The Job Ladder: Inflation vs. Reallocation*

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Abstract

We introduce on-the-job search frictions in an otherwise standard monetary DSGE model. Heterogeneity in productivity across jobs generates a job ladder. Wages are set by Bertrand competition, as in the Sequential Auctions protocol of Postel-Vinay and Robin (2002). We study the effects of aggregate shocks to TFP and money supply. The ability of firms to commit to wage contracts, until outside offers arrive and trigger renegotiation, insulates wages from unemployment altogether. Outside job offers to employed workers, when accepted, reallocate employment up the productivity ladder, and are socially beneficial; when matched by the employer, thus declined, they cause sudden increases in production costs and, due to nominal price rigidities, decreases in mark-ups, building inflationary pressure. When employment is concentrated at the bottom of the job ladder, typically after recessions, the reallocation effect prevails; as employment climbs the job ladder, the inflation effect takes over. The job-to-job transition rate is a better predictor of wage pressure than unemployment, as we showed empirically in previous work. Because this transition rate is low, the economy takes time to absorb cyclical misallocation, hence features strong propagation in the response of job creation, unemployment and wage inflation to aggregate shocks.

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1 Introduction

The Phillips curve, a negative relationship between the rates of unemployment and inflation, is still a guiding principle of monetary policy. Microfoundations of this relationship build on price-setting frictions, due either to explicit costs of price adjustment or to incomplete information about the nature of demand shocks faced by producers. In this body of work, the labor market is typically modeled as competitive, and features no unemployment; the relevant measure of slack is an output gap. Nominal, thus real, wage rigidity can generate classical unemployment associated with such a gap (Erceg, Henderson and Levine (2000), Gertler and Trigari (2009)). But the canonical model of unemployment builds on search frictions, supported by a vast arsenal of empirical evidence on labor market flows. In the so-called DMP framework, wages are set by Nash Bargaining, with the value of unemployment representing the worker's outside option. When the economy is expanding and firms post many vacancies, the unemployed have an easy time finding a new job, hence unemployment declines, while employed workers have a strong threat and bargaining power, and real wages rise. This view seems to capture well the original idea behind the Phillips curve: low unemployment signals scarcity of labor, hence pressure on its price. Combining with product nominal price rigidities, Christiano et al. (2016) show that the modeling the labor market in this DMP tradition significantly improves the empirical fit of an otherwise standard monetary DSGE model.

In this paper, we advocate shifting emphasis away from unemployment to the "job ladder" as the relevant indicator of slack to predict inflation. In Moscarini and Postel-Vinay (2017), using microdata from the SIPP to control for composition effects, we provide empirical evidence that neither the unemployment rate nor the job-finding rate from unemployment have any significant comovement over time with nominal wage inflation. In contrast, the rate at which workers move from job to job (or employer to employer, EE) has a significant positive relationship with nominal wage inflation. This is not surprising, in light of an alternative view of labor markets characterized by search frictions, one where wages are not subject to bargaining but are offered unilaterally by firms, and workers bargaining power derives from their ability to receive outside offers. Such offers can either be accepted, moving the worker up a job ladder, or matched and declined, pushing wages closer to marginal product and representing, for the employer, a cost-push shock. The latter outcome is more likely when workers have been moving up the ladder for a while, after a sufficiently long aggregate expansion, and so are difficult to poach away. In this case cost pressure builds and, with a lag due to nominal price rigidities, eventually manifests itself as price inflation.

Our claim is that competition for employed, not unemployed, workers transmits aggregate shocks to wages, and that the distinction is important because the intensity of these two forces have different cyclical patterns. As Shimer (2012) showed, cyclical movements in the unemployment rate are driven to a large extent by those in the job-finding rate from unemployment, which in turn reflect closely the vacancy/unemployment rate, thus job creation. The latter is a very volatile variable, that unemployment tracks closely because job finding rates in the US are high, negating much propagation. In contrast, the EE transition rate is low, the reallocation of employment up the job ladder is a very slow process, and the propagation of aggregate shocks through the poaching/outside offers channel is strong. Because firms cannot perfectly target their pool of job applicants, they create more jobs and post more vacancies when either there are many unemployed or when the employed are poorly matched and easy to poach (or both). Thus, independently of the state of unemployment, the distribution of employment on the job ladder, a very slow-moving state variable, determines job creation and thus, ultimately, also the pace of job finding from unemployment. Wages do not respond, despite falling unemployment, for quite some time, until few workers are left at the bottom of the ladder, and competition for employed workers takes off. The relevant measure of slack or tightness is not fully summarized by the vacancy/unemployment ratio, but must also take misallocation into account. In terms of observables, monetary authorities should pay attention to the EE transition rate, which predicts wage inflation, particularly with a lag.

To formalize and quantitatively investigate this hypothesis, we introduce search in the labor market, both on- and off-the job, and endogenous entry/job creation into an otherwise standard monetary DSGE model with complete financial markets, a representative risk-averse household, Calvo pricing. Wages are set by Postel-Vinay and Robin (2002)'s Sequential Auctions protocol: firms make unilateral offers that can be renegotiated only by mutual consent, when outside offers arrive. We are interested in business cycles and monetary shocks, hence we must move from the steady state analysis that is common in search models to allow for aggregate uncertainty. Accordingly, we allow firms to offer and commit to contracts that are state-contingent wages, and to Bertrand-compete in such contracts for already employed workers. To the best of our knowledge, we are the first to introduce on the job search and ex post competition in a business cycle, g.e. macro model with risk-averse agents and nominal price rigidities. We review the literature in the paper.

Section 2 describes the model, Section 3 its equilibrium, Section 4 preliminary quantitative results, the Appendix equilibrium computation.

2 The Economy

Agents, goods, endowments and technology. Time $t = 0, 1, 2...$ is discrete. There are three vertically integrated sectors in the economy, each producing a different kind of non-storable output. From upstream down: Service, Intermediate inputs and Final good.

Firms in the Service sector produce with linear technology using only labor. Each unit of labor ("job match") produces x units of the Service, which is then sold on a competitive market at price q_t. Productivity x is specific to each match and is drawn, once and for all, when the match forms, in a iid manner from a cdf Γ.

This Service is used to produce differentiated Intermediate inputs, indexed by $i \in [0, 1]$. Each input is produced by a single firm, also indexed by i, with a linear technology that turns each unit of the homogeneous Service into z_t units of variety i, then sold at price $p_t(i)$

in a monopolistic competitive market.[1]

Firms in the Final good sector buy quantities $c_t(i)$ of the Intermediate inputs and use them in a CES technology, with elasticity of substitution $\eta > 1$, to produce a homogeneous Final good

$$Q_t = \left(\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} \, di \right)^{\frac{\eta}{\eta-1}} \tag{1}$$

which trades at price P_t.

A representative household is a collection of agents $j \in [0,1]$. Each household member has an indivisible unit endowment of time per period, and the household is collectively endowed with ownership shares of all firms in all three sectors. We indicate whether household member j is employed at time t by the indicator $e_t(j) \in \{0,1\}$.

Preferences. The household has concave preferences over consumption C_t of the Final good and linear preferences for leisure

$$U(C_t) + b \int_0^1 (1 - e_t(j)) \, dj$$

where $U' > 0 > U''$, $b \geq 0$. The household maximizes the present value of expected utility discounted with factor $\beta \in (0,1)$.

Search frictions in the labor market. Service sector producers can advertise vacancies by using κ units of the Final good per vacancy, per period. Previously unemployed workers search for these vacancies. Previously employed workers are separated from their jobs with probability $\delta \in (0,1]$ and become unemployed, in which case they have to wait until next period to search; if not, they also receive this period, with probability $s \in (0,1]$, an

[1] For ease of notation, we assume that Intermediate inputs are varieties of the same good, so they are measured in the same units, and we can add up quantities of different varieties to obtain a total demand for the input. If these inputs were instead different goods altogether, measured in different units (lbs, hours, gallons etc.), technology would contain a i-dependent unit conversion $\chi(i)$ factor multiplying z_t, with scale (#units of i)/(#units of Service), and production would be $\chi(i)z_t$ units of Intermediate good i per unit of Service. In this case, we could add up together the demand for different inputs only after dividing them by by $\chi(i)$, thus expressing them in units of the Service.

opportunity to search for a vacant job (a new match). Let

$$\theta = \frac{v}{u + s(1-\delta)(1-u)}$$

be effective job market tightness, the ratio between vacancies and total search effort by (previously) unemployed and (remaining) employed. A homothetic meeting function gives rise to a probability $\phi(\theta) \in [0,1]$ for a searching worker of locating an open vacancy, increasing in θ, and a probability $\frac{\phi(\theta)}{\theta}$ for an open vacancy of meeting a worker, decreasing in θ. Therefore, the Service produced in this sector can be thought of as a bundle of efficiency units of labor, assembled by Service sector firms in a frictional labor market, and leased to Intermediate good producers in a competitive market at unit price q_t. Service sector firms are essentially labor market intermediaries, solving the hiring problems of Intermediate good producers, and making zero profits on average, due to free entry in vacancy creation.

Price determination. Service and Final good trade in competitive markets. Intermediate good producers $i \in [0,1]$ are monopolistic competitors. Each firm i draws every period with probability $\nu \in (0,1)$ in an iid fashion an opportunity to revise its price $p_t(i)$. Given the price, either revised or not, the firm immediately serves the resulting demand $c_t(i)$ for Intermediate good i by buying the required quantity $c_t(i)/z_t$ of (labor) services at unit price q_t.

Finally, we describe wage setting. A Service-producing firm can commit to a state-contingent wage (a "contract") and renegotiate it only by mutual consent. The firm's commitment is limited, in that it can always unilaterally separate, so firms' profits cannot be negative. Same for the worker: the utility value from staying in the contract cannot fall below the value of unemployment, else the worker will quit. When an employed worker contacts an open vacancy, the prospective poacher and the incumbent employer observe each other's match qualities with the worker, and engage in Bertrand competition over contracts. The worker chooses the contract that delivers the larger value.

Financial assets. There is a numéraire money. Households have access to a nominal bond, which costs $(1 + R_t)^{-1} \leq 1$ units of money to buy, and delivers one unit of money for sure one period later. The monetary authority controls the nominal interest rate R_t according to some (typically Taylor) rule. The bond is in zero net supply.

Households trade, in competitive financial markets, ownership shares of all firms: Final good (F), Intermediate goods (I) and Service (S) producers. To eliminate idiosyncratic risk associated to infrequent pricing of Intermediate goods and to search frictions in the labor market, the household combines these shares in mutual funds that own a representative cross-section of all firms. The competitive prices of these mutual fund share are (resp.) p_t^F, p_t^I and p_t^S.

Timing of events within a period.

1. Nature draws the innovation to TFP z_t in the Final good sector, and simultaneously the monetary authority chooses the nominal interest rate R_t;

2. firms and households produce and exchange Final good, Intermediate goods and Service, and employers in the Service sector pay their workers wages according to the current contracts that they are committed to;

3. households trade nominal bonds and shares of all firms with each other, and nominal bonds with the monetary authority;

4. previously unemployed workers receive utility from leisure b;

5. existing matches break up, both exogenously with probability δ and endogenously;

6. firms post vacancies;

7. previously unemployed and (still) employed workers search for those vacancies;

8. upon meeting, a vacancy and a worker draw a permanent match quality x, and then the firm posting the vacancy makes the worker an offer; if the worker is already employed, his current employer makes a counteroffer;

9. if the worker accepts the new offer, he becomes employed in the new match, otherwise he remains in his current state, either unemployed or employed in a previous match.

3 Equilibrium

3.1 Household optimization

Statement of the problem. The household chooses stochastic processes for Final good consumption C_t, holdings of bonds B_t and ownership shares of firms in all three sectors $(\xi_t^F, \xi_t^I, \xi_t^S)$, given their prices, resp. $P_t, R_t, p_t^F, p_t^I, p_t^S$. The household does not freely choose its member j's labor supply $e_t(j)$, because of search frictions: rather, the household chooses the probability $a_t(j)$ that member j accept any new job offer he might have received at the end of period t. Thus, the household solves:

$$\max_{\left\{C_t, B_t, \xi_t^F, \xi_t^I, \xi_t^S, a_t(j)\right\}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t \left[U\left(C_t\right) + b \int_0^1 \left(1 - e_t(j)\right) dj \right]$$

subject to:

- the budget constraint (in nominal terms)

$$\begin{aligned} &P_t C_t + \frac{B_{t+1}}{1+R_t} + \xi_{t+1}^F p_t^F + \xi_{t+1}^I p_t^I + \xi_{t+1}^S p_S^I \\ &\leq \xi_t^F \left(\Pi_t^F + p_t^F\right) + \xi_t^I \left(\int_0^1 \Pi_t^I\left(i\right) di + p_t^I\right) + \xi_t^S \left(\Pi_t^S + p_t^S\right) + \int_0^1 e_t\left(j\right) w_t\left(j\right) dj + B_t \end{aligned}$$

  where Π_t^F are the profits earned by each Final good producer, $\Pi_t^I\left(i\right)$ by the only firm producing Intermediate input i, and Π_t^S by each Service producer (after paying the vacancy costs $P_t \kappa$ ex ante), and $\int_0^1 e_t\left(j\right) w_t\left(j\right) dj$ are the household's nominal earnings, the sum of wages $w_t\left(j\right)$ paid to employed workers $j \in [0,1]$ within the household by Service producers; because of search frictions, different workers will receive different wages;

- the law of motion of individual employment:

$$e_{t+1}(j) = e_t(j)(1-\delta) + (1 - e_t(j))\,\phi(\theta_t)\,a_t(j) \tag{2}$$

- the stochastic process for equilibrium wages $w_t(j)$, to be determined;

- the No Ponzi Game condition

$$\Pr\left(\lim_{t\to\infty} B_t \prod_{s=0}^{t-1}(1+R_s)^{-1} = 0\right) = 1.$$

Household decisions. We solve the household's maximization problem in steps: consumption and asset allocations first, then labor market turnover decisions. Denoting the (current-value) Lagrange multiplier on the date-t budget constraint by λ_t, the FOC for the optimal demand of the consumption good yields $U'(C_t) = \lambda_t P_t$. Combined with the FOC for the optimal demand of bonds B_{t+1} , this yields the standard Euler equation

$$(1+R_t)\,\beta\mathbb{E}_t\left[\frac{U'(C_{t+1})}{U'(C_t)}\frac{P_t}{P_{t+1}}\right] = 1 \tag{3}$$

which discounts the real interest rate $(1+R_t)\,E_t[P_t/P_{t+1}]$ at the pricing kernel $\beta U'(C_{t+1})/U'(C_t)$.

The FOC for the optimal number of new shares ξ^k_{t+1} of the mutual fund that invests in producers in sector $k = F, I, S$ is

$$\lambda_t p^k_t = \beta\mathbb{E}_t\left[\lambda_{t+1}\left(\Pi^k_{t+1} + p^k_{t+1}\right)\right] \Leftrightarrow p^k_t = \beta\mathbb{E}_t\left[\frac{U'(C_{t+1})}{U'(C_t)}\frac{P_t}{P_{t+1}}\left(\Pi^k_{t+1} + p^k_{t+1}\right)\right]$$

where $\Pi^I_{t+s} = \int_0^1 \Pi^I_{t+s}(i)\,di$ are the aggregate profits from all Intermediate good producers. Substituting forward and ruling out bubbles implies:

$$p^k_t = \sum_{s=1}^{+\infty}\mathbb{E}_t\left[D^{t+s}_t \Pi^k_{t+s}\right]$$

where

$$D^{t+s}_t = \beta^s \frac{U'(C_{t+s})}{U'(C_t)}\frac{P_t}{P_{t+s}}$$

is the nominal stochastic discount factor between dates t and $t+s$. Firms maximize the value to their owners, or consumption value of the share price of each mutual fund, which is the present value of real profits, discounted by pricing kernel

$$\frac{p_t^k}{P_t} = \sum_{s=1}^{+\infty} \mathbb{E}_t \left[\beta^s \frac{U'(C_{t+s})}{U'(C_t)} \frac{\Pi_{t+s}^k}{P_{t+s}} \right]$$

We now turn to labor market turnover decisions $a_t(j)$. The only objects in the household's maximization problem that depend on those decisions are the value of leisure $b \int_0^1 (1 - e_t(j))\, dj$, household labor income $\int_0^1 e_t(j)\, w_t(j)\, dj$ through the laws of motion of $e_t(j)$, namely (2), and of $w_t(j)$. Thus, when deciding upon $a_t(j)$, the household solves the sub-problem:

$$\begin{aligned} &\max_{\{a_t(j)\}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t \left[b \int_0^1 (1 - e_t(j))\, dj + \lambda_t \int_0^1 e_t(j)\, w_t(j)\, dj \right] \\ &= \max_{\{a_t(j)\}} \int_0^1 \left\langle \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t \left[b (1 - e_t(j)) + \lambda_t e_t(j)\, w_t(j) \right] \right\rangle dj \end{aligned} \tag{4}$$

subject to (2) and the stochastic process for equilibrium wages $w_t(j)$.

The next key remark is that job acceptance decisions are taken independently by different household members, because they do not affect each other's employment prospects: the household is one of many, and does not internalize congestion externalities in the search labor market, not even the externalities that its own members create on each other. The only interaction between household members is through income pooling, which explains the λ_t weight on income, independent of each member's identity j and employment status $e_t(j)$. The household's labor turnover problem (4) separates into two types, one for each currently unemployed member ($e_t(j) = 0$), which can be written in recursive form as

$$\begin{aligned} \lambda_t V_{ut}^j &= b + \max_{\{a_s(j)\}} \mathbb{E}_t \sum_{s=t+1}^{+\infty} \beta^{s-t} \left[b (1 - e_s(j)) + \lambda_s w_s(j)\, e_s(j) \right] \\ &= b + \beta \max_{\{a_t(j)\}} \mathbb{E}_t \left[\phi(\theta_t)\, a_t(j)\, \lambda_{t+1} V_{e,t+1}^j + (1 - \phi(\theta_t)\, a_t(j))\, \lambda_{t+1} V_{u,t+1}^j \mid e_t(j) = 0 \right] \end{aligned}$$

and one for each currently employed member ($e_t(j) = 1$) paid $w_t(j)$ in a match of quality $y_t(j)$,

$$\begin{aligned} \lambda_t V_{et}^j (w_t(j), y_t(j)) &= \lambda_t w_t(j) + \max_{\{a_s(j)\}} \mathbb{E}_t \sum_{s=t+1}^{+\infty} \beta^{t-s} \left[b (1 - e_s(j)) + \lambda_s w_s(j)\, e_s(j) \mid w_t(j), y_t(j) \right] \\ &= \lambda_t w_t(j) + \beta \max_{\{a_s(j)\}} \mathbb{E}_t \left[\delta \lambda_{t+1} V_{u,t+1}^j + (1 - \delta)\, \lambda_{t+1} V_{e,t+1}^j (w_{t+1}(j), y_{t+1}(j)) \mid e_t(j) = 1, w_t(j), y_t(j) \right] \end{aligned}$$

In this notation, V_{ut}^{j} and V_{et}^{j} represent the household's *nominal* value of having its jth member (resp.) unemployed or employed at date t, with $\lambda_t V_{ut}^{j}$ and $\lambda_t V_{et}^{j}$ representing the corresponding utility values. We can now write the recursive representations of those two problems in a form that is common in equilibrium models with on-the-job search:

$$V_{ut}^{j} = \frac{b}{\lambda_t} + \mathbb{E}_t \left\langle D_t^{t+1} \left[\phi\left(\theta_t\right) a_t\left(j\right) V_{e,t+1}^{j} + \left(1 - \phi\left(\theta_t\right) a_t\left(j\right)\right) V_{u,t+1}^{j} \right] \right\rangle$$

and:

$$\begin{aligned} V_{et}^{j}\left(w_t\left(j\right), y_t\left(j\right)\right) &= w_t\left(j\right) \\ &+ \mathbb{E}_t \left\langle D_t^{t+1} \left[\delta V_{u,t+1}^{j} + \left(1-\delta\right) V_{e,t+1}^{j}\left(w_{t+1}\left(j\right), y_{t+1}\left(j\right)\right) \mid e_t\left(j\right) = 1, w_t\left(j\right), y_t\left(j\right) \right] \right\rangle \end{aligned}$$

3.2 Final good producers' optimization

Each perfectly competitive Final good producer takes as given the price P_t of its output Q_t and the prices $p_t(i)$ of its inputs, and chooses the quantity of each input $c_t(i)$ to maximize static profits $P_t Q_t - \int_0^1 p_t\left(i\right) c_t\left(i\right) di$ period by period. Using the production function (1) to express the aggregate supply Q_t of the Final good in terms of inputs:

$$\Pi_t^F = \max_{c_t(i), i \in [0,1]} P_t \left(\int_0^1 c_t\left(i\right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} - \int_0^1 p_t\left(i\right) c_t\left(i\right) di.$$

The FOC for the optimal demand $c_t\left(i\right)$ of input i is:

$$P_t \left(\int_0^1 c_t\left(i\right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}-1} c_t\left(i\right)^{-\frac{1}{\eta}} = p_t\left(i\right). \tag{5}$$

We now manipulate this equation in three ways as is standard. First, multiplying by $c_t\left(i\right)$ and integrating over i both sides

$$P_t \left(\int_0^1 c_t\left(i\right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} = \int_0^1 p_t\left(i\right) c_t\left(i\right) di$$

so that Final good producers make zero profits. Second, raising both sides of (5) to the power $1-\eta$ and integrating them over i yields the equilibrium relationship between Intermediate input prices and Final output price:

$$P_t = \left(\int_0^1 p_t\left(i\right)^{1-\eta} di \right)^{\frac{1}{1-\eta}} \tag{6}$$

Finally, solving (5) for $c_t(i)$ and using (1) yields the Final good producer's isoelastic demand for input i as a function of its output Q_t and of the relative input/output price

$$c_t(i) = Q_t \left(\frac{p_t(i)}{P_t} \right)^{-\eta} \tag{7}$$

3.3 Intermediate good producers' optimization

Firm i producing Intermediate good i chooses its price $p_t(i)$ to serve the resulting input demand $c_t(i)$ from the isoelastic demand function, and maximizes profits, given the linear technology that turns $y_t(i)$ units of the homogeneous Service, purchased at given unit price q_t, into $c_t(i) = z_t y_t(i)$ units of Intermediate input i.[2] Each Intermediate good producer is allowed to revise its price with probability ν each period. When this revision opportunity arises, firm i solves:

$$\Pi_t^I(i) = \max_{p_{t+s}(i)} \mathbb{E}_t \sum_{s=0}^{+\infty} (1-\nu)^s D_t^{t+s} Q_{t+s} \left(\frac{p_{t+s}(i)}{P_{t+s}} \right)^{-\eta} \left(p_{t+s}(i) - \frac{q_{t+s}}{z_{t+s}} \right).$$

The optimal reset price at date t is then independent of i:

$$p_t^* = \frac{\eta}{\eta - 1} \frac{\mathbb{E}_t \sum_{s=0}^{+\infty} (1-\nu)^s D_t^{t+s} Q_{t+s} P_{t+s}^{\eta} \frac{q_{t+s}}{z_{t+s}}}{\mathbb{E}_t \sum_{s=0}^{+\infty} (1-\nu)^s D_t^{t+s} Q_{t+s} P_{t+s}^{\eta}} \tag{8}$$

Because the selection of firms that get to reset their price is random, using (6) the Final good price P_t then solves:

$$P_t^{1-\eta} = \nu \left(p_t^* \right)^{1-\eta} + (1-\nu) P_{t-1}^{1-\eta} \tag{9}$$

This price adjustment technology causes dispersion in the prices of Intermediate inputs. Specifically, in each period t prices are geometrically distributed across inputs, with a fraction $\nu(1-\nu)^s$ of the inputs being priced at p_{t-s}^*, for $s \in \mathbb{N}$. Total demand for the Service is then:

$$\frac{1}{z_t} \int_0^1 c_t(i) di = \frac{1}{z_t} Q_t \nu \sum_{s=0}^{+\infty} (1-\nu)^s \left(\frac{p_{t-s}^*}{P_t} \right)^{-\eta}.$$

[2] Note that shocks to TFP z_t could similarly affect the production of the Service, i.e. all quantities of y could scale up and down.

3.4 Service producer's optimization and labor market equilibrium

Match values. In the Service sector, firms hire workers in a frictional labor market to assemble a (labor) Service that they sell in a competitive market to downstream, Intermediate good producers. Service sector firms can commit to pay their workers streams of wages, and can only renegotiate the deal following an outside offer if the worker agrees, so only if the worker's value of employment rises. We now drop the individual-member superscript j from labor market values and investigate said values further. Because employers extract the full match rent from unemployed workers, the value of unemployment simplifies to:

$$V_{ut} = \frac{b}{\lambda_t} + \mathbb{E}_t \left[D_t^{t+1} V_{u,t+1} \right] = \frac{b}{\lambda_t (1 - \beta)} \tag{10}$$

Assuming that b is small enough, no matches will break up endogenously, so all separations will be exogenous, with probability δ.

Next, let $\overline{V}_t(y)$ denote the date-t expected (dollar) PDV to a worker of accepting the offer of a firm where he can produce a flow y of Service, and which commits to pay the worker the value of all Service output $q_t y$ at all times until separation. Since the firm has a zero outside option by free entry, this employment "contract" generates the maximum value that the firm can credibly promise to deliver to the worker. When a worker who is currently promised continuation value $V_{t+1}(y)$ in a match of quality y, where $V_{ut+1} \leq V_{t+1}(y) \leq \overline{V}_{t+1}(y)$ by limited commitment of either party, meets an open vacancy and draws a new match quality y', Bertrand competition yields one of three possible outcomes: (i) $V_{t+1}(y) \geq \overline{V}_{t+1}(y')$, in which case the incumbent employer needs to do nothing to retain the worker, and the offer is irrelevant; (ii) $V_{t+1}(y) < \overline{V}_{t+1}(y') \leq \overline{V}_{t+1}(y)$, in which case the incumbent employer retains the worker by renegotiating the offer, for a raise to $\overline{V}_{t+1}(y')$; (iii) and finally $V_{t+1}(y) \leq \overline{V}_{t+1}(y) < \overline{V}_{t+1}(y')$, in which case the worker is poached with an offer worth $\overline{V}_{t+1}(y)$. In any case, the worker moves if and only if $\overline{V}_{t+1}(y) < \overline{V}_{t+1}(y')$, and turnover decisions depend uniquely on the full-rent extraction values $\overline{V}_{t+1}$ of the matches. This value solves

$$\overline{V}_t(y) = q_t y + \mathbb{E}_t \left\langle D_t^{t+1} \left[\delta V_{u,t+1} + (1 - \delta) \overline{V}_{t+1}(y) \right] \right\rangle$$

because, if the worker remains employed, he receives $\overline{V}_{t+1}(y)$ either from the incumbent employer (as part of the current contract) or from a poacher. Subtracting (10) from both sides of this equation:

$$\overline{V}_t(y) - V_{ut} = q_t y - \frac{b}{\lambda_t} + \mathbb{E}_t\left[(1-\delta)D_t^{t+1}\left(V_{t+1}(y) - V_{u,t+1}\right)\right].$$

The fact that households value a marginal dollar of profit as much as a dollar of labor income (namely, λ_t) ensures that value is perfectly transferrable between individual workers and firms. Therefore, a worker's value $\overline{V}_t(y)$ of extracting full rents from a type-y job is also the value of said job to the firm-worker pair under any sharing rule. We can therefore define a type-y job *surplus* as $S_t(y) = \overline{V}_t(y) - V_{ut}$. Solving forward:

$$S_t(y) = \mathbb{E}_t\left[\sum_{s=0}^{+\infty}(1-\delta)^s D_t^{t+s}\left(q_{t+s}y - \frac{b}{\lambda_{t+s}}\right)\right].$$

Because the value of unemployment is the same for all workers, turnover decisions depend on a comparison of such surplus function. Crucially, $S_t(y)$ is affine increasing in y. Therefore, the firm with the higher y wins the auction, and we draw the main conclusion of this subsection: the equilibrium is Rank Preserving (RPE), and the direction of reallocation is efficient, always from less to more productive matches.

Evolution of worker stocks. Due to the RP property of equilibrium, the law of motion of the measure of workers in type-y matches is

$$\ell_{t+1}(y) = (1-\delta)\left\{\left[1 - s\phi(\theta_t)\overline{\Gamma}(y)\right]\ell_t(y) + s\phi(\theta_t)\gamma(y)\int_{\underline{y}}^{y}\ell_t(y')\,dy'\right\} + \phi(\theta_t)\gamma(y)u_t \tag{11}$$

Integration w.r.t. y yields the law of motion of unemployment:

$$u_{t+1} = \left[1 - \phi(\theta_t)\right]u_t + \delta(1-u_t) \tag{12}$$

Labor demand. By the time a firm and a worker who have met on the search market must decide whether to consummate the match or not, they know the quality of the potential match, y', which yields an expected surplus $\mathbb{E}_t\left[D_t^{t+1}S_{t+1}(y')\right]$. The surplus in the worker's previous situation is known, too: it is zero if the worker was unemployed, and $\mathbb{E}_t\left[D_t^{t+1}S_{t+1}\left(y\right)\right]$ if the worker was employed in a type-y match. From the analysis above, we note that the expected surplus is affine in match quality y:

$$\mathbb{E}_t\left[D_t^{t+1}S_{t+1}(y)\right] = \mathbb{E}_t\left[\sum_{s=1}^{+\infty}(1-\delta)^{s-1}D_t^{t+s}\left(q_{t+s}y - \frac{b}{\lambda_{t+s}}\right)\right]$$

Using the definition of the SDF D_t^{t+s}, this rewrites as:

$$\mathbb{E}_t\left[D_t^{t+1}S_{t+1}(y)\right] = \frac{\beta P_t}{U'(C_t)}\left[\mathcal{Z}_t y - \frac{b}{1-\beta\left(1-\delta\right)}\right]$$

where we define the expected PDV of a unit flow of Service, in units of the Final good:

$$\mathcal{Z}_t = \mathbb{E}_t\left[U'(C_{t+1})\frac{q_{t+1}}{P_{t+1}} + \beta(1-\delta)\mathcal{Z}_{t+1}\right] \tag{13}$$

The free-entry condition then writes as:

$$\frac{\kappa U'(C_t)}{\beta}\frac{\theta_t}{\phi(\theta_t)} = \frac{\left[\mathcal{Z}_t\mathbb{E}_\Gamma(y) - \frac{b}{1-\beta(1-\delta)}\right]u_t + \mathcal{Z}_t(1-\delta)s\int_{\underline{y}}^{\overline{y}}\gamma(y)\int_{\underline{y}}^{y}\ell_t\left(y'\right)\left(y-y'\right)dy'dy}{u_t + \left(1-\delta\right)s\left(1-u_t\right)} \tag{14}$$

On the LHS are flow vacancy costs, on the RHS the expected profits earned by Service sector producers, namely the expected PDV of Π_t^S, all in units of the Final good.

3.5 Market-clearing

Financial markets. The representative household holds all shares of all firms, $\xi_t^F = \xi_t^I = \xi_t^S = 1$, and $B_t = 0$ as bonds are in zero net supply. Households neither borrow nor save, but spend all income on the Final good.

Final good market. The supply Q_t by Final good producers equals the demand of Final good by households for final consumption and by Service producers for vacancy advertising:

$$Q_t = C_t + \kappa\theta_t\left[u_t + s\left(1-\delta\right)\left(1-u_t\right)\right] \tag{15}$$

Intermediate good markets. They clear by construction, because Intermediate good producers set prices and serve all demand. Given prices $p_t(i)$ quoted by Intermediate good producers, including those that just reset their price, and the resulting equilibrium price P_t of the Final good, as well as the isoelastic demand for input i in (7), we can write the aggregate supply and equilibrium quantity of Intermediate inputs as a function of Final good production and prices only

$$\int_0^1 c_t(i)\, di = Q_t P_t^{\eta} \int_0^1 p_t(i)^{-\eta}\, di$$

Let

$$\hat{P}_t = \left(\int_0^1 p_t(i)^{-\eta} di \right)^{-\frac{1}{\eta}} \tag{16}$$

so

$$\int_0^1 c_t(i)\, di = Q_t \left(\frac{P_t}{\hat{P}_t} \right)^{\eta} \tag{17}$$

Service market. The supply by Service producers equals the demand by Intermediate good producers, which is the supply of Intermediate goods divided by TFP:

$$\int_{\underline{y}}^{\overline{y}} y \ell_t(y) dy = \frac{1}{z_t} \int_0^1 c_t(i)\, di \tag{18}$$

It is easy to verify that, by Walras law, (15), (17), (18) and zero profits in competitive markets imply that the budget constraint of the representative household is satisfied.

Combining the last two equations eliminates the equilibrium production of Intermediate goods and links directly aggregate Service output from all active job matches to Final good output

$$z_t \int_{\underline{y}}^{\overline{y}} y \ell_t(y) dy = Q_t \left(\frac{P_t}{\hat{P}_t} \right)^{\eta} \tag{19}$$

3.6 Equilibrium

The economy enters period t with a set of variables that are pre-determined. These imply the employment distribution $\ell_t(\cdot)$, hence unemployment $u_t = \int_{\underline{y}}^{\overline{y}} \ell_t(y) dy$, the distribution of

Intermediate good prices $p_{t-1}(\cdot)$ and, at the beginning of the period, the new realizations of TFP z_t and monetary policy instrument R_t. The first two are endogenous, infinitely-dimensional state variables. TFP has an exogenous law of motion. Monetary policy is assumed to follow a rule that makes R_t a function of the other three.

A key observation is that the price distribution $p_{t-1}(\cdot)$ enters equilibrium conditions only through the two price indexes $P_{t-1}, \hat{P}_{t-1}$, which have known laws of motion: P_t follows (9) and, by the same reasoning, $\hat{P}_t$ follows

$$\hat{P}_t^{-\eta} = \nu \left(p_t^* \right)^{-\eta} + (1-\nu)\hat{P}_{t-1}^{-\eta} \tag{20}$$

where note that the reset price p_t^* that updates these two price indexes only depends on the processes $C_{t+s}, Q_{t+s}, P_{t+s}, q_{t+s}$ through (8).

Definition 1 *A* ***Recursive Rational Expectations Equilibrium*** *is a collection of measurable functions* $\{C, Q, \theta, q\}$ *of the state vector* $\langle P_{-1}, \hat{P}_{-1}, \ell(\cdot), z\rangle$*, and a monetary policy rule* R*, a given function of the same state vector, that solve the consumption Euler equation* (3)*, the optimal reset price equation* (8)*, the free entry condition* (14)*, market-clearing in the Final good market* (15)*, and the combined market-clearing condition in the Intermediate good and Service markets* (19)*, and which induce a Markov process for each endogenous component of the state vector:* (9) *for* P*,* (20) *for* $\hat{P}$*,* (11) *for* $\ell(\cdot)$*. The exogenous component* z *evolves according a predetermined Markov process.*

3.7 Discussion

The wage mark-up and the labor wedge. From the free entry condition (14), vacancy creation θ_t depends on the weighted average of two expected returns, from hires from unemployment and employment, with weights given by the shares of these two groups in the job searching pool. The expected returns from an unemployed hire are

$$\frac{\beta}{U'(C_t)}\left(\mathcal{Z}_t \mathbb{E}_\Gamma(y) - \frac{b}{1-\beta(1-\delta)}\right) = \mathbb{E}_t\left[\sum_{s=1}^{+\infty}(1-\delta)^{s-1}\beta^s\frac{U'(C_{t+s})}{U'(C_t)}\left(MPL_{t+s} - MRS_{t+s}\right)\right]$$

the expected PDV of the difference between the Marginal Product of Labor in units of the Final good

$$MPL_{t+s} = \frac{q_{t+s}\mathbb{E}_\Gamma(y)}{P_{t+s}}$$

and the Marginal Rate of Substitution between consumption of the Final good and leisure:

$$MRS_{t+s} = \frac{b}{U'(C_{t+s})}$$

Indeed, the term labeled MPL_t is the flow Service output of an extra unit of work (averaged across possible match outcomes) $\mathbb{E}_\Gamma(y)$, converted into consumption goods by the relative price $\frac{q_t}{P_t}$, thus a measure of the Marginal Product of Labor. The term labeled MRS_t is the ratio between the additional utility b from one less unit of work and the marginal utility of consumption of the Final good, namely the MRS between consumption and leisure.

The Business Cycle accounting literature (Chari, Kehoe and McGrattan, Ecta 2007) defines the "labor wedge" as the ratio between the MRS and the MPL. They find that, measured in the data through the lens of a neoclassical growth model with balanced growth preferences, this labor wedge is countercyclical and plays a key role in amplifying business cycle fluctuations. In our model, the expected returns to hiring an unemployed worker in (3.7) equal the expected present value of the MPL times one minus the labor wedge. A countercyclical labor wedge makes the returns to hiring unemployed workers procyclical. The MRS, however, contributes a procyclical component to the labor wedge: in recessions, when consumption is low, workers value income more, so they are willing to work for less. So the countercyclical movement in the labor wedge required to account for business cycle must originate from a strongly procyclical MPL, or relative price q_{t+s}/P_{t+s}.

An alternative interpretation of the "Service" in our model is a composite quantity of labor, with Service producers acting as labor market intermediaries, or temp-agencies, that hire workers in a frictional labor market and sell their services to good producers in a competitive market. Therefore, q_{t+s}/P_{t+s} is the average cost of labor to good producers, and the firm discounts the difference between this real wage index and the MRS between consumption and leisure. Estimated New-Keynesian models (Smets and Wouters AER 2007) define the "wage markup" as the ratio between the real wage and the MRS, and find that changes in this mark-up are key to explain inflation and output dynamics. Lacking a mechanism to generate endogenous changes in the wage mark-up, they attribute them to shocks, that they estimate to be procyclical. Erceg, Henderson and Levine (JME 2000) generate wage mark-ups by assuming sticky nominal wages. Gali (2011 JEEA) calls for a theory of an endogenous wage mark-up. Our model delivers just that. The expected returns to hiring an unemployed worker in (3.7) equal the expected present value of the MRS times the wage mark-up minus one. Thus, in our model the labor wedge is the reciprocal of the wage mark up. If both input (labor) and output (Service) markets were competitive, both the labor wedge and the wage mark-up would be identically equal to one, with workers on their labor supply curve and firms on their labor demand curve. If the labor market was competitive but the output (Service) market was monopolistically competitive, with Service providers charging a constant mark-up over the marginal cost of labor, the labor wedge would be less than one and the wage mark-up larger than one, but both would be constant. With our frictional labor market, the labor wedge is smaller than one and the wage mark-up is larger than one, to compensate for hiring costs, and, crucially, both are endogenous and time-varying.

A novel propagation mechanism. Our model contains an additional, novel transmission mechanism of aggregate shocks to job creation, which is not present in either strand of the literature. Service providers, when posting vacancies, also mind the expected return from an *employed* hire, the second term in the numerator of (14). This is independent of the MRS, and depends entirely on the distribution of employment ℓ_{t+s}, which is a slow-moving aggregate state variable. This term introduces an additional, time-varying component to labor demand, with a complex cyclical pattern. At a cyclical peak, workers have had time and opportunities to climb the ladder, so poaching is difficult/expensive and the returns to hiring employed workers are weak. After a recession, as the unemployed regain employment, they restart from random rungs on the match quality ladder, which are worse than the employment distribution at the cyclical peak. Hence, early in a recovery many recent hires are easily "poachable". The transition of cheap unemployed job applicants into low-quality jobs makes these workers more expensive, but still profitable, to hire. As time goes by, and unemployment declines, employment reallocation up the ladder through job-to-job quits picks up, employed workers become more and more expensive to hire, ultimately putting pressure on wages, until we are back to a cyclical peak. This second component of the returns to job creation delivers a procyclical wage mark-up, or countercylical labor wedge, but only as long as employment is still misallocated and "poachable".

In the US economy, the transition rate from job to job is fairly small, similar to the separation rate into unemployment, both an order of magnitude smaller than the transition rate from unemployment to employment. Therefore, movements in the employment distribution up the job ladder are slow. An important implication is that, in our model, job market-tightness, thus the unemployment rate, have sluggish transitional dynamics. This stands in contrast to the canonical model with only job search from unemployment, where tightness is a jump variable, with no transitional dynamics, and the unemployment rate converges very quickly to its new steady state. This is important, because the slow decline in the unemployment rate that we witnessed from 2010 to 2016 can only be explained in the canonical model by a long (and implausible) sequence of small, consecutive, positive aggregate shocks. A slowly mean-reverting process for the aggregate driver of business cycles will not do, because the free entry condition is forward-looking and would incorporate the expected recovery. In contrast, our model has a built-in, slowly moving endogenous propagation mechanism of temporary aggregate shocks.

DSGE models with search frictions (Andolfatto (1996), Mertz (1996), Gertler and Trigari (2009), Christiano et al (2016)) typically focus on unemployment and abstract from on the job search. Within the linear-utility labor market search tradition, Robin (2011 Ecta) adopts the Sequential Auction model of a labor market with on the job search, but stresses permanent worker heterogeneity. Firms are identical, thus the job ladder has only two steps.

Only unemployed hires generate profits for firms. An employed job searcher extracts all rents from both incumbent and prospective employer. Therefore, our additional term does not appear in returns to job creation. This stochastic job ladder mechanism appears in Moscarini and Postel-Vinay (2013), which relies on wage-posting contracts without renegotiation, but cannot easily accommodate nominal price stickiness, and in Lise and Robin (AER 2017), who allow for ex ante worker and firm heterogeneity and sorting. The latter, although still cast in a linear utility framework, is the closest comparison. Our simpler model of a job ladder is meant to flesh out the propagation mechanism of aggregate shocks that poaching introduces, and to be embedded in a full-fledged general equilibrium model to be able to study monetary policy.

3.8 Special cases

Having defined an equilibrium and connected our model to the business cycle literature, we now characterize its properties. Our model features four important "frictions": risk aversion in consumer preferences for the Final good, monopoly power and nominal price rigidity in the Intermediate input market, and search frictions in the labor market. To gain understanding about the response of the economy to aggregate shock, we first shut down one or more of these frictions. In the next section, we will compute numerically a calibrated version of the full model.

3.8.1 Frictionless labor market

Barring search frictions in the labor market, every worker is employed at the highest possible match quality, the labor market is competitive, the Service price q_t is the nominal wage, and the model reduces to a standard New Keynesian model.

3.8.2 Homogeneous Intermediate inputs

Barring monopoly power in the Intermediate input market, namely assuming that $\eta = \infty$ and inputs are perfect substitutes and sold in a perfectly competitive market, implies that only firms that are tapped to reset their price today will be active and selling inputs, at a common equilibrium price $p_t(i) = P_t$ equal to the marginal cost q_t/z_t of producing those inputs. This marginal cost is independent of the quantity of Intermediate input produced and thus of the demand for services necessary for that production, because the Service market is perfectly competitive. To see why only firms that get to reset their price today will be able to sell, note that firms that had set their price before at a higher (lower) level than the current q_t/z_t and cannot revise it now will be temporarily driven out of the market because their old price is too high and will be undercut by the new price vintage (resp., the old price is too low and will not cover costs at the new level of marginal cost q_t/z_t). Therefore, prices are, for all practical purpose, flexible, and for this case we can refer to the following analysis.

3.8.3 Flexible prices

Barring nominal rigidities, namely assuming $\nu = 1$, we obtain the most interesting benchmark, the flexible price economy. In this case, monopoly power has no impact on the business cycle properties of the model, because mark-ups are constant. Therefore, any conclusions that we reach in this particular will extend, with minor modifications, to the economy with both flexible prices and perfectly competitive input producers.

Intermediate good producers that face no pricing frictions all choose the same optimal static mark-up price

$$p_t = P_t = \frac{\eta}{\eta - 1} \frac{q_t}{z_t} \tag{21}$$

and supply the same quantity, that we denote by Q_t^n, which is then also the equilibrium quantity of Final good produced. The subscript "n" stands for "natural", thus Q_t^n is the natural rate of output, the quantity of final goods produced in the absence of nominal price rigidities. In the canonical New Keynesian model, this level of economic activity is not first-best because of the monopoly distortion, but this distortion can be and is usually undone with an appropriate tax and subsidy scheme. In this model, there is a second, unavoidable distortion due to search frictions, and no presumption that the natural rate is constrained efficient. Nonetheless, we maintain the nomenclature for ease of comparison.

The demand for Service is Q_t^n/z_t, and market-clearing in the Intermediate and Service good markets pin down the natural rate of Final good output, given the supply of Service:

$$Q_t^n = z_t \int_{\underline{y}}^{\overline{y}} y\ell_t^n(y)dy.$$

Using this expression eliminates Q_t^n from market-clearing in the Final good market (Eq. (15)) and determines the natural rate of consumption given the state of the economy, now summarized only by TFP z_t and the employment distribution $\ell_t^n(\cdot)$, and given the current natural rate of market tightness θ^n which determines the absorption of final goods for hiring purposes:

$$C_t^n = z_t \int_{\underline{y}}^{\overline{y}} y\ell_t^n(y)dy - \kappa\theta_t^n \left[u_t^n - s\left(1-\delta\right)\left(1-u_t^n\right)\right] \tag{22}$$

where of course the natural rate of unemployment is $u_t^n = 1 - \int_{\underline{y}}^{\overline{y}} \ell_t^n(y)dy$. Finally, we study free entry in vacancy creation. Mark-up pricing (21) implies that

$$\mathcal{Z}_t^n = \mathbb{E}_t \left[z_{t+1} U'\left(C_{t+1}^n\right) \frac{\eta - 1}{\eta} + \beta\left(1-\delta\right) \mathcal{Z}_{t+1}^n\right] \tag{23}$$

is the expected PDV of the utility from producing an extra unit of Service each period, until match dissolution, essentially a risk-adjusted discounted marginal product of labor, which depends on the covariance between returns to production and consumption. Note that the monopoly distortion in the Intermediate goods market manifests itself in a lower $\mathcal{Z}_t^n$ on the RHS, i.e. a lower marginal value of services. This has a direct, negative impact on job creation through the free entry condition (14).

Flex price equilibrium is a collection of stochastic processes for Final good consumption C_t^n, job market tightness θ_t^n, risk-adjusted value of services $\mathcal{Z}_t^n$ and employment distribution $\ell_t^n(\cdot)$ (with implied natural rate of unemployment u_t^n) which solve (22), (23), the free entry condition (14) and the law of motion of employment (11).

Money is neutral and nominal variables are determined by monetary policy. From the Euler equation, given the equilibrium process for C_t^n and a nominal interest rate policy rule that generates a process for R_t, the inflation rate $\pi_{t+1} = P_{t+1}/P_t - 1$ is a process that satisfies

$$\mathbb{E}_t \left[\frac{U'\left(C_{t+1}^n\right)}{U'\left(C_t^n\right)} \frac{1+R_t}{1+\pi_{t+1}} \right] = \frac{1}{\beta}.$$

The stochastic flexible price equilibrium also includes two important special cases. Removing consumer risk aversion, the model boils down to the standard case analyzed in search model of the labor market, linear utility and competitive output market, a business cycle version of the Sequential Auction model of Postel-Vinay and Robin (2002). Maintaining all frictions but assuming no aggregate uncertainty, we obtain the steady state equilibrium allocation of the full model, as we will explain shortly.

3.8.4 Risk-neutral consumers

When the marginal utility of consumption of the Final good is constant, and normalized to one for convenience, from Equation (23) the present value of additional Service $\mathcal{Z}_t^n$ reduces to the expected present discounted value of future TFP, scaled by the constant mark-up, an exogenous process which is known before solving for equilibrium. Given the current state of TFP z_t, thus of $\mathcal{Z}_t^n$, and the current employment distribution $\ell_t^n(\cdot)$, Equations (22) and (14) still apply, and uniquely determine C_t^n, θ_t^n, which allows to update $\ell_{t+1}^n(\cdot)$ using (11). So an equilibrium realization is constructed exactly and recursively moving forward.

3.8.5 Steady state

Another special case, and an important benchmark for stochastic equilibrium computation, is the steady state equilibrium. Absent aggregate shocks to TFP z_t and nominal interest rate R_t, price rigidity is irrelevant, because prices never need to change. Therefore, the steady state of the full, frictional economy closely resembles the stochastic equilibrium of the flex price benchmark. Specifically, Equation (23) yields a simple expression for the present value of additional Service

$$\mathcal{Z} = \frac{zU'(C)}{1-\beta(1-\delta)} \frac{\eta - 1}{\eta}$$

while Equations (22) and (14) still apply, without time subscript and n superscript. Let $L(y) = \int_{\underline{y}}^{y} \ell(y')\,dy'$. The stationary employment distribution $\ell(y) = L'(y)$ solves the following ordinary linear differential equation:

$$L'(y) = (1-\delta)\left[1 - s\phi(\theta)\overline{\Gamma}(y)\right] L'(y) + s\phi(\theta)\gamma(y)L(y) + \phi(\theta)\gamma(y)u$$

The solution can be found in closed form:

$$L(y) = \frac{\phi(\theta)\Gamma(y)u}{\delta + (1-\delta)s\phi(\theta)\overline{\Gamma}(y)}$$

so

$$u = \frac{\delta}{\delta+\phi(\theta)} \quad \text{and} \quad \ell(y) = \frac{\phi(\theta)\left[\delta + (1-\delta)s\phi(\theta)\right]\gamma(y)u}{\left[\delta + (1-\delta)s\phi(\theta)\overline{\Gamma}(y)\right]^2} \tag{24}$$

4 Equilibrium computation

In any version of this model with labor market frictions, independently of price rigidities, monopoly power, and risk aversion, the distribution of employment $\ell_t(\cdot)$ is a state variable, which makes equilibrium computation difficult in the presence of aggregate shocks. We propose an approximation algorithm based on parameterized expectations, described in Appendix A. In this section, we apply that algorithm and compare the properties and predictions of different versions of the model.

4.1 Specifications and parameter values

We calibrate the model at a monthly frequency. We specify the TFP process as a 51-point discrete Markov chain approximation to the following AR(1) process:

$$\ln z_t = (1-\rho_z)\mu_z + \rho_z \ln z_{t-1} + \varepsilon_t^z, \qquad \varepsilon_t^z \overset{\text{i.i.d.}}{\sim} \mathcal{N}\left(0, \sigma_z^2\right)$$

The distribution Γ of match quality is a discrete approximation to a Pareto distribution with parameter α_y over an evenly space 50-point grid with bounds $\left[\underline{y}, \overline{y}\right]$. The utility function is specified as CRRA: $U(C) = \sigma C^{1-1/\sigma}/(\sigma-1)$. The matching function is specified as

TFP process				
ρ_z	σ_z	μ_z		
0.96	0.0028	−5E−5		
Monetary policy rule/pricing frictions				
ρ_R	σ_R	ψ_π	ψ_Q	ν
0.88	0.0016	1.25	0.05	0.1111
Match quality				
α_y	$\underline{y}$	$\overline{y}$		
1.5	1	50		
Preferences				
σ	η	β	b	
0.5	6	0.9957	0.033	
Matching/hiring/job destruction				
ϕ_0	ξ	s	δ	κ
0.5036	0.8	0.12	0.02	80

Table 1: Parameter values

Cobb-Douglas, so that $\phi(\theta) = \phi_0\theta^\xi$. Finally, the monetary policy rule is specified as:

$$\ln(1+R_t) = \rho_R \ln(1+R_{t-1}) + (1-\rho_R)\left[\psi_\pi \ln(1+\pi_{t-1}) + \psi_Q \ln\left(\frac{Q_t}{Q_{t-1}}\right) - \ln\beta\right] + \varepsilon_t^R \quad (25)$$

where $\varepsilon_t^R \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_R^2)$. We ignore for now the Zero Lower Bound.

All parameter values are gathered in Table 1.

4.2 Results: Impulse Response Functions (in progress)

4.2.1 TFP shock

Figure 1 plots the responses of average labor productivity ALP $= z\int y\ell(y)dy/(1-u)$, mean match quality $\int y\ell(y)dy/(1-u)$, the job finding rate JFR=$\phi(\theta) = \phi_0\theta^\xi$, the EE transition rate $(1-\delta)s\phi(\theta)\int_{\underline{y}}^{\overline{y}}\gamma(y)\int_{\underline{y}}^{y}\ell_t(y')\,dy'dy$, unemployment u and inflation to a one-time shock to TFP z, starting from a situation where TFP is constant at its deterministic mean μ_z, when monetary policy follows the Taylor rule. Figure 2 shows the same information under an interest rate peg. In both cases, the size of the negative TFP shock is one standard deviation of the ergodic distribution of TFP. Time (on the horizontal axis) is measured in months. Each impulse response is plotted for three different versions of the model: the full sticky-price model (labeled NK on the graphs), the flexprice model (FL), and the flexprice

	NK (Taylor)	**NK** (Peg)	**FL**	**RN**
Half-life of log TFP (months)	17	17	17	17
Half-life of log ALP (months)	46	42	40	23
Half-life of log JFR (months)	48	42	40	34
Half-life of log unemployment (months)	49	43	40	36
Autocorrelation of log TFP	0.960	0.960	0.960	0.960
Autocorrelation of log ALP	0.979	0.979	0.982	0.973
$\frac{\text{StD}(\ln\theta)}{\text{StD}(\ln \text{ALP})}$	1.675	1.654	1.525	0.792

Table 2: Comparison between models

model with linear utility (RN). The top rows of Table 2 further report the half-lives relating to a selection of the impulse responses plotted on Figures 1 and 2.[3]

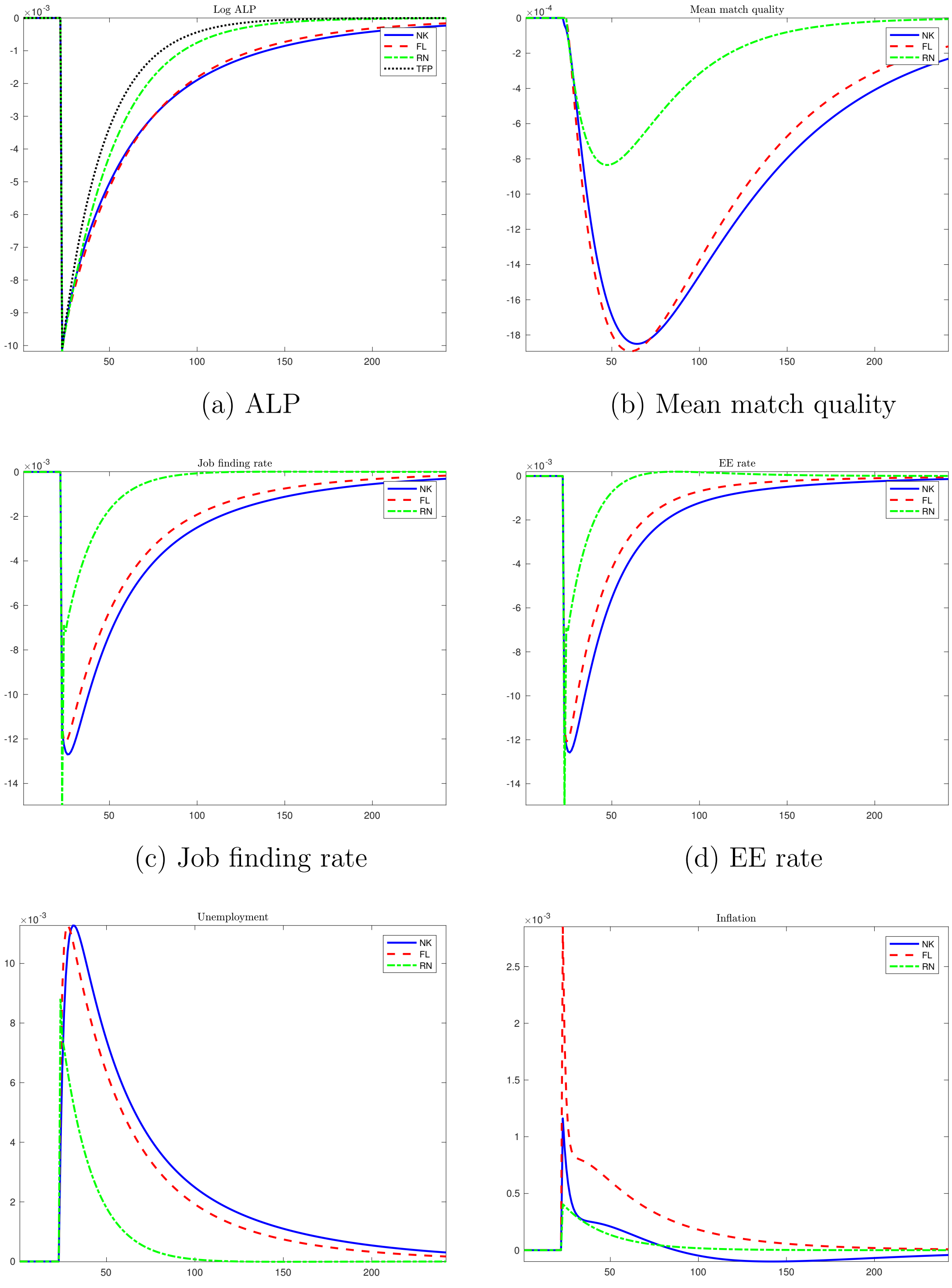
The three versions of our model can clearly be ranked in terms of propagation, with the NK model offering the most propagation, and the RN model the least. The RN case is interesting in that it isolates the impact of the job ladder on propagation. In that case, ALP has a half-life which is about 33% longer than TFP. This is due to the slow movement of employed workers up and down the job ladder, which causes the mean match quality component of ALP to adjust only very gradually (see Fig. 1b).

4.2.2 Monetary shock

4.3 Results: Full time-series simulation

Figure 3 shows various simulated monthly time series over a period of twenty years (240 months), produced from the full sticky-price (NK) version of our model under application of the Taylor rule. In addition, Figure 3e plots the inflation rate against the unemployment

[3]Because the variables plotted on Figures 1 and 2 do not decay exponentially, their half-lives vary over the course of their adjustment to steady state. The "half-lives" reported in Table 2 are averages, computed as follows: for a variable x_t, the reported half-life is $\frac{1}{T}\sum_{t=t_0}^{t_0+T} -\ln 2/\left(\ln x_{t+1} - \ln x_t\right)$, where t_0 is the time when x_t reaches its extremum after the shock, and T is a horizon of 20 years.



(a) ALP

(b) Mean match quality

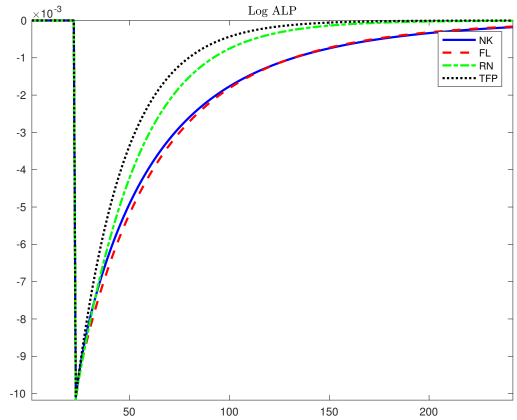
(c) Job finding rate

(d) EE rate

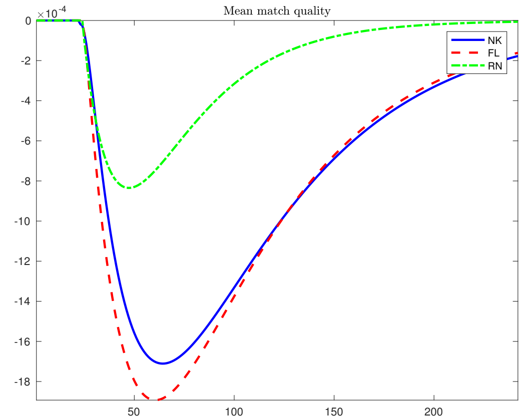
(e) Unemployment

(f) Inflation

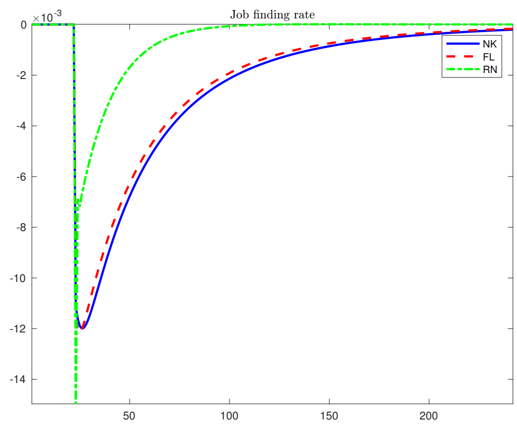
Fig. 1: Responses to a negative TFP shock under Taylor rule



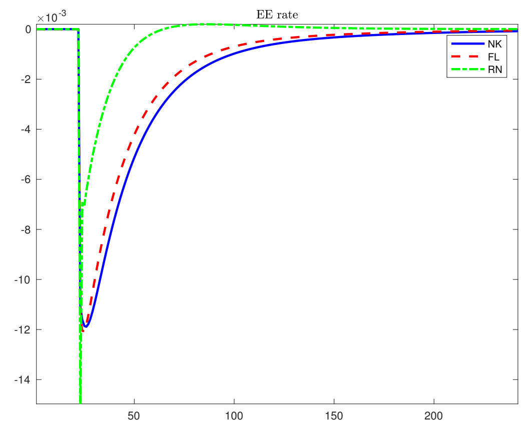
(a) ALP



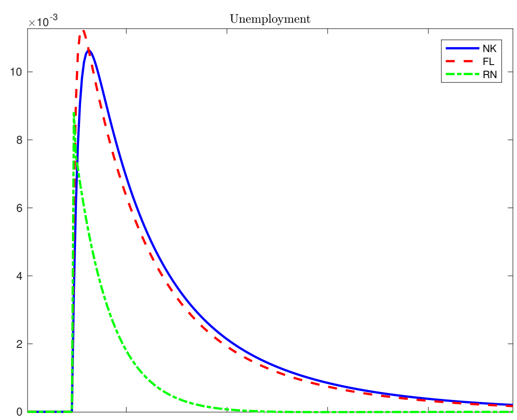
(b) Mean match quality



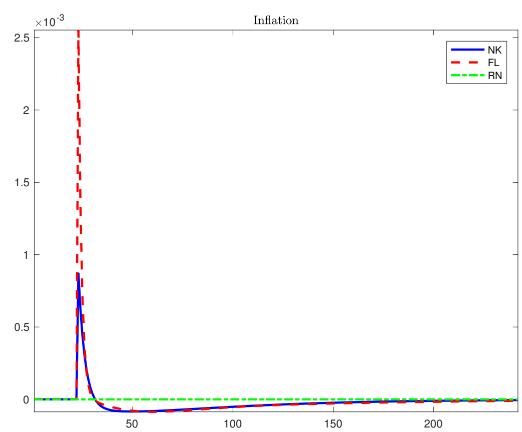
(c) Job finding rate



(d) EE rate



(e) Unemployment



(f) Inflation

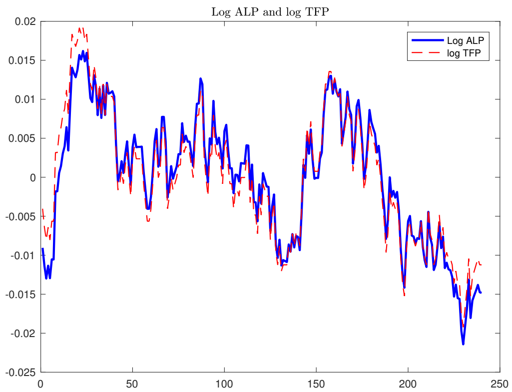
Fig. 2: Responses to a negative TFP shock under interest rate peg

rate (the Phillips curve), and Figure 3f plots the vacancy rate against the unemployment rate (the “empirical Beveridge curve” or U/V-curve). While the U/V-curve does not have a direct theoretical interpretation in the context of a model with on-the-job search such as ours, the attention it has received in the empirical literature makes it an object of empirical interest. Figure 4 replicates Figure 3, only under an interest rate peg.

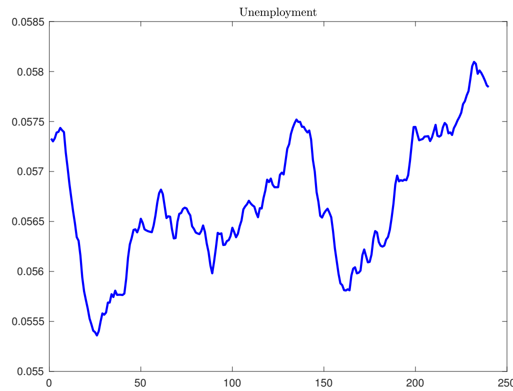
For brevity, we do not show time series plots obtained from the FL and RN versions of the model. However, the three versions can be compared in terms of various measures of amplification and propagation. Table 2 provides a summary.

Interestingly, the estimated elasticity of a misspecified matching function which would omit employed job seekers, i.e. the coefficient in an OLS regression of the log-job finding rate on $\ln(V/U)$, is 0.53, much lower than the structural elasticity of the true matching function (0.8 — see Table 1), but well in line with findings from the empirical literature.

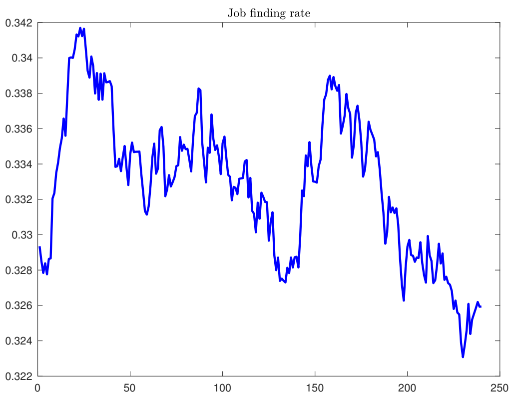
Most notable is the strong relationship between inflation and the lagged EE rate. High EE reallocation requires both strong job creation (to explain frequent contacts with open vacancies) and pronounced misallocation of employment down the ladder (to explain why these contacts often result in new matches). Over time, the former force persists with TFP, but the latter force wanes, as workers climb the job ladder: at that point, inflationary pressure builds, as outside offers are increasingly matched.



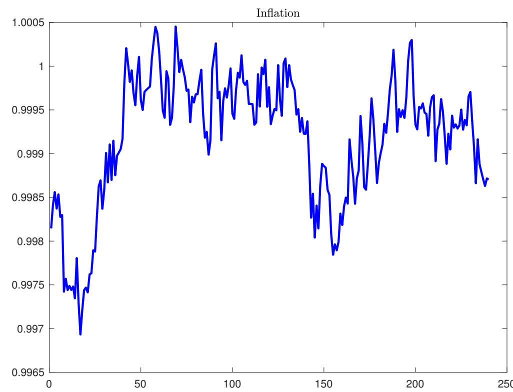
(a) ALP



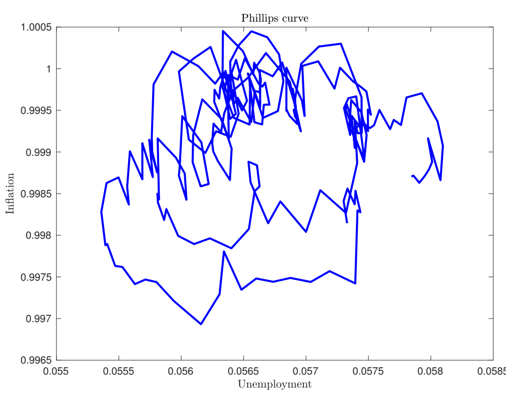
(b) Unemployment



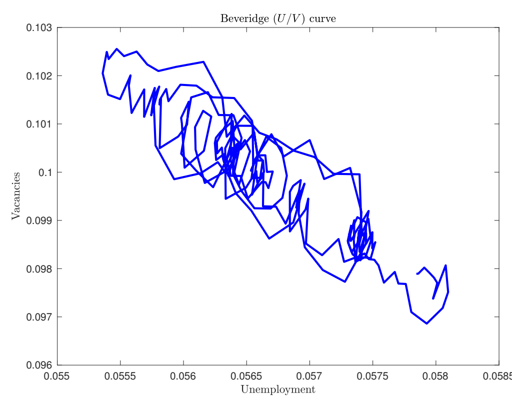
(c) Job finding rate



(d) Inflation

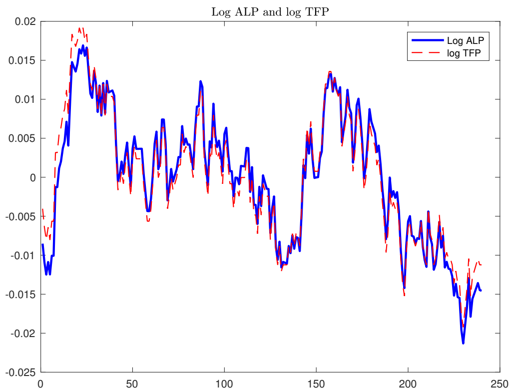


(e) Phillips curve

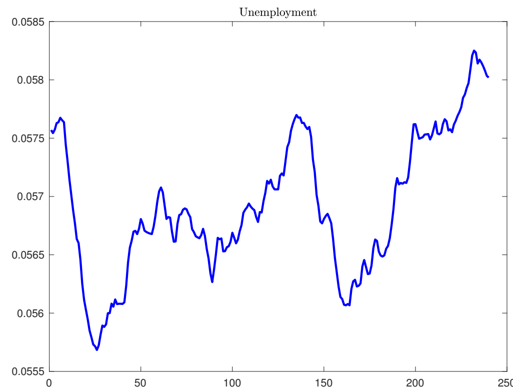


(f) Beveridge (U/V) curve

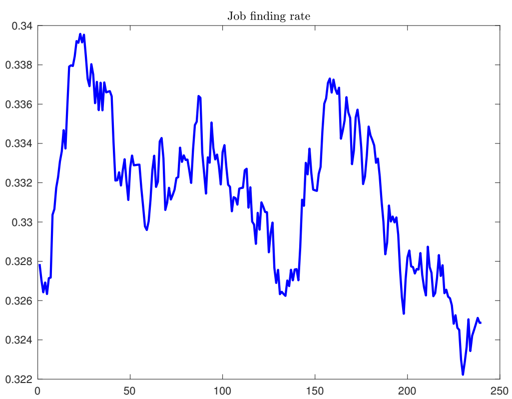
Fig. 3: Simulated time series under Taylor rule



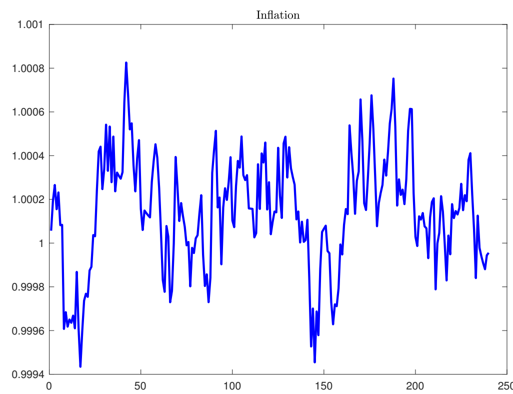
(a) ALP



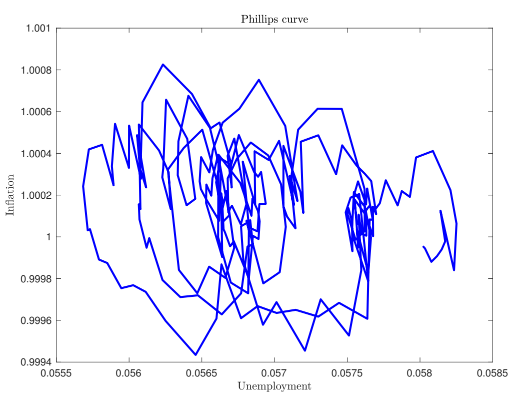
(b) Unemployment



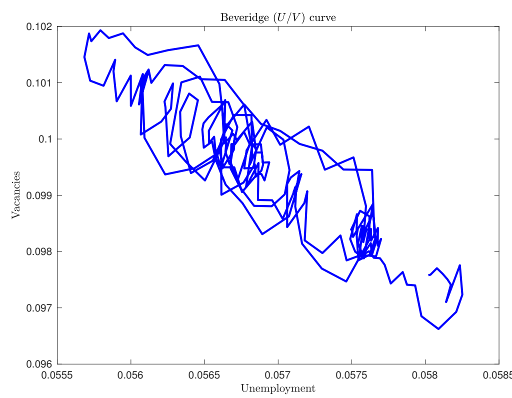
(c) Job finding rate



(d) Inflation

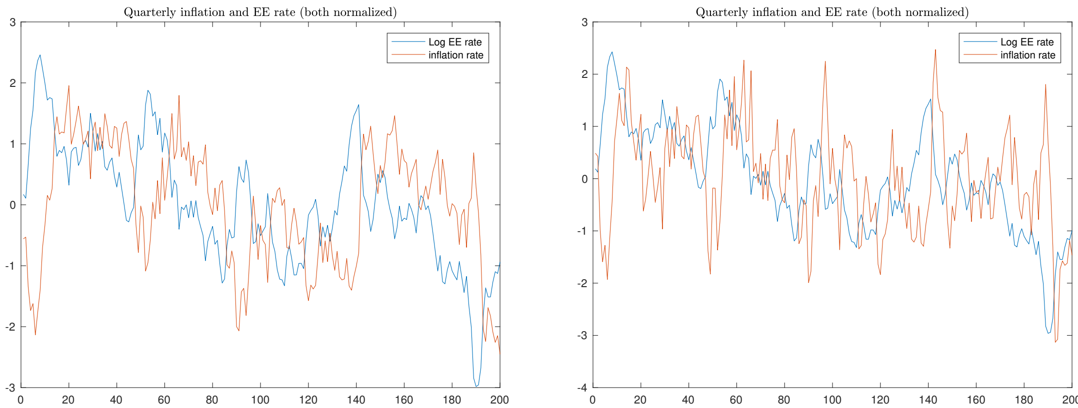


(e) Phillips curve



(f) Beveridge (U/V) curve

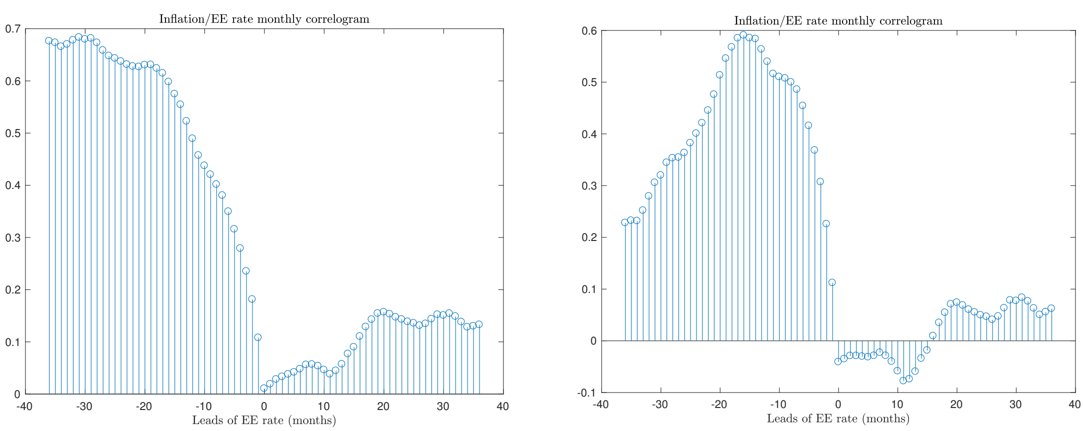
Fig. 4: Simulated time series under interest rate peg



(a) Taylor rule

(b) Interest rate peg

Fig. 5: Quarterly inflation and the EE rate



(a) Taylor rule

(b) Interest rate peg

Fig. 6: Inflation/EE cross-correlogram

APPENDIX

A Computation algorithms

In this appendix, we first present the details of the algorithm we use to simulate the full sticky-price model presented in the main body of the paper. We then explain how the algorithm is adjusted to simulate simpler versions of the model (i.e. the flexprice model, with or without curvature in the utility function).

A.1 Main algorithm: simulating the sticky-price model

A.1.1 Preliminaries

The model is simulated at monthly frequency using approximations in the spirit of "parameterized expectations". To define the main objects used in the algorithm, we begin by collating the equations that characterize equilibrium (equation numbers refer to equations in the main body of the paper).

Euler equation:

$$\mathbb{E}_t \left[\frac{U'(C_{t+1})}{U'(C_t)} \frac{1}{1+\pi_{t+1}} \right] = \frac{1}{(1+R_t)\beta} \tag{3}$$

Free-entry condition:

$$\frac{\kappa U'(C_t)}{\beta} \frac{\theta_t}{\phi(\theta_t)} = \frac{u_t \left[\mathcal{Z}_t \mathbb{E}_\Gamma(y) - \frac{b}{1-\beta(1-\delta)} \right] + \mathcal{Z}_t (1-\delta) s \int_{\underline{y}}^{\overline{y}} \gamma(y) \int_{\underline{y}}^{y} \ell_t(y')(y-y')\, dy' dy}{u_t + (1-\delta) s (1-u_t)} \tag{14}$$

Definition of $\mathcal{Z}$:

$$\mathcal{Z}_t = \mathbb{E}_t \left[U'(C_{t+1}) \frac{q_{t+1}}{P_{t+1}} + \beta(1-\delta)\mathcal{Z}_{t+1} \right] \tag{13}$$

Pricing decisions of Intermediate good producers:

$$\frac{p_t^*}{P_t} = \frac{\eta}{\eta-1} \frac{\frac{q_t}{z_t P_t} U'(C_t) Q_t + \mathbb{E}_t \sum_{\tau=1}^{+\infty} (1-\nu)^\tau \beta^\tau U'(C_{t+\tau}) Q_{t+\tau} \left(\frac{P_{t+\tau}}{P_t} \right)^\eta \frac{q_{t+\tau}}{z_{t+\tau} P_{t+\tau}}}{U'(C_t) Q_t + \mathbb{E}_t \sum_{\tau=1}^{+\infty} (1-\nu)^\tau \beta^\tau U'(C_{t+\tau}) Q_{t+\tau} \left(\frac{P_{t+\tau}}{P_t} \right)^{\eta-1}} \tag{8}$$

Goods market equilibrium:

$$C_t + \kappa\theta_t \left(u_t + (1-\delta)\, s\, (1-u_t)\right) = Q_t \tag{15}$$

Service market equilibrium:

$$Q_t = \left(\frac{\hat{P}_t}{P_t}\right)^{\eta} z_t \int_{\underline{y}}^{\overline{y}} y \ell_t(y) dy \tag{18}$$

Definition of $\hat{P}_t$:

$$\left(\frac{\hat{P}_t}{P_t}\right)^{-\eta} = \nu \left(\frac{p_t^*}{P_t}\right)^{-\eta} + (1-\nu)\,(1+\pi_t)^{\eta} \left(\frac{\hat{P}_{t-1}}{P_{t-1}}\right)^{-\eta} \tag{??}$$

Final good price:

$$P_t^{1-\eta} = \nu\; p_t^{*1-\eta} + (1-\nu) P_{t-1}^{1-\eta} \tag{9}$$

Law of motion of employment:

$$\ell_{t+1}(y) = (1-\delta) \left\{ \left[1 - s\phi(\theta_t)\, \overline{\Gamma}(y)\right] \ell_t(y) + s\phi(\theta_t)\, \gamma(y) \int_{\underline{y}}^{y} \ell_t(y')\, dy' \right\} + \phi(\theta_t)\, \gamma(y)\, u_t \tag{11}$$

Monetary policy rule:

$$\ln\left(\frac{1+R_t}{1+\overline{R}}\right) = \rho_R \ln\left(\frac{1+R_{t-1}}{1+\overline{R}}\right) + (1-\rho_R)\left[\psi_\pi \ln\left(\frac{1+\mathbb{E}_t\pi_{t+1}}{1+\overline{\pi}}\right) + \psi_Q \ln\left(\frac{Q_t}{\overline{Q}}\right)\right] + \varepsilon_t^R \tag{25}$$

We now define a number of auxiliary variables for use in the algorithm. First, let $\mathcal{D}_t$ denote the denominator in equation (8). It solves:

$$\mathcal{D}_t = U'(C_t)\, Q_t + \beta(1-\nu)\mathbb{E}_t\left[(1+\pi_{t+1})^{\eta-1}\mathcal{D}_{t+1}\right] \tag{26}$$

Next, define the auxiliary variable $\mathcal{Q}_t := \frac{\eta}{\eta-1}\frac{q_t}{z_t P_t} U'(C_t)\, Q_t$ and let $\mathcal{N}_t$ denote the numerator in equation (8). The latter solves:

$$\mathcal{N}_t = \mathcal{Q}_t + \beta(1-\nu)\mathbb{E}_t\left[(1+\pi_{t+1})^{\eta}\mathcal{N}_{t+1}\right] \tag{27}$$

Finally, equation (13) rewrites in terms of $\mathcal{Q}_t$ as:

$$\mathcal{Z}_t = \mathbb{E}_t\left[\frac{z_{t+1}\mathcal{Q}_{t+1}}{Q_{t+1}}\frac{\eta-1}{\eta} + \beta(1-\delta)\mathcal{Z}_{t+1}\right] \tag{28}$$

A.1.2 Algorithm

Preliminary step. Choose a number K of moments of the employment distribution $\ell(\cdot)$, $\{m^1, \cdots m^K\}$ (those can include the unemployment rate, mean match quality, etc., or simply quantiles of $\ell(\cdot)$). Collect those moments (including a constant term) in a vector $\mathbf{M}$.

Parameterize auxiliary variables $\mathcal{Z}$, $\mathcal{N}$, $\mathcal{D}$ and $\mathcal{Q}$ as follows:

$$\ln \mathcal{Z}_t = \mathcal{P}\left(z_t, \mathbf{M}_t | \tau_{\mathcal{Z}}\right)$$

$$\ln \mathcal{N}_t = \mathcal{P}\left(z_t, \mathbf{M}_t | \tau_{\mathcal{N}}\right)$$

$$\ln \mathcal{D}_t = \mathcal{P}\left(z_t, \mathbf{M}_t | \tau_{\mathcal{D}}\right)$$

$$\ln \mathcal{Q}_t = \mathcal{P}\left(z_t, \mathbf{M}_t | \tau_{\mathcal{Q}}\right)$$

where $\mathcal{P}(\cdot|\tau)$ is a polynomial function, parameterized by a vector of parameters $\tau \in \mathbb{R}^{K+1}$.

Simulate a history of TFP shocks z_t^*, $t = 0, 2, \cdots T$ and fix initial conditions for the employment distribution ℓ_1 — implying initial conditions for the elements of $\mathbf{M}_1$ — and for $\hat{P}_0/P_0$ and R_0. (Note that the initial condition for ℓ has a subscript indicating date $t = 1$ rather than $t = 0$ because, with our timing convention, ℓ_t is the employment distribution at the end of period $t - 1$.)

Step 1. Fix values $\left\{\tau_{\mathcal{Z}}^{(i)}, \tau_{\mathcal{N}}^{(i)}, \tau_{\mathcal{D}}^{(i)}, \tau_{\mathcal{Q}}^{(i)}\right\}$ for the parameters of $\mathcal{Z}$, $\mathcal{N}$, $\mathcal{D}$ and $\mathcal{Q}$. At time $t = 1, 2, \cdots T$, let

$$\mathcal{Z}_t^{(i)}(z) = \exp\left[\mathcal{P}\left(z, \mathbf{M}_t | \tau_{\mathcal{Z}}^{(i)}\right)\right]$$
$$\mathcal{N}_t^{(i)}(z) = \exp\left[\mathcal{P}\left(z, \mathbf{M}_t | \tau_{\mathcal{N}}^{(i)}\right)\right]$$
$$\mathcal{D}_t^{(i)}(z) = \exp\left[\mathcal{P}\left(z, \mathbf{M}_t | \tau_{\mathcal{D}}^{(i)}\right)\right]$$
$$\mathcal{Q}_t^{(i)}(z) = \exp\left[\mathcal{P}\left(z, \mathbf{M}_t | \tau_{\mathcal{Q}}^{(i)}\right)\right]$$

Given those values of $\mathcal{Z}$, $\mathcal{N}$, $\mathcal{D}$ and $\mathcal{Q}$, simulate the model recursively as follows. At date t, update the ratio p_t^*/P_t in all possible TFP states z according to (8) as $p_t^*(z)/P_t(z) =$

$\mathcal{N}_t^{(i)}(z)/\mathcal{D}_t^{(i)}(z)$. From there, infer the inflation rate in all possible TFP states z using (9):

$$1+\pi_t(z)=\left(\frac{1-\nu\left(p_t^*(z)/P_t(z)\right)^{1-\eta}}{1-\nu}\right)^{1/(\eta-1)}$$

Next, use (**??**), $p_t^*(z)/P_t(z)$, $\pi_t(z)$ and $\left(\hat{P}_{t-1}/P_{t-1}\right)^{-\eta}$ from previous period to update $\left(\hat{P}_t/P_t\right)^{-\eta}$ in all states:

$$\left(\frac{\hat{P}_t(z)}{P_t(z)}\right)^{-\eta}=\nu\left(\frac{p_t^*(z)}{P_t(z)}\right)^{-\eta}+(1-\nu)\left(1+\pi_t(z)\right)^{\eta}\left(\frac{\hat{P}_{t-1}}{P_{t-1}}\right)^{-\eta}$$

Next, jointly solve for job market tightness $\theta_t(z)$, output $Q_t(z)$, and Final good consumption $C_t(z)$ using equations (14), (15) and (18). This can be performed as follows: given $\left(\hat{P}_t(z)/P_t(z)\right)^{-\eta}$ just computed and (z,ℓ_t), use (18) to calculate $Q_t(z)$. Given u_t (from ℓ_t), $\mathcal{Z}_t^{(i)}(z)$ (from the parameterized guess), use (14) and (15) to calculate $(C_t(z),\theta_t(z))$. To that end, solve for $C_t(z)$ from (15) and replace into (14), which then becomes a nonlinear equation in $\theta_t(z)$ alone, and can be solved by bisection.

Finally, use $\theta_t\left(z_t^*\right)$ it to update the employment distribution to $\ell_{t+1}(\cdot)$ and thus its moments $\mathbf{M}_{t+1}$. Use $\pi_t\left(z\right)$ and $Q_t\left(z\right)$ in the monetary policy rule to calculate R_{t-1}.

Step 2. Except when $t=0$, calculate:

$$\begin{aligned}
\widehat{\mathcal{Z}}_{t-1}&=\mathbb{E}\left[\frac{z_t\mathcal{Q}_t^{(i)}(z_t)}{Q_t(z_t)}\frac{\eta-1}{\eta}+\beta(1-\delta)\mathcal{Z}_t^{(i)}(z_t)\mid z_{t-1}=z_{t-1}^*\right]\\
\widehat{\mathcal{N}}_{t-1}&=\mathcal{Q}_{t-1}^{(i)}(z_{t-1}^*)+\beta(1-\nu)\mathbb{E}\left[(1+\pi_t(z_t))^{\eta}\mathcal{N}_t^{(i)}(z_t)\mid z_{t-1}=z_{t-1}^*\right]\\
\widehat{\mathcal{D}}_{t-1}&=U'\left(C_{t-1}\left(z_{t-1}^*\right)\right)Q_{t-1}\left(z_{t-1}^*\right)+\beta(1-\nu)\mathbb{E}\left[(1+\pi_t(z_t))^{\eta-1}\mathcal{D}_t^{(i)}(z_t)\mid z_{t-1}=z_{t-1}^*\right]\\
\widehat{\mathcal{E}}_{t-1}&=\beta\left(1+R_{t-1}\right)\mathbb{E}\left[\frac{U'\left(C_t(z_t)\right)}{U'\left(C_{t-1}\left(z_{t-1}^*\right)\right)}\frac{1}{1+\pi_t(z_t)}\mid z_{t-1}=z_{t-1}^*\right]
\end{aligned}$$

Final step. Use steps 1 and 2 in a minimizer to find the vector of parameters $\left\{\tau_{\mathcal{Z}}^{(i)},\tau_{\mathcal{N}}^{(i)},\tau_{\mathcal{D}}^{(i)},\tau_{\mathcal{Q}}^{(i)}\right\}$ that minimizes the distance:

$$\sum_{t=0}^{T}\left[\left(\mathcal{Z}_t^{(i)}\left(z_t^*\right)-\widehat{\mathcal{Z}}_t\right)^2+\left(\mathcal{N}_t^{(i)}\left(z_t^*\right)-\widehat{\mathcal{N}}_t\right)^2+\left(\mathcal{D}_t^{(i)}\left(z_t^*\right)-\widehat{\mathcal{D}}_t\right)^2+\left(1-\widehat{\mathcal{E}}_t\right)^2\right]$$

A.1.3 Approximation specification

The choices of the degree of the polynomial $\mathcal{P}$ and of how many moments to put in the vector $\mathbf{M}$ is informed by the quality of the approximations to $\mathcal{Z}$, $\mathcal{D}$, $\mathcal{Q}$ and $\mathcal{R}$. Said quality is measured by the maximum relative error made in approximating $\mathcal{Z}$, $\mathcal{D}$, etc., namely:

$$\max_{t=1,\cdots,T} \frac{\left|\mathcal{Z}_t^{(i)}\left(z_t^*\right) - \widehat{\mathcal{Z}}_t\right|}{\left|\mathcal{Z}_t^{(i)}\left(z_t^*\right)\right|}, \ \max_{t=1,\cdots,T} \frac{\left|\mathcal{N}_t^{(i)}\left(z_t^*\right) - \widehat{\mathcal{N}}_t\right|}{\left|\mathcal{N}_t^{(i)}\left(z_t^*\right)\right|}, \ \max_{t=1,\cdots,T} \frac{\left|\mathcal{D}_t^{(i)}\left(z_t^*\right) - \widehat{\mathcal{D}}_t\right|}{\left|\mathcal{D}_t^{(i)}\left(z_t^*\right)\right|}, \ \max_{t=1,\cdots,T} \left|1 - \widehat{\mathcal{E}}_t\right|$$

which we monitor as we implement the algorithm. We find that those maximum relative errors are all below 1% two-dimensional $\mathbf{M}$ that includes only the unemployment rate and mean match quality $\int y\ell(y)dy$, and with $\mathcal{P}$ specified as a linear function of $(z, \mathbf{M})$.

A.2 Algorithm variant 1: simulating the flexprice model

The flexprice model coincides with the sticky-price model in the special case $\nu = 1$, and so the algorithm described in A.1 continues to apply. Yet the case $\nu = 1$ allows important simplifications. In that case, $p_t^* = P_t$ at all dates, which obviates the need to parameterize $\mathcal{N}$ and $\mathcal{D}$, and implies that $\mathcal{Q}_t$ is a straightforward function of date-t equilibrium quantities. Moreover, the inflation rate is irrelevant to equilibrium real quantities. Thus the only variable that needs parameterization in this case is $\mathcal{Z}$, whose characterization now reads:

$$\mathcal{Z}_t = \mathbb{E}_t \left[z_{t+1} U'\left(C_{t+1}\right) \frac{\eta - 1}{\eta} + \beta(1-\delta)\mathcal{Z}_{t+1} \right].$$

A.3 Algorithm variant 2: simulating the flexprice model with linear utility

This case is simply the flexprice model under the restriction $U'(C) = 1$, which obviates the need to parameterize even $\mathcal{Z}$, as in this case it equals the expected present discounted value of future TFP (adjusted for the monopoly distortion):

$$\mathcal{Z}_t = \mathbb{E}_t \left[z_{t+1} \frac{\eta - 1}{\eta} + \beta(1-\delta)\mathcal{Z}_{t+1} \right]$$

which can be computed exactly before running any simulation. With the specifications used in the simulation:

$$\mathcal{Z}_t = \frac{\eta - 1}{\eta} \sum_{\tau=1}^{+\infty} [\beta(1-\delta)]^{\tau-1} \exp\left(\mu_z + \frac{\sigma_z^2}{2} \frac{1-\rho_z^{2\tau}}{1-\rho_z^2}\right) z_t^{\rho_z^\tau}$$