International R&D Spillovers and Asset Prices

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Abstract

We provide new empirical evidence of a relationship between international R&D spillovers through trade and asset prices. We find that country pairs that share more R&D have more correlated stock market returns and less volatile exchange rates. We develop an endogenous growth model of innovation and international technology diffusion that rationalizes our empirical findings. A calibrated version of the model matches several important asset pricing and quantity moments, thus alleviating some of the classical quantity-price puzzles of the international macroeconomic literature.

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1 Introduction

Technological innovation is a fundamental source of sustained economic growth (Romer (1990)). Asset prices reflect changes in the future growth prospects of the economy, and hence capture variations in technological innovation (Kung and Schmid (2011)). In an international setting, technology may diffuse across countries through trade in the products that embody such technology. Hence a country's growth rate depends not only on its own innovation, but also on the innovative efforts of its trading partners (Coe and Helpman (1995) and Keller (1998)). Accordingly, the dynamics of technological innovation both *within* and *across* countries may inform us about the comovements of international asset returns.

In this paper, we investigate the link between trade in varieties — our measure of how technological innovation diffuses across countries — and comovements in asset prices. Our first contribution is empirical. Using highly disaggregated bilateral trade data, we document the following empirical regularities. First, country pairs that share more R&D by trading a higher number of varieties with each other have more correlated stock market returns. Second, country pairs that share more R&D have less volatile exchange rates. These patterns are robust to controlling for alternative measures of R&D and trade across each country-pair, suggesting that international R&D spillovers play a distinct role in capturing the relation between the international diffusion of technological innovation and asset prices.[1]

Our next contribution is theoretical. We build a two-country endogenous growth model of innovation and international technology diffusion through trade in varieties that rationalizes our empirical findings. Growth in each country is driven by the accumulation of technology through endogenous innovation. Our assumption is that the technology embodied in intermediate goods spreads across countries through international trade. As a result, the productivity level of a country depends not only on its own innovation, but also on foreign innovations that are embodied in imported intermediate products. Prefer-

[1]The international trade literature has argued in favor of trade in varieties as a channel through which R&D diffuses across countries (Broda, Greenfield, and Weinstein (2006), Bøler, Moxnes, and Ulltveit-Moe (2012), and Santacreu (2015)). Other channels include the effect of multinationals in spreading the benefits of R&D across countries (Guadalupe, Kuzmina, and Thomas (2010), Ramondo (2009)) and the effect of knowledge spillovers through international networks (Cai and Li (2012)). The analysis of these additional channels is beyond the scope of the paper.

ences are recursive, so that consumers care for the timing of resolution of uncertainty and fear variation in long-run future growth prospects of the economy. International financial markets are complete. Endogenous innovation, together with recursive preferences, makes the equilibrium growth path risky, through its effect on the present discounted value of future profits of all the firms in the economy.

Our endogenous growth mechanism works as follows. R&D drives a small and persistent component in equilibrium growth rates. International diffusion through trade in varieties makes this component common across countries. The intuition is that a technology shock in the domestic country affects not only the incentive to innovate in that country, but also the incentives to innovate abroad and in turn impacts the prospects of global growth. Therefore, a short-run technology shock in the domestic country has a long-run effect on the dynamics of future growth rates both in the domestic and in the foreign economy.

With recursive preferences, variations in the future prospects of the economy have a significant impact on asset prices: agents require a large risk premium for holding assets that are exposed to such variations. As technological innovation diffuses across countries through trade in varieties, it generates a sizeable common component in asset returns that drives up the correlation in stock market returns and reduces the volatility of exchange rates. Notably, our model can replicate these international asset pricing facts together with a sensible calibration of macroeconomics quantities. In particular, consistently with the data, realized output growth and realized consumption growth are only mildly correlated across countries because they are mostly driven by exogenous technology shocks with low cross-country correlation.

We calibrate the model to match our empirical findings. Consistently with our predictions, we find that asset prices are largely driven by the long-run future prospects of the economy, while international quantities are mostly driven by current technological levels. With endogenous growth, stock market returns are highly correlated across countries, and more so for stronger international R&D spillovers. We further investigate our mechanism and decompose the stock market return into two components. The first component, which we label return on *tangible* capital, captures the return on installed physical capital and is the standard measure of stock market return in a real business cycle model with trade in intermediate goods. The second component is the return on *intangible* capital and captures the effects of endogenous innovation and international diffusion of technologies. We find that the return on tangible capital is only mildly correlated across countries, so that the large cross-country correlation that we observe in the overall stock market returns is largely driven by the intangible component. Furthermore, in our model, the exchange rate is as volatile as in the data and its volatility increase for weaker international R&D spillovers.

Our model provides a novel set of testable implications. In particular, it predicts that both domestic innovation and foreign innovations embodied in imported intermediate goods have a predictive power on future domestic productivity growth. We test this theoretical restriction and provide empirical evidence in favour of our mechanism.

Our paper is related to several strands of the literature. First, the macroeconomic mechanism is related to the literature on endogenous growth through innovation. In our model, technological progress increases with the number of intermediate goods that embody technology. Kung and Schmid (2011) extend Romer (1990) to include recursive preferences and reproduce asset prices dynamics that are consistent with the empirical literature. We develop our model along these lines, and extend it to an international setting to capture our novel empirical findings on the relation between trade, R&D, and asset prices.[2]

The second strand of literature is the one on technology adoption and innovation through international trade in varieties, as in Broda, Greenfield, and Weinstein (2006) and Santacreu (2015). Using highly disaggregated trade data, these papers find that adoption of foreign innovations through trade in varieties has an effect on the growth rate of a country. However, they do not discuss the asset pricing implications of their mechanism, which is one of the main contributions of our work.

Finally, the paper is related to the literature on asset pricing with long-run risk, starting from the seminal one-country model of Bansal and Yaron (2004), and later applied to the international setting by Bansal and Shaliastovich (2009), Colacito and Croce (2011), and Colacito and Croce (2013). While these papers specify global long-run risk exogenously, our model shows how such risk — highly persistent within countries and highly correlated

[2] A number of recent papers have examined the link between technological growth and prices. Examples include (Pastor and Veronesi (2009), Garleanu, Panageas, and Yu (2012), and Garleanu, Kogan, and Panageas (2012)). In these papers, technology growth is assumed to be exogenous. In contrast, and consistently with our empirical findings, we focus on the relation between international asset prices and endogenous growth through R&D.

across countries — arises endogenously through innovation and international diffusion of R&D. Methodologically, our paper is related to Croce, Nguyen, and Schmid (2013) who examine the role of fiscal policy in an international endogenous growth model with recursive preferences. Unlike them, we focus on the empirical link between trade, R&D, and asset prices in the context of an endogenous growth model with recursive preferences.

The rest of the paper is organized as follows. Section 2 shows our main empirical findings and Section 3 describes our baseline model. Section 4 provides a description of the main mechanism at work, while Section 5 presents the calibration and quantitative results. In Section 6 we investigate the predictions of the model. Finally, Section 7 concludes.

2 Innovation, Trade, and Asset Prices: Empirical Evidence

One stark feature of international macroeconomics data is the low cross-country correlation of consumption growth relative to the correlation of stock market returns. The first two rows of Table 1 show that the average cross-country correlation of consumption growth is 0.23 whereas the cross-country correlation of stock market returns is three times higher, around 0.71.[3] We also find that the average volatility of the exchange rate depreciation for our sample of countries and time period of analysis is around 9.6%. Standard international macroeconomic models have had a hard time reconciling these empirical asset pricing and quantity moments.

We argue that international spillovers of R&D through trade in varieties are a significant driver of the dynamics of international asset prices. To the extent that the technology created by investing in R&D is embodied in a particular good, movements of goods across borders help diffuse those technologies (Coe and Helpman (1995), Keller (1998)).

In this Section, we investigate the empirical relevance of the mechanism that we propose. We collect data on asset prices, international trade, and R&D, and we proceed in two steps. First, we look at the correlation of broad measures of asset pricing moments with international trade and R&D. Then, we follow Liao and Santacreu (2015) and per-

[3]The average is taken over a sample of 20 countries and the period 1993-2009. Details on the specific data are left in Appendix A.

Table 1: Summary Statistics

Variable Moment	Value
Corr. Consumption Growth	0.23
Corr. Stock Market Returns Growth	0.71
Volat. exchange rate	9.60

form a regression analysis to investigate the main driving forces behind this relationship. The main sources for our data are: i) the UN COMTRADE database for international trade; ii) the World Development Indicators Database of the World Bank for R&D; iii) Ken French's data and Global Financial Data for asset prices.[4] Our main dataset covers the 1993-2009 period for a sample of 20 countries. The choice of countries and time period has been determined based on the availability of asset pricing, trade and R&D data. Details on the sources and the construction of the measures we use in our analysis can be found in Appendix A.[5]

2.1 Correlation between Asset Pricing Moments, R&D, and International trade

Here, we analyze the correlation of broad measures of asset prices with measures on international trade and R&D. The goal is to explore whether the mechanism that we propose (i.e., R&D embodied in trade in varieties) has something to say about asset prices. We consider two statistics for asset prices: the cross-country correlation in stock market returns and the volatility of the currency depreciation rate. We start with monthly observations and construct twelve-month non-overlapping measures of cross-country correlation and volatility. Specifically, at the end of each year, we look at the previous twelve monthly observations and calculate their cross correlation - for stock market returns - and volatility - for the currency depreciation rate. We then take the time series average of these annual correlation and volatility measures, so that we are left with two statistics per each (i,j)

[4] Ken French's website: $http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html$.

[5] The sample of countries includes Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Singapore, Spain, Sweden, Switzerland, United Kingdom, and United States.

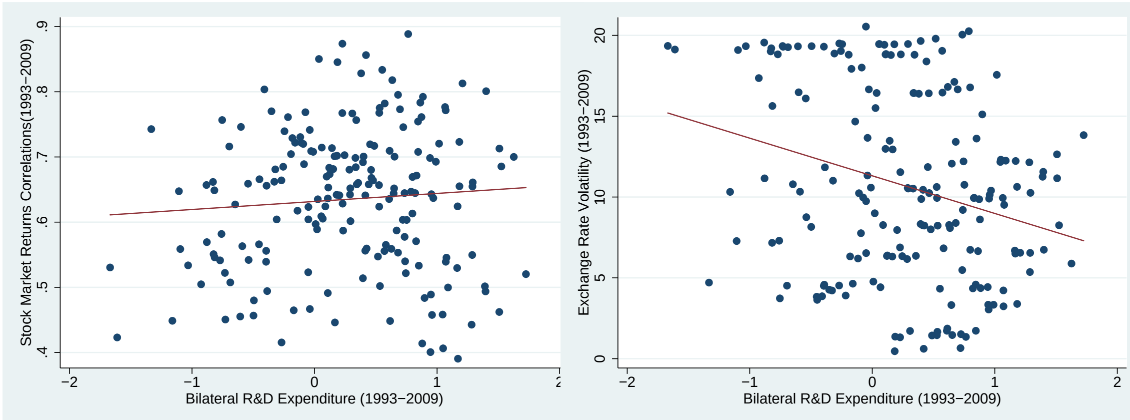

Figure 1: R&D intensity and asset prices

ordered country pair: the average cross-country correlation in stock market returns, and the average volatility of the currency depreciation rate.[6]

We then use data on R&D and international trade to construct several measures that capture our proposed mechanism. First we construct a measure of innovation between each country pair in our sample. We use R&D intensity, computed as R&D expenditures over GDP and then compare this measure with those for asset prices to determine whether there is any relation between them.[7] Figure 1 shows that country pairs that do more R&D have more correlated stock market returns and less volatile exchange rate fluctuations. While a clear pattern emerges, Figure 1 does not provide any insight on how R&D spreads across countries. Next, we show whether there is any relationship between the strength of international trade and our measures of asset prices. In particular, we construct a measure of overall bilateral trade as the value of trade between each pair of countries in our sample. We then decompose this measure into the so-called extensive and intensive margins of trade, that is, the number of products that are traded between each pair of countries and how much of each product is traded, respectively. These measures are constructed so that overall trade is equal to the product of the extensive and the intensive margins of trade. To facilitate comparison, we normalize trade by GDP, and then compare these measures with those for asset prices to determine whether there is any correlation between them. Figures

[6]We also consider 60-month overlapping measures, with a 12-month overlap. The results are virtually unchanged.

[7]As a robustness check, we also use a measure of R&D intensity computed as R&D expenditures over the stock of R&D. We use the perpetual inventory method to compute the stock of R&D and assume a depreciation rate of 15% a year, as it is standard in the literature (e.g.,Coe and Helpman (1995), Nishioka and Ripoll (2012)). The results are very similar.

2 and 3 report the results. We find that countries that trade more with each other have more correlated stock market returns and less volatile exchange rate movements. To the extent that R&D is embodied in the products that are traded internationally, these results suggest that international R&D spillovers through trade may be an important channel to understand moments of asset pricing.

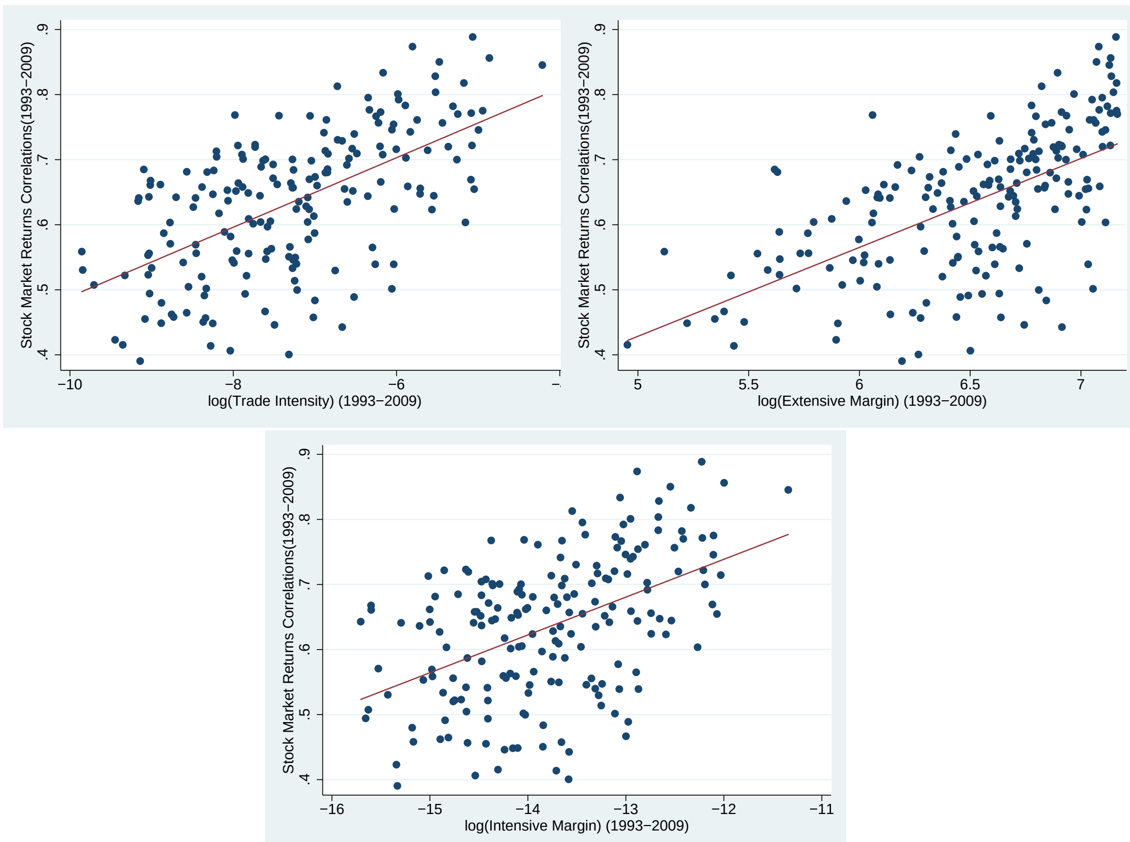

Figure 2: International trade and the correlation of stock market returns

So far, we have shown that both R&D and international trade are related to asset pricing moments. Ideally, we would like to have *direct* measures of how much R&D is embedded in international trade. Unfortunately, this data does not exist at a very disaggregated level. Our strategy, then, is to construct an *indirect* measure that weights the bilateral trade of each country-pair by the R&D of the exporter. This measure puts more weight on imported intermediate products from more innovative exporters.[8] Figures 4 and 5 show that pairs of countries with higher R&D content of international trade have more correlated stock market returns and less volatile exchange rates.

[8]The details on how we construct this variable are in Appendix A.

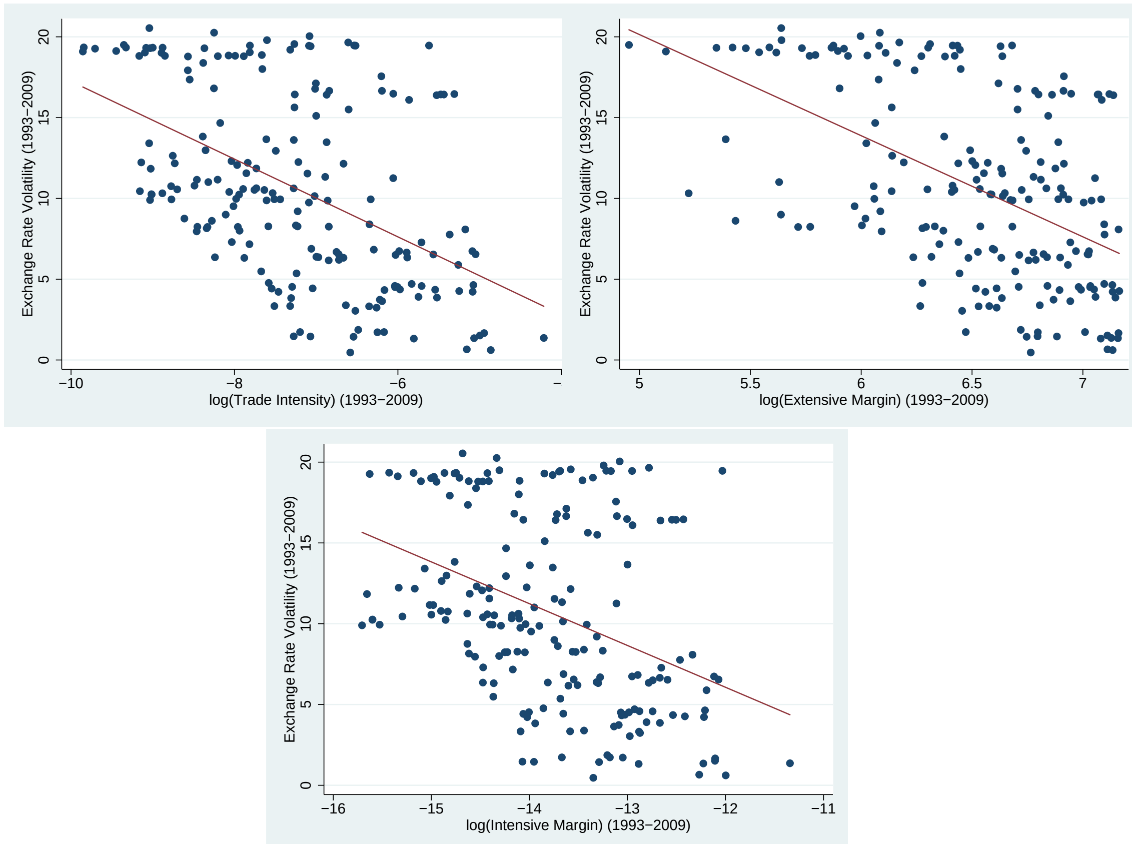

Figure 3: International trade and the volatility of exchange rates

Taken together, the graphs suggest that R&D and international trade are relevant to understand the joint dynamics of asset prices.

2.2 Regression Analysis

We now address the economic and statistical significance of the results presented in the previous section through a formal regression analysis. In particular, we regress the asset pricing moments of interest — the correlation in stock market returns and the volatility of exchange rate depreciation — on our measures of R&D, international trade, and R&D embodied in trade.

In Table 2, we regress our asset pricing moments on the margins of international trade for each country pair. We find that both coefficients are statistically significant. However, the extensive margin accounts for most of the change in asset pricing moments. In particular, Table 1 shows that, holding the intensive margin constant, a 1% increase in the extensive margin increases the correlation of asset returns by 0.094% (first column)

Figure 4: R&D embodied trade and the correlation of stock market returns

Figure 5: R&D embodied trade and the volatility of exchange rates

and it decreases the volatility of the depreciation rate by 3.53% (second column); instead, if we hold the extensive margin constant, a 1% increase in the intensive margin increases the correlation of stock market returns by 0.032% and it decreases the volatility of the exchange rate depreciation by 1.384%. The effect of the intensive margin is weaker than that of the extensive margin of trade. These findings are consistent with the mechanism that we propose. Indeed, if trade in varieties is the channel through which R&D spreads across countries, variations in the extensive margin of trade (i.e. the number of varieties) rather than the intensive margin of trade should account for most of the effect of asset pricing moments.

In Table 3, we perform the regression analysis using our indirect measure of R&D embodied in trade. We find that higher R&D content of trade is associated with a larger cross-country correlation in stock market returns and a lower volatility of the exchange

Table 2: International trade and asset prices

	(1) Stock Market Correlations	(2) Exchange Rate Volatility
$\log(EM)_{ij}$	0.094***	-3.523***
	(0.006)	(0.160)
$\log(IM)_{ij}$	0.032***	-1.384***
	(0.003)	(0.085)
Constant	0.422***	13.443***
	(0.068)	(1.818)
Observations	6460	6460
R^2	0.075	0.149

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

rate depreciation. Importantly, when we consider the effect of the R&D content of the extensive and intensive margins of trade, respectively, on the volatility of the exchange rate depreciation, only the former remains statistically significant, providing additional support to the main mechanism of our model: international spillovers of R&D though trade in varieties have a significant impact on international asset prices.

Table 3: R&D embodied in trade and asset prices

	(1) Stock Market Correlations	(2) Exchange Rate Volatility
$\log(EM_{ij}^{R\&D})$	0.031***	-1.810***
	(0.007)	(0.183)
$\log(IM_{ij}^{R\&D})$	0.016***	-0.192
	(0.004)	(0.122)
Constant	0.747***	3.081***
	(0.013)	(0.343)
Observations	6422	6422
R^2	0.029	0.059

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Finally, in Table 4, we provide additional empirical evidence that R&D matters for international asset prices. In particular, we find that country-pairs that do more R&D have more correlated stock market returns and less volatile exchange rates.

Table 4: R&D intensity and asset prices

	(1) Stock Market Correlations	(2) Exchange Rate Volatility
$\log(\frac{R\&D_i}{GDP_i}) + \log(\frac{R\&D_j}{GDP_j})$	0.054***	-1.183***
	(0.004)	(0.122)
Constant	0.577***	9.900***
	(0.003)	(0.095)
Observations	6384	6384
R^2	0.024	0.014

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

In Appendix D we show that these results are robust to adding time fixed effects and time and country fixed effects.

3 Model

In this Section, we present a model of innovation and international diffusion of R&D through trade in varieties that captures our empirical findings. Each country has a representative household with recursive preferences and consume a final good. A final producer uses labor, capital and a composite of intermediate goods that we call materials to produce a non-tradable final good that is used for consumption, investment in capital and investment in R&D. Materials are produced with traded intermediate goods (varieties), both domestic and foreign, which are produced by monopolistic competitive firms. The production of materials features a love-for-variety effect so that, holding expenditure constant, a higher number of varieties increases the productivity of the country. New varieties are introduced in each country through an endogenous process of innovation, and then spread exogenously across countries through a slow process of adoption. Endogenous innovation and adoption together with recursive preferences are the new features at the core of our mechanism. The model is closed with an international risk sharing condition.

Below we describe the domestic economy d. The foreign economy f is defined analogously.

3.1 Households

The domestic representative household has Epstein and Zin (1989) recursive preferences over consumption:

$$U_{d,t} = \left\{(1-\beta)C_{d,t}^{\theta} + \beta\left(E_t\left(U_{d,t+1}^{1-\gamma}\right)\right)^{\frac{\theta}{1-\gamma}}\right\}^{\frac{1}{\theta}} \quad , \tag{1}$$

where γ is the CRRA, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $\psi \equiv \frac{1}{1-\theta}$ is the intertemporal elasticity of substitution. We assume that $\psi > \frac{1}{\gamma}$, so that the representative agent has a preference for early resolution of uncertainty and fears variations in the long-run prospects of the economy. The stochastic discount factor is given by

$$M_{d,t+1} = \beta\left(\frac{C_{d,t+1}}{C_{d,t}}\right)^{\theta-1}\left(\frac{U_{d,t+1}}{E_t\left(U_{d,t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{1-\gamma-\theta} \quad , \tag{2}$$

where the last term captures the agent's concerns over uncertainty in future growth. The household consumes, supplies labor to the final producers, and makes investment/saving decisions participating in complete international financial markets. Accordingly, her budget constraint is

$$C_{d,t} + E_t\Big[M_{d,t+1}\mathcal{A}_{d,t+1}\Big] = W_{d,t}L_{d,t} + \mathcal{A}_{d,t} \quad ,$$

where $W_{d,t}$ is the wage rate, $L_{d,t}$ denotes hours worked, and $\mathcal{A}_{d,t}$ is the state contingent value of the household's financial wealth. Since there is no disutility of labor, the household supplies her entire endowment, which is normalized to one.

3.2 Final Good Producers

Domestic final producers are perfectly competitive, and use capital, $K_{d,t}$, labor, $L_{d,t}$, and a composite of domestic and foreign intermediate goods, $G_{d,t}$, to produce a non-traded final good $Y_{d,t}$ according to the following Cobb-Douglas production function:

$$Y_{d,t} = \left(K_{d,t}^{\alpha} \left(\Omega_{d,t} L_{d,t}\right)^{(1-\alpha)}\right)^{(1-\xi)} G_{d,t}^{\xi} \quad . \tag{3}$$

The composite good $G_{d,t}$ is defined as

$$G_{d,t} = \left[\sum_{i=1}^{N_{dt}^d} (X_{d,i,t}^d)^{\nu} + \sum_{i=1}^{N_{ft}^d} (X_{f,i,t}^d)^{\nu}\right]^{\frac{1}{\nu}} \quad , \tag{4}$$

where $X_{d,i,t}^d$ is the amount of domestically produced intermediate good i that is used for final production in the domestic economy, $X_{f,i,t}^d$ is the amount of foreign-produced intermediate good i that is used for final production in the domestic economy, N_{dt}^d (N_{ft}^d) is the mass of domestic (foreign) intermediate goods that is used by domestic final producers, and $\frac{1}{1-\nu}$ is the elasticity of substitution across intermediate goods with $\nu < 1$. The parameter α governs the physical capital share, and ζ is the share of materials on final production (which we will refer later as intangible capital). Throughout the paper, the subscript of a variable refers to the origin country and the superscript refers to the destination country. Note that intermediate goods are aggregated according to a CES production function a la Ethier (1979) which implies that, holding expenditures constant, a higher number of varieties increases the productivity of the final producers. In this set-up, the larger is the elasticity of substitution between intermediate varieties the higher is the effect of varieties on productivity.

The exogenous process $\Omega_{d,t}$ is the source of exogenous uncertainty in our model. We assume that $\Omega_{d,t} = e^{a_{d,t}}$, where $a_{d,t}$ follows the following AR(1) process:

$$a_{d,t} = \varphi \; a_{d,t-1} + \epsilon_{d,t} \quad ,$$

with $\epsilon_{d,t} \sim N(0, \sigma^2)$. We allow for cross-country correlation in the exogenous technology shocks and denote $\rho = corr(\epsilon_{d,t}, \epsilon_{f,t})$.[9]

Final producers choose capital, labor, investment, and intermediate goods to maximize

[9]When we solve the model, we augment the AR(1) process for exogenous technology with an error correction term in the spirit of Colacito and Croce (2013). This error correction term ensures stability of our solution method and has virtually no impact on our results.

shareholder value subject to the production technology (3). Formally,

$$\max_{\{I_{d,t},\ L_{d,t},\ K_{d,t+1},\ X^d_{d,i,t},\ X^d_{f,i,t}\}_{t\geq 0, i\in\Omega, j\in\Omega^*}} E_0\left[\sum_{t=0}^{\infty} M_{d,t} D_{d,t}\right] \quad , \tag{5}$$

where firm's dividends are given by

$$D_{d,t} = Y_{d,t} - I_{d,t} - W_{d,t}L_{d,t} - \sum_{i=1}^{N^d_{dt}} P^d_{d,i,t} X^d_{d,i,t} - \sum_{i=1}^{N^d_{ft}} P^d_{f,i,t} X^d_{f,i,t} \quad . \tag{6}$$

Here, $M_{d,t}$ is the stochastic discount factor, $W_{d,t}$ is the wage rate, $I_{d,t}$ is investment in physical capital, $P^d_{d,i,t}$ is the price of a domestically produced intermediate good, and $P^d_{f,i,t}$ is the price of a foreign produced intermediate good that is used for domestic production. Both prices are expressed in units of the domestic producer's final good.

The law of motion for physical capital is given by

$$K_{d,t+1} = (1-\delta)K_{d,t} + \Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right) K_{d,t} \quad , \tag{7}$$

where $\delta \in (0,1)$ is the depreciation rate and $\Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right)$ captures convex capital adjustment costs.[10]

3.3 Intermediate Good Producers

In each country, a set of monopolistic competitive firms produces a differentiated good using final output according to a CRS production function (one unit of final output is used to produce one unit of the intermediate good). All intermediate producers produce with the same efficiency. Intermediate producers produce both for the domestic and foreign market. To sell the good abroad, they face an iceberg transport cost τ.

Every period, each domestic intermediate producer i solves the following static profit maximization problem:

[10]Specifically, $\Lambda_{d,t} \equiv \Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right) = \frac{\alpha_1}{\zeta}\left(\frac{I_{d,t}}{K_{d,t}}\right)^{\zeta} + \alpha_2$, as in Jermann (1998). The parameters α_1 and α_2 are chosen so that there are no adjustments costs in the steady state, and $\frac{1}{1-\zeta}$ is the elasticity of the investment rate with respect to Tobin's Q.

$$
\begin{aligned}
\max_{P^d_{d,i,t},P^{f*}_{d,i,t}} \Pi_{d,i,t} &\equiv \max_{P^d_{d,i,t},P^{f*}_{d,i,t}} (\pi^d_{d,i,t} + \pi^f_{d,i,t}) \\
&= \max_{P^d_{d,i,t}} \quad P^d_{d,i,t} \quad X^d_{d,i,t}(P^d_{d,i,t}) - X^d_{d,i,t} \quad (P^d_{d,i,t}) \\
&+ \max_{P^{f*}_{d,i,t}} \quad \underbrace{(P^f_{d,i,t}Q_t)}_{P^{f*}_{d,i,t}} \quad X^f_{d,i,t}\underbrace{(P^f_{d,i,t}Q_t)}_{P^{f*}_{d,i,t}} - X^f_{d,i,t}\underbrace{(P^f_{d,i,t}Q_t)}_{P^{f*}_{d,i,t}} \quad ,
\end{aligned} \tag{8}
$$

where $\pi^d_{d,i,t}$ ($\pi^f_{d,i,t}$) are the profits from selling the domestic product at home (abroad), $P^{f*}_{d,i,t} = P^f_{d,i,t}Q_t$ is the price, in domestic good units, of a domestically produced intermediate good that is being exported, and Q_t is the real exchange rate, defined as the number of domestic final goods per one unit of foreign final good.[11]

3.4 Innovation and Adoption

3.4.1 Innovation

In each country innovators invest resources (final output) to introduce new prototypes of a product. If an innovator is successful, it starts producing the new good as an intermediate producer. Each domestic innovator i chooses $S_{d,i,t}$ units of final output to maximize the present discounted value of future profits that it expects to obtain from selling the good to both domestic and foreign producers.

The law of motion of new prototypes is

$$
N^d_{d,t+1} = \vartheta_{d,t} S_{d,t} + (1-\phi) N^d_{d,t} \quad , \tag{9}
$$

where $N^d_{d,t}$ is the mass of new technologies that arrive to country d at time t, ϕ is the exogenous probability that a new variety becomes obsolete and $\vartheta_{d,t}$ is the productivity of innovation. We assume it takes the following functional form:

$$
\vartheta_{d,t} = \frac{\chi N^d_{d,t}}{S^{1-\eta}_{d,t}(N^d_{d,t})^\eta} \tag{10}
$$

[11] We express real prices of the intermediate goods in units of the importers' final good. In particular, when the domestic (foreign) intermediate good is used for the production of the foreign (domestic) final output, we have $P^f_{d,t} \equiv \frac{1}{\nu}\tau Q_t^{-1}$ and $P^d_{f,t} \equiv \frac{1}{\nu}\tau Q_t$.

as in Comin and Gertler (2006). The process $S_{d,t} = \sum_{i=1}^{N_{dt}^d} S_{d,i,t}$ describes the total R&D expenditure in the domestic country (in terms of the domestic final good). In this specification, $\vartheta_{d,t}$ is an externality and it is taken as given when the innovators choose their optimal investment into R&D.

3.4.2 International Adoption

We assume that international adoption is exogenous and that, in every period, only a fraction ϑ_d^f of foreign intermediate goods from country d can be used by the domestic final producer in country f. This parameters governs the speed of adoption and is crucial in our mechanism. The law of motion of domestic intermediate goods that can be used by the foreign final producer evolves according to

$$N_{d,t+1}^f = \vartheta_d^f(1-\phi)(N_{d,t}^d - N_{d,t}^f) + (1-\phi)N_{d,t}^f \quad , \tag{11}$$

where $N_{d,t}^f$ is the number of domestic goods imported by the foreign economy. Therefore $(N_{d,t}^d - N_{d,t}^f)$ is the mass of domestic varieties that has not been yet adopted by the foreign country.

3.4.3 Value Functions

We assume that every innovation that is produced in a country can immediately be used by the final producer of that country. However, a new intermediate product can be sold abroad only with probability ϑ_d^f, so that $N_{dt}^f/N_{dt}^d < 1$ is the fraction of domestically produced intermediate goods that are used by the final producers in the foreign country.

The value of a domestic innovation, $V_{d,i,t}$ is given by the present discounted value of the profits that innovator i expects to obtain from selling the good domestically and abroad. Let the value of the domestic innovations that are immediately sold to the domestic (foreign) final producers be $V_{d,i,t}^d$ ($V_{d,i,t}^f$), and the value of the innovations that can potentially being adopted by country f be $J_{d,i,t}^f$. We have

$$V_{d,i,t} = V_{d,i,t}^d + J_{d,i,t}^f \quad , \tag{12}$$

with

$$V_{d,i,t}^d = \pi_{d,i,t}^d + (1-\phi)E_t[M_{d,t+1}V_{d,i,t+1}^d] \quad , \tag{13}$$

$$V^f_{d,i,t} = \pi^f_{d,i,t} + (1-\phi)E_t[M_{d,t+1}V^f_{d,i,t+1}] \quad , \tag{14}$$

and

$$J^f_{d,i,t} = (1-\phi)E_t\left[M_{d,t+1}\left(\vartheta^f_d\pi^f_{d,i,t+1} + (1-\vartheta^f_d)J^f_{d,i,t+1}\right)\right] \quad , \tag{15}$$

so that, with probability ϑ^f_d, the firm can sell the product abroad at $t+1$ and with probability $(1-\vartheta^f_d)$ the product remains unadopted.

Discounted future profits on patents are the payoff to innovators. Since the R&D sector is competitive, the free entry condition for R&D investment in the symmetric equilibrium in which all firms are identical is

$$S_{d,t} = E_t\left[M_{d,t+1}V_{d,t+1}\right](N^d_{d,t+1} - (1-\phi)N^d_{d,t}) \tag{16}$$

or, equivalently,

$$\frac{1}{\vartheta_{d,t}} = E_t\left[M_{d,t+1}V_{d,t+1}\right] \quad . \tag{17}$$

3.5 Resource Constraint

Final output is used for consumption, intermediate goods production and investment in R&D. Thus the resource constraint is

$$Y_{d,t} = C_{d,t} + I_{d,t} + S_{d,t} + N^d_{d,t}X^d_{d,t} + N^f_{d,t}X^f_{d,t} \quad .$$

3.6 Equilibrium and steady-state

We define a symmetric equilibrium as a set of equations according to which all firms within a country behave symmetrically.

For each country $i=(d,f)$, a general symmetric equilibrium is defined as an exogenous stochastic sequence of technology shocks $\{\Omega_{i,t}\}^\infty_{t=0}$, an initial vector $\{N^d_{d,0}, N^f_{d,0}, N^f_{f,0}, N^d_{f,0}, K_{d,0}, K_{f,0}\}$, a set of parameters $\{\beta,\theta,\gamma,\psi,\alpha,\xi,\varphi,\sigma,\rho,\delta,\theta,\nu,\chi,\phi,\eta,\tau,\vartheta^f_d\}$, a sequence of aggregate prices $\{W_{i,t}, V_{i,t}, Q_t, q_{it}\}^\infty_{t=0}$, value functions $\{V^i_{it}, J^f_{dt}, J^d_{ft}, V^f_{dt}, V^d_{ft}\}^\infty_{t=0}$, a sequence of intermediate good prices $\{P^d_{d,t}, P^f_{d,t}, P^f_{f,t}, P^d_{f,t}\}^\infty_{t=0}$, a sequence of aggregate quantities $\{Y_{i,t}, G_{i,t}, C_{i,t}, I_{i,t}, L_{i,t}, S_{i,t}\}^\infty_{t=0}$, quantities of intermediate goods $\{X^d_{d,t}, X^f_{d,t}, X^f_{f,t}, X^d_{f,t}\}^\infty_{t=0}$, a sequence of profits $\{\Pi_{d,t}, \Pi_{f,t}, \pi^d_{d,t}, \pi^f_{d,t}, \pi^f_{f,t}, \pi^d_{f,t}\}^\infty_{t=0}$, and laws of motion $\{N^d_{d,t+1}, N^f_{d,t+1}, N^f_{f,t+1}, N^d_{f,t+1}, K_{i,t+1}\}^\infty_{t=0}$ so that

- The state variables satisfy the law of motion;
- The endogenous variables solve the producers', innovators', and households problems;
- The resource constraint is satisfied;
- Prices are such that all markets clear.

The equilibrium conditions are reported in Appendix B.

3.7 Asset Prices

We assume that stocks are claims to all the production sectors, namely the final good sector, the intangible sector, as well as the innovation sector. Accordingly, we define the aggregate dividend as the net payout from the production sector

$$\mathcal{D}_{d,t} = D_{d,t} + N^d_{d,t}\pi^d_{d,t} + N^d_{f,t}\pi^d_{f,t} - S_{d,t} \quad . \tag{18}$$

Optimality implies the following asset pricing condition:

$$\mathcal{P}_{d,t} \quad = \quad E_t[M_{d,t+1}(\mathcal{P}_{d,t+1} + \mathcal{D}_{d,t+1})] \quad ,$$

where $\mathcal{P}_{d,t}$ is the domestic stock market price, and $\mathcal{D}_{d,t}$ is the aggregate market dividend.

Given complete financial markets, exchange rate depreciation is pinned down by the ratio of the domestic and foreign stochastic discount factors, that is

$$\frac{Q_{t+1}}{Q_t} \quad = \quad \frac{M_{f,t+1}}{M_{d,t+1}} \quad . \tag{19}$$

Because of recursive preferences, the risk sharing mechanism is non-standard as agents fear not only current shocks but also variation in future utility. Formally, let $\Upsilon_t = Q_t \left(\frac{C_{d,t}}{C_{f,t}}\right)^{\theta-1}$. Using the expression for the stochastic discount factor in (2) together with the no arbitrage condition (19), we can express Υ_t recursively as

$$\Upsilon_{t+1} = \Upsilon_t \, \frac{M_{f,t+1}}{M_{d,t+1}} \, \frac{e^{(\theta-1)\Delta c_{d,t+1}}}{e^{(\theta-1)\Delta c_{f,t+1}}} \quad . \tag{20}$$

Notice that, in the CRRA case, Υ_t is constant. Instead, with Epstein-Zin recursive preferences, it evolves over time depending on the cross-country realizations of the agents' continuation utilities. Colacito and Croce (2013) provides a thorough analysis of this mechanism.

4 The Mechanism: Aggregate Productivity and the Stock Market

In this section we present the expression for aggregate productivity that is central to our mechanism. Domestic TFP can be expressed as

$$Z_{d,t} \equiv \Omega_{d,t} \left(\overline{A}\right)^{\frac{1}{1-\alpha}} \left[N_{d,t}^d + (\tau Q_t)^{\frac{\nu}{\nu-1}} \, N_{f,t}^d\right] \quad , \tag{21}$$

where $\overline{A} \equiv (\xi\nu)^{\frac{\xi}{1-\xi}}$. Taking logs,

$$\log Z_{d,t} = \log \Omega_{d,t} + \log \left\{ \left(\overline{A}\right)^{\frac{1}{1-\alpha}} \left[N_{d,t}^d + (\tau Q_t)^{\frac{\nu}{\nu-1}} \, N_{f,t}^d\right] \right\} \quad ,$$

Hence, TFP has both an exogenous and an endogenous component, that is

$$\log(TFP_{d,t}) = \log(TFP_{d,t}^{EXO}) + \log(TFP_{d,t}^{ENDO}) \quad ,$$

with

$$\log(TFP_{d,t}^{EXO}) \equiv \log \Omega_{d,t} \quad ,$$

and

$$\log(TFP_{dt}^{ENDO}) \equiv \log \left\{ \left(\overline{A}\right)^{\frac{1}{1-\alpha}} \left[N_{d,t}^d + (\tau Q_t)^{\frac{\nu}{\nu-1}} \, N_{f,t}^d\right] \right\} \quad .$$

The exogenous component of TFP is given by the stochastic process $\Omega_{d,t}$; the endogenous component, which plays a crucial role in our mechanism, depends on the number of varieties that have been produced domestically, $N_{d,t}^d$, and the number of varieties that have been produced in the foreign country and are already adopted at home, $N^d_{f,t}$. We refer to $N^d_{d,t}$ as the domestic component of endogenous TFP and $(\tau Q_t)^{\frac{\nu}{\nu-1}} N^d_{f,t}$ as the foreign component of endogenous TFP. From the process of innovation and adoption explained in Section 3.4 it can be shown that endogenous TFP is positively affected by the R&D done in the domestic country and the foreign R&D embodied in imports. Thus, foreign R&D diffuses across countries through trade in varieties, generating a positive comovement of TFP across countries. This process of innovation and international diffusion makes a positive productivity shock have a persistent effect on the productivity of a country and its trading partner, which helps to explain the positive correlation of quantities.[12] For this channel to have a quantitatively relevant effect on asset prices we need to understand the role of recursive preferences.

For asset prices, the main mechanism works as follows. Risky growth through endogenous innovation and recursive preferences determines the optimal level of R&D, and therefore the level of current and future expected growth. Innovations then spread across countries through a process of technology adoption that we measure with trade in varieties. Risky growth has a first order impact on the stock market and governs its international correlation structure. To see this, recall that aggregate dividends in each country (Equation (18)) are given by the present discounted value of the future profits of all the firms operating in that country. In Appendix C, we show that we can decompose the return of the stock market into four components:

1. Price of installed capital: $q^k_{d,t} K_{d,t+1}$

2. Value of adopted domestic technologies (adopted by both domestic and foreign final producers)
$$N^d_{d,t}(V^d_{d,t} - \pi^d_{d,t}) + N^f_{d,t}(V^f_{d,t} - \pi^f_{d,t})$$
with $V^d_{d,t} = J^d_{d,t}$ and $V^f_{d,t} = \pi^f_{d,t} + (1-\phi)E_t\left[M_{d,t+1}V^f_{d,t}\right]$ is the present discounted value of the future profits of the firms that are established in the market and are also selling abroad.

3. Value of existing not-yet adopted technologies
$$(1-\phi)(N^d_{d,t} - N^f_{d,t})E_t\left[M_{d,t+1}J^f_{d,t+1}\right]$$

[12]See Liao and Santacreu (2015) for a cross-sectional analysis of the mechanism.

where

$$J_{d,t}^f = (1-\phi)E_t\left[M_{d,t+1}(\vartheta_d^f V_{d,t+1}^f + (1-\vartheta_d^f)J_{d,t+1}^f)\right]$$

4. Value of all the technologies that we expect to develop in the future.

$$\sum_{i=0}^{\infty} E_t\left[M_{d,t+i+1}\left((N_{d,t+i+1}^d - N_{dt+i}^d(1-\phi))V_{d,t+i+1} - S_{d,t+i}\right)\right] = 0$$

We refer to the first element as the price of the tangible component of the stock market and the remaining three elements as the price of the intangible component of the stock market. Accordingly, we define $r_{d,t}$ as the log return of the overall stock market, $r_{d,t}^{tan}$ as the log return of its tangible component, and $r_{d,t}^{int}$ as the log return of its intangible component.

5 Quantitative implications

In this section we present the quantitative implications of our model and explore its ability to replicate key international moments for both macroeconomic quantities, stock market returns, and exchange rate dynamics. Our baseline model is calibrated at a quarterly frequency.

5.1 Calibration

We need to specify a total of sixteen parameters. The parameter values are reported in Table 5. We start by discussing the more standard parameters. The preference parameters are set in the spirit of the long-run risk literature (see Bansal and Yaron (2004) and Colacito and Croce (2013)). In particular, we set the coefficient of relative risk aversion γ equal to 10, and the coefficient θ equal to 1/3, implying an elasticity of intertemporal substitution of 1.5. Note that with this calibration of the preference parameters, agents in the economy dislike shocks to expected future growth. The subjective discount factor is chosen to pin down the mean of the risk free rate, which implies a $\beta = 0.984^{1/4}$.

The parameters relating to the final goods production technology are obtained from Kung and Schmid (2011). The capital share α is set to 0.35 to match the average capital share, and the share of intangible capital ξ is set to 0.5 as in Comin and Gertler (2006).

Description	Parameter	Value
Preferences:		
Risk Aversion	γ	10
IES	$\psi = 1/(1-\theta)$	1.5
Subjective Discount Factor	β^4	0.984
Final Production:		
Capital Share	α	0.35
Share of Materials	ξ	0.5
Autocorrelation of $\Omega = e^a$	φ	0.95
Volatility of exogenous shock ϵ	σ	1.08%
Cross-correlation of exogenous shock	ρ	0.35
Depreciation of capital stock	δ	0.02
Investment Adjustment Cost Parameter	ζ	0.33
Inverse Markup	ν	0.5
Innovation and International Adoption:		
Scale Parameter	χ	0.4240
Innovation Obsolence Rate	ϕ	0.0375
Elasticity of Innovation wrt R&D	η	0.60
Shipping Cost	τ	1.5
International Adoption Parameter	ϑ_d^f	0.01

Table 5: The table reports the parameters for the baseline quarterly calibration.

The depreciation rate of physical capital is set to 0.02 and ζ, which pins down the elasticity of the investment rate with respect to Tobin's Q, is set to 1/3. The parameter ν is set to 0.5, also consistent with the literature. This parameter is related to the elasticity of substitution across intermediate goods and pins down the intermediate goods markup in our model.

We set the autocorrelation of the exogenous technology shock to 0.95 and the volatility parameter σ to obtain a sensible volatility for consumption and output growth. Finally, we fit an AR(1) process to the TFP of each of the 20 countries in our sample and compute the cross-country correlation of the error term, which give us a cross-country correlation in the exogenous TFP shocks of 0.35.

We now move to the non-standard parameters that govern the process of innovation and technology adoption. The parameter χ is a pure scaling parameter. We choose it such that the steady state growth rate of consumption has an annualized mean of 1.9%.

Together with ν, it gives us a value of $\overline{A}$ which is consistent with the balanced growth restriction. The parameter η governs the elasticity of new varieties with respect to R&D and is set to 0.6, a number within the range of estimates from Griliches (1990). Finally, we set $\phi = 0.0375$, which corresponds to an annualized depreciation rate of the R&D stock of 15%, as is standard in the literature.

The last parameters that we calibrate are those associated with international trade costs: τ and ϑ_d^f. We set the iceberg cost parameter τ to 1.5 to match international trade flows, as in Santacreu (2015). Finally, we calibrate ϑ_d^f, which governs the strength of the international adoption process and is crucial for the mechanism of our model. We calibrate this parameter using the law of motion of newly adopted varieties in equation (11), as follows

$$\vartheta_d^f = \frac{\phi N_{d,t}^f}{(1-\phi)\left(N_{d,t}^d - N_{d,t}^f\right)} \quad .$$

Then, we use annual data on R&D and disaggregated bilateral trade data to measure the right hand side of the above expression. Specifically, $N_{d,t}^f$ is measured using the number of varieties that are exported from country d to country f, which is what we call the extensive margin of trade in the empirical section; $N_{d,t}^d$ is measured as the stock of R&D, computed with the perpetual inventory method using R&D expenditures and an annual depreciation rate of 15% (i.e., $\phi = 0.0375$)). We obtain a value for each pair of countries and each time period. Averaging across time we obtain that ϑ_d^f is between 0.048 and 0.084, which corresponds to a quarterly average value of $\vartheta_d^f = 0.01$. This is the value that we use in our baseline calibration.

Given these parameters, we use perturbation methods to solve our system of equations. We compute an approximation of the third order of our policy functions using the Dynare++ package. All variables included in our code are expressed in log-units.[13]

5.2 Results

Table 6 reports simulated moments of four different calibrations: Baseline, CRRA, Fast Adoption, and EXO. For the CRRA calibration, we impose $\psi = 1/\gamma$ and leave all other parameters unaltered. For the calibration with Fast Adoption we increase ϑ_d^f to 0.02. The

[13]For additional details concerning the solution and the approximation of recursive economies with multiple agents see Colacito and Croce (2012, 2013) and Rabitsch et al. (2015).

EXO calibration corresponds to a model in which innovation is exogenous.[14] All results shown are averages of 1000 simulations of a 100 quarters each.

In terms of moments of macroeconomic quantities, our baseline model with recursive preferences can generate sensible means and standard deviations for both output and consumption growth. The mean growth is 1.90% for both variables and standard deviations are 1.42% and 1.21%, respectively. Note that our model endogenously generates a very high autocorrelation in the conditional mean of future consumption growth and total TFP growth, which suggests the existence of a slow moving component governing future growth prospects, in the spirit of Bansal and Yaron (2004). We also report moments of the properties of innovation and R&D expenditure. The cross-country correlation of R&D intensity is around 0.34 in our calibration, and around 0.4 in the data. Similar values are obtained for the cross-country correlation of the growth rate in the number of varieties.

The model generates a cross-country correlation of consumption growth of 0.17. This relative low correlation is in line with the empirical estimates in Section 2 and previous studies. Notice that this value is lower than the calibrated cross-country correlation of exogenous TFP shocks. This is a consequence of the risk sharing mechanism implied by recursive preferences. Similarly to Colacito and Croce (2013), when a positive long-run shock hits the domestic economy, agents experience a sharp drop in their marginal utility resulting in a substantial reallocation of resources towards the foreign country and, ultimately, in a lower cross-country correlation in consumption growth. The novel feature of our model is that long-run shocks are the endogenous outcome of the innovation mechanism.

As for the asset pricing moments, our model generates a volatility of the depreciation rate of 9.62% and a cross-country correlation in stock market returns, r^s, as high as 0.49. This high correlation is a manifestation of the mechanism at work in our model. Indeed, focusing on the different components of the stock market, we note that the cross-country correlation in the returns on tangible capital is low and equal to 0.28. On the other hand, the return on intangible capital is very highly correlated across countries (0.78), suggesting that the international adoption of foreign innovation is a significant driver of comovements in international asset prices. Finally, note that our model can generate a

[14] In this version of the model, new prototypes arrive exogenously according to a Poisson process, so that the steady state growth rate of consumption remains the same. Growth, hence, is exogenous.

	Baseline	CRRA	EXO	Fast Adoption
Macro Quantities:				
$\mathrm{E}(\Delta c)$	1.900	1.900	1.900	1.900
$\mathrm{Std}(\Delta c)$	1.213	1.106	1.325	1.211
$\mathrm{ACF}_1\ \mathrm{E}_t(\Delta c_{t+1})$	0.899	0.901	0.903	0.892
$\mathrm{E}(\Delta y)$	1.900	1.900	1.900	1.900
$\mathrm{Std}(\Delta y)$	1.416	1.379	1.378	1.409
$\mathrm{E}(\Delta z)$	1.900	1.949	1.899	1.901
$\mathrm{Std}(\Delta z)$	2.185	2.129	2.126	2.174
$\mathrm{ACF}_1\ \mathrm{E}_t(\Delta z_{t+1})$	0.900	0.900	0.899	0.899
$\mathrm{Corr}(\Delta c_d, \Delta c_f)$	0.169	0.427	0.497	0.194
$\mathrm{Corr}(\Delta y_d, \Delta y_f)$	0.322	0.388	0.435	0.332
$\mathrm{Corr}(\Delta z_d, \Delta z_f)$	0.322	0.388	0.435	0.332
$\mathrm{Corr}\ (\Delta s_d, \Delta s_f)$	0.338	0.320		0.318
$Corr(\Delta n_d, \Delta n_f)$	0.302	0.262		0.286
Asset prices:				
$\mathrm{E}(r^f)$	2.818	20.440	2.872	2.823
$\mathrm{ACF}_1\ (r^f)$	0.899	0.896	0.903	0.898
$\mathrm{Std}(\Delta q)$	9.621	11.839	1.083	8.481
$\mathrm{Corr}(r^s_d, r^s_f)$	0.491	0.376	0.371	0.520
$\mathrm{Corr}(r^{tan}_d, r^{tan}_f)$	0.277	0.350	0.461	0.271
$\mathrm{Corr}(r^{int}_d, r^{int}_f)$	0.782	0.405	0.288	0.828

Table 6: Simulated moments for macroeconomic quantities and asset prices. Results shown are averages of 1000 simulations of 100 quarters. The subscript d (domestic) and f (foreign) are suppressed when there is no ambiguity. 'Baseline' refers to our baseline calibration in Table 5. 'CRRA' refers to the constant relative risk aversion case and is obtained by setting $\psi = 1/\gamma$. 'EXO' refers to the model with exogenous growth. 'Fast Adoption' refers to a calibration with $\vartheta^f_d = 0.02$.

low and persistent risk-free rate, as we observe in the data.[15]

Why recursive preferences? The CRRA case. Recursive preferences are crucial to our mechanism as they allow for realistic dynamics of asset prices without compromising the performance of the model for macroeconomic quantities. In the third column of Table 6, we show the results we obtain in the standard CRRA case. The dynamics of macroeconomic quantities within each country are only marginally affected. However, the model with CRRA preferences suffers from several important drawbacks. Specifically, it

[15]In the current baseline calibration, the level and the volatility of the stock market returns are smaller than what we observe in the data. A similar issue arises in Kung and Schmid (2011) and Colacito, Croce, Ho, and Howard (2012). Introducing leverage in the spirit of Boldrin, Christiano, and Fisher (2001) substantially alleviates this problem.

cannot account for the sizeable wedge between cross-country correlations in consumption growth and stock market returns that we observe in the data. Cross-country correlation in consumption growth is too high (0.43) and cross-country correlation in stock market returns is too low (0.38). With CRRA, agents in the economy do not fear variation about future prospects of the economy, as their marginal rates of substitution solely reflects current consumption growth realizations. The quantitative effect of this mechanism is evident from the cross-country correlation of the intangible part of the stock market returns, which drops to 0.40 from 0.72 in our Baseline calibration. Also, notice that the average of the risk free rate is too high, a manifestation of the well known risk-free rate puzzle.

Why endogenous growth? The EXO case. When growth is exogenous, the propagation mechanism of risk that is at the core of our baseline calibration is muted. Put simply, short-run risk, which comes from the exogenous TFP process, does not have long-run effects. In this case, the power of recursive preferences is limited. If there is little variation about the future prospects of the economy, agents will not attach a sizeable price of risk to it, and the dynamics of asset prices will resemble the ones obtained in the standard CRRA case. Quantitatively, Table 6 shows that the cross-country correlation of consumption growth increases to 0.50, while the cross correlation of stock market returns drops to 0.37.

The Role of International Adoption. The last column of Table 6 shows the results we obtain when we increase the ϑ_d^f parameter to 0.02. This value remains within the range of our empirical estimates and it implies Fast Adoption of foreign innovations. According to our mechanism, faster adoption of foreign R&D has a significant impact of the dynamics of asset prices. In particular, we see that relative to our baseline calibration stock market returns are more correlated and exchange rates are less volatile. This results is consistent with our empirical findings which highlight the effects of international diffusion of R&D on asset prices.

6 Predictability Regressions

Our model implies that domestic R&D and foreign R&D embodied in imported intermediate products should predict domestic TFP growth at lower frequencies. We provide empirical evidence for this mechanism for our sample of 20 countries and the time period

Table 7: Band-pass filtered TFP forecast

	(1) t=0	(2) t=1	(3) t=2	(4) t=3	(5) t=4	(6) t=5
$\log(\sum_{j\neq i} EM_{ij} \frac{R\&D_j}{GDP_j})$	0.086***	0.146***	0.163***	0.214***	0.228**	0.310**
	(0.014)	(0.028)	(0.042)	(0.059)	(0.076)	(0.095)
$\log(\frac{R\&D_i}{GDP_i})$	0.075***	0.152***	0.233***	0.326***	0.421***	0.515***
	(0.004)	(0.008)	(0.013)	(0.018)	(0.023)	(0.028)
Constant	-0.217	-0.191	0.274	0.412	0.906	0.739
	(0.136)	(0.273)	(0.418)	(0.577)	(0.745)	(0.929)
Observations	6099	6099	5757	5396	5035	4674
R^2	0.064	0.061	0.061	0.067	0.071	0.076

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

1993-2009. TFP is computed as the Solow residual in the following way. For each country i,

$$\log(z_{it}) = \log(y_{it}) - \alpha \log(n_{it}) - (1-\alpha)\log(k_{it});$$

here z_{it} denotes the aggregate productivity, y_{it} the real income, n_{it} the total employment, and k_{it} the real physical capital stock. The nominal GDP data (annual index in national currency) are collected from IMF IFS. We take the gross fixed capital formation (GFCF) data from IFS and take the employment index from IFS and the OECD database. For OECD countries, the GFCF data are given by a series named VOBARSA (millions of national currency, volume estimates, OECD reference year, annual levels, seasonally adjusted); the employment data are from the OECD Labour Force Statistics (MEI, Main Economic Indicators) dataset (all persons, index OECD base year 2005 = 100, seasonally adjusted). For other countries, data are from the IFS database. The GFCF data are deflated by a GDP deflator (2005 = 100, also from the IFS database) to obtain the real capital formation data. For countries and periods when quarterly data are not available, we interpolate the annual index while assuming a constant volume every quarter within a year. As a robustness check, we exclude the periods when quarterly data are not available; this does not affect our results.

Physical capital is constructed using the perpetual inventory method with a constant quarterly depreciation of 2.5% and assuming that the initial capital stock is zero. We follow the literature in setting α, the labor share of income in GDP, to 0.64 for all countries.[16]

We then compute quarter-to-quarter growth rates of our measure of TFP by computing the log-difference of the series just computed. Because we are interested in capturing low-frequency movements of this variable, we apply a Band-Pass filter that removes frequencies higher than 32 quarters. Then, we run the following regression to the filtered data for 1993-2009, using annual data for R&D and international trade:

$$\Delta TFP_{i,t+p} = a + b * log\left(R\&D_{i,t}\right) + c * log\left(\sum_j EM_{ij,t} R\&D_{j,t}\right) + u_{i,t}$$

with $p = 0, 1, 2, 3, 4, 5$ years. The first term in the right hand side corresponds to domestic R&D intensity and the second term corresponds to total foreign R&D intensity that is embodied in imported intermediate goods.

The results are reported in Table 7. There are two main findings. First, the coefficients on domestic R&D are all positive and statistically significant and they increase as the horizon increases. This result is consistent with Kung and Schmid (2011). Domestic R&D intensity forecasts the medium-term component of TFP over horizons of 1 to 5 years. Second, we test whether foreign R&D embodied in imported intermediate goods can forecast the medium-term component of TFP. This is the novel mechanism in our paper, that is, there is a common component in the TFP of countries that trade with each other that is driven by their R&D and it is weighted by how much they trade with each other. As in the case for domestic R&D, we find that the coefficients of the foreign component of R&D are positive and statistically significant and their value increases for larger horizons. Hence, consistent with the predictions of our model, the data show that both domestic and foreign R&D can predict TFP over longer horizons. Furthermore, the R^2 of the regression lies between 0.064 and 0.076 and it increases with the horizon.

In the Appendix we add two additional sets of predictability regressions in which we consider the effect of domestic R&D and foreign R&D embodied in trade separately. Consistent with the results that we have just presented, both innovations have a predictive power over the medium-term component of TFP. Interestingly, the R^2 on the predictability regression of foreign R&D is six times larger than in the case of domestic R&D.

[16]As a robustness check, we also calculate aggregate productivity for emerging markets while setting α = 0.5; this does not affect our results.

7 Conclusion

We have provided a quantitative analysis of a symmetric two-country endogenous growth model of innovation and international adoption of foreign innovations through trade in varieties. We have shown, both theoretically and with a calibration exercise, that recursive preferences, together with our endogenous growth channels, are key to match the lower cross-country correlation of quantities relative to the larger cross-country correlation of asset prices.

In the paper, we provide empirical evidence of our mechanism. First we show that country-pairs with a higher R&D content of international trade have more correlated stock market returns and less volatile exchange rates. Second, we find that both domestic and foreign R&D embodied in traded intermediate goods drive a predictable component of TFP at lower frequencies and over log horizons of time.

Our model can be extended to tackle other international asset pricing puzzles. Relaxing our symmetry assumption, for instance, we can analyze the role of our mechanism in explaining deviations of the uncovered interest parity condition and the profitability of the currency carry trade. We leave these issues for future research.

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Appendix

A Trade Data, Asset Prices, and Comovements

In this note, we describe trade data and asset prices and construct the measures relevant to our analysis.

Trade Data

The source of our trade data is UN COMTRADE. We collect product data at the 6-digit level of disaggregation. The data is annual and covers the 1985-2009 period. We focus on the trade that occurs between the importer i (identified with its IISCODE) and the exporter j (identified by its EISCODE), and collect data on the kind of product that is traded (the 6 digits identifying it) and the dollar value of the trade in each product (the per-product trade value).

Preliminary stats: calculate the fraction of world trade and world GDP that is accounted for by the countries in our sample. For the entire list, refer to the paper.

From this data, we construct the following measures:

Step 1:

- Trade Intensity (i, j): $TI_{i,j}$, i.e. the sum of the trade value of all the products
- Extensive margin (i, j): $EM_{i,j}$, i.e. the number (the“count") of different kinds of good imported by country i from country j
- Intensive Margin (i, j): $IM_{i,j} = TI_{i,j}/EM_{i,j}$, i.e. “how much”, in dollars, country i is trading on average for each product imported from country j

In order to compare these numbers across-country pairs, we normalize them taking into account each country’s GDP. In particular, we define the normalized measures as

$$\tilde{TI}_{i,j} = \frac{TI_{i,j}}{GDP_i + GDP_j}$$

$$\tilde{EM}_{i,j} = EM_{i,j}$$

$$\tilde{IM}_{i,j} = \frac{\tilde{TI}_{i,j}}{\tilde{EM}_{i,j}}$$

Note: this country pair is ordered: i is the importer and j is the exporter, i.e., $\tilde{TI}_{i,j}$ will usually be different from $\tilde{TI}_{j,i}$

Aside: we want to make sure that the relationship $\tilde{TI}_{i,j} = \tilde{EM}_{i,j}\tilde{IM}_{i,j}$ holds, so that, taking logs, we can easily run linear regressions.

Step 2:

For the measures above, we calculate their R&D *intensity*. In order to do so, we collect data on the percentage of each country's GDP that comes from expenditure in R&D. We obtain the R&D *intensity* of the trade intensity, of the extensive margin, and of the intensive margin as follows (k indexes the countries from which country i is importing):

$$\tilde{TI}_{i,j}^{R\&D} = \frac{TI_{i,j}\%R\&D_{GDP(j)}}{\sum_k TI_{i,k}\%R\&D_{GDP(k)}}$$

$$\tilde{EM}_{i,j}^{R\&D} = \frac{EM_{i,j}\%R\&D_{GDP(j)}}{\sum_k EM_{i,k}\%R\&D_{GDP(k)}}$$

$$\tilde{IM}_{i,j}^{R\&D} = \frac{IM_{i,j}\%R\&D_{GDP(j)}}{\sum_k IM_{i,k}\%R\&D_{GDP(k)}}$$

Asset Prices

We consider two main statistics for asset prices: the cross-country correlation in stock market returns and the volatility of the currency depreciation rate. We use monthly observations for the 1993/2009 period from the following sources: i) Ken French's website for stock market data, and ii) Global Financial Data exchange rates. From this data, we construct the following measures:

Stock Market

- Cross-country stock market return correlations between country i and country j: $corr(r^s_{i,t}, r^s_{j,t})$, for the entire sample. The return r^s is per quarter.

Exchange Rate

- Quarterly log depreciation rate for currency i w.r.t. currency j: $\Delta q^j_{i,t} = q^j_{i,t} - q^j_{i,t-1}$, where $q^j_{i,t}$ is the log exchange rate level at time t for country i (in units of currency i per one unit the j currency)

- Volatility of currency i depreciation rate w.r.t. currency j: $vol(\Delta q^j_{i,t})$ for the entire sample.

B Model Equations

Here, we present the equations for the domestic econmy. The foreign economy is represented by similar equations.

Preferences

$$U_{d,t} = \left\{(1-\beta)C_{d,t}^{\theta} + \beta\left(E_t\left(U_{d,t+1}^{1-\gamma}\right)\right)^{\frac{\theta}{1-\gamma}}\right\}^{\frac{1}{\theta}}$$

Stochastic discount factor

$$M_{d,t+1} = \beta\left(\frac{C_{d,t+1}}{C_{d,t}}\right)^{\theta-1}\left(\frac{U_{d,t+1}}{E_t\left(U_{d,t+1}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}\right)^{1-\gamma-\theta}$$

Final producers

$$Y_{d,t} = (Z_{d,t}L_{d,t})^{1-\alpha}K_{d,t}^{\alpha}$$

Labor

$$L_{d,t} = 1$$

Aggregate productivity

$$Z_{d,t} \equiv \Omega_{d,t}\left(\overline{A}\right)^{\frac{1}{1-\alpha}}\left[N_{d,t}^{d} + (\tau Q_t)^{\frac{\nu}{\nu-1}}N_{f,t}^{d}\right]$$

$$\overline{A} = (\xi\nu)^{\frac{\xi}{1-\xi}}$$

$$\Omega_{d,t} = e^{a_{d,t}}$$

$$a_{d,t} = \varphi\ a_{d,t-1} + \epsilon_{d,t}$$

First order condition of labor

$$W_{d,t} = (1-\alpha)(1-\xi)\frac{Y_{d,t}}{L_{d,t}}$$

First order condition of investment

$$q_{d,t} = \frac{1}{\Lambda'_{d,t}}$$

$$1 = E_t\left[M_{d,t+1}\left\{\frac{1}{q_{d,t}}\left(\alpha(1-\xi)\frac{Y_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}(1-\delta) - \frac{I_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}\Lambda_{d,t+1}\right)\right\}\right]$$

Law of motion of capital

$$K_{d,t+1} = (1-\delta)K_{d,t} + \Lambda_{d,t}K_{d,t}$$

Investment adjustment costs

$$\Lambda_{d,t} \equiv \Lambda\left(\frac{I_{d,t}}{K_{d,t}}\right) = \frac{\alpha_1}{\zeta}\left(\frac{I_{d,t}}{K_{d,t}}\right)^{\zeta} + \alpha_2$$

$$\Lambda'_{d,t} = \alpha_1\left(\frac{I_{d,t}}{K_{d,t}}\right)^{\zeta-1}$$

Demand for domestic intermediate goods

$$X^d_{d,t} = \left(\xi\nu Y_{d,t}G^{-\nu}_{d,t}\right)^{\frac{1}{1-\nu}}$$

Demand for foreign intermediate goods (imports)

$$X^d_{f,t} = \left(\xi\nu Y_{d,t}G^{-\nu}_{d,t}\right)^{\frac{1}{1-\nu}}(\tau Q_t)^{\frac{1}{\nu-1}} = X^d_{d,t}(\tau Q_t)^{\frac{1}{\nu-1}}$$

Materials (intermediate goods)

$$G_{d,t} = \xi\nu Y_{d,t}\left[N^d_{d,t} + (\tau Q_t)^{\frac{\nu}{\nu-1}}N^d_{f,t}\right]^{\frac{1-\nu}{\nu}}$$

Profits of intermediate producers

$$\Pi_{d,t}N^d_{d,t} = \pi^d_{d,t}N^d_{d,t} + \pi^f_{d,t}N^f_{d,t}$$

Profits of domestic producers in the domestic market

$$\pi^d_{d,t} = \left(\frac{1}{\nu} - 1\right) X^d_{d,t}$$

Profits of domestic producers in the foreign market

$$\pi^f_{d,t} = \left(\frac{\tau}{\nu} - 1\right) X^f_{d,t}$$

Present Discounted Value (PDV) of a domestic producers selling in the domestic market

$$V^d_{d,t} = \pi^d_{d,t} + (1-\phi)E_t[M_{d,t+1}V^d_{d,t+1}]$$

PDV of a domestic producers selling in the domestic market

$$V^f_{d,t} = \pi^f_{d,t} + (1-\phi)E_t[M_{d,t+1}V^f_{d,t+1}]$$

PDV of a domestic producers not-yet selling in the domestic market

$$J^f_{d,t} = (1-\phi)E_t\left[M_{d,t+1}\left(\vartheta^f_d\pi^f_{dt+1} + (1-\vartheta^f_d)J^f_{d,t+1}\right)\right]$$

Value of an innovation

$$V_{d,t} = V^d_{d,t} + J^f_{d,t}$$

Law of motion of new technologies

$$N^d_{d,t+1} = \vartheta_{d,t}S_{d,t} + (1-\phi)N^d_{d,t}$$

$$\vartheta_{d,t} = \frac{\chi N^d_{d,t}}{S^{1-\eta}_{d,t}(N^d_{d,t})^\eta}$$

Free entry condition of innovation

$$S_{dt} = E_t\left[M_{d,t+1}V_{d,t+1}\right]\left(N^d_{d,t+1} - (1-\phi)N^d_{d,t}\right)$$

Law of motion of adopted technologies

$$N^f_{d,t+1} = \vartheta^f_d(1-\phi)(N^d_{d,t} - N^f_{d,t}) + (1-\phi)N^f_{d,t}$$

Resource Constraint

$$Y_{d,t} = C_{d,t} + I_{d,t} + S_{d,t} + N^d_{d,t}X^d_{d,t} + N^f_{d,t}X^f_{dt}$$

International risk sharing

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{f,t+1}}{M_{d,t+1}}$$

C Deriving the Stock Market

Dividends are generated by the final producers, the intermediate producers, and the innovators. The stock market is the present discounted value of the future dividends generated by all the firms in the economy. Optimality implies the following asset pricing condition:

$$\mathcal{P}_{d,t} \quad = \quad E_t[M_{d,t+1}(\mathcal{P}_{d,t+1} + \mathcal{D}_{d,t+1})] \quad ,$$

where $\mathcal{P}_{d,t}$ is the domestic stock market price, and $\mathcal{D}_{d,t}$ is the aggregate market dividend. Substituting forward, we have

$$\mathcal{P}_{d,t} = E_t \sum_{i=0}^{\infty} M_{d,t+i+1}\mathcal{D}_{d,t+i+1}$$

Total dividends are

$$\mathcal{D}_{d,t} = D_{d,t} + N^d_{d,t}\pi^d_{d,t} + N^d_{f,t}\pi^d_{f,t} - S_{d,t} \quad ,$$

with the dividends of the final producers, $D_{d,t}$, evolving according to

$$D_{d,t} = Y_{d,t} - I_{d,t} - W_{d,t}L_{d,t} - N^d_{d,t}P^d_{d,t}X^d_{d,t} - N^d_{f,t}P^d_{f,t}X^d_{f,t} \quad .$$

Consider the present discounted value of the dividends of the final producers, $\mathcal{P}^{tan}_{d,t}$. We have

$$\mathcal{P}^{tan}_{d,t} = E_t\left[\sum_{i=0}^{\infty} M_{d,t+i+1}D_{d,t+i+1}\right]$$

or, in recursive form, $\mathcal{P}^{tan}_{d,t} = E_t \left[M_{d,t+1} \left(\mathcal{P}^{tan}_{d,t+1} + D_{d,t+1} \right) \right]$.

Result. $\mathcal{P}^{tan}_{d,t} = q_{d,t} K_{d,t+1}$.

Proof. Consider the following expression:

$$E_t(M_{d,t+1} D_{d,t+1}) = E_t \left[M_{d,t+1} \left(Y_{d,t+1} - I_{d,t+1} - W_{d,t+1} L_{d,t+1} - N^d_{d,t+1} P^d_{d.t+1} X^d_{d,t+1} - N^d_{f,t+1} P^d_{f,t+1} X^d_{f,t+1} \right) \right] \quad .$$

From the FOC of labor, we have

$$W_{d,t+1} L_{d,t+1} = (1-\alpha)(1-\varepsilon) Y_{d,t+1} \quad .$$

Use the first order conditions for intermediate producers to rewrite the expression

$$N^d_{d,t} P^d_{dt} X^d_{d,t} + N^d_{f,t} P^d_{f,t} X^d_{f,t}$$

or, substituting for the prices of intermediate goods,

$$N^d_{d,t} \frac{1}{\nu} X^d_{d,t} + N^d_{f,t} \frac{\tau}{\nu} Q_t X^d_{f,t} \quad .$$

Using

$$X^d_{f,t} = X^d_{d,t} (\tau Q_t)^{\frac{1}{\nu-1}} \text{“},$$

we have

$$\left(N^d_{d,t} \frac{1}{\nu} + N^d_{f,t} \frac{\tau}{\nu} Q_t (\tau Q_t)^{\frac{1}{\nu-1}} \right) X^d_{d,t} = \left(N^d_{d,t} + N^d_{f,t} (\tau Q_t)^{\frac{\nu}{\nu-1}} \right) \frac{1}{\nu} X^d_{d,t} \quad .$$

Similarly, using

$$X^d_{d,t} = \left(\varepsilon \nu Y_{d,t} G^{-\nu}_{d,t} \right)^{\frac{1}{1-\nu}}$$

and substituting $G_{d,t} = \varepsilon \nu Y_{d,t} \left(N^d_{d,t} + N^d_{f,t} (\tau Q_t)^{\frac{\nu}{\nu-1}} \right)$, we have

$$X^d_{d,t} = \varepsilon \nu Y_{d,t} \left(N^d_{d,t} + N^d_{f,t} (\tau Q_t)^{\frac{\nu}{\nu-1}} \right)^{-1} \quad .$$

Plugging this expression into the total spending for intermediate producers, we have

$$\left(N^d_{d,t} \frac{1}{\nu} + N^d_{f,t} \frac{\tau}{\nu} Q_t (\tau Q_t)^{\frac{1}{\nu-1}} \right) X^d_{d,t} = \left(N^d_{d,t} + N^d_{f,t} (\tau Q_t)^{\frac{\nu}{\nu-1}} \right) \frac{1}{\nu} X^d_{d,t} =$$

$$= \left(N^d_{d,t} + N^d_{f,t} (\tau Q_t)^{\frac{\nu}{\nu-1}} \right) \frac{1}{\nu} \varepsilon \nu Y_{d,t} \left(N^d_{d,t} + N^d_{f,t} (\tau Q_t)^{\frac{\nu}{\nu-1}} \right)^{-1} = \varepsilon Y_{d,t} \quad .$$

Finally, consider the FOC for investment and rearrange it to obtain

$$q_{d,t} K_{d,t+1} = E_t \left[M_{d,t+1} \left(\alpha(1-\varepsilon) Y_{d,t+1} - I_{d,t+1} \right) \right] + E_t \left[M_{d,t+1} q_{d,t+1} \left((1-\delta) + \Lambda_{d,t+1} \right) K_{d,t+1} \right] \quad .$$

From the law of motion of capital, we have

$$\left(\frac{(1-\delta) + \Lambda_{d,t+1}}{K_{d,t+1}} \right) = \frac{K_{d,t+2}}{K_{d,t+1}}$$

and, substituting in the previous expression, we obtain

$$q_{d,t}K_{d,t+1} = E_t\left[M_{d,t+1}\left(\alpha(1-\varepsilon)Y_{d,t+1} - I_{d,t+1}\right)\right] + E_t\left[M_{d,t+1}q_{d,t+1}K_{d,t+2}\right] \quad .$$

Letting $\hat{q}_t = q_t K_{t+1}$, solving the expression above recursively, and imposing the standard transversality condition, we have

$$\hat{q}_t = E_t\left[\sum_{i=0}^{\infty} M_{t+i+1}\left(\alpha(1-\varepsilon)\,Y_{t+i+1} - I_{t+i+1}\right)\right] \quad .$$

Combining these results together, we have

$$\begin{aligned} \mathcal{P}^{tan}_{d,t} &= E_t\left[M_{d,t+1}\left(\mathcal{P}^{tan}_{d,t+1} + D_{d,t+1}\right)\right] \\ &= E_t\left[\sum_{i=0}^{\infty} M_{d,t+i+1}\left(-I_{d,t+i+1} + (\alpha)(1-\varepsilon)Y_{d,t+i+1}\right)\right] = \hat{q}_{d,t} \end{aligned}$$

□

We now need to compute the present discounted values of the remaining terms in the expression for the market dividends. To compute the present discounted value of the profits of all the existing intermediate producers, we need take into account that there are two types of intermediate producers in the economy. First, there are firms that are selling both to the domestic and the foreign market. Second, there are firms that are not yet selling to the foreign market but have the possibility of doing that in the future.

The present discounted value of each firm that sells both to the domestic and foreign market today. These firms keep selling to both markets unless they do disappear with probability ϕ. Let's call the present discounted value of the dividends of these firms as $\pi_{d,t}$, which are given in recursive form by

$$\pi_{d,t} = \pi^d_{d,t} + \pi^f_{d,t} + (1-\phi)E_t\left[M_{d,t+1}\pi_{d,t+1}\right] = V^d_{d,t} + V^f_{d,t}$$

where the last equality uses the definition of the value function for one firm that is selling in the domestic market and the value funtion of one firm that is already selling in the foreign market. From the previous expression, the expected present discounted value of the future dividends for one firm that sells both in the domestic and in the foreign market, which is the component that we need to compute the stock market and we call V_{1t}, is

$$V_{1t} = (1-\phi)E_t\left[M_{d,t+1}\pi_{d,t+1}\right] = \left(V^d_{d,t} - \pi^d_{d,t}\right) + \left(V^f_{d,t} - \pi^f_{d,t}\right)$$

Finally, the present dicounted value of the dividends firms that only sell today in the domestic market but have a chance to sell tomorrow to the export market is given, in recursive form, by

$$\pi^d_{d,t} + (1-\phi)E_t\left[M_{d,t+1}\left(\pi^d_{d,t+1} + J^f_{d,t+1}\right)\right] = V^d_{d,t} + (1-\phi)E_t\left[M_{d,t+1}J^f_{d,t+1}\right]$$

From the previous expression, the expected present discounted value of the future dividends for one firm that sells only in the domestic market, which is the component that we need to compute the stock market and we call V_{2t}, is

$$V_{2t} = (1-\phi)E_t\left[M_{d,t+1}\left(\pi^d_{d,t+1} + J^f_{d,t+1}\right)\right] = \left(v^d_{d,t} - \pi^d_{d,t}\right) + (1-\phi)E_t\left[M_{d,t+1}J^f_{d,t+1}\right]$$

Since there are $N_{d,t}^{f}$firms selling both in the domestic and in the foreign market and $\left(N_{d,t}^{d}-N_{d,t}^{f}\right)$ firms selling only in the foreign market, aggregating the previous expressions we can obtain the component of the stock market that is driven by the already established intermediate producers as

$$N_{d,t}^{f}V_{1t}+(N_{d,t}^{d}-N_{d,t}^{f})V_{2t}=$$
$$=N_{d,t}^{f}\left[\left(V_{d,t}^{d}-\pi_{d,t}^{d}\right)+\left(V_{d,t}^{f}-\pi_{d,t}^{f}\right)\right]+(N_{d,t}^{d}-N_{d,t}^{f})\left[\left(V_{d,t}^{d}-\pi_{d,t}^{d}\right)+(1-\phi)E_{t}\left[M_{d,t+1}J_{d,t+1}^{f}\right]\right]$$
$$=N_{d,t}^{d}\left(V_{d,t}^{d}-\pi_{d,t}^{d}\right)+N_{d,t}^{f}\left(V_{d,t}^{f}-\pi_{d,t}^{f}\right)+(N_{d,t}^{d}-N_{d,t}^{f})\left((1-\phi)E_{t}\left[M_{d,t+1}J_{d,t+1}^{f}\right]\right)$$

Therefore, there are three components to the stock market

1. Price of installed capital
$$q_{d,t}K_{d,t+1}$$
2. Value of intangible capital
$$N_{d,t}^{d}\left(V_{d,t}^{d}-\pi_{d,t}^{d}\right)+N_{d,t}^{f}\left(V_{d,t}^{f}-\pi_{d,t}^{f}\right)$$
3. Value of intangible capital that can potentially be sold abroad
$$(1-\phi)(N_{d,t+1}^{d}-N_{d,t+1}^{f})E_{t}\left[M_{d,t+1}J_{d,t+1}^{f}\right]$$

D Additional Empirical Results

Table 8: International trade and the correlation of stock market returns

	(1)	(2)	(3)
$\log(EM)_{ij}$	0.094***	0.085***	0.018
	(0.006)	(0.005)	(0.009)
$\log(IM)_{ij}$	0.032***	0.038***	0.029***
	(0.003)	(0.003)	(0.004)
Constant	0.422***	0.432***	0.841***
	(0.068)	(0.057)	(0.102)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6460	6460	6460
R^2	0.075	0.361	0.442

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 9: R&D embodied trade and the correlation of stock market returns

	(1)	(2)	(3)
$\log(EM_{ij}^{R\&D})$	0.031***	0.021***	0.004
	(0.007)	(0.006)	(0.009)
$\log(IM_{ij}^{R\&D})$	0.016***	0.019***	0.036***
	(0.004)	(0.004)	(0.004)
Constant	0.747***	0.848***	0.812***
	(0.013)	(0.015)	(0.044)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6422	6422	6422
R^2	0.029	0.310	0.441

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 10: R&D intensity and the correlation of stock market returns

	(1)	(2)	(3)
$\log(\frac{R\&D_i}{GDP_i}) + \log(\frac{R\&D_j}{GDP_j})$	0.054^{***}	0.025^{***}	0.019
	(0.004)	(0.004)	(0.014)
Constant	0.577^{***}	0.695^{***}	0.392^{***}
	(0.003)	(0.012)	(0.030)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6384	6384	6384
R^2	0.024	0.288	0.431

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 11: Domestic and foreign R&D intensity and the correlation of stock market returns

	(1)	(2)	(3)
$\log(\frac{R\&D_i}{GDP_i})$	0.051^{***}	0.030^{***}	0.055^{*}
	(0.008)	(0.007)	(0.027)
$\log(\frac{R\&D_j}{GDP_j})$	0.061^{***}	0.016	-0.019
	(0.010)	(0.009)	(0.028)
Constant	0.577^{***}	0.697^{***}	0.401^{***}
	(0.005)	(0.017)	(0.047)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	3192	3192	3192
R^2	0.024	0.288	0.434

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 12: International trade and the volatility of the exchange rate

	(1)	(2)	(3)
$\log(EM)_{ij}$	-3.523***	-3.476***	-3.731***
	(0.160)	(0.148)	(0.233)
$\log(IM)_{ij}$	-1.384***	-1.354***	-1.217***
	(0.085)	(0.079)	(0.088)
Constant	13.443***	15.295***	25.743***
	(1.818)	(1.697)	(2.514)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6460	6460	6460
R^2	0.149	0.278	0.565

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 13: R&D embodied trade and the volatility of the exchange rate

	(1)	(2)	(3)
$\log(EM_{ij}^{R\&D})$	-1.810***	-1.949***	-2.011***
	(0.183)	(0.170)	(0.229)
$\log(IM_{ij}^{R\&D})$	-0.192	-0.134	-1.678***
	(0.122)	(0.113)	(0.109)
Constant	3.081***	7.725***	-0.938
	(0.343)	(0.449)	(1.091)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6422	6422	6422
R^2	0.059	0.198	0.561

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 14: R&D intensity and the volatility of the exchange rate

	(1)	(2)	(3)
$\log(\frac{R\&D_i}{GDP_i}) + \log(\frac{R\&D_j}{GDP_j})$	-1.183***	-1.611***	0.782*
	(0.122)	(0.117)	(0.363)
Constant	9.900***	15.315***	27.581***
	(0.095)	(0.359)	(0.765)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	6384	6384	6384
R^2	0.014	0.163	0.515

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 15: Domestic and foreign R&D intensity and the volatility of the exchange rate

	(1)	(2)	(3)
$\log(\frac{R\&D_i}{GDP_i})$	-1.044***	-1.367***	1.082
	(0.215)	(0.201)	(0.689)
$\log(\frac{R\&D_j}{GDP_j})$	-1.427***	-2.067***	0.444
	(0.283)	(0.270)	(0.708)
Constant	9.916***	15.388***	26.640***
	(0.135)	(0.509)	(1.187)
Time fixed effects	No	Yes	Yes
Country fixed effects	No	No	Yes
Observations	3192	3192	3192
R^2	0.015	0.164	0.540

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 16: Band-pass filtered TFP forecast: Domestic R&D intensity

	(1)	(2)	(3)	(4)	(5)	(6)
	t=0	t=1	t=2	t=3	t=4	t=5
$\log(\sum_{j\neq i} EM_{ij} \frac{R\&D_j}{GDP_j})$	0.140***	0.254***	0.340***	0.472***	0.580***	0.762***
	(0.014)	(0.028)	(0.043)	(0.060)	(0.077)	(0.096)
Constant	-0.740***	-1.234***	-1.443***	-2.106***	-2.525***	-3.671***
	(0.138)	(0.277)	(0.425)	(0.587)	(0.758)	(0.946)
Observations	6137	6137	5776	5415	5054	4693
R^2	0.016	0.013	0.011	0.011	0.011	0.013

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 17: Band-pass filtered TFP forecast (foreign R&D instensity embodied in trade

	(1)	(2)	(3)	(4)	(5)	(6)
	t=0	t=1	t=2	t=3	t=4	t=5
$\log(\frac{R\&D_i}{GDP_i})$	0.080***	0.159***	0.242***	0.339***	0.436***	0.536***
	(0.004)	(0.008)	(0.013)	(0.017)	(0.022)	(0.028)
Constant	0.627***	1.252***	1.883***	2.516***	3.150***	3.784***
	(0.002)	(0.005)	(0.007)	(0.010)	(0.012)	(0.015)
Observations	6099	6099	5757	5396	5035	4674
R^2	0.058	0.057	0.059	0.065	0.069	0.073

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$