Identifying Incomplete Information Discrete Games without Bayesian Nash Equilibrium

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Background

- Game theoretical model is a standard tool in studying economic phenomenons when people interact with each other
- In reality, asymmetric information is prevalent and researchers model it as a game with incomplete information
- Bayesian Nash Equilibrium (BNE) is a commonly used solution concept in estimation of empirical games
- BNE enables researchers to recover player's payoff from player's choice data

BNE Restrictions

BNE places two behavioral restrictions

- Each player maximizes his expected payoff given his belief
- Each player forms an equilibrium/unbiased belief (i.e. each player's belief is other players' actual choice probabilities given available information)

Potential Misspecification of Unbiased Belief

- Each player has to figure out other player's equilibrium strategy and integrate it over the distribution of other player's private information
- In games with multiple equilibria, a player has to know which equilibrium strategy is used by other player
- Learning other player's behavior through repeated interactions or similar past experience is also complicated when economic environment and market conditions vary dramatically
- Empirical evidence from both laboratory and field show that equilibrium is inconsistent with players' behaviors in many games (i.e. Georee and Holt (2001) and Aguirregabiria and Magesan (2016))
- Falsely imposing equilibrium yields biased estimation for interactive effect

A More General Model

In this paper, I relax the equilibrium belief assumption

- I assume each player chooses an action that maximizes his expected payoff given his subjective belief
- This subjective belief is allowed to be any probability distribution over other player's action set
- This framework nests BNE as a special case when player has equilibrium/unbiased belief
- It also permits non-equilibrium behaviors and attribute them to non-equilibrium/biased belief
- Player's both payoff and belief are treated as unknown non-parametric functions

Identification Result

In a game that player 1 has more than two actions and player 2 has binary choice

- With an exclusion restriction that only affects player 2's payoff, player 1's interactive effect ratio is identified without imposing BNE
- With another type of exclusion restriction that only affect player's interactive effect, player 1's non-interactive payoff and his subjective expectation of payoff impacted by player 2 is identified
- Similar identification results are generalized to the case when player 2 has more than two actions but still smaller than player 1's actions
- However, there is no identification result for player 2

Generalization of Identification Results

In an ordered-action game with N players and each player has $J + 1$ actions

- Suppose interactive effect is multiplicative separable between player's own action and other players' actions
- Each player's identification problem is conceptually equivalent to the one for player 1 in previous game with asymmetric number of actions
- Identification results for player 1 in asymmetric actions game trivially holds for ***each*** player in this ordered-action game
- Conventional two-step estimator can be applied in estimation; moreover, when payoff and belief are smooth functions, standard MLE or GMM can be applied to reduce finite sample bias

Identification Intuition

Suppose player 1 has $J_1 + 1$ actions and player 2 has $J_2 + 1$ actions with $J_1 > J_2$

- Let Z_2 be a variable that only affects player 2's payoff
- As Z_2 varies, player 2's payoff changes and he is likely to alter his behaviors
- If player 1 anticipate this, he will adjust his belief and also alter his behaviors
- A new realization of Z_2 introduces J_2 unknowns (i.e. player 1's belief) but imposes J_1 restrictions (i.e. player 1's choice probabilities)
- The variation of Z_2 enables us to establish an over-identification restrictions for a function of player 1's payoff

Relation to Literature

Aradillas-Lopez and Tamer (2008) replace BNE with rationality assumption in an incomplete information game

- They show for each level of rationality (Bernheim (1984) and Pearce (1984)), there is an identified set of payoff parameters
- Such identified set shrinks as the level of rationality increases
- I do not assume player's level of rationality and proves point identification of non-interactive payoff and subjective expectation of impact

Relation to Literature

Aguirregabiria and Magesan (2016) study player's biased belief in dynamic game

- They show that Markov Perfect Equilibrium (MPE) is testable and they attribute the failure of MPE to player's biased belief
- To identify player's payoff, they need to assume that player has equilibrium belief in at least two realizations of state variables
- Similar idea has been applied to static experimental games with incomplete information by Aguirregabiria and Xie (2016)
- This paper achieves identification in another class of games without assuming equilibrium belief in any realization of state variable

Empirical Application

I study KFC and McDonald's store type competition in China

- In an isolated market, each fast food chain possesses multiple stores
- Some of stores open 24 hours while others only open during day time
- I model this store type decision as an entry game such that each chain simultaneously chooses how many stores to open in the night
- Compared with other static entry games, entry cost is small and retractable in this application
- Potential entrants are clearly defined

Roadmap

- Model
- Identification Results
  - Review of identification under BNE
  - Identification in game with asymmetric number of actions
  - Identification in game with ordered actions
- Possible Extensions
  - Relaxation of known distribution of private information
  - Allowing unobserved heterogeneity
- Empirical Application
  - Preliminary data
- Conclusions

Model

- Two players indexed by $i \in \{1, 2\}$ and $-i$ indexes other player
- Let $A_i = \{a_i^0, a_i^1, \cdots, A_i^{J_i}\}$ denote player i's action set; assume $J_1 > J_2$
- Cartesian product $A = A_1 \times A_2$ represents the space of action profile
- Each player i simultaneously chooses an action $a_i \in A_i$

Payoff Function

When realized outcome is $\mathbf{a} = (a_1, a_2) \in A$, player i's payoff is

$$\Pi_i[X, Z_i, \epsilon_i, \mathbf{a}] = \pi_i(X, Z_i, a_i) + \delta_i[X, Z_i, (a_i, a_{-i})] \cdot \mathbb{1}(a_{-i} \neq a^0_{-i}) + \epsilon_i(a_i)$$

- $X \in \mathbb{R}^{L_X}$ is a vector of variables that affect both players' payoff
- $Z_i \in \mathbb{R}$ is a variable that only affects player i's payoff
- $\pi_i(X, Z_i, a_i)$ represents player i's payoff of action a_i when player $-i$ chooses action a^0_{-i}
- $\delta_i[X, Z_i, (a_i, a_{-i})]$ measures the change of player i's payoff of action a_i when player $-i$'s action varies from a^0_{-i} to a_{-i}
- π_i is referred as non-interactive payoff (base return in De Paula and Tang (2012)) and δ_i is called as interactive payoff
- Even though they are additive, it is actually non-parametrically specified Details

Assumption on Private Information

$\epsilon_i(a_i)$ is a variable affects player i's payoff of action a_i and it is player i's private information

Assumption

(a) for each player $i = 1, 2$, $\epsilon_i = (\epsilon_i(a_i^0), \cdots, \epsilon_i(a_i^{J_i}))'$ *follows a CDF* $G_i(\cdot)$ *that is absolutely continuous with respect to Lebesgue measure in* $\mathbb{R}^{J_i+1}$. $G_i(\cdot)$ *is known by both players and econometrician.*
(b) ϵ_i *is independently distributed across players and independent of common information* X, Z_1 *and* Z_2.

Belief and Best Response

- $\mathbf{b}_i(X, Z_1, Z_2) = (b_i^0(X, Z_1, Z_2), \cdots, b_i^{J_{-i}}(X, Z_1, Z_2))'$ is a vector of player i's belief
- $b_i^j(X, Z_1, Z_2)$ represents player i's belief about the probability that player $-i$ will choose action a_{-i}^j
- No more restrictions imposed on this belief vector except: $0 \leq b_i^j(X, Z_1, Z_2) \leq 1 \ \forall \ j$ and $\sum_{j=0}^{J_{-i}} b_i^j(X, Z_1, Z_2) = 1$
- Player i's expected payoff of action i is

$$\pi_i(X, Z_i, a_i) + \sum_{j=1}^{J_{-i}} \delta_i[X, Z_i, (a_i, a_{-i})] \cdot b_i^j(X, Z_1, Z_2) + \epsilon_i(a_i)$$

- Each player i chooses an action that maximizes above expected payoff and denote such strategy by $\sigma_i(X, Z_i, Z_{-i}, \epsilon_i)$

Conditional Choice Probability

Let $\mathbf{p}_i(\mathbf{a}_i|X, Z_1, Z_2) = (p_i(a_i^0|X, Z_1, Z_2), \cdots, p_i(a_i^{J_i}|X, Z_1, Z_2))'$ represent a vector of player i's conditional choice probability

$$p_i(a_i^j|X, Z_1, Z_2) = \int \mathbb{1}\{\sigma_i(X, Z_i, Z_{-i}, \epsilon_i) = a_i^j\} dG_i(\epsilon_i)$$

I use upper letter (X, Z_1, Z_2) to denote random variables and lower letter (x, z_1, z_2) to represent their realizations

BNE as a Special Case

Definition

Observed data is consistent with Bayesian Nash Equilibrium if each player's belief is other player's actual choice probability, i.e. $p_i(a_i^j|X, Z_1, Z_2) = b_{-i}^j(X, Z_1, Z_2)\ \forall\ 0 \leq j \leq J_i$ and $i = 1, 2$.

Data Generating Process

- Researchers have a data set that contains M independent games played by same two players and each game is indexed by m
- Each player i observes state variables $(x_m, z_{1,m}, z_{2,m})$ and his private shock $\epsilon_{i,m}$ and chooses an optimal action based on his belief $\mathbf{b}_i(x_m, z_{1,m}, z_{2,m})$
- Researchers observe $(x_m, z_{1,m}, z_{2,m})$ and players' choices $(a_{1,m}, a_{2,m})$ for each game m
- The asymptotics comes from $M \to \infty$; in this case, $\hat{p}_i(X, Z_1, Z_2)$ can be consistently estimated
- For identification illustration, I assume $\mathbf{p}_i$ is known by researcher
- Researchers want to use this data set to do inference on player i's payoff without imposing BNE

Normalization and CCP Inversion

Assumption

For player $i = 1, 2$, *the payoff for action* a_i^0 *is normalized to zero. That is* $\pi_i(x, z_i, a_i^0) = 0$ *and* $\delta_i[x, z_i, (a_i^0, a_{-i})] = 0 \; \forall \; x, z_i, a_{-i}$

Hotz and Miller (1993) CCP inversion

- Given previous normalization and distributional assumption on ϵ_i, there is a one-to-one mapping $F_i(\cdot) : \mathbb{R}^{J_i+1} \Rightarrow \mathbb{R}^{J_i+1}$ between player i's conditional choice probability and his expected payoff

$$\pi_i(x, z_i, a_i^k) + \sum_{j=1}^{J_{-i}} \delta_i[x, z_i, (a_i^k, a_{-i}^j)] \cdot b_i^j(x, z_i, z_{-i}) = F_i^k[\mathbf{p}_i(x, z_i, z_{-i})]$$

Identification Under BNE

Under BNE assumption, $b_i^j(x, z_i, z_{-i})$ can be replaced by its counter-part $p_{-i}^j(x, z_i, z_{-i})$

$$\pi_i(x, z_i, a_i^k) + \sum_{j=1}^{J_{-i}} \delta_i\left[x, z_i, (a_i^k, a_{-i}^j)\right] \cdot p_{-i}^j(x, z_i, z_{-i}) = F_i^k\left[\mathbf{p}_i(x, z_i, z_{-i})\right]$$

- Conditional on (x, z_i), π_i and δ_i is fixed
- p_{-i}^j has exogenous variation as z_{-i} varies
- It can be seen as a regression of $F(\cdot)$ on $\mathbf{p}_{-i}$ where π_i is the coefficient for constant and δ_i is the coefficient on the regressors

Identification without BNE

- I focus on player 1 and consider a simple case that player 2 has binary choice; i.e. $A_2 = (a_2^0, a_2^1)$
- (x, z_1) are suppressed as arguments since the identification relies on exogenous variation of Z_2 conditional on (x, z_1)
- For an action a_1^k, we have following equation

$$\pi_1(a_1^k) + \delta_1(a_1^k, a_2^1)b_1^1(z_2) = F_1^k[\mathbf{p}_1(z_2)]$$

- Suppose Z_2 has two realizations, say z_2^1 and z_2^2; we can plug them into above equation and cancel $\pi_1(a_1^k)$

$$\delta_1(a_1^k, a_2^1)[b_1^1(z_2^1) - b_1^1(z_2^2)] = F_1^k[\mathbf{p}_1(z_2^1)] - F_1^k[\mathbf{p}_1(z_2^2)]$$

Identification without BNE

- For any two actions a_1^j and a_1^k, we then have

$$\delta_1(a_1^j, a_2^1)[b_1^1(z_2^1) - b_1^1(z_2^2)] = F_1^j[\mathbf{p}_1(z_2^1)] - F_1^j[\mathbf{p}_1(z_2^2)]$$

$$\delta_1(a_1^k, a_2^1)[b_1^1(z_2^1) - b_1^1(z_2^2)] = F_1^k[\mathbf{p}_1(z_2^1)] - F_1^k[\mathbf{p}_1(z_2^2)]$$

- In case that $b_1^1(z_2^1) \neq b_1^1(z_2^2)$, $\frac{\delta_1(a_1^j, a_2^1)}{\delta_1(a_1^k, a_2^1)}$ can be identified by

$$\frac{\delta_1(a_1^j, a_2^1)}{\delta_1(a_1^k, a_2^1)} = \frac{F_1^j[\mathbf{p}_1(z_2^1)] - F_1^j[\mathbf{p}_1(z_2^2)]}{F_1^k[\mathbf{p}_1(z_2^1)] - F_1^k[\mathbf{p}_1(z_2^2)]}$$

- Even though we assume BNE, player's payoff is typically non-identified without Z_i

Economic Interpretation of $\frac{\delta_1(a_1^j,a_2^1)}{\delta_1(a_1^k,a_2^1)}$

- Typically, δ_1 receives most interest in empirical games since it measures the interactive effect
- $\frac{\delta_1(a_1^j,a_2^1)}{\delta_1(a_1^k,a_2^1)}$ measures the relative impact of player 2's behavior on player 1's payoff of two actions
- It sheds light on player 1's choice incentive and competitive effect
- Suppose in a duopoly competition, we have estimated that compared with action a_1^k, the payoff for a_1^j is less sensitive to player 2's behavior
- We can conclude that at least part of the reason that player 1 chooses a_1^j is to alleviate the negative impact of player 2's action

Another Type of Exclusion Restriction

- X can be partitioned by two subvectors $\tilde{X} \in \mathbb{R}^{Lx-1}$ and $S \in \mathbb{R}$
- Non-interactive payoff does not depend on S; for instance

$$\pi_i(X, Z_i, a_i) = \pi_i(\tilde{X}, Z_i, a_i)$$

- Interactive payoff depends on S; for instance

$$\delta_i[X, Z_i, (a_i, a_{-i})] = \delta_i[\tilde{X}, S, Z_i, (a_i, a_{-i})]$$

- In KFC and McDonald's store type example, S can be a measure of two chains' network; for instance, my store's distance from my competitor's store

Identification

- Suppress $(\tilde{x}, z_1, z_2)$ as identification relies on S
- As shown above, we have

$$\pi_1(a_1^j) + \delta_1[s, (a_1^j, a_2^1)]b_1^1(s) = F_1^j[\mathbf{p}_1(s)]$$

$$\pi_1(a_1^k) + \delta_1[s, (a_1^k, a_2^1)]b_1^1(s) = F_1^k[\mathbf{p}_1(s)]$$

- Simple algebra yields

$$\pi_1(a_1^j) - \frac{\delta_1[s, (a_1^j, a_2^1)]}{\delta_1[s, (a_1^k, a_2^1)]}\pi_1(a_1^k) = F_1^j[\mathbf{p}_1(s)] - \frac{\delta_1[s, (a_1^j, a_2^1)]}{\delta_1[s, (a_1^k, a_2^1)]}F_1^k[\mathbf{p}_1(s)]$$

- Note the coefficient on $\pi_1(a_1^k)$ and terms on right hand side are identified

Identification

Given two realizations of S, say s^1 and s^2, we then have following two equations

$$\pi_1(a_1^j) - \frac{\delta_1[s^1,(a_1^j,a_2^1)]}{\delta_1[s^1,(a_1^k,a_2^1)]}\pi_1(a_1^k) = F_1^j[\mathbf{p}_1(s^1)] - \frac{\delta_1[s^1,(a_1^j,a_2^1)]}{\delta_1[s^1,(a_1^k,a_2^1)]}F_1^k[\mathbf{p}_1(s^1)]$$

$$\pi_1(a_1^j) - \frac{\delta_1[s^2,(a_1^j,a_2^1)]}{\delta_1[s^2,(a_1^k,a_2^1)]}\pi_1(a_1^k) = F_1^j[\mathbf{p}_1(s^2)] - \frac{\delta_1[s^2,(a_1^j,a_2^1)]}{\delta_1[s^2,(a_1^k,a_2^1)]}F_1^k[\mathbf{p}_1(s^2)]$$

- This is a linear equation system with two equations and two unknowns
- $\pi_1(a_1^j)$ and $\pi_1(a_1^k)$ are identified
- $\delta_1[s,(a_1^j,a_2^1)] \cdot b_1^1(s)$ is identified for every a_1^j thereafter
- All results are generalized to the case that player 2 has more than two actions Details

Economic Interpretation

- In KFC and McDonald's store type decision game, $\pi_1(\tilde{X}, Z_1, a_1^j)$ can be interpreted as player 1's "monopolistic profit"; i.e. firm 1's profit of opening j stores during the night if firm 2 opens no store
- $\delta_1[\tilde{X}, S, Z_1, (a_1^j, a_2^1)] \cdot b_1^1(\tilde{X}, S, Z_1, Z_2)$ measures player 1's subjective expectation about player 2's impact on him
- It implies interactive effect δ_1 is identified up to a scale of player 1's belief
- If there is just one realization of Z_2, say z_2^1, such that player 1 has unbiased belief; then δ_1 is also point identified
- Which state to justify unbiased belief can be guided by the unbiased belief test proposed by Aguirregabiria and Magesan (2016) and Aguirregabiria and Xie (2016)

Game with Ordered-Action

- Suppose $A_i = \{a_i^0, a_i^1, \cdots, a_i^{J_i}\}$ has a natural order interpretation; i.e. how many stores to open during the night
- Let $J_i > 1$; no further restrictions on J_i or relationship between J_1 and J_2
- Suppose interactive effect can be decomposed in two functions

$$\delta_i[X, Z_i, (a_i, a_{-i})] = \tilde{\delta}_i(X, Z_i, a_i) \cdot \eta_i(X, Z_i, a_{-i})$$

- Where $\eta_i(X, Z_i, a_{-i}^1) = 1$
- Commonly used parametric assumption in ordered-action game

Parametric Interpretation

Given that $\delta_i\left[X, Z_i, (a_i, a_{-i})\right] = \tilde{\delta}_i(X, Z_i, a_i) \cdot \eta_i(X, Z_i, a_{-i})$

- $\tilde{\delta}_i(X, Z_i, a_i) = \delta_i\left[X, Z_i, (a_i, a^1_{-i})\right]$, it measures the impact of player $-i$'s action a^1_{-i} on player i's payoff of action a_i
- $\eta_i(X, Z_i, a_{-i})$ measures additional multiplicative impact when player 2 increases his action

$$\eta_i(X, Z_i, a_{-i}) = \frac{\delta_i\left[X, Z_i, (a_i, a_{-i})\right]}{\delta_i\left[X, Z_i, (a_i, a^1_{-i})\right]}$$

- Aradillas-Lopez and Gandhi (2016) refer η_i as strategic index and $\tilde{\delta}_i$ as the overall scale of interactive effect

Identification in Games with Ordered-Action

Player i's expected payoff of action a_i is

$$
\begin{aligned}
&\pi_i(X, Z_i, a_i) + \sum_{j=1}^{J_{-i}} \tilde{\delta}_i(X, Z_i, a_i) \cdot \eta_i(X, Z_i, a_{-i}) \cdot b_i^j(X, Z_1, Z_2) \\
=&\pi_i(X, Z_i, a_i) + \tilde{\delta}_i(X, Z_i, a_i)\{\sum_{j=1}^{J_{-i}} \eta_i(X, Z_i, a_{-i}) \cdot b_i^j(X, Z_1, Z_2)\} \\
=&\pi_i(X, Z_i, a_i) + \tilde{\delta}_i(X, Z_i, a_i) \cdot g_i(X, Z_1, Z_2)
\end{aligned}
$$

Compared with player 1's expected payoff of a_1 in game with asymmetric number of actions

$$
\pi_1(X, Z_1, a_1) + \delta_1[X, Z_1, (a_1, a_2^1)] \cdot b_1^1(X, Z_1, Z_2)
$$

Identification in Games with Ordered-Action

- All identification results for player 1 in a game with asymmetric actions hold for both players in this ordered-action game
- Results are generalized to an ordered-action game with more than two players Details

A Weaker Assumption on Private Information

- Previous identification results assume researchers know the distribution of ϵ_i
- A weaker distributional assumption can still achieve identification
- Consider following assumption such the distribution depends on a vector of unknown parameters

Assumption

$\epsilon_i = (\epsilon_i(a_i^0), \cdots, \epsilon_i(a_i^{J_i}))'$ follows a CDF $G(\cdot; \boldsymbol{\beta}_i)$ where $\boldsymbol{\beta}_i$ is a vector of parameters with L_i dimensions

Identification Results

Suppress $(\tilde{x}, z_1)$ and suppose there exist $k \geq 2$ realizations of S, say s^1 up to s^k, and $h \geq 2$ realizations of Z_2, say z_2^1 up to z_2^h

$$F_1^1\left[\mathbf{p}_1(s^1, z_2^1); \boldsymbol{\beta}_1\right] = \pi_1(a_1^1) + \delta_1\left[s^1, (a_1^1, a_2^1)\right] \cdot b_1^1(s^1, z_2^1)$$

$$\vdots$$

$$F_1^{J_1}\left[\mathbf{p}_1(s^1, z_2^1); \boldsymbol{\beta}_1\right] = \pi_1(a_1^{J_1}) + \delta_1\left[s^1, (a_1^{J_1}, a_2^1)\right] \cdot b_1^1(s^1, z_2^1)$$

$$F_1^1\left[\mathbf{p}_1(s^2, z_2^1); \boldsymbol{\beta}_1\right] = \pi_1(a_1^1) + \delta_1\left[s^2, (a_1^1, a_2^1)\right] \cdot b_1^1(s^2, z_2^1)$$

$$\vdots$$

$$F_1^{J_1}\left[\mathbf{p}_1(s^k, z_2^h); \boldsymbol{\beta}_1\right] = \pi_1(a_1^{J_1}) + \delta_1\left[s^k, (a_1^{J_1}, a_2^1)\right] \cdot b_1^1(s^k, z_2^h)$$

Identification Results

- This is an equation system with khJ_1 equations
- Unknowns contain following:

Parameters	# of Unknowns
$\pi_1(\cdot)$	J_1
$\delta_1[s,(a_1^1,a_2^1)]b_1^1(s,z_2)$	kh
$\frac{\delta_1\left[s,(a_1,a_2^1)\right]}{\delta_1\left[s,(a_1^1,a_2^1)\right]}$	$(J_1-1)k$
$\boldsymbol{\beta}_1$	L_1

- Order condition satisfies if $khJ_1 > J_1 + kh + (J_1-1)k + L_1$ which yields $k(J_1-1)(h-1) \geq J_1 + L_1$

Identification Results

- Let $\mathbf{F}_1(\boldsymbol{\beta}) = (F_1^1[\mathbf{p}_1(s^1, z_2^1); \boldsymbol{\beta}_1], \cdots, F_1^{J_1}[\mathbf{p}_1(s^k, z_2^h); \boldsymbol{\beta}_1])'$
- The Jacobian matrix $\frac{\partial \mathbf{F}_1(\boldsymbol{\beta}_1)}{\partial \boldsymbol{\beta}_1}$ has full column rank
- Since both order and rank conditions are satisfied, then $\boldsymbol{\beta}_1$ is identified
- Note that the column rank condition for Jacobian matrix is a generic assumption; without it, $\boldsymbol{\beta}_1$ is non-identified even though researchers know perfectly about player's belief and payoff

Unobserved Heterogeneity

- Previous analysis assumes researchers observe all common information
- In reality, players typically observe some variables which are unobserved by econometricians
- In existence of unobserved heterogeneity, player i's payoff function turns

$$\pi_i(X, W, Z_i, a_i) + \delta_i[X, W, Z_i, (a_i, a_{-i})] \cdot \mathbb{1}(a_{-i} \neq a^0_{-i}) + \epsilon_i(a_i)$$

- $W \in \mathbb{R}^{L_W}$ is a vector of state variables observed by both players but not by researchers
- Please note that if researchers can consistently estimate $\mathbf{p}_i(X, W, Z_i, Z_{-i})$, then previous identification results trivially hold

Unobserved Heterogeneity

- I discuss how to apply recent development of unobserved heterogeneity in games into my framework
- In general, the identifying restrictions for unobserved heterogeneity when researchers do not assume BNE are not stronger than the ones when BNE is imposed

Control Function Approach

Bajari et al. (2010) and Ellickson and Misra (2012) discuss a control function approach

- Suppose researchers observe a vector of variables $V \in \mathbb{R}^{L_V}$ such that unobserved heterogeneity W is a smooth function of (X, V); i.e. $W = f(X, V)$
- In McDonald's and KFC example, W can represent Chinese consumers' taste towards western style food in a market and V can be the experience of western fast food chain in such market (Shen and Xiao (2014))
- Therefore, instead of controlling for (X, W, Z_1, Z_2), researchers only need to control (X, V, Z_1, Z_2) and $\mathbf{p}_1(X, V, Z_1, Z_2)$ can be consistently estimated as V are observables

Finite Mixture

Aguirregabiria and Mira (2015) study the case when W is discrete and has finite support

- They have shown that in a game with more than two players, every player's choice probability conditional on W is identified
- Their results can be directly applied into my framework as my identification results have been generalized to an ordered-action game with more than two players

Parametric Assumption

Grieco (2014) study an empirical game with flexible information structure

- He considers a linear payoff function and assumes unobserved heterogeneity is additive separable to payoff
- He then establishes the identification result through an identification at infinity approach
- In my framework, if W is assumed to enter payoff linearly and further assume player believes other players will not choose strictly dominated action (i.e. level-1 rationality); then Grieco's identification result directly holds

Industry Background

Two western fast food chains start their competition in China from 1990

- KFC opened its first store in Beijing in 1987 and operates 5,051 stores in 2016
- McDonald's opened its first store in Shenzhen, Guangdong Province in 1990 and operates 2,232 stores in 2016
- Most stores are chains for both firms
  - At end of 2014, about 15% of McDonald's stores are franchised (Sina News)
  - By 2012, less than 10% of KFC stores are franchised (China Times)
- Burger King only operates more than 300 stores by 2014
- Subway operates about 600 stores and almost every store is closed during night

Industry Background

Some consider Dicos as a competitor of KFC and McDonald's in China

- It is a restaurant brand owned by a Chinese company (Ting Hsin International Group) and operates more than 2,000 stores in mainland China
- The restaurant sell similar products as KFC and McDonad's (i.e. hamburger, French fries, etc.) and has similar decoration
- Most of stores are franchised and rarely open 24 hours store

Preliminary Data

From both firms' official website, I obtain following information for every store in 28 April, 2016

- Each store's address and store type: 24 Hours, drive through, delivery, breakfast
- For KFC, I also know whether each store offers birthday party, self-service machine
- For McDonald's, I also know whether each store offers a separate ice cream stand

From google map, baidu map and China Yellow page, I obtain address of two firms' distribution centers

- 16 distribution centers for KFC and 7 for McDonald's (I may miss one distribution center for McDonald's)
- The distance from a particular market to its nearest distribution center is used as an exclusion restriction

Preliminary Data

I am collecting demographic data and merging it with previous store data

- Demographic data is from China Data Center by University of Michigan
- Observe population, land size, unemployment, GDP, retail sales, educational measure etc.

Conclusion

- This paper investigates the identification of incomplete information game without Bayesian Nash Equilibrium
- The framework allows player to have biased belief so that non-equilibrium play is permissible
- In a game when player 1 has more action than player 2, I show that player 1's non-interactive payoff and his subjective expectation of player 2's impact are point identified
- This identification results generalize to an ordered-action game with multiple players

Non-Parametric Representation

Let $u_i\left[X, Z_i, (a_i, a_{-i})\right]$ denote player i's payoff for realized outcome (a_i, a_{-i}), this is a non-parametric specification. π_i and δ_i can be defined by following:

- $\pi_i(X, Z_i, a_i) = u_i\left[X, Z_i, (a_i, a^0_{-i})\right]$
- $\delta_i\left[X, Z_i, (a_i, a_{-i})\right] = u_i\left[X, Z_i, (a_i, a_{-i})\right] - u_i\left[X, Z_i, (a_i, a^0_{-i})\right]$
- By construction $\delta_i\left[X, Z_i, (a_i, a^0_{-i})\right] = 0$ and therefore it is suppressed Go Back

Identification when $J_2 > 1$

For some $k \le J_1 - J_2$, define following $J_2 \times J_2$ matrix of interactive effect $\mathbf{\Delta}_1^{k:J_2+k-1}(x, z_1)$ as

$$\begin{bmatrix} \delta_1\left[x, z_1, (a_1^k, a_2^1)\right], & \cdots, & \delta_1\left[x, z_1, (a_1^k, a_2^{J_2})\right] \\ \delta_1\left[x, z_1, (a_1^{k+1}, a_2^1)\right], & \cdots, & \delta_1\left[x, z_1, (a_1^{k+1}, a_2^{J_2})\right] \\ \vdots & \ddots & \vdots \\ \delta_1\left[x, z_1, (a_1^{k+J_2-1}, a_2^1)\right], & \cdots, & \delta_1\left[x, z_1, (a_1^{k+J_2-1}, a_2^{J_2})\right] \end{bmatrix}$$

Then for any $k', k \le J_1 - J_2$, the function of interactive effect $\mathbf{\Delta}_1^{k':J_2+k'-1}(x, z_1) \cdot [\mathbf{\Delta}_1^{k:J_2+k-1}(x, z_1)]^{-1}$ is identified

Proof

Suppress (x, z_1) and define following matrices

$$\ddot{\mathbf{B}}_1(\mathbf{z}_2^{1:J_2+1}) = \begin{bmatrix} b_1^1(z_2^2) - b_1^1(z_2^1), & \cdots, & b_1^1(z_2^{J_2+1}) - b_1^1(z_2^1) \\ \vdots & \ddots & \vdots \\ b_1^{J_2}(z_2^2) - b_1^{J_2}(z_2^1), & \cdots & b_1^{J_2}(z_2^{J_2+1}) - b_1^{J_2}(z_2^1) \end{bmatrix}$$

$$\ddot{\mathbf{F}}_1^k(\mathbf{z}_2^{1:J_2+1}) = \begin{bmatrix} F_1^k[\mathbf{p}_1(z_2^2)] - F_1^k[\mathbf{p}_1(z_2^1)], & \cdots, & F_1^k[\mathbf{p}_1(z_2^{J_2+1})] - F_1^k[\mathbf{p}_1(z_2^1)] \\ \vdots & \ddots & \vdots \\ F_1^{k+J_2-1}[\mathbf{p}_1(z_2^2)] - F_1^{k+J_2-1}[\mathbf{p}_1(z_2^1)], & \cdots, & F_1^{k+J_2-1}[\mathbf{p}_1(z_2^{J_2+1})] - F_1^{k+J_2-1}[\mathbf{p}_1(z_2^1)] \end{bmatrix}$$

Proof

For any $k', k \leq J_1 - J_2$, we then have following two equations

$$\ddot{\mathbf{B}}_1(\mathbf{z}_2^{1:J_2+1}) = [\mathbf{\Delta}_1^{k:J_2+k-1}]^{-1} \cdot \ddot{\mathbf{F}}_1^{k}(\mathbf{z}_2^{1:J_2+1})$$

$$\ddot{\mathbf{B}}_1(\mathbf{z}_2^{1:J_2+1}) = [\mathbf{\Delta}_1^{k':J_2+k'-1}]^{-1} \cdot \ddot{\mathbf{F}}_1^{k'}(\mathbf{z}_2^{1:J_2+1})$$

Equating previous equations will yield

$$\mathbf{\Delta}_1^{k':J_2+k'-1}(x, z_1) \cdot [\mathbf{\Delta}_1^{k:J_2+k-1}(x, z_1)]^{-1} = [\ddot{\mathbf{F}}_1^{k'}(\mathbf{z}_2^{1:J_2+1})] \cdot [\ddot{\mathbf{F}}_1^{k}(\mathbf{z}_2^{1:J_2+1})]^{-1}$$

Proof

Suppose there exists another type of exclusion S such that it only affects interactive effect without impact on non-interactive payoff

- Non-interactive payoff $\pi_1(\tilde{x}, z_1, a_1)$ and perceived interactive effect $\sum_{j=1}^{J_2} \delta_1 [\tilde{x}, s, z_1, (a_1, a_2^j)] \cdot b_1^j(\tilde{x}, s, z_1, z_2)$ are point identified for every $(\tilde{x}, s, z_1, z_2)$ and $a_1 \in A_1$ Go Back

Model of Multi-Player Ordered-Action Game

- There are N players indexed by $i, n \in \{1, 2, \cdots, N\}$ and $-i$ indexes players other than i
- Each player i simultaneously chooses an action a_i from his choice set $A_i = \{a_i^0, a_i^1, \cdots, a_i^{J_i}\}$
- Cartesian product $A = A_1 \times A_2 \cdots \times A_N$ denote the space of action profile; assume $N \leq \min\{J_1, J_2, \cdots, J_N\}$
- Given a realized outcome $\mathbf{a} = (a_1, a_2, \cdots, a_N) \in A$ in this game, player i's payoff is

$$\pi_i(\tilde{X}, Z_i, a_i) + \sum_{n=1, n\neq i}^{N} \delta_{i,n}[\tilde{X}, S, Z_i, (a_i, a_n)] \cdot \mathbb{1}(a_n \neq a_n^0) + \epsilon_i(a_i)$$

Identification

Assumption

$$\delta_{i,n}[\tilde{X}, S, Z_i, (a_i, a_n)] = \tilde{\delta}_{i,n}(\tilde{X}, S, Z_i, a_i) \cdot \eta_{i,n}(\tilde{X}, S, Z_i, a_n)$$

Under above assumption, player i's expected payoff of action a_i is

$$\pi_i(\tilde{X},Z_i,a_i)+\sum_{n=1,n\neq i}^{N} \tilde{\delta}_{i,n}(\tilde{X},S,Z_i,a_i)\cdot\Big[\sum_{j=1}^{J_n}\eta_{i,n}(\tilde{X},S,Z_i,a_n)\cdot b_{i,n}^{j}(\tilde{X},S,Z_i,Z_{-i})\Big]+\epsilon_i(a_i)$$
$$=\pi_i(\tilde{X},Z_i,a_i)+\sum_{n=1,n\neq i}^{N} \tilde{\delta}_{i,n}(\tilde{X},S,Z_i,a_i)\cdot g_{i,n}(\tilde{X},S,Z_i,Z_{-i})+\epsilon_i(a_i)$$

This expected payoff has same structure as player 1 who has more actions in a two-player asymmetric number of actions game; therefore, all identification results hold for every player in this ordered-action game Go Back