), and computer systems design services (541512). Total capitalized investments in these industries by the manufacturing sector only amount to 0.64% of gross manufacturing output. I do not use input-output table data on intangible input expenditures after 1997 because of subsequent changes to accounting rules that generate a lot of time variation in the data series.

**Industry Definition.** I construct a unified industry nomenclature, BEAX, that is time-invariant over the period 1997 and 2012 and concordable with HS, NAICS, and BEA industry codes in each year. There are 206 BEAX industries in manufacturing. I use the HS-NAICS concordance in US Census Bureau data provided by Schott (2008) and Pierce and Schott (2012) to convert import and export HS codes (at the 10-digit and 6-digit levels) in each year to NAICS. I use the concordances provided by US Census Bureau and BEA to go between NAICS codes and BEA codes in each year. I use an iterative

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<sup>83</sup>My model does not speak to the various mechanisms explored in the corporate finance literature, such as internal capital markets (Stein, 1997) or corporate socialism (Scharfstein and Stein, 2000). Instead, I soak up these effects under estimate properties of residual inputs in my quantitative framework, and under the  $h_{fj}^{SYM}$  control variable in the reduced-form.

Table 7: Definition of Intangible Inputs and their Use in Manufacturing in 1997

$m \in \mathcal{M}^{INT}$	Description	Share of Gross Output (%)		
		Mean	25th pctile	75th pctile
550000	Management of companies and enterprises	3.54	2.60	4.94
541700	Scientific research and development services	0.62	0.25	0.96
541300	Architectural, engineering, and related services <sup>†</sup>	0.62	0.31	0.96
5419A0	All other professional, scientific, and technical services	0.61	0.61	0.63
541511	Custom computer programming services <sup>†</sup>	0.58	0.21	0.86
541800	Advertising, public relations, and related services	0.48	0.14	0.62
541610	Management consulting services	0.28	0.28	0.30
541100	Legal services	0.28	0.09	0.30
541200	Accounting, tax prep., bookkeeping, & payroll services	0.15	0.08	0.21
541400	Specialized design services	0.09	0.01	0.02
541512	Computer systems design services <sup>†</sup>	0.07	0.02	0.06
54151A	Other computer related services	0.04	0.02	0.04
5416A0	Environmental and other technical consulting services	0.04	0.01	0.02
541940	Veterinary services	0.00	0.00	0.00
541920	Photographic services	0.00	0.00	0.00
533000	Lessors of nonfinancial intangible assets	0.69	0.06	0.34
5111A0	Wired telecommunications carriers	0.34	0.17	0.37
511200	Software publishers <sup>†</sup>	0.33	0.05	0.19
518200	Data processing, hosting, and related services	0.20	0.17	0.26
512100	Motion picture and video industries	0.03	0.00	0.03
512200	Sound recording industries	0.00	0.00	0.00
Total Intangible Expenditures		9.01	6.38	11.48

Source: BEA Input-Output & Capital Flow Tables, 1997. Mean refers to the weighted average across all 206 BEAX manufacturing industries, with industry gross output as weights. 25th and 75th pctile refers to expenditure shares of the corresponding percentiles (unweighted) across the 206 manufacturing industries. Codes in the first column refer to BEAX codes that are hand-developed; they roughly correspond to codes available in BEA I/O tables but are aggregated to ensure consistency over time.

<sup>†</sup> Industries where data on capitalized investments from the capital flow tables are used to compute expenditures. This makes up only 0.64% of gross manufacturing output.

algorithm to aggregate over m:m splits over years and in each cross section so that in any given year, each NAICS code and HS10 code is entirely contained within a BEAX code.

## A.2 The Non-Manufacturing Sector

Statistics shown in Table 1 are limited to the manufacturing sector since this paper limits attention to economies of scope in manufacturing. Here I briefly extend the reported statistics on multi-industry firm activity using a different classification of multi-industry firms: whether their sales span two or more industries in *any* sector. In the non-manufacturing sector, I find that 52% of sales come from multi-industry firms in 1997, while this number rises to 57% in 2012. The main difference compared to manufacturing is the long tail of small firms in non-manufacturing industries. The share of firms that are multi-industry in non-manufacturing is only 1% (compared to 20% in manufacturing). However, the lack of comparable product-trailer data for the non-manufacturing sector makes an apples-to-apples comparison difficult. In the non-manufacturing sector, industry variation in the data comes exclusively from the span of plants owned by a firm and the extent to which these plant classifications differ.

## A.3 Export Demand Shocks

I leverage both the LFTTD and CMF sources of data on firm-industry exports to construct demand shocks,  $\Delta \log S_{fjt}$ . First, among LFTTD data, I compute export shares of each industry of each firm across destinations  $n$  and HS6 products  $h$ . I exclude destination-product markets whenever the firm's exports in those markets exceed 10% of the market's imports from the rest of the world. I use these shares as  $s_{fnh|fj,t-1}$  in the analysis. Data on export intensity,  $s_{fj,t-1}^*$ , come from the CMF export shipment response variable. This is a firm-industry level variable as described in the preceding section. If the firm has no reported exports in an industry by manufacturing plants producing in that industry, it is likely that customs data is an instance of carry-along trade, made by the firm's wholesale / retail arm. These demand shocks are unlikely to affect the firm's manufacturing sales (reported by its plants) any more than other firms in the industry. Export intensity helps to discipline the customs-derived export demand shocks. I also set export intensity to zero for instances where carry-along trade of the firm (customs exports less census exports) in an industry exceeds its total external shipments in the CMF. After purging these edge cases, I am left with two measures of export intensity: (i) census exports divided by census sales in an industry, and (ii) customs exports divided by census sales in an industry. I take the average of these two measures as my measure of  $s_{fj,t-1}^*$ .

Table 8: Regression Sample Summary Statistics

Description			
<i>By firm-industry:</i>			
Sales Growth	$\Delta \log X_{fjt}$	Mean	Std. Dev.
Has Export Demand Shock?		0.15	0.99
Export Intensity		0.68	0.47
Own-Industry Demand Shock	$s_{fj,t-1}^*$	0.06	0.10
Cross-Industry Demand Shock, sales-weighted	$\Delta \log S_{fjt}$	0.028	0.082
Cross-Industry Demand Shock, intangible-input-weighted	$h_{fj}^{SYM}$	0.025	0.063
	$h_{fj}^{INT}$	0.002	0.007
Initial Period Sales (millions)	$X_{fj,t-1}$	165	1225
Initial Period Employment		522	2245
<i>Other Statistics</i>			Value
Number of firms in 1997-2002			5000
Number of firms in 2002-2007			4700
Share of U.S. manuf. output accounted for by sample			0.51
Share of U.S. manuf. employment accounted for by sample			0.37

This table reports sample statistics for the particular sample of multi-industry firms and industries used in the reduced-form regression (Table 2). The selection criteria is any firm-industry with continuing sales over a 5-year period, and belonging to a firm with at least one industry exporting (so that at least one out of the own-industry and cross-industry demand shock variables will be non-zero).

## A.4 Regression Analysis

### A.4.1 Summary Statistics

Table 8 displays summary statistics on common variables that appear in the regressions, for the regression sample. The regression sample consists of all continuing firm-industries (across 5-year periods) of firms that have at least one industry with a non-zero export demand shock.

### A.4.2 Export Demand Shock

Before proceeding to the main test of spillovers, I verify that demand shifters are indeed able to shift firm sales in the same industry by running the following regression for only the sample of firm-industries that have non-zero own-industry export demand shocks:

$$\Delta \log X_{fjt} = \alpha \Delta \log S_{fjt} + Controls_{jt}(s_{fj,t-1}^*) + FE_{jt} + \epsilon_{fjt},$$

where  $Controls_{jt}(s_{fj,t-1}^*)$  refers to various ways of controlling for the export intensity scaling variable, to make sure that variation in the export demand shock is not driven by firms with different export intensities being on different growth trends. Results are presented in Table 9. Across all three columns (that vary in terms of the control for export intensity used), the coefficient on the shock variable is



Table 9: Relevance of Export Demand Shocks for Predicting Sales Growth

$\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)
$\Delta \log S_{fjt}$	0.59*** (0.10)	0.36*** (0.12)	0.34*** (0.11)	0.32*** (0.11)
Industry-year-FE	✓	✓	✓	✓
$s_{fj,t-1}^* \times \text{year-FE}$		✓	✓	
$s_{fj,t-1}^* \times \text{Industry-year-FE}$				✓
Control for pre-period sales, $\log X_{fj,t-1}$			✓	✓
Observations	14,500	14,500	14,500	14,500
$R^2$	0.08	0.08	0.12	0.15

This table displays responses of firm-industry sales to demand shocks the firm receives in the same industry, in 5-year differences over the period 1997-2007. Standard errors in parentheses are clustered at the firm level. Observations are at the firm-industry-year level, for continuing industries of all multi-industry firms that have an export demand shock in that same industry. The control  $s_{fj,t-1}^*$  is the firm's export intensity (exports over sales) in industry  $j$  in the initial census year.

positive and ranges from 0.32 to 0.59. Without controls for export intensity (column 1), the impact of the demand shock is statistically higher, consistent with selection on export intensity.<sup>84</sup>

#### A.4.3 Control Variables

The regression results in Table 2 are robust to including an exhaustive set of controls, as described in the main text. I define the pre-period size of the firm as the log of the sum of sales over all the firm's industries. The pre-period industry size is the log of the firm's sales in that industry. I define export status as a dummy variable equal to 1 if the firm has a non-zero export intensity in that industry,  $s_{jt}^* > 0$ .

In column (1) of Table 10, I re-estimate specification (5) of Table 2 but including all the controls discussed in the body of the text. I find that—aside from an increase in  $R^2$ —there are no changes to the magnitude or significance of spillover coefficients.

#### A.4.4 Other Input Linkages

The remaining columns (2)-(4) of Table 10 estimates a variant of the main regression equation (2) where instead of using sales weights to form the  $h_{fj}^{SYM}$  function, I focus only on input proximity and separate out intangible inputs from the remaining inputs in the BEA I/O tables. I call the remaining set of inputs tangible, denoted by  $TAN$ , and construct  $h_{fj}^{TAN}$  in the same way as  $h_{fj}^{INT}$ . Column (2)

<sup>84</sup>I also run a placebo test where I assign firms in a given industry  $j$  a random  $\Delta SHK_{fjt}$  drawn from the empirical distribution of shocks received by all firms active in industry  $j$ . The placebo tests return false positives in column (1) but not columns (2) and (3). This suggests that linear controls for export intensity control adequately for selection on export intensity.

Table 10: Cross-Industry Spillovers within the Firm: Additional Robustness Specifications

Sales growth, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)
Own-industry shock, $\Delta \log S_{fjt}$ $\psi^{OWN}$	0.51*** (0.10)		0.47*** (0.09)	0.47*** (0.09)
Cross-industry shocks, $\{\Delta \log S_{fkt}\}_{k \neq j}$				
Intangible expenditure weighted $\psi^{INT}$	8.26*** (2.22)	6.54*** (2.08)	7.02*** (2.11)	8.14*** (2.22)
Tangible expenditure weighted $\psi^{TAN}$		-0.75*** (0.26)	-0.86*** (0.26)	-0.44 (0.44)
Sales weighted $\psi^{SYM}$	-0.81*** (0.26)			-0.48 (0.39)
Industry-year-FE	✓	✓	✓	✓
Full Set of Controls	✓			
Observations	21,500	21,500	21,500	21,500
$R^2$	0.12	0.05	0.06	0.06

This table displays responses of firm-industry sales to demand shocks across the firm's range of industries, in 5-year differences over the period 1997-2007. Standard errors are clustered at the firm level. Observations are at the firm-industry-year level, for continuing observations of a sample of multi-industry firms with at least one industry directly exporting. Results are unweighted but robust to weighting by the inverse within-firm share of sales of industry  $j$ . Results are robust to deflating outcomes and also shocks. The magnitude and significance of coefficients  $\psi$  are robust to inclusion of a host of control variables, including: initial period firm size, firm-industry size, export status, export intensity, as well as controls for the shares in the functional forms used to collapse shocks in other industries, and the interaction of these shares with other initial-period firm-industry variables.

shows that they pull in opposite directions within the firm, strongly suggestive that industry spillovers differ in the intangible input proximity dimension. Column (3) adds the own-industry shock to the regression, and column (4) includes both  $h_{fj}^{TAN}$  and  $h_{fj}^{SYM}$ . The negative coefficients on  $h_{fj}^{TAN}$  and  $h_{fj}^{SYM}$  are imprecisely estimated and at roughly half their respective magnitudes when appearing individually, which is consistent with the presence of collinearity (their unconditional correlation is 98%).

To further disentangle whether it is certain tangible inputs or simply sales presence in other industries that are driving the negative spillovers, I conduct a placebo exercise based on column (5) where I use different proximity measures constructed based on bilateral similarity in use over different sets of inputs (in lieu of intangibles,  $h_{fj}^{INT}$ ). Table 11 displays the regression table counterpart to coefficients shown in Figure 2. The numbers next to the description in parentheses display the BEAX subroot (1, 2 or 3 digits) among 6-digit BEAX industry inputs that make up the sector. Taxes, government sector inputs, and the two types of value-added are specific BEA categories that have no corresponding numeric BEAX code.

#### A.4.5 Firm-level Responses

I construct weights,  $\eta_{fkt}$ , in the firm-level regression equation (4) for the various outcome categories as follows:

$$\eta_{fkt} = \sum_k \frac{\beta_{y,k} X_{fkt}}{\sum_{k'} \beta_{(i),k} X_{fk't}},$$

where  $X_{fkt}$  is firm sales and  $\beta_{y,k}$  takes on the following values depending on the outcome variable:

- (i) Purchased professional services:  $\beta_{y,k} = \beta_{INT,k}$ , the share of gross output by industry  $k$  on intangible inputs.
- (ii) Sales:  $\beta_{y,k} = 1$  (so  $\eta$  are simply sales weights).
- (iii) Capex:  $\beta_{y,k} = \beta_{CAP,k}$ , the share of gross output by industry  $k$  on capital value added.
- (iv) Payroll:  $\beta_{y,k} = \beta_{LAB,k}$ , the share of gross output by industry  $k$  on labor value added.

The quantitative appendix discusses the extent to which the weight for intangible input expenditures, (i), approximate theoretically-relevant weights.

Data on purchased professional services at the firm level come from aggregating responses of plants of the firm to the following ASM survey expense line items: expenses on legal, accounting, management, communication, advertising, and computer software and data processing services. Firms that do not have plants respond to these questions in the ASM and firms that have sales outside of the manufacturing sector account for larger than 5% of overall sales are dropped for this particular regression. Data on firm-wide capex come from summing up plant-level capital expenditures, and data on payroll come from summing up plant-level production worker payroll.

#### A.4.6 Vertical Explanations

There are four general reasons a demand shock in industry  $k$  may propagate within a firm to generate increased sales in industry  $j$ : (i)  $j$  supplies  $k$ , (ii)  $k$  supplies  $j$ , (iii)  $k, j$  use similar inputs, and (iv)  $k, j$  are demand-complementary and have similar buyers. My focus is on story (iii). The discussion in the body of the paper (Section 2.5) rules out reason (iv), demand-complementarity. The following points discuss the first two, vertical stories.

- (i) I use the external sales growth in industry  $j$  as my main outcome variable, so the reaction is precisely in the increased outward sales in industry  $j$ . It could still be true that external sales growth is driven by productivity effects induced by intra-firm sales growth. I check specifically whether intra-firm sales growth in  $j$  occurs in response to demand shocks in  $k$ . I find that they do not respond (even among only the tiny fraction of  $j$  industries that have any inter-plant shipments at all).
- (ii) For this to occur there must first be an increase in internal shipments in the shocked industry  $k$ . Then the story would be that increased quality of shipments (as measured by increased internal sales) drives growth in industry  $j$ . I use growth in inter-plant (intra-firm) shipments as an outcome variable across the specifications in Table 9. I find that they do not respond (even among the tiny fraction of  $k$  industries that have any inter-plant shipments at all).

#### A.4.7 Threats to Identification

Related to the discussion on threats to identification in Section 2.5, I test directly and reject the hypothesis that the import growth patterns across industries within a destination are positively correlated among intangible-input intensive industries. I aggregate imports of each destination to the industry level (across products),  $IMP_{nk,t-1}^{US}$ , and construct spillover functions corresponding to the same functional forms used in the main firm-industry regression table:

$$h_{njt}^{BLK} \equiv \psi^{BLK} \sum_{k \neq j} \sum_{m \in \mathcal{M}^{BLK}} \beta_{jm} \left( \frac{\beta_{km} IMP_{nk,t-1}^{US}}{\sum_{k \neq j} \beta_{km} IMP_{nk,t-1}^{US}} \right) \Delta \log IMP_{nkt}^{US},$$

for  $BLK = \{INT, TAN\}$  to separate out intangible inputs from remaining inputs in the input-output tables. I then run the following regression, at the level of destination-industries, over the same time period (in 5-year differences):

$$\Delta \log IMP_{njt}^{US} = \psi^{TAN} h_{njt}^{TAN} + \psi^{INT} h_{njt}^{INT} + FE_{jt} + FE_{nt}.$$

I do not find that  $\psi^{INT}$  is positive, either with or without destination-year fixed effects.

#### A.4.8 Deflating

Even though the main regressions specifications all include industry-year fixed effects, whether variables are nominal or deflated could make a difference in terms of the relative sizes of export shares and expenditure shares. All the reduced-form results are virtually unchanged when the following variables are deflated with industry-level price deflators from the NBER: demand shocks (import growth at destinations), outcomes (external shipments of a firm-industry), as well as ‘initial-period’ variables in the formulation of weights, for example, behind the  $h_{fj}$  functions.

Table 11: Intensive Margin Cross Industry Spillovers: Placebo Input Linkages

Sales growth, $\Delta \log X_{fjt}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Own-industry shock	0.48***	0.45***	0.46***	0.46***	0.46***	0.45***	0.45***	0.45***	0.45***	0.46***	0.45***	0.45***	0.45***	0.45***
Sales-weighted linkage	-0.26**	-0.68**	-0.49**	-0.82*	-0.18	-0.45*	-0.12	-0.15	-0.29	-0.00	-0.07	0.25	-0.05	0.19
$h_{fj}(\cdot)$ based on the following input linkages														
Leasing of Intangibles (533)	27.73***													
Headquarter Services (55)		15.82**												
Professional Services and Information (54, 51)			8.262*	40.26										
Finance, Insurance, and Real Estate (52, 531)														
Leasing of Tangibles (532)					24.92									
Transportation, Wholesale, and Retail (4)						14.94*								
Taxes and Government							3.603							
Utilities and Construction (2)								3.413						
Capital Value-Added (Gross Operating Surplus)									1.766					
Labor Value-Added										-0.325				
Agriculture (1)											-0.715			
Manufacturing (3)												-0.889		
Administrative Services (56)													-4.305	
All Other Services (6, 7, 8, 9)														-20.8
Industry-year-FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Observations	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500	21,500
R <sup>2</sup>	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06

Standard errors clustered at the firm level. Observations at the firm-industry-year level, for continuing observations of a sample of multi-industry firms with at least one industry directly exporting. Results for 5-year stacked long differences, 2007-2002, and 2002-1997.

## B Theory Appendix

In this section I prove any claims and derive the expressions found in Propositions and Lemmas in the main body of the paper. I also provide commentary on a few relevant extensions of the results of the model.

### B.1 Proposition 1: Identification Benchmark

Consider the partial equilibrium problem of a firm that chooses own quantities of production  $\{q_j\}_j$  to maximize profits facing inverse residual demand  $p_j(q_j)$ :<sup>85</sup>

$$\pi \equiv \max_{\{q_j\}_j} \sum_{j \in J} p_j q_j - C(\vec{q}; \vec{w}).$$

The first order conditions relate marginal revenue  $MR_j$  to marginal cost  $MC_j$  in each industry:

$$MR_j(q_j) = MC_j(\vec{q}; \vec{w}) \quad \forall j.$$

Let  $S_j$  be exogenous and relevant shifters of marginal revenue from the point of view of the firm, so that  $\frac{\partial C}{\partial S_j} = 0$  and  $\frac{\partial MR_j}{\partial S_j} \neq 0$ . First consider the case where marginal revenue is only a function of own-industry quantities and demand shifters. Take a total derivative to yield

$$\frac{\partial MR_j}{\partial S_j} dS_j + \frac{\partial MR_j}{\partial q_j} dq_j = \sum_k \frac{\partial^2 C}{\partial q_j \partial q_k} dq_k, \quad \forall j.$$

Rearranging and expressing in matrix form:

$$dq = \Gamma^{-1} \text{diag} \left( \frac{\partial MR_j}{\partial S_j} \right) dS,$$

where elements of  $\Gamma$  are given by

$$[\Gamma]_{jk} = \frac{\partial^2 C}{\partial q_j \partial q_k} - \frac{\partial MR_j}{\partial q_j},$$

and a sufficient condition for the invertibility of  $\Gamma$  is that the second order conditions of the firm's profit maximization problem are satisfied, implying that the matrix  $\Gamma$  is positive definite. Using the

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<sup>85</sup>When the demand function is known, identification is robust to arbitrary demand-side complementarities (where inverse demand can be a function of quantities in all industries). Of course, in the CES-monopolistic competition equilibrium introduced in the model, this is ruled out. This assumption is not unreasonable in my empirical setting with only 206 industries. For example, results in [Flaen et al. \(2019\)](#) suggest significant demand-side complementarities between washers and dryers, which fall within the same industry.



relationship between sales and quantities we can re-express the matrix as

$$d \log X = \underbrace{\left( \Gamma^{-1} \text{diag} \left( \frac{MR_j}{X_j} \frac{\partial MR_j}{\partial \log S_j} \right) + \text{diag} \left( \frac{\partial \log p_j}{\partial \log S_j} \right) \right)}_{=\psi_{jk}} d \log S,$$

so that knowledge of quantities and the demand function is enough to identify the Hessian of the cost function,  $\frac{\partial^2 C}{\partial q_j \partial q_k}$ , from  $\psi_{jk}$ .

Finally, working out the more general case where inverse demand  $p_j$  is a function of the full vector of quantities  $\vec{q}$  and demand shifters  $\vec{S}$  is logically trivial since we assume knowledge of the demand function but simply algebraically more cumbersome to show.

## B.2 Lemma 1: Microfoundation for Non-rivalry

Combine the expression in Assumption 2 with the expression for firm-industry profits in equation (6) to compute the expected impact on firm profits from the arrival of an idea  $i_m$  generated by specialized input  $m$ :

$$\frac{\Delta_{fm}}{Z} \equiv \mathbb{E} \left[ \max_j \tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fjm,i} \right],$$

where  $\phi_{fjm,i}$  is an independent random draw from a Fréchet distribution. The expected impact is the change in profits in the industry in which the idea (given its distribution of qualities in different industries) generates the highest improvement in profitability. The remainder of this proof simply relies on properties of the Fréchet distribution popularized by Eaton and Kortum (2002). I can re-express the profit contribution as:

$$\frac{\Delta_{fm}}{Z} = \mathbb{E} \left[ \max_j \tilde{\phi}_{fjm,i} \right],$$

where  $\tilde{\phi}_{fjm,i}$  is an independent random draw from a different Fréchet distribution that absorbs the multiplicative shifters:

$$Pr(\tilde{\phi}_{fjm,i} \leq x) = e^{-(\tilde{\alpha}_{mj} B_j \xi_{fj})^{\theta_m} x^{-\theta_m}}, \quad \forall j,$$

and it follows that

$$\begin{aligned} \frac{\Delta_{fm}}{Z} &= \left( \sum_j (\tilde{\alpha}_{mj} B_j \xi_{fj})^{\theta_m} \right)^{\frac{1}{\theta_m}} \Gamma(1 - 1/\theta_m), \\ \Delta_{fm} &= \left( \sum_j \delta_{fmj}^{\theta_m} \right)^{\frac{1}{\theta_m}}. \end{aligned}$$

where  $\Gamma$  is the gamma function and  $\delta_{fmj} \equiv \xi_{fj} \alpha_{mj} B_j Z$ .

### B.3 Lemma 2: The Firm's Solution

Given the timing assumptions, the firm's problem is solved in two steps. During the production step, the firm decides expenditures on generic inputs knowing values of accumulated knowledge capital,  $\varphi_{fj}$ . This yields the expression for gross profits of the firm (sales less generic input expenses) in equation (6).

The remaining problem of the firm is to solve expenditures on specialized inputs at the beginning of time and decide the industries in which to deploy the ideas that arrive during the incubation period. Given Assumption 2, expected firm net profits  $\Pi_f$  can be written as:

$$\mathbb{E}[\Pi_f] = \max_{I_{fm}} \mathbb{E} \left[ \sum_j \sum_m B_j \xi_{fj} \tilde{\alpha}_{mj} \sum_i^{A_{fm}} \phi_{fjm,i} \mathbf{1}_{fjm,i} \right] - \sum_m \frac{\rho_m - 1}{\rho_m} w \left( \frac{I_{fm}}{Z} \right)^{\frac{\rho_m}{\rho_m - 1}}.$$

The first half of the expression denotes the expected gross profits of the firm given choices of the Poisson arrival rate  $I_{fm}$  and deployment decisions  $\mathbf{1}_{fjm,i}$ . The remaining half of the expression relates to the costs (in terms of expenditures on specialized inputs) spend on acquiring the chosen Poisson arrival rate  $I_{fm}$  over all specialized inputs  $m$ . Given the independence of the Poisson and Fréchet distributions, this problem is additively separable, so that the contribution and deployment decisions of each idea that arrives is independent of past and future decisions.

More formally, the deployment decision  $\mathbf{1}_{fjm,i}$  has the following expectational properties inherited from Fréchet (Section B.2):

$$\begin{aligned} Pr(\mathbf{1}_{fjm,i} = 1) &= Pr(\tilde{\phi}_{fjm,i} = \max_k \tilde{\phi}_{fkm,i}) \\ &= \frac{\delta_{fjm}^{\theta_m}}{\Delta_{fm}^{\theta_m}} \equiv \mu_{fjm}, \end{aligned}$$

where  $\mu_{fjm}$  are deployment probabilities for any given idea generated by specialized input  $m$  and

$$\mathbb{E}[\tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fjm,i} | \mathbf{1}_{fjm,i} = 1] = \Delta_{fm}.$$

Recalling that  $A_{fm}$  is distributed independently with Poisson mean  $I_{fm}$ , expected firm net profits  $\Pi_f$  can be re-written as

$$\begin{aligned} \mathbb{E}[\Pi_f] &= \max_{I_{fm}} \sum_m \sum_j \mathbb{E}[A_{fm} | I_{fm}] \mathbb{E}[\tilde{\alpha}_{mj} B_j \xi_{fj} \phi_{fjm,i} | \mathbf{1}_{fjm,i} = 1] Pr(\mathbf{1}_{fjm,i} = 1) - \sum_m \frac{\rho_m - 1}{\rho_m} w \left( \frac{I_{fm}}{Z} \right)^{\frac{\rho_m}{\rho_m - 1}} \\ &= \max_{I_{fm}} \sum_j \sum_m I_{fm} \frac{\Delta_{fm}}{Z} \mu_{fjm} - \sum_m \frac{\rho_m - 1}{\rho_m} w \left( \frac{I_{fm}}{Z} \right)^{\frac{\rho_m}{\rho_m - 1}} \\ &= \max_{I_{fm}} \sum_m I_{fm} \frac{\Delta_{fm}}{Z} - \sum_m \frac{\rho_m - 1}{\rho_m} w \left( \frac{I_{fm}}{Z} \right)^{\frac{\rho_m}{\rho_m - 1}}. \end{aligned}$$

This is a simple convex optimization problem separable across specialized inputs  $m$ , with solution given by

$$I_{fm} = \Delta_{fm}^{\rho_m-1} w^{1-\rho_m} Z, \quad \forall m,$$

and thus net profits are equal to

$$\begin{aligned} \mathbb{E}[\Pi_f] &= \sum_m \Delta_{fm}^{\rho_m} w^{1-\rho_m} - \sum_m \frac{\rho_m - 1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m} \\ &= \sum_m \frac{1}{\rho_m} \Delta_{fm}^{\rho_m} w^{1-\rho_m}. \end{aligned}$$

Likewise, expected gross profits in industry  $j$  are given by

$$\mathbb{E}[\pi_{fj}] = \sum_m I_{fm} \frac{\Delta_{fm}}{Z} \mu_{fmj} = \sum_m \mu_{fmj} \Delta_{fm}^{\rho_m} w^{1-\rho_m}.$$

The probability that a firm is active in industry  $j$ , denoted  $\chi_{fj} = 1$ , is one minus the probability that no ideas are chosen to be deployed to that industry. Since deployment of one idea is independent of another, and the total arrival of ideas generated by any input  $m$  is a Poisson process with rate  $I_{fm}$ , the arrival of ideas *deployed to industry  $j$*  is also a Poisson process, with different rate  $I_{fm} \mu_{fmj}$ . The probability that no ideas are chosen to be deployed is thus the probability that there are no arrivals from the joint Poisson processes of deployed ideas to industry  $j$  over all specialized inputs:

$$Pr(\chi_{fj} = 1) = 1 - \exp\left(-\sum_m \mu_{fmj} I_{fm}\right) = 1 - \exp\left(-Z \sum_m \mu_{fmj} \Delta_{fm}^{\rho_m-1} w^{1-\rho_m}\right),$$

and is independent across industries. Similarly, an inactive firm is a firm with no ideas arrive at all. The probability that a firm is inactive is thus given by

$$Pr(\chi_f = 1) = 1 - \exp\left(-\sum_m I_{fm}\right) = 1 - \exp\left(-Z \sum_m \Delta_{fm}^{\rho_m-1} w^{1-\rho_m}\right).$$

## B.4 Proposition 2: Firm-level Elasticities

Log-differentiating equation (8) with respect to shifters of firm profitability in industries  $k$ , holding factor prices  $w$  constant, yields

$$\begin{aligned} d \log \mathbb{E}[X_{fj}] &= d \log \mathbb{E}[\pi_{fj}] \\ &= \sum_m \lambda_{fjm} \left( \theta_m \mathbf{1}_{k=j} d \log \xi_{fk} B_k + (\rho_m - \theta_m) \sum_k \mu_{fmk} d \log \xi_{fk} B_k \right), \end{aligned}$$

where  $\mu_{fjm}$  are choice shares given in Lemma 2, and  $\lambda_{fjm}$  denote utilization shares: the share of gross profits of industry  $j$  attributable to knowledge capital contributions by input  $m$ :

$$\lambda_{fjm} \equiv \frac{\mu_{fjm} \Delta_{fm}^{\rho_m} w^{1-\rho_m}}{\sum_{m'} \mu_{fm'j} \Delta_{fm'}^{\rho_{m'}} w^{1-\rho_{m'}}}.$$

## B.5 Connecting Firm-level Elasticities in the Model and Reduced-Form

The firm-level cross-industry elasticity from Proposition 2 combines responses on both intensive and extensive margins ( $\mathbb{E}[X_{fj}]$  includes the non-trivial probability of zero sales).<sup>86</sup> But for sufficiently large firms (high in  $\xi_f$ ), all the responses load on the intensive margin. The intuition is that the largest firms choose a level of expenditures on specialized inputs so high to start with that the likelihood of cross-industry shocks affecting the extensive margin vanishes.<sup>87</sup> With a high enough arrival rate, the expectation operator becomes exact thanks to the law of large numbers.<sup>88</sup> The following Lemma clarifies this point and motivates the focus of the reduced-form regressions on the intensive margin (given that the regression sample comprises large firms):

**Lemma 3 (Intensive Margin Spillovers for Large Firms)** *For a firm with non-zero latent productivities  $\xi_{fj}$  and  $\xi_{fk}$ , cross-industry elasticities characterized by Proposition 2 load completely onto the intensive margin as the average firm productivity  $\xi_f$  become arbitrarily high:*

$$\lim_{\xi_f \rightarrow \infty} \frac{d \log \mathbb{E}[X_{fj}]}{d \log \xi_{fk} B_k} = \frac{d \log \mathbb{E}[X_{fj} | X_{fj} > 0]}{d \log \xi_{fk} B_k}.$$

*As a corollary, the share of the cross-industry elasticity in Proposition 2 explained by extensive margin changes within the firm ranges from 1 (for the lowest  $\xi$  firms) to 0 (for the highest  $\xi$  firms).*

**Proof.** Separate the expected gross sales into intensive margin and extensive margins:

$$\log \mathbb{E}[X_{fj}] = \log \mathbb{E}[X_{fj} | X_{fj} > 0] + \log \Pr(X_{fj} > 0).$$

Differentiate the extensive margin:

$$\frac{d \log \Pr(X_{fj} > 0)}{d \log \xi_{fk} B_k} = \frac{\exp(-\Sigma_{fj}) \Sigma_{fj}}{1 - \exp(-\Sigma_{fj})} \sum_m s_{mj} (\theta_m \mathbf{1}_{k=j} + (\rho_m - \theta_m) \mu_{fmk}),$$

<sup>86</sup>It is easy to log-differentiate equation (9) to derive purely extensive margin predictions and thus decompose the action.

<sup>87</sup>For intuition, consider how demand shocks for MRI machines might affect General Electric's intensive margin sales of jet engines but is unlikely to affect whether the company is active at all in jet engines. Of course, the Lemma does not mean that we should expect big firms to have no adjustments on the extensive margin: adjustments can still occur whenever firm-industry demand and supply shifters ( $\xi_{fj}$ ) change (i.e. from 0 to some positive value) for reasons outside of the model.

<sup>88</sup>All the Frechet idiosyncratic errors wash out. This large firm limit also corresponds to the framework pioneered in Tintelnot (2016) and Antràs et al. (2017), whereby outcomes are smoothed across a continuum within the firm instead of being granular.

where

$$s_{mj} \equiv \frac{Z \mu_{f m j} \Delta_{f m}^{\rho_m - 1} w^{1 - \rho_m}}{\Sigma_{f j}},$$

and  $\Sigma_{f j} \equiv Z \sum_m \mu_{f m j} \Delta_{f m}^{\rho_m - 1} w^{1 - \rho_m}$ . Let  $\bar{\xi}_f$  denote the average of the profitability shifters in two industries of the firm  $j, k$  in which the firm has non-zero sales. The term to the right of the  $\Sigma_m$  in the derivative of the extensive margin is bounded (weighted average of elasticities),

$$\lim_{\xi_f \rightarrow \infty} \frac{d \log \Pr(X_{f j} > 0)}{d \log \xi_{f k} B_k} = \lim_{\Sigma_{f j} \rightarrow \infty} \frac{\exp(-\Sigma_{f j}) \Sigma_{f j}}{1 - \exp(-\Sigma_{f j})} = 0,$$

where the last equality makes use of L'hospital's rule. ■

## B.6 Proposition 3: Aggregate Economies of Scale and Scope

Equation (12) is a system of goods market clearing conditions linking residual profits  $B_j$  to industry gross output,  $X_k$ . Log-differentiating this system of equations yields

$$d \log \mathbf{B} = \Upsilon^{-1} d \log \mathbf{X}.$$

Equation (7) is a system of equations defining residual profits  $B_j$  in terms of industry price indices and gross output. This system of equations is separable yielding

$$d \log P_j = \frac{1}{\sigma_j - 1} (\sigma_j (1 - \varsigma_j) d \log B_j - d \log X_j), \quad \forall j.$$

Combining the two equations yields the result in Proposition 3.

Technically speaking, economies of scale and scope relate to properties of a cost function. In monopolistic competition with product differentiation, industry-level economies of scale and scope can still be represented with reduced-form industry-level aggregate production functions that take into account firms' endogenous responses to competition. I provide such a representation below. Let  $Q_j$  denote an aggregate, industry-level production technology for a composite good in the industry:

$$Q_j \equiv \left( N \int_f q_{f j}^{\frac{\sigma_j - 1}{\sigma_j}} df \right)^{\frac{\sigma_j}{\sigma_j - 1}},$$

where  $q_{f j}$  is the quality-adjusted output by firm  $f$  in industry  $j$  given in Assumption 1.

The total cost of producing  $Q_j$  units is given by  $C_j(Q_j) = P_j Q_j$  where the industry price index  $P_j$  is defined as:

$$P_j^{1 - \sigma_j} \equiv N \int_f p_{f j}^{1 - \sigma_j} df,$$

and corresponds to the  $P_j$  term inside residual profits in equation (7).  $P_j$  can be expressed as a function of  $Q_j$ , underlying industry quantities of production, through re-defining  $X_k$  in equations

(12) and (7) as  $X_k = P_k Q_k$ . Carrying through the same sets of derivatives but now using  $Q_k$  as the exogenous shifter rather than  $S_k$  yields

$$d \log \mathbf{B} = \Upsilon^{-1} (d \log \mathbf{Q} + d \log \mathbf{P}).$$

$$d \log P_j = \frac{1}{\sigma_j - 1} (\sigma_j(1 - \varsigma_j) d \log B_j - d \log Q_j - d \log P_j), \quad \forall j.$$

These can be combined to give an alternative formulation of Proposition 3 in terms of a matrix of responses of average cost to quantities produced:

$$d \log \mathbf{P} = \left( \Upsilon \operatorname{diag} \left( \frac{\sigma_j}{\sigma_j(1 - \varsigma_j)} \right) - \mathbb{I} \right)^{-1} \left( \mathbb{I} - \Upsilon \operatorname{diag} \left( \frac{1}{\sigma_j(1 - \varsigma_j)} \right) \right) d \log \mathbf{Q}. \quad (15)$$

When there are no economies of scope within the firm ( $\rho_m = \theta_m \forall m$ ) we recover certain well-known cases. It is easy to check that as  $\rho \rightarrow \infty$ , we reach the limit where  $d \log P_j = -\frac{1}{\sigma_j} d \log Q_j$ . And as  $\rho \rightarrow 1$ , and generic inputs are constant returns to scale in production,  $d \log P_j = 0$ .

Here is where, not being concerned with market structure, I differ from Baumol et al. (1982). The aggregate production function generates changes in the sets of prices through the behavior of *other* firms, rather than any particular firm itself. It is awkward to ask the question of what would happen if aggregate quantity indices  $Q_j, Q_k$  were produced jointly instead of separately (i.e. tests of sub-additivity in the aggregate cost function  $C = \sum_j C_j(Q_j)$ ), since they are already composites from the output of other firms, and an economy rarely has zero production in any given industry.

The more relevant question is what would happen to price indices in any given industry (or average costs of the aggregate production function) were quantities of production in another industry changed. The above matrix of derivatives, equation (15), answers that question.

In counterfactual scenarios,  $Q_j$  does not change for exogenous reasons; rather, exogenous shifters of market size induce changes in equilibrium  $Q_j$ . In monopolistic competition this is well-captured by iso-elastic shifters of market size,  $S_k$ . It is easy to check that plugging in  $d \log Q_j + d \log P_j = d \log X_j$  back into equation (15) yields the result in Proposition 3.

## B.7 Extension of Proposition 3 with Exogenous Productivity Shocks

The framework is general enough to also handle exogenous productivity shocks at the industry level. Suppose productivity shocks are Hicks neutral. They can be represented in the micro production framework as shifters of  $\xi_{fj}$  for all firms  $f$ . Call these shocks  $d \log G_j$ . Proposition 3 can be amended to incorporate the effect of these shocks on equilibrium objects: prices and sales.

Log-differentiating equation (12) yields

$$d \log \mathbf{B} + d \log \mathbf{G} = \Upsilon^{-1} d \log \mathbf{X}.$$

The derivative to equation (7) does not change. Combining the two equations yields a modified

version of Proposition 3:

$$d \log P_j = \frac{1}{\sigma_j - 1} ([\Psi]_{jk} d \log X_k - \sigma_j(1 - \varsigma_j) d \log G_j), \quad \forall j, k.$$

The above derivations also imply that if sales  $d \log X$  is invariant to prices on the demand side (i.e. if demand is Cobb-Douglas across industries) then industry productivity shocks under monopolistic competition do not generate spillovers, even when there are economies of scope.

## C Quantitative Appendix

### C.1 Input Classification

I use the 1997 BEA I/O table to estimate the model. This provides me with a matrix of input expenditure shares by industry  $j$ . I define the set of generic inputs ( $l_j$  in the theory) to correspond to NAICS sectors 1, 2, 3, 4, and labor value-added. I take  $\varsigma_j \equiv \gamma_j \frac{\sigma_j - 1}{\sigma_j}$  to be the industry-level expenditures on these input categories as a share of gross output.

I group expenditures into three separate intangible input categories corresponding to the first three rows of Figure 2: headquarters (55), professional services and information (51 and 54), and the leasing of intangible assets (533). I group all remaining inputs into one residual input. No expenditure information is needed on this remaining input category, as the derivations below will make clear. This residual category is set up to also absorb payments to latent factors (venture capital, sweat equity) that are paid rents from out of gross operating surplus in the I/O tables.

This choice of classification yields four specialized inputs. The three intangible inputs share the same scale and rivalry elasticities ( $\rho^{INT}, \theta^{INT}$ ), and the remaining latent input category has a separate set of scale and rivalry elasticities ( $\rho^{RES}, \theta^{RES}$ ). I let  $\Theta$  denote the parameter set containing all four elasticities.

### C.2 Estimation Details

#### C.2.1 Inversion

Conditional on  $\Theta, \gamma_0, \gamma_1$ , I describe how to use the aggregate predictions of the parametrized model to invert for macro technology coefficients  $\alpha_{mj}$ , residual profits in each year,  $B_{jt}$ , and the technology index in the cost function,  $Z_t$ .

First, although expenditures on specialized inputs in the model are at the firm-level, aggregate data in the BEA tables has these expenditures separated by industry. I assume that, for multi-industry firms, accounting over these expenditures is proportionalized across industries with respect to choice shares  $\mu_{f mj}$  for the deployment of ideas.<sup>89</sup> Under such a case, the model predicts aggregate

<sup>89</sup>Alternatively, one can assume that the BEA computes specialized input expenditures based off of the first industry in which the firm deployed a successful idea generated by  $m$ . This yields the same aggregate

expenditures of industry  $j$  on specialized input  $m$ :

$$\frac{M_{mj}}{N} = \frac{\rho^{INT} - 1}{\rho^{INT}} \int_f \mu_{f mj} \Delta_{fm}^{\rho^{INT}} dG(\xi_f), \quad \forall j, \forall m \in INT, \quad (16)$$

where  $G(\xi_f)$  is the distribution of  $\xi$  given by Assumption 3. To get  $\alpha_{mj}$  from base period ( $t = 1$ ) data on  $M_{mj}$ , I solve equation (16) separately for each of the three  $m$  inputs that I classify as an intangible. I solve for the  $J$  vector of unknowns  $\alpha_{mj} B_{j,t=1} Z_t$  (grouped together) given data on expenditures  $\{M_{mj}\}_j$ .

The mean of  $\alpha_{mj}$  across  $m$  for each  $j$  is isomorphic to shifts in  $B_{j,t=1}$ . Thus, I am free to normalize  $\alpha_{RES,j} = 1$ . To find base period  $B_{j,t=0}$ , note that gross profits (sales minus expenditures on generic inputs) among all firms in an industry equal

$$\frac{\pi_j}{N} = \sum_m \int_{\xi} \mu_{f mj} \Delta_{fm}^{\rho_m} dG(\xi), \quad \forall j, \quad (17)$$

I subtract equations (16) from equation (17) to yield

$$\frac{\pi_j - \sum_{m \in INT} \frac{\rho^{INT}}{\rho^{INT} - 1} M_{mj}}{N} = \int_{\xi} \mu_{f, RES, j} \Delta_{f, RES}^{\rho^{RES}} dG(\xi),$$

where the left-hand-side is data contained in BEA input-output tables, and the right-hand side contain a  $J$  vector of unknowns  $B_{j,t=1} Z$  that can be inverted. Note that this equation imposes a non-negativity restriction which manifests as a lower bound on the value of  $\rho^{INT}$  according to the model:

$$\begin{aligned} \pi_j &> \sum_m \frac{\rho^{INT}}{\rho^{INT} - 1} M_{mj}, \quad \forall j \\ \iff (1 - \varsigma_j) &> \frac{\rho^{INT}}{\rho^{INT} - 1} \sum_{m \in INT} \gamma_{mj}, \quad \forall j \\ \iff \frac{\rho^{INT} - 1}{\rho^{INT}} &> \max_j \frac{\gamma_{INT,j}}{1 - \varsigma_j} = \max_j \frac{\gamma_{INT,j}}{\gamma_{INT,j} + \gamma_{OTH,j}}, \end{aligned}$$

for I/O expenditure shares  $\gamma_{mj}$ . In the IO data, this restriction corresponds roughly to imposing that  $\rho^{INT} > 3$ .

I use the values of  $B_{j,t=1} Z_{t=1}$ , and  $\alpha_{mj} B_{j,t=1} Z_{t=1}$  to back out  $\alpha_{mj}$ . I hold  $\alpha_{mj}$  constant over all three time periods, for lack of I/O table expenditure data on intangibles in subsequent years.

To find future-period residual profits  $\{B_{j,2}, B_{j,3}\}_j$  (jointly with  $Z_t$ ), I invert for  $B_{jt} Z_t$  using equation (17) with data from that corresponding year directly and values of  $\alpha_{mj}$  from the above two steps.

Finall, given the full set of  $B_{ft} Z_t$  and  $\alpha$ , I can solve for  $Z_t$  such that the share of single-industry prediction as equation 16.



firms matches 0.8 in the data:

$$\frac{\sum_j \left( Pr(\chi_{fjt} = 1) \prod_{k \neq j} (1 - Pr(\chi_{fkt} = 1)) \right)}{Pr(\chi_{ft} = 1)} = 0.8. \quad (18)$$

where the probabilities of entry by industry ( $\chi_{fjt}$ ) and by firm ( $\chi_{ft}$ ) are given in Section B.3.

### C.2.2 Inference

First, I show that at true parameter values  $\Theta, \gamma$ , the following  $J \times J$  structural moment conditions hold true:

$$\mathbf{E}_f \left[ (\epsilon_{fjt} - \hat{\epsilon}_{fjt-1}) \widetilde{\Delta \log S_{fkt}} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] = 0, \quad \forall j, k, \forall t = \{2, 3\}$$

where  $\widetilde{\Delta \log S_{fkt}}$  is the de-meaned shock among shocks received by all firms that are active in industry  $k$ , and  $\hat{\epsilon}_{fj,t-1}$  is the structural error conditional on the firm being active in industry  $j$ :

$$\hat{\epsilon}_{fj,t-1} \equiv X_{fj,t-1} - \mathbb{E}[X_{fj,t-1} \mid \xi_{f,t-1}, \chi_{fj,t-1} = 1].$$

By the law of iterated expectations, the moment condition for any pair of industries  $j, k$  in any year  $t = \{2, 3\}$  can be written as

$$\mathbf{E}_{\xi_{f,t-1}, \Delta \log S_{ft}} \left[ \widetilde{\Delta \log S_{fkt}} \mathbf{E}_f \left[ (\epsilon_{fjt} - \hat{\epsilon}_{fjt-1}) \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1, \xi_{f,t-1}, \Delta \log S_{ft} \right] \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right],$$

where the  $(\epsilon_{fjt} - \hat{\epsilon}_{fjt-1})$  terms inside the inner expectation are zero in expectation because

$$\mathbf{E}_f \left[ X_{fjt} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1, \xi_{f,t-1}, \Delta \log S_{ft} \right] = \mathbb{E} \left[ X_{fjt} \mid \xi_{f,t} \right],$$

using Assumption 4 (relevance), so that  $\xi_{f,t}$  can be expressed as a known function of  $(\xi_{f,t-1}, \Delta \log S_{ft})$ , and

$$\mathbf{E}_f \left[ X_{fj,t-1} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1, \xi_{f,t-1}, \Delta \log S_{ft} \right] = \mathbb{E} \left[ X_{fj,t-1} \mid \xi_{f,t-1}, \chi_{fj,t-1} = 1 \right],$$

using Assumption 5 (conditional independence), so that  $\Delta \log S_{ft}$  can be dropped conditional on the industry presence  $\chi$  and unobserved profitability shifters. Note that it is important for  $\Delta \log S_{ft}$  to be independent of realized outcomes  $X_{fj,t-1}$ , so what Assumption 5 rules out is for firms that do unexpectedly well (conditional on  $\xi$ ) to receive higher demand shocks.

Next, I show that the inference can proceed off of computable sample analogs of the moment conditions. One component of the moment condition is pure data (the terms inside structural residuals  $\epsilon$  that correspond to realized sales  $X_{ft}$ ). I label the set of firms that are active in any

pair of industries  $j, k$  in year  $t - 1$  by  $n_{fj,t-1}$  and construct

$$\Xi_{jkt}^o \equiv \frac{1}{n_{jk,t-1}} \sum_{f \in n_{jk,t-1}} (X_{fjt} - X_{fj,t-1}) \Delta \widetilde{\log S_{fkt}}, \quad \forall j, k, \forall t = \{2, 3\}.$$

The remaining components of the structural residuals in the moment condition is model-implied sales (i.e.  $\mathbb{E}[X_{fjt}| \xi_{f,t-1}, \chi_{fj,t-1} = 1]$ ). I re-write this as:

$$\begin{aligned} \Xi_{jkt}^m &\equiv \mathbf{E}_f \left[ \left( \mathbb{E}[X_{fjt}| \xi_{ft}] - \mathbb{E}[X_{fj,t-1}| \xi_{f,t-1}, \chi_{fj,t-1} = 1] \right) \Delta \widetilde{\log S_{fkt}} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] \\ &= \mathbf{E}_{\xi_{f,t-1}, \Delta \log S_{ft}} \left[ g_{jk}(\xi_{fj,t-1}, \Delta \log S_{ft}) \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] \\ &= \mathbf{E}_{\Delta \log S_{ft}, \chi_{f,t-1}} \left[ \int_{\xi_{f,t-1}} g_{jk}(\xi_{fj,t-1}, \Delta \log S_{fkt}) \Pr(\xi_{f,t-1} \mid \chi_{f,t-1}) d\xi_{f,t-1} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] \\ &= \mathbf{E}_{\Delta \log S_{ft}, \chi_{f,t-1}} \left[ \int_{\xi} g_{jk}(\xi, \Delta \log S_{fkt}) \frac{\Pr(\chi_{f,t-1} \mid \xi)}{\int_{\xi} \Pr(\chi_{f,t-1} \mid \xi) dG(\xi)} dG(\xi) \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] \end{aligned}$$

where the second line uses the law of iterated expectations and introduces a  $g_{jk}$  function to capture the inner expectation terms:

$$g_{jk}(\xi_{fj,t-1}, \Delta \log S_{ft}) \equiv \Delta \widetilde{\log S_{fkt}} \mathbf{E}_f \left[ \left( \mathbb{E}[X_{fjt}| \xi_{ft}] - \mathbb{E}[X_{fj,t-1}| \xi_{f,t-1}, \chi_{fj,t-1} = 1] \right) \mid \Delta \log S_{ft}, \xi_{f,t-1}, \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right],$$

which is an analytical function of  $(\xi_{fj,t-1}, \Delta \log S_{ft})$  given the solution properties of the model and again, given Assumption 4 (relevance), so that  $\xi_{f,t}$  is known. The third line applies Assumption 5 (conditional independence) to get  $\Pr(\xi_{f,t-1} \mid \Delta SHK_{ft}, \chi_{f,t-1}) = \Pr(\xi_{f,t-1} \mid \chi_{f,t-1})$  and the forth line applies Bayes rule to transform  $\Pr(\xi_{f,t-1} \mid \chi_{f,t-1})$  into known analytical extensive margin probabilities given the model.

The sample analog of the last line is given by

$$\Xi_{jkt}^m \equiv \frac{1}{n_{jk,t-1}} \sum_{f \in n_{jk,t-1}} \sum_{s \in S} \omega_{fs} m_{fjk}(\xi_s, \Delta \log S_{ft}) \Delta \widetilde{\log S_{fkt}},$$

where  $\omega_{fs}$  are probability weights that stand for the probability that a simulated firm  $s$  has fundamentals  $\xi$  that belong to firm  $f$  in the data (with industry presence  $\chi_{f,t-1}$ ) relative to other simulated firms  $s' \in S$ ,

$$\omega_{fs} \equiv \frac{\prod_j \Pr(\chi_{j,t-1} = \chi_{fj,t-1} | \xi_s)}{\sum_{s'} \prod_j \Pr(\chi_{j,t-1} = \chi_{fj,t-1} | \xi_{s'})},$$

By the law of large numbers,

$$\lim_{n_{jk,t-1} \rightarrow \infty} \lim_{s \rightarrow \infty} \Xi_{jkt}^o - \Xi_{jkt}^m = \mathbf{E}_f \left[ (\epsilon_{fjt} - \epsilon_{fj,t-1}) \Delta \widetilde{\log S_{fkt}} \mid \chi_{fj,t-1} = 1, \chi_{fk,t-1} = 1 \right] = 0.$$

Table 12: Scope Distribution in the Data and Model, 1997

Number of Industries	Share of Firms (%)		Share of Sales (%)	
	Data	Model	Data	Model
1	80.99	80.15	26.13	25.00
2	13.01	13.54	10.80	15.74
3	3.32	3.29	7.34	9.43
4	1.33	1.07	5.51	6.33
5	0.61	0.44	4.08	5.48
6	0.28	0.23	2.89	5.76
7	0.14	0.15	3.54	6.71
8	0.08	0.10	3.63	4.05
9 +	0.25	1.03	36.05	21.49

This Table shows the distribution of firms by scope, in the data and in the model (with the six estimated parameters,  $\Theta, \gamma_0, \gamma_1$ ). Sales of firms with 9 or more industries could not be simulated via brute force due to memory issues when simulating the discrete Poisson process. Instead, it is backed out from the fact that the share of sales by firms with one industry was set to equal 0.25 in the estimation.

### C.3 External Validity

Table 12 displays the distribution of firms and sales over firm scope behind Figure 3. These are computed by simulating the actual outcomes of firms in the model when matched to data on output and input expenditure by industry in 1997 (which is done for estimation).

### C.4 Counterfactuals

#### C.4.1 Equilibrium Definition and Mapping to Data

I introduce some more notation used to characterize the open economy equilibrium. Let  $D$  denote the US manufacturing trade deficit (exports of capital services) vis-à-vis the rest of the world. Let  $\bar{Y}_{r,j}$  and  $\bar{Y}_{c,j}$  denote the total market size faced by US firms in each industry  $j$  in the rest of the world and China, and suppose that all firms are common exporters.<sup>90</sup> Let  $\bar{W}P_j, \bar{C}P_j$  represent indices of price competitiveness in each of the two foreign markets by all foreign firms. Let  $\bar{C}M_j$  and  $\bar{W}M_j$  represent indices of price competitiveness in the US market by firms from China and from the rest of the world in industry  $j$ . For example, an increase in  $\bar{C}M_j$  indicates that Chinese prices have been lowered (become more competitive) in the US market.

<sup>90</sup>The selection into exporting margin is not a margin that is addressed in this paper. But in fact, the model setup here is isomorphic to a setup where firms have common probability  $\chi_j$  of adapting each specialized idea successively to a foreign market (at the same time as for domestic sale). I show this in the Online Supplementary Appendix.

**Definition 2 (General Equilibrium)** *Given fixed foreign price competitiveness abroad and at home,  $\bar{W}P_j$ ,  $\bar{C}P_j$ ,  $\bar{C}M_j$ ,  $\bar{W}M_j$ , foreign expenditures  $\bar{Y}_{r,j}$ ,  $\bar{Y}_{c,j}$ , general equilibrium for a given country is described by a wage,  $w$ , and a vector of domestic price competitiveness  $\{PD_j\}_j$  such that the following equations and related definitions hold:*

(i) *Total industry expenditures in the US (from domestic and foreign) is given by*

$$Y_j = \sum_k \gamma_{jk} X_k + \beta_j (1 - \beta_S) (wL),$$

*where  $X_j$  stands for domestic industry gross output,  $\gamma_{jk}$  is the share of expenditures on generic inputs by industry  $k$  on inputs from industry  $j$ ,  $\varsigma_j$  is the share of generic input expenditures in gross output of industry  $j$ ,  $\beta_j$  reflects Cobb-Douglas final consumption shares within manufacturing, and  $\beta_S$  is the share of final consumption by private households on non-manufacturing.*

(ii) *Goods market clearing yields a system of  $J$  equations in  $J$  residual profit shifters  $B_j$ : output produced over all firms have to equal total domestic industry output, which has to equal output consumed at home plus output exported to foreign markets:*

$$\begin{aligned} X_j &= N \int_f \mathbb{E}[X_{fj}(\vec{B})] dG(\xi_f) \\ &= Y_j \frac{PD_j}{PD_j + \bar{C}M_j + \bar{W}M_j} + \bar{Y}_{c,j} \frac{PD_j}{PD_j + \bar{C}P_j} + \bar{Y}_{r,j} \frac{PD_j}{PD_j + \bar{W}P_j}, \quad \forall j = 1, \dots, J, \end{aligned} \quad (19)$$

*where the index of domestic competitiveness,  $PD_j$  is endogenous:*

$$PD_j \equiv N \int_f \mathbb{E} \left[ p_{fj}^{1-\sigma_j} \right] dG(\xi_f).$$

(iii) *Domestic competitiveness can be related to residual profit shifters  $B_j$  (through combining equation 19 with an open-economy version of equation 7):*

$$B_j = (1 - \varsigma_j) \left( \frac{c_j}{\varsigma_j} \right)^{\frac{\varsigma_j}{\varsigma_j - 1}} \left( \frac{X_j}{PD_j} \right)^{\frac{1}{\sigma_j(1 - \varsigma_j)}}, \quad (20)$$

*where  $c_j$  stands for the unit price index of a bundle of generic inputs assembled using a Cobb-Douglas intermediate production function:*

$$c_j \equiv w^{\gamma_{lj}} \prod_{m \in J} P_m^{\gamma_{mj}},$$

*where  $\gamma_{mj}$ ,  $\gamma_{lj}$  is the Cobb-Douglas share of expenditures (among expenditures on generic inputs) of industry  $j$  on input  $m \in J$  or labor value-added  $l$ , and the domestic industry price index  $P_j$  (in both final and intermediate consumption) is given by:*

$$P_j^{1-\sigma_j} \equiv PD_j + \bar{C}M_j + \bar{W}M_j. \quad (21)$$

(iv) The trade balance condition equates manufacturing imports with manufacturing exports plus net exports in a residual service sector,  $D$  (the manufacturing trade deficit):

$$\sum_j Y_j \frac{\bar{C}M_j + \bar{W}M_j}{PD_j + \bar{C}M_j + \bar{W}M_j} = D + \sum_j \bar{Y}_{c,j} \frac{PD_j}{PD_j + \bar{C}P_j} + \bar{Y}_{r,j} \frac{PD_j}{PD_j + \bar{W}P_j}. \quad (22)$$

(v) The residual service sector is produced with constant returns to scale using labor under perfect competition. Domestic value-added in the residual sector is given by

$$wL_s = D + \Pi + T + \beta_s(wL),$$

where  $T$  is government tariffs (assumed 0 in initial equilibrium), and  $\Pi$  is net profits in the manufacturing sector given by equation (10).

(vi) Manufacturing sector payroll is

$$wL_m = \sum_k X_k \left( 1 - \sum_j \gamma_{jk} \varsigma_k \right) - \Pi,$$

where  $L_m + L_s = L$  satisfies labor market clearing.

**Data in the Initial Equilibrium.** These equilibrium definitions allow me to impute consumption expenditure shares  $\beta_j$ , the manufacturing deficit  $D$ , all price competitiveness indices, and foreign expenditures  $\bar{Y}_{c,j}$ ,  $\bar{Y}_{r,j}$  given US and world trade and industry level data in 2017. I introduce the data I use from public data in 2017:

1. Data on gross output by manufacturing industry,  $X_j$  come from the BEA in 2017.
2. I hold the number of total manufacturing firms,  $N$ , fixed, at 318,000.
3. Data on  $\gamma_{jk}$  and  $\varsigma_k$  come from the 2012 BEA I/O tables (the 2017 tables are not yet available).<sup>91</sup>
4. Trade data in 2017 on US imports and exports by country and industry (after mapping HS10 to BEAX) come from the US Census Bureau (made available by Schott (2008)).<sup>92</sup>
5. World trade data in 2017 by industry and country come from BACI Comtrade.

**Variables in the Model.** Using the trade data, I compute  $\lambda_{dj}^s$  is the share of US firms' total sales in industry  $j$  going to destination  $d \in \{u, r, c\}$  (the US, rest-of-world, and China, respectively),  $\lambda_{odj}^c$  is the share of destination  $d \in \{u, r, c\}$ 's consumption of industry  $j$  on goods sold by the origin  $o \in \{u, r, c\}$ .

<sup>91</sup>There are a few industries where implied input-output use shares are so large that final use is predicted to be negative. I adjust input-output shares downward by a proportional factor for that industry until final use is at least 2% of gross consumption.

<sup>92</sup>There are a few industries where US exports is higher than measures of gross output in BEA data. I harmonize the two data sources by adjusting gross output,  $X_j$ , to be at least 1% higher than gross exports.

These trade shares are important; note that all the ratios of price competitiveness in the definition of equilibrium can be written as these observable trade shares.

I compute industry gross expenditures as  $Y_j = \frac{X_j \lambda_{uj}^s}{\lambda_{uu}^c}$ .

I compute final consumption shares in manufacturing,  $\beta_k$ , as:

$$\beta_k = \frac{Y_k - \sum_j \gamma_{jk} \varsigma_k X_k}{\sum_k (Y_k - \sum_j \gamma_{jk} \varsigma_k X_k)}.$$

Using the estimated parameters, I use the same macro inversion steps as in the structural estimation (equations 16, 17, and 18) to compute  $\alpha, B, Z$  in 2017. I use 1997 expenditure shares on intangible inputs  $m$  by industry  $j$  combined with 2017 output data to impute expenses on intangibles  $M_{mj}$  used in the inversion. With these variables on hand I compute net profits in the manufacturing sector  $\Pi$  using equation (10).

I compute the manufacturing deficit as the difference between total consumption and total output:  $D = \sum_j Y_j - \sum_j X_j$ . I normalize the wage  $w$  to 1 by choosing an appropriate unit in which to measure efficiency-adjusted labor, whereby

$$L = GDP - \Pi,$$

where GDP is 19.4 trillion in 2017. The share of consumption on residual services is then given by:

$$1 - \beta_s = \frac{\sum_j Y_j - \sum_j \sum_k \gamma_{jk} \varsigma_k X_k}{L}.$$

Foreign demand in the model is given by  $\bar{Y}_{r,j} \lambda_{urj}^c = EX_{urj}$  where  $EX_{urj}$  is US exports to destination  $r$  in industry  $j$ . An identical expression pins down  $\bar{Y}_{c,j}$ .

### C.4.2 Exogenous Shocks

The equilibrium set-up in Definition 2 accommodates different assumptions on how  $D$  adjusts to external shocks. Assuming that  $D$  is constant allows the use of the trade balance equation (22) to solve wages,  $w$ . On the other hand, assuming that  $D$  is generated by a perfectly elastic foreign demand for residual services pins down  $w$  across counterfactuals;  $D$  is simply computed from subtracting new manufacturing imports and exports under fixed  $w$ , from the same trade balance equation (22). The results I present in this paper follow this second approach in order to not have wage effects generate spillovers.

I consider three types of counterfactual shocks. I map these shocks to changes in exogenous variables under Definition 2.

1. A change in the domestic labor force, given by  $\hat{L}$ .
2. A change in foreign market size, given by  $\hat{Y}_{r,j}, \hat{Y}_{c,j}$
3. I consider new tariffs imposed by the US on imports from China, denoted by  $\tau_{cu,j}$ , and import

tariffs imposed by China on imports from the US, denoted by  $\tau_{ucj}$ . I model tariffs  $\tau \geq 1$  as ad-valorem, so that

- (a) The change in Chinese price competitiveness in the US is  $\hat{C}\hat{M}_j = \tau_{cuj}^{1-\sigma_j}$ .
- (b) The change in US price competitiveness in China can be modeled as  $\hat{C}\hat{P}_j = \tau_{ucj}^{\sigma_j-1}$ . Tariffs also cause take-home revenues of firms to fall to  $\frac{1}{t_j}$  of tax-inclusive sales. This can be reflected by a change in  $\hat{Y}_{cj} = \tau_{ucj}^{-1}$ .
- (c) US tariff revenues are given by

$$T' = \sum_j \frac{\tau_{cuj} - 1}{\tau_{cuj}} \tau_{cuj}^{1-\sigma_j} \lambda_{cuj}^c \hat{P}_j^{\sigma_j-1},$$

I assume that pre-existing tariffs on Chinese imports are zero. If they are non-zero, the new tariffs change infra-marginal tariff revenues and the calculation needs to be revised. I assume that Chinese tariffs on US goods are taken out of the system and do not go towards increasing market demand  $\tilde{Y}_{c,j}$ .

### C.4.3 Proposition 5: Arbitrary Propagation in Open Economy with Input-Output Linkages under Economies of Scale and Scope

I log differentiate the system of equations in Definition 2 to derive a unified system of equations linking endogenous equilibrium variables (price indices, sales, etc) to exogenous shocks described above (changes to  $L$ ,  $\tilde{Y}$ ,  $CM$ ,  $CP$ ). These equations yield analytical propagation matrices. First, log-differentiating the second line of equation (19) yields the following demand-side relationship between sales and changes in gross output  $X$  and domestic productivity  $PD$ , in matrix algebra:

$$d \log X = (\mathbb{I} - \Omega^R)^{-1} \left( d \log S + \text{diag}(\lambda^{cpt}) d \log PD + \text{diag}(\lambda_{uj}^s) d \log PF \right), \quad (23)$$

where  $\lambda_{dj}^s$ ,  $\lambda_{odj}^c$  are defined earlier, and  $\lambda_j^{cpt}$  is a measure of foreign competition in the US given by

$$\lambda_j^{cpt} \equiv \sum_{d \in \{u,r,c\}} \lambda_{dj}^s (1 - \lambda_{udj}^c),$$

the matrix  $\Omega^R$  contains external input-output revenue-weighted coefficients denoting the extent to which changes in gross output in other industries  $k'$  affect gross output in  $j$ :

$$[\Omega^R]_{jk} \equiv \lambda_{uj}^s (1 - \lambda_{fin,j}^c) \frac{\gamma_{jk} \zeta_k X_k}{\sum_{k'} \gamma_{jk'} \zeta_{k'} X_{k'}},$$

where  $\lambda_{fin,j}^c$  is the share of final use in consumption in industry  $j$ , and exogenous shocks are grouped under changes to the scale of output,  $d \log S$ , and changes to domestic competition,  $d \log PF$ :

$$d \log S_j \equiv \lambda_{rj}^s d \log \bar{Y}_{rj} + \lambda_{cj}^s d \log \bar{Y}_{cj} + \lambda_{uj}^s \lambda_{fin,j}^c d \log L - \lambda_{cj}^s \lambda_{ucj}^c d \log \bar{C}P_j - \lambda_{rj}^s \lambda_{urj}^c d \log \bar{W}P_j, \\ d \log PF_j \equiv -\lambda_{cu,j}^c d \log \bar{C}M_j - \lambda_{ru,j}^c d \log \bar{W}M_j.$$

Next, I turn to the supply-side relationship between market size (residual profits), prices and productivities. The first line of equation (19) can be log-differentiated to yield

$$\Upsilon^{-1} d \log X = d \log B,$$

where  $\Upsilon$  is the aggregate matrix over individual firm spillovers given by equation (13). I log-differentiate the expression for residual profits  $B_j$  in equation (20), open up the generic cost index  $c_j$  to reflect intermediate input purchases from manufacturing industries, replace  $d \log B$  with  $d \log X$  given the above equation to get

$$\Psi d \log X = -(\mathbb{I} - \Omega^P \text{diag}(\lambda_{uu}^c)) d \log PD - \Omega^P d \log PF, \quad (24)$$

where  $\Psi$  is an inverse spillover matrix (due to economies of scale and scope) given by

$$[\Psi]_{jk} \equiv \sigma_j(1 - \varsigma_j)[\Upsilon^{-1}]_{jk} - \mathbf{1}_{j=k},$$

and  $\Omega^P$  is matrix of external input-output price-related coefficients given by

$$[\Omega^P]_{jk} \equiv \frac{\gamma_{kj} \varsigma_j \sigma_j}{\sigma_k - 1}.$$

Equations (24) and (23) represent two systems of equations in two vectors of unknowns ( $X$  and  $PD$ ). I combine them to express domestic productivity  $PD$  changes in terms of external shocks  $PF$  and  $S$ :

$$d \log PD = -\left(\mathbb{I} - \Omega^P \text{diag}(\lambda_{uuk}^c) + \Psi (\mathbb{I} - \Omega^R)^{-1} \text{diag}(\lambda^{cpt})\right)^{-1} \times \\ \times \left(\Psi (\mathbb{I} - \Omega^R)^{-1} d \log S + \left(\Omega^P + \Psi (\mathbb{I} - \Omega^R)^{-1} \text{diag}(\lambda_u^s)\right) d \log PF\right). \quad (25)$$

Proposition 5 follows from setting  $d \log PF = 0$  in the above expression (the only exogenous shock is to market size, not to competitiveness). The main results described in Figure 5 are computed as follows. I express the productivity effects as:

$$d \log PD = -\Xi d \log S,$$

where  $\Xi$  is a matrix defined below, taking on different values across the scenarios described in the



text.

$$\Xi = - \left( \mathbb{I} - \Omega^P \text{diag}(\lambda_{uuk}^c) + \Psi (\mathbb{I} - \Omega^R)^{-1} \text{diag}(\lambda^{cpt}) \right)^{-1} \Psi (\mathbb{I} - \Omega^R)^{-1}$$

For each scenario (associated with a  $\Xi$ ), I decompose  $\Xi$  into diagonal and off-diagonal elements, respectively, as

$$\Xi = \Xi^{DIAG} + \Xi^{CROSS}.$$

The values of  $\lambda_j^{prod} \equiv X_j/X$  are shares of industry  $j$  output among total manufacturing output  $X$ . I use these shares to collapse the vector of price (productivity) effects  $d \log PD_j$  into an aggregate productivity index. I compute, for example, the diagonal effect as:

$$d \log PPI = \left( \frac{\lambda^{prod}}{1 - \sigma} \right)' \times \Xi^{DIAG} \times \text{diag}(\lambda_r^s + \lambda_c^s), \quad (26)$$

which yields a  $1 \times J$  vector describing the effect of a proportional change in foreign market size in a column industry on the manufacturing PPI.

Table 13 describes the numbers behind producing Figure 5. I compute the effects of a 1% proportional rise in foreign demand (across all industries) on US firm productivity and gross output. I decompose these effects into direct (own-industry) and spillover effects using equations (24), (25), and (26). The change in gross output is computed using the same  $\lambda^{prod}$  weights. I compute the share of spillovers generated by scope economies by differencing the productivity effects between scenarios (b) and (a), and likewise for scenarios (d) and (c) and expressing this difference as a share of the total productivity effect (in (b) and (d), respectively).

Table 14 presents results for the main counterfactual, scenario (b), industry-by-industry, rather than collapsed into an aggregate productivity index. It shows the top and bottom industries by the proportion of productivity response that occurs as a result of spillovers relative to the total effect whenever the industry receives an industry-specific demand shock. Note that this is a relative result (spillovers as a share of total). The top industries listed do not indicate the industries where demand shocks generate the highest total aggregate productivity gains, nor industries where the total elasticity of productivity to gross output is highest.<sup>93</sup>

#### C.4.4 Robustness to Alternative values of $\sigma$

I explore the sensitivity of the key results in Table 13 to alternative values of  $\sigma_j$ , against the benchmark of  $\sigma_j = 5 \forall j$ . I consider three different exercises. First, I vary the absolute value of  $\sigma$  but continue to hold it constant across industries. Figure 7 shows how the elasticity of productivity with respect to gross output varies with values of  $\sigma \in (3, 10)$ , decomposing the aggregate productivity response to the direct, own-industry effect versus net spillovers. While the direct effect is, as expected, decreasing with the value of  $\sigma_j$ , the indirect effect is much less so, contributing between 3 and 5 percentage points

<sup>93</sup>The top 5 industries by total aggregate productivity gains are aircraft manufacturing, petroleum refineries, other motor vehicle parts, light truck and utility vehicles, and broadcast and wireless communications equipment.

Table 13: Effect of a 1% Increase in Foreign Demand on US Productivity and Output

Scenario	(a)	(b)	(c)	(d)
Total Effect on Productivity (%)	0.066	0.082	0.636	1.410
Direct	0.066	0.066	0.067	0.134
Positive Spillovers	0	0.018	0.526	1.275
Negative Spillovers	0	-0.002	0	0.000
Total Effect on Gross Output (%)	0.387	0.416	1.976	3.922
Direct	0.387	0.392	0.515	0.581
Positive Spillovers	0	0.032	1.460	3.342
Negative Spillovers	0	-0.003	0	0.000
Elasticity of Productivity with respect to Gross Output	0.17	0.20	0.32	0.36
share due to Scope Economies	0	0.20	0	0.55

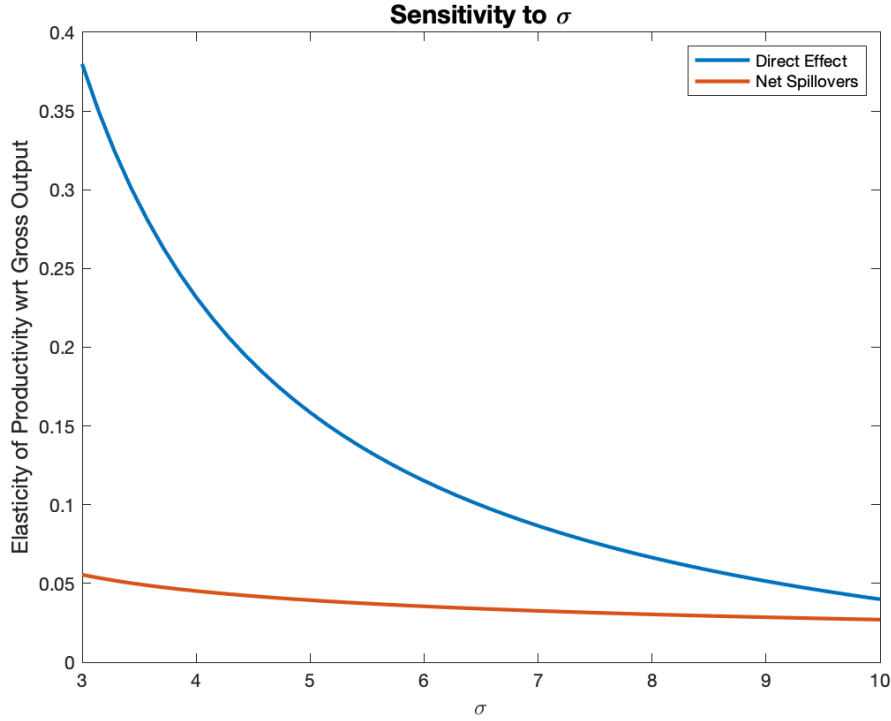
This table depicts the effect on aggregate productivity and output in the US due to a proportional 1% foreign demand shock (across all industries) used to compute the elasticities in Figure 5. The sub-rows decompose the total effects into those accruing due to direct, own-industry responses versus spillovers across industries. The four columns depict four scenarios corresponding to different versions of the underlying economy: (a) only scale economies (along the own-industry), (b) both scale and scope economies, (c) only scale economies with external I/O linkages, (d) both scale and scope economies with external I/O linkages. These effects are computed using equations (24), (25), and (26). The elasticities in Figure 5 correspond to the direct and spillover productivity effects scaled by the total effect on gross output. They sum to the penultimate row, the elasticity of productivity with respect to gross output. To compute the last row, the contribution of scope economies in scenario (b), I take the difference in productivity between (b) and (a) divided by the productivity response in (b). I do the analogous computation for scenario (d) by comparing to (c).

Table 14: Top and Bottom Industries by Positive Spillovers Generated

BEAX	Description	Share
334514	Totalizing fluid meter and counting device	0.71
3122A0	Tobacco product manufacturing	0.71
334510	Electromedical and electrotherapeutic apparatus manufacturing	0.60
334516	Analytical laboratory instrument	0.58
33451B	Watch, clock, and other measuring and controlling device manufacturing	0.57
33641A	Propulsion units and parts for space vehicles and guided missiles	0.52
33461X	Manufacturing and reproducing magnetic and optical media	0.50
339115	Ophthalmic goods manufacturing	0.46
334511	Search, detection, and navigation instruments	0.45
325620	Toilet preparation manufacturing	0.44
31161A	Animal (except poultry) slaughtering, rendering, and processing	0.06
326160	Plastics bottle manufacturing	0.06
314110	Carpet and rug mills	0.06
311920	Coffee and tea manufacturing	0.06
312140	Distilleries	0.05
337215	Showcase, partition, shelving, and locker manufacturing	0.05
311221	Wet corn milling	0.03
33142X	Copper rolling, drawing, extruding and alloying	0.02
327992	Ground or treated mineral and earth manufacturing	0.02
311930	Flavoring syrup and concentrate manufacturing	-0.06

This table depicts the top and bottom 10 industries in terms of the proportion of aggregate productivity response (due to a marginal demand shock in that industry) that accrues due to net cross-industry spillovers. These effects are evaluated using equation (14); the numbers here correspond to the scatterplot in Figure 6. Results are generated as an average of 100 monte carlo simulations of the model's draws of firm productivities to wash out sampling variation.

Figure 7: Robustness: Elasticity of Productivity to Gross Output under different values of  $\sigma$



This graph explores the sensitivity of the results in Figure 5 to alternative common values of  $\sigma_j$  across industries. I decompose the aggregate elasticity productivity to output in scenario (b) into the direct effect in the blue line (driven by scale economies) and an indirect effect in the orange line (driven by scope economies).

to the elasticity aggregate productivity to gross output across the entire range of values of  $\sigma$ .

In the next two calibrations, I allow  $\sigma_j$  to vary across industries  $j$  by: (i) assuming that profit shares (gross operating profits) in each industry are equal to  $\frac{1}{\sigma_j}$  (as would be true in a case with constant returns to scale, monopolistically competitive firms, and sunk entry costs paid in some pre-period), and (ii) targeting the same own-sector returns to scale as in Bartelme et al. (2019). While assumption (i) is ad-hoc (and not consistent with the model), the values of  $\sigma_j$  serve as a useful benchmark as they appear in other papers. Assumption (ii) is however model-consistent. In this scenario, I first generate a mapping from BEAX to the two-digit manufacturing sectors in Table 1 of their paper, denoted by  $s$ . I allow for as many differences across  $\sigma_j$  as there are sectors  $s$ , so all  $\sigma_j$  is the same within a sector but different across sectors. I then solve for the values of  $\sigma_s$  that would generate the following relationship between sectoral price indices and sectoral size (from combining equations 26 and 24).

$$\frac{d \log PPI_s}{d \log X_s} = -\gamma_s^{BCDR} = - \sum_{j \in s} \frac{\lambda_j^{prod}}{1 - \sigma_s} \sum_{k \in s} \Psi_{jk}, \quad \forall s,$$

where  $\gamma_s^{BCDR}$  are estimates of scale elasticities in Table 1 of Bartelme et al. (2019).<sup>94</sup>

<sup>94</sup>Note that this procedure does attribute (correctly) the cross-industry spillovers across industries within

Table 15: Effect of a 1% Increase in Foreign Demand on US Productivity and Output: Robustness under different values of  $\sigma_j$

Value of $\sigma_j$ Scenario	Baseline (=5)		Profit Share		BCDR	
	(b)	(d)	(b)	(d)	(b)	(d)
Total Effect on Productivity (%)	0.082	1.410	0.085	0.417	0.064	1.073
Direct	0.066	0.134	0.069	0.094	0.050	0.101
Positive Spillovers	0.018	1.275	0.018	0.323	0.016	0.972
Negative Spillovers	-0.002	0.000	-0.002	0.000	-0.002	0.000
Total Effect on Gross Output (%)	0.416	3.922	0.408	1.639	0.395	3.299
Direct	0.387	0.581	0.372	0.454	0.365	0.524
Positive Spillovers	0.032	3.342	0.039	1.185	0.032	2.775
Negative Spillovers	-0.003	0.000	-0.004	0.000	-0.003	0.000
Elasticity of Productivity w.r.t. Gross Output	0.20	0.36	0.21	0.25	0.16	0.33
share due to Scope Economies	0.20	0.55	0.19	0.43	0.23	0.52

This table explores the sensitivity of results in Table 13 and Figure 5 to alternative calibrations of  $\sigma_j$  that vary across industries. Across the rows of each column I compute the effect on aggregate productivity and output in the US due to a proportional 1% foreign demand shock (across all industries). The sub-rows decompose the total effects into those accruing due to direct, own-industry responses versus spillovers across industries. These effects are computed using equations (24), (25), and (26). The penultimate row divides the total response of productivity by the total effect on output. To compute the last row, the contribution of scope economies in, say, scenario (b), I take the difference in productivity between (b) and (a) divided by the productivity response in (b). I do the analogous computation for scenario (d) by comparing to (c). Across the columns, I alter the values of  $\sigma$  and the scenario. The first two columns display results obtained under the baseline assumption of  $\sigma_j = 5 \forall j$ , for scenarios (b) (no I/O linkages) and (d) (incl. I/O linkages). The next set of two columns repeat the baseline calculations but using  $\sigma_j$  calibrated to match the profit share in each industry assuming constant returns to scale, and the final set of two columns repeat the baseline calculations but using  $\sigma_s$  calibrated (at the 2-digit manufacturing level) to match sectoral level returns to scale in Bartelme et al. (2019).

Table 15 displays how these baseline results change with respect to the two different calibrations, both with and without I/O linkages. The different calibrations with heterogeneous  $\sigma_j$  do not alter the main quantitative message that spillovers due to scope are large. In the benchmark model set-up without input-output links (b), scope economies account for between 19% (Profit Share) and 23% (BCDR) of the total elasticity of productivity with respect to gross output. Whereas own-industry effects under the BCDR calibration, scenario (b), generate an aggregate scale elasticity of 0.13 (=0.050/0.395), accounting for spillovers induced by scope economies brings this up to 0.16 (=0.064/0.395).

#### C.4.5 Solving the Model in Exact Changes

For any set of counterfactual exogenous shocks, the system of equations admits a new solution for  $PD_j$  and  $w$ . I solve the system of equations in terms of exact hat changes. Specifically, for any guess

a sector to sectoral economies of scale. The only productivity response that would be missed in BCDR are cross-sectoral spillovers.

of  $\hat{P}\hat{D}_j$  and  $\hat{w}$ , I can compute

$$\hat{B}_j = \hat{c}_j^{\frac{\varsigma_j}{\varsigma_j-1}} \left( \frac{\hat{X}_j}{\hat{P}\hat{D}_j} \right)^{\frac{1}{\sigma_j(1-\varsigma_j)}},$$

where  $\hat{c}_j$  is given by

$$\hat{c}_j = \hat{w}^{\gamma_{lj}} \prod_{m \in J} \hat{P}_m^{\gamma_{mj}},$$

and  $\hat{P}_j$  is the change in the domestic price index given by

$$\hat{P}_j^{1-\sigma_j} = \hat{P}\hat{D}_j \lambda_{uu,j}^c + \hat{C}\hat{M}_j \lambda_{cu,j}^c + \hat{W}\hat{M}_j \lambda_{ru,j}^c,$$

and  $\hat{X}_j$  is given by

$$\hat{X}_j X_j = Y_j' \hat{P}\hat{D}_j \lambda_{uu,j}^c \hat{P}_j^{\sigma_j-1} + \bar{Y}_{c,j} \hat{Y}_{c,j} \hat{P}\hat{D}_j \lambda_{uc,j}^c \hat{P}_{chn,j}^{\sigma_j-1} + \bar{Y}_{r,j} \hat{Y}_{r,j} \hat{P}\hat{D}_j \lambda_{ur,j}^c \hat{P}_{row,j}^{\sigma_j-1},$$

and  $\hat{P}_{row,j}$  is the change in the rest-of-world consumption price index given by

$$\hat{P}_{row,j}^{1-\sigma_j} = \hat{P}\hat{D}_j \lambda_{ur,j}^c + \hat{W}\hat{P}_j(1 - \lambda_{ur,j}^c),$$

$\hat{P}_{chn,j}$  is the change in the consumption price index in China given by

$$\hat{P}_{chn,j}^{1-\sigma_j} = \hat{P}\hat{D}_j \lambda_{uc,j}^c + \hat{C}\hat{P}_j(1 - \lambda_{uc,j}^c),$$

and finally the new vector of gross expenditures  $Y_j'$  can be inverted from

$$Y_j' = \sum_k \gamma_{jk} \varsigma_k (\hat{X}_k X_k) + \beta_j (1 - \beta_S) (\hat{w} L \hat{L}),$$

where  $T'$  is tariff revenues defined above.

To evaluate the guess I use a system of  $J$  equations equal to deviations between industry sales as computed above,  $X_j'$ , and the implied industry sales (by solving the firm's problem) given by equation 12 under the new  $B_j'$ . I also use the trade balance condition, expressed (succinctly) as

$$\sum_j Y_j' = D' + \sum_j \hat{X}_j X_j,$$

to either pin down  $D'$  when  $\hat{w} = 1$  (foreign demand for residual services is assumed to be perfectly elastic), or to solve for  $\hat{w}$  when  $D$  is held exogenous (as is more typical in trade counterfactuals). A gradient based optimization algorithm is found to work very well with this system of equations.

**Equilibrium Changes.** Throughout counterfactuals presented in Table 6, I compute several changes in macroeconomic variables of interest:

1. The change in the manufacturing CPI (consumer price deflator) is

$$\prod_j \hat{P}_j^{\beta_j}$$

2. The change in the manufacturing CPI excluding the domestic response of productivity is

$$\prod_j \left( \hat{P}_j^{CTF} \right)^{\beta_j},$$

where

$$\left( \hat{P}_j^{CTF} \right)^{1-\sigma_j} = \lambda_{uu_j}^c + \hat{C}\hat{M}_j \lambda_{cu_j}^c + \hat{W}\hat{M}_j \lambda_{ru_j}^c.$$

3. Expressions for the change in imports, US output and US exports in each industry, tariff revenues and the deficit can also be computed directly given the equations above.

#### C.4.6 Alternative Tariffs

I compute the vector of alternative tariffs as follows. First, local effects of external shocks on the CPI can be computed from log-differentiating the price index equation (21):

$$d \log CPI = \sum_j \beta_j d \log P_j = \sum_j \frac{\beta_j}{1-\sigma_j} \left( \lambda_{uu_j}^c d \log PD_j - d \log PF_j \right),$$

where the change in foreign prices in my setting is given by  $d \log PF_j = \lambda_{cu_j}^c (\sigma_j - 1) d \log \tau_j$ . Finally, I use equation (25) to replace  $d \log PD_j$  above with its exogenous source,  $\{d \log \tau_j\}_j$ .

The change in imports from China is given by

$$d \log IM_c = \sum_j \lambda_{cj} \left( -\lambda_{uu_j}^c d \log PD_j - (1 - \lambda_{cu_j}^c) (\sigma_j - 1) d \log \tau_j + d \log Y_j \right),$$

where  $\lambda_{cj}$  is the share of imports from China in industry  $j$ , and again  $d \log Y_j$  and  $d \log PD_j$  can be solved for using the equilibrium relationships surrounding equation (25).

A marginal tariff in each industry thus generates a cost (higher consumer prices) versus a benefit (reducing imports from China, assuming that it is a policy-making objective). I manipulate the matrix of relationships above to yield

$$\hat{\tau}_k = \frac{\frac{-d \log IM_c}{d \log \tau_k}}{\frac{d \log CPI}{d \log \tau_k}}, \quad \forall k,$$

which expresses the effect of a marginal tariff in industry  $k$  towards achieving the policy target (reducing imports from China) per unit of increase in the CPI. Industries  $k$  with a higher  $\hat{\tau}_k$  should have higher tariffs (due to their higher benefit / cost ratio), but exactly how much higher is not exact given the non-linear structure of the model. I use this starting vector of alternative tariffs,  $\hat{\tau}$ , as the

basis for a search over various scaling parameters  $\beta \geq 0$ :

$$\tau_j = \max(1, \alpha \hat{\tau}_j^\beta),$$

where the  $\alpha$  is chosen so that the alternative tariffs  $\tau_j$  reduce Chinese imports by 41%, the same amount as uniform tariffs of 20% would. Intuitively, the values of  $\beta$  adjust the skew / bias that is applied to the tariffs identified by the local propagation matrix as less adverse on the CPI.<sup>95</sup>

Figure 8 displays the impact of alternative tariffs on the CPI and domestic productivity, across varying degrees of industry bias in tariffs on the  $x$ -axis (induced by values of  $\beta$ ). For each value of  $\beta$ , the parameter  $\alpha$  is chosen to scale the set of tariffs so that the overall impact on reducing Chinese imports is 41%. The left-most point, where tariffs of  $\tau = 1.2$  are symmetric across industries, corresponds to Column (2) of Table 6. Intuitively, as bias increases, more concentrated tariffs are required in order to achieve the same reduction in Chinese imports. I hold the lower bound of tariffs at 1 (so that import subsidies are ruled out).

#### C.4.7 China Import Tariff Counterfactual: Model with Input-Output Linkages

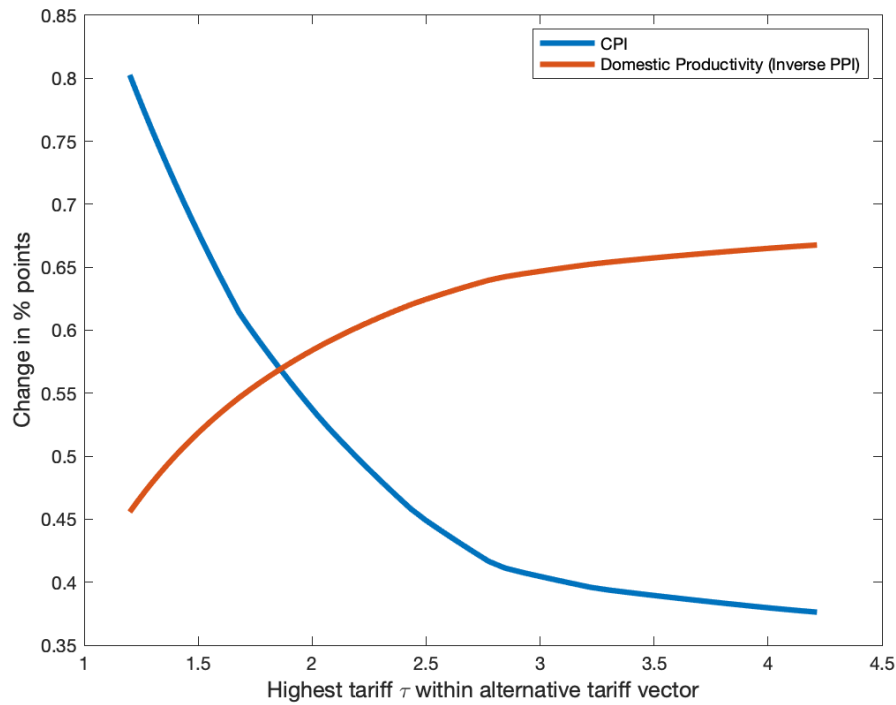
Finally, in Table 16, I repeat the China import tariff counterfactuals under the extension of the model with input-output linkages. First, in the neoclassical setting under constant returns to scale (Column 1), I find that input-output linkages generate higher increases in the CPI relative to the case without I/O linkages. Not only are prices of directly imported goods higher, but domestic goods prices also increase due to higher input costs. Column (2), which displays results from my model, highlights additional competing forces coming from scale and scope economies. While import taxes raise the cost of foreign imported inputs, substitution towards the use of domestic intermediate goods generates a scale effect which lowers the cost of domestic inputs. These domestic price effects are amplified by further success in export markets (the foreign demand for US exports is elastic), so that the net effect on upstream input prices faced by US producers in any industry is theoretically ambiguous. I find, in this calibration, that the domestic scale effect dominates. The results in Column (2) under input-output linkages actually reduce welfare losses of tariffs compared to the main results presented in column (2) of Table 6 (without input-output linkages). The improved success of domestic producers is reflected in an increase in net profits (equal to 0.28% of gross manufacturing output) and a much larger reduction in the trade deficit. Finally, column (3) displays results from applying a similar process of searching for alternative import tariffs that mitigate the reduction in the CPI while achieving the same reduction in imports from China. The results demonstrate that there are pure gains from import protection in this environment with scale and scope economies, exogenous foreign prices, and wages pinned down by a residual (services) sector. The results are sensitive, however, to assumptions on the foreign import demand elasticity. Inelastic foreign demand curtails the amplification of domestic scale effects and would turn the impact on the CPI back to positive territory.

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<sup>95</sup>While the values  $\hat{\tau}$  are identified using a local propagation matrix, the computed effects shown in Figure 8 and Table 6 are fully non-linear and not local approximations.



Figure 8: Impact of Alternative Tariff Policies on US Manufacturing CPI and Productivity



This graph displays the impact on the manufacturing CPI (in blue) and domestic productivity (in orange) from a range of tariff policies that are designed to achieve the same reduction in US imports from China (of 41%). Tariff policies range on the  $x$ -axis in terms of their degree of bias towards the vector  $\hat{\tau}$  identified by the model's propagation matrix. The bias is summarized as the highest value of  $\tau$  among the vector. A highest value of 1.2 (the initial point) reflects the benchmark counterfactual of blanket 20% tariffs.

Table 16: Model with external I/O links: Effects on the US Economy from US Tariffs on Chinese Imports

Model (with input-output linkages) Import tariffs on Chinese imports	(1) Neoclassical CRS 20% Uniform	(2) Scale + Scope 20% Uniform	(3) Scale + Scope Alternative
<i>Change (%)</i>			
Consumer Price Index (CPI)	1.69	0.11	-2.68
Producer Price Index (PPI)	0.58	-2.12	-6.10
Imports from China	-38.7	-40.8	-40.8
Imports from Rest of World	8.9	4.6	-4.2
US Output	0.6	7.4	20.2
US Exports	2.6	7.5	26.0
Manufacturing Trade Deficit	-3.0	-32.5	-86.3
<i>As Share of Manufacturing Output (%)</i>			
New Manufacturing Profits	0	0.28	0.72
New Tariff Revenues	0.89	0.86	0.15

This table extends the results in Table 6 to an environment with external input-output linkages. It presents estimates of the impact of two different sets of tariffs on outcomes in the US economy, under two different model settings calibrated to match US national industry-level aggregates (industry output and trade patterns) in 2017. Column (1) presents results from a 20% increase in tariffs on Chinese imports across all industries, under a neoclassical model with constant returns to scale and perfect competition. Column (2) compares the impact of the same set of tariffs to my model with economies of scale and scope. Column (3) presents results from alternative tariffs predicted by my model that seek to minimize the CPI impact while achieving the same reduction in total imports from China. See Definition 2 for a characterization of the open economy equilibrium and Appendix Section C.4.6 for details on how I compute alternative tariffs.