

# THE GROWTH OF EMERGING ECONOMIES AND GLOBAL MACROECONOMIC INSTABILITY

Vincenzo Quadrini\*  
University of Southern California and CEPR

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## Abstract

This paper studies how the unprecedent growth within emerging countries during the last two decades has affected macroeconomic stability in both emerging and industrialized countries. The paper develops a two-country model where financial intermediaries play a central role in the intermediation of funds and crises emerge from self-fulfilling expectations about the liquidity of the banking sector. By increasing the worldwide ‘net demand’ of financial assets, the growth of emerging countries raises the incentive of banks to leverage, which in turn contributes to higher financial and macroeconomic instability in both industrialized and emerging economies.

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## 1 Introduction

During the last two decades we have witnessed unprecedented growth within emerging countries. As a result of the sustained growth, the size of these economies has increased dramatically compared to industrialized countries. The top panel of Figure 1 shows that, in PPP terms, the GDP of emerging countries at the beginning of the 1990s was 46 percent the GDP of industrialized countries. This number has increased to 90 percent by 2011. When the GDP comparison is based on nominal exchange rates, the relative size of emerging economies has increased from 17 to 52 percent.

During the same period, emerging economies have increased their net holdings of foreign financial assets. As the second panel of Figure 1 shows, starting in the second half of 1990s, emerging countries have experienced current account surpluses compared to current account deficits for industrialized countries. Also significant is the fact that the foreign assets acquired by emerging countries were concentrated in safer asset classes.

It is customary to divide foreign assets in four classes: (i) debt instruments and international reserves; (ii) portfolio investments; (iii) foreign direct investments; (iv) other investments (see Gourinchas and Rey (2007) and Lane and Milesi-Ferretti (2007)). The net foreign position in the first class of assets—debt and international reserves—is plotted in the bottom panel of Figure 1, separately for industrialized and emerging countries. As can be seen from the figure, since the early 1990s emerging countries have accumulated ‘positive’ net positions in this particular asset class while industrialized countries have accumulated ‘negative’ net positions.

There are several theories proposed in the literature that explain why emerging economies accumulate financial assets issued by industrialized countries.<sup>1</sup> The primary goal of this paper, however, is not to explain why emerg-

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<sup>1</sup>One explanation posits that emerging countries have pursued policies aimed at keeping their currencies undervalued and, to achieve this, they have purchased large volumes of foreign financial assets, especially securities issued by foreign governments. Another explanation is based on differences in the characteristics of financial markets. The idea is that lower financial development in emerging countries impairs the ability of these countries to create viable saving instruments for intertemporal smoothing (Caballero, Farhi, and Gourinchas (2008)) or for insurance purpose (Mendoza, Quadrini, and Ríos-Rull (2009)). Because of this limitations, they turn to industrialized countries for the acquisition of these assets. A third explanation is based on greater idiosyncratic uncertainty faced by consumers and firms in emerging countries due, for example, to higher idiosyncratic risk or lower safety net provided by the public sector. Theoretical contributions in the global

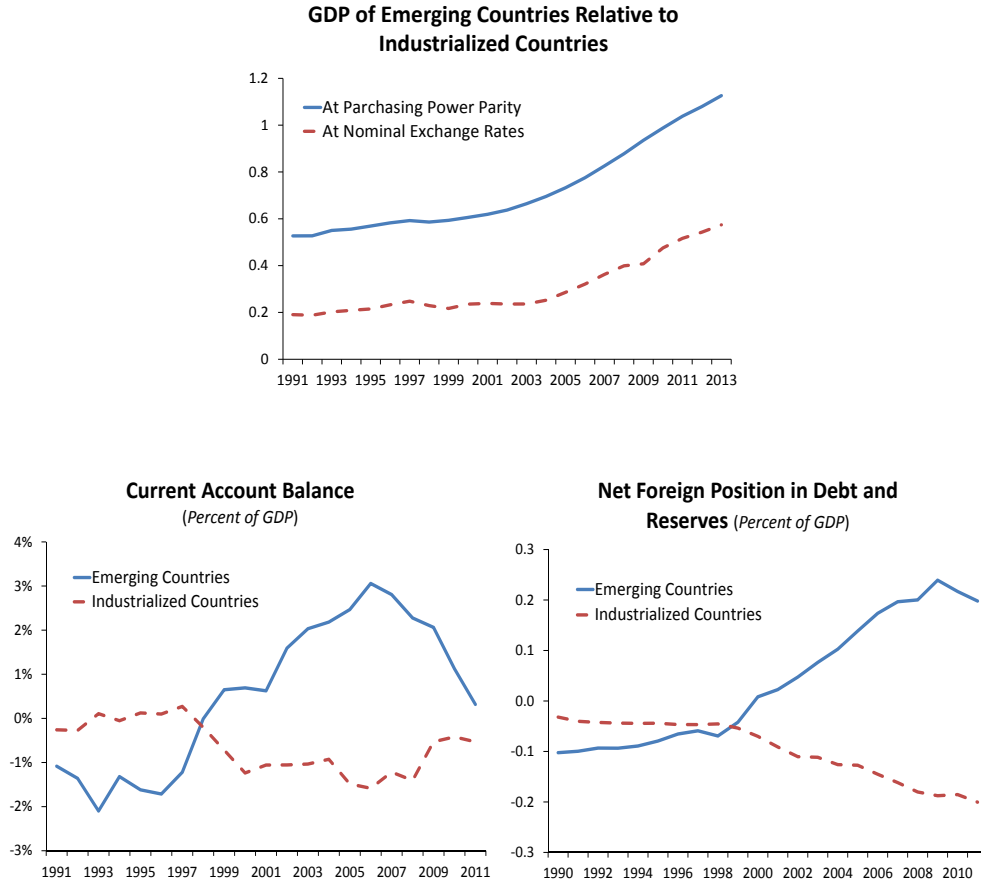


Figure 1: Gross domestic product and net foreign positions in debt instruments and international reserves of emerging and industrialized countries. **Emerging countries:** Argentina, Brazil, Bulgaria, Chile, China, Hong.Kong, Colombia, Estonia, Hungary, India, Indonesia, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Industrialized countries:** Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United.Kingdom, United.States. **Sources:** World Development Indicators (World Bank) and External Wealth of Nations Mark II database (Lane and Milesi-Ferretti (2007)).

ing countries acquire net financial assets issued by industrialized countries.

imbalance literature include Carroll and Jeanne (2009), Angeletos and Panousi (2011), Song, Storesletten, and Zilibotti (2011), Sandri (2014), Bacchetta and Benhima (2015), Fogli and Perri (2015).

Instead, it investigates how the increase in net demand for financial assets by emerging countries (global imbalances) affects financial and macroeconomic stability in both emerging and industrialized countries. Thus, the goal is not to explain global imbalances but the consequences of global imbalances.

To understand how the growth of emerging economies impacts global financial and macroeconomic stability, I develop a two-country model—one representative of emerging countries and the other representative of industrialized countries—where financial intermediaries play a central role in transferring resources from agents in excess of funds (lenders) to agents in need of funds (borrowers). Differently from recent macroeconomic models proposed in the literature,<sup>2</sup> I emphasize the central role of banks in issuing liabilities (or facilitating the issuance of liabilities) rather than its lending role for macroeconomic dynamics.

In the model, bank liabilities play an important role as insurance instruments. When the stock of bank liabilities held by other sectors of the economy increases, agents are better insured and willing to engage in activities that are individually risky. In aggregate, this allows for sustained employment, production and consumption. However, when banks issue more liabilities, they also create the conditions for a liquidity crisis. A crisis generates a drop in the volume of intermediated funds and with it a decline in the stock of bank liabilities held by the nonfinancial sector. As a consequence, the non-financial sector is less willing to engage in risky activities and this generates a macroeconomic contraction.

A central feature of the model is that the probability and macroeconomic consequences of a liquidity crisis depend on the leverage chosen by banks, which in turn depends on the interest rate paid on their liabilities (funding cost). When the interest rate is low, banks have higher incentives to leverage, which in turn increases the likelihood of a crisis and/or the economic consequences of a crisis. It is then easy to see how the growth of emerging countries could contribute to global economic instability. As the world share of these countries increases, the worldwide demand for financial assets (bank liabilities in the model) rises relatively to the supply. This drives down the interest rate paid by banks on their liabilities, increasing the incentives to leverage. But as the banking sector becomes more leveraged, a crisis be-

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<sup>2</sup>See, for example, Van den Heuvel (2008), Meh and Moran (2010), Brunnermeier and Sannikov (2014), Gertler and Kiyotaki (2010), Mendoza and Quadrini (2010), De Fiore and Uhlig (2011), Gertler and Karadi (2011), Boissay, Collard, and Smets (2010), Corbae and D’Erasmus (2012), Rampini and Viswanathan (2012), Adrian, Colla, and Shin (2013).

comes more likely and/or bigger. As long as the crisis does not materialize, the economy enjoys sustained levels of financial intermediation, asset prices and economic activity. Eventually, however, a crisis materializes, inducing a reversal in financial intermediation with consequent contractions in asset prices and overall economic activity.

The organization of the paper is as follows. Section 2 describes the model and characterizes the equilibrium. Section 3 uses the model to study the central question addressed in the paper, that is, how the growth of emerging economies affects financial and macroeconomic stability in both emerging and industrialized countries. Section 4 discusses the importance of international capital markets integration and Section 5 concludes.

## 2 Model

There are two countries indexed by  $j \in \{1, 2\}$ . The first is representative of industrialized economies and the second is representative of emerging economies. In each country there are two non-financial sectors: the entrepreneurial sector and the household sector. In addition, there is an intermediation sector populated by profit-maximizing banks that operate globally in a regime of international capital mobility. The role of banks is to facilitate the transfer of resources between entrepreneurs and households and across countries. The ownership of banks by country 1 or country 2 is irrelevant as it will become clear later. What is important is that banks operate globally, that is, they can issue liabilities and make loans in both countries.

Countries are heterogeneous in two dimensions: (i) economic size captured by differences in aggregate productivity  $A_{j,t}$ ; and (ii) financial market development captured by the parameter  $\eta_j$ . While productivity changes over time, financial market development remains constant. This explains the time subscript in  $A_{j,t}$  but not in  $\eta_j$ . Although changes in the relative size of countries could also be generated by other factors besides productivity (for example population growth, investment, real exchange rates), in the model these additional changes are isomorphic to productivity changes. This will become clear in the quantitative section.

The assumption that only cross-country productivity (as a proxy for economic size) changes over time while differences in financial markets development remain constant, is justified by the main question addressed in this paper, that is, how the economic growth of emerging countries impacts financial and macroeconomic stability. In order to isolate the effect of the change

in economic size from other factors, I keep everything else constant, including the financial characteristics of these countries. As we will see, financial heterogeneity plays an important role for the growth of emerging countries to affect global macroeconomic stability.

## 2.1 Entrepreneurial sector

In each country there is a unit mass of atomistic entrepreneurs indexed by  $i$ . Entrepreneurs are individual owners of firms with lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_{j,t}^i),$$

where  $c_{j,t}^i$  is the consumption of entrepreneur  $i$  in country  $j$  at time  $t$ .

Entrepreneurs are business owners producing a single good with the production technology described below. Although the model is presented as if all firms are privately owned, we should think of the entrepreneurial sector more broadly and including public companies. In this case entrepreneurial consumption corresponds to the dividends paid by the firm and the concavity of the utility function captures the risk aversion of managers or major shareholders. It can also be interpreted as capturing, in reduced form, the possible costs associated with financial distress: even if shareholders and managers are risk-neutral, a convex cost of financial distress would make the objective of the firm concave.

Each firm operates the production function

$$y_{j,t}^i = z_{j,t}^i h_{j,t}^i,$$

where  $h_{j,t}^i$  is the input of labor supplied by households in country  $j$  at the market wage  $w_{j,t}$ , and  $z_{j,t}^i$  is productivity.

Productivity is equal to  $z_{j,t}^i = A_{j,t} \pi_{j,t}^i$ . The first component,  $A_{j,t}$ , is the aggregate country-specific productivity. This is the same for all entrepreneurs of the same country but differs across countries. I allow  $A_{j,t}$  to change over time to capture the changing economic size of the two countries. In particular, I assume that  $A_{j,t}$  follows some stochastic process that will be specified later. The second component,  $\pi_{j,t}^i \in [\underline{\pi}, \bar{\pi}]$ , is an idiosyncratic shock independently and identically distributed among entrepreneurs and over time with probability distribution  $\Gamma(\pi)$ . Notice that the distribution of the idiosyncratic

shock is the same in the two countries. Therefore, the production technology differs between the two countries only in the aggregate productivity  $A_{j,t}$ .

As in Arellano, Bai, and Kehoe (2011) the input of labor  $h_{j,t}^i$  is chosen before observing  $z_{j,t}^i$  and its components  $A_{j,t}$  and  $\pi_{j,t}^i$ . This implies that the choice of labor is risky. To insure consumption smoothing, entrepreneurs have access to a market for non-contingent bonds at price  $q_t$ . As we will see, bonds held by entrepreneurs are the liabilities issued by banks. Notice that the market price of bonds does not have the subscript  $j$  because capital mobility implies that the price is equalized across countries. Since the bonds cannot be contingent on the realization of productivity  $z_{j,t}^i$ , they provide only partial insurance.

Appendix A provides a micro-foundation for the limited insurability of productivity shocks and shows that the economy studied here is equivalent to an economy where entrepreneurs have access to state contingent claims but financial contracts are not perfectly enforceable.

An entrepreneur  $i$  in country  $j$  enters period  $t$  with financial wealth  $b_{j,t}^i$ . At the beginning of the period the entrepreneur may incur financial losses proportional to their wealth, as a result of a financial crisis. Denoting by  $\delta_t$  the unit loss, the residual wealth will be denoted by  $\tilde{b}_{j,t}^i = (1 - \delta_t)b_{j,t}^i$ . The variable  $\delta_t$  is a stochastic aggregate variable which is endogenous in the model and it is the same in both countries (which explains the absence of the country-specific subscript  $j$ ). The determination of this variable in the general equilibrium will be described later with the introduction of the financial intermediation sector.

Given the residual wealth  $\tilde{b}_{j,t}^i$ , the entrepreneur chooses the input of labor  $h_{j,t}^i$ . After observing  $z_{j,t}^i$ , he/she chooses consumption  $c_{j,t}^i$  and next period bonds  $b_{j,t+1}^i$ . The budget constraint, after the realization of productivity, is

$$c_{j,t}^i + q_t b_{j,t+1}^i = (z_{j,t}^i - w_{j,t})h_{j,t}^i + \tilde{b}_{j,t}^i. \quad (1)$$

Because labor  $h_{j,t}^i$  is chosen before the realization of  $z_{j,t}^i$ , while the saving decision is made after the observation of  $z_{j,t}^i$ , it will be convenient to define  $a_{j,t}^i = \tilde{b}_{j,t}^i + (z_{j,t}^i - w_{j,t})h_{j,t}^i$  the entrepreneur's wealth after production. Given the timing assumption, the input of labor  $h_{j,t}^i$  depends on  $\tilde{b}_{j,t}^i$  while the saving decision  $b_{j,t+1}^i$  depends on  $a_{j,t}^i$ . To further clarify the decision timing, it would be convenient to think of a period as divided in three subperiods:

1. **Subperiod 1:** Entrepreneurs enter the period with financial wealth

$b_{j,t}^i$  and observe the aggregate variable  $\delta_t$ . The realization of financial losses brings the residual wealth of entrepreneurs to  $\tilde{b}_{j,t}^i = (1 - \delta_t)b_{j,t}^i$ .

2. **Subperiod 2:** Given  $\tilde{b}_{j,t}^i$ , entrepreneurs choose the input of labor  $h_{j,t}^i$ . At this stage entrepreneurs do not know neither the idiosyncratic productivity  $\pi_{j,t}^i$  nor the aggregate productivity  $A_{j,t}$ .
3. **Subperiod 3:** Productivity  $z_{j,t}^i = A_{j,t}\pi_{j,t}^i$  is realized and the end-of-period wealth is  $a_{j,t}^i = \tilde{b}_{j,t}^i + (z_{j,t}^i - w_{j,t})h_{j,t}^i$ . This is in part used for consumption,  $c_{j,t}^i$ , and in part is saved in new bonds at cost  $q_t b_{j,t+1}^i$ .

The following lemma characterizes the entrepreneur's policies.

**Lemma 2.1** *Let  $\phi_{j,t}$  satisfy the condition  $\mathbb{E}_{z_{j,t}^i} \left\{ \frac{z_{j,t}^i - w_{j,t}}{1 + (z_{j,t}^i - w_{j,t})\phi_{j,t}} \right\} = 0$ . The optimal entrepreneur's policies are*

$$\begin{aligned} h_{j,t}^i &= \phi_{j,t} \tilde{b}_{j,t}^i, \\ c_{j,t}^i &= (1 - \beta)a_{j,t}^i, \\ q_t b_{j,t+1}^i &= \beta a_{j,t}^i. \end{aligned}$$

**Proof 2.1** *See Appendix B.*

The demand for labor, which is chosen before observing the realization of productivity, is linear in the financial wealth of the entrepreneur  $\tilde{b}_{j,t}^i$ . The proportional factor  $\phi_{j,t}$  is defined by the condition  $\mathbb{E}_{z_{j,t}^i} \left\{ \frac{z_{j,t}^i - w_{j,t}}{1 + (z_{j,t}^i - w_{j,t})\phi_{j,t}} \right\} = 0$ , which is the same for all entrepreneurs of a country (but could differ across countries because of different aggregate productivities).

The factor  $\phi_{j,t}$  captures the role played by risk aversion in determining the demand for labor. Because productivity is unknown when an entrepreneur chooses the scale of production, labor is risky and entrepreneurs require a premium in order to produce. As a result, the expected marginal product of labor is bigger than the wage rate, that is,  $\mathbb{E}_t z_{j,t}^i > w_{j,t}$ . Furthermore, higher is the expected unit profit and higher is the scale of production  $\phi_{j,t}$ . On the other hand, if we fix the expected unit profit, the scale of production decreases with the volatility of productivity, that is, it decreases with risk.

Since the distribution of productivity (aggregate and idiosyncratic) is fixed in the model, the only 'endogenous' variable that affects  $\phi_{j,t}$  is the



wage rate  $w_{j,t}$ . Therefore, I denote this variable by the function  $\phi_{j,t}(w_{j,t})$ , which is strictly decreasing in the (country) wage rate. The dependence of this function on aggregate productivity is captured by the subscripts  $j$  and  $t$ . The aggregate demand for labor in country  $j$  is derived by aggregating individual demands and can be written as

$$H_{j,t} = \phi_{j,t}(w_{j,t}) \int_i \tilde{b}_{j,t}^i = \phi_{j,t}(w_{j,t}) \tilde{B}_{j,t},$$

where capital letters denote aggregate variables.

The aggregate demand for labor depends negatively on the wage rate—which is a standard property—and positively on the aggregate financial wealth of entrepreneurs even if they are not financially constrained—which is a special property of this model. Since hiring is risky, entrepreneurs are willing to hire more labor when they hold more financial wealth. Also linear is the consumption policy which follows from the logarithmic utility.

## 2.2 Household sector

In each country there is a unit mass of atomistic households with utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( c_{j,t} - \alpha A_{j,t} \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right),$$

where  $c_{j,t}$  is consumption and  $h_{j,t}$  is the supply of labor. Households are homogeneous and they do not face idiosyncratic shocks.

The assumption that households have linear utility in consumption simplifies the characterization of the equilibrium (with some of the results derived analytically) without affecting the key properties of the model. As I will discuss below, as long as households do not face idiosyncratic risks (or the idiosyncratic risk faced by households is significantly lower than entrepreneurs), the model displays similar properties even if households are risk averse.

Another special feature of the utility function is that the dis-utility from working depends on country-specific productivity  $A_{j,t}$ . This is necessary for the model to display balanced growth. Without this assumption labor would increase without bound as productivity increases.

Households hold an asset which is available in fixed supply  $\bar{K}$ . Each unit of the asset produces  $A_{j,t}$  units of consumption goods to households

but not to entrepreneurs. The productivity of the asset increases with the country-specific productivity, which guarantees balanced growth. The asset is divisible and can be traded at the market price  $p_{j,t}$  by households.<sup>3</sup> I will interpret the fixed asset as residential houses and its production as housing services.

Households can borrow at the gross interest rate  $R_t$  and face the budget constraint

$$c_{j,t} + l_{j,t} + (k_{j,t+1} - k_{j,t})p_{j,t} = \frac{l_{j,t+1}}{R_t} + w_{j,t}h_{j,t} + A_{j,t}k_{j,t},$$

where  $l_{j,t}$  is the loan contracted in the previous period  $t - 1$  and due in the current period  $t$ , and  $l_{j,t+1}$  is the new loan that will be repaid in the next period  $t + 1$ . The interest rate on loans does not have the country subscript  $j$  because of cross-country equalization (given capital mobility).

Household debt is constrained by a borrowing limit which derives from the limited enforceability of debt contracts. I will consider two specifications of the borrowing limit. The first specification takes the form

$$l_{j,t+1} \leq \eta_j A_{j,t}, \quad (2)$$

where  $\eta_j$  is a parameter that differs across countries.

This specification allows me to characterize the equilibrium analytically with simple intuitions for the key results of the paper. However, the equilibrium price  $p_{j,t}$  will be only a function of the exogenous productivity  $A_{j,t}$  and will not be affected by changing financial market conditions. To have a more interesting dynamics of asset prices I will then consider a second specification that takes the form

$$l_{j,t+1} \leq \eta_j \mathbb{E}_t p_{j,t+1} k_{j,t+1}. \quad (3)$$

The dependence of the borrowing limit from the collateral value of assets introduces a mechanism through which borrowing affects the equilibrium price  $p_{j,t}$ , which responds to changing financial market conditions. With this specification, however, the equilibrium cannot be fully characterized analytically. I will then provide a numerical characterization.

As for entrepreneurs, households' decisions are made in two steps with different information sets. The supply of labor is chosen before the realization

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<sup>3</sup>Entrepreneurs could also trade this asset. However, since it provides services only to households, in equilibrium entrepreneurs choose not to hold it.

of aggregate productivity while the borrowing decisions are made after the realization of  $A_{j,t}$ . Appendix C describes the households' problem and derives the first order conditions. When the borrowing limit takes the form specified in (2), the optimality conditions are

$$\alpha \mathbb{E}_t A_{j,t} h_{j,t}^{\frac{1}{\nu}} = w_{j,t}, \quad (4)$$

$$1 = \beta R_t (1 + \mu_{j,t}), \quad (5)$$

$$p_{j,t} = \beta \mathbb{E}_t (A_{j,t+1} + p_{j,t+1}), \quad (6)$$

where  $\beta \mu_{j,t}$  is the Lagrange multiplier associated with the borrowing constraint. Since the supply of labor is chosen before the observation of productivity, the aggregate supply depends on the expected value of productivity, that is,  $\mathbb{E}_t A_{j,t}$ . As can be seen from equation (6), the housing price  $p_{j,t}$  depends only on aggregate productivity.

When the borrowing limit takes the form specified in (3), the optimality conditions with respect to  $h_{j,t}$  and  $l_{j,t+1}$  are still (4) and (5) but the first order condition with respect to  $k_{j,t+1}$  becomes

$$p_{j,t} = \beta \mathbb{E}_t \left[ A_{j,t+1} + (1 + \eta_j \mu_{j,t}) p_{j,t+1} \right]. \quad (7)$$

In this case the price  $p_{j,t}$  also depends on the multiplier  $\mu_{j,t}$ , which captures the tightness of the borrowing constraint. Thus, changes in financial market conditions now affect also the market price of houses.

### 2.3 Equilibrium with direct borrowing and lending

Before introducing the financial intermediation sector it would be instructive to characterize the equilibrium with direct borrowing and lending. In equilibrium, the worldwide bonds purchased by entrepreneurs are equal to the household debt, that is,

$$B_{1,t+1} + B_{2,t+1} = L_{1,t+1} + L_{2,t+1},$$

where capital letters denotes the aggregation of individual variables. In equilibrium, the interest rate on bonds is equal to the interest rate on loans, that is,  $1/q_t = R_t$ . Because of capital mobility and cross-country heterogeneity, the net foreign asset positions of the two countries could be different from zero, that is,  $B_{j,t+1} \neq L_{j,t+1}$ .

**Proposition 2.1** *Suppose that  $A_{j,t}$  is constant and there are no financial losses for entrepreneurs, that is,  $\tilde{B}_{j,t} = B_{j,t}$ . The economy converges to a steady state where households borrow from entrepreneurs and  $q = 1/R > \beta$ .*

**Proof 2.1** *See Appendix D*

The fact that the steady state interest rate is lower than the intertemporal discount rate is a consequence of the uninsurable risk faced by entrepreneurs. If  $q = \beta$ , entrepreneurs would continue to accumulate bonds without limit as an insurance against the idiosyncratic risk. The supply of bonds from households, however, is limited by the borrowing limit. To insure that entrepreneurs do not accumulate an infinite amount of bonds, the interest rate has to fall below the intertemporal discount rate.

The equilibrium in the labor market in each country is determined by the intersection of the demand and supply of labor as shown in Figure 2. The aggregate demand in country  $j$  was derived in the previous subsection and takes the form  $H_{j,t}^D = \phi_{j,t}(w_{j,t})\tilde{B}_{j,t}$ . It depends negatively on the wage rate  $w_{j,t}$  and positively on the aggregate wealth of entrepreneurs,  $\tilde{B}_{j,t}$ . The supply of labor is derived from the households' first order condition (4) and takes the form  $H_{j,t}^S = \left(\frac{w_{j,t}}{\alpha\mathbb{E}_t A_{j,t}}\right)^\nu$ .

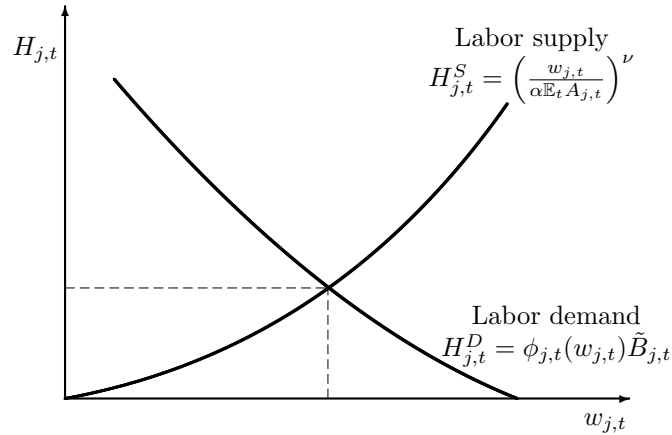


Figure 2: Labor market equilibrium.

The dependence of the demand of labor from the financial wealth of entrepreneurs is an important property of this model. When entrepreneurs

hold a lower value of  $\tilde{B}_{j,t}$ , the demand for labor declines and in equilibrium there is lower employment and production. Importantly, the reason lower entrepreneurial wealth reduces the demand for labor is not because employers lack the funds to finance hiring or because they face a higher financing cost. In the model, employers do not need any financing to hire and produce. Instead, the transmission mechanism is based on the lower insurance of the production risk. This mechanism is clearly distinct from the typical ‘credit channel’ where firms are in need of funds to finance employment (for example, because wages are paid in advance) or to finance investment.

The next step is to introduce financial intermediaries and show that a decline in entrepreneurial wealth could be the consequence of a crisis that originates in financial markets. More specifically, a financial crisis will generate financial losses  $\delta_t > 0$ , and therefore,  $\tilde{B}_{j,t} < B_{j,t}$ .

**Discussion and remarks** Before proceeding with the description of the financial intermediation sector, it would be helpful to clarify the importance of some modeling assumptions and the associated properties.

The equilibrium of this model is characterized by producers (entrepreneurs) that are net savers and households that are net borrowers. This equilibrium structure differs from other models proposed in the literature where, typically, producers are net borrowers. Although having producers with positive net financial wealth might appear counterfactual at first, it is not inconsistent with the recent changes in the financial structure of US corporations. It is well known that during the last two and half decades, US corporations have increased their holdings of financial assets. This suggests that the proportion of financially dependent firms has declined significantly over time, consistent with the empirical findings of Shourideh and Zetlin-Jones (2012) and Eisfeldt and Muir (2012).

The large accumulation of financial assets by firms (often referred to ‘cash’) is related to the significance of business savings. Busso, Fernández, and Tamayo (2016) document the share of savings done by firms both in advanced and emerging countries and present evidence that in Latin America this share is even larger than in industrialized economies. The importance of business savings is also documented in Bebczuk and Cavallo (2015). Using data for 47 countries over 1995–2013 they show that the contribution of businesses to national savings is on average more than 50%. The model developed here captures the growing importance of firms that are not heavily

dependent on external financing.

The second remark is that the equilibrium property for which firms are net lenders does not rely on the assumption that households are risk neutral. What is crucial for the model to generate this property is that only entrepreneurs are exposed to uninsurable idiosyncratic risks. As long as entrepreneurs face more risk than households, entrepreneurs would continue to be net lenders in equilibrium even if households were risk averse.

The third remark relates to the assumption that the idiosyncratic risk faced by entrepreneurs cannot be insured away fully (market incompleteness). Since households are risk neutral, it would be optimal to offer wages that are contingent on the output of the firm. However, as we show in Appendix A, the idiosyncratic shock is the residual risk that cannot be insured away because of agency issues. The micro-foundation provided in the appendix also explains why wages cannot be state-contingent.

## 2.4 Financial intermediation sector

If direct borrowing is not feasible or efficient, financial intermediaries would play an important role for transferring funds from lenders to borrowers and to create financial assets that could be held for insurance purposes. By specializing in financial intermediation, financial firms have a comparative advantage (lower cost) in transferring funds from lenders to borrowers. It is under this premise that I introduce the financial intermediation sector.

Financial intermediaries are infinitely lived, profit-maximizing firms owned by households. The assumption that financial firms are owned by households, as opposed to entrepreneurs, is motivated by two considerations. The first is for analytical simplicity. The risk neutrality of shareholders implies that the operation of banks is not affected by the ownership structure (domestic versus foreign households). The second consideration is more substantive and relates to the redistributive consequences of a financial crisis. As we will see, a financial crisis generates wealth losses for the holders of bank liabilities (entrepreneurs) and wealth gains for the owners of banks, that is, households. If entrepreneurs were also the shareholders of banks, they would not experience any financial losses (the losses from holding  $b_{j,t}^i$  would be compensated by the reduction in the debt of banks they own). Similar properties could be obtained by assuming that banks are owned by risk-neutral investors that are distinct from both households and entrepreneurs.

Banks operate globally, that is, they sell liabilities and make loans to

domestic and foreign households. As observed above, the ownership of banks by domestic or foreign households is irrelevant for the equilibrium. Also notice that, even if I use the term ‘banks’, it should be clear that the financial sector is representative of all financial firms, not only commercial banks or depository institutions.

A bank starts the period with loans made to households,  $l_t$ , and liabilities held by entrepreneurs,  $b_t$ . These loans and liabilities were issued in the previous period  $t - 1$ . Since the interest rates on loans will be equalized across countries, banks are indifferent about the nationality of their borrowers as long as the borrowing constraints, which are country-specific, are not violated. Similarly, the interest rate paid by banks on their liabilities will be equalized across countries. Therefore, I will use the notation  $l_t$  and  $b_t$  without subscript  $j$  to denote the loans and liabilities of an individual bank. The difference between loans and liabilities is the bank’s equity  $e_t = l_t - b_t$ .

**Renegotiation of bank liabilities** Given the beginning of period balance sheet position, banks could default on their liabilities. In case of default, creditors have the right to liquidate the assets of the bank  $l_t$ . However, they may not recover the full value of the liquidated assets. The recovery fraction, denoted by  $\xi_t \leq 1$ , is an endogenous aggregate variable (same for all banks) that is realized at the beginning of period  $t$ . Therefore,  $\xi_t$  was unknown at time  $t - 1$  when the bank issued the liabilities  $b_t$  and made the loans  $l_t$ .

Since  $\xi_t$  is an aggregate stochastic variable, to characterize the optimal policies of individual banks we can take  $\xi_t$  as given. The next period value  $\xi_{t+1}$  will be drawn from a distribution function  $f_t(\xi_{t+1})$ . This distribution is an endogenous object that will be determined in the general equilibrium. The subscript  $t$  indicates its dependence on the aggregate states of the economy (as specified later).

Once the value of  $\xi_t$  becomes known at the beginning of period  $t$ , banks could use the threat of default to renegotiate the outstanding liabilities  $b_t$ . Under the assumption that banks have the whole bargaining power, the outstanding liabilities could be renegotiated to the liquidation value of assets  $\xi_t l_t$ . Of course, banks will renegotiate only if the liabilities are bigger than the liquidation value, that is,  $b_t > \xi_t l_t$ . Therefore, after renegotiation, the

residual liabilities of a bank are

$$\tilde{b}_t(b_t, l_t) = \begin{cases} b_t, & \text{if } b_t \leq \xi_t l_t \\ \xi_t l_t & \text{if } b_t > \xi_t l_t \end{cases}. \quad (8)$$

Renegotiation carries a cost that increases with leverage. The cost takes the form  $(\omega_t - \xi_t)^2 b_t$ , where  $\omega_t = b_t/l_t$  is the leverage of the bank. The cost is incurred only if the bank renegotiates, which arises when  $\omega_t > \xi_t$ . In addition, the bank faces an operation cost  $\tau b_t$ . The sum of the operation and renegotiation costs will be denoted by  $\varphi_t(\omega_t)b_t$ , with the unit cost function taking the form

$$\varphi_t(\omega_t) = \tau + \begin{cases} 0 & \text{if } \omega_t \leq \xi_t \\ (\omega_t - \xi_t)^2 & \text{if } \omega_t > \xi_t \end{cases} \quad (9)$$

When banks renegotiate their liabilities, there is a redistribution of wealth from entrepreneurs (the creditors of the bank) to households (the shareholders of banks). This property may appear counterfactual. However, at the cost of some additional analytical complexity, the model could be modified by assuming that in a crisis banks realize capital losses because households renegotiate their debt. The losses are then transferred first to shareholders and then to the bank creditors (entrepreneurs). This alternative model would have similar redistributive implications since the bank losses, which are transferred to entrepreneurs, represent gains for households in the form of lower repayments of the loans. Therefore, even if in the model the redistribution to households takes the form of higher dividends, we could think of them as lower repayments of household debt. See Quadrini (2016) for a formalization of this alternative model.

**Price of liabilities** The possibility that a bank renegotiates its liabilities is fully anticipated and will be reflected in their market price.

Denote by  $\bar{R}_t^b$  the expected gross return from holding the market portfolio of bank liabilities issued in period  $t$  and repaid in period  $t + 1$ . Since banks are competitive, the expected return on the liabilities issued by an individual bank must be equal to the aggregate expected return  $\bar{R}_t^b$ . The price for the liabilities issued at time  $t$  by an individual bank, denoted by  $q_t(b_{t+1}, l_{t+1})$ ,



satisfies

$$q_t(b_{t+1}, l_{t+1})b_{t+1} = \frac{1}{\bar{R}_t^b} \mathbb{E}_t \tilde{b}_{t+1}(b_{t+1}, l_{t+1}). \quad (10)$$

The left-hand-side is the payment incurred by investors to purchase  $b_{t+1}$ . The term on the right-hand-side is the expected repayment in the next period, discounted by  $\bar{R}_t^b$  (the expected market return for the liabilities of the whole banking sector). Since an individual bank could renegotiate in the next period if  $\omega_{t+1} > \xi_{t+1}$ , the actual repayment  $\tilde{b}_{t+1}(b_{t+1}, l_{t+1})$  could differ from  $b_{t+1}$ . Arbitrage requires that the cost of purchasing  $b_{t+1}$  for investors (the left-hand-side of (10)) is equal to the discounted value of the expected repayment (the right-hand-side of (10)).

**Bank problem** The budget constraint of the bank after renegotiation can be written as

$$\tilde{b}_t(b_t, l_t) + \varphi_t \left( \frac{b_t}{l_t} \right) b_t + \frac{l_{t+1}}{R_t^l} + d_t = l_t + q_t(b_{t+1}, l_{t+1})b_{t+1}. \quad (11)$$

The left-hand-side of the budget constraint contains the residual liabilities after renegotiation, the operation/renegotiation cost, the cost of making new loans, and the dividends paid to shareholders (households). The right-hand-side contains the initial loans and the funds raised by issuing new liabilities. Using the arbitrage condition (10), the funds raised with new liabilities are equal to  $\mathbb{E}_t \tilde{b}_{t+1}(b_{t+1}, l_{t+1}) / \bar{R}_t^b$ .

The optimization problem of the bank can be written recursively as

$$V_t(b_t, l_t) = \max_{d_t, b_{t+1}, l_{t+1}} \left\{ d_t + \beta \mathbb{E}_t V_{t+1}(b_{t+1}, l_{t+1}) \right\} \quad (12)$$

subject to (8), (10), (11).

The leverage chosen by the bank will never exceed 1 since this will trigger renegotiation with certainty. Once the probability of renegotiation is 1, a further increase in  $b_{t+1}$  does not increase the borrowed funds but raises the renegotiation cost. Therefore, the optimization problem of the bank is also subject to the constraint  $b_{t+1} \leq l_{t+1}$ .

The first order conditions with respect to  $b_{t+1}$  and  $l_{t+1}$ , derived in Appendix E, can be expressed as

$$\frac{1}{\bar{R}_t^b} \geq \beta \left[ 1 + \Phi(\omega_{t+1}) \right] \quad (13)$$

$$\frac{1}{R_t^l} \geq \beta \left[ 1 + \Psi(\omega_{t+1}) \right], \quad (14)$$

where  $\Phi(\omega_{t+1})$  and  $\Psi(\omega_{t+1})$  are increasing functions of the leverage  $\omega_{t+1}$ . These conditions are satisfied with equality if  $\omega_{t+1} < 1$  and with inequality if  $\omega_{t+1} = 1$  (given the constraint  $\omega_{t+1} \leq 1$ ).

Conditions (13) and (14) make clear that it is the leverage of the bank  $\omega_{t+1} = b_{t+1}/l_{t+1}$  that matters, not the scale of operation,  $b_{t+1}$  or  $l_{t+1}$ . This follows from the linearity of the intermediation technology and the risk neutrality of banks. It implies that in equilibrium all banks choose the same leverage (although they could chose different scales of operation).<sup>4</sup>

Further exploration of the first order conditions (13) and (14) reveals that the funding cost  $\bar{R}_t^b$  is smaller than the interest rate on loans  $R_t^l$ , which is necessary to cover the operation and renegotiation cost of the bank. This property is stated formally in the following lemma.

**Lemma 2.2** *If  $\tau > 0$ , then  $\bar{R}_t^b < R_t^l$ . Furthermore, if  $\int_0^{\omega_{t+1}} f_t(\xi) > 0$ , then  $\bar{R}_t^b < R_t^l < \frac{1}{\beta}$ .*

**Proof 2.2** *See Appendix F*

The positive spread between the lending rate and the cost of funds is necessary to cover the operation cost  $\tau$  and the renegotiation cost if the bank defaults. The term  $\int_0^{\omega_{t+1}} f_t(\xi)$  is the probability that a bank with leverage  $\omega_{t+1}$  will renegotiate at  $t + 1$  (since renegotiation arises when  $\omega_{t+1} > \xi_{t+1}$ ). Thus, the lemma states that, if the probability of renegotiation is positive, the lending rate is smaller than the intertemporal discount rate.

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<sup>4</sup>Because the first order conditions (13) and (14) depend only on one individual variable—the leverage  $\omega_{t+1}$ —there is no guarantee that these conditions are both satisfied for arbitrary values of  $\bar{R}_t^b$  and  $R_t^l$ . In the general equilibrium, however, these rates adjust to clear the markets for bank liabilities and loans and both conditions will be satisfied.

Banks' renegotiation generates a loss of financial wealth for entrepreneurs, causing a macroeconomic contraction through the 'bank liabilities channel' as described earlier. For this to happen, the recovery fraction  $\xi_t$  must fall below the leverage  $\omega_t$  chosen by banks. The next subsection describes the determination of  $\xi_t$ .

## 2.5 Banking liquidity and liquidation value of bank assets

The recovery fraction  $\xi_t$  is made endogenous by assuming that it represents the equilibrium market price for the liquidated assets of banks. The operational structure of the market for liquidated capital is specified by two assumptions.

**Assumption 1** *If a bank is liquidated, its assets  $l_t$  are divisible and can be sold either to other banks or to other sectors (households and entrepreneurs). However, other sectors can recover only a fraction  $\underline{\xi} < 1$ .*

This assumption implies that it is more efficient to sell the assets of a liquidated bank to other banks since they have the ability to recover the whole value  $l_t$  while other sectors can recover only  $\underline{\xi}l_t$ . This is a natural assumption since banks have a comparative advantage in the management of financial investments. However, a bank can purchase the liquidated assets of other banks only if the bank itself does not have an incentive to default.

**Assumption 2** *Banks can purchase the assets of a liquidated bank only if  $b_t < \xi_t l_t$ .*

If a bank starts with a stock of liabilities bigger than the liquidation value of its assets, that is,  $b_t > \xi_t l_t$ , the bank will be unable to raise additional funds to purchase the liquidated assets of other banks. Potential investors know that the new liabilities (as well as the outstanding liabilities) are not collateralized and the bank will renegotiate immediately after issuing the new liabilities. I refer to a bank that satisfies the condition  $b_t < \xi_t l_t$  as a 'liquid' bank.<sup>5</sup>

To better understand Assumptions 1 and 2, consider the condition for not renegotiating,  $b_t \leq \xi_t l_t$ . The variable  $\xi_t \in \{\underline{\xi}, 1\}$  is the liquidation price of

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<sup>5</sup>The bank is liquid because it has the ability to raise extra funds by issuing liabilities.

bank assets at the beginning of the period. If this condition is satisfied, banks have the ability to raise funds to purchase the assets of a defaulting bank. This insures that the market price for the liquidated assets is  $\xi_t = 1$ . However, if  $b_t > \xi_t l_t$  for all banks, there will be no bank capable of buying the liquidated assets. As a result, the liquidated assets can only be sold to non-banks. But then the price will be  $\xi_t = \underline{\xi}$ . Therefore, the value of the liquidated assets depends on the financial decision of banks, which in turn depends on the expected liquidation value of their assets. This interdependence creates the conditions for multiple self-fulfilling equilibria.

**Proposition 2.2** *There exists multiple equilibria if and only if the leverage of banks is within the two liquidation prices, that is,  $\underline{\xi} \leq \omega_t \leq 1$ .*

**Proof 2.2** *See appendix G.*

When multiple equilibria are possible, the equilibrium is selected through the random draw of sunspot shocks.

Let  $\varepsilon_t$  be a variable that takes the value of 0 with probability  $\lambda$  and 1 with probability  $1 - \lambda$ . If both prices are possible in equilibrium, agents coordinate their expectations on the low liquidation price when  $\varepsilon_t = 0$  and on the high liquidation price when  $\varepsilon_t = 1$ . Thus, the probability distribution of the liquidation price is

$$f_{t-1}(\xi_t = \underline{\xi}) = \begin{cases} 0, & \text{if } \omega_t < \underline{\xi} \\ \lambda, & \text{if } \underline{\xi} \leq \omega_t \leq 1 \end{cases}$$

$$f_{t-1}(\xi_t = 1) = \begin{cases} 1, & \text{if } \omega_t < \underline{\xi} \\ 1 - \lambda, & \text{if } \underline{\xi} \leq \omega_t \leq 1 \end{cases}$$

If the leverage is sufficiently small ( $\omega_t < \underline{\xi}$ ), banks remain liquid even if the (expected) liquidation price is  $\xi_t = \underline{\xi}$ . But then the liquidation price cannot be low and the realization of the sunspot shock is irrelevant for the equilibrium. Instead, when the leverage is between the two liquidation prices ( $\underline{\xi} \leq \omega_t \leq 1$ ), the liquidity of banks depends on the expectation of this price. The realization of the sunspot shock  $\varepsilon_t$  then becomes important for selecting one of the two equilibria. When  $\varepsilon_t = 0$ —which happens with probability  $\lambda$ —the market expects that the liquidation price is  $\xi_t = \underline{\xi}$ , making the banking sector illiquid. On the other hand, when  $\varepsilon_t = 1$ —which happens with

probability  $1 - \lambda$ —the market expects that the liquidation price is  $\xi_t = 1$ , and the banking sector remains liquid, validating the expectation of the high liquidation price.

## 2.6 General equilibrium

At the beginning of the period, the aggregate states of the economy are given by the bank liabilities held by entrepreneurs in both countries,  $B_{1,t}$  and  $B_{2,t}$ , the loans made by banks to households in both countries,  $L_{1,t}$  and  $L_{2,t}$ , aggregate productivity in both countries,  $A_{1,t-1}$  and  $A_{2,t-1}$ . Since aggregate productivity is still unknown at the beginning of the period, the set of state variables includes lagged productivity. The vector of aggregate states is denoted by  $\mathbf{s}_t = (B_{1,t}, B_{2,t}, L_{1,t}, L_{2,t}, A_{1,t-1}, A_{2,t-1})$ .

Given the states, the worldwide liabilities and loans of banks are equal to  $B_t = B_{1,t} + B_{2,t}$  and  $L_t = L_{1,t} + L_{2,t}$ . The leverage, common to all banks, is equal to  $\omega_t = B_t/L_t$ . The equilibrium in each period is determined sequentially in three subperiods:

1. **Subperiod 1:** The sunspot shock  $\varepsilon_t$  is realized and agents form self-fulfilling expectations about the liquidation price  $\xi_t$  based on the realization of the sunspot shock. As described above, this implies that the equilibrium price is  $\xi_t = \underline{\xi}$  if  $\omega_t \geq \underline{\xi}$  and  $\varepsilon_t = 0$ . Otherwise,  $\xi_t = 1$ . Given the liquidation price banks choose whether to default and renegotiate. The renegotiated liabilities are

$$\tilde{B}_t = \begin{cases} \underline{\xi} L_t, & \text{if } \omega_t \geq \underline{\xi} \text{ and } \varepsilon_t = 0 \\ B_t & \text{otherwise} \end{cases}.$$

The post-renegotiation liabilities held by each country are proportional to their pre-renegotiated holdings, that is,  $\tilde{B}_{1,t} = \tilde{B}_t (B_{1,t}/B_t)$  and  $\tilde{B}_{2,t} = \tilde{B}_t (B_{2,t}/B_t)$ .

2. **Subperiod 2:** Given the residual wealth  $\tilde{B}_{j,t}$ , entrepreneurs choose the demand for labor and workers choose the supply. Market clearing in the labor market determines the wage rate  $w_{j,t}$  and employment  $H_{j,t}$  in each country. At this stage the idiosyncratic productivity  $\pi_{j,t}^i$  and the aggregate productivity  $A_{j,t}$  are unknown. Therefore, decisions are based on their probability distributions.

3. **Subperiod 3:** Idiosyncratic and aggregate productivities  $\pi_{j,t}^i$  and  $A_{j,t}$  are realized. The wealth of entrepreneurs becomes  $\tilde{B}_{j,t} + (A_{j,t} - w_{j,t})H_{j,t}$  which is in part consumed and in part saved in new bank liabilities,  $q_t B_{j,t+1}$ . Households choose borrowing  $L_{j,t+1}/R_t^l$  and the holding of the productive asset. Banks choose the new leverage  $\omega_{t+1} = B_{t+1}/L_{t+1}$ . Market clearing determines the price for bank liabilities  $q_t$ , the interest rate on loans  $R_t^l$ , and the new stocks of bank liabilities and loans in each country, that is,  $B_{1,t+1}$ ,  $B_{2,t+1}$ ,  $L_{1,t+1}$ ,  $L_{2,t+1}$ . The prices for productive assets  $p_{1,t}$  and  $p_{2,t}$  are also determined.

In the rest of this section I will focus on the version of the model in which the borrowing limit takes the form specified in (2). This allows me to characterize the equilibrium analytically. The characterization of the general equilibrium proceeds in three steps:

1. I first derive the aggregate ‘demand’ for bank liabilities from the optimal saving decision of entrepreneurs.
2. I then derive the aggregate ‘supply’ of bank liabilities by consolidating the demand of loans from households with the optimal policies of banks.
3. Finally, I derive the general equilibrium by combining the demand and supply of bank liabilities derived in the first two steps.

**Step 1: Demand for bank liabilities.** As shown in Lemma 2.1, the optimal savings of entrepreneurs takes the form  $q_t b_{j,t+1}^i = \beta a_{j,t}^i$ , where  $a_{j,t}^i$  is the end-of-period wealth  $a_{j,t}^i = \tilde{b}_t^i + (z_{j,t}^i - w_{j,t})h_{j,t}^i$ .

Since  $h_{j,t}^i = \phi_j(w_{j,t})\tilde{b}_{j,t}^i$  (see Lemma 2.1), the end-of-period wealth can be rewritten as  $a_{j,t}^i = [1 + (z_{j,t}^i - w_{j,t})\phi_j(w_{j,t})]\tilde{b}_{j,t}^i$ . Substituting into the optimal saving and aggregating over all entrepreneurs we obtain

$$q_t B_{j,t+1} = \beta \left[ 1 + (A_{j,t} - w_{j,t})\phi_j(w_{j,t}) \right] \tilde{B}_{j,t}. \quad (15)$$

This equation defines the aggregate demand for bank liabilities in country  $j$  as a function of its price  $q_t$ , the wage rate  $w_{j,t}$ , and the wealth of entrepreneurs  $\tilde{B}_{j,t}$ .

Using the equilibrium condition in the labor market, we can express the wage rate as a function of  $\tilde{B}_t$ . In particular, equalizing the demand for labor,

$H_{j,t}^D = \phi_j(w_{j,t})\tilde{B}_{j,t}$ , to the supply from households,  $H_{j,t}^S = (w_{j,t}/\alpha A_{j,t})^\nu$ , the wage can be expressed only as a function of  $\tilde{B}_{j,t}$ . We can then use this function to rewrite equation (15) more compactly as  $q_t B_{j,t+1} = s_j(\tilde{B}_{j,t})$ .

The total (worldwide) demand for bank liabilities is the sum of the demands in both countries, that is,

$$B_{t+1} = \left[ s_1(\tilde{B}_{1,t}) + s_1(\tilde{B}_{2,t}) \right] \frac{1}{q_t}. \quad (16)$$

Figure 3 plots this function for given values of  $\tilde{B}_{1,t}$  and  $\tilde{B}_{2,t}$ . The demand for  $B_{t+1}$  increases when its price  $q_t$  declines. The slope depends (positively) on the entrepreneurs' wealth  $\tilde{B}_{1,t}$  and  $\tilde{B}_{2,t}$ .

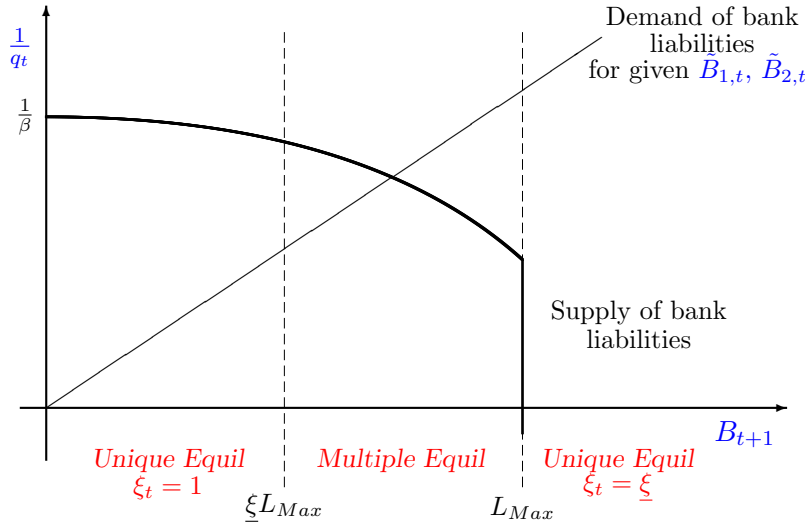


Figure 3: Demand and supply of bank liabilities.

**Step 2: Supply of bank liabilities** The supply of bank liabilities is derived from consolidating the borrowing decisions of households with the investment and funding decisions of banks.

According to Lemma 2.2, when banks are leveraged, the interest rate on loans must be smaller than the intertemporal discount rate, that is,  $R_t^l < 1/\beta$ . From the households' first order condition (5) we can see that the lagrange multiplier associated with the borrowing constraint  $\mu_{j,t}$  is greater than zero

if  $R_t^l < 1/\beta$ . This implies that the borrowing constraint of households is binding and the loans received by households are equal to the borrowing limits, that is,  $L_{j,t+1} = \eta_j A_{j,t}$ . The total loans made by banks is the sum of the loans made in both countries, that is,  $L_{t+1} = \eta_1 A_{1,t} + \eta_2 A_{2,t}$ .

By definition,  $B_{t+1} = \omega_{t+1} L_{t+1}$ . We can then express the total supply of bank liabilities as

$$B_{t+1} = \omega_{t+1}(\eta_1 A_{1,t} + \eta_2 A_{2,t}). \quad (17)$$

So far I have derived the supply of bank liabilities as a function of leverage  $\omega_{t+1}$ . However, leverage is endogenously chosen by banks and depends on the cost of borrowing  $\bar{R}_t^b$  (see the optimality condition (13)). The expected return  $\bar{R}_t^b$  is in turn related to the price of bank liabilities  $q_t$  through the condition

$$q_t = \frac{1}{\bar{R}_t^b} \left[ 1 - \theta(\omega_{t+1}) + \theta(\omega_{t+1}) \left( \frac{\xi}{\omega_{t+1}} \right) \right], \quad (18)$$

where  $\theta(\omega_{t+1})$  denotes the probability that banks default.

The term in square brackets on the right-hand-side is the expected payment received at time  $t + 1$  from holding one unit of bank liabilities. With probability  $1 - \theta(\omega_{t+1})$  banks do not renegotiate and pay back 1. With probability  $\theta(\omega_{t+1})$  banks renegotiate and investors receive only the fraction  $\xi/\omega_{t+1}$ . The current value of the expected repayment, discounted by the market return  $\bar{R}_t^b$ , must be equal to the price  $q_t$ .

Using (18) to replace  $\bar{R}_t^b$  in equation (13), we obtain a function that relates the price of bank liabilities  $q_t$  to leverage  $\omega_{t+1}$ . Finally, using (17) to eliminate  $\omega_{t+1}$ , we obtain the supply of liabilities as a function of  $q_t$ . The derived supply, plotted in Figure 3, is decreasing in  $1/q_t$  until it reaches a maximum. This is the maximum loans that can be made to households, that is,  $L_{Max} = \eta_1 A_{1,t} + \eta_2 A_{2,t}$ . If banks issue more liabilities than loans, their leverage would be bigger than 1 triggering default with certainty.

**Step 3: Demand and supply together.** I can now characterize the general equilibrium as the intersection of the aggregate demand and supply for bank liabilities derived above. As shown in Figure 3, the supply (from banks) is decreasing in  $1/q_t$  while the demand (from entrepreneurs) is increasing in  $1/q_t$ . The demand is plotted for a particular value of outstanding post-renegotiation liabilities  $\tilde{B}_t = \tilde{B}_{1,t} + \tilde{B}_{2,t}$ . By changing the outstanding



liabilities, the slope of the demand function would also change and would result in different equilibrium price and liabilities.

The figure also indicates the regions with unique and multiple equilibria. When the liabilities exceed  $\underline{\xi}L_{Max}$ , multiple equilibria are possible. In this case the economy is subject to stochastic fluctuations induced by the realization of the sunspot shock. Whether the economy is in the region with unique or multiple equilibria depends on the initial state  $\tilde{B}_t$ , which evolves endogenously.

The model generates a simple dynamics. Given the initial aggregate wealth of entrepreneurs  $\tilde{B}_t$ , we can solve for  $q_t$  and  $B_{t+1}$  by equalizing the aggregate demand and supply of bank liabilities as shown in Figure 3. This in turn allows us to determine the next period wealth  $\tilde{B}_{t+1}$ . In absence of renegotiation we have  $\tilde{B}_{t+1} = B_{t+1}$ , where  $B_{t+1}$  is determined by equation (16). In the event of renegotiation (if in the region with multiple equilibria) we have  $\tilde{B}_{t+1} = (\underline{\xi}/\omega_{t+1})B_{t+1}$ . The new  $\tilde{B}_{t+1}$  will determine a new slope for the demand of bank liabilities, and therefore, new values of  $q_t$  and  $B_{t+1}$ .

In absence of aggregate productivity shocks (constant values of  $A_{j,t}$ ), the economy may or may not reach a steady state. In order to reach a steady state the economy must converge to a state  $B_t < \underline{\xi}L_{Max}$  (region with a unique equilibrium). However, if the economy does not converge to this region, it will experience stochastic fluctuations associated with the realization of the sunspot shock. The renegotiation and operation cost  $\varphi(\omega_{t+1})$  plays an important role in determining the type of long-run equilibria (unique or not).

Figure 3 is also helpful for understanding the dynamics and severity of crises. When banks are more leveraged, the economy switches from a state in which the equilibrium is unique (no crises) to a state with multiple equilibria (with the possibility of financial crises). But even if the economy was already in a state with multiple equilibria, the increase in leverage implies that the consequences of a crisis are more severe. In fact, when the economy switches from the non-renegotiation equilibrium (no crisis) to the equilibrium with renegotiation (financial crisis), bank liabilities are renegotiated to  $\underline{\xi}L_{Max}$ . Therefore, bigger are the liabilities  $B_t$  issued by banks and larger are the losses incurred by entrepreneurs holding these liabilities. Larger financial losses incurred by entrepreneurs then imply larger declines in the demand for labor in both countries (larger macroeconomic contractions).

## 2.7 The growth of emerging countries and bank leverage

Before moving to the quantitative section, it would be helpful to provide some intuition of how the growth of emerging countries effects the equilibrium.

A central variable for the characterization of the equilibrium is the leverage of banks defined as

$$LEVERAGE = \frac{B_{1,t} + B_{2,t}}{L_{1,t} + L_{2,t}}.$$

This can be rewritten as

$$LEVERAGE = \frac{\left(\frac{B_{1,t}}{L_{1,t}}\right) L_{1,t} + \left(\frac{B_{2,t}}{L_{2,t}}\right) L_{2,t}}{L_{1,t} + L_{2,t}}.$$

Suppose that the two countries are homogeneous, and therefore,  $B_{1,t}/L_{1,t} = B_{2,t}/L_{2,t}$ . Furthermore, assume that, if country 2 grows in size by  $g_t$ , both  $B_{2,t}$  and  $L_{2,t}$  grow by the same rate so that the ratio of these two variables does not change. This implies that the leverage of banks also does not change and will be equal to  $B_{1,t}/L_{1,t} = B_{2,t}/L_{2,t}$ .

Now consider the case in which the two countries are heterogeneous with  $B_{1,t}/L_{1,t} < B_{2,t}/L_{2,t}$ . If we think of country 1 as representative of industrialized economies and country 2 as representative of emerging economies, this would capture the idea that the domestic savings of emerging countries exceed domestic credit. Now suppose that the size of country 2 increases at rate  $g_t$ . Furthermore, suppose that  $B_{2,t}$  (domestic savings) and  $L_{2,t}$  (domestic credit) also grow by the same rate so that  $B_{2,t}/L_{2,t}$  does not change. The leverage of banks would then become

$$LEVERAGE = \frac{\left(\frac{B_{1,t}}{L_{1,t}}\right) L_{1,t} + \left(\frac{B_{2,t}}{L_{2,t}}\right) L_{2,t}(1 + g_t)}{L_{1,t} + L_{2,t}(1 + g_t)}.$$

Since we started with the assumption that  $B_{1,t}/L_{1,t} < B_{2,t}/L_{2,t}$ , higher is the growth rate of country 2 and higher will be the leverage of banks.

In the general equilibrium, of course, the growth of  $B_{2,t}$  and  $L_{2,t}$  could differ and the ratios  $B_{1,t}/L_{1,t}$  and  $B_{2,t}/L_{2,t}$  would change. To the extent that these ratios do not change too much, the growth of country 2 will lead to higher banking leverage. The higher leverage implies larger financial losses for entrepreneurs when a crisis materializes which in turn generates higher financial and macroeconomic instability.

In general, however, since the ratios  $B_{2,t}/L_{2,t}$  could decline when country 2 grows, a higher value of  $g_t$  could also result in lower banking leverage. In fact, even if the growth in productivity of country 2 leads to higher values of  $B_{2,t}$  and  $L_{2,t}$ , the second variable (household debt) could grow more than the second variable (entrepreneurial savings). This does not happen if the increase in  $B_t$  is sufficiently large, which in turn requires that entrepreneurs experience large increases in income. The assumption that aggregate productivity is observed with delay plays a crucial role in generating a large increase in  $B_{2,t}$ . Since  $A_{2,t}$  is still unknown when workers are hired and wages clear the labor market, an unexpected growth in productivity will generate an unexpected increase in entrepreneurial profits. If the increase in  $g_t$  was anticipated, the higher productivity would generate an immediate increase in wages with only a small effect on profits. In this case households would borrow more but the savings of entrepreneurs increase only marginally. Then, the higher household debt would be mostly financed with bank equities and the overall leverage of banks could decline. This point will be further discussed in the quantitative section of the paper.

### 3 Quantitative analysis

In this section I calibrate the model to study the quantitative impact of the growth of emerging countries on financial and macroeconomic stability. The calibration uses data for the period 1991-2013 under the assumption that country 1 is representative of industrialized economies and country 2 is representative of emerging economies. Starting in 1991, I simulate the model until 2013. The list of industrialized and emerging countries is provided in Figure 1.

For the quantitative exercise I use the borrowing limit specified in equation (3). This specification implies that the price of the fixed asset  $p_t$  responds to changing financial market conditions.

#### 3.1 Calibration

The model is calibrated annually. Changes in the relative economic size of the two countries are captured by the relative productivity  $A_{2,t}/A_{1,t}$ . Therefore, an important element of the quantitative exercise is the construction of the sequence productivity.

Total production is the sum of entrepreneurial output,  $A_{j,t}H_{j,t}$ , and housing services,  $A_{j,t}\bar{K}$ . Therefore, aggregate production in country  $j$  is equal to  $Y_{j,t} = A_{j,t}(H_{j,t} + \bar{K})$ . Because in the model there is no capital accumulation, the empirical counterpart of aggregate output is Gross Domestic Product minus Investment.

The relative productivity  $A_{2,t}/A_{1,t}$  are chosen to replicate the relative economic size of the two (groups of) countries over the period 1991-2013 measured at nominal exchange rates, not PPP. This is consistent with the goal of the quantitative exercise whose aim is to study how the change in the relative size of the two countries affects the world demand for financial assets. Since movements in nominal exchange rates affect the purchasing power of a country in the acquisition of foreign assets, the relative productivity  $A_{2,t}/A_{1,t}$  should also reflect these movements. Another factor that contributes to generate differences in the overall economic size of the two countries is population growth. Since population differences are not explicitly modelled,  $A_{2,t}/A_{1,t}$  should also reflect changes in population.

Define the nominal output of country  $j$  as

$$P_{j,t}Y_{j,t} = P_{j,t}\hat{A}_{j,t}(H_{j,t} + \bar{K})N_{j,t},$$

where  $\hat{A}_{j,t}$  is actual productivity,  $H_{j,t}$  is labor supply per household,  $\bar{K}$  is the endowment of houses per household,  $N_{j,t}$  is population, and  $P_{j,t}$  is the nominal price of country  $j$  expressed in the same currency units for all countries. For example, using US dollars as the common denominator, prices are calculated by multiplying local currency units by the dollar exchange rate. Notice that the above definition of output assumes that the endowment of houses increases with population. This is necessary to preserve balanced growth.

The economic size of country 2 relative to the size of country 1 is then

$$\frac{P_{2,t}Y_{2,t}}{P_{1,t}Y_{1,t}} = \frac{P_{2,t}\hat{A}_{2,t}N_{2,t}}{P_{1,t}\hat{A}_{1,t}N_{1,t}} \left( \frac{H_{2,t} + \bar{K}}{H_{1,t} + \bar{K}} \right) \equiv \frac{A_{2,t}}{A_{1,t}} \left( \frac{H_{2,t} + \bar{K}}{H_{1,t} + \bar{K}} \right). \quad (19)$$

The equation shows that the productivity ratio in the model,  $A_{2,t}/A_{1,t}$ , also reflects cross-country differences in population and prices.

Before I can use Equation (19) to back up  $A_{2,t}/A_{1,t}$ , I need to pin down the value of  $\bar{K}$ . This is done by using the share of housing services in net GDP (net of investment), which in the model is equal to  $\bar{K}/(H_{j,t} + \bar{K})$ . Unfortunately, data for the share of housing services is not available for many countries. To obviate this problem, I impose that all countries have

the same share of housing services in output (GDP minus investment in the data) and use the US share as the calibration target for both countries. Using data from NIPA, the average share of housing services on net GDP over the period 1991-2013 is 12.2%. Therefore,  $\bar{K}$  is calibrated using the condition

$$\frac{\bar{K}}{\bar{H} + \bar{K}} = 0.122.$$

The variable  $\bar{H}$  is set to the average employment-to-population ratio over the period 1991-2013 for all countries (emerging and industrialized). Using data from the World Development Indicators (WDI), this ratio is equal to 0.449.

Given the value of  $\bar{K}$ , I then compute the sequence of  $A_{2,t}/A_{1,t}$  with equation (19). The variable  $P_{j,t}Y_{j,t}$  is measured in the data as GDP minus investment in current US dollars from the WDI. The variable  $H_{j,t}$  is measured as the ratio of employment over total population also from the WDI. The resulting sequence is plotted in Figure 4.

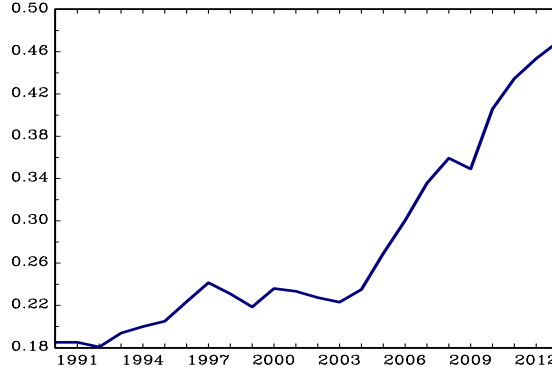


Figure 4: Relative productivity of emerging vs. industrialized countries, 1990-2013.

For the simulation of the model I also need to specify the stochastic processes for the aggregate productivity of the two groups of countries. I assume that  $A_{j,t}$  follows the process

$$\ln(A_{j,t}) = \rho + \ln(A_{j,t-1}) + \epsilon_{j,t},$$

where  $\epsilon_{j,t}$  is a shock that is independently and identically distributed across countries and over time. Notice that the expected growth rate of productivity  $\rho$  does not have the country subscript  $j$ . Thus, the expected growth rate

of productivity is assumed to be the same in the two countries. Even if emerging and industrialized countries have experienced very different growth rates during the last two decades, this is not the case when we look at a longer horizon. Furthermore, it is not obvious that the growth differential experienced by the two groups of countries during the last two decades was anticipated. I further assume that  $\epsilon_{1,t}$  and  $\epsilon_{2,t}$  are independently drawn from the same normal distribution with zero mean and standard deviation  $\sigma_\epsilon$ . I set  $\sigma_\epsilon$  to the average standard deviation of GDP growth for emerging and industrialized countries over the period 1970-2013 (which is the period for which WDI data is available).<sup>6</sup>

**Remaining parameters** The discount factor is set to  $\beta = 0.94$ , implying an annual intertemporal discount rate of about 6%. The parameter  $\nu$  in the utility function of households is the elasticity of labor supply. In order to capture possible wage rigidities, I set the elasticity to the high value of 50. The alternative would be to model explicitly downward wage rigidities but this requires an additional state variable and would make the computation of the model much more demanding. The utility parameter  $\alpha$  is chosen so that the average labor in the model is equal to the average ratio of employment over population during the period 1991-2013 for all sample countries (industrialized and emerging). Using WDI data, the ratio is 0.449.

The parameter  $\eta_j$  determines the fraction of the fixed asset used as a collateral in country  $j$ . Cross-country differences in this parameter captures differences in the ability of countries to create financial assets (in the spirit of Caballero et al. (2008)) and it is calibrated by targeting the ratio of credit over output. More specifically, I choose  $\eta_1$  and  $\eta_2$  so that the average values of  $L_{1,t}/Y_{1,t}$  and  $L_{2,t}/Y_{2,t}$  are equal to the calibration targets.

The targets are constructed using the ratio of Private Domestic Credit over Net GDP in 1991, which is the first year in the sample, from the WDI database. The 1991 ratio for industrialized countries was 145.7 and for emerging countries was 49.6. However, in the data, only some of the liabilities issued by banks are held by the business sector. Some of these liabilities are held by households. This implies that the ‘net’ debt for the whole house-

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<sup>6</sup>The stochastic processes for the productivity of the two countries imply that the relative productivity  $x_t = A_{2,t}/A_{1,t}$  follows the process  $x_t = x_{t-1}e^{\bar{\epsilon}_t}$ , where  $\bar{\epsilon}_t = \epsilon_{2,t} - \epsilon_{1,t}$  is the stochastic growth differential between emerging and industrialized countries. This is normally distributed with zero mean and standard deviation  $\sqrt{2}\sigma_\epsilon$ .

hold sector is smaller than domestic credit. As a compromise, I impose that  $L_{1,t}/Y_{1,t}$  and  $L_{1,t}/Y_{1,t}$  are half the values of domestic credit in the data.

I now move to the calibration of the banking sector which is characterized by three parameters: the operation cost  $\tau$ , the recovery rate outside the banking sector  $\xi$ , and the sunspot probability  $\lambda$ . The probability that the sunspot takes the value  $\varepsilon = 0$  is set to  $\lambda = 0.03$ . Therefore, provided that the economy is in a state that admits multiple equilibria, a crisis is a low probability event that arises, on average, every 33 years. Similar numbers have been used in the literature (see for example ??). Next I set  $\xi = 0.85$ . This implies that, in a crisis, the liquidation value of bank assets is 85%. The operation cost  $\tau$  is set to 0.04. This implies, approximately, that the interest spread between loans and liabilities is in the order of 4%.

The final calibration step is the specification of the stochastic process for the uninsurable idiosyncratic productivity  $\pi$  which is assumed to follow a truncated normal distribution with zero mean and standard deviation  $\sigma_\pi$ . I set  $\sigma_\pi$  so that, in the period that preceded the growth of emerging economies, the average leverage of banks is slightly above the renegotiation threshold. In the model there is a positive relation between  $\sigma_\pi$  and the leverage of banks. Higher values of  $\sigma_\pi$  increase the demand for bank liabilities leading to lower interest rates, which in turn increases the incentive to leverage. I choose the calibration target  $B_t/L_t = 0.86$  which is achieved by setting  $\sigma_\pi = 0.15$ . This implies that the standard deviation of wealth for entrepreneurs is about 15%.

### 3.2 Quantitative results

I simulate the model for 100+23 years using a random sequence of draws of the sunspot shock ( $\varepsilon = 0$  with probability  $\lambda = 0.03$  and  $\varepsilon = 1$  with probability  $1 - \lambda = 0.97$ ). During the first 100 periods the aggregate productivities of the two countries are assumed to be constant at their 1991 level. The goal of the first 100 simulated periods is to eliminate the impact of initial conditions. Starting at period 101, corresponding to year 1991, the productivities of the two countries follow the actual series shown in Figure 4.

In absence of sunspot shocks, the dynamics of the economy would be solely driven by changes in productivity. The presence of sunspot shocks adds another source of fluctuation. The simulated dynamics would then depend on the actual realizations of these shocks. To better illustrate how the sunspot shocks affect the stochastic properties of the model, I repeat the simulation 1,000 times, with each simulation over 100+23 years.

**Simulation results** Figure 5 plots the average as well as the 5th and 95th percentiles of the 1,000 repeated simulations. The range of variation between the 5th and 95th percentiles indicates the potential volatility at any point in time (for given productivity).

The first panel shows the relative productivity  $A_{2,t}/A_{1,t}$  constructed from the data as described earlier (and separately plotted in Figure 4). The next three panels plot the leverage of banks, the interest rates paid on liabilities and the return earned on loans. The remaining panels show the dynamics of asset prices (the prices for houses) and labor in both countries.

Following the increase in the ‘relative’ productivity of emerging countries, the interval delimited by the 5th and 95th percentiles for the repeated simulations widens significantly. This means that financial and macroeconomic volatility increases substantially as we move to the 2000s. In this particular simulation the probability of a bank crisis is always positive, even before the structural break in 1991 when the relative productivity of emerging countries starts to change. However, after the structural break, the consequence of a bank crisis could be much bigger since the distance between the 5th and 95th percentiles widens. This is especially true in the second half of 2000s.

Notice that, without the growth of emerging countries (relatively to industrialized countries), the 5th and 95th percentile band would not change after 1991. Therefore, the comparison of the band before and after 1991 shows how the growth of emerging countries contributed to increasing financial and macroeconomic volatility in both groups of countries.

Besides the increase in financial and macroeconomic volatility, Figure 5 reveals other interesting patterns. First, as the relative size of emerging countries increases, banks raise their leverage while the interest rate on liabilities declines. The economy also experiences a decline in the interest rate on loans made to households which in turn allows for a boom in housing prices. This is because the decline in the interest rate makes the financing of houses cheaper raising their demand.

Figure 5 also shows that labor declines on average. This is because, as the share of emerging countries in the world economy increases, the global demand for bank liabilities increases. Banks respond by raising the supply but not enough to compensate the increase in demand. Thus, entrepreneurs in both countries hold less financial assets relatively to the scale of production. This implies lower insurance and, therefore, less demand for labor.



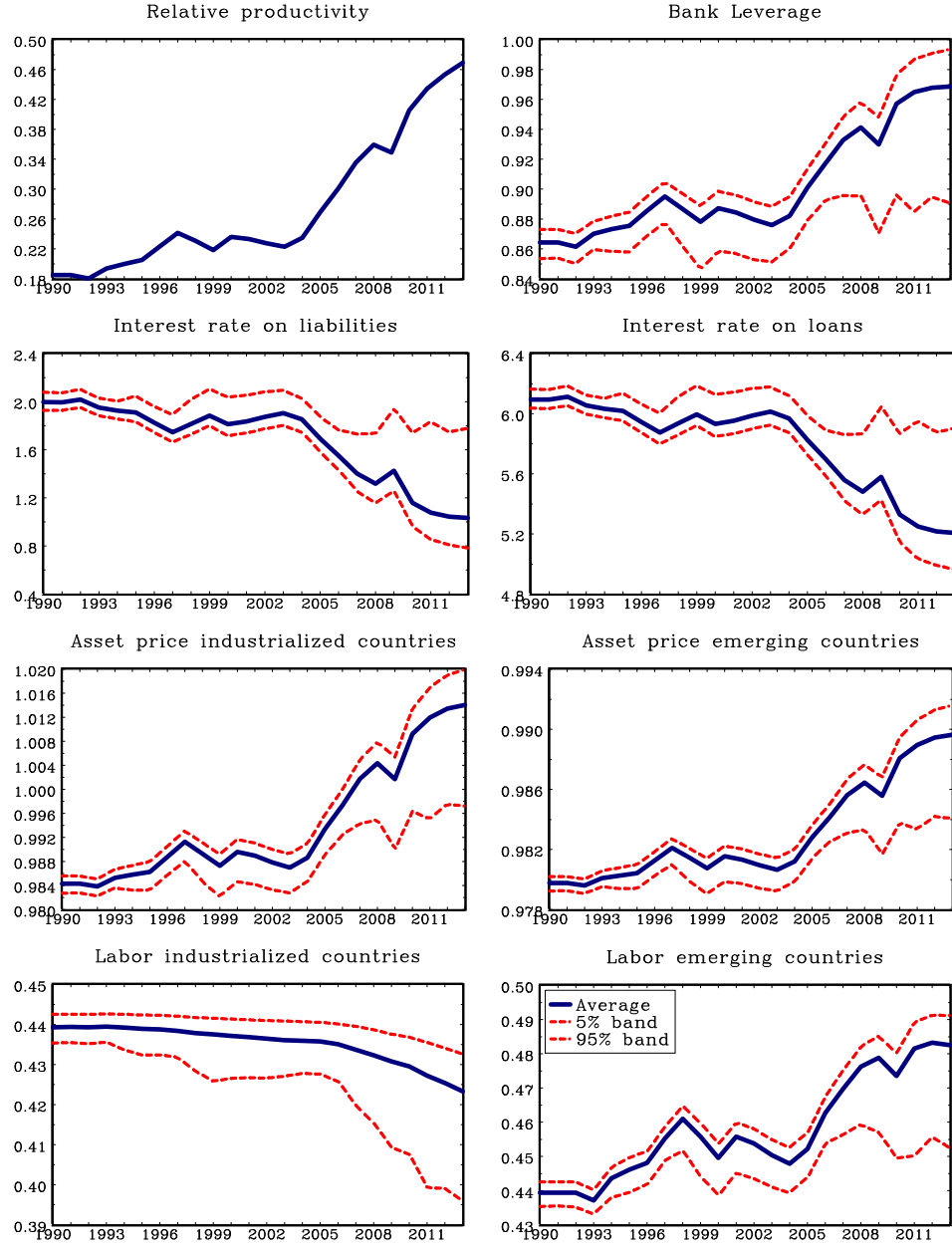


Figure 5: Change in productivity of emerging countries relative to industrialized countries, 1992-2013. Responses of 1,000 repeated simulations.

**Central mechanism** The dynamics shown above follow from the increase in the demand for bank liabilities ‘relatively’ to the demand for bank loans. It is important to stress the term ‘relative’. As emerging countries become bigger, the demand for bank liabilities increases in absolute value. But also the demand for loans from emerging countries increases. However, since households in emerging countries face a tighter borrowing limit than in industrialized countries, the world demand for bank liabilities increases more than the demand for bank loans.

This point can be illustrated more precisely using the definition of bank leverage shown in Subsection 2.7,

$$LEVERAGE = \frac{B_{1,t} + B_{2,t}}{L_{1,t} + L_{2,t}} = \frac{\left(\frac{B_{1,t}}{L_{1,t}}\right) L_{1,t} + \left(\frac{B_{2,t}}{L_{2,t}}\right) L_{2,t}}{L_{1,t} + L_{2,t}}.$$

Since the model is calibrated so that the borrowing constraint for industrialized countries is less stringent than emerging countries, that is,  $\eta_1 > \eta_2$ , the equilibrium is characterized by  $B_{1,t}/L_{1,t} < B_{2,t}/L_{2,t}$ . Now suppose that emerging countries grow in size by  $g_t$ . Furthermore, suppose that  $B_{2,t}$  (savings) and  $L_{2,t}$  (credit) also grow by the same rate so that  $B_{2,t}/L_{2,t}$  does not change in emerging countries. The leverage of banks would be

$$LEVERAGE = \frac{\left(\frac{B_{1,t}}{L_{1,t}}\right) L_{1,t} + \left(\frac{B_{2,t}}{L_{2,t}}\right) L_{2,t}(1 + g_t)}{L_{1,t} + L_{2,t}(1 + g_t)}.$$

Since the initial equilibrium was characterized by  $B_{1,t}/L_{1,t} < B_{2,t}/L_{2,t}$ , the growth of country 2 generates an increase in the leverage of banks (as shown in the second panel of Figure 5).

The above argument relies on the assumption that  $B_{2,t}/L_{2,t}$  does not change or at least does not decline much, which is the case in the quantitative simulation of the model. On the household sector, a higher  $A_{2,t}$  relaxes the borrowing constraint and leads to higher household borrowing, that is, higher  $L_{2,t}$ . Therefore, in order for the ratio  $B_{2,t}/L_{2,t}$  not to decline,  $B_{2,t}$  must also increase. The assumption that aggregate productivity is observed with delay plays a crucial role in generating a sufficiently large increase in  $B_{2,t}$ .

Since  $A_{2,t}$  is still unknown when workers are hired and wages clear the labor market, an unexpected growth in productivity generates an unexpected increase in entrepreneurial profits. Since entrepreneurs save a fraction of their end-of-period wealth, higher profits imply higher savings, that is, higher

value of  $B_{2,t+1}$ . If the increase in  $g_t$  was anticipated, instead, the higher productivity would generate an immediate increase in wages with only a small effect on profits. In this case  $B_{2,t+1}$  increases only marginally and the ratio  $B_{2,t}/L_{2,t}$  would decline. The lagged observation of productivity is also important for generating a positive correlation between the growth of a country and its current account balance. This point will be explained shortly after showing the dynamics of the current account.

**Simulation for a particular sequence of shocks** Although the model predicts that financial and macroeconomic volatility has increased in both industrialized and emerging countries, it is difficult to detect it from the empirical data. This is because crises are very low probability events: in the model a crisis could materialize when the draw of the sunspot shock is low, an event that has been calibrated to arise on average every 33 years.

Because of the low probability of negative sunspot shocks, the recent crisis could have been the only ‘global’ crisis experienced during the last 20 years. Thus, the macroeconomic dynamics observed prior to 2008 would appear quite stable even if the underlying volatility has increased. Because the probability of a negative sunspot shock is very small (calibrated to 3% per year), the probability of a sequence of positive realizations from 1992 to 2008 is about 50 percent. Therefore, the hypothesized scenario is quite plausible. It also fits with anecdotal evidence according to which 2008 is the only truly worldwide financial crisis observed during the last 20 years.

To illustrate the dynamics under the hypothesized scenario, I simulate the model for a particular sequence of sunspot shocks. Starting in 1991, I assume that the economy experiences a sequence of positive sunspot draws  $\varepsilon = 1$  until 2008. Then, in 2009, the sunspot draw becomes  $\varepsilon = 0$  and reverts back to  $\varepsilon = 1$  afterwards. This particular sequence of sunspot shocks captures the idea that expectations may have turned pessimistic in the fourth quarter of 2008. Since the model has been calibrated annually, the negative sunspot shock is assumed to arise in 2009 even if in the data the crisis materialized toward the end of 2008. The simulated variables are plotted in Figure 6.

As we can see, as long as the draws of the sunspot shock are  $\varepsilon = 1$ , housing prices continue to increase and the input of labor does not drop. However, a single realization of  $\varepsilon = 0$  can trigger a large decline in labor and, therefore, in output. Furthermore, even if the negative shock is only for one period and there are no crises afterwards, the recovery in the labor market

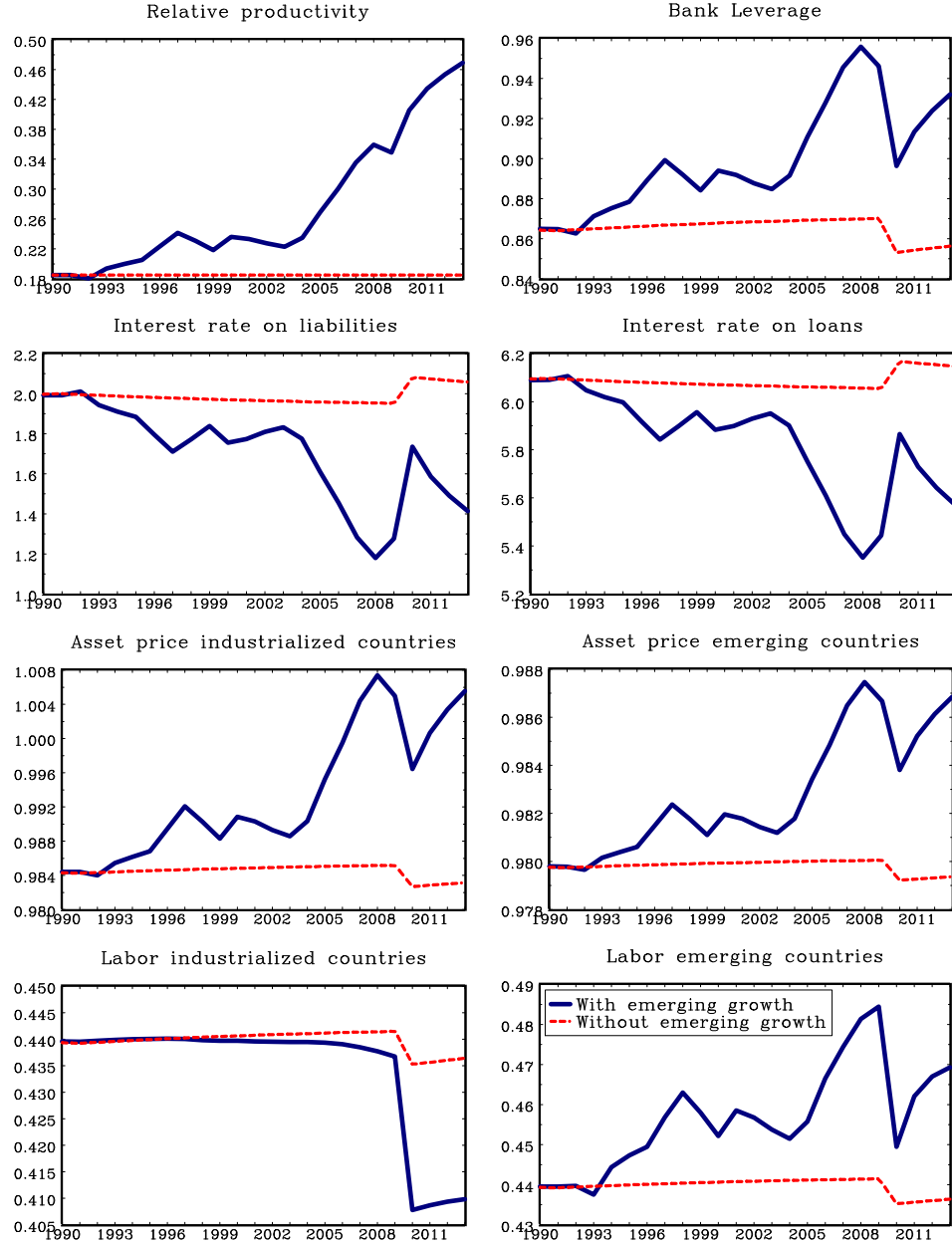


Figure 6: Change in productivity of emerging countries relative to industrialized countries, 1992-2013. Responses of 1,000 repeated simulations with same draws of positive sunspot shocks starting in 1992 and negative draw in 2009.

is slow. This is because the crisis generates a large decline in the financial wealth of employers and it will take a long time to rebuilt the lost wealth through savings.

The magnitude of the housing price boom and the subsequent contraction that follows the crisis is not big. However, qualitatively, it captures the dynamics of housing prices observed in most of the industrialized countries with an acceleration during the 2003-2007 period.<sup>7</sup>

Another way of showing the importance of the growth of emerging countries for macroeconomic volatility is by conducting a counterfactual exercise. I repeat the simulation under the assumption that the ‘relative’ productivity of emerging countries does not grow but remains at the 1991 level for the whole simulation period. Essentially, emerging countries experience the same growth rate as industrialized countries. This counterfactual exercise tells us how the macroeconomic dynamics would have changed in response to the same sequence of sunspot shocks if emerging countries had not experienced growth (again, relatively to industrialized countries). The resulting simulation is shown in Figure 6 by the dashed line.

Without the growth of emerging countries, the same sequence of sunspot shocks would have generated a smaller financial expansion before 2009 as well as a much smaller contraction in 2009. Therefore, the increase in demand for financial assets could have contributed to the observed expansion of the financial sector but it also created the conditions for greater financial and macroeconomic fragility. However, the fragility became evident only after the crisis materialized.

**Global imbalances** As showed in Figure 1, the 2000s are also characterized by large imbalances between industrialized and emerging countries. Starting in 2000, industrialized countries have experienced current account deficits while the current account of emerging countries has been in surplus. The surplus of emerging countries was especially large in the 2003-2007 period, only partially corrected by the crisis. Figure 7 shows that, during the same period, the simulated model predicts large current account surpluses for emerging countries.

In order to compute the current account balance in the model, I need to specify the owners of banks since the dividends paid by banks to foreigners

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<sup>7</sup>Housing prices reported in the figure are normalized by the aggregate productivity of the country and proxies for the price-GDP ratio.

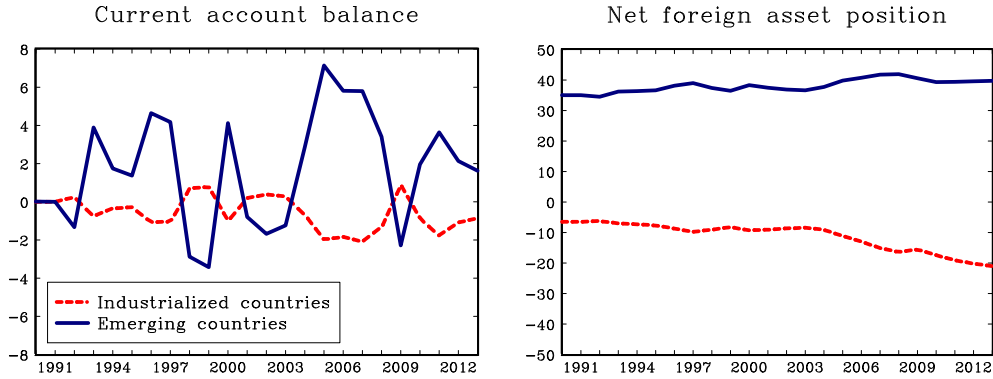


Figure 7: Current account balance and net foreign asset position as a percentage of net output. Responses of 1,000 repeated simulations with draws of positive sunspot shocks starting in 1992 and negative draw in 2009.

contribute to the current account balance. To simplify the analysis I assume that banks are owned by households in industrialized countries.

The simulated current account balance of emerging countries is more volatile than in the data. However, it is positive on average and this leads to an increase in net foreign asset positions during the simulation period (as shown in the right panel of Figure 7). The model also generates a widening surplus prior to the crisis and the contraction after the crisis. The current account balance of industrialized countries displays the same pattern but with the reversed sign.

The fact that fast growing countries tend to have current account surpluses while countries with moderate growth tend to experience current account deficits has been shown in Gourinchas and Jeanne (2013). This paper also shows that this empirical fact is inconsistent with the predictions of many macro models where higher growth is associated with higher net foreign borrowing. The model developed in this paper, instead, can generate a positive correlation under the same conditions it generates an increase in bank leverage as described earlier. Again, the lagged realization of productivity is important for generating this result. In fact, the higher profits of entrepreneurs in fast growing countries allow for higher entrepreneurial savings (higher  $B_{2,t}$ ) in excess of the increase in household borrowing (higher  $L_{2,t}$ ). When the savings of emerging countries exceed borrowing, these countries experience a current account surplus as shown in Figure 7.

**Intermediation cost** One prediction of the model is that the leverage of banks increases with the growth of emerging countries, which in turn raises the expected renegotiation cost. This will be reflected in a higher spread between the interest rate paid on bank liabilities and the interest rate earned on loans. The increasing spread may seem inconsistent with recent evidence suggesting that the financial intermediation cost has not changed significantly during the last two decades, at least in the United States (Philippon (2015)).

A closer examination of the calibrated model, however, shows that this conclusion is not accurate. The left panel of Figure 8 plots the response of the interest rate spread defined as  $R_t^l/\bar{R}_t^b - 1$ . During the period that preceded the 2008-2009 crisis we see that the interest rate spread increased somewhat. However, the increase is so small that a similar change in the data would be difficult to detect using statistical methods (that is, to find it statistically significant). Therefore, I can conclude that the interest rate spread predicted by the model does not change significantly during the simulation period, and this is consistent with empirical evidence.

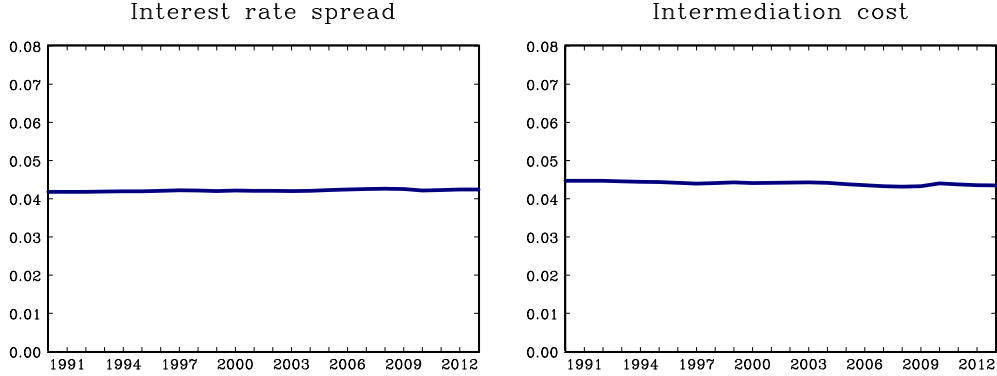


Figure 8: Interest rate spread and intermediation cost. Responses of 1,000 repeated simulations with same draws of positive sunspot shocks starting in 1992 and negative draw in 2009.

I could also compute a measure of the intermediation cost defined as the ratio of net interest revenues of banks over their investments. Formally,

$$\frac{\left(L_{t+1} - \frac{L_{t+1}}{R_t^l}\right) - \left(B_{t+1} - \frac{B_{t+1}}{\bar{R}_t^b}\right)}{\frac{L_{t+1}}{R_t^l}}.$$

The first term in parenthesis in the numerator is the interest revenue of banks received on their loans  $L_{t+1}/R_t^l$ . The second term, also in parenthesis, is the interest payment on their liabilities  $B_{t+1}/\overline{R}_t^b$ . The denominator is the value of loans  $L_{t+1}/R_t^l$  (bank investment). This can be interpreted as the intermediation margin for each unit of investment. As can be seen from the second panel of Figure 8, the intermediation cost does not increase. In fact, it decreases slightly. This derives from the fact that, even if the interest rate spread increases somewhat, banks finance a larger fraction of investments with debt, which is cheaper than equity. Therefore, the model does not predict that the intermediation cost has increased during the sample period.

Another prediction of the model is that the potential losses from holding bank liabilities increase with the growth of emerging countries. Even if the probability of a crisis stays constant, higher leverage implies that the creditors of banks incur higher losses when a crisis arises. This should be reflected in the market price of securities that insure the default risk. For example, credit default swaps.

The problem, however, is that default swaps provide insurance only for the idiosyncratic risk of bank liabilities (which is absent in the model), not for systemic or economy-wide risks (the only risk in the model). As the recent financial crisis has taught us, when the crisis involves the whole banking sector, even the suppliers of insurance (for example, the issuers of credit default swaps) could be at risk of default. Thus, we would not expect that the price of credit default swaps truly reflects the risk of a crisis that involves the whole banking sector (as formalized in the model).

#### 4 The importance of capital mobility

Starting in the early 1980s, international capital markets have become more integrated as many restrictions on the international flows of capital have been lifted. Since the paper focused on the last two decades, it considered only the version of the model with capital mobility. Still, it would be interesting to study how capital mobility affects financial and macroeconomic volatility.

To investigate the impact of capital mobility, I conduct a counterfactual simulation in which each of the two countries operate in the autarky regime. In autarky there is an intermediation sector in each country that sells liabilities only to domestic entrepreneurs and makes loans only to domestic households. Countries continue to be heterogeneous in productivity  $A_{j,t}$  and



financial structure as captured by the parameter  $\eta_j$ .

Figure 9 plots some simulation statistics for industrialized countries in response to 1,000 repeated simulations. The left panels are for the autarky regime while the right panels are for the mobility regime (already shown in the previous Figure 5).

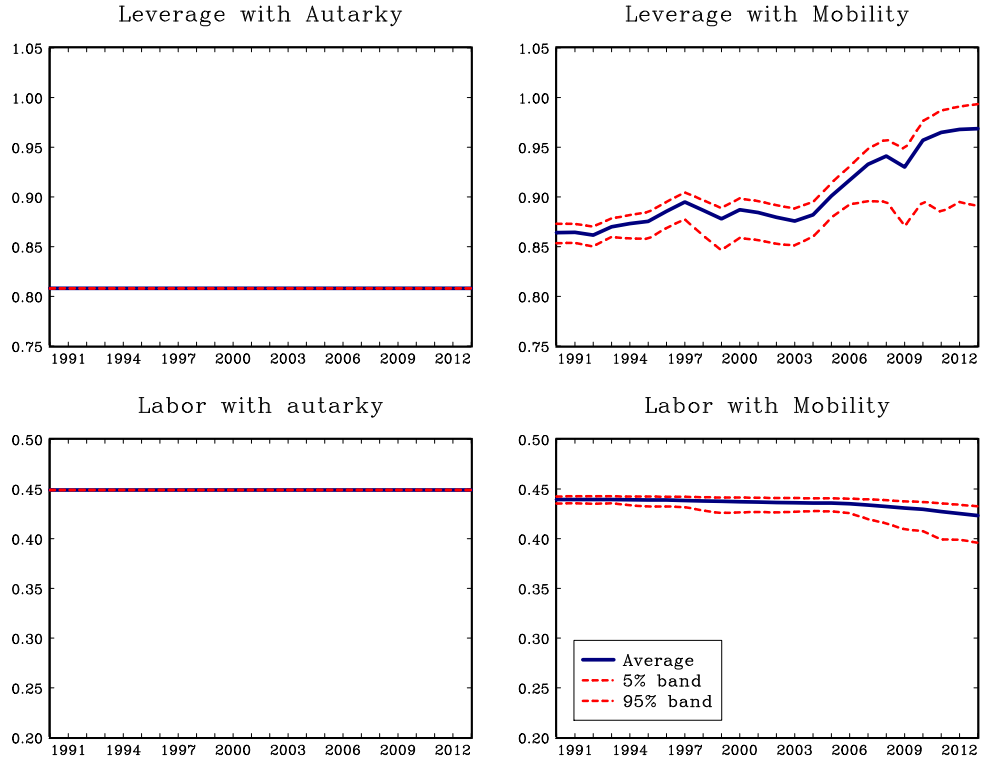


Figure 9: Dynamics of bank leverage and labor for Industrialized Countries. Responses of 1,000 repeated simulations.

When financial markets are not integrated in industrialized countries, the demand for bank liabilities ‘relatively’ to their supply is lower. This implies that the equilibrium interest rate is higher and, therefore, banks have lower incentive to leverage. But with lower leverage the economy shifts to a state without multiple equilibria. This explains why in the left panels of Figure 5) the lines denoting the 5th and 95th percentiles overlap with the average.

In addition to ‘volatility’, capital mobility also affects the ‘level’ of the economy. As financial markets become integrated, entrepreneurs in indus-

trialized countries hold less liabilities as they get crowded out by the purchases of these liabilities by entrepreneurs in emerging countries. As a result, entrepreneurs in industrialized countries hold less financial wealth, causing lower employment and production. Therefore, financial integration tends to increase macroeconomic volatility and to reduce the level of aggregate output in industrialized countries. Of course, this does not necessarily mean that industrialized countries are worse-off after becoming financially integrated. Mobility also allows households in industrialized countries to borrow at a lower interest rate, which is beneficial for them.

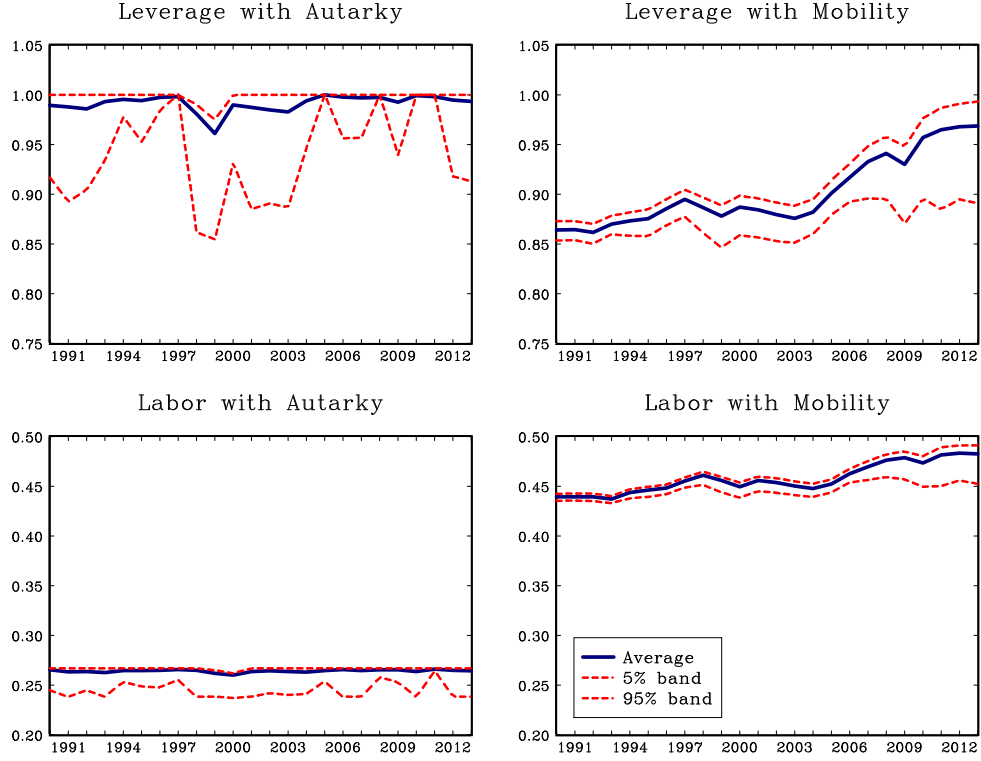


Figure 10: Dynamics of bank leverage and labor for Emerging Countries. Responses of 1,000 simulations.

Figure 10 plots the same variables plotted in Figure 9 but for emerging countries. Comparing the left panels (autarky) to the right panels (mobility), we can see that financial integration reduces the volatility of emerging countries and increases their employment (and therefore, production). The lower

level of lending in emerging countries implies that the supply of bank liabilities to entrepreneurs is also lower when financial markets are not integrated. This reduces the interest rate on (local) bank liabilities and creates an incentive for banks to leverage. It also implies that in equilibrium entrepreneurs do not hold as many liabilities as in industrialized countries, which reduces the demand for labor. With mobility, instead, entrepreneurs in emerging countries have access to foreign markets and can purchase foreign liabilities at a more favorable price (higher interest). They will then increase the holding of financial assets and with it the demand for labor. According to the simulation, the quantitative impact of financial integration on the macroeconomic level and volatility of emerging countries is quite large. Therefore, financial integration could be an important mechanism for these countries to stabilize their economy and speed up growth.

#### **4.1 Additional impact of capital mobility**

In the above simulations I have assumed that the capital regime (autarky or mobility) does not affect the probability of the sunspot shock. Therefore, provided that the country is in a region with multiple equilibria, the probability of a crisis does not depend on the capital regime. However, the probability of crises could also change when countries switch from autarky to mobility.

The idea underlying sunspot equilibria is that agents coordinate on a particular belief. For a crisis to emerge, all agents must believe (expect) that the liquidation value of banks' assets is low. In a closed economy where banks operate locally, only the expectations of agents in the local market matters. For example, a crisis in emerging countries would arise if all agents in emerging countries form pessimistic expectations. The expectations of agents in industrialized countries are irrelevant for the equilibrium of the emerging economies. Similarly, a crisis in industrialized countries would arise if all agents in industrialized countries form pessimistic expectations. The expectations of agents in emerging countries are irrelevant for the equilibrium of industrialized economies. This is because international arbitrage cannot take place in autarky and the liquidation prices of bank liabilities could differ across countries.

When financial markets become integrated, however, arbitrage guarantees the international equalization of prices. But a single price requires that agents in both countries have the same expectations about the liquidation

price for banks assets. This requires the coordination of a larger number of agents: with mobility agents in emerging countries must form the same expectations as agents in industrialized countries. Obviously, the likelihood of coordination declines as the number of agents increases.

To formalize this idea I follow Perri and Quadrini (2011) and I assume that the two countries draw, independently, their own sunspot shock:  $\varepsilon_1$  in country 1 and  $\varepsilon_2$  in country 2, both with probability  $\lambda$ . If financial markets are not integrated, then country 1 will experience a crisis only if  $\varepsilon_1 = 0$  and country 2 will experience a crisis only if  $\varepsilon_2 = 0$ . Therefore, the probability of a crisis in each of the two countries is  $\lambda$ . Since the draws of the sunspot shock in the two countries are independent, financial crises are not internationally correlated in the autarky regime.

Now let's consider the regime with capital mobility. In this case a crisis can only emerge if both countries draw low values of the sunspot shock, that is,  $\varepsilon_1 = 0$  and  $\varepsilon_2 = 0$ . Since the draws remain independent even if the two countries are financially integrated, the probability of a crisis is now  $\lambda^2$ .

To understand why consider the case in which the draws are  $\varepsilon_1 = 1$  and  $\varepsilon_2 = 0$ . This implies that country 1 forms optimistic expectations while country 2 forms pessimistic expectations about the liquidation price. As a result, banks in country 2 renegotiate their liabilities but banks in country 1 do not renegotiate. But then the liquidated assets in country 2 can be sold to banks in country 1 at full price, invalidating the rationality of the expectation for a low liquidation price in country 2. This demonstrates why a crisis can emerge only if both countries draw a negative value of the sunspot shock. Since this happens with probability  $\lambda^2$ , the probability of crises is smaller when financial markets are integrated.

The probability of a crisis has important implications for the optimal policy of banks. Recall that the renegotiation cost takes the form  $(\omega_t - \xi_t)^2 b_t$ , which is increasing in the leverage  $\omega_t = b_t/l_t$ . The cost is incurred only if the bank defaults and renegotiates. Therefore, what matters for the optimal choice of  $\omega_t$  is the 'expected' cost, which in turn depends on the probability of crises. More specifically, denoting by  $\theta_{t+1}(\omega_{t+1})$  the probability that a bank defaults, the expected renegotiation cost is

$$\theta(\omega_{t+1})(\omega_{t+1} - \xi_{t+1})^2 b_{t+1}.$$

When  $\omega_{t+1} \in (\underline{\xi}, 1)$  for all banks, the probability of crises in autarky is  $\theta_{t+1}(\omega_{t+1}) = \lambda$ . With capital mobility, instead, the probability is  $\theta_{t+1}(\omega_{t+1}) =$

$\lambda^2$ . Thus, capital mobility decreases the (expected) cost of leveraging which increases the incentive of banks to take more leverage. As a result, crises could be less frequent but they imply larger macroeconomic contractions.

To illustrate this property I re-simulate the model under autarky and under mobility but taking into account that crises can emerge only if both countries draw a low realization of the sunspot shock. I continue to assume that the probability of crises in the mobility regime is 3 percent, that is,  $\lambda^2 = 0.03$ . Consequently, the probability of crises in the autarky regime is  $\lambda = \sqrt{0.03} = 0.173$ .

## 5 Discussion and conclusion

An implication of the sustained high growth of emerging countries and their larger share in the world economy, is that the economic performance of these countries has become more important for the economies of industrialized countries. The view that emerging countries are a collection of small open economies whose dynamics is negligible for industrialized countries is no longer a valid approximation.

There are many channels through which emerging countries could affect the industrialized world. In this paper I emphasized one of these channels: the increased demand for financial assets traded in globalized capital markets. In particular, I have shown that the worldwide increase in the demand for financial assets raises the incentives of financial intermediaries to leverage. On the one hand, this allows for the expansion of the financial sector with positive effects on real macroeconomic variables. On the other, it increases the fragility of the financial system, raising the probability and/or the consequences of a crisis.

These results are illustrated using a model in which the banking sector plays a central role in the intermediation of funds and, therefore, in the creation of financial assets. The paper emphasizes a special channel through which banks can affect the real sector of the economy: the issuance of liabilities held by the nonfinancial sector for insurance purposes. When the supply of bank liabilities or their value are low, agents are less willing to engage in risky activities and this causes a macroeconomic contraction.

The analysis of the paper also shows that booms and busts in financial intermediation can be driven by self-fulfilling expectations about the liquidity of the banking sector. When the economy expects the banking sector to be liquid, banks have an incentive to leverage and this allows for an economic

boom. But as leverage increases, the banking sector becomes vulnerable to pessimistic expectations about the liquidity of the overall banking sector, creating the conditions for a financial crisis. The increase in the demand for financial assets from emerging economies amplifies this mechanism because, by reducing the funding cost, it increases the incentive of banks to leverage. The result is an increase in underlying financial and macroeconomic volatility.

In reality, financial assets held for precautionary reasons are also created directly in nonfinancial sectors. For example, firms and governments issue liabilities that are directly held by nonfinancial sectors. Still, financial intermediaries play an important role in the direct issuance of these securities. Financial intermediaries also play a role in the secondary market for these securities. Therefore, difficulties in financial intermediation is likely to affect the functioning and valuation of all financial securities independently of the issuer. It is for this reason that in this paper I focused on financial intermediaries.

An important feature of the model is that the expansion of the financial sector improves allocation efficiency. This is because the issuance of bank liabilities provides insurance instruments for entrepreneurs, encouraging them to hire labor. Effectively, the creation of financial assets that can be used for insurance purposes reduces the wedge in the demand for labor. However, the creation of more financial assets is often associated with higher leverage, making the financial system more vulnerable to crises. From a policy perspective there is a trade-off: the benefit of an expanded financial system versus the potential cost of deeper crises. A similar mechanism also arises in models with asset price bubbles and borrowing constraints as in Miao and Wang (2011). I leave the study of optimal policies for future research.

## Appendix

### A Limited enforcement and transformed problem

The micro-foundation for the lack of insurance is based on the assumption that entrepreneurs have the ability to divert part of their revenues. The diverted revenues are observable but cannot be verified legally. If an entrepreneur diverts  $x_t$ , he/she retains  $(1 - \zeta)x_t$  and the remaining part,  $\zeta x_t$ , will be lost. Therefore,  $\zeta$  parameterizes the cost of diversion.

Entrepreneurs can purchase financial claims  $n_{j,t}^i(z_{j,t}^i)$  that are contingent on the realization of productivity in a competitive market. Because the counterparts of these claims are households who are risk-neutral, the prices for the claims are the probabilities associated with the realizations of  $z_{j,t}^i$ . The probabilities are denoted by  $G_{j,t}(z)$ . Notice that  $z_{j,t}^i = A_{j,t}\pi_{j,t}^i$  includes both aggregate and idiosyncratic productivities. While the limited verifiability of the idiosyncratic component is a plausible assumption, the same cannot be said for the aggregate component. However, as will become clear below, if I assume that the minimum value of the idiosyncratic shock is zero, that is,  $\underline{\pi} = 0$ , then the results obtained here are also valid under the assumption that the aggregate productivity  $A_{j,t}$  is verifiable. Therefore, in the rest of this section I assume that  $\underline{\pi} = 0$ . Below I will also discuss what would change if  $\underline{\pi} \neq 0$ .

Financial contracts are not exclusive, meaning that agents can always switch to another supplier of these claims in the subsequent period. The financial contract must be incentive-compatible.

For notational simplicity, in the analysis that follows I will ignore the agent superscript  $i$  and the country subscript  $j$ . After the realization of  $z_t$  the entrepreneur could divert part of the revenue. By claiming that the realization of productivity is the lowest value,  $\underline{z} = 0$ , the entrepreneur would divert  $(z_t - \underline{z})h_t$ . Because part of the diverted revenue is lost, the entrepreneur retains  $(1 - \zeta)(z_t - \underline{z})h_t$  and, from the financial contract, he/she receives  $n_t(\underline{z})$ . The net worth after diversion is then

$$b_t + (\underline{z} - w_t)h_t + (1 - \zeta)(z_t - \underline{z})h_t + n_t(\underline{z}) = a_t(\underline{z}) + (1 - \zeta) \cdot (z_t - \underline{z})h_t.$$

Denoting by  $\tilde{\Omega}_t(a_t(z_t))$  the value function at the end of the period before consumption when the net worth is  $a_t(z_t)$ , the value of diversion is  $\tilde{\Omega}_t(a_t(\underline{z}) + (1 - \zeta) \cdot (z_t - \underline{z})h_t)$ . Incentive-compatibility requires

$$\tilde{\Omega}_t(a_t(z_t)) \geq \tilde{\Omega}_t(a_t(\underline{z}) + (1 - \zeta) \cdot (z_t - \underline{z})h_t),$$

which must hold for all  $z_t$ .

The entrepreneur's problem can be written as

$$\Omega_t(b_t) = \max_{h_t, n(z_t)} \mathbb{E}_t \tilde{\Omega}_t(a_t(z_t)) \quad (20)$$

subject to

$$a_t(z_t) = \tilde{b}_t + (z_t - w_t)h_t + n_t(z_t)$$

$$\tilde{b}_t = (1 - \delta_t)b_t$$

$$\int_{z_t} n_t(z_t) G_t(z_t) = 0$$

$$\tilde{\Omega}_t(a_t(z_t)) \geq \tilde{\Omega}_t(a_t(\underline{z}) + (1 - \zeta) \cdot (z_t - \underline{z})h_t)$$

$$\tilde{\Omega}_t(a_t(z_t)) = \max_{b_{t+1}} \left\{ \ln(c_t) + \beta \mathbb{E}_t \Omega_{t+1}(b_{t+1}) \right\} \quad (21)$$

subject to

$$c_t = a_t - q_t b_{t+1}$$

The optimization problem has been divided in two sub-problems because the information set changes from the beginning of the period to the end of the period. In sub-problem (24) the entrepreneur chooses the input of labor and the contingent claims before knowing the productivity  $z_t$ . Remember that, even if the entrepreneur starts the period with financial wealth  $b_t$ , he/she may incur some financial losses that brings the residual wealth to  $\tilde{b}_t$ . The losses are captured by the variable  $\delta_t$ . This is an aggregate endogenous variable that will be derived in the general equilibrium. For the characterization of the individual problem, however, we can take it as an exogenous stochastic variable since it cannot be affected by an individual (atomistic) entrepreneur.

The last constraint on Problem (24) is the cost to purchase the contingent claims. Imposing that the total cost is equal to zero is a normalization. In sub-problem (25) the entrepreneur allocates the end of period wealth in consumption and savings.

It is important to emphasize that the contractual party knows whether the entrepreneur is diverting. However, there is no legal procedure that can be used to enforce the payment because it is not possible to legally verify the diverted funds. The assumption that financial contracts are not exclusive and entrepreneurs can switch to other intermediaries from one period to the other is important because it limits the punishment for diversion. Notice that, although the new level of wealth after diversion is verifiable when a new contract is signed, this does not allow for the verification of diversion because the additional resources could derive from lower consumption in previous periods, which is not verifiable. The fraction of revenue lost,  $\zeta$ , can be interpreted as the cost for hiding (making non-verifiable)



the diverted revenue and for hiding consumption.

Using standard arguments for recursive problems, we can prove that the solution is unique and the functions  $\Omega_t$  and  $\tilde{\Omega}_t$  are strictly increasing and concave. The strict monotonicity of the value functions implies that the incentive-compatibility constraint can be written as

$$a_t(z_t) \geq a_t(\underline{z}) + (1 - \zeta) \cdot (z_t - \underline{z})h_t.$$

The concavity of  $\tilde{V}_t$  implies that it is optimal for the entrepreneur to choose the contingent claims so that the above inequality is always satisfied with equality. What this says is that the entrepreneur will choose as much insurance as possible. Since  $a_t(z_t) = \tilde{b}_t + (z_t - w_t)h_t + n_t(z_t)$ , we have

$$\tilde{b}_t + (z_t - w_t)h_t + n_t(z_t) = a_t(\underline{z}) + (1 - \zeta) \cdot (z_t - \underline{z})h_t,$$

which we can solve for  $n_t(z_t)$ ,

$$n_t(z_t) = a_t(\underline{z}) + (1 - \zeta) \cdot (z_t - \underline{z})h_t - (z_t - w_t)h_t - \tilde{b}_t. \quad (22)$$

Multiplying both sides by the probabilities  $G_t(z_t)$  and integrating over  $z_t$  we obtain

$$\int_{z_t} n_t(z_t)G_t(z_t) = a_t(\underline{z}) + (1 - \zeta) \cdot (\mathbb{E}z_t - \underline{z})h_t - (\mathbb{E}z_t - w_t)h_t - \tilde{b}_t. \quad (23)$$

Subtracting (23) to (22) and taking into account that  $\int_{z_t} n_t(z_t)\Gamma(z_t) = 0$ , we obtain

$$n_t(z_t) = -\zeta(z_t - \mathbb{E}z_t)h_t.$$

Substituting in the law of motion for end-of-period assets we have

$$a_t(z_t) = \tilde{b}_t + \left[ \mathbb{E}z_t + (1 - \zeta)(z_t - \mathbb{E}z_t) - w_t \right] h_t.$$

Therefore, if we define  $\tilde{z}_t = \mathbb{E}z_t + (1 - \zeta)(z_t - \mathbb{E}z_t)$ , we would have the same problem as the one without contingent claims but with the transformed productivity  $\tilde{z}_t$ . When  $\zeta = 1$ , which corresponds to perfect enforcement, the volatility of  $\tilde{z}_t$  becomes zero. Thus, entrepreneurs can perfectly insure the production risk. When  $\zeta = 0$ , any insurance of the production risk is unfeasible.

The assumption  $\underline{\pi} \neq 0$  implies that the minimum value of revenues is always zero independently of the realization of the aggregate shock. This is because  $\underline{z} = A_t \underline{\pi} = 0$  whatever the value of  $A_t$ . If  $\underline{\pi} \neq 0$ , however, the minimum value of  $z_t$  will depend on aggregate productivity  $A_t$  and, to the extent that  $A_t$  is verifiable, the entrepreneur would be able to insure some of the volatility associated with aggregate productivity. Still, the risk associated with the idiosyncratic productivity cannot be fully insured and, therefore, the transformed model would have a similar structure with regards to the idiosyncratic productivity shock.

## B Proof of Lemma 2.1

Ignoring the agent superscript  $i$  and the country subscript  $j$ , the optimization problem of an entrepreneur can be written recursively as

$$\Omega_t(b_t) = \max_{h_t} \mathbb{E}_t \tilde{\Omega}_t(a_t) \quad (24)$$

subject to

$$a_t = \tilde{b}_t + (z_t - w_t)h_t$$

$$\tilde{b}_t = (1 - \delta_t)b_t$$

$$\tilde{\Omega}_t(a_t) = \max_{b_{t+1}} \left\{ \ln(c_t) + \beta \mathbb{E}_t \Omega_{t+1}(b_{t+1}) \right\} \quad (25)$$

subject to

$$c_t = a_t - q_t b_{t+1}$$

Since the information set changes from the beginning of the period to the end of the period, the optimization problem has been separated according to the available information. In sub-problem (24) the entrepreneur chooses the input of labor before knowing the productivity  $z_t$ . The variable  $\delta_t$  is an aggregate stochastic variable that denotes the possible losses incurred by the entrepreneur at the beginning of the period. This is taken as given by an individual entrepreneur. In sub-problem (25) the entrepreneur allocates the end of period wealth in consumption and savings after observing  $z_t$ .

The first order condition for sub-problem (24) is

$$\mathbb{E}_t \frac{\partial \tilde{\Omega}_t}{\partial a_t} (z_t - w_t) = 0.$$

The envelope condition from sub-problem (25) gives

$$\frac{\partial \tilde{\Omega}_t}{\partial a_t} = \frac{1}{c_t}.$$

Substituting in the first order condition we obtain

$$\mathbb{E}_t \left( \frac{z_t - w_t}{c_t} \right) = 0. \quad (26)$$

At this point we proceed by guessing and verifying the optimal policies for employment and savings. The guessed policies take the form:

$$h_t = \phi_t \tilde{b}_t \quad (27)$$

$$c_t = (1 - \beta)a_t \quad (28)$$

Since  $a_t = \tilde{b}_t + (z_t - w_t)h_t$  and the employment policy is  $h_t = \phi_t \tilde{b}_t$ , the end of period wealth can be written as  $a_t = [1 + (z_t - w_t)\phi_t]\tilde{b}_t$ . Substituting the guessed consumption policy we obtain

$$c_t = (1 - \beta) \left[ 1 + (z_t - w_t)\phi_t \right] \tilde{b}_t. \quad (29)$$

This expression is used to replace  $c_t$  in the first order condition (26) to obtain

$$\mathbb{E}_t \left[ \frac{z_t - w_t}{1 + (z_t - w_t)\phi_t} \right] = 0, \quad (30)$$

which is the condition stated in Lemma 2.1.

To complete the proof, we need to show that the guessed policies (27) and (28) satisfy the optimality condition for the choice of consumption and saving. This is characterized by the first order condition of sub-problem (25), which is equal to

$$-\frac{q_t}{c_t} + \beta \mathbb{E}_t \frac{\partial \Omega_{t+1}}{\partial b_{t+1}} = 0.$$

From sub-problem (24) we derive the envelope condition  $\partial \Omega_t / \partial b_t = \mathbb{E}_t[(1 - \delta_t)/c_t]$  which can be used in the first order condition to obtain

$$\frac{q_t}{c_t} = \beta \mathbb{E}_t \frac{1 - \delta_{t+1}}{c_{t+1}}.$$

We have to verify that the guessed policies satisfy this condition. Using the guessed policy (28) and equation (29) updated one period, the first order condition can be rewritten as

$$\frac{q_t}{a_t} = \beta \mathbb{E}_t \frac{1 - \delta_{t+1}}{[1 + (z_{t+1} - w_{t+1})\phi_{t+1}]\tilde{b}_{t+1}}.$$

Notice that  $(1 - \delta_{t+1})/\tilde{b}_{t+1} = 1/b_{t+1}$ . Using the guessed policy (28), this can also be written as  $(1 - \delta_{t+1})/\tilde{b}_{t+1} = q_t/(\beta a_t)$ . Substituting in the first order condition and rearranging we obtain

$$1 = \mathbb{E}_t \left[ \frac{1}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}} \right]. \quad (31)$$

The final step is to show that, if condition (30) is satisfied, then condition (31) is also satisfied. Let's start with condition (30), updated by one period. Multiplying both sides by  $\phi_{t+1}$  and then subtracting 1 in both sides we obtain

$$\mathbb{E}_{t+1} \left[ \frac{(z_{t+1} - w_{t+1})\phi_{t+1}}{1 + (z_{t+1} - w_{t+1})\phi_{t+1}} - 1 \right] = -1.$$

Multiplying both sides by -1 and taking expectations at time  $t$  we obtain (31).

## C First order conditions for households

Ignoring country subscript  $j$ , the optimization problem of a household is

$$\begin{aligned}
W_t(l_t, k_t) &= \max_{h_t} \mathbb{E}_t \tilde{W}_t(a_t) \\
&\text{subject to} \\
&a_t = w_t h_t + (A_t + p_t)k_t - l_t \\
\\
\tilde{W}_t(a_t) &= \max_{l_{t+1}, k_{t+1}} \left\{ c_t - \alpha A_t \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} + \beta \mathbb{E}_t W_{t+1}(l_{t+1}, k_{t+1}) \right\} \\
&\text{subject to} \\
&c_t = a_t + \frac{l_{t+1}}{R_t} - k_{t+1} p_t \\
&l_{t+1} \geq \eta A_t.
\end{aligned}$$

Also for households, the optimization problem has been separated before the realization of the aggregate productivity (when labor supply is decided) and after (when consumption, housing and borrowing are decided).

Given  $\beta\mu_t$  the lagrange multiplier associated with the borrowing constraint, the first order conditions with respect to  $h_t$ ,  $l_{t+1}$ ,  $k_{t+1}$  are, respectively,

$$\begin{aligned}
-\alpha \mathbb{E}_t A_t h_t^{\frac{1}{\nu}} + w_t &= 0, \\
\frac{1}{R_t} + \beta \mathbb{E}_t \frac{\partial W_{t+1}(l_{t+1}, k_{t+1})}{\partial l_{t+1}} - \beta \mu_t &= 0, \\
-p_t + \beta \mathbb{E}_t \frac{\partial W_{t+1}(l_{t+1}, k_{t+1})}{\partial k_{t+1}} &= 0.
\end{aligned}$$

The envelope conditions are

$$\begin{aligned}
\frac{\partial W_t(l_t, k_t)}{\partial l_t} &= -1, \\
\frac{\partial W_t(l_t, k_t)}{\partial k_t} &= A_t + p_t.
\end{aligned}$$

Updating by one period and substituting in the first order conditions we obtain (4), (5), (6). When the borrowing constraint takes the form  $\eta \mathbb{E}_t p_{t+1} k_{t+1} \geq l_{t+1}$ , the first order condition with respect to  $k_{t+1}$  becomes

$$-p_t + \beta \mathbb{E}_t \frac{\partial W_{t+1}(l_{t+1}, k_{t+1})}{\partial k_{t+1}} + \eta \beta \mu_t \mathbb{E}_t p_{t+1} = 0,$$

Substituting the envelope condition we obtain (7).

## D Proof of Proposition 2.1

As shown in Lemma 2.1, the optimal saving of entrepreneurs takes the form  $q_t b_{j,t+1}^i = \beta a_{j,t}^i$ , where  $a_{j,t}^i$  is the end-of-period wealth  $a_{j,t}^i = \tilde{b}_{j,t}^i + (z_{j,t}^i - w_{j,t})h_{j,t}^i$ . Since  $h_{j,t}^i = \phi(w_{j,t})\tilde{b}_{j,t}^i$  (see Lemma 2.1), the end-of-period wealth can be rewritten as  $a_{j,t}^i = [1 + (z_{j,t}^i - w_{j,t})\phi_j(w_{j,t})]\tilde{b}_{j,t}^i$ . Substituting into the optimal saving and aggregating over all entrepreneurs of country  $j$  we obtain

$$B_{j,t+1} = \frac{\beta}{q_t} \left[ 1 + (A_j - w_{j,t})\phi_j(w_{j,t}) \right] \tilde{B}_{j,t}. \quad (32)$$

This equation defines the aggregate demand for bonds in country  $j$  as a function of the price  $q_t$ , the wage rate  $w_{j,t}$ , and the aggregate wealth of entrepreneurs  $\tilde{B}_{j,t}$  after the realization of financial losses. Notice that the term in square brackets is bigger than 1. Therefore, in a steady state equilibrium  $B_{j,t+1} = B_{j,t}$  and the condition  $\beta < q_t$  must be satisfied. Remember that we are considering the case without financial losses, and therefore,  $\tilde{B}_{j,t} = B_{j,t}$ .

Using the equilibrium condition in the labor market, I can express the wage as a function of  $B_{j,t}$ . In particular, equalizing the demand for labor,  $H_{j,t}^D = \phi_j(w_{j,t})B_{j,t}$ , to the supply from households,  $H_{j,t}^S = (w_{j,t}/\alpha A_j)^\nu$ , the wage can be expressed as a function of only  $B_{j,t}$ . We can then use this function to replace  $w_{j,t}$  in (32) and express the demand for bonds as a function of only  $B_{j,t}$  and  $q_t$  as follows

$$B_{j,t+1} = \frac{s_j(B_{j,t})}{q_t}. \quad (33)$$

The function  $s_j(B_{j,t})$  is strictly increasing in the wealth of entrepreneurs,  $B_{j,t}$ .

Consider now the supply of bonds from households. For simplicity I assume that the borrowing constraint takes the form specified in equation (2), that is,  $l_{j,t+1} \leq \eta_j A_j$ . Using this limit together with the first order condition (5), we have that either the price of bonds satisfies  $q_t = \beta$  or households are financially constrained, that is,  $L_{j,t+1} = \eta_j A_j$ . When the price of bonds is equal to the inter-temporal discount factor (first case), we can see from (32) that  $B_{j,t+1} > B_{j,t}$ . So eventually, the global demand of bonds will reach the global supply, that is,  $B_{1,t+1} + B_{2,t+1} = \eta_1 A_1 + \eta_2 A_2$ . At this stage the borrowing constraint of households is binding in both countries and, therefore, the multiplier  $\mu_{j,t}$  is positive. Condition (5) then implies that the price of bonds is bigger than the inter-temporal discount factor. So the economy has reached a steady state. The steady state price of bonds is determined by condition (33) after setting  $B_{j,t} = B_{j,t+1}$  and  $B_{1,t} + B_{2,t} = \eta_1 A_1 + \eta_2 A_2$ . This is the only steady state equilibrium.

When the borrowing constraint takes the form (3), the proof is more involved but the economy also reaches a steady state with  $\beta < q_t$ .

## E First order conditions for problem (12)

Define  $\beta\gamma_t$  the Lagrange multiplier associated to the constraint  $b_{t+1} \leq l_{t+1}$ . The first order conditions for problem (12) with respect to  $b_{t+1}$  and  $l_{t+1}$  are

$$\begin{aligned} \frac{1}{\bar{R}_t^b} \mathbb{E}_t \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}} - \beta \mathbb{E}_t \left[ \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}} + \frac{\partial \varphi_{t+1}}{\partial b_{t+1}} b_{t+1} + \varphi_{t+1} + \gamma_t \right] &= 0, \\ -\frac{1}{\bar{R}_t^l} + \frac{1}{\bar{R}_t^b} \mathbb{E}_t \frac{\partial \tilde{b}_{t+1}}{\partial l_{t+1}} + \beta \mathbb{E}_t \left[ 1 - \frac{\partial \tilde{b}_{t+1}}{\partial l_{t+1}} - \frac{\partial \varphi_{t+1}}{\partial l_{t+1}} b_{t+1} + \gamma_t \right] &= 0. \end{aligned}$$

These conditions can be re-arranged as

$$\frac{1}{\bar{R}_t^b} = \beta \left[ 1 + \frac{\mathbb{E}_t \frac{\partial \varphi_{t+1}}{\partial b_{t+1}} b_{t+1} + \mathbb{E}_t \varphi_{t+1} + \gamma_t}{\mathbb{E}_t \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}}} \right], \quad (34)$$

$$\frac{1}{\bar{R}_t^l} = \beta \left[ 1 + \left( \frac{1}{\beta \bar{R}_t^b} - 1 \right) \mathbb{E}_t \frac{\partial \tilde{b}_{t+1}}{\partial l_{t+1}} - \mathbb{E}_t \frac{\partial \varphi_{t+1}}{\partial l_{t+1}} b_{t+1} + \gamma_t \right]. \quad (35)$$

I now use the definition of  $\tilde{b}_{t+1}$  provided in (8) and the specification of  $\varphi_{t+1}(\omega_{t+1})$  provided in (9) to derive the terms that enter equations (34) and (35). Define  $f_t(\xi_{t+1})$  the probability density of the recovery fraction  $\xi_{t+1}$ . The probability density is endogenously derived in the general equilibrium but individual agents will take it as given. We can then derive

$$\mathbb{E}_t \varphi_{t+1} = \int_0^{\omega_{t+1}} (\omega_{t+1} - \xi)^2 f_t(\xi), \quad (36)$$

$$\mathbb{E}_t \frac{\partial \varphi_{t+1}}{\partial b_{t+1}} = 2 \left[ \omega_{t+1} - \int_0^{\omega_{t+1}} \xi f_t(\xi) \right] \frac{1}{l_{t+1}}, \quad (37)$$

$$\mathbb{E}_t \frac{\partial \varphi_{t+1}}{\partial l_{t+1}} = -2 \left[ \omega_{t+1} - \int_0^{\omega_{t+1}} \xi f_t(\xi) \right] \omega_{t+1} \frac{1}{l_{t+1}^2}, \quad (38)$$

$$\mathbb{E}_t \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}} = \int_{\omega_{t+1}}^1 f_t(\xi), \quad (39)$$

$$\mathbb{E}_t \frac{\partial \tilde{b}_{t+1}}{\partial l_{t+1}} = \int_0^{\omega_{t+1}} \xi f_t(\xi), \quad (40)$$

Substituting these terms in (34) and (35) we can verify that the right-hand-side terms are only functions of  $\omega_{t+1}$ . Furthermore, since the multiplier  $\gamma_t$  is zero if  $\omega_{t+1} < 1$  and positive if  $\omega_{t+1} = 1$ , the first order conditions can be written as in (13) and (14).

## F Proof of Lemma 2.2

I first show that  $\bar{R}_t^b < 1/\beta$  using condition (34). From equations (36), (37) and (39) we can verify that  $\mathbb{E}_t \varphi_{t+1} > 0$ ,  $\mathbb{E}_t \frac{\partial \varphi_{t+1}}{\partial b_{t+1}} > 0$  and  $\mathbb{E}_t \frac{\partial \tilde{b}_{t+1}}{\partial b_{t+1}} > 0$ . Substituting in condition (34) we obtain  $1/\bar{R}_t^b > \beta$  or, equivalently,  $\bar{R}_t^b < 1/\beta$ .

I now show that  $R_t^l < 1/\beta$  using condition (35). We have already shown that  $1/(\beta \bar{R}_t^b) - 1 > 0$  and from condition (40) we can see that  $\mathbb{E}_t \frac{\partial \tilde{b}_{t+1}}{\partial l_{t+1}} > 0$ . Furthermore, from equation (38) we can verify that  $-\mathbb{E}_t \frac{\partial \tilde{b}_{t+1}}{\partial l_{t+1}} b_{t+1} > 0$ . Substituting in condition (35) we obtain  $1/R_t^l > \beta$  or, equivalently,  $R_t^l < 1/\beta$ .

## G Proof of Proposition 2.2

Banks make decisions at two different stages. At the beginning of the period they choose whether to renegotiate the debt and at the end of the period they choose the funding and lending policies. Given the initial states,  $b_t$  and  $l_t$ , the renegotiation decision boils down to a take-it or leave-it offer made by each bank to its creditors for the repayment of the debt. Denote by  $\tilde{b}_t = \psi(b_t, l_t, \xi_t^e)$  the offered repayment. This depends on the individual liabilities  $b_t$ , individual assets  $l_t$ , and the expected liquidation price of assets  $\xi_t^e$ . The superscript  $e$  is to make clear that the bank decision depends on the expected price in the eventuality of liquidation. The best repayment offer made by the bank is

$$\psi(b_t, l_t, \xi_t^e) = \begin{cases} b_t, & \text{if } b_t \leq \xi_t^e l_t \\ \xi_t^e l_t, & \text{if } b_t > \xi_t^e l_t \end{cases}, \quad (41)$$

which is accepted by creditors if they have the same expectation about the liquidation price  $\xi_t^e$ .

After the renegotiation stage, banks choose the funding and lending policies,  $b_{t+1}$  and  $l_{t+1}$ . These policies depend on the two interest rates,  $\bar{R}_t^b$  and  $R_t^l$ , and on the probability distribution of the next period liquidation price  $\xi_{t+1}$ . Since we could have multiple equilibria, the next period price could be stochastic. Suppose that the price takes two values,  $\underline{\xi}$  and 1, with the probability of the low value defined as

$$\theta(\omega_{t+1}) = \begin{cases} 0, & \text{if } \omega_{t+1} < \underline{\xi} \\ \lambda, & \text{if } \underline{\xi} \leq \omega_{t+1} \leq 1. \end{cases}$$

The variable  $\omega_{t+1} = b_{t+1}/l_{t+1}$  represents the leverage of all banks in a symmetric equilibrium. For the moment the symmetry of the equilibrium is an assumption.

I will then show below that in fact banks do not have incentives to deviate from the leverage chosen by other banks.

Given the above assumption about the probability distribution of the liquidation price, the funding and lending policies of the bank are characterized in Lemma 2.2 and depend on  $\bar{R}_t^b$  and  $R_t^l$ . In short, if  $\bar{R}_t^b/(1 - \tau) = R_t^l$ , then the optimal policy of the bank is to choose a leverage  $\omega_{t+1} \leq \underline{\xi}$ . If  $\bar{R}_t^b/(1 - \tau) < R_t^l$ , the optimal leverage is  $\omega_{t+1} > \underline{\xi}$ .

Given the assumption that the equilibrium is symmetric (all banks choose the same leverage  $\omega_{t+1}$ ), multiple equilibria arise if the chosen leverage is  $\omega_{t+1} \in \{\underline{\xi}, 1\}$ . In fact, once we move to the next period, if the market expects  $\xi_{t+1}^e = \underline{\xi}$ , all banks are illiquid and they choose to renege on their liabilities (given the renegotiation policy (41)). As a result, there will not be any bank that can buy the liquidated assets of other banks. Then the only possible price that is consistent with the expected price is  $\xi_{t+1} = \underline{\xi}$ . On the other hand, if the market expects  $\xi_{t+1}^e = 1$ , banks are liquid and, if one bank reneges, creditors can sell the liquidated assets to other banks at the price  $\xi_{t+1} = 1$ . Therefore, it is optimal for banks not to renegotiate, consistently with the renegotiation policy (41).

The above proof, however, assumes that the equilibrium is symmetric, that is, all banks choose the same leverage. To complete the proof, we have to show that there is no incentive for an individual bank to deviate from the leverage chosen by other banks. In particular, I need to show that, in the anticipation that the next period liquidation price could be  $\xi_{t+1} = \underline{\xi}$ , a bank do not find convenient to chose a lower leverage so that, in the eventuality that the next period price is  $\xi_{t+1} = \underline{\xi}$ , the bank could purchase the liquidated asset at a price lower than 1 and make a profit (since the unit value for the bank of the liquidated assets is 1).

If the price at  $t + 1$  is  $\xi_{t+1} = \underline{\xi}$ , a liquid bank could offer a price  $\underline{\xi} + \epsilon$ , where  $\epsilon$  is a small but positive number. Since the repayment offered by a defaulting bank is  $\underline{\xi}_{t+1}$ , creditors prefer to sell the assets rather than accepting the repayment offered by the defaulting bank. However, if this happens, the expectation of the liquidation price  $\xi^e = \underline{\xi}$  turns out to be incorrect ex-post. Therefore, the presence of a single bank with liquidity will raise the expected liquidation price to  $\underline{\xi} + \epsilon$ . But even with this new expectation, a bank with liquidity can make a profit by offering  $\underline{\xi} + 2\epsilon$ . Again, this implies that the expectation turns out to be incorrect ex-post. This mechanism will continue to raise the expected price to  $\xi_{t+1}^e = 1$ . At this point the liquid bank will not offer a price bigger than 1 and the ex-post liquidation price is correctly predicted to be 1. Therefore, as long as there is a single bank with liquidity, the expected liquidation price must be 1. But then a bank cannot make a profit in period  $t + 1$  by choosing a lower leverage in period  $t$  with the goal of remaining liquid in the next period. This proves that there is no incentive to deviate from the policy chosen by other banks.



Finally, the fact that multiple equilibria cannot arise when  $\omega_t < \underline{\xi}$  is obvious. Even if the price is  $\underline{\xi}$ , banks remain liquid.

## H Numerical solution

I describe the numerical procedure to solve the model with the endogenous borrowing constraint specified in (3). I first describe the procedure when the relative productivity  $A_{2,t}/A_{1,t}$  remains constant. I will then describe the numerical procedure when the relative productivity changes over time.

### H.1 Stationary equilibrium without structural break

The states of the economy are given by the bank liabilities held in both countries,  $B_{1,t}$  and  $B_{2,t}$ , the bank loans,  $L_{1,t}$  and  $L_{2,t}$ , and the realization of the sunspot shock  $\varepsilon_t$ . These five variables are important in determining the renegotiation liabilities  $\tilde{B}_{1,t}$  and  $\tilde{B}_{2,t}$ . However, once we know the renegotiated liabilities  $\tilde{B}_{1,t}$  and  $\tilde{B}_{2,t}$ , they become the sufficient states. Therefore, in the computation I will solve for the recursive equilibrium using  $\tilde{B}_{1,t}$  and  $\tilde{B}_{2,t}$  as state variables.

The equilibrium is the solution to the following systems of equations:

$$H_{j,t} = \phi_j(w_{j,t})\tilde{B}_{j,t}, \quad (42)$$

$$q_t B_{j,t+1} = \beta \bar{a}_{j,t}, \quad (43)$$

$$\bar{a}_{j,t} = \tilde{B}_{j,t} + (A_j - w_{j,t})H_{j,t} \quad (44)$$

$$\alpha H_{j,t}^{\frac{1}{\nu}} = w_{j,t}, \quad (45)$$

$$1 = \beta R_t^l (1 + \mu_{j,t}), \quad (46)$$

$$p_{j,t} = \beta \mathbb{E}_t \left[ A_j + (1 + \eta_j \mu_{j,t}) p_{j,t+1} \right], \quad (47)$$

$$L_{j,t+1} = \eta_t \mathbb{E}_t p_{j,t+1}, \quad (48)$$

$$\frac{1}{\bar{R}_t^b} \geq \beta \left[ 1 + \Phi(\omega_{t+1}) \right], \quad (49)$$

$$\frac{1}{R_t^l} \geq \beta \left[ 1 + \Psi(\omega_{t+1}) \right], \quad (50)$$

$$\bar{R}_t^b = \left[ 1 - \theta(\omega_{t+1}) + \theta(\omega_{t+1}) \left( \frac{\underline{\xi}}{\omega_{t+1}} \right) \right] \frac{1}{q_t}, \quad (51)$$

$$\omega_{t+1} = \frac{B_{1,t+1} + B_{2,t+1}}{L_{1,t+1} + L_{2,t+1}} \quad (52)$$

Equations (42)-(44) are derived from aggregating the optimal policies of entrepreneurs (labor demand, savings, end of period wealth). Equations (45)-(48) derive from the optimization problem of households (labor supply, optimal borrowing, optimal holding of the fixed asset, borrowing constraint). Notice that the borrowing constraint of households (equation (48) is not always binding. However, when it is not binding and the multiplier is  $\mu_t = 0$ , households' borrowing is not determined. Therefore, without loss of generality I assume that in this case households borrow up to the limit. This explains why the borrowing constraint is always satisfied with equality. Equations (49)-(50) are the first order conditions for banks. They are satisfied with equality if  $\omega_{t+1} < 1$  and with inequality if  $\omega_{t+1} = 1$ . Equation (51) defines the expected return on bank liabilities given their price  $q_t$ . The final equation (52) defines the leverage of banks.

One complication in solving the dynamic system is that the expectation of the next period prices for the fixed asset,  $\mathbb{E}_t p_{j,t+1}$ , is unknown. All we know is that the next period prices are functions of  $\tilde{B}_{1,t+1}$  and  $\tilde{B}_{2,t+1}$ , that is,  $p_{1,t+1} = P_1(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  and  $p_{2,t+1} = P_2(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$ . If I knew these two functions, for any given states  $\tilde{B}_{1,t}$  and  $\tilde{B}_{2,t}$  the above conditions would be a system of 18 equations in 18 unknowns:  $H_{j,t}$ ,  $\bar{a}_{j,t}$ ,  $\mu_{j,t}$ ,  $w_{j,t}$ ,  $p_{j,t}$ ,  $B_{j,t+1}$ ,  $L_{j,t+1}$ ,  $q_t$ ,  $R_t^l$ ,  $\bar{R}_t^b$ ,  $\omega_{t+1}$ . Notice that  $\tilde{B}_{j,t+1}$  is a known function of  $B_{j,t+1}$ ,  $L_{j,t+1}$  and the realization of the sunspot shock  $\varepsilon_{t+1}$ . Therefore, I can compute the expectation of the next period prices  $p_{1,t+1}$  and  $p_{2,t+1}$  if I know the price functions  $P_1(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  and  $P_2(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$ . We can then solve the 18 equations for the 18 variables and this would provide a solution for any given state  $\tilde{B}_{1,t}$  and  $\tilde{B}_{2,t}$ .

Unfortunately, the price functions  $P_1(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  and  $P_2(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  are unknown. Thus, the procedure will be based on a parametrization of these two functions. In particular, I approximate  $P_1(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  and  $P_2(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  with piece-wise linear functions over a grid for the state variables  $\tilde{B}_{1,t}$  and  $\tilde{B}_{2,t}$ . I then solve the above system of equations at all grid points for  $\tilde{B}_{1,t}$  and  $\tilde{B}_{2,t}$ . As part of the solution I obtain the current prices  $p_{1,t}$  and  $p_{2,t}$ . I then use the current prices to update the approximated functions  $P_1(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  and  $P_2(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  at the grid points. I repeat the iteration until convergence, that is, the values guessed for  $P_1(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  and  $P_2(\tilde{B}_{1,t+1}, \tilde{B}_{2,t+1})$  at all grid points will be equal (up to a small rounding number) to the values of  $p_{1,t}$  and  $p_{2,t}$  obtained by solving the model (given the guesses for the price functions).

## H.2 Equilibrium with changing relative productivity

When the relative productivity  $A_{2,t}/A_{1,t}$  changes over time, the economy transits from a stochastic equilibrium to a new stochastic equilibrium. Therefore, I need to solve for the transition. Since I assume that the changes in aggregate productivity

are unexpected, the solution method consists in solving for the decision rules at any date under the assumption that  $A_{2,t}/A_{1,t}$  stays constant at the current level. For example, when I solve for the decision rules in 2003, I assume that the relative productivity is  $A_{2,2003}/A_{1,2003}$  and this stay the same in 2004, 2005, etc. The decision rules are then found using the procedure described in the previous subsection. Next a compute the decision rules for 2004. In doing so I assume that the relative productivity is  $A_{2,2004}/A_{1,2004}$  and this stay the same in 2005, 2006, etc. Once I have the decisions rules for all transition dates, I simulate the model using these rules.

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