

Income Growth and the Distributional Effects of Urban Spatial Sorting*

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Abstract

We explore the link between rising nominal incomes at the top of the income distribution, within-city spatial sorting, and real income inequality. We develop and quantify a spatial model of a city with heterogeneous agents and non-homothetic preferences for endogenous differentiated private neighborhood amenities (e.g., restaurants and entertainment). As the rich get richer, their increased demand for such luxury amenities drives housing prices up in downtown areas, where amenity development is fueled by economies of density. The poor are made worse off, either being displaced or paying higher rents for amenities that they do not value as much. Using our model, we find that the neighborhood change within urban areas during the last two decades increased the welfare of richer households relative to that of poorer households by an additional two percentage points above and beyond the differential income growth. We conclude that welfare estimates of increased income inequality are understated if within-city spatial sorting responses are ignored.

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1 Introduction

Over the last three decades income inequality in the United States has grown sharply, with income growth being disproportionately concentrated at the top of earnings distribution. During this same time period, American cities have attracted more college educated and higher income individuals (Couture and Handbury (2017); Baum-Snow and Hartley (2017)). This latter trend has accompanied a renewed discussion of neighborhood change within many U.S. cities.¹

In this paper, we explore the link between rising incomes of the rich and the net in-migration of richer households to the downtown areas of American cities since 1990 and study the effects of this sorting behavior on real income inequality. The paper reflects two empirical regularities. First, demand system estimates suggest that many privately provided local amenities are relative luxury goods.² Second, downtown areas of major cities have a higher density of such amenities.

We build a model of residential sorting with heterogeneous agents and heterogeneous neighborhoods within a city that embeds the above two empirical regularities. As the incomes of the rich increase, their demand for urban amenities increases, resulting in them choosing to reside in downtown urban areas so as to be closer to these amenities. As the income composition of downtown urban areas changes, the provision of high quality urban amenities increases endogenously there, further fueling the in-migration of the rich. The in-migration of higher income individuals drives up downtown rents, which imposes a pecuniary externality on low income residents of downtown urban areas. Given the empirical fact that most poor residents in downtown urban areas are renters, they do not reap the capital gains of increasing house prices. Poorer residents have the choice between paying higher rents for a bundle of amenities they do not value as much and moving out of the downtown urban area.

We quantify the model, and find that increased incomes of the rich are, in part, causing a phenomenon that looks like urban gentrification. Some neighborhoods that were initially populated by poorer residents see the in-migration of higher income residents causing the amenity mix of the neighborhood to endogenously change.³ Moreover, we estimate the

¹For instance, in response to the influx of higher income residents into downtown areas, some municipalities, like New York City, have implemented policies to slow down neighborhood gentrification. See “New York Passes Rent Rules to Blunt Gentrification”, New York Times, March 22, 2016.

²For example, Aguiar and Bils (2015) estimate that restaurant meals and non-durable entertainment are among the goods with the highest income elasticities.

³Throughout the paper, we often use “neighborhood change” for low income neighborhoods and “gentrification” interchangeably. We realize that gentrification is a complex process with many potential definitions and drivers. Our interpretation is closest to the definition in the Merriam-Webster dictionary that defines gentrification as “the process of renewal and rebuilding accompanying the influx of middle-class or affluent people into deteriorating areas that often displaces poorer residents.” Our paper is not intended to explore

economic impact of this spatial sorting response resulting from increased incomes of the rich on residents of downtown areas with differing income levels. One of the paper’s main findings is that, because of externalities imposed by the in-migration of the rich, welfare estimates of increased income inequality are *understated* when spatial sorting responses are ignored. Preliminary estimates suggest that welfare differences between those at the 90th percentile of the income distribution and those at the 10th percentile of the income distribution increased by an additional 2.3 percentage points once accounting for the changing nature of spatial sorting of the rich into downtown urban areas during the 1990 to 2013 period. As way of comparison, in our sample of large cities, the measured income gap between the 10th and 90th percentiles of the income distribution increased by about 18 percentage points during this time period. Moreover, we are finding that the resulting neighborhood change within downtown areas in response to the rising incomes of the rich reduced the well-being of those at the 10th percentile of the income distribution by about 2.5 percent, in consumption equivalent terms.

Our paper proceeds in four parts. In the first part of the paper, we document a set of facts highlighting cross-sectional and time series variation in the nature of spatial sorting within urban areas during the last few decades. We focus our attention on the 100 largest Census Bureau Statistical Areas (CBSA’s) in the United States in 1990. We begin by showing that, within these CBSAs, the propensity to live in downtown areas is U-shaped in resident income. Poorer residents are more likely to live in downtown urban areas relative to middle-income residents. However, as resident family income increases above \$100,000 (in 1999 dollars), the relationship between resident income and the propensity to live downtown becomes monotonically increasing. This fact persists across different survey years of the U.S. Census, for a variety of definitions of downtown urban areas, for a variety of income measures, and conditioning on household type, race, and age. While stable in prior years, the relationship between income and the share of households living downtown has become steeper for the richest households by 2013. This suggests an increasing propensity for the rich to live in downtown urban in recent years, even conditional on a given income level. Consistent with our model, this urbanization of the rich was more pronounced in CBSAs that saw greater aggregate income growth, which also saw more downtown neighborhood change.

In the next part of the paper, we formalize the link between income inequality growth and urban gentrification. To do so, we build a model with heterogeneous agents and het-

all potential underlying causes of neighborhood gentrification. Rather, we wish to focus on the dimension of gentrification that follows the rise in incomes. Specifically, we focus on the interaction of rising incomes, non-homothetic preferences for urban amenities, and endogenous spatial responses.

erogeneous types of land. Individuals differ by their level of income. Land is comprised of downtown areas and suburban (non-downtown) areas which differ from each other in their bundle of fixed amenities, their proximity to jobs, and their housing supply elasticity. The suburbs have a higher endowment of fixed public amenities. Housing and associated endogenous amenities are also supplied more elastically in the suburbs. We model different quality neighborhoods within both the downtown and suburban areas. The quality of neighborhoods within each area indexes the quality of both the housing and amenity bundles, which are provided endogenously by profit maximizing developers. Furthermore, within each area-quality pair, neighborhoods are horizontally differentiated. Households choose a neighborhood where to live. They also visit other neighborhoods for the consumption of local amenities. Therefore, households benefit from living in areas with a high density of privately provided residential amenities. In the model, individuals at different income levels make different location choices on average. The model lends itself naturally to welfare analysis, as it provides a unique welfare measure for each level of income, that accounts for these location choices.

The key mechanism in the model is that high quality endogenous residential amenities are relative luxury goods. Given this, the model implies that urban areas of cities are disproportionately populated by both low income households who want to minimize commuting costs to jobs and reside in low-quality neighborhoods, and by higher income households who want to be close to the density of high quality neighborhoods. Middle income individuals disproportionately locate in the suburbs to take advantage of the higher fixed publicly provided amenities there. One important assumption in our model is that economies of density fuel the attractiveness of urban amenities. Higher density allows for a greater variety of neighborhoods and urban amenities in a given geographic area. While Princeton, New Jersey may have a handful of dining and entertainment options within a one-mile square radius, Manhattan has an order of magnitude more options within a similarly defined area. Given our love of variety preferences, this makes downtown urban areas attractive for consuming such endogenous amenities.⁴ In the model, downtowns have lower housing supply elasticities than do the suburbs, hence they grow through density rather than sprawl. As a result, as the city grows, downtown increases its comparative advantage compared to the suburbs in providing endogenous residential amenities.

We show that within our model, as incomes of richer households increase, richer house-

⁴Within the model, we do not take a stand on what particular endogenous amenity is contributing to the non-homotheticity. Restaurants and entertainment options could be one component of these amenities. These consumption goods are shown to have among the highest expenditure elasticities with respect to income out of all different consumption goods. But our model is broad enough to encompass any differentiated endogenous amenity that individuals consider to be a relative luxury good (including high-quality schooling).

holds become more likely to move downtown where the neighborhood landscape becomes higher quality, on average. Downtown neighborhoods that were populated by poorer individuals tend to be replaced by new high quality neighborhoods, chosen by richer households. This increases the density of high quality neighborhoods available to residents of these neighborhoods downtown making these locations even more attractive to richer households. In that sense, increasing income levels of the rich is one factor leading to neighborhood gentrification in downtown areas. The influx of richer residents into downtown areas causes rents to increase, in both high and low quality neighborhoods, and the supply of low quality neighborhoods to shrink. Poorer incumbent households who remain in low quality neighborhoods downtown see their rents increase, while others choose to migrate out. On average, even accounting for the small fraction that own their own home, they are made worse off.

In the third part of the paper, we take the model to the data. In a first stage, we estimate the key elasticities of the model. Important among these are the elasticity of substitution across neighborhoods of differing types (high vs. low quality, downtown vs. suburbs), the elasticity of substitution across neighborhoods within a given type, and the elasticity of substitution of amenity consumption across different neighborhoods. Using novel data and identification strategies, we estimate these key elasticities. In a second stage, armed with these estimated elasticities, we then calibrate the full model by a method of moments procedure. The procedure targets the whole U-shaped distribution of the propensity to reside downtown by income, as well as the relative housing prices between different neighborhoods types and location, both in 1990. We show that the model can replicate these salient facts from the data.

We then use the quantified model for welfare and counterfactual analysis. First, we assess how much of observed changing in sorting patterns across neighborhood types by different income groups during the 1990 to 2013 period can be explained by the rising incomes of the rich through the lens of the model. To that end, we take the observed change in the income distribution from 1990 to 2013, feed it into the model and compute the corresponding new spatial equilibrium. We then use the model to compute the corresponding welfare effects for different income groups, inclusive of spatial sorting responses. We find that increases in the incomes of high income individuals contributed to part of the changing within-city spatial sorting patterns by income levels in the U.S. during the last three decades. Furthermore, we find that the changing spatial sorting patterns of the rich resulting from their increased incomes made poorer residents worse off. In our base estimates, we find that the well-being of individuals at the 10th percentile of the income distribution was reduced by about 2.5 percent in consumption equivalent units during the 1990-2013 period as a result of higher income individuals moving into downtown urban areas. Almost all of that decline was due

to the increased housing prices that the poor had to pay in response to influx of the rich. While the model is stylized in many dimensions, it does suggest that this aspect of neighborhood change that looks like gentrification has been welfare reducing for the poor. Overall, we find that the change in the relative welfare of the 90th percentile of the income distribution, compared with that of the 10th percentile, is 2.3 percentage points higher when accounting for the spatial sorting response of the rich as a result of their rising incomes during the 1990-2013 period.

Finally, we use the model to perform additional counterfactuals. We explore how residential sorting and gentrification is altered in response to changes in policy, and in response to further changes in income inequality. First, we use the model to assess the effect of implementing policies that aim at limiting gentrification of downtowns. Specifically, we simulate a policy in which high-quality neighborhoods downtown are taxed, and the tax levied is used to subsidize housing downtown for the poor. We find that such policies can be effective in shaping the income mix of urban residents. Even though tax policies can help mitigate the rise in well-being inequality qualitatively, we find that their well-being effects are quantitatively limited and are far from overturning the increase in well-being inequality that we found for 1990-2013 in our quantification exercise. Finally, we ask how the well-being of the poor and the downtown composition of residents would change if incomes of the rich were to continue to rise. Our model suggests that in this case, gentrification would continue to be a prominent feature of urban landscapes in the years to come.

2 Related Literature

Our paper draws from and contributes to a number of related literatures, in particular to recent empirical work highlighting the importance of urban centers as a consumption good for residents. Additionally, our paper contributes to the large literature on the causes and consequences of neighborhood change and gentrification. Finally, our quantitative spatial model with income sorting contributes to a large theoretical literature on spatial location choice.

Historically, cities were viewed as being centers of production. Recently, however, a literature has emerged documenting that cities also serve as centers of consumption. Glaeser et al. (2001) puts forth the notion of a “consumer city” where urban density facilitates the provision a larger variety of consumption opportunities. To that end, both Glaeser et al. (2001) and Couture and Handbury (2017) document the increased prevalence of individuals residing in downtown areas and commuting to the suburbs for work. Diamond (2016) shows that residents value local amenities and documents that local amenity values endogenously

improve as higher income residents populate a neighborhood.

There is also an emerging literature demonstrating the importance of non-tradable services in making urban area desirable places to live. Murphy (2017) provides theoretical and empirical support for the notion that urban residents consume non-tradable services as a substitute for home production (i.e., urban residents eat at restaurants while suburban residents cook at home). Couture (2016) measures gains from restaurant density and suggests that non-tradable service variety accounts for most of the consumption value of cities. Couture and Handbury (2017) quantify the importance of non-tradable services as a driver of the urbanization of the young and college-educated in the 2000s, and find such services to be more important than jobs, house prices, tradable retail amenities, or public amenities like school and crime. Baum-Snow and Hartley (2017) also find that local urban amenities drive recent urban gentrification.

A large literature documents the pace of neighborhood change and explores its causes. Early contributions include Rosenthal (2008), who documents that neighborhoods frequently move from one income quartile to another, and Guerrieri et al. (2013), who document wide variation in house price growth across neighborhoods of the same city.⁵ A number of papers document the rapid rise in the socio-economic status of downtown neighborhoods in the 2000s. Baum-Snow and Hartley (2017) and Couture and Handbury (2017) argue that rising values of local amenities drive this recent urban revival. Edlund et al. (2016) and Su (2017) argue instead that high-skilled workers moved downtown in search of shorter commutes, as a consequence of longer work hours for high-skill workers since the 1990s. Ellen et al. (2017) highlight the role of crime, and show that central cities that experienced faster violent crime decline in the 1990s had faster income and college growth in the 2000s. Our quantitative model is rich enough to nest all these hypotheses on the drivers of recent downtown gentrification. In the next section we provide evidence for our preferred gentrification mechanism, which combines rising top income and luxury urban amenities.

The existing literature on the consequences of gentrification focuses on identifying evidence of displacement. These studies find little evidence that individuals with low socio-economic status exit gentrifying neighborhood at a faster rate than they exit non-gentrifying neighborhoods.⁶ Our paper’s focus, however, is not on short-run displacement, but on long-run welfare change. We document a rapid decline in the absolute number of poor households

⁵Rosenthal (2008) highlight the role of aging housing stock and redevelopment in moving neighborhoods across the income distribution, while Lee and Lin (2018) show that natural amenities like beaches anchor high-income neighborhood. Guerrieri et al. (2013) show that the fastest house price appreciation occurs in poor neighborhoods in close proximity to rich neighborhoods.

⁶See, for example, Vigdor et al. (2002); Lance Freeman (2005); McKinnish et al. (2010); Ellen and ORegan (2010); Ding et al. (2016); Brummet and Reed (2018). Waights (2014), however, finds evidence the poor renters in the UK are more likely to exit neighborhoods with rapidly rising educational achievement.

in gentrifying tracts, but our welfare results do not depend on whether these households are displaced or replaced. In fact, our model shows that poorer residents (most of whom are renters) who stay in gentrifying neighborhoods also experience lower welfare overall due to rising rents.

A limited number of papers study consequences of gentrification other than displacement. In gentrifying neighborhoods, Lester and Hartley (2014) find an overall gain in jobs with a shift away from manufacturing, Meltzer and Ghorbani (2017) find rising commute times, and Autor et al. (2017) find a decline in crime. Su (2017) is probably closest to our work in that he explores the welfare impact of rising value of time for high-skilled worker. Using a structural model, Su (2017) shows that the desire to minimize commuting costs draws richer residents downtown, bidding up land prices and causing adverse effects on the poorer residents who populate city centers. Berkes and Gaetani (2018) study the rise in income segregation within cities. They find that higher patent (innovation) intensity at the city level accounts for 20% of the rise in income segregation from 1990 to 2010, but that endogenous residential amenities again play the most important role. We differ from Su (2017) and Berkes and Gaetani (2018) in that we model the development of endogenous amenities, we focus on rising top income as a driving force, and we have luxury urban amenities play a direct role in gentrification.

A recent body of empirical work documents the rapid rise in the share of income accruing to the top of the income distribution (Piketty et al., 2018). This shift in high incomes lies at the core of the gentrification mechanism in our paper. Many papers explore the causes of this growth in income inequality, for instance Smith et al. (2017), but fewer papers explore its consequences. Two exceptions are Bertrand and Morse (2016) who show that rising incomes at the top of the distribution induced households lower in the income distribution to save less and consume a larger share of their income, and Gyourko et al. (2013) who show that the rising number of rich households nationally explains some of the rapid rise in house prices and income in "superstar" cities with low housing supply elasticities, like San Francisco, New York, and Boston. Our contribution is to use a quantitative structural model to measure the welfare consequences of an exogenous rise in high incomes, with a focus on endogenous spatial income sorting within cities.

Finally, our paper contributes to a theoretical literature on the location of different income groups within a city. These models are generally extensions of the monocentric city model, reviewed in Duranton and Puga (2015). In the basic version of this model, richer individuals consume more land, and therefore choose to live away from the city center, where land is cheaper. Brueckner et al. (1999) show that adding amenities at the city center can reverse this income sorting pattern. Gaigne et al. (2017) propose a linear polycentric city model with jobs and amenities at different locations with non-homothetic Stone-Geary preferences.

Their model, like ours, can match the non-monotonic income sorting patterns observed in reality. Important contributions outside the linear city framework include LeRoy and Sonstelie (1983) and Glaeser et al. (2008). They show that transit availability in cities and the high cost of commuting by cars to suburbs make downtown attractive to poor individuals. Our model draws from this insight in that downtowns attract poor individuals due to higher commute costs in suburbs.

Instead of using the linear city framework, our model builds on recent developments in quantitative spatial economics, reviewed in Redding and Rossi-Hansberg (2017). We add individual income heterogeneity to a quantitative spatial model of a city. More specifically, we extend Fajgelbaum et al. (2011)’s model of international trade with non-homothetic preferences to an urban context. Closest to our work is Tsivanidis (2018) that uses a spatial sorting model with heterogeneous residents to analyze the distributional effects of urban transit networks.

3 Motivating Facts

In this section, we provide a set of motivating facts that will help guide our modeling choices. To contextualize these facts, we first outline the data used and how we characterize an “urban” area empirically. First, we document the propensity to live in urban centers of CBSAs across income levels and over time. These are the patterns that we seek to explain with our model. Then, we use cross-CBSA variation to show that these changing sorting patterns are associated with income growth.

3.1 Data

Our primary data comes from the 1970, 1990, and 2000 *U.S. Censuses* and from the 2011-2015 *American Community Surveys* (ACS). When measuring variables at the census tract level, we use published tables from the *National Historical Geographic Information System* (NHGIS). When measuring variables at the Core-Based Statistical Area (CBSA) level, we either aggregate census tract level variables from the NHGIS, or we aggregate microdata from the 1% *Integrated Public Use Micro-data Series* (IPUMS) in 1970, the 5% IPUMS sample of the 1990 and 2000 Censuses and the 5% IPUMS sample from the 2011-2015 ACS surveys. From henceforth, we refer to the 2011-2015 pooled ACS data as the 2013 ACS. All data are interpolated to constant 2010-boundary tracts and constant 2013-boundary CBSAs using the *Longitudinal Tract Data Base* (LTBD). We limit our analysis to the 100 largest CBSAs based on population in 1990. All dollar values are adjusted to 1999 dollars. We use

the Census and ACS data to measure the location choice of households with differing levels of income, housing prices at different levels of location aggregation, and location specific neighborhood characteristics such as the share of residents with a bachelor’s degree.

Central to our motivating facts and model is the notion of the dense urban center of a CBSA. When defining urban centers, we focus on the downtown area of the CBSA’s main city. We often refer to downtowns, urban areas, and urban centers interchangeably in what follows. For example, for the Chicago CBSA, we define the downtown area as the set of census tracts surrounding the center of the city of Chicago.⁷ For our primary measure of a downtown area, we use the set of tracts closest to the city center accounting for 10 percent of a CBSA’s population in 2000. We fix this definition of downtown area across all other years. Note that our notion of the downtown area of a city is measured in population units as opposed to distance given that CBSAs differ in their size and density. However, our key motivating facts are robust to many alternate definitions of downtown areas including those census tracts with centroids within a three mile radius of the city center as in Baum-Snow and Hartley (2017). For each CBSA, we refer to all remaining non-downtown tracts as being suburban tracts. Our definition of suburban is quite broad in that it simply reflects non-downtown tracts. In all work that follows, all tracts in a CBSA are either classified as downtown (D) or suburban (S). In Appendix C, we show the tracts for New York, Chicago, Philadelphia, San Francisco, Boston, and Las Vegas CBSAs that are classified as downtown and suburban based on our definition.

3.2 Downtown Residential Propensity and Household Income

Figure 1 shows the relative propensity of families to reside downtown by income over time. Each point plots the share of families in a given census income bracket that resides downtown in each year normalized by the share of all families that reside downtown in that year against the median CPI-adjusted family income for that bracket in the same year.^{8,9}

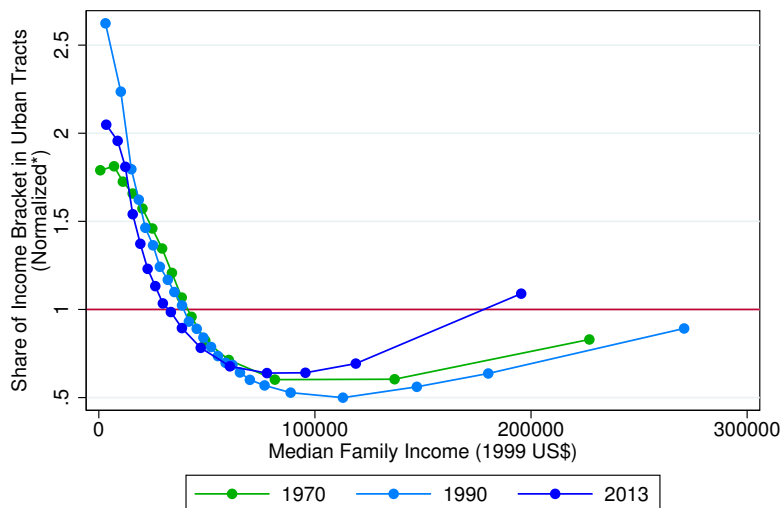
The figure reveals two facts that motivate our model. The first is cross-sectional. The propensity to reside downtown is a U-shaped function of income. Lower income families are much more likely to live in urban areas. For example, families earning \$25,000 a year in 1970, 1990, and 2013 were between 1.5 and 2 times more likely to live in downtown urban areas

⁷We define the city center of each CBSA using the locations provided by Holian and Kahn (2012), obtained by entering the name of each CBSA’s principal city into Google Earth and recording the returned coordinates.

⁸To abstract from the suburbanization of the population as a whole over this period, we plot the share of families in a each income bracket that resides in urban census tracts normalized by the aggregate population share that resides in urban census brackets in each year. Given our primary definition of downtown areas, the average share of families that live downtown is 0.1 in 2000, but 0.17 in 1970 and 0.07 in 2013

⁹We calculate the median income for each bracket using IPUMS micro data.

Figure 1: Downtown Residential Income Propensity by Income



Note: Table uses Census data on family income for the 100 largest CBSAs in 1970, 1990, and 2013. Urban tracts consists of all tracts closest to the city center that account for 10% of a CBSA's population in 2000. Each dot in the Figure corresponds, on the x-axis, to the median family income within each Census bracket. We compute this median using IPUMS microdata for the corresponding year in the 100 largest CBSAs. All incomes are in real 1999 dollars.

than other households. As their income increases, residents are increasingly more likely to live in the suburbs. However, when income gets above roughly \$100,000, households start to move back downtown. Importantly, this U-shaped sorting pattern is not a new phenomenon, it is present in 1970, 1990, and 2013.

The second fact is about the variation in this U-shaped pattern over time. Though the U-shaped sorting pattern itself is not new, it has become more pronounced between 1990 and 2013, with a rising propensity of high income families to reside downtown.

Both of these facts are robust to the definition of an urban area, CBSA sub-samples, and the use of household instead of family income. Additionally, they are almost invariant to controls for demographics (such as age and race) and household composition indicating that the propensity of the rich to reside downtown and, in particular, their further urbanization between 1990 and 2013 cannot be explained by demographic shifts that might also explain the general tendency for households to reside downtown or in the suburbs.

We will calibrate the model that we develop below to match the U-shape pattern in 1990. Given that, our model will need mechanisms to explain both why the poor and the rich are more likely to live downtown (relative to middle income groups). We will also ask, through the lens of the model, how much of the change in the U-shape pattern between 1990 and 2013 can be explained by the change in the income distribution that occurred during that

time period.

3.3 CBSA Income Growth and Changing Spatial Sorting by Income

We now exploit cross-CBSA variation to assess the association between income growth and the changing sorting patterns documented above. Specifically, we ask whether rising income at the city level has a more positive impact on the propensity of rich households to move downtown than on that of poor households.

Figure 2 plots CBSA-level growth in the relative propensity of households with incomes greater than \$70,000 to reside downtown between 1990 and 2013 against CBSA-level growth in average household income over the same period.¹⁰ The figure shows that CBSAs with higher aggregate income growth saw higher increases in the share of high income households residing downtown (relative to their overall urbanization rate) between 1990 and 2013. A 10 percent increase in CBSA income during the 1990-2013 period was associated with a 13 percent increase in the fraction of individuals with at least \$70,000 of household income living downtown relative to the average CBSA resident.

This correlation suggests that shifts in the income distribution observed during the 1990-2013 period may be a quantitatively important factor in drawing higher income individuals into downtown areas of major cities. Below, we formalize a model that does just that. The pattern that we document in this subsection will also be useful in helping to discipline some of the model’s structural parameters.

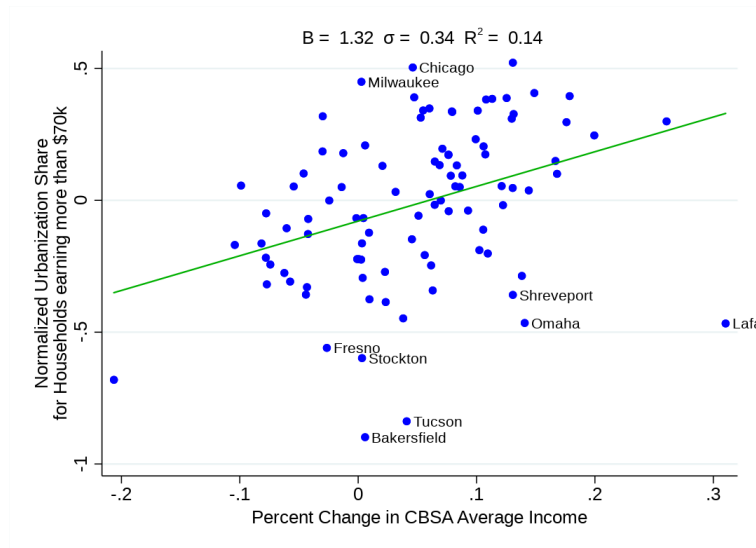
3.4 Discussion

We acknowledge that non-homothetic preferences for urban amenities and rising incomes are only one potential cause of urban neighborhood change during the last few decades. Our goal is to use a calibrated structural model to see how much of the changes in within city spatial sorting patterns by income observed during the last few decades can be potentially explained by our mechanism.

Before introducing this model, we want to specifically address two other forces in the literature that have been discussed as a potential cause of urban neighborhood change, otherwise known as gentrification. First, Edlund et al. (2016) and Su (2017) provide evidence that longer work hours for high-skilled workers drove them into urban areas in the 1990s, as

¹⁰We choose income greater than \$70,000 as the cut off because that is roughly the inflection point of the U-shape in Figure 1. Household income levels at the tract level come in bracketed form, so we use all brackets containing individuals with income greater than \$70,000.

Figure 2: Richer Household Propensity to Live Downtown in Response to a CBSA Level Change in Income 1990-2013



Note: Each observation in the figure is one of the largest CBSAs. On the x-axis is the average CBSA real household income growth between 1990 and 2013. On the y-axis is change in the share of individuals earning \$70,000 or more residing downtown relative to the average individual between 1990 and 2013. A simple weighted regression through the scatter plot (where the weights are the CBSA 1990 population) yields a slope coefficient of 1.32 with a standard error of 0.34.

a way to reduce their travel cost to jobs. Our model allows for work commute costs to vary between suburban and downtown residents and to increase with income. So, as the income of the rich increases, it becomes more expensive for them to commute from the suburbs. However, we do not focus on this mechanism as a primary motive of urban gentrification because empirically commuting times have not fallen for the rich during this period. Specifically, Couture and Handbury (2017) show that the average commute distance for high-wage workers actually *increased* slightly from 2002 to 2011, with the most positive percentage change in commute distance for workers living closest to the city center.¹¹ The reason for this is that, while the rich are more likely to move downtown, they are reverse commuting to the suburbs with increased propensity. This is consistent with downtown amenities being more important than commuting costs in determining the patterns of sorting by income within a CBSA.¹² In order to simplify the model, we abstract from reverse commuting. Given that commuting costs for individuals of differing income levels did not change much (on net), we feel that we are not missing too much by abstracting from this force in our model.

Another potential story for recent urban gentrification was put forth by Ellen et al. (2017) who show that central cities with a faster decline in crime in the 1990s experienced rising shares of high-income and college-educated residents. Again, our model has an amplification mechanism that captures endogenous amenities that the rich value more than the poor generally, such as crime. However, unlike urbanized luxury amenities like restaurants and entertainment, crime cannot explain the relatively high propensity of rich families to live downtown even during the high urban crime eras of 1970 and 1990 in Figure 1. It could be that a policy change occurred that reduced crime in downtown urban areas in the 1990s, and that this attracted some high income individuals back to downtown urban areas. We find such a story plausible although it is not a force that we focus on in our model. Our goal is not to explain all factors that generated gentrification during the 1990s and 2000s. Instead, we focus on one important force and discipline that force using moments of the micro data that we think are connected to the mechanism that we highlight. Accordingly, while our mechanism explains much of the change in urban sorting by income during the last few decades in the US, it does not explain it all. Given this, there is room for these other

¹¹We performed our own analysis of the relationship between changes in commuting time and household income of residents who were 25-54 years old living in metropolitan areas using commuting data from the Censuses and ACS. Our findings reinforce the patterns in Couture and Handbury (2017). In particular, we find that both median and 95th percentile commute times are stable for workers within all income deciles during the 1990-2013 period except for the top two deciles that experienced an *increase* in commute times over the last two decades.

¹²Recent findings in (Gaigne et al., 2017) support the view that richer individuals' desire to minimize commuting costs can only explain a small fraction of the increase in urban gentrification during the 2000s (holding the sorting based on amenities fixed). Likewise, Glaeser et al. (2001) has documented the rise of reverse commuting in U.S. cities since 1980.

forces to have contributed to the large amount of urban gentrification that occurred within the U.S. during the last few decades.

4 Model

We propose here a model of a city which is flexible enough to capture the salient feature of the data, yet stylized enough to be a model of a *representative* city rather than matching quantitatively one specific city. A city is comprised of two parts, a central district which we call “Downtown”, and the rest of the city which we call the “Suburbs.” Households with different income levels choose their location of residence. Downtown offers easier access to jobs, while the suburbs have nicer public amenities, at the cost of a longer commute. In both areas, private developers develop neighborhoods featuring housing and retail outlets. The development of a richer variety of private urban amenities is fueled, downtown, by economies of density. Non-homotheticities in the consumption of urban amenities and in transportation leads to the sorting of heterogeneous households in different parts of the city. Downtown has an over-representation of both extremes of the income distribution. The model builds on Fajgelbaum et al. (2011), which we adapt to an urban context and extend to feature two sources of non-homotheticities.

4.1 Model setup

4.1.1 Choice of neighborhood

The city is comprised of neighborhoods, indexed by r . Households choose a neighborhood where to live. A neighborhood r is characterized by the part of the town where it is located, downtown or the suburbs, as indexed by $n \in \{D, S\}$. Within these two broad areas, neighborhoods also differ by the quality of their housing stock and private amenities. Within each level of quality, neighborhoods r are further differentiated horizontally.¹³

4.1.2 Preferences

Households who live in neighborhood r of type (n, j) derive utility from the consumption of a freely traded composite good c , private urban amenities a consumed in different parts of the city, as detailed further below, as well as directly from the enjoyment of the non-rival

¹³The model can be readily extended to include a greater range of neighborhood qualities, but we find that two levels of quality are sufficient to capture quantitatively the non-monotonic U-shaped patterns of location choice observed in the data (see section 5).

amenities of their neighborhood, which require renting one unit of housing in r . The utility of household ω who lives in neighborhood r of type (n, j) is:

$$U_r(\omega) = Q_{j(r)} A_{n(r)} \left(\frac{a}{\alpha}\right)^\alpha \left(\frac{c}{1-\alpha}\right)^{1-\alpha} b_r(\omega). \quad (1)$$

In this expression, A_n is a shifter summarizing quality of life in downtown vs the suburbs (e.g., their differences in public amenities such as parks or schools) while Q_j is a shifter that summarizes the quality level of a neighborhood, in terms of the housing stock and of urban amenities. The shock $b_r(\omega)$ captures the idiosyncratic preference worker ω has for living in neighborhood r . Specifically, each household draws a vector $\{b_r(\omega)\}_r$ of idiosyncratic preference shocks, following a Generalized Extreme Value distribution:

$$F(\{b_r\}) = \exp \left(- \left[\sum_{n,j} \left(\sum_{r \in \mathcal{B}(n,j)} b_r^{-\gamma} \right)^{-\frac{\rho}{\gamma}} \right] \right),$$

where $\mathcal{B}(n, j)$ is the set of neighborhoods of quality j in part of the city n . With this nested structure of idiosyncratic preferences, the preferences of a given household are more correlated for neighborhoods of the same quality and located in the same part of the city, than they are for neighborhoods of different types. Specifically, the parameter ρ governs the variance of draws across types of neighborhoods (*across* n, j pairs) and γ governs the variance of idiosyncratic preference draws for neighborhoods of the same type (*within* n, j pairs), where consistency with utility maximization requires $\gamma > \rho > 1$.

Households consume amenities in different locations in the city. We assume that private amenities – restaurants and retail options – are differentiated across neighborhoods, and that households have CES preferences over amenities located in various neighborhoods. Specifically, a household that resides in location r consumes $a_{rr'}$ amenities in neighborhood r' and choosing where to consume amenities so as to maximize their bundle of amenity consumption:

$$a_r = \left(\sum_{r'} (\beta_{rr'})^{\frac{1}{\sigma}} (a_{rr'})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution between amenities from different neighborhoods and the term $\beta_{rr'}$ captures utility costs that are pair of neighborhood specific. We decompose these shifters into two components. The first one measures the utility cost of travelling to another neighborhood to consume amenities there, which we assume increases with distance between neighborhoods $d_{rr'}$ with elasticity $\tilde{\delta}$. The second one depends on dissimilarity in

quality between a household's own neighborhood and the destination neighborhood. It captures the notion that people value horizontal differentiation within a given quality range that corresponds to their preferred quality level, but might not value as much amenity options of a different quality. Specifically, we write:

$$\beta_{rr'} = (d_{rr'})^{\bar{\delta}} \beta_{j(r)j'(r')}^q,$$

where $\beta_{j,j'}^q = 1$ if $j = j'$. The price index for amenities consumption for a household who lives in r is therefore:

$$P_r^a = \left(\sum_{r'} \beta_{rr'}^q \left((d_{rr'})^{\bar{\delta}} p_{r'}^a \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}},$$

where $p_{r'}^a$ is the price of amenities sold in r' . Given that we model a representative city, we will make the assumption that neighborhoods of a given type (n, j) are all symmetric in size and location, so that all neighborhoods of type (n, j) have the same price index for amenity consumption P_{nj}^a , and the same local price of amenities p_{nj}^a .¹⁴ Denote with N_{nj} the number of neighborhoods within a n, j pair. The price index for amenities in a given neighborhood can therefore be re-written as:

$$P_{nj}^a = \left(\sum_{n', j'} N_{n'j'} \beta_{jj'} \left(p_{n'j'}^a (d_{nn'})^{\bar{\delta}} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \quad (2)$$

where $d_{n'n}$ is the representative distance between two neighborhoods, one located in location n and one in location n' , while $\beta_{jj'}$ is the disutility shifter associated with shopping at locations whose quality is different than one's own residential type.¹⁵ Note that under our symmetry assumption, we can write distance frictions as a constant utility function of space, such that

$$(d_{nn'})^{\bar{\delta}} = \Delta (K_{n'})^{\delta}, \quad (3)$$

where $\Delta = 1$ if $n = n'$ and $\Delta > 1$ if $n \neq n'$. The term Δ captures the border friction, assumed to be symmetric, between downtown and the suburbs.¹⁶

¹⁴While this assumption does not allow us to speak to the actual detailed spatial patterns of gentrification within a location in a given city, it allows us to capture the salient features of neighborhood change in a representative city.

¹⁵We assume that $\beta_{jj} = 1$ while typically $\beta_{jj'} < 1$ if $j \neq j'$.

¹⁶The expression for the price index (2) therefore simplifies to

$$P_{nj}^a = p_{nj}^a (N_{nj})^{\frac{1}{1-\sigma}} K_n^{\delta}$$

under the assumption that households only consume amenities in neighborhoods of their own type.

4.1.3 Labor Income and Net Income

Households supply labor inelastically and are heterogeneous in skill. We assume that labor income is an increasing function of skill, so that we can summarize the heterogeneity of households with their wage w . The number of households with income w who live in the city is given by $L(w)$. Our focus is to study how heterogeneous households sort into different neighborhoods in the city, given the overall distribution of skills $L(w)$. We take this aggregate distribution as a primitive of the model.¹⁷

Workers commute to work. We assume that commuting costs depend on the part of the city where one lives, summarized by n and that the cost of commuting is proportional to labor income. It is captured by commuting costs τ_n , so that net labor income is $(1 - \tau_n)w$ for a household with wage w living in n . We assume that $\tau_D < \tau_S$ to capture the fact that households living downtown have an easier access to jobs compared to those living in the suburbs.

Households also own a share of the city-wide real estate portfolio. We allow this share to vary systematically by worker type, as captured by $\chi(w)\Pi$ where Π is the total returns to the real estate portfolio and $\chi(w)$ is the fraction owned by a worker of type w . Market clearing for asset ownership requires that $0 \leq \int \chi(w)L(w)dw \leq 1$.¹⁸ Overall, the net income m of a household w who lives in n is given by:

$$m_n(w) = (1 - \tau_n)w + \chi(w)\Pi \quad (4)$$

Households choose their neighborhood of residence r by maximizing (1) subject to the budget constraint. Given the specification of income and the utility function, the indirect utility of a household ω whose wage is w is given by:¹⁹

$$\max_r (m_{n(r)}(w) - p_r^h) (P_r^a)^{-\alpha} A_n Q_j b_r(\omega)$$

The price p_r^h is the price of the unit of housing in neighborhood r that one must rent to live there, while the price of the freely traded good is taken as the numeraire. There are two sources of non-homotheticities in the model. First, the unit-housing requirement in the model generates non-homothetic demand. Low-income workers can only afford to live in low housing costs neighborhoods. In contrast, higher income households self-select into

¹⁷Specifically, we study how spatial sorting patterns in the city change (and in turn, how house prices and urban amenities change) when this primitive income distribution changes.

¹⁸The total share can be less than one, the rest being owned by absentee landlords.

¹⁹In what follows, we simply use subscripts n and j when it is clear to do so. They are really functions of the neighborhood chosen: $j = j(r)$; $n = n(r)$.

high quality-high price neighborhoods. This force leads to an over-representation of the rich households in the highest-quality neighborhoods, typically the ones downtown.

Second, commuting costs generate another source of non-homotheticity. Living in the suburbs is nicer than downtown with respect to fixed amenities, but more costly in terms of commuting time. Higher income workers are willing to take this trade-off, while it is too costly for low-income workers who spend most of their income on housing rents and are close to a subsistence level. This generates an over-representation of the lower income households downtown. Taken together, these two mechanisms can generate a U-shaped distribution of the probability of living downtown as a function of income, provided that high-quality neighborhoods downtown are sufficiently attractive.

We now turn to describing the endogenous provision of differentiated neighborhoods by developers.

4.1.4 Land Markets and Developers

Neighborhoods are developed by private developers who use land to develop neighborhoods that feature housing units and retail amenities. They rent out housing units and operate retail stores and restaurants, which are marketed to households living in the neighborhood as well as in other parts of the city. The number of neighborhoods of each type is an endogenous outcome of the model.

Land is provided competitively by atomistic absentee landowners. Downtown and the suburbs differ in their elasticity of land supply ϵ_n that is typically lower downtown. We posit the following reduced-form land-supply equation:

$$K_n = K_n^0 (r_n)^{\epsilon_n}, \quad (5)$$

where r_n and K_n are respectively rents and land supply in location n , and K_n^0 is a n -specific exogenous shifter, which controls the relative size of downtown vs the suburbs. Developers use land in location n to build $H_{n,j}^h$ housing units of quality j as well as $H_{n,j}^a$ retail areas of quality j following:²⁰

$$H_{n,j}^h = \frac{K_{n,j}^h}{h_{n,j}^h} \quad \text{and} \quad H_{n,j}^a = \frac{K_{n,j}^a}{h_{n,j}^a}, \quad (6)$$

where a higher quality space is more expensive to build, as captured by $h_{n,H}^i > h_{n,L}^i$, for

²⁰Land is understood here to be equipped land. The model can be easily extended to feature a production function for housing that relies on land and capital. Given that the calibration relies on matching the resulting housing supply elasticity, this extension does not affect the results.

$i \in \{h, a\}$. Land market clearing pins down the rental price in location n :

$$r_n = \left(\sum_j (h_{n,j}^h H_{n,j} + h_{n,j}^a A_{n,j}^h) \right)^{\frac{1}{\epsilon_n}} \quad (7)$$

Developers pay a fixed cost $f_{n,j}$ to develop a differentiated neighborhood r of type (n, j) . We assume that developers price housing, as well as amenities, according to monopolistic competition. Finally, the number of developers is pinned down by free entry. The number of neighborhoods of type (n, j) , $N_{n,j}$, adjusts so that developers' profits are driven to zero, where a developer profit in an (n, j) is:

$$\pi_{n,j}^h + \pi_{n,j}^a - f_{n,j} = 0 \quad (8)$$

4.2 Equilibrium

4.2.1 Workers residential choice

Among workers with labor income w , the share of workers who locate in a particular neighborhood r of type (n, j) is:

$$\lambda_{n,j,r}(w) = \lambda_{n,j}(w) \lambda_{r|n,j}(w),$$

where the notation $\lambda_{r|n,j}$ indicates the share of workers who choose neighborhood r conditional on choosing a neighborhood of quality j in location n . Given the structure of the idiosyncratic preference shocks, the conditional probability of choosing r among other (n, j) choices is:²¹

$$\lambda_{r|n,j}(w) = \frac{V_r(w)^\gamma}{\sum_{r' \in \mathcal{B}(n,j)} V_{r'}(w)^\gamma}, \quad (9)$$

where $V_r(w)$ is the inclusive value of neighborhood r :

$$V_r(w) = (m_n(w) - p_r^h) (P_r^a)^{-\alpha} A_{n(r)} Q_{j(r)} \quad (10)$$

Second, the probability that the neighborhood chosen is of type (n, j) is:

$$\lambda_{n,j}(w) = \frac{V_{n,j}^\rho(w)}{\sum_{n',j'} V_{n',j'}^\rho(w)} \quad (11)$$

where $V_{n,j}$ is the inclusive value of all neighborhoods of type (n, j) . Note that the inclusive

²¹In equilibrium, all neighborhoods are symmetric within type, so that $\lambda_{r|n,j}(w) = \frac{1}{N_{n,j}}$.

value of living in any neighborhood n, j is:

$$V_{n,j}(w) = \left(\sum_{r' \in \mathcal{B}(n,j)} V_{r'}(w)^\gamma \right)^{\frac{1}{\gamma}} = A_n Q_j N_{nj}^{\frac{1}{\gamma}} (P_r^a)^{-\alpha} (m_n(w) - p_r^h). \quad (12)$$

To drive intuition, we can specialize the equations for a moment to the case where households only consume amenities in their own type of neighborhood (n, j) . In this case, we get:

$$V_{n,j}(w) \propto Q_j A_n \frac{N_{nj}^{\frac{1}{\gamma} + \frac{\alpha}{\sigma-1}} K_n^{-\alpha\delta}}{(p_{nj}^a)^\alpha} (w - p_r^h).$$

We see that the number of neighborhoods N_{nj} acts as an agglomeration, for two reasons. First, because it drives a love of variety effect in the choice of residential neighborhoods, as mitigated by the elasticity $\frac{1}{\gamma}$. Second, because it drives a love of variety effect in the choice of neighborhoods to visit to consume urban amenities, as mitigated by $\frac{\alpha}{\sigma-1}$. This dual benefit of having more variety in neighborhood choice is dampened by distance. In the suburbs, where the extension in the number of neighborhoods leads to sprawl, K_n increases faster than in downtown, mitigating the welfare impact of expanding the number of neighborhood options in the suburbs. The same forces are at play in the general case, where, in addition, welfare depends on amenity options in other neighborhood types, as captured by the price index P_r^a defined in equation (2).

Finally, the model lends itself naturally to welfare analysis. The average welfare of a household with wage w is the same irrespective of his location choice:

$$V(w) = \left(\sum_{n',j'} V_{n',j'}^\rho(w) \right)^{1/\rho}. \quad (13)$$

Note that higher-income households will be over-represented in costly neighborhoods, as income and housing prices are complement in equation (11).²² Since higher quality neighborhoods will endogenously, in equilibrium, have higher demand hence higher housing prices, they will attract high-income households disproportionately. This first feature of the model can rationalize why higher income households disproportionately locate in high quality downtown neighborhoods, where the quality of urban amenities is reinforced by economics of

²²The complementarity can be seen from $\frac{\partial \log \lambda_{n,j}(w,p)}{\partial w \partial p} > 0$, which means that higher income workers are disproportionately located in higher p neighborhoods.

density. Second, rewriting the same expression as:

$$V_{n,j}(w) = A_{n(r)} Q_{j(r)} N_{n,j}^{\frac{1}{\gamma}} (P_r^a)^{-\alpha} ((1 - \tau_n)) \left[w + \frac{\chi(w)\Pi}{(1 - \tau_n)} - \frac{p_r}{(1 - \tau_n)} \right]$$

shows that commuting cost makes the real cost of living in the suburbs higher ($\frac{p_r}{(1-\tau_n)} > p_r$). Provided that the quality of life in the suburbs is high enough to justify such a commute, it will be so only for higher-income workers as, again, income and prices are complement in this expression. This force will generate the disproportionate sorting of lower income workers away from the suburbs and into downtown housing units.

4.2.2 Developers

Given CES demand for amenities, developers price amenities at a constant markup over marginal costs, that is:

$$p_r^a = \frac{\sigma}{\sigma - 1} h_{n(r),j(r)}^a r_{n(r)}, \quad (14)$$

so that in equilibrium, operational profits made on the amenities market by a developer of type (n, j) is:

$$\pi_{n,j}^a = \frac{\alpha^a}{\sigma} \int_w \frac{\lambda_{n,j}(w) (w - p_{n,j}^h)}{N_{n,j}} dL(w)$$

and the land used by amenities of type (n, j) is:

$$r_n K_{n,j}^a = \frac{\sigma - 1}{\sigma} \alpha^a \int_w \lambda_{n,j}(w) (w - p_{n,j}^h) dL(w) \quad (15)$$

Similarly, the land used by housing of type (n, j) is:

$$r_n K_{n,j}^h = \int_w \lambda_{n,j}(w) h_{n,j}^h r_n dL(w) \quad (16)$$

and the price of housing is pinned down by profit maximization of developers on the housing market given demand:

$$\pi_r^h = \left[\int_w \lambda_r(w) dL(w) \right] (p_r^h - h_{n,j}^h r_n) \quad (17)$$

Using (10), (9) and (11) leads to the following pricing formula:

$$p_r^h = \frac{\gamma}{\gamma + 1} h_{n,j}^h r_n + \frac{1}{\gamma + 1} \mathcal{I}_{n,j}(p_r^h), \quad (18)$$

where the term $\mathcal{I}_{n,j}(p_r^h)$ is a measure of demand for neighborhood r .²³ By symmetry, all neighborhoods of type (n, j) have the same price in equilibrium, which we denote as $p_{n,j}^h$.

This leads to:

$$N_{n,j} = \frac{1}{f_{n,j}} \left[\int_w \lambda_{n,j}(w) \left(p_{n,j}^h - h_{n,j}^h r_n + \frac{\alpha^a}{\sigma} (w - p_{n,j}^h) \right) dL(w) \right] \quad (19)$$

A few comments are in order here. First, note that free entry creates a feedback loop. Take for instance neighborhoods of high quality downtown. The higher the demand for living there, the more developers enter and offer horizontally differentiated high quality neighborhoods - i.e., $N_{D,H}$ in (19) increases. This, in turn, raises the demand for this type of neighborhood by a love of variety effect (captured by equations (11) and (12)). Second, the intensity of this feedback loop depends on the elasticity of substitution between neighborhoods where to live, γ , and the elasticity of substitution between urban amenities, σ . The lower these elasticities, the larger the entry feedback loop - as neighborhoods are less substitutable, for housing or for amenity consumption. Third, the intensity of this feedback loop also depends on whether the density of neighborhoods increases or not in response to higher demand, as captured by the distance friction in (2). The idea that the model captures here is that when new neighborhoods are developed through an increase in density, households living in the location have an easy access the corresponding urban amenities, even outside of their own neighborhood. The presence of nearby differentiated neighborhoods increases the appeal of residing in a given location. This is the case downtown, where land supply is fixed. The number of neighborhoods expands by filling up a constant space, so that accessibility to these varieties is high. In contrast, new neighborhoods developed in the suburbs are built, in part, over new land, as land is supplied more elastically there. Sprawl limits the amenity value of expanding the number of neighborhoods in the suburbs, as it reduces the accessibility to amenities.

This concludes the set up of the model. An equilibrium of the model is a distribution of location choices by income $\lambda_{n,j}(w)$, housing and amenity prices $p_{n,j}^i$, land rents $r_{n,j}$, number of neighborhoods $N_{n,j}$ such that (i) households maximize their utility (ii) developers maximize profits and (iii) land and housing markets clear.

Given the structure of the model, it is straightforward to show that an equilibrium of the model can be expressed in relative changes compared to another reference equilibrium, with different primitives (e.g., different city-level distribution of income, or different exogenous levels of amenities). We detail this approach and leverage it in section 6.1, where we analyze a series of counterfactual equilibria, starting from an initial one calibrated from the data.

²³ Specifically, $\mathcal{I}_{n,j}(p) = \frac{\int_w \Lambda_{n,j}(p,w) [(1-\tau_n)w + \chi(w)\Pi] dF(w)}{\int_w \Lambda_{n,j}(p,w) dF(w)}$ with $\Lambda_{n,j}(p,w) = \frac{\lambda_{n,j,r}(w)L(w)}{[(1-\tau_n)w + \chi(w)\Pi - p]}$.

We now describe this calibration.

5 Model Parameterization

In this section, we take the model to the data. We first describe how we match the model concepts of quality and location to their empirical counterparts. We then detail how we parameterize the model in two stages. In a first stage, we estimate key parameters and calibrate others using existing estimates from the literature. In a second stage, we use method of moments to fully calibrate the model.

5.1 Model Notions of Space

There are three notions of space within our model. First, as discussed above, a representative metropolitan area is divided into two areas indexed by n : downtown (D) and the suburbs (S). Second, within each area n , there are two levels of neighborhood quality indexed by j : high quality (H) and low quality (L). Finally, within each $\{n, j\}$ pair, there is a continuum of r differentiated neighborhoods. As discussed in section 3, we empirically define neighborhoods as census tracts and the downtown (D) area as all census tracts surrounding the center city of a CBSA that contain 10 percent of the CBSA population in 2000. We take two approaches to segmenting high and low quality census tracts within the downtown and suburban areas. First, we define high quality neighborhoods based on the demographic composition of residents. We draw from Diamond (2016), who shows that the college-educated share can proxy for endogenous amenities. Specifically, we define a high quality neighborhood as a neighborhood where at least 40 percent of residents between the ages of 25 and 65 have at least a bachelor's degree. Under this definition, 15, 22 and 31 percent of census tracts in the downtown areas of the top 100 CBSAs are respectively classified as high quality in 1990, 2000 and 2013.

As a second measure, we define high quality neighborhoods based on the quality of amenities provided in the neighborhood. We measure the quality of local amenities using the *Market Potential Index* (MPI) of 61 local restaurant chains combined with the geo-coded location of these restaurants from the National Establishment Time-Series (NETS). We use the *MPI* calculated by Esri ArcGIS which uses data from the *Survey of American Consumers* to measure the propensity of residing in different neighborhoods to visit a given chain relative to the average American. We can use this data to measure the propensity of individuals residing in high-income neighborhoods to visit a given chain.²⁴ To compute

²⁴These measures are as in Couture and Handbury (2017). Specifically, we use the MPIs in segments with

our neighborhood quality measure using this data, we take the average MPI of restaurant chains within walking distance of a tract centroid.²⁵ The five restaurant chains with the highest MPI are California Pizza Kitchen, Cheesecake Factory, Panera Bread, Chipotle, and Starbucks. Amenity quality and household location are determined in equilibrium, so these MPIs capture both the propensity of high income individuals to visit and locate near a given chain and the type of chains that choose to locate near them (given that propensity to visit a chain declines with travel time). We define a high quality census tract as being those census tracts with average MPI greater than 1. Under this definition, 18 and 39 percent of census tracts with non-missing quality in the top 100 CBSAs are classified as high quality, respectively, in 2000 and 2010, the two years for which this measure is available.

Defining high quality tracts inherently involves some judgment. Given that, we pursue multiple approaches to measure high quality neighborhoods. Despite the two methods being conceptually different, we find very similar estimates of our key parameters across both methods for segmenting tracts into quality tiers. Finally, we have also explored segmenting high quality neighborhoods based on the value of housing. Given that below we assess the fit of our calibrated model by comparing the model’s prediction of house price changes over time for each $\{n, j\}$ pair with the actual data, we have chosen not to use house prices to segment neighborhoods into quality tiers.

5.2 Parameterization: First Stage

In this section, we discuss our parameterization of the model’s key elasticities. Specifically, we discuss how we estimate or calibrate the 9 parameters highlighted in Table 1 using several micro data sources. The role played by these parameter in driving sorting patterns and welfare results is discussed in section 6.4.4.

5.2.1 Estimation of Elasticity of Demand Between Neighborhood Types (ρ)

We begin by discussing how we estimate ρ . Our identification strategy relies on the spatial sorting response of individuals with different levels of income to a city wide income shock. These spatial sorting responses that we identify also helps provide empirical validation for our model.

Specifically, we estimate ρ from the following equation implied by equation 9 of the average income above \$100,000 to identify the restaurants that high-income individuals are most likely to visit

²⁵In Appendix Appendix A we discuss the construction of this measure in greater detail.

Table 1: Key Model Parameters

Parameter	Description	Value	Source
<u>Location Substitution Elasticities</u>			
ρ	Between-type neighborhood substitution elasticity	2.7	Estimated
γ	Within-type neighborhood substitution elasticity	5.7	Assumption
<u>Amenity Demand</u>			
α	Amenity share	0.15	CEX
σ	Substitution elasticity across neighborhoods	5.7	Estimated
δ	Distance elasticity across neighborhoods	0.22	Calibrated to Couture (2016)
<u>Land and Housing Supply Elasticities</u>			
ϵ_D	Downtown land supply elasticity	0.6	Calibrated to Saiz (2010)
ϵ_S	Suburban land supply elasticity	1.3	Calibrated to Saiz (2010)
<u>Transportation Costs</u>			
τ_D^C	Commute costs as share of labor income downtown	0.044	Authors' calculation
τ_S^C	Commute costs as share of labor income suburbs	0.059	Authors' calculation

model:²⁶

$$\Delta \ln \left(\frac{\lambda_{cDj}(w)/\lambda_{cD}}{\lambda_{cSj}(w)/\lambda_{cS}} \right) = \overbrace{\rho \Delta \ln \left[\frac{A_{cD} N_{cDj}^{\left(\frac{1}{\gamma} + \frac{\alpha}{\sigma-1}\right)} / \left(p_{cDj}^a K_{cD}^\delta\right)^\alpha}{A_{cS} N_{cSj}^{\left(\frac{1}{\gamma} + \frac{\alpha}{\sigma-1}\right)} / \left(p_{cSj}^a K_{cS}^\delta\right)^\alpha} \right]}^{\psi_{cj}} + \rho \Delta \ln \left(\frac{w - p_{cDj}}{w - p_{cSj}} \right)$$

The first term measures any change public amenities in either the downtown or suburban areas and any change in the size of the downtown and suburban areas within a CBSA over time. This is constant for all residents within a given CBSA. The second term measures the change in the number of neighborhoods N within any nj within a given CBSA over time. This is constant for all residents within a given CBSA quality tier. Both the first and second term, therefore, can be proxied with a CBSA-quality fixed effect, ψ_{cj} . With the CBSA/quality fixed-effect ψ_{cj} , all the identifying variation for ρ comes from exploiting how house price variation effects location choices across neighborhood types for different income groups within a CBSA. We estimate a larger ρ if richer households tend to locate in areas with higher housing prices, conditional on neighborhood quality j . Given the above, our key estimation comes from estimating:

²⁶To derive equation (20), we start with (9), drop the commuting cost and mutual fund terms for simplicity, take logs, difference over time and across the two areas in a CBSA.

$$\Delta \ln \left(\frac{\lambda_{cDj}(w)/\lambda_{cDj}}{\lambda_{cSj}(w)/\lambda_{cSj}} \right) = \delta_{cj} + \rho \Delta \ln \left(\frac{w - p_{cDj}}{w - p_{cSj}} \right) + \epsilon_{cj}(w) \quad (20)$$

where ϵ_{cj} captures measurement error and any time-varying variables with a differential impact across income groups missing from the model. Equation (20) takes our model literally and interprets the relationship between changes in the propensity to live downtown and changes in house prices as being driven by non-homothetic preferences for urban amenities (broadly defined). Other forces, however, could be driving house prices and location choices outside of our model. ψ_{cj} captures the changes in demand for area/quality pairs that are common across income groups, but any downtown public amenity change that appeals to the the rich more than the poor could confound our estimates. For example, if local public officials start creating amenities downtown that appeal to the rich, this could both drive up housing prices downtown and draw the rich downtown. Such variation would not be helpful in identifying ρ .

To deal with this potential concern, we instrument $\Delta \ln((w - p_{cDj})/(w - p_{cSj}))$ with a CBSA-level shift-share (e.g., Bartik) per-capita income shock interacted with individual income bracket dummies. The CBSA-level shift-share shock interacts variation across CBSA's in their industry mix interacted with national trends (excluding that CBSA) in average earnings in that industry. Basically, we are asking when a CBSA gets an exogenous shock to labor demand which increases per-capita income, how do residents of differing incomes in that CBSA adjust their location choice across neighborhoods of different type. Our exclusion restriction is that, conditional on ψ_{cj} , the shift-share shock changes the relative propensity of a given income group to live in Dj relative to Sj only through its impact on relative real incomes driven by differential house price variation across the n, j pairs. This would imply, for example, that the extent to which city planners change amenities downtown that could be valued more by the rich such changes are orthogonal to our CBSA level shift-share shock. Our instrument isolates variation in relative house prices in downtown versus suburbs associated with plausibly exogenous movements in CBSA-level income that are at the heart of our model's gentrification mechanism.²⁷

To understand the variation that facilitates our identification, our instrument offers a useful reduced-form representation of the structural equation (20) in which we estimate the direct impact of CBSA-level Bartik per capita income shocks on the urbanization of household within each income bracket. To illustrate this variation, we run the following equation:

²⁷There is a growing literature discussing the potential pros and cons of using a shift-share (Bartik) instrument to isolate parameters exploiting cross-region variation. See, for example, Ado et al. (2018), Borusyak et al. (2018), and Goldsmith-Pinkham et al. (2018). Many of these papers discuss the potential that initial industry mix is also correlated with other CBSA specific factors that may also be driving variation in the dependent variable of interest. These papers, however, are less relevant for our analysis. Given our estimation includes CBSA fixed effects (which proxies for other CBSA specific factors), we are using the shift-share instrument to exploit within CBSA variation across different income groups. It is this interaction that facilitates our identification.

$$\Delta \ln \left(\frac{\lambda_{cD}(w)/\lambda_{cD}}{\lambda_{cS}(w)/\lambda_{cS}} \right) = \mu_w^0 + \mu_w^1 \widehat{\text{Income}_c^{\text{Bartik}}} + \epsilon_{cw}. \quad (21)$$

The above regression equation measures the extent to which individuals residing in income bracket w change their relative propensity to live downtown (as opposed to the suburbs) as CBSA predicted income increases. We estimate 21 separately for each of our bracketed income groups. Finally, we estimate this regression both for changes between 2000 and 2013 and for changes between 1990 and 2013. To estimate equation (21), we use the same definition of λ as in our U-shape plot of section 3, which is consistent with the model above. $\mu_w^1 > 0$ implies that income group w is more likely to have an increased propensity to live downtown as opposed to the suburbs relative to the average CBSA resident when CBSA income increases. Our model implies that μ_w^1 should be increasing as w increases. Moreover, given that the relationship is in shares, if higher income individuals have $\mu_w^1 > 0$, lower income individuals must have $\mu_w^1 < 0$. Finally, we wish to stress that there is nothing tautological about these regressions. If spatial sorting responses are unrelated to income, μ_w^1 could equal zero for all income groups.

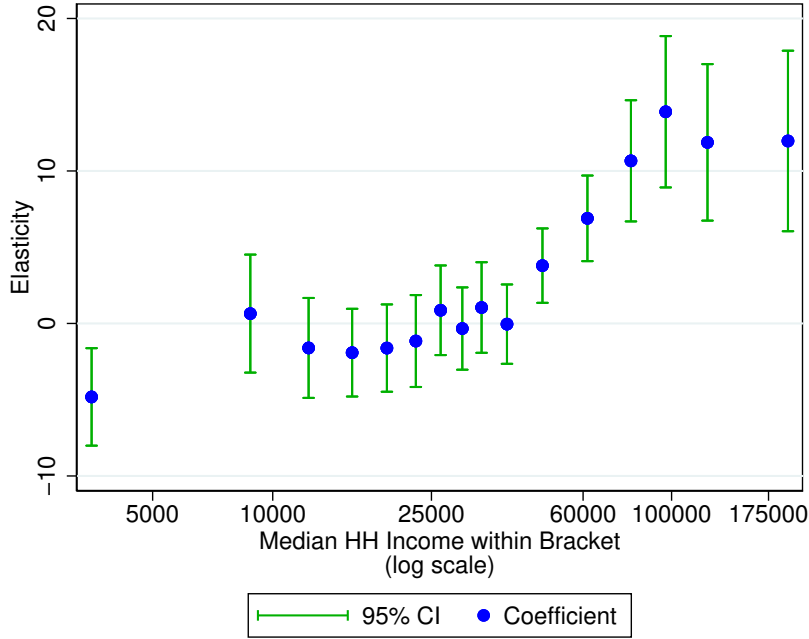
Figure 3 depicts the reduced-form estimates from equation (21), along with their 95 percent confidence bounds, where all changes are defined over the 2000 to 2013 period. The results show that CBSA per-capita income shocks are associated with differential spatial sorting patterns of the rich and the poor, and provide empirical validation for our model. For all of the top six income bins, $\mu_w > 0$ and all are statistically significant at the 5 percent level. Conversely, all but one of the bottom six income bins have estimates of $\mu_w < 0$ (but they are often not statistically significant). We find the same patterns of relative urbanization of the rich and suburbanization of the poor following an income shock for the estimation period of changes between the 1990 and 2013 period, and in regressions replacing the Bartik shock with actual per-capita income growth.

Our main estimation results of ρ from equation (20) are in Table ???. We again show results for changes during the 2000 to 2013 period. To compute $\Delta \ln((w - p_{cDj})/(w - p_{cSj}))$, we again set w as the median household income within each constant CPI-adjusted census bracket. We calculate this median using 2000 IPUMS microdata from the 100 largest CBSA. As a result, w is fixed over time and across CBSAs. We use the Zillow all home price index in 2000 and 2013 as well as the prevailing mortgage rate in a given year to compute annual housing costs p_{cnj} associated with the median house price in each $\{n, j\}$ pair within a CBSA.²⁸ To reiterate, our instrument is our predicted CBSA level shift share shock interacted with the individual level income brackets. In total, our estimation includes at most 2000 observations (100 CBSAs, 10 income groups, and 2 quality tiers).²⁹ In many specifications, we have less than 2000 observations given the presence

²⁸The Zillow Home Value Index for all homes in 2000 and 2013 measures the median home value at the zip code level. We first map it to census tracts, and then compute a population-weighted median house price over all census tracts within $\{n, j\}$. Finally, we compute the first annual mortgage payment associated with this house price for a standard 30 year-fixed rate mortgage with a 20% downpayment at the prevailing mortgage rate in a given year.

²⁹We have 10 income groups in this estimation, instead of 16 as in Figure 21. This is because we drop all

Figure 3: Reduced-form: Elasticity of Change Urban Share on Bartik Income Shock for each Income Bracket, 2000 to 2013



Note: Plot depicts income bracket-specific coefficients from equation (21).

of missing data at the CBSA-income group-quality triplet.³⁰ We provide additional details on all variable construction in Appendix A.

Columns (1) and (2) segment neighborhood quality using whether a census tract has at least 40 percent of residents with at least a bachelor's degree. Columns (3) and (4) segment tracts based on the restaurant quality index. Within each quality measure, we show both the OLS estimates and the IV estimates where we use our Bartik instrument for $\Delta \ln \left(\frac{w - p_{cDj}}{w - p_{cSj}} \right)$. As seen in Table ??, our IV estimates of ρ are very stable across our two measures of neighborhood quality. Our IV estimate using the college share to segment quality is our preferred estimates. Across both measures of neighborhood quality, our estimates of ρ are close to 2.7. For our base specification, we set $\rho = 2.7$. As we show later, ρ is an important parameter determining our welfare results. Therefore, in our counterfactual exercises we show the sensitivity of our results a value of $\rho = 2, 4$,

households with income smaller than \$25,000 per year from our model calibration. Given the presence of public housing, such households are not well represented by our model. This does not effect our estimation of ρ but does have some effect on our method of moments calibration described below. For consistency, we exclude them from this regression.

³⁰We also remove any observation with $w - p_{enj} < 0$ (1.4% of our sample). Within each year and quality type, we then censor the top and bottom 1% of $w - p_{enj}$. Censoring the top and bottom 5% instead leads to similar estimates.

Table 2: Estimation of ρ

	Tract College Share > 40%		Local Restaurant Chain Quality	
	(1)	(2)	(3)	(4)
	OLS	IV	OLS	IV
$\hat{\rho}$	1.35 (0.09)	2.73 (0.17)	1.01 (0.12)	2.62 (0.23)
δ_{cj}	Yes	Yes	Yes	Yes
R2	0.79		0.82	
KP F-Stat		60.48		53.14
Obs	1,370	1,370	1,441	1,441

Notes: Data from 100 largest CBSAs in 2000 and 2013. Observations are CBSA-population weighted. Standard errors, in parentheses, are clustered at CBSA-level. Standard error on $\hat{\gamma}$ computed with the delta method. KP F-Stat = Kleinberger-Papp Wald F statistic.

and 6.

Finally, it is worth noting that we have explored the robustness of the results in Table ?? to additional controls. One potential concern with our instrument is that predicted rising average income in some CBSAs comes from a concentration of typically urbanized industries that recently experienced high national wage growth (e.g., technology or FIRE). The growth of these urbanized industries could then drive both the income growth at the CBSA level and the shift of the wealthy downtown in the cities where these industries were overrepresented in 1990. Specifically, if the technology sector was booming during the 2000s and technology jobs are more likely to be located downtown and employ high income people, we may observe high income people moving downtown in response to the positive shift share shock to minimize their commute as opposed to consume urban amenities. To investigate whether such a concern is warranted, we compute Bartik instruments that leave out technology or FIRE industries. Our results from these leave out Bartiks are very similar to those from Figure 3.³¹

5.2.2 Estimation and Parameterization of Amenity Demand (α, δ, σ)

We begin this section by discussing how we can estimate the amenity demand elasticities (δ and σ) using a model-implied gravity equation. We then talk about how we can use expenditure data to parameterize α , the share of expenditure households allocate to residential amenities above and

³¹These results are consistent with Couture and Handbury (2017), Baum-Snow and Hartley (2017) and Su (2017) who all find evidence that spatial job sorting plays little to no role in explaining the recent movement of high income individuals downtown.

beyond what they pay for housing. Throughout, we define residential amenities as non-tradable services including expenditures at restaurants, bars, entertainment venues (movie theater, shows, etc), gyms, and other personal services. When thinking of residential amenities, we exclude retail consumption at apparel, grocery, and other merchandise stores. Non-tradable services like restaurants and entertainment venues most closely match our model’s amenity that are luxurious, endogenous, locally-provided, and subject to strong economies of density.³²

The model delivers the following gravity equation for amenity demand:

$$\ln \left(\frac{a_{rr'}}{a_{rr}} \right) = \overbrace{\ln \left(\frac{\beta_{rr'}^q}{\beta_{rr}^q} \right)}^{\beta_{HtoL} + \beta_{LtoH}} + \sigma \delta \ln \left(\frac{d_{rr'}}{d_{rr}} \right) + \overbrace{\sigma \ln \left(\frac{p_{r'}^a}{p_r^a} \right)}^{\theta_r + \theta_{r'}}. \quad (22)$$

The first term captures the possibility that people place less value on consuming amenities of a quality type other than their preferred type. We proxy this with a dummy variable β_{HtoL} equal to 1 when the home neighborhood r is of quality type H and amenities are consumed in a neighborhood of quality type L . β_{LtoH} likewise is equal to 1 when the home neighborhood r is of quality type L and amenities are consumed in a neighborhood of quality type H . The second term captures the travel distance required to access amenities in neighborhood r' relative to the travel distance required to consume within the home neighborhood r . The third term captures the relative amenity prices in neighborhood r' and r . Given that amenity price is constant within neighborhood, we can proxy for this third term with an origin r and a destination r' fixed-effect, θ_r and $\theta_{r'}$. Importantly, these fixed-effects can absorb any unobserved tract characteristics. We then obtain the following estimating equation:

$$\ln \left(\frac{a_{rr'}}{a_{rr}} \right) = \beta_{HtoL} + \beta_{LtoH} + \sigma \delta \ln \left(\frac{d_{rr'}}{d_{rr}} \right) + \theta_r + \theta_{r'} + \epsilon_{rr'}, \quad (23)$$

Estimating (23) requires information on the origin and destination of a large number of trips to consume amenities, which is not available in conventional travel surveys like the National Household Travel Survey. To circumvent this issue, we use new smartphone movement data from Couture, Dingel, Green and Handbury (work in progress). The data comes from aggregating GPS geolocation from multiple apps locational services used by a given smartphone device. Each amenity trip corresponds to the intersection (with high enough confidence) of this movement data with a basemap of polygons describing the location of commercial establishments. This smartphone data allows us to identify billions of visits to commercial establishments that we classify as non-tradable services (e.g., restaurants, gyms, theaters), from tens of millions of devices.³³ We provide additional details

³²As discussed before, Aguiar and Bils (2015) identify non-tradable services like restaurant and entertainment as having steep Engel curves, Couture and Handbury (2017) identify non-tradable services in particular as the key driver of the downtown location choice of the college-educated.

³³Athey et al. (2018) and Chen and Rohla (2018) use similar smartphone data from a different provider. Athey et al. (2018) models restaurant demand in San Francisco, while Chen and Rohla (2018) investigate the impact of Donald Trump’s election on the length of Thanksgiving dinners in the United States.

on the dataset and variable construction in Appendix A.³⁴

In order to isolate the choice of consuming amenities from other considerations of travelers (e.g., eating during lunch at work), we study the robustness of our estimates to restricting the sample to only trips starting from home or trips that take place on weekends. We define the device’s home location as being the place the phone resides for most of the night.³⁵ We again define neighborhoods as census tracts, so $a_{rr'}$ is the number of trips by devices living in tract r to non-tradable service establishments located in tract r' . We define $d_{rr'}$ as the haversine distance from the centroid tract r to r' and δ_{rr} as half the radius of the home tract. Each observation in our regression is a tract pair rr' , and we limit the choice set of each device to tracts available within its CBSA. Note that $\delta\sigma$ is large if devices make few trips far from home, either because the cost of distance δ is large or because amenities are highly substitutable (i.e., σ is large).³⁶

Table 3 shows the estimation results, for both the college and restaurant quality definition, for all trips, only trips originating from home, and only weekend trips. The coefficients $\delta\sigma$ are stable and remain within -1.09 and -1.49 across all estimations. Interestingly, our amenity trip gravity coefficient is similar, albeit somewhat larger, to those from the trade literature, which center around -1.

The coefficients on β_{HtoL} are consistently large, negative and significant, indicating a distaste of households living in high quality tract for visiting low quality tracts, and providing some evidence that our quality definition capture relevant features of household’s preference for amenities. In fact, using our preferred quality definition based on college-educated share, devices living in high quality tracts make 76 percent of their trips to other high quality tracts, and devices living in low quality tract make 66 percent of trips to other low quality tracts. The coefficient on β_{LtoH} are smaller, generally negative, and not always significant. We also verified that our coefficient are robust to adding additional controls for tract pair characteristics, such as an index of racial dissimilarity (using the Euclidian Demographic Distance measure developed in Davis et al. (Forthcoming) and median age difference.

³⁴We refer to Couture, Dingel, Green, and Handbury (work in progress) for evidence that the spatial distribution of smartphone devices provides a balanced representation of the US population along a number of dimensions (CBSA, income, race, education), that smartphones visit distance resembles that from the NHTS travel survey, and that the basemap of commercial establishments in the data is relatively complete. We note that the ratio of devices to Census population declines somewhat in very low density rural areas, but such areas are not in our sample of the 100 largest CBSAs.

³⁵We may not observe all movements for a given device if a user shuts off their phone or does not bring it with them when consuming certain amenities. Additionally, our geocoding data does not map office buildings, schools, and hospitals so we miss these trips. Given this, we are not able to distinguish easily a cellphone’s place of work versus any other retail establishment. To remedy this, we define a trip from home as occurring when the last location prior to the amenity visit is home, and there is at most one hour between the last observation at home and the first in the consumption venue.

³⁶The fixed-effect θ_r ensures that our estimates are not biased by the fact that Americans generally live in mostly residential tracts, relatively far from commercial destinations. In practice, removing the fixed-effect does strongly bias our estimates of $\delta\sigma$ downward. In a few cases, the home tract has no commercial establishments, so we take the ratio of trips and distances relative to the closest tract from home with a positive number of trips.

Table 3: Estimation of $\sigma\delta$

	Tract College Share > 40%			Local Restaurant Chain Quality		
	All (1)	Home (2)	Weekend (3)	All (4)	Home (5)	Weekend (6)
$\hat{\delta}\sigma$	-1.48 (0.00)	-1.22 (0.00)	-1.09 (0.00)	-1.49 (0.00)	-1.22 (0.00)	-1.10 (0.00)
β_{HtoL}	-0.25 (0.04)	-0.13 (0.06)	-0.15 (0.03)	-0.10 (0.00)	-0.06 (0.01)	-0.06 (0.00)
β_{LtoH}	-0.02 (0.04)	-0.05 (0.06)	-0.04 (0.03)	0.00 (0.00)	0.00 (0.01)	0.00 (0.00)
R2	0.92	0.87	0.89	0.92	0.87	0.89
Obs	19,657,245	3,670,979	9,669,575	19,657,245	3,670,979	9,669,575

The above gravity equation estimates $\sigma\delta$. However, for our calibration, we also need estimates of σ and δ separately. We are not aware of existing estimates of δ in the literature. In our model, δ governs how the share of travel cost in the total price of consuming amenities rises with travel distance d . Couture (2016) uses a different parametrization of the cost of distance, but combining data on restaurant trips, prices, and expenditures with existing empirical estimates of value of travel time, he finds that a significant majority (59%) of trips to a restaurant from home take between 5 and 15 minutes, and that over this range of travel times, the total price of amenity rises by 27% due to travel costs. If we similarly calibrate our δ such that tripling distance increases travel cost by 27% percent, we obtain:³⁷ :

$$\frac{d_{rr'}^\delta p_{r'}^a}{d_{rr'}^\delta p_{r'}^a} = \left(\frac{15}{5}\right)^\delta = 1.27$$

and recover $\delta = 0.22$. We can then also recover $\sigma = 1.25/0.22 = 5.7$, which we use in our preferred calibration. This estimate of the elasticity of substitution across neighborhoods for amenity trips is between $\sigma = 3.9$ that Atkin et al. (2018) find for retail stores in Mexico, and $\sigma = 8.8$ that Couture (2016) finds for restaurants in the US. Finally, we highlight that our estimates of the gravity coefficient $\delta\sigma$ are quite robust, but we have much less confidence in our parametrization of δ . Our robustness checks will reflect this uncertainty.

Finally, we use data from the Consumer Expenditure Survey (CEX) to discipline α . Setting $\chi(w) = 0$, α is the share of expenditures net of housing costs and transportation to work $((1 - \tau)w - p^h)$ that is spent on residential amenities such as restaurants, bars, entertainment venues,

³⁷This result is not reported by Couture (2016), but can be computed with the data reported in that paper.

gym memberships, and other personal services. Within the CEX, food away from home represents roughly 6 percent of spending out of the average individuals total expenditures. Given that housing is about twenty percent of total expenditures and transportation to work is about five percent of total expenditures (see below), restaurant spending alone 8 percent of expenditure net of housing costs and transportation ($6\%/(1-25\%)$). Adding in other residential amenities such as movie tickets, tickets to theater and sporting events, gym memberships, and other personal services yields another 5 to 7 percent of expenditures net of housing and transportation. As a result, our base calibration uses $\alpha = 0.15$. Given there is some ambiguity about what could be included in our measure of luxury residential amenities, we investigate the robustness of our results to $\alpha \in [0.08, 0.25]$. The lower bound makes the narrow assumption that our residential amenities only includes restaurants. The upper bound allows for the fact that there are other luxury residential amenities (like a reduction in crime) that households are willing to pay for and that also evolves endogenously. As we show below, the higher the value of α , the larger the amplification in welfare differences between the rich and the poor predicted from the model due to the spatial sorting response that results from increasing income inequality. Appendix A provides detailed definitions.

5.2.3 Parameterization of Elasticity of Demand Between Neighborhoods (γ)

In our base parameterization, we assume a value of $\gamma = 5.7$. This value is chosen purposefully. Given the assumptions on idiosyncratic preference shocks, $\rho < \gamma$. This bounds γ from below. Second, existing research finds that there is less socio-economic diversity within census tracts than there is within retail establishments such as grocery stores and restaurants.³⁸ Through the lens of our model, this implies the substitution elasticity for residential amenities (σ) is an upper bound on our estimate of γ . Given our above estimation, this implies that γ will lie in the range between 2.7 and 5.7. The lower the value of γ the larger the endogenous response of amenities to the changing income distribution. For our base assumption, we set $\gamma = 5.7$. However, as a robustness exercise, we show the sensitivity of our results to $\gamma = 4$ and $\gamma = \infty$.

5.2.4 Parameterization of Land and Housing Supply Elasticities

To calibrate the elasticity of land supply ϵ_n , we first note that in our model it equals the elasticity of housing supply, because only land enters the production of housing.³⁹

Saiz (2010) provides housing supply elasticity estimates ϵ_c for 95 large Metropolitan Statistical Areas, based on geographical constraints and housing regulations. We can therefore assign an elasticity to 83 of the largest 100 CBSAs in our sample, and rely on this cross-CBSA variation

³⁸Handbury et al. (2015) find that Nielsen panelists who are from college- and non-college educated households are more likely to co-locate in grocery stores than in census tracts. This is consistent with Davis et al. (Forthcoming) who find a higher rate of racial segregation across residential neighborhoods than restaurants within NYC.

³⁹We experimented with a model that includes a non unitary land share and found little impact on our calibration results.

to infer values of ϵ_D and ϵ_S . In particular, we find that the Saiz housing elasticities decline with average household density ($density_c$). This suggests that $\epsilon_D < \epsilon_S$.

To formalize this argument, we estimate the following log-linear regression of ϵ_c on $density_c$:

$$\ln(\xi_c) = 1.97 - \underset{(0.07)}{0.30} \ln(density_c) + \epsilon_c^\xi, R^2 = 0.21 \quad (24)$$

We then define $\hat{\epsilon}_D$ and $\hat{\epsilon}_S$ as the fitted values from equation (24) computed at typical density of D and S neighborhoods in the 100 largest CBSAs. We find $\hat{\epsilon}_D = 0.60$ and $\hat{\epsilon}_S = 1.33$.⁴⁰ We use these values in our baseline calibration and test the sensitivity of our results to alternative parameter values.

5.2.5 Parameterization of Commuting Costs

To estimate the commuting costs, we use data on trip time to work by car from the geo-coded 2009 National Household Travel Survey to estimate area-specific τ'_n s. Specifically, the average daily commute time for drivers living in the suburbs of the top 100 CBSAs is 64 minutes, while it is 47 minutes for those living downtown. We compute τ_n by assuming that each worker allocates 9 hours per day to working and commuting, and by valuing an hour of commuting at half of the hourly wage as recommended by Small et al. (2007). This implies a per labor hour commute cost of $\tau_n = 0.5 * \text{CommuteTime}_n / 9$, or $\tau_D = 0.0435$ and $\tau_S = 0.059$.⁴¹

5.3 Second Stage: Method of Moments

Armed with estimates for the key elasticities of the model, we then conclude the calibration of the model using a method of moments. That is, we set the remaining parameters that allow, conditional on the model elasticities, to minimize the distance between the model moments and their empirical counterparts. The model is flexible enough to exactly match some of these moments, while others will be targeted without being fully matched.

Specifically, we exactly match three shares taken from the equilibrium read from the data. The first, which we denote S_{nj}^{mk} , is the share of amenity expenditures of households living in a neighborhood of type (n, j) spent on amenities consumed in a neighborhood of type (m, k) . The second is the share s_n^Π of city-wide land rents that correspond to location n . The last one is the share of equipped land $s_{n,j}^i$, within each location, used by activity $i \in a, h$ of quality j .⁴² Beyond

⁴⁰In our downtowns, the average CBSA population-weighted household density is 4,300 households per square mile, versus 300 in the suburbs. The highest density CBSA, New York, has 850 households per square mile, so the average density in D is out-of-sample. However, $\hat{\epsilon}_D = 0.60$ turns out to equal the elasticity of housing supply in Miami, which is the metropolitan area with the most inelastic housing supply in Saiz (2010).

⁴¹In our base parameterization, we assume that all housing rents in the city (land rents and fixed costs of development) accrue to an absentee landlord and none are transferred to the city residents, i.e., that $\chi(w) = 0$ for all w .

⁴²We have $\sum_n s_n^\Pi = 1$, $\sum_{i,j} s_{n,j}^i = 1$ for $n = D$ or S and $\sum_{m,k} S_{nj}^{mk} = 1$, for $n = D, S$ and $j = H, M, L$.

these shares that the model exactly replicates, we target the following set of moments: (i) the whole distribution, by income level, of the share of workers living downtown in 1990, as described in the stylized facts section 3.2, and (ii) the median house price by neighborhood type, also in 1990. These two moments summarize the key economic concepts we aim to capture.

As described below, the method of moments allows us to back out two key composite model variables: (i) the relative values of effective neighborhood quality in each location for each neighborhood type in the baseline equilibrium:

$$\tilde{q}_{n,j} = A_n Q_j N_{nj}^{\frac{1}{\gamma}} (P_r^a)^{-\alpha}, \quad (25)$$

without separately identifying the individual terms, and (ii) the price of housing in each location for each neighborhood type ($p_{n,j}^h$). Combined with the model elasticities, these variables pin down the calibrated value for location choice $\lambda_{n,j}(w)$ in the model. Furthermore, using the equilibrium relationship (18), these composite variables also pin down $h_{n,j}^h r_n$, the land prices of the model, and Π the value of the real estate portfolio.

Importantly, we note that this calibration does not identify all of the deep parameters of the model separately. Rather, it identifies a set of composite variables $\{\lambda_{n,j}(w), L_{n,j}, p_{n,j}^h, S_{nj}^{mk}, s_{n,j}^i, s_n^\Pi\}$ that is just sufficient, conditional on estimates for the model elasticities $\{\rho, \gamma, \epsilon_n, \sigma, \alpha, \tau_n\}$, to compute any counterfactual equilibrium of the same model that relies on different primitives, using exact hat algebra following the method popularized by Dekle et al. (2007). Intuitively, these composite variables embed just the right level of information on the deep parameters of the model, such as the exogenous quality of different neighborhoods A_n and Q_j or the fixed cost of building neighborhoods f_{nj} that ultimately determines $N_{n,j}$, to compute relevant counterfactuals. This step is described in more detail in section Appendix B.1 below.

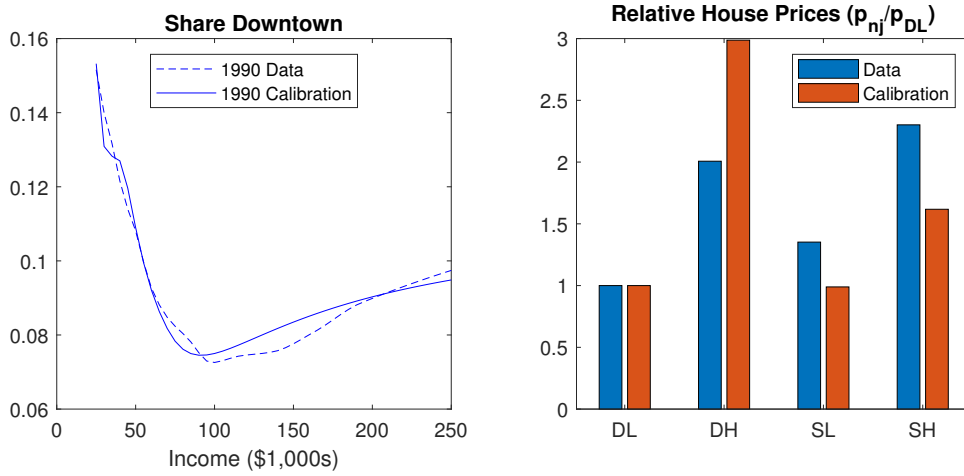
The identification of the model in this second stage is quite straightforward. First, it is clear how the house price moments is directly informative for the calibration of $p_{n,j}^h$. Note though that since the model is over-identified, the price moment cannot be perfectly matched. Depending on the weight put on moments (i) and (ii), the procedure trades-off a better fit of the U-shape for location choices against a better fit for housing prices. Then, conditional on prices, the U-shape pattern of the location choice data helps identify the relative quality of different types of neighborhoods ($\tilde{q}_{n,j}$). This is a quite intuitive revealed preference approach, applied to our non-homothetic demand function: the same level of price and quality of a neighborhood generates different demand patterns at different levels of income. Of course, this reasoning is conditional on prices that are in part informed by the other moment. Concretely, the identification relies on the following equation for all (n, j) :

$$\frac{\lambda_{n,j}(w)}{\lambda_{S,L}(w)} = \frac{\tilde{q}_{n,j}^\rho \left[(1 - \tau_n)w + \chi(w)\Pi - p_{n,j}^h \right]^\rho}{\tilde{q}_{S,L} \left[(1 - \tau_S)w + \chi(w)\Pi - p_{S,L}^h \right]^\rho}$$

Knowing τ_n , w , and Π , which is set by a fixed point of the procedure as it depends on prices,⁴³ the calibration backs out the $\tilde{q}_{n,j}$ and $p_{n,j}^h$ that allow to match best the data distribution of location choices, $\frac{\lambda_{n,j}(w)}{\lambda_{L,S}(w)}$.⁴⁴

The moment fit is presented in Figure 4. Despite a sparse specification, the calibrated model is able to match the rich non-monotonic U-shape patterns of location choice by households of various incomes. To be able to match these sorting patterns, the model calls for housing prices in the suburbs that are below those in the data, and for high quality housing prices downtown that are above those in the data.

Figure 4: Calibration to 1990 Urban Shares and Neighborhood Prices



6 Counterfactual Analysis

In this section, we use the calibrated model to analyze the impact of spatial sorting on welfare inequality. We first describe how we use the structure of the model and its 1990 calibration to compute counterfactual equilibria. We then describe and analyze a series of counterfactual exercises.

6.1 Computing Counterfactuals

We summarize here how to compute a counterfactual equilibrium for a different income distribution $L'(w)$ in the city, conditional on (i) an initial calibration corresponding to the income distribution $L(w)$, described above, and (ii) on the model elasticities $\{\rho, \gamma, \epsilon_n, \sigma, \alpha, \tau_n\}$. A similar approach can

⁴³Specifically, the real estate portfolio is given $\Pi = \sum_{n,j,i} r_n K_{n,j}^i$ with $r_n K_{n,j}^i$ for $i \in \{a, h\}$ respectively given by equations (15) and (16).

⁴⁴These vectors are pinned down up to a normalization level, whose value does not impact the counterfactuals of section 6.1.

be used for solving for a counterfactual equilibrium where, for instance, the exogenous value of A_D/A_S changes, or one that implements a specific tax and transfer policy. Details of the procedure as well as the corresponding set of equations are given in Appendix Appendix B. Specifically, the information necessary to perform this step are the model elasticities and the calibrated values at the initial equilibrium for $\{\lambda_{n,j}(w), L_{n,j}, p_{n,j}^h, S_{n,j}^{mk}, s_{n,j}^i, s_n^\Pi\}$, where $L_{n,j}$ is the total population living in neighborhoods of type $\{n, j\}$, i.e., $L_{n,j} = \int L(w)\lambda_{n,j}(w)dw$

We write a counterfactual equilibrium in changes relative to the initial equilibrium, denoting by $\hat{x} = \frac{x'}{x}$ the relative change of the variable x between the two equilibria. We show in Appendix Appendix B that given the structure of the model, the counterfactual equilibrium is the solution to the set of equations (26)-(31) for $\{(p_{n,j}^h)'\!, \lambda'_{n,j}(w), L'_{n,j}\}$ (or, equivalently, their “hat” values) as well as all the auxiliary variables $\{\hat{P}_{n,j}^a, \hat{r}_n, \hat{N}_{n,j}, \hat{\Pi}\}$, which is fully determined conditional on the model elasticities and $\{\lambda_{n,j}(w), L_{n,j}, p_{n,j}^h, S_{n,j}^{mk}, s_{n,j}^i, s_n^\Pi\}$.

6.2 Computing Welfare Measures

Before turning to describing the outcome of the counterfactual analysis, we detail here the measure of welfare inequality we use in what follows. A goal of our analysis is to compare changes in well-being inequality, which take into account changes in prices and spatial sorting, to simple measures of changes in income inequality. Since the welfare measure delivered by the model is hard to interpret as is, we instead compute a measure of compensating variation (CV) to guide our well-being analysis.

Let i denote a percentile in the income distribution. Let $m_1(i)$ denote the corresponding income in the initial equilibrium (and $m_2(i)$ in the second equilibrium). More generally, subscript 2 indicates the new equilibrium (e.g., a 2013 counterfactual), and subscript 1 the initial one (the 1990 calibration).

The CV measure is defined as follows:

$$\begin{aligned} CV(i) &= e(p_2, V_2(i)) - e(p_2, V_1(i)) \\ &= m_2(i) - V_2^{-1}(V_1(m_1(i))) \end{aligned}$$

This measures tells us, in 2013 dollars (i.e., at 2013 prices), how much additional income a worker at percentile i of the income distribution would need to go from his 1990 welfare level to his 2013 welfare level (where welfare is captured by V , defined in (13)).

We compare this measure to the change in income of a worker at this percentile of the income distribution, by computing the percentage point difference between the two relative income (or CV) changes:

$$\Delta\mathcal{W}^c(i) = \frac{CV(i) - (m_2(i) - m_1(i))}{m_1(i)}$$

Note that we also compute a measure of equivalent variation (EV), that is how much income would a worker i have needed in period 1 to get the utility they get in period 2. This leads to similar insights. We report the corresponding results in the Appendix.⁴⁵

6.3 Counterfactual 1: Changing Both Population and Income

We start by studying the model’s prediction for the change in the spatial distribution of the population within a CBSA and resident welfare, following a change in total CBSA population and a shift in the income distribution that mimics the observed changes between 1990 and 2013. This counterfactual is what drives the spatial sorting responses of residents over this period as depicted in Figure 1. This counterfactual is done for illustrative purposes to understand the forces in our model; however, it will not be our base counterfactual. Our preferred counterfactual is instead one where we isolate the effect of changes to the shape of the income distribution holding total CBSA population fixed. We discuss that counterfactual in section 6.4.

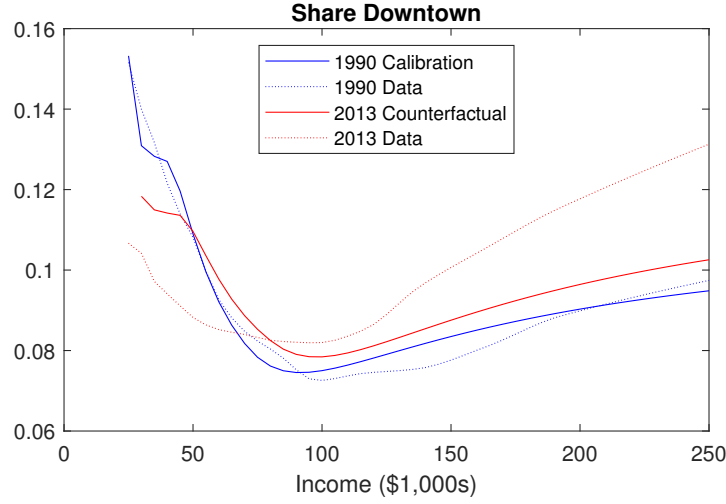
To perform the counterfactual in this subsection, we recompute the equilibrium of the quantified model that corresponds to the new income distribution of 2013 and the new population in 2013, holding all exogenous parameters of the model constant. The 2013 income distribution is characterized, in particular, by a fatter right tail than the 1990 income distribution. There was also 16 percent population growth between 1990 and 2013 within our sample the top 100 CBSAs, which had associated price effects that will also drive the sorting response in this initial counterfactual. The love of variety effects and love of density effects within our model increase either when a given set of individuals experience income gains or when there is population growth increasing the number of individuals at a given level of income.

Figure 5 presents the model’s prediction for the share of workers living downtown by income level for this new distribution of income. The figure also shows, for reference, the calibrated share of workers living downtown in 1990 and the actual shares living downtown in 2013. Our quantified model predicts that the growth in income at the upper-end of the distribution between 1990 and 2013 results in a spatial redistribution of higher-income households downtown, as well as the displacement of lower-income households to the suburbs. Qualitatively, these patterns are overall consistent with that observed in the data. Of course, other changes - beyond a change in the income distribution and the population - happened over the last two decades. We interpret the predictions of our model as being indicative of what *part* of the spatial shifts of the past two decades are plausibly attributable to the growth in population and income inequality that took place over this time period.

⁴⁵ Equivalent variations are measured as:

$$\begin{aligned} EV(i) &= e(p_1, W_2(i)) - e(p_1, W_1(i)) \\ &= W_1^{-1}(W_2(m_2(i))) - m_1(i) \end{aligned}$$

Figure 5: Illustrative Counterfactual



Quantitatively, we find that the model predicts only a portion of the shift in the propensity to live downtown for individuals with income above \$100,000. According to our model, about one-quarter of the shift in the downtown propensity for individuals earning \$150,000 and about one-fifth of the shift in the downtown propensity for individuals earning \$200,000 can be explained by the growing number of richer households living in large CBSAs. Turning to the bottom of the income distribution, the model predicts some displacement of lower income households to the suburbs, but again less so than in the data. Specifically, the poorest households are displaced outside of downtown, both in the data and in the model, but the model predicts that households at intermediate levels of income remain downtown, more so than in the data.

Overall, we conclude from this analysis that the change in income distribution together with aggregate population growth between 1990 and 2013 has the potential to explain a substantive portion of the shift in the composition of downtowns over the same period. We are highlighting one important force that can explain neighborhood change within metropolitan areas. Our results, however, also suggest that there are room for other stories as well.

6.4 Counterfactual 2: Changing Only the Income Distribution

The results from the first counterfactual summarize the joint effects of the observed growth in the aggregate population and the income distribution and are useful for testing the predictive power of the model. We now turn to isolating the contribution of changes in the income distribution, separate from population growth. It is in this exercise that we compute our welfare effects from the changing nature of spatial sorting. To do so, we run the same counterfactual as above, but normalize the counterfactual population to its 1990 level.

In particular, our goal is to estimate the welfare implications of the changing nature of spatial sorting in response to the rising incomes of the rich. To that end, we compute compensating

variation (CV) for households at each percentile of the income distribution in 1990, as explained in section 6.2. This measure reflects changes in well-being associated not only with changing income, but also with changing housing costs, and changes in endogenous amenity quality. We refer to this CV measure as "welfare" or "well-being", to simply contrast it with the simple change in income over time.

6.4.1 Results

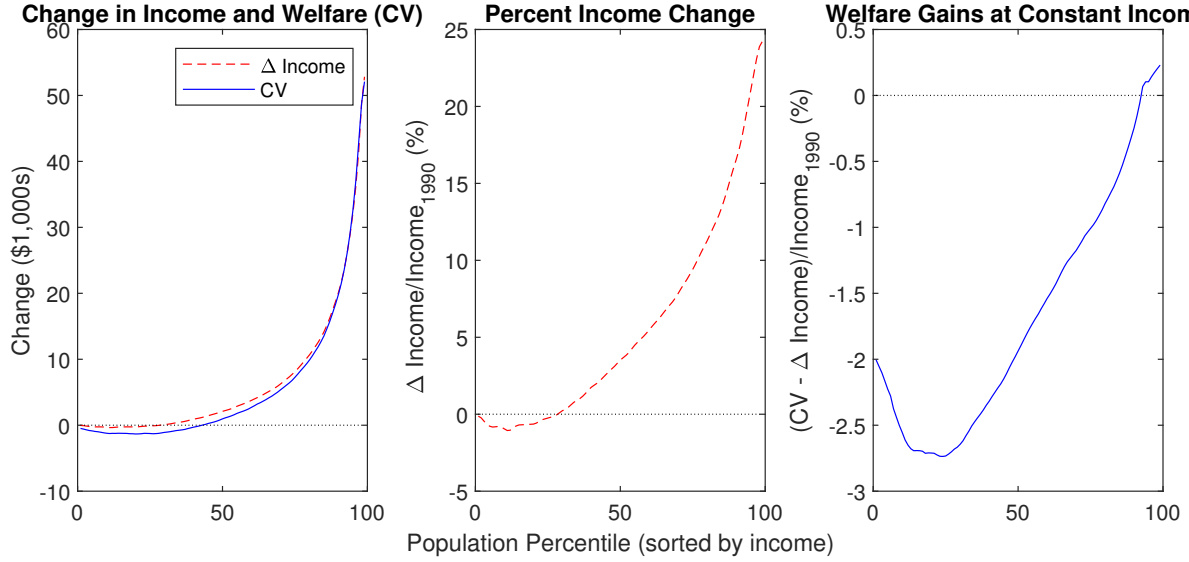
Figure 6 shows our headline results. Specifically, the left hand panel of Figure 6 represents the change in well-being measure in actual dollars, in solid blue, together with the raw change in income, in dotted red, at all percentiles of the income distribution. Note that the welfare measure is slightly tilted, compared to the change in income: the changes in well-being inequality represented in the figure are *larger* than the underlying changes in income. Households at lower levels of income are actually made worse off from the corresponding spatial sorting response associated with the rich moving downtown as they get richer.

To see that point more clearly, we represent two additional panels of results. In the middle panel, we show the percentage change in income that occurred between the two time periods at each percentile of the income distribution. As has been documented extensively within the literature for the economy as a whole, income inequality has increased within the top 100 largest CBSAs over the last 25 years in the United States. For the 10th percentile of the income distribution for our sample, income actually fell slightly by 1 percent. For the 90th percentile, income increased by 17 percent. Overall, there was an 18 percentage point increase in the income gap between the 10th and 90th percentile within our sample since 1990.

The right panel shows the actual change in well-being above and beyond the income change due to the spatial sorting responses within our model. As discussed above, the spatial sorting response amplifies the differences in well-being between the rich and the poor during this time period. In particular, as the rich got richer, they moved into downtown urban areas making the poor worse off. The welfare for households at the 90th percentile of the income distribution is essentially unchanged from the associated spatial sorting response. In particular, their welfare only fell by -0.25 percentage points. As they move into downtown areas, amenities which they value endogenously respond making them better off. House prices increase making them slightly worse. These two effects roughly cancel out for higher income households. Households at the 10th percentile of the income distribution, however, are made worse off in absolute terms. Specifically, households at the 10th percentile have 2.6 percentage points lower utility from the movement of higher income individuals into urban areas as a response to their rising incomes. Their rents are increasing downtown and while amenities are increasing, they do not value those amenities as much.

In total, the spatial sorting response increases inequality as measured by the 90-10 gap by an additional 2.3 percentage points. In other words, the true difference in well-being between the 10th and the 90th percentile increased by 20.3 percentage points (18 percentage points plus 2.3

Figure 6: Welfare Changes From Spatial Sorting Response to Changing Income Distribution



percentage points). The key finding of our paper is that ignoring spatial sorting substantively understates the welfare differences between the rich and the poor of rising income inequality.

6.4.2 Mechanisms

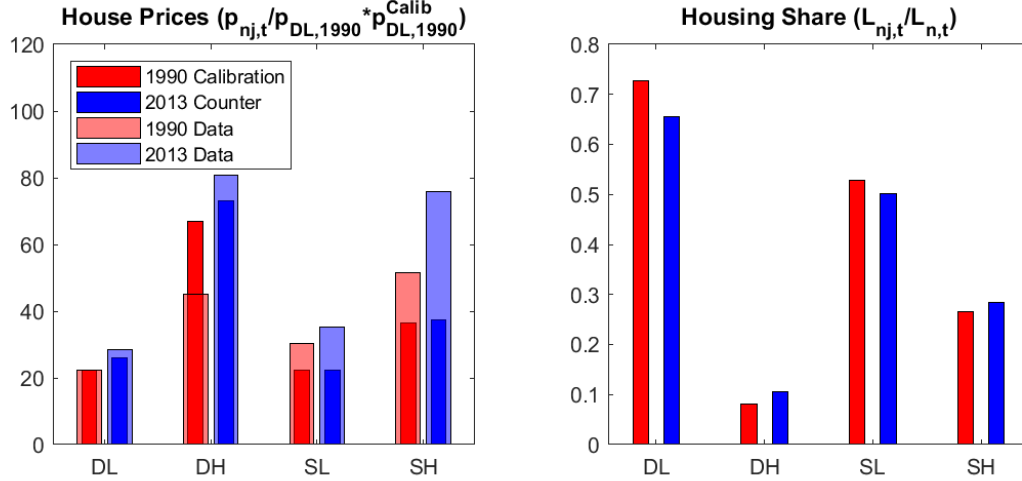
What drives these results in the model? There are three important components here. First, as the rich get richer, they move downtown to consume urban amenities. This puts upward pressure on housing prices downtown. Given that the poor initially live downtown, this makes the poor worse off if they remain downtown. Second, as the rich move downtown, the number of high quality neighborhoods increases. Some of this entry is at the cost of exit (or gentrification) of poorer neighborhoods. Given the love of variety and love of density preferences, the additional entry of high quality neighborhoods downtown makes high income households better off and the associated gentrification makes low income households worse off. Finally, some of the poor may choose to relocate to lower quality neighborhoods in the suburbs. In doing so, the poor will incur additional commute costs and, if there is some autocorrelation in the idiosyncratic neighborhood utility draws, moving costs as they move from their preferred downtown locations.

The left-hand panel of Figure 7 shows the prediction of house prices within our model from the changing income distribution for different n, j pairs. The darker colored bars show the predictions of the model. We also show the actual change in house prices from the data in the lighter colored bars in the background. A few things are of note. First, the change in the income distribution only explains some of the change in house prices in the data. This is not surprising given that a lot of the population growth in all areas is determined by the net increase in population within CBSAs during this period. Given we have shut down population growth in this counterfactual, it is not surprising that we are not matching all of house price growth. Second, our model is predicting

that the shift in the income distribution alone is having a much larger effect on house prices in downtown areas (both high and low quality) relative to the suburban areas. It is this increase in house prices generated by the shift in incomes of the rich that contribute to the welfare losses of the poor who remain downtown.

The right hand side of Figure 7 shows the share of high and low quality housing in each area. The change in this share is a model analog to gentrification. Within a given area (downtown or suburbs), housing is part of either a high or a low quality neighborhood. Our model predicts a decline in low quality downtown neighborhoods and a corresponding increase in high quality neighborhoods in response to the shift in the income distribution. As the number of high quality neighborhoods increase, this increases the utility of all downtown residents given our love of variety preferences.

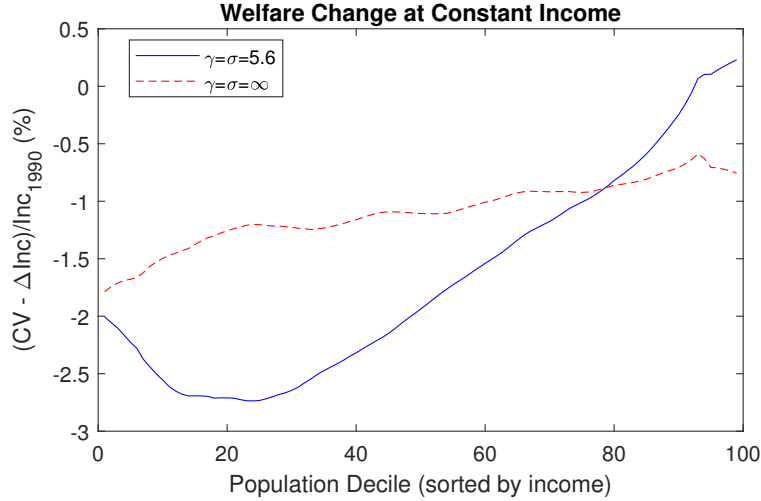
Figure 7: Mechanisms



To tease out the respective roles played by endogenous neighborhood entry and price changes in driving the welfare results at various levels of the income distribution, we study the changing welfare measures in a calibration that assumes zero developer profits by setting the two between-neighborhood substitution elasticities, γ and σ , to infinity. γ governs the elasticity of substitution across neighborhoods within an n, j pair while σ governs the elasticity of substitution of consuming amenities from other neighborhoods. These results are shown in Figure 8. For ease of comparison, we redisplay our base welfare results in the figure (blue line). The red dashed line is the well-being results from the counterfactual that shuts down the neighborhood variety effects. In this counterfactual, prices and land area respond to changes in the income distribution, but there is no increase in neighborhood (or associated amenity) variety. Comparing solid blue line with the dashed red line, we see that the welfare gap across income groups is mitigated substantially. Specifically, the well-being growth gap between households at the 90th and 10th percentiles of the income distribution in 1990, holding income constant, is approximately a third as large when we shut down

neighborhood change (0.79 percentage points vs. 2.31 percentage points) suggesting that two-thirds of the welfare gap in our base results stems from the endogenous amenity response.

Figure 8: Shutting Down Amplification



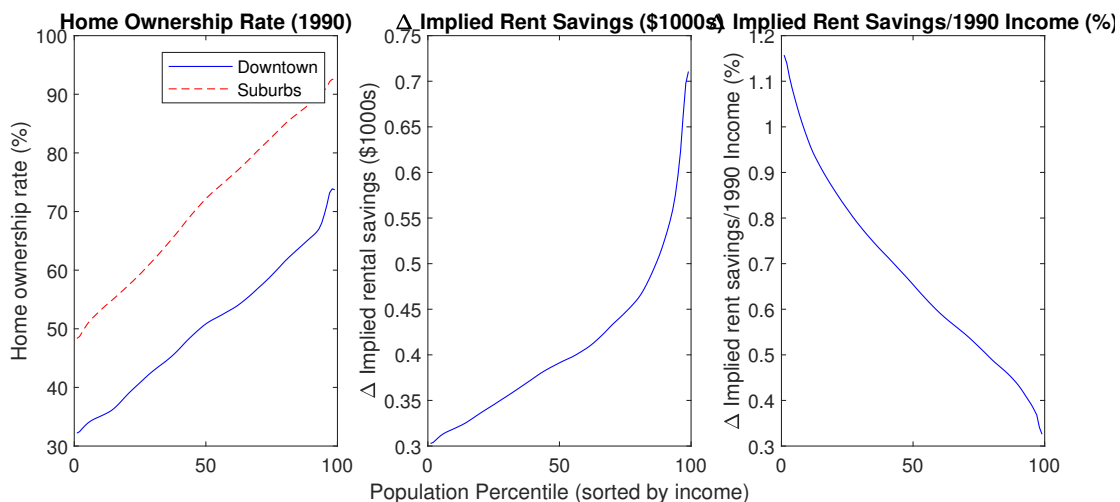
6.4.3 Homeownership

In the baseline calibration and counterfactual exercise above, we have abstracted from home ownership, setting Π to equal zero by assuming absentee landlords. One way to quantify the distributional implications of house price appreciation observed in our model is to calculate the returns to ownership implied by our model for households of different income levels based on 1990 ownership rates and residential locations. The thought experiment here is to suppose that each household that owned a home in 1990 is an absentee landlord of a home in the neighborhood where they resided in that year and, between 1990 and 2013, saw an increase in their non-labor income stream equal to the rental appreciation in that neighborhood. For simplicity, we assume that (i) this rental income does not enter the landlords' budget constraint for housing, amenities, or outside good consumption and (ii) landlords have quasi-linear utility in these goods and their rental income stream. Under these assumptions we can get a sense of how accounting for home ownership might impact our welfare results.

We first calculate the expected average rental appreciation that would accrue to households at each 1990 income percentile, weighting local price growth $\Delta p_{n,j}$ by the share of households that reside at each location in 1990 $\lambda_{n,j,1990}(w)$ and the 1990 home ownership rate in that area $\text{ownshare}_{n,1990}(w)$, both as a function of labor income. The results from these calculations are shown in Figure 9. The left-hand panel shows the share of households at each percentile of the income distribution that owned their home in 1990, separately for households that reside downtown (in blue) and in the suburbs (in dashed red). Unsurprisingly, home ownership rates are around 20 percent higher in the suburbs and increasing in income. At the lower end of the income distribution,

only about 30 percent of residents own their home. That number rises to about 70 percent for richer downtown residents. So, while low-income households are much more likely than high-income households to live downtown, where prices appreciated by more, they are also less likely to own. As a result, the nominal rental income growth that accrues to average household at the 10th percentile of the labor income distribution is only \$320 (annualized) compared with the \$530 (annualized) that accrues to the average household at the 90th percentile of the labor income distribution. While smaller in levels, this implied rental income growth amounts to a greater share of the low-income households' nominal income, 0.97 percent versus 0.43 percent.

Figure 9: Homeownership Rates and Returns

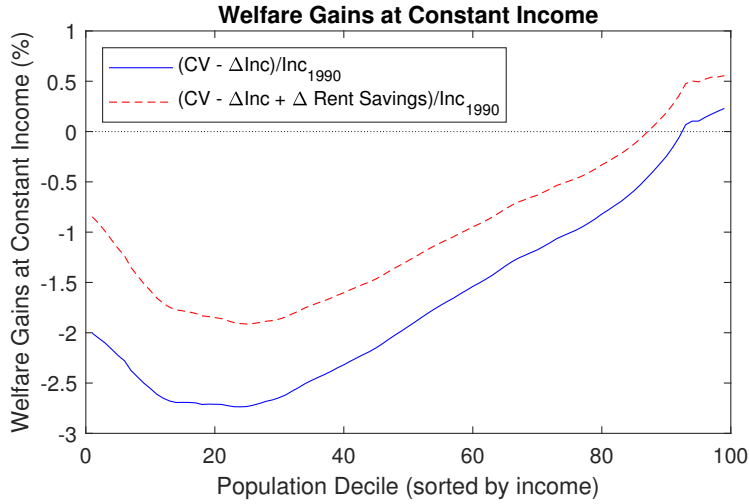


Note: The left-hand panel shows 1990 home ownership rates by 1990 income percentile in urban and suburban PUMAs, calculated from IPUMS data. The middle panel shows the increase in rents that households at each 1990 income percentile were saved by their homeownership status and location in 1990. That is, the predicted average increase in rents in each nj area between 1990 and 2013 weighted by the share of the income percentile that resided in that nj area and multiplied by the share of households in that income percentile-area n that owned in 1990. The right panel presents these implicit savings as a share of 1990 income.

In Figure 10, we add the expected average rental income growth as a fraction of resident income for each income percentile (right panel of Figure 9) to our baes welfare estimate (right panel of Figure ??). Adding in the returns to house price appreciation mitigates the loss in welfare to low income households. Specifically, residents at the 10th percentile of the income distribution are now only 1.6 percentage points worse off from the spatial sorting response as opposed to being 2.5 percentage points worse off in our base case. However, accounting for house price appreciation also makes richer households better off. Households at the 90th percentile of the income distribution are now 0.2 percentage points *better off* as opposed to being 0.25 percentage points worse off in our base case. As shown in Figure 10, the welfare gains at all income levels shifts up after accounting for the profits from house price appreciation. On net, accounting for the profits from rising housing prices results in a 1.8 percentage point understatement of 90-10 inequality measures when spatial

sorting responses are ignored.

Figure 10: Welfare Changes including Implicit Returns to Homeownership



6.4.4 Robustness

Which key elasticities are important in driving the magnitudes of our distributional welfare results? To explore this question, we examine the sensitivity of our welfare results to alternate parameter values. Table 4 summarizes the sensitivity of our two main results to variation in key parameter values. Specifically, we focus on the sensitivity of (1) the absolute change in welfare that results from spatial sorting in response to the change in the income distribution at the 10th and 90th percentiles of the income distribution and (2) the relative change in welfare between the 10th and 90th percentiles. This table highlights the key mechanisms that are driving our welfare estimates.

ρ is the parameter we feel we identify best using the underlying variation in spatial sorting across neighborhood types in response to exogenous CBSA-wide income shocks. Our base estimate is $\rho = 2.7$. Table 4 shows the sensitivity of our results when we set $\rho = 2$ and $\rho = 4$. At lower levels of ρ , neighborhoods are less substitutable with each other. As individuals get richer, they are more likely to move downtown when ρ is lower. Additionally, the poor are less likely to migrate out - despite the price increase associated with rich moving downtown - as ρ is lower. Therefore, lower values of ρ amplify our welfare results. But, it is interesting to note that even when $\rho = 4$, the increase in the 90-10 welfare gaps are large when spatial sorting responses are ignored.

As discussed above, the elasticities of substitution between neighborhoods for housing and non-tradable amenity consumption, γ and σ , respectively, are important parameters governing the endogenous response of amenities. As seen in Figure 8, even if we shut off all the love of variety ($\gamma = \infty$) and love of density ($\sigma = \infty$) effects, our key welfare results only fall by two-thirds. In Table 4, we show the sensitivity of our results to different $\gamma - \sigma$ pairs. As discussed above, our model and existing empirical work suggest that γ should be bound between ρ and σ . We chose a

Table 4: Robustness of Welfare Estimates to Key Parameters

	$(\Delta CV - \Delta Inc)/Inc_{1990}$			$(\Delta CV - \Delta Inc + \Delta Rent)/Inc_{1990}$		
	90th Pct	10th Pct	90/10 Gap	90th Pct	10th Pct	90/10 Gap
Base Specification	-0.25	-2.55	2.31	0.19	-1.58	1.77
Elasticity of Substitution between Neighborhood Types						
$\rho = 2.7$	-0.25	-2.55	2.31	0.19	-1.58	1.77
$\rho = 2$	-0.39	-3.73	3.34	0.10	-2.45	2.55
$\rho = 4$	-0.52	-0.94	0.42	-0.32	-0.58	0.26
Elasticity of Substitution between Neighborhoods (within type)						
$\gamma = \sigma = 5.6$	-0.25	-2.55	2.31	0.19	-1.58	1.77
$\gamma = 4, \sigma = 5.6$	0.71	-2.79	3.51	1.20	-1.58	2.78
$\gamma = \infty, \sigma = 5.6$	-1.02	-1.89	0.87	-0.63	-1.10	0.46
$\gamma = \sigma = \infty$	-0.71	-1.50	0.79	-0.35	-0.76	0.41
Distance Elasticity of Amenity Consumption						
$\delta = 0.22$	-0.25	-2.55	2.31	0.19	-1.58	1.77
$\delta = 0.1$	-0.22	-2.52	2.30	0.21	-1.55	1.76
$\delta = 0.4$	-0.30	-2.62	2.32	0.13	-1.64	1.78
Amenity Expenditure Share						
$\alpha = 0.15$	-0.25	-2.55	2.31	0.19	-1.58	1.77
$\alpha = 0.08$	0.27	-1.60	1.87	0.58	-0.90	1.48
$\alpha = 0.3$	-0.63	-3.41	2.78	-0.13	-2.27	2.14
Housing/Land Supply Elasticities						
$\epsilon_D = 0.6, \epsilon_S = 1.33$	-0.25	-2.55	2.31	0.19	-1.58	1.77
$\epsilon_D = 0, \epsilon_S = 1.33$	-0.33	-2.72	2.39	0.13	-1.70	1.82
$\epsilon_D = \epsilon_S = 1.33$	-1.00	-2.39	1.40	-0.60	-1.71	1.10
$\epsilon_D = \epsilon_S = 2$	0.32	-1.72	2.04	0.63	-1.02	1.65

conservative baseline by assuming $\gamma = \sigma$. If we chose a lower value of γ , there is even more love of neighborhood variety effects and even more endogenous amplification of amenities downtown. This draws even more high income people downtown imposing an even higher pecuniary externality on the poor. Lower values of γ only amplify our welfare results.

Note, conditional on $\gamma = \infty$ setting $\sigma = \infty$ has very little effect on our welfare results. Additionally, changing δ has essentially no effect on our welfare estimates. Part of the reason why these amenity demand parameters have such a small effect on our welfare calculations is because the share of household non-housing spending on amenities is relatively small ($\alpha = 0.15$ in our base case). The higher the value of α the larger our welfare results and the more that σ and δ matter for our welfare results. Even if we only restaurant expenditures in our amenity share ($\alpha = 0.08$) we still find substantial welfare effects through the spatial sorting channel resulting in the shifting income distribution.

Finally, the elasticity of housing supply downtown vs. the suburbs play an important role in the welfare effects. This is not surprising. Much of the welfare effect on the poor stems from them paying higher rents downtown as the rich move in. The more inelastic is the downtown housing supply, the more house prices move and the larger the welfare results.

Overall, this variation in our welfare estimates to different parameter values is useful for understanding the forces driving our results. However, over reasonable parameter ranges, our welfare results are fairly stable. Our main qualitative results are not reversed by any of these perturbations: the poor are worse off in both absolute terms and relative to the wealthy from the spatial sorting response to top income growth between 1990 and 2013.

6.5 Counterfactual 3: Redistributive Policy (preliminary)

We finally turn to analyzing the impact of policies that shape the spatial sorting of heterogeneous households within the city. Specifically, we use the model to study what would be the impact of systematically taxing the high-quality housing developments downtown, while subsidizing poorer households who choose to live downtown. This is a stylized “anti-gentrification” policy. It aims to limit the development of high-quality neighborhoods downtown, while helping poorer households to remain located in the city. Specifically, we implement the following policy. We assume that the local government imposes a tax t on housing prices downtown, for houses in high-quality neighborhoods. The tax levied is then redistributed as a place-quality-specific subsidy for households who choose to locate downtown, but in low-quality neighborhoods. That is, the price perceived by household for D,H houses is $p_{DH}(1 + t)$ while the one perceived for D,L houses is $p_{DL} - \delta$, where:

$$\delta = \frac{\int L(w)\lambda_D(w)\lambda_{DH}(w)tp_{DH}dF(w)}{\int L(w)\lambda_D(w)\lambda_DL(w)dF(w)}.$$

We recompute the spatial equilibrium of the city, with the distribution of household income that corresponds in shape to that in 2013 but holds population fixed at its 1990 level, implement-

ing the policy with a tax of 7.5%. Figure 11 shows that the policy has the effect of stemming the gentrification of downtown neighborhoods (the transfer in housing share from low to high quality neighborhoods downtown). Post-transfer, the policy also prevents land price increases in downtown neighborhoods, where effective housing costs are unchanged for low quality housing residents downtown who receive the redistribution of the tax revenues. Turning to the well-being effect of this policy, we find that, unsurprisingly, it stems some of the losses to the poorest households and reverses some of the (relative) gains for higher income households (figure 12). Note though that middle income households are also worse off under this policy. This is because the development of their preferred housing option (low quality suburban neighborhoods) is crowded out by an increase in high quality suburban housing development, the preferred tax-free alternative for high-income households to high quality neighborhoods downtown. That is, the policy generates a form of gentrification of the suburbs.

Contrasting the welfare results to the ones in Figure 6, however, we see that the welfare implications of such a large policy are insufficient to prevent the redistributive impacts of the neighborhood change associated with growing nominal income inequality. Though the illustrative policy prevents urban gentrification, the effects are transferred to the suburbs where they still hurt the poor vis-a-vis the rich: the policy reduces the growth in well-being for the 90th income percentile relative to the 10th income percentile by less than a third. So, even if such a policy could in principle contain all of the gentrification (and income composition changes) of downtowns, its re-distributive effects would be quantitatively limited.

Figure 11: Mechanisms under “Anti-Gentrification” Policy

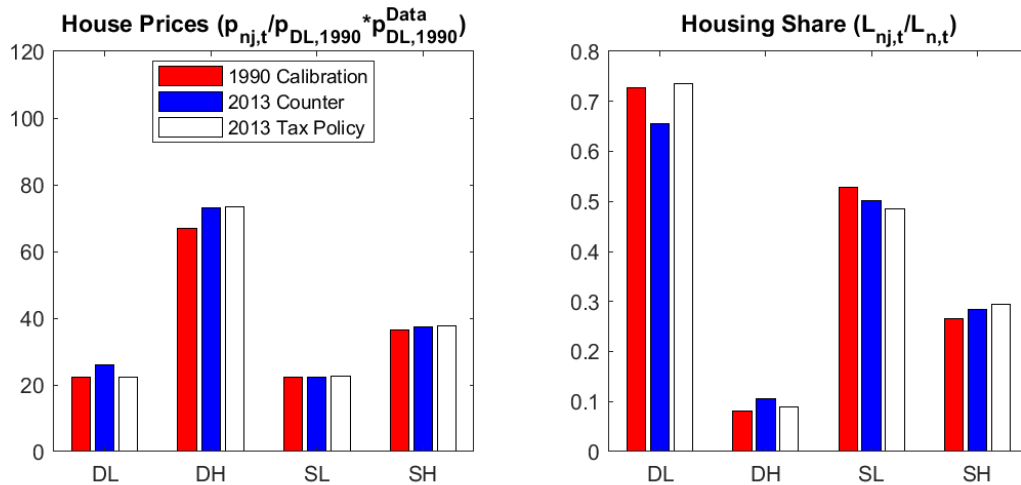
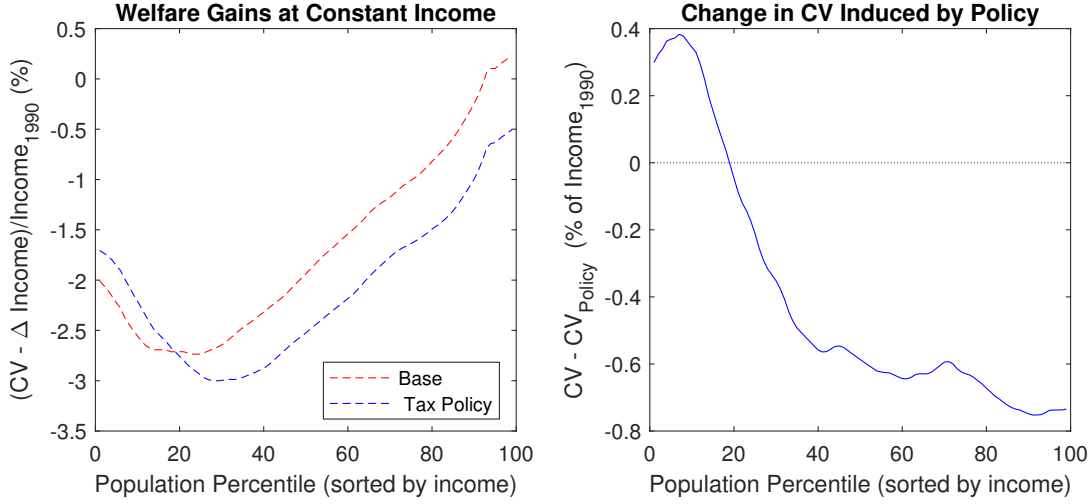


Figure 12: Well-Being Effects of “Anti-Gentrification” Policy



6.6 Counterfactual 4: Looking Backward, Looking Forward and Alternate Income Distributions

To be completed.

7 Conclusion

In order to explore the link between rising incomes at the top of the income distribution and urban gentrification, we write down a spatial model of a city with heterogeneous agents and non-homothetic preferences. We quantify the model using detailed location and income data, at the tract level, on the the top 100 CBSA’s in the US. We then use the quantified model to tease out how much of the change in spatial sorting over time can be plausibly traced back to changes in the income distribution, tilted towards higher incomes.

In the model, as the rich get richer, their increased demand for urban amenities drives up housing prices in downtown areas, where the development of these amenities is fueled by economies of density. The poor are either displaced or end up paying higher rents, making them worse off. Our preliminary estimates suggest that increases in the incomes of high income individuals was a substantive contributor to increased urban neighborhood change during the last 25 years within the U.S. and that the neighborhood change resulting from the increased incomes of the rich did, in fact, make poorer residents worse off. We explore possible policy responses to the rise in gentrification, and find that policies that contain gentrification seem to only lead to a very modest mitigation of the increase in well-being inequality, which could arguably be targeted more efficiently by direct redistribution.

References

- Ado, Rodrigo, Michal Kolesr, and Eduardo Morales**, “Shift-Share Designs: Theory and Inference,” Working Paper 24944, National Bureau of Economic Research August 2018.
- Aguiar, Mark and Mark Bils**, “Has consumption inequality mirrored income inequality?,” *American Economic Review*, 2015, *105* (9), 2725–56.
- Athey, Susan, David Blei, Robert Donnelly, Francisco Ruiz, and Tobias Schmidt**, “Estimating Heterogeneous Consumer Preferences for Restaurants and Travel Time Using Mobile Location Data,” *AEA Papers and Proceedings*, 2018, *108*, 64–67.
- Atkin, David, Benjamin Faber, and Marco Gonzalez-Navarro**, “Retail globalization and household welfare: Evidence from mexico,” *Journal of Political Economy*, 2018, *126* (1), 1–73.
- Autor, David H., Christopher J. Palmer, and Parag A. Pathak**, “Gentrification and the Amenity Value of Crime Reductions: Evidence from Rent Deregulation,” Working Paper 23914, National Bureau of Economic Research 2017.
- Baum-Snow, Nathaniel and Daniel Hartley**, “Accounting for Central Neighborhood Change, 1980-2010,” Working Paper 2016-09, Federal Reserve Bank of Chicago 2017.
- Berkes, Enrico and Ruben Gaetani**, “Income segregation and rise of the knowledge economy,” *Unpublished Manuscript*, 2018.
- Bertrand, Marianne and Adair Morse**, “Trickle-Down Consumption,” *The Review of Economics and Statistics*, 2016, *98* (5), 863–879.
- Borusyak, Kirill, Peter Hull, and Xavier Jaravel**, “Quasi-Experimental Shift-Share Research Designs,” Working Paper 24997, National Bureau of Economic Research September 2018.
- Brueckner, Jan K., Jacques-Francois Thisse, and Yves Zenou**, “Why is Central Paris Rich and Downtown Detroit Poor?: An Amenity-Based Theory,” *European Economic Review*, 1999, *43* (1), 91–107.
- Brummet, Quentin and Davin Reed**, “The Effects of Gentrification on Original Neighborhood Residents: Evidence from Longitudinal Census Microdata,” Technical Report 2018.
- Chen, M Keith and Ryne Rohla**, “The effect of partisanship and political advertising on close family ties,” *Science*, 2018, *360* (6392), 1020–1024.
- Couture, Victor**, “Valuing the Consumption Benefits of Urban Density,” *University of California, Berkeley. Processed*, 2016.

- and **Jessie Handbury**, “Urban Revival in America, 2000 to 2010,” Working Paper 24084, National Bureau of Economic Research 2017.
- Davis, Donald R, Jonathan I Dingel, Joan Monras, and Eduardo Morales**, “How Segregated is Urban Consumption?,” *Journal of Political Economy*, Forthcoming.
- Dekle, Robert, Jonathan Eaton, and Samuel Kortum**, “Unbalanced trade,” *American Economic Review*, 2007, *97* (2), 351–355.
- Diamond, Rebecca**, “The Determinants and Welfare Implications of US Workers’ Diverging Location Choices by Skill: 1980-2000,” *American Economic Review*, 2016, *106* (3), 479–524.
- Ding, Lei, Jackelyn Hwang, and Eileen Divringi**, “Gentrification and Residential Mobility in Philadelphia,” *Regional Science and Urban Economics*, 2016, *61*, 38–51.
- Duranton, Gilles and Diego Puga**, “Chapter 8 - Urban Land Use,” in J. Vernon Henderson and William C. Strange Gilles Duranton, ed., *Handbook of Regional and Urban Economics*, Vol. 5 of *Handbook of Regional and Urban Economics*, Elsevier, 2015, pp. 467–560.
- Edlund, Lena, Cecilia Machado, and Michaela Sviatchi**, “Bright Minds, Big Rent: Gentrification and the Rising Returns to Skill,” Working Paper 21729, National Bureau of Economic Research 2016.
- Ellen, Ingrid Gould and Katherine ORegan**, “Crime and Urban Flight Revisited: The Effect of the 1990s Drop in Crime on Cities,” *Journal of Urban Economics*, 2010, *68* (3), 247–259.
- , **Keren Mertens Horn, and Davin Reed**, “Has Falling Crime Invited Gentrification?,” Technical Report CES 17-27, Center for Economic Studies 2017.
- Fajgelbaum, Pablo, Gene M. Grossman, and Elhanan Helpman**, “Income Distribution, Product Quality, and International Trade,” *Journal of Political Economy*, 2011, *119* (4), 721–765.
- Gaigne, Carl, Hans Koster, Fabien Moizeau, and Jacques-Francois Thisse**, “Amenities and the Social Structure of Cities,” HSE Working paper WP BRP 162/EC/2017, National Research University Higher School of Economics 2017.
- Glaeser, Edward, Jed Kolko, and Albert Saiz**, “Consumer City,” *Journal of Economic Geography*, 2001, *1*.
- Glaeser, Edward L., Matthew E. Kahn, and Jordan Rappaport**, “Why Do the Poor Live in Cities? The Role of Public Transportation,” *Journal of Urban Economics*, 2008, *63* (1), 1–24.
- Goldsmith-Pinkham, Paul, Isaac Sorkin, and Henry Swift**, “Bartik Instruments: What, When, Why, and How,” Working Paper 24408, National Bureau of Economic Research March 2018.

- Guerrieri, Veronica, Daniel Hartley, and Erik Hurst**, “Endogenous Gentrification and Housing Price Dynamics,” *Journal of Public Economics*, 2013, 100, 45–60.
- Gyourko, Joseph, Christopher Mayer, and Todd Sinai**, “Superstar cities,” *American Economic Journal: Economic Policy*, 2013, 5 (4), 167–99.
- Handbury, Jessie, Ilya Rahkovsky, and Molly Schnell**, “Is the focus on food deserts fruitless? Retail access and food purchases across the socioeconomic spectrum,” Technical Report, National Bureau of Economic Research 2015.
- Holian, Matthew J. and Matthew E. Kahn**, “The Impact of Center City Economic and Cultural Vibrancy on Greenhouse Gas Emissions from Transportation,” MTI Report 11-13, Mineta Transportation Institute 2012.
- Lance Freeman**, “Displacement or Succession?: Residential Mobility in Gentrifying Neighborhoods,” *Urban Affairs Review*, 2005, 40 (4), 463–491.
- Lee, Sanghoon and Jeffrey Lin**, “Natural Amenities, Neighbourhood Dynamics, and Persistence in the Spatial Distribution of Income,” *The Review of Economic Studies*, 2018, 85 (1), 663–694.
- LeRoy, Stephen F. and Jon Sonstelie**, “Paradise Lost and Regained: Transportation Innovation, Income, and Residential Location,” *Journal of Urban Economics*, 1983, 13 (1), 67–89.
- Lester, T. William and Daniel A. Hartley**, “The Long Term Employment Impacts of Gentrification in the 1990s,” *Regional Science and Urban Economics*, 2014, 45 (1), 80–89.
- McKinnish, Terra, Randall Walsh, and T. Kirk White**, “Who Gentrifies Low-Income Neighborhoods?,” *Journal of Urban Economics*, 2010, 67 (2), 180–193.
- Meltzer, Rachel and Pooya Ghorbani**, “Does Gentrification Increase Employment Opportunities in Low-Income Neighborhoods?,” *Regional Science and Urban Economics*, 2017, 66 (C), 52–73.
- Murphy, Daniel**, “Home Production, Expenditure, and Economics Geography,” *University of Virginia Darden School of Business*, 2017.
- Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman**, “Distributional National Accounts: Methods and Estimates for the United States,” *The Quarterly Journal of Economics*, 2018, *Forthcoming*.
- Redding, Stephen J. and Esteban Rossi-Hansberg**, “Quantitative Spatial Economics,” *Annual Review of Economics*, 2017, 9 (1), 21–58.

- Rosenthal, Stuart S.**, “Old Homes, Externalities, and Poor Neighborhoods: A Model of Urban Decline and Renewal,” *Journal of Urban Economics*, 2008, 63 (3), 816–840.
- Saiz, Albert**, “The geographic determinants of housing supply,” *The Quarterly Journal of Economics*, 2010, 125 (3), 1253–1296.
- Small, Kenneth A, Erik T Verhoef, and Robin Lindsey**, *The economics of urban transportation*, Routledge, 2007.
- Smith, Matthew, Danny Yagen, Owen Zidar, and Eric Zwick**, “Capitalists in the Twenty-First Century,” Technical Report 2017.
- Su, Yichen**, “The Rising Value of Time and the Origin of Urban Gentrification,” *Stanford University*, 2017.
- Tsivanidis, Nick**, “The Aggregate and Distributional Effects of Urban Transit Infrastructure: Evidence from Bogotá TransMilenio,” Technical Report, Working Paper, University of Chicago Booth School of Business 2018.
- Vigdor, Jacob L., Douglas S. Massey, and Alice M. Rivlin**, “Does Gentrification Harm the Poor?,” *Brookings-Wharton Papers on Urban Affairs*, 2002, pp. 133–182.
- Wights, Sevrin**, “Gentrification and Displacement in English Cities,” Technical Report, European Regional Science Association 2014.

Appendix A Data Appendix

To be completed.

Appendix B Model Appendix

Appendix B.1 Computing Counterfactuals

We describe here how to compute a counterfactual equilibrium for a different income distribution $L'(w)$, conditional on (i) an initial calibration corresponding to the income distribution $L(w)$, and (ii) on the model elasticities $\{\rho, \gamma, \epsilon_n, \sigma, \alpha, \tau_n\}$. The information necessary to perform this step are the calibrated values at the initial equilibrium for $\{\lambda_{n,j}(w), L_{n,j}, p_{n,j}^h, S_{nj}^{mk}, s_{n,j}^i, s_n^\Pi\}$, where $L_{n,j}$ is the total population living in neighborhoods of type $\{n, j\}$ in the initial equilibrium, i.e.:

$$L_{n,j} = \int L(w) \lambda_{n,j}(w) dw,$$

and where we have defined three shares measured in the initial equilibrium. First, S_{nj}^{mk} is the share of amenity expenditures of households living in a neighborhood of type nj spent on amenities consumed in a neighborhood (m, k) . Second, s_n^Π is the share of city-wide land rents that correspond to location n . Third, within location n , $s_{n,j}^i$ is the share of equipped land used by activity $i \in a, h$ of quality j in location n .⁴⁶

We write a counterfactual equilibrium in changes relative to the initial equilibrium, denoting by $\hat{x} = \frac{x'}{x}$ the relative change of the variable x between the two equilibria. The counterfactual equilibrium is the solution to the following set of equations for $\{(p_{n,j}^h)'\}$, $\lambda'_{n,j}(w)$, $L'_{n,j}$ (or, equivalently, their “hat” values).

First, given (7), changes in housing costs are given by:

$$\hat{r}_n = \left(\sum_j s_{n,j}^h \hat{r}_n \hat{L}_{n,j} + s_{n,j}^a \hat{r}_n \hat{K}_{n,j}^a \right)^{\frac{1}{1+\epsilon_n}}, \quad (26)$$

where we have used the notation $s_{n,j}^i$ to denote the shares of land used by usage $i \in \{h, a\}$ and quality j within location n in the initial equilibrium, that is:

$$s_{n,j}^i = \frac{K_{nj}^i}{\sum_{j',i'} K_{nj'}^{i'}}.$$

Note that $\hat{L}_{n,j} = \frac{\int \lambda'_{n,j}(w) dL'(w)}{\int \lambda_{n,j}(w) dL(w)}$ while $\hat{r}_n \hat{K}_{n,j}^a = \frac{\int \lambda'_{n,j}(w) (w - (p_{n,j}^h)') dL'(w)}{\int \lambda_{n,j}(w) (w - p_{n,j}^h) dL(w)}$, where $\lambda'_{n,j}(w)$ is unknown and a solution of the system of equations described here, while the counterfactual distribution

⁴⁶We have $\sum_n s_n^\Pi = 1$, $\sum_{i,j} s_{n,j}^i = 1$ for $n = D$ or S and $\sum_{m,k} S_{nj}^{mk} = 1$, for $n = D, S$ and $j = H, M, L$.

of income $L'(w)$ is taken as given.

Second, the housing prices in the new equilibrium are defined by⁴⁷:

$$\left(p_{n,j}^h\right)' = \frac{\gamma}{\gamma+1} h_{n,j}^h r_n \hat{r}_n + \frac{1}{\gamma+1} \mathcal{I}'_{n,j} \left(\left(p_{n,j}^h\right)'\right), \quad (27)$$

where the function $\mathcal{I}'_{n,j}(p)$ is defined by:

$$\mathcal{I}'_{n,j}(p) = \frac{\int_w \Lambda'_{n,j}(p, w) [(1 - \tau_n)w + \chi(w)\Pi'] L'(w) dw}{\int_w \Lambda'_{n,j}(p, w) L'(w) dw}, \quad (28)$$

with $\Lambda'_{n,j}(p, w) = \frac{\lambda'_{n,j,r}(w)}{[(1 - \tau_n)w + \chi(w)\Pi - p]}$. Note here that τ_n and $\chi(w)$ are assumed constant between the two equilibria. The value of the real estate portfolio Π' , on the other hand, is endogenous and defined, given equation 5, by:

$$\hat{\Pi} = \sum_n \hat{r}_n^{1+\epsilon_n} s_n^\Pi, \quad (29)$$

where s_n^Π is the share of the city-wide real estate portfolio that correspond to location n :⁴⁸

$$s_n^\Pi = \frac{K_n r_n}{\sum_n K_n r_n}.$$

Third, the change in overall neighborhood quality $\tilde{q}_{n,j}$ is driven in particular by changes in number of neighborhoods of different types $\hat{N}_{n,j}$ and the change in density $\hat{K}_n$. Starting from (25), simple algebraic manipulations lead to:

$$\hat{\tilde{q}}_{n,j} = \hat{N}_{n,j}^{\frac{1}{\gamma}} \left(\hat{P}_{n,j}^a\right)^{-\alpha} \quad (30)$$

In this expression, the change in the number of neighborhoods is given by:

$$\hat{N}_{n,j} = s_{n,j}^{\pi,h} \hat{L}_{n,j} \frac{\left(p_{n,j}^h\right)' - h_{n,j}^h r_n'}{p_{n,j}^h - h_{n,j}^h r_n} + \left(1 - s_{n,j}^{\pi,h}\right) \frac{X'_{n,j} - \left(p_{n,j}^h\right)' L'_{n,j}}{X_{n,j} - p_{n,j}^h L_{n,j}},$$

where we define $X_{n,j}$ to be total income in n, j :

$$X_{n,j} = \int_w \lambda_{n,j}(w) w dL(w),$$

⁴⁷Note that $h_{n,j}^h r_n$ is known in the initial equilibrium using equation 18 and the known variables $p_{n,j}^h, \lambda_{n,j,r}(w), L(w)$

⁴⁸Note that the initial value of Π is given by: $\Pi = \sum_{n,j,i} r_n K_{n,j}^i$ with $r_n K_{n,j}^i$ for $i \in \{a, h\}$ respectively given by equations (15) and (16), which are both fully characterized given the initial calibration.

and we have defined the initial shares in profits made on the housing (vs amenities) market:

$$s_{n,j}^{\pi,h} = \frac{(p_{n,j}^h - h_{n,j}^h r_n) L_{n,j}}{(p_{n,j}^h - h_{n,j}^h r_n) L_{n,j} + \frac{\alpha}{\sigma} (X_{n,j} - p_{n,j}^h L_{n,j})}.$$

Furthermore, the change in the price index for amenities in a neighborhood of type n, j is found combining 2, 3 and 14:

$$\left(\hat{P}_{nj}^a\right)^{1-\sigma} = \sum_{j'n'} S_{nj}^{n'j'} \hat{N}_{n'j'} \left(\hat{r}_{n'} \hat{K}_{n'}^\delta\right)^{1-\sigma},$$

where S_{nj}^{mk} is the share of expenditure on amenities spent on neighborhood of type m, k for households living in neighborhood of type n, j :

$$S_{nj}^{mk} = \frac{N_{m,k} \beta_{jk} \left(p_{mk'}^a (d_{nm})^{\tilde{\delta}}\right)^{1-\sigma}}{\sum_{n',j'} N_{n',j'} \beta_{jj'} \left(p_{n'j'}^a (d_{nn'})^{\tilde{\delta}}\right)^{1-\sigma}}.$$

Finally, the counterfactual location choice of workers can be simply expressed as a function of initial location choices $\lambda_{n,j}$, changes in neighborhood quality and prices defined above, and changes in income, which we take as an exogenous input to the counterfactual. Specifically, changes in location choices are given by:

$$\hat{\lambda}_{n,j}(w) = \frac{\widehat{q_{n,j}^\rho}}{\widehat{V}^\rho(w)} \frac{[w(1 - \tau_n) + \chi(w)\Pi' - p_r']^\rho}{[w(1 - \tau_n) + \chi(w)\Pi - p_r]^\rho}, \quad (31)$$

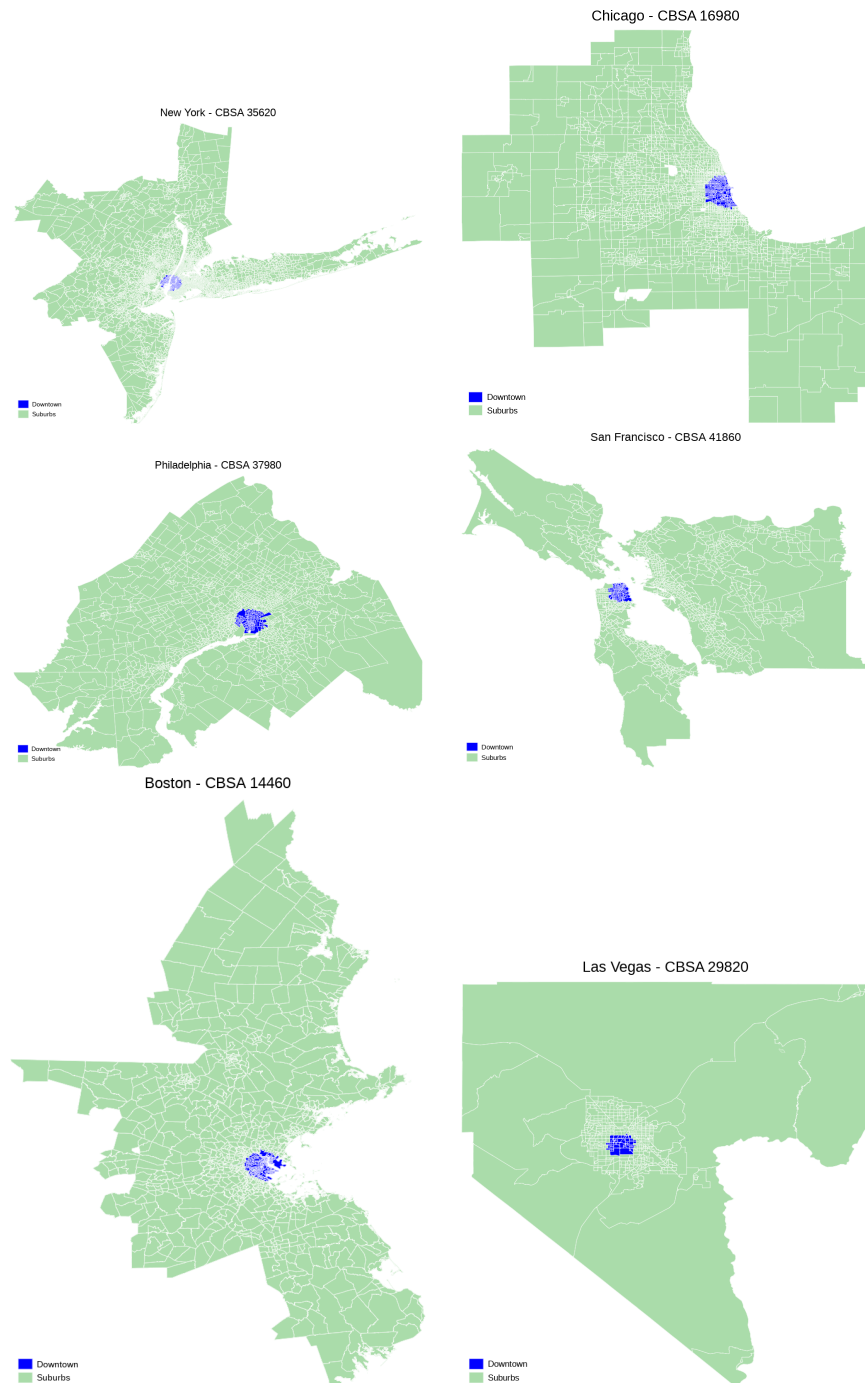
In parallel, we get the change in welfare given by:

$$\widehat{V}^\rho(w) = \sum_{n,j} \widehat{q_{n,j}^\rho} \frac{[(1 - \tau_n)w + \chi(w)\Pi' - p_r']^\rho}{[(1 - \tau_n)w + \chi(w)\Pi - p_r]^\rho} \lambda_{n,j}(w), \quad (32)$$

Values for $\{p'_{n,j}, \lambda'_{n,j}(w), L'_{n,j}, R'_{n,j}, \Pi'\}$ are the solutions of equations (26)-(31) that define a counterfactual equilibrium of the economy corresponding to an alternative distribution of income $L'(w)$ in the city.

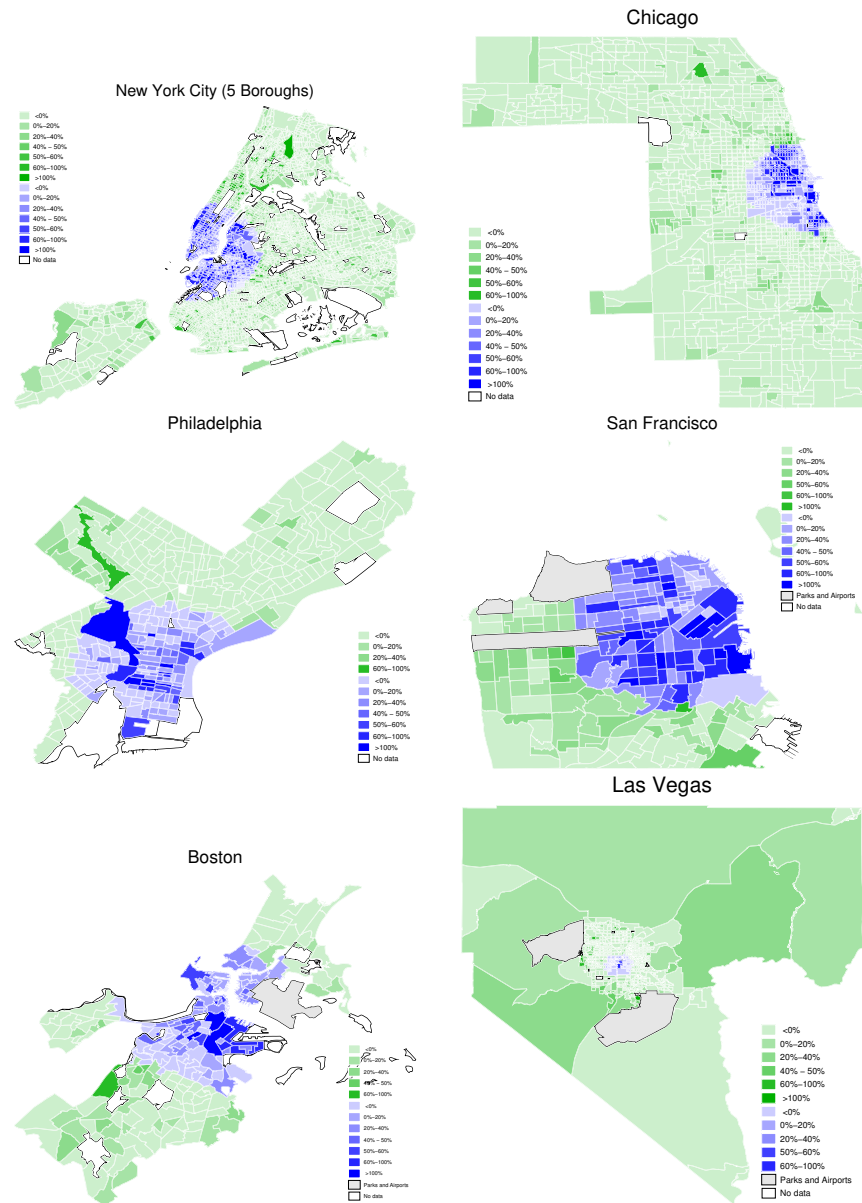
Appendix C Additional Figures and Tables

Figure 13: Downtown and Suburban Tracts in Selected CBSAs.



Note: Downtown tracts in dark blue consists of all tracts closest to the city center and accounting for 10% of total CBSA population in 2000.

Figure 14: Gentrifying Tracts in Central County of Selected CBSA



Note: Each map shows the central county of a given CBSA, except for New York which shows the five counties (boroughs) of New York City. Downtown tracts in blue consists of all tracts closest to the city center and accounting for 10% of total CBSA population in 2000. The shading of each tract shows the percent growth in tract household income between 1990 and 2013.