Informed Trading with a Short-Sale Prohibition

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Abstract

Using a rational expectations equilibrium framework, I evaluate the effects of a short-sale prohibition in an economy with asymmetrically-informed investors who are identical except for their information sets. Relative to an economy in which short sales are permitted, I find that a short-sale ban decreases aggregate welfare for informed and uninformed investors. Under a short-sale ban, the market is less informationally efficient. This alters the risk-sharing environment and reduces welfare for uninformed investors. Informed investors' welfare falls because more investors acquire information, which increases competition among informed investors. A short-sale prohibition may slightly increase the potential for market crashes.

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1 Introduction

Short-sale constraints exist in many forms. They include search costs to locate securities to borrow, fees paid to securities lenders, and margin accounts maintained with broker-dealers. Numerous institutional and legal restrictions on short selling exist, as well. For example, Almazan et al. (2004) report that a majority of mutual funds restrict themselves from taking short positions, and financial regulations designed to curb short selling in the United States date back to 1934, including an outright ban on short sales in 2008 during the Great Recession.[1] Because short sellers are often well informed and account for a large fraction of trading volume (Dechow et al., 2001; Bris, Goetzmann, and Zhu, 2007; Chang, Cheng, and Yu, 2007; Diether, Lee, and Werner, 2009; Saffi and Sigurdsson, 2011; Beber and Pagano, 2013; Boehmer, Jones, and Zhang, 2013; Boehmer and Wu, 2013), any constraints that impede short sales can extensively impact both markets and market participants. Yet, despite their prevalence, the economic implications of short-sale constraints are not well understood. Much of the extant theoretical and empirical literature is either silent or conflicted regarding the impact of short-sale constraints on various economic attributes such as prices, welfare, and the potential for market crashes.

To evaluate the economic effects of short-sale constraints, I construct a rational expectations equilibrium model a la Grossman and Stiglitz (1980) in which short sales are prohibited. Informed traders, uninformed traders, and liquidity traders all trade a single stock and a single bond. All three types of traders are prohibited from shorting the stock but not the bond. Informed traders, who are identical to uninformed traders in every respect except for their information sets, possess an information advantage over uninformed and liquidity traders regarding the future payoff of the stock, and they can trade on the basis of this information advantage. Some of the informed traders' private information is revealed to

[1] A report by the Division of Economic and Risk Analysis (2014) of the U.S. Securities and Exchange Commission (SEC) describes a history of financial regulations related to short selling in the United States. Saffi and Sigurdsson (2011), Beber and Pagano (2013), and Boehmer, Jones, and Zhang (2013) discuss the 2008 short-sale ban. Furthermore, Section 16(c) of the Securities Exchange Act of 1934 currently prohibits officers, directors, and 10% beneficial owners from shorting stock in their firm.

uninformed traders through trading.

Relative to an economy in which short sales are permitted, I find that barring short sales decreases the amount of private information conveyed to uninformed traders through trading because a short-sale ban may censor informed traders' demand and thereby lower the precision of the price signal. As a result, the market is less informationally efficient under a short-sale prohibition. The drop in market efficiency increases uninformed traders' perceived risk associated with the stock payoff, which in turn has important risk-sharing and welfare implications.

A short-sale prohibition can have different effects on welfare depending on whether information acquisition is exogenous or endogenous. When the mass of informed traders is exogenous and fixed, a short-sale ban decreases welfare for uninformed traders but increases welfare for informed traders. In this case, the decline in market efficiency creates additional uncertainty for uninformed traders, and informed traders share some of this risk by holding a greater amount of stock on average. For informed traders to hold more stock, though, the price must fall. This means that informed traders tend to hold more stock in equilibrium but pay a lower price per share, whereas uninformed traders tend to hold fewer shares and receive less per share. Because the lower stock price amply compensates informed traders for sharing some of the additional risk generated by a short-sale prohibition, informed traders experience an increase in welfare. Conversely, uninformed traders are not sufficiently compensated, and their welfare declines. Aggregate welfare of informed and uninformed traders also falls under a short-sale ban.

Alternatively, when traders can choose whether to acquire information, a short-sale restriction leads to more traders becoming informed. The increase in the number of informed traders mitigates the detrimental effect on market efficiency caused by a short-sale ban because more information is revealed through trading when more traders are informed. Although welfare still falls for uninformed traders, the extent of the welfare loss is not as severe. However, informed traders' welfare also declines in this case because they face greater competition from other informed traders and are therefore unable to profit as much from their information advantage.

To the best of my knowledge, this article is the first to model a short-sale ban using a rational expectations equilibrium framework in which the only difference between traders is their information sets. A few models analyze the effects of a short-sale constraint when investors have heterogeneous beliefs. For example, Miller (1977) argues that a short-sale prohibition should increase prices when investors' opinions about asset payoffs diverge because equilibrium prices should reflect the views of optimists but not those of pessimists, who are prevented from shorting. Chen, Hong, and Stein (2002) echo this view. However, Jarrow (1980) and Gallmeyer and Hollifield (2008) show that stock prices can either rise or fall when investors have heterogeneous beliefs. Empirically, some studies find that short-sale constraints lead to overvaluation (Figlewski, 1981; Jones and Lamont, 2002; Boehme, Danielsen, and Sorescu, 2006; Chang, Cheng, and Yu, 2007), whereas other studies do not (Beber and Pagano, 2013; Boehmer, Jones, and Zhang, 2013). I find that a short-sale prohibition may either increase or decrease the average stock price, but such a prohibition tends to result in a higher price when the ban is binding for informed traders, which is consistent with many empirical analyses (Jones and Lamont, 2002; Asquith, Pathak, and Ritter, 2005; Boehme, Danielsen, and Sorescu, 2006; Beber and Pagano, 2013; Boehmer, Jones, and Zhang, 2013).

A separate class of models evaluates the effects of a short-sale constraint on bid-ask spreads. Diamond and Verrecchia (1987) demonstrate that a short-sale prohibition decreases both market efficiency and liquidity because it results in larger spreads. Liu and Wang (2014) also show that a short-sale constraint leads to larger bid-ask spreads. Empirically, several studies report that short-sale restrictions result in larger spreads (Beber and Pagano, 2013; Boehmer, Jones, and Zhang, 2013) as well as decrease market efficiency (Senchack and Starks, 1993; Bris, Goetzmann, and Zhu, 2007; Saffi and Sigurdsson, 2011; Beber and Pagano, 2013; Boehmer and Wu, 2013). As discussed above, I find that banning short sales decreases the informational efficiency of the market. I also find that a short-sale prohibition increases the liquidity, or depth, of the market. Because informed traders may be bound by the short-sale constraint and hold zero shares of stock in several states of the world when short selling is prohibited, there is a greater likelihood that a marginal change in aggregate demand is due to a change in liquidity trader demand (rather than informed trader demand) under a short-sale ban as compared to when short selling is permitted. Therefore, uninformed traders' beliefs about the stock payoff, and hence the equilibrium stock price, are less influenced by changes in demand when short sales are prohibited. Consequently, liquidity trades have a smaller impact on the stock price, and the market has greater depth.

Other working papers by Bai, Chang, and Wang (2006), Yuan (2006), and Cao, Zhang, and Zhou (2007) explore the effects of short-sale constraints on asset prices using rational expectation equilibrium frameworks. Like me, these authors find that a short-sale ban may decrease market efficiency and result in a lower stock price. However, none evaluate the effect on welfare or information acquisition.[2]

Finally, there are differing views in the literature regarding the relation between short sales and market crashes. Some authors find that the ability to sell short can lead to market crashes (Bris, Goetzmann, and Zhu, 2007; Chang, Cheng, and Yu, 2007), other authors show that short-sale restrictions can result in market crashes (Hong and Stein, 2003; Marin and Olivier, 2008), and still others find little to no evidence of a link between short-sale constraints and crashes (Diether, Lee, and Werner, 2009; Boehmer and Wu, 2013). Using the negative skewness of the price as a measure of the potential for a market crash, I find that a

[2]Furthermore, some of their modeling assumptions are difficult to justify. For example, Bai, Chang, and Wang (2006) assume that all informed traders receive an identical endowment shock whereas all uninformed traders receive a different identical endowment shock, but they provide no economic reason why all informed traders, of which there is a continuum, should receive the same endowment shock that differs from the endowment shock received by all uninformed traders, of which there is also a continuum. Yuan (2006) assumes that only a fraction of informed traders are subject to a short-sale constraint but that no uninformed traders are. Similarly, Cao, Zhang, and Zhou (2007) assume that short-sale constraints do not apply to uninformed traders. Moreover, the distributional assumptions regarding the asset payoffs in some of these models are inconsistent with most of the rational expectations equilibrium literature. In this article, I assume that the only difference between informed and uninformed traders is their information sets and that asset payoffs are normally distributed, which is consistent with the literature.

short-sale prohibition may slightly increase crash potential because uninformed traders are more pessimistic about the stock payoff when informed traders' demand is censored.

The remainder of this article proceeds as follows. I first describe the model in Section 2 and present results for the case where the mass of informed traders is exogenous in Section 3. Next, I consider the case where information acquisition is endogenous in Section 4. I then discuss the effect of a short-sale ban on market crashes in Section 5. Last, Section 6 concludes.

2 Model

The model is an extension of Grossman and Stiglitz (1980) that incorporates a short-sale constraint. Three types of agents exist: informed traders, uninformed traders, and liquidity traders. All three types of traders may be prohibited from taking a short position in a risky asset, depending on the regulatory regime. I consider two different regimes. Under the first regime, short sales are permitted. This environment serves as a benchmark for evaluating the effects of a short-sale ban. Under the second regime, short sales are prohibited.[3] The only difference between the two regulatory regimes is the permissibility of short sales.

There are two assets in the financial economy: a risk-free asset, which I refer to as a bond, and a risky asset, which I refer to as a stock. The bond has an exogenous interest rate that, for simplicity, is set equal to zero. The stock pays a random amount,

$$\tilde{z} = \tilde{x} + \tilde{y}, \tag{1}$$

[3] I evaluate the effects of a strict prohibition against short selling for simplicity. In reality, many short-sale constraints are less restrictive (e.g., search costs and lending fees), though outright bans on short selling do exist (e.g., the short-sale prohibition in 2008 during the Great Recession and the prohibition for statutory insiders).

where $\tilde{x} \sim \mathcal{N}(0, \varepsilon^2)$ and $\tilde{y} \sim \mathcal{N}(0, \sigma^2)$.[4] The stock price is endogenous and denoted by p. The supply of the bond is elastic, but the supply of the stock is normalized to one share. Traders are always permitted to short the bond, but, depending on the regulatory regime, they may be prohibited from shorting the stock.

There are continuums of informed and uninformed traders. For tractability, I assume that both types of traders have identical mean-variance preferences with risk-aversion coefficient γ. Other models in the asymmetric-information literature that assume mean-variance preferences include, for example, Leland (1992) and Hong, Scheinkman, and Xiong (2006).[5] Both types of traders receive identical endowments of the stock and bond, which are denoted by w_s and w_b, respectively. The mass of informed traders is exogenous and denoted by $\lambda \in (0, 1)$. Accordingly, the mass of uninformed traders is $1-\lambda$. I later relax this assumption and allow the mass of informed traders to be determined endogenously in section 4.

Time is indexed by $t \in \{1, 2\}$. Trading occurs at $t = 1$, and consumption occurs at $t = 2$. Informed traders observe x before trading at $t = 1$, but uninformed traders do not. Neither type of agent observes y until $t = 2$. Let s_i (s_u) and b_i (b_u) denote the respective quantities of the stock and bond held by informed (uninformed) traders from $t = 1$ to $t = 2$. The amount of stock held by each type of trader must be non-negative when short sales are prohibited but can take any value on the real number line when short sales are permitted.

In addition to informed and uninformed traders, there also exist liquidity traders who demand a random amount of stock at $t = 1$ denoted by v. As argued by Diamond and Verrecchia (1987), liquidity traders are unlikely to short a stock for liquidity reasons in practice because shorting tends to be more expensive. I therefore assume that liquidity-trader demand is drawn from a normal distribution truncated at zero and one, $\tilde{v} \sim \mathcal{TN}(0, \eta^2, 0, 1)$. The lower truncation point, zero, prevents liquidity traders from shorting the stock. The upper truncation point, one, ensures that liquidity-trader demand cannot exceed the outstanding

[4] Throughout this article, a tilde ($\tilde{}$) denotes a random variable whose value is yet to be realized or observed.

[5] An analytical expression for an uninformed trader's demand function is not attainable with some other types of preferences, e.g., constant absolute risk aversion.

stock supply. Without an upper bound on liquidity-trader demand, aggregate demand could exceed supply when short sales are prohibited, and thus the market would not clear.[6] The assumption that liquidity-trader demand is drawn from a truncated normal distribution, as opposed to some other type of distribution, aligns the model as closely as possible with the tradition of the extant rational expectations literature, which generally assumes that liquidity trades are normally distributed. In any event, the role of liquidity traders in the model is merely to provide camouflage so that uninformed traders cannot perfectly infer the informed traders' private information. Hence, the distribution of liquidity trades is not particularly germane to the analysis. I assume that x, y, and v are mutually independent.

2.1 Equilibrium without a Short-Sale Prohibition

In this subsection, I describe the equilibrium derivation in an economy without a short-sale prohibition. This setting serves as a benchmark for evaluating the effects of a short-sale ban. To obtain the equilibrium, I first derive the demand functions for the informed and uninformed traders. I then impose a market-clearing condition.

The problem facing informed traders is standard. They must allocate their wealth between the stock and bond to maximize their utility from consumption, c_i, subject to a budget constraint:

$$\max_{s_i} \quad \mathbb{E}[\tilde{c}_i|x] - \tfrac{1}{2}\gamma\mathbb{V}[\tilde{c}_i|x] \tag{2}$$

$$\text{s.t.} \quad \tilde{c}_i = b_i + s_i\tilde{z} \tag{3}$$

$$b_i = w_b + (w_s - s_i)p, \tag{4}$$

where $\mathbb{E}$ and $\mathbb{V}$ are the expectation and variance operators, respectively. An informed trader's demand function is derived by first substituting (1), (3), and (4) into (2). Because informed

[6] For consistency, I assume that liquidity-trader demand is doubly truncated even when short sales are permitted. This ensures that any differences between the equilibrium with a short-sale prohibition and the equilibrium without a short-sale prohibition are not driven by the distribution of liquidity-trader demand.

traders observe x, their objective function can be rewritten as

$$\max_{s_i} \quad w_b + w_s p + s_i(x-p) - \tfrac{1}{2}\gamma s_i^2 \sigma^2. \tag{5}$$

Solving the first-order condition gives an informed trader's demand function,

$$s_i = \frac{x-p}{\gamma\sigma^2}. \tag{6}$$

Uninformed traders do not directly observe x, but they can infer conditional moments of x given the price. For the market to clear, aggregate demand must equal supply. This implies

$$\lambda s_i + (1-\lambda)s_u + v = 1. \tag{7}$$

Because uninformed traders know their own demand as well as the proportion of informed traders, uninformed traders indirectly observe

$$k \equiv \frac{\lambda(x-p)}{\gamma\sigma^2} + v, \tag{8}$$

which follows from substituting (6) into (7).[7] Conditional on observing k, which is a noisy signal of x, uninformed traders infer a posterior distribution of x. This posterior distribution is truncated normally distributed because v is truncated normally distributed. Uninformed traders can then infer conditional moments of x from this signal using well-known properties of truncated normal random variables. The following lemma describes these moments.

[7] Alternatively, substituting (6) into (7) and rearranging gives

$$p = \frac{\gamma\sigma^2}{\lambda} - \frac{1-\lambda}{\lambda}\gamma\sigma^2 s_u + x + \frac{\gamma\sigma^2}{\lambda}v.$$

Thus, the stock price and k contain the same information about x.

Lemma 1. *The expectation and variance of x conditional on k are*

$$\mathbb{E}[x|k] = \mu + \frac{\phi\left(\frac{\alpha-\mu}{\Sigma}\right) - \phi\left(\frac{\beta-\mu}{\Sigma}\right)}{\Phi\left(\frac{\beta-\mu}{\Sigma}\right) - \Phi\left(\frac{\alpha-\mu}{\Sigma}\right)}\Sigma \tag{9}$$

and

$$\mathbb{V}[x|k] = \left[1 + \frac{\frac{\alpha-\mu}{\Sigma}\phi\left(\frac{\alpha-\mu}{\Sigma}\right) - \frac{\beta-\mu}{\Sigma}\phi\left(\frac{\beta-\mu}{\Sigma}\right)}{\Phi\left(\frac{\beta-\mu}{\Sigma}\right) - \Phi\left(\frac{\alpha-\mu}{\Sigma}\right)} - \left(\frac{\phi\left(\frac{\alpha-\mu}{\Sigma}\right) - \phi\left(\frac{\beta-\mu}{\Sigma}\right)}{\Phi\left(\frac{\beta-\mu}{\Sigma}\right) - \Phi\left(\frac{\alpha-\mu}{\Sigma}\right)}\right)^2\right]\Sigma^2, \tag{10}$$

where

$$\alpha \equiv p + \frac{\gamma\sigma^2(k-1)}{\lambda} \tag{11}$$

$$\beta \equiv p + \frac{\gamma\sigma^2 k}{\lambda} \tag{12}$$

$$\mu \equiv \frac{\lambda\varepsilon^2(\lambda p + \gamma\sigma^2 k)}{\theta^2} \tag{13}$$

$$\Sigma \equiv \frac{\sqrt{\gamma^2\sigma^4\eta^2\varepsilon^2}}{\theta} \tag{14}$$

$$\theta^2 \equiv \gamma^2\sigma^4\eta^2 + \lambda^2\varepsilon^2 \tag{15}$$

and $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal probability density and cumulative distribution functions.

Proof. See Appendix. □

The problem for uninformed traders then is to maximize their utility from consumption, c_u, by allocating their wealth between the stock and bond subject to a budget constraint:

$$\max_{s_u} \quad \mathbb{E}[\tilde{c}_u|k] - \tfrac{1}{2}\gamma\mathbb{V}[\tilde{c}_u|k] \tag{16}$$

$$\text{s.t.} \quad \tilde{c}_u = b_u + s_u\tilde{z} \tag{17}$$

$$b_u = w_b + (w_s - s_u)p. \tag{18}$$

An uninformed trader's objective function can be rewritten as

$$\max_{s_u} \quad w_b + w_s p + s_u(\mathbb{E}[x|k] - p) - \tfrac{1}{2}\gamma s_u^2(\mathbb{V}[x|k] + \sigma^2) \tag{19}$$

after substituting (1), (17), and (18) into (16). Then, solving the first-order condition yields an uninformed trader's demand function,

$$s_u = \frac{\mathbb{E}[x|k] - p}{\gamma(\mathbb{V}[x|k] + \sigma^2)}. \tag{20}$$

In equilibrium, supply must equal demand, i.e., (7) must hold. Because closed-form expressions for $\mathbb{E}[x|k]$ and $\mathbb{V}[x|k]$ do not exist, however, a closed-form expression for the uninformed traders' demand does not exist. Consequently, a closed-form expression for the equilibrium price does not exist.[8] Therefore, I solve for the price numerically, as discussed in Section 2.3 below.

2.2 Equilibrium with a Short-Sale Prohibition

Here, I describe the equilibrium derivation in an economy with a short-sale prohibition. The only difference between this setting and the one analyzed in the previous subsection is that informed and uninformed traders are now prohibited from shorting the stock. To distinguish this setting from the other one, I add a circumflex ($\hat{}$) to some of the variables.

Informed traders maximize their utility subject to a budget constraint and a prohibition

[8]Unlike in a standard Grossman and Stiglitz (1980) setup, where closed-form solutions for equilibrium prices and allocations exist, the truncation of liquidity-trader demand in this setting inhibits the acquisition of closed-form solutions. Because liquidity traders are unable to demand a negative quantity of stock when short sales are prohibited, I assume that liquidity traders do not demand a negative quantity of stock when short sales are permitted to ensure that any differences between the equilibrium in the two settings are not driven by changes in the distribution of liquidity-trader demand.

against short sales:

$$\max_{\hat{s}_i} \quad \mathbb{E}[\tilde{\hat{c}}_i|x] - \tfrac{1}{2}\gamma\mathbb{V}[\tilde{\hat{c}}_i|x] \tag{21}$$

$$\text{s.t.} \quad \tilde{\hat{c}}_i = \hat{b}_i + \hat{s}_i\tilde{z} \tag{22}$$

$$\hat{b}_i = w_b + (w_s - \hat{s}_i)\hat{p} \tag{23}$$

$$\hat{s}_i \geq 0. \tag{24}$$

Substituting (1), (22), and (23) into (21) allows the objective function to be rewritten as

$$\max_{\hat{s}_i \geq 0} \quad w_b + w_s\hat{p} + \hat{s}_i(x - \hat{p}) - \tfrac{1}{2}\gamma\hat{s}_i^2\sigma^2. \tag{25}$$

Solving the first-order condition then gives the demand function for an informed trader,

$$\hat{s}_i = \max\left\{\frac{x - \hat{p}}{\gamma\sigma^2}\,,\, 0\right\}. \tag{26}$$

This demand function is similar to an informed trader's demand in the absence of a short-sale ban, except that here the demand is bounded from below by zero.

Like in the case without a short-sale prohibition, demand must equal supply for the market to clear, which implies

$$\lambda\hat{s}_i + (1 - \lambda)\hat{s}_u + v = 1. \tag{27}$$

The market-clearing condition enables uninformed traders to indirectly observe a noisy signal of x,

$$\hat{k} \equiv \max\left\{\frac{x - \hat{p}}{\gamma\sigma^2}\,,\, 0\right\}\lambda + v. \tag{28}$$

Because uninformed traders do not know whether the short-sale constraint is binding for informed traders, however, the information-inference problem faced by uninformed traders is more complicated when short sales are prohibited. If informed traders are short-sale constrained, then uninformed traders can infer that $(x - \hat{p})/(\gamma\sigma^2) < 0$, which implies that $x < \hat{p}$. Conversely, if informed traders are not short-sale constrained, then uninformed traders can infer that $(x - \hat{p})/(\gamma\sigma^2) \geq 0$, which implies that $x \geq \hat{p}$. The complication arises because uninformed traders cannot observe whether informed traders are short-sale constrained. Nonetheless, uninformed traders can use $\hat{k}$ to update their beliefs about x, taking into account their uncertainty regarding whether informed traders are short-sale constrained, as described in the following lemma.

Lemma 2. *The expectation and variance of x conditional on $\hat{k}$ are*

$$\mathbb{E}[x|\hat{k}] = \frac{-\phi\big(\frac{\hat{p}}{\varepsilon}\big)\phi\big(\frac{\hat{k}}{\eta}\big)\theta\lambda^2\varepsilon^3 + \gamma\sigma^2\varepsilon^2\big[\phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\lambda^2\hat{\beta}\eta - \frac{1}{\sqrt{2\pi}\varepsilon}\phi\big(\frac{\hat{\beta}}{\varepsilon}\big)\theta\gamma\sigma^2\eta^2\big]}{\theta^2\big[\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)\theta + \phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2\eta\big]} \tag{29}$$

and

$$\begin{aligned}\mathbb{V}[x|\hat{k}] = {} & \frac{\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)\theta\big(\mathbb{E}[x|\hat{k}]^2 + \varepsilon^2\big) + \big[\big(\mathbb{E}[x|\hat{k}] - \frac{\lambda^2\varepsilon^2\hat{\beta}}{\theta^2}\big)^2 + \frac{\gamma^2\sigma^4\varepsilon^2\eta^2}{\theta^2}\big]\phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2\eta}{\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)\theta + \phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2\eta} \\ & + \frac{\phi\big(\frac{\hat{p}}{\varepsilon}\big)\phi\big(\frac{\hat{k}}{\eta}\big)\varepsilon\big(2\lambda^2\varepsilon^2\mathbb{E}[x|\hat{k}] - \theta^2\hat{p}\big)}{\theta\big[\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)\theta + \phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2\eta\big]} \\ & + \frac{\gamma^2\sigma^4\varepsilon\eta^2\big[\phi\big(\frac{\hat{p}}{\varepsilon}\big)\phi\big(\frac{\hat{k}}{\eta}\big)(\lambda^2\varepsilon^2\hat{\beta} + \theta\hat{p}) + \frac{1}{\sqrt{2\pi}}\phi\big(\frac{\hat{\beta}}{\varepsilon}\big)\big(2\theta^2\mathbb{E}[x|\hat{k}] - \hat{\beta}(\theta + \lambda^2\varepsilon^2)\big)\big]}{\theta^3\big[\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)\theta + \phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2\eta\big]},\end{aligned} \tag{30}$$

where

$$\hat{\beta} \equiv \hat{p} + \frac{\gamma\sigma^2\hat{k}}{\lambda} \tag{31}$$

$$\hat{\mu} \equiv \frac{\lambda\varepsilon^2(\lambda\hat{p} + \gamma\sigma^2\hat{k})}{\theta^2} \tag{32}$$

$$\hat{\Omega} \equiv \Phi\left(\frac{\hat{\beta} - \hat{\mu}}{\Sigma}\right) - \Phi\left(\frac{\hat{p} - \hat{\mu}}{\Sigma}\right). \tag{33}$$

Proof. See Appendix. □

Uninformed traders maximize their utility subject to a budget constraint and a prohibition against short sales:

$$\max_{\hat{s}_u} \quad \mathbb{E}[\tilde{\hat{c}}_u|\hat{k}] - \tfrac{1}{2}\gamma\mathbb{V}[\tilde{\hat{c}}_u|\hat{k}] \tag{34}$$

$$\text{s.t.} \quad \tilde{\hat{c}}_u = \hat{b}_u + \hat{s}_u\tilde{z} \tag{35}$$

$$\hat{b}_u = w_b + (w_s - \hat{s}_u)\hat{p} \tag{36}$$

$$\hat{s}_u \geq 0. \tag{37}$$

Again, an uninformed trader's objective function can be rewritten as

$$\max_{\hat{s}_u \geq 0} \quad w_b + w_s\hat{p} + \hat{s}_u(\mathbb{E}[x|\hat{k}] - \hat{p}) - \tfrac{1}{2}\gamma\hat{s}_u^2(\mathbb{V}[x|\hat{k}] + \sigma^2) \tag{38}$$

after substituting (1), (35), and (36) into (34). An uninformed trader's demand function,

$$\hat{s}_u = \max\left\{\frac{\mathbb{E}[x|\hat{k}] - \hat{p}}{\gamma(\mathbb{V}[x|\hat{k}] + \sigma^2)}, 0\right\}, \tag{39}$$

is obtained by solving the first-order condition.

Similar to the setting without a prohibition against short sales, a closed-form expression for aggregate demand does not exist when short sales are prohibited. Thus, a closed-form expression for the price does not exist. The equilibrium price is obtained using numerical methods, as discussed in the following subsection.

2.3 Numerical Solution Method

In the previous subsections, I derive analytical expressions for the traders' demand functions both with and without a short-sale prohibition. While the demand functions for an informed trader are in closed form, the demand functions for uninformed traders are not.

A closed-form expression for the price in each setting does not exist because an uninformed trader's demand in each setting depends on the moments of the conditional distribution of x, and the moments are functions of normal cumulative distribution and probability density functions that have the stock price as an argument. Consequently, the conditional moments, and thus the market-clearing condition, cannot be inverted to obtain an expression for the price. I therefore numerically compute the equilibrium price and allocations. Yuan (2005) and Gallmeyer and Hollifield (2008), for instance, also use numerical methods to study situations where traders have portfolio constraints.[9]

The algorithm is as follows. First, I draw values of x and v from their respective distributions. Next, I conjecture a price and use it to compute informed traders' demand with either (6) or (26). I then compute the signal observed by uninformed traders with either (8) or (28), which I subsequently use to compute the conditional moments of x with the equations in either Lemma 1 or 2. Next, I compute uninformed traders' demand with either (20) or (39). Last, I iterate until a price is found such that excess demand equals zero. I repeat this algorithm for 30,000 different pseudo-random draws of x and v and values of λ ranging from 0.05 to 0.95.[10]

The parameters used in the numerical analysis are summarized in Table I.[11] For robustness, I consider three different calibrations. Informed and uninformed traders are identical in every respect except their information sets. In all of the calibrations, I set the traders' risk aversion coefficient, γ, equal to 1. Informed and uninformed traders each receive a stock endowment of 1 and a bond endowment of 0. Because the mass of informed traders equals λ and the mass of uninformed traders equals $1 - \lambda$, the aggregate endowment of stock and

[9] Alternatively, a closed-form expression for the stock price is obtainable under different distributional assumptions. For example, Bai, Chang, and Wang (2006) derive an expression for the price of a risky asset when its payoff is uniformly distributed. I nevertheless assume in this article that asset payoffs are normally distributed because (i) normally-distributed returns are more reflective of reality, (ii) normally-distributed payoffs are consistent with the vast majority of the extant literature, and (iii) normally-distributed payoffs provide a much richer learning environment for uninformed traders.

[10] With this number of draws, the standard error of the mean price is less than 0.001% for all of the calibrations.

[11] The parameter values are selected to illustrate the qualitative effects of a short-sale ban. The model is not calibrated to match moments in reality.

bonds in the economy is 1 and 0, respectively. I set the variance of the second random component of the stock payoff, y, equal to 1 but consider values for the variance of the first random component of the stock payoff, x, which is the informed traders' private information, ranging from 3 to 5. I also consider variances of liquidity-trader demand ranging from 0.04 to 0.09.

2.4 Existence and Uniqueness

As a preliminary matter, I examine whether the equilibrium in each setting exists and whether it is unique. Because the conditional moments of x are composed of non-monotonic functions of the stock price, an analytical fixed point is not readily attainable. I therefore demonstrate existence and uniqueness numerically.

First, note that the informed traders' demand functions, which are given by (6) and (26), are continuous and monotonically non-increasing in the stock price, i.e., both $\partial s_i/\partial p$ and $\partial \hat{s}_i/\partial \hat{p}$ are less than or equal to zero. Provided that the short-sale constraint is not binding, these functions are monotonically decreasing in the stock price.

Because the analytical expressions for the uninformed traders' demand functions, which are given by (20) and (39), are not directly interpretable, I plot the demand functions, and their partial derivatives with respect to price, over a range of values for the stock price and the uninformed traders' signal. Figures 1(a) and 1(b) plot the demand functions without and with a short-sale prohibition, and Figures 1(c) and 1(d) plot the respective derivatives.[12] These figures indicate that an uninformed trader's demand function is continuous and monotonically non-increasing in the stock price and monotonically decreasing when the short-sale constraint is not binding.

[12]The figures are generated using the parameter values for calibration #1 with $\lambda = 0.5$. Using other calibrations and values for $\lambda \in (0, 1)$ produce similar results. The demand is reported for $\hat{k} \in [0, 1]$ because, in equilibrium, $\hat{k} \equiv \hat{s}_i + v$ must be within the unit interval. Although it is feasible for k take values outside of the unit interval in equilibrium when short sales are permitted, the same relations between s_u, k, and p hold for reasonable values of k outside of the unit interval. Over all of the calibrations and the 30,000 pseudo-random draws, the maximum equilibrium price is approximately 8, while the minimum equilibrium price is approximately -10.

Taken together the continuous and monotonic nature of the informed and uninformed traders' demand functions indicate that aggregate stock demand is continuous and monotonically non-increasing in the stock price. Further, aggregate demand appears to be monotonically decreasing in the price, provided that the short-sale constraint is not binding for both informed and uninformed traders. This suggests that an equilibrium exists both with and without a short-sale prohibition. Moreover, the equilibrium seems to be unique when at least one type of trader is not short-sale constrained. Although there are multiple equilibria when $v = 1$, as any price that results in both types of traders holding zero shares of stock can support an equilibrium in this case, the probability that $v = 1$ is zero because v is drawn from $\mathcal{TN} \sim (0, \eta^2, 0, 1)$. Thus, the equilibrium appears to be almost surely unique.

3 Results

In this section, I discuss the impact of a short-sale prohibition when the mass of informed traders is exogenous and fixed. Such a setting is relevant for understanding the economic effects of barring short sales in cases where traders may not be able to easily or quickly augment their information sets, such as the temporary ban on short selling imposed by the SEC in 2008. I evaluate the effects of a short-sale ban when information acquisition is endogenous in Section 4.

Before discussing the economic implications of preventing short sales in the following subsections, I note that a short-sale prohibition is seldom binding for informed traders. As shown in Figure 2(a), informed traders' stock demand is constrained in roughly 0% – 7.5% of cases, depending on the calibration and the fraction of traders that are informed. A short-sale ban tends to bind when the stock price is higher and informed traders' private information is more negative, i.e., when the stock is "overvalued." However, because the traders are risk averse, the desire to share risk usually dissuades informed traders from selling all of their shares, even when the stock is somewhat overvalued. Thus, the constraint is binding in only a small fraction of cases.

The likelihood of informed traders being bound by a short-sale constraint in equilibrium depends on the degree of information asymmetry as well as the volatility of liquidity trades. Comparing the different calibrations indicates that a short-sale ban is more likely to be binding when informed traders possess less private information (calibration #1 vs. #2) and when liquidity trades have a larger variance (calibration #1 vs. #3). As discussed below in Section 3.2, the market is more liquid when informed traders possess a smaller information advantage or when liquidity trades are more volatile. Because informed traders can trade more shares of stock with a smaller impact on the price in a more-liquid market, informed traders more frequently sell all of their shares in these cases. Therefore, the short-sale constraint binds more frequently when informed traders possess less private information or when liquidity trades are more volatile.

The frequency of a binding constraint also depends on the mass of informed traders. As shown in Figure 2(a), there is a non-monotonic relation between the mass of informed traders and the likelihood of the constraint binding. On the one hand, as λ increases, uninformed traders' beliefs about the stock payoff are more affected changes in the aggregate demand of informed traders plus liquidity traders, as suggested by (8) and (28), because a larger portion of the aggregate demand stems from informed traders. This increases the potential for high overvaluations of the stock. On the other hand, the price tends to more accurately reflect the fundamental value of the stock when more traders are informed, which decreases the potential for high overvaluations. Depending on which effect dominates, the likelihood of the constraint binding may either increase or decrease with the mass of informed traders. When λ is small, increasing the mass of informed traders increases the likelihood of a short-sale ban binding because the relatively few informed traders take advantage of the more frequent high overvaluations brought about by the greater influence of changes in aggregate demand on uninformed traders' beliefs. Once λ is sufficiently large, however, further increasing the mass of informed traders lowers the likelihood of the constraint binding because although changes in aggregate demand have an even greater effect on uninformed traders' beliefs, the price tends to more accurately reflect the fundamental value of the stock, and the likelihood of there being an overvaluation high enough to persuade informed traders to sell all of their shares declines.

Notably, a short-sale ban is not always binding in cases where informed traders prefer to hold a short position in the absence of a ban, as can be seen by comparing Figure 2(a) with Figure 2(b), which plots the frequency with which informed traders hold a short position when short selling is permitted. Traders hold a short position in the absence of a restriction more often than they hold zero shares when they are prohibited from shorting the stock. This is because, as shown below, a short-sale ban increases uncertainty for uninformed traders, which results in a lower stock price and greater demand for informed traders on average.

Finally, the frequencies of short sales generated by the model are compatible with empirical evidence reported by Dechow et al. (2001) and Asquith, Pathak, and Ritter (2005), who report that short interest tends to be small in practice. Yet, despite the fact that a short-sale ban is usually non-binding and short interest in the absence of a ban tends to be small, prohibiting short sales has a considerable impact on equilibrium outcomes, as discussed in the following subsections.

3.1 Market Efficiency

A short-sale prohibition may alter uninformed traders' beliefs about the stock payoff. This can occur even when a ban is non-binding because the possibility that informed traders' demand is censored affects how uninformed traders update their beliefs. Consequently, a short-sale ban can change the level of market efficiency, which measures the degree to which informed traders' private information is conveyed to uninformed traders through trading. In this subsection, I discuss how a short-sale prohibition affects uninformed traders beliefs about the stock payoff and the informational efficiency of the market.

Figure 3(a) plots the difference between the conditional distribution of x given $\hat{k}$ (short sales prohibited) and the conditional distribution of x given k (short sales permitted) over a range of values for λ.[13] As indicated by the figure, a short-sale ban shifts some of the mass of the conditional distribution to the left, thereby increasing the perceived likelihood of a lower stock payoff. Hence, uninformed traders are more pessimistic under a short-sale ban. Uninformed traders become more pessimistic because the potential censoring of informed traders' demand makes it more difficult for uninformed traders to disentangle moderately bad news from extremely bad news.

Figure 3(b) plots the difference between the average conditional expectation of x when short sales are prohibited and when they are permitted. Figure 3(c) plots the difference between the average conditional variance. As shown in the figures, the leftward shift of some of the mass of the conditional distribution results in a lower expected stock payoff but greater variability on average under a short-sale ban. Therefore, when market efficiency is measured as the precision (or inverse of the variance) of uninformed traders' conditional beliefs about informed traders' private information, as in Spiegel and Subrahmanyam (1992) and Lenkey (2014), a short-sale ban tends to result in a less efficient market. That is, uninformed traders observe a less precise signal of the informed traders' private information and, as a result, face greater uncertainty regarding the stock payoff under a short-sale ban.

The effect on the conditional moments, and thus market efficiency, is more pronounced when informed traders possess more private information and when liquidity trades are more volatile. In the former case, censoring informed traders' demand has a bigger impact when there is a greater degree of information asymmetry because a greater amount of information is potentially censored. In the latter case, prohibiting short sales has a greater effect when liquidity trades are more volatile because a short-sale ban binds much more frequently, which means that informed traders' demand is censored more often, than when liquidity trades are less volatile.

[13] Figure 3(a) plots the difference between the conditional distributions for calibration #1 for the average realization of x, v, and the corresponding equilibrium stock price. The other calibrations generate similar differences between the distributions.

The magnitude of the effect of a short-sale prohibition also depends on the mass of informed traders. When λ is small, a short-sale ban has only a minor effect on efficiency because there is little information revealed through trading when there are few informed traders. As λ increases up to a certain point, there is a greater impact from imposing a short-sale ban because there are more informed traders whose trading would reveal a greater amount of information if their demand were uncensored as in the absence of a ban. When λ is sufficiently large, the effects of a short-sale prohibition become less substantial because the short-sale constraint rarely binds and, therefore, informed traders' demand is seldom censored.

3.2 Liquidity

Market liquidity, or market depth, measures the extent to which the price of an asset changes in response to a change in demand. In other words, liquidity measures the market's ability to absorb changes in demand with only a minimal impact on the price. Because a short-sale prohibition alters the distribution of informed traders' demand, which influences the price, the price reacts differently to changes in aggregate demand depending on whether short sales are allowed. Hence, imposing a short-sale ban alters the level of market liquidity.

I measure market liquidity as the inverse of the change in price in response to a marginal increase in liquidity-trader demand, i.e., $(\partial p/\partial v)^{-1}$. Under this measure, which is also utilized by Kyle (1985) and Leland (1992), a market is more liquid if a liquidity trade has a smaller impact on the equilibrium price. Because a closed-form expression for the price is unattainable, I compute the derivative numerically.

Figure 4 plots the difference between average market liquidity when short sales are forbidden and when short sales are permitted. In contrast with much of the extant literature, which measures liquidity with the sizes of bid-ask spreads, the figure indicates that liquidity, measured as market depth, tends to increase under a short-sale ban. Because there are several states in which informed traders hold zero shares of stock when short-sales are prohibited, there is a greater likelihood that a marginal change in aggregate demand stems from a change in liquidity trader demand as compared to when short sales are allowed. Consequently, uninformed traders' beliefs about the stock payoff are less affected by changes in aggregate demand, which results in smaller changes in the stock price. Thus, the market is more liquid on average when short sales are prohibited.

Barring short sales has a greater effect on liquidity when there is a lower degree of information asymmetry or when liquidity trades are more volatile. Because a short-sale constraint is more likely to be binding in these cases, there is a greater increase in the likelihood that a marginal change in aggregate demand is due to a change in liquidity-trader demand when a short-sale ban is imposed. Therefore, the extent to which a short-sale prohibition affects a liquidity trade's impact on the stock price tends to be greater in these cases.

Additionally, there is a non-monotonic relation between the mass of informed traders and the magnitude of the effect of banning short sales on liquidity. As mentioned above, changes in aggregate demand have a greater effect on uninformed traders' beliefs about the stock payoff when the mass of informed traders is larger. However, a short-sale prohibition lessens the effect because there is a greater likelihood that a change in aggregate demand is due to a change in liquidity trader demand when short sales are prohibited. When λ is small, marginally increasing the mass of informed traders amplifies the effect of a short-sale ban on liquidity because the influence of changes in aggregate demand over uninformed traders' beliefs, which is greater in the absence of a ban when λ is larger, is reduced to a greater extent by the relatively high frequency with which the prohibition binds. Once λ reaches a certain level, though, further increasing the mass of informed traders diminishes the effects of a short-sale ban. Even though uninformed traders' beliefs are more influenced by changes in aggregate demand when λ is larger and short sales are permitted, a ban reduces this influence to a lesser extent because informed traders are bound by the constraint much less frequently.

3.3 Prices and Allocations

A short-sale prohibition alters equilibrium prices and allocations, as well. Figure 5(a), which plots the difference between the average stock price with a short-sale prohibition and the average price without such a prohibition, shows that the average price is lower when short-sales are banned. As discussed in Section 3.1, a short-sale prohibition increases uninformed traders' uncertainty regarding x. The increased uncertainty results in a lower price because uninformed traders face greater risk and are therefore willing to pay less to hold a given amount of stock. For the market to clear, the price must fall on average. Additionally, the volatility of the stock price decreases when traders are prohibited from taking a short position, as shown in Figure 5(b).

Figure 6 plots the differences between the traders' average allocations with and without a short-sale prohibition. The figure shows that informed traders tend to hold more shares of stock when short sales are banned whereas uninformed traders tend to hold fewer shares. Again, the shift in allocations is due to the increase in uninformed traders' uncertainty about x. Note that the stock payoff contains two sources of risk: x and y. Informed traders face uncertainty only with respect to y (because they observe x before trading), but uninformed traders face uncertainty with respect to both x and y. Because barring short sales increases uninformed traders' uncertainty regarding x, they face more risk when short selling is prohibited. This raises the aggregate level of perceived risk in the economy. Although informed traders do not face additional uncertainty, they share in some of the increased risk faced by uninformed traders by holding additional shares of stock. By increasing their exposure to the stock, informed traders take on extra risk associated with y but reduce risk associated with both x and y for uninformed traders, thereby enabling the perceived risk in the economy to be shared more efficiently. Thus, optimal risk sharing under a short-sale prohibition entails an increase in risk exposure for both informed and uninformed traders.

A short-sale ban tends to have a greater effect on the equilibrium price and allocations in cases where uninformed traders experience a greater increase in uncertainty, i.e., when there is more information asymmetry or higher volatility of liquidity trades. The relation between the mass of informed traders and the magnitude of the impact of a short-sale prohibition is non-monotonic because, as discussed above, the extent to which a short-sale ban affects other equilibrium attributes—such as efficiency, liquidity, and the frequency with which the short-sale prohibition binds—is also non-monotonic.

3.4 Welfare

The effects of a short-sale prohibition on market efficiency, liquidity, prices, and allocations ultimately affect the welfare of the traders. I measure the welfare of informed and uninformed traders as their expected utility.[14] Similar approaches are used by, for example, Van Nieuwerburgh and Veldkamp (2009) and Mondria (2010) to characterize ex ante utility for agents with mean-variance preferences. I compute the unconditional expectations of the traders' utility functions using the corresponding stock prices and allocations computed in Section 2. The respective expected utilities of informed an uninformed agents in the absence of a short-sale ban are given by

$$U_i = \mathbb{E}\big[\mathbb{E}[\tilde{c}_i|x] - \tfrac{1}{2}\gamma\mathbb{V}[\tilde{c}_i|x]\big] \tag{40}$$

and

$$U_u = \mathbb{E}\big[\mathbb{E}[\tilde{c}_u|k] - \tfrac{1}{2}\gamma\mathbb{V}[\tilde{c}_u|k]\big]. \tag{41}$$

Analogous expressions apply to the case where short sales are barred.

Figures 7(a) and 7(b) plot the differences between welfare when short sales are prohibited and when short sales are permitted for informed and uninformed traders, respectively. These differences can be construed as certainty equivalents of a short-sale prohibition. The plots indicate that welfare tends to increase for informed traders but decrease for uninformed

[14] Although it is well known that mean-variance preferences can lead to time-inconsistent portfolio decisions (see, e.g., Basak and Chabakauri, 2010), traders in the model do not make dynamic portfolio decisions.

traders when short sales are forbidden. Furthermore, aggregate welfare drops under a short-sale ban, as shown in Figure 7(c). I measure the change in aggregate welfare as the difference between the weighted sum of the welfare of informed and uninformed traders when short sales are prohibited and when they are permitted,

$$\lambda(\hat{U}_i - U_i) + (1 - \lambda)(\hat{U}_u - U_u), \tag{42}$$

where $\hat{U}_i$ and $\hat{U}_u$ denote the ex ante expected utilities of informed and uninformed traders, respectively, when short sales are prohibited.

The effects of a short-sale ban on welfare are driven by the shift in risk sharing discussed in Section 3.3. Both informed and uninformed traders bear more risk when short sales are forbidden, but the nature of the additional risk borne by each type of trader is different. Informed traders bear more "fundamental" risk because they tend to hold a greater amount of stock in equilibrium. In contrast, uninformed traders bear more "perceived" risk because they face greater uncertainty regarding x, but they bear less fundamental risk because they tend to hold fewer shares of stock. Informed traders are amply compensated for bearing more fundamental risk in the form of a lower stock price, as the lower price they pay to buy shares of stock under a short-sale ban reflects not only the amount of fundamental risk in the economy but also the greater amount of perceived risk, to which informed traders are not exposed, that is due to the rise in uninformed traders' uncertainty about x. Thus, welfare rises for informed traders. Conversely, uninformed traders experience a loss of welfare because they receive less compensation from the lower stock price even though they bear more (total) risk.

4 Endogenous Information Acquisition

The preceding analysis assumes that the mass of informed traders is fixed. I now evaluate the effects of a short-sale prohibition when the mass of informed traders is determined endogenously, i.e., when traders can choose whether to become informed. I first explain the additional assumptions used to determine the mass of traders who acquire information. I then describe the algorithm employed to compute the mass of informed traders in Section 4.1. Finally, I report results in Section 4.2.

As shown in Figures 8(a) and 8(b), which plot the differences between informed and uninformed traders' expected utilities, informed traders attain a greater expected utility than uninformed traders when information acquisition is exogenous for any given λ, regardless of whether a short-sale ban exists. Consequently, all traders will choose to become informed unless they incur a sufficiently large cost to do so. I therefore assume that traders incur a fixed cost to become informed, which could be thought of as, for example, a search cost or a monetary cost to acquire information. However, I do not explicitly model this cost of information acquisition because assigning a numerical value for the cost would be arbitrary. Instead, for any given value of $\lambda \in (0, 1)$, I assume that the information market is in equilibrium. This means that the expected utility of informed traders, after accounting for the cost of information acquisition, equals the expected utility of their uninformed counterparts. If their expected utilities were not equal, then some uninformed traders would choose to become informed, or vice versa, until the expected utilities of each group were equal. Thus, the assumption implies that there is some fixed cost incurred to acquire information and, depending on the size of the cost, a different value of λ will be realized in equilibrium.

I also assume that the cost of acquiring information is independent of whether short sales are allowed. Therefore, in equilibrium the mass of informed traders when short sales are prohibited, $\hat{\lambda}$, must be such that the difference between informed and uninformed traders' expected utilities when short sales are prohibited is the same as the difference when short sales are permitted.

4.1 Algorithm

For each given mass of traders who are informed when short sales a permitted, λ, I use a bisection method to compute the mass of traders who become informed when they are prohibited from shorting the stock, $\hat{\lambda}$. I first compute the difference between informed and uninformed traders' expected utilities when short sales are permitted. I then compute $\hat{\lambda}$ such that the difference between informed and uninformed traders' expected utilities when shorts sales are prohibited is the same as when short sales are permitted, i.e.,

$$\hat{U}_i(\hat{\lambda}) - \hat{U}_u(\hat{\lambda}) = U_i(\lambda) - U_u(\lambda). \tag{43}$$

To compute $\hat{\lambda}$, I set lower and upper bounds for $\hat{\lambda}$ equal to 0 and 1, respectively, and conjecture a value of $\hat{\lambda}$ equal to the midpoint of the two bounds. Using this conjectured value of $\hat{\lambda}$, I compute the equilibrium stock price and allocations, as described in Section 2.3, along with the traders' expected utilities. If the difference between informed and uninformed traders' expected utilities with a short-sale ban is greater (less) than the difference without a ban, I replace the lower (upper) bound on $\hat{\lambda}$ with the conjectured value and repeat the process until either the differences are the same or the conjectured value of $\hat{\lambda}$ converges to 0 or 1.

There appears to be a unique equilibrium in the information market for any given λ. The difference between an informed trader's expected utility and an uninformed traders' expected utility when short sales are permitted and when they are prohibited is plotted in Figures 8(a) and 8(b), respectively. In both cases, the difference between the traders' expected utilities is monotonically decreasing in the fraction of informed traders, as shown in Figures 8(c) and 8(d), which plot the numerically-computed partial derivatives of the utility differences with respect to λ. Thus, for any given λ, there appears to be a unique $\hat{\lambda}$ such that either the difference between an informed and uninformed trader's expected utility in the presence of a short-sale ban is the same as the difference in the absence of a ban or the difference is such that all of the traders choose to become informed.

4.2 Results

A short-sale prohibition leads to more traders becoming informed, as illustrated by Figure 9, which plots the fraction of informed traders when short sales are banned for each given fraction of informed traders when short sales are permitted. More traders acquire information when short sales are forbidden because, *ceteris paribus*, a short-sale prohibition raises welfare for informed traders but lowers welfare for uninformed traders, as discussed in Section 3.4. Because more traders become informed under a short-sale ban when they are given a choice of whether to acquire information, the effects of prohibiting short sales may depend on whether or not the mass of informed traders is fixed. In the remainder of this subsection, I evaluate the effects of a short-sale ban when information acquisition is endogenous. For the sake of brevity, I focus on the results that differ from those described in Section 3 and that are useful in explaining how a short-sale prohibition affects welfare.

Because there is a greater mass of informed traders when information acquisition is endogenous, more information is revealed through trading. Consequently, relative to the setting where information acquisition is exogenous, a short-sale ban tends to have a smaller impact on market efficiency when information acquisition is endogenous, which can be seen by comparing Figures 3(c) and 10(a).[15] As a result, uninformed traders' uncertainty regarding x increases to a lesser extent, which means that there is less additional perceived risk to be shared. Furthermore, when λ is sufficiently large, the aggregate uncertainty regarding x decreases under a short-sale ban because many more traders become informed and face no uncertainty about x. Thus, a short-sale prohibition results in a lower average stock price when information acquisition is endogenous and λ is small but a higher average price when λ is large, as shown in Figure 10(b).

[15]Figure 10 is truncated because no traders are uninformed under a short-sale ban when λ is sufficiently large, as shown in Figure 9. The plots illustrating the effects of a short-sale ban on uninformed traders' average stock allocation and welfare also are truncated for the same reason.

The average equilibrium stock allocations are also affected by endogenous information acquisition, which is evident from comparing Figures 6 and 11. When information acquisition is endogenous, there is a greater increase (decrease) in the average amount of stock held by informed (uninformed) traders as a group when a short-sale ban is imposed because more traders choose to become informed. Similar to the case where information acquisition is exogenous, individual uninformed traders tend to hold less stock under a short-sale prohibition because they face additional uncertainty. However, individual informed traders may hold more or less stock on average. When λ is small, informed traders tend to hold more shares, though the increase in the allocation is smaller than when information acquisition is exogenous. When λ is large, individual informed traders tend to hold fewer shares because many more traders become informed, which allows the risk associated with the stock payoff to be spread thinner over the larger mass of informed traders.

Ultimately, a short-sale prohibition leads to a loss of welfare for both informed and uninformed traders when information acquisition is endogenous, as shown in Figure 12. Each type of trader experiences an identical loss of welfare, which is implied by (43). Like in the case with a fixed fraction of informed traders, welfare decreases for uninformed traders because they are not adequately compensated for bearing risk. Informed traders experience a drop in welfare because there is increased competition from the larger mass of informed traders, which reduces their ability to capitalize on their information advantage.

5 Crashes

As discussed above, the literature is conflicted regarding the relation between short-sale constraints and market crashes. Although some studies find that short-sale restrictions are positively related to market crashes, some studies find that short sales themselves are related to crashes, and other studies find no relation. In this section, I examine how a short-sale prohibition affects the potential for crashes within the context of the model. Because the potential for a short-sale constraint to cause a large price decline in a single stock may differ from the potential to cause a decline in the entire market (Bris, Goetzmann, and Zhu, 2007), the results presented here should not necessarily be extrapolated to the market as a whole.

Following Bates (1991), Chen, Hong, and Stein (2001), and others, I measure crash potential as the skewness of the stock price. Figure 13(a) plots the difference between the stock price skewness when short sales are prohibited and when they are permitted, holding fixed the mass of informed traders. The figure indicates that a short-sale ban increases negative skewness. This occurs because uninformed traders are more pessimistic about the stock payoff when informed traders may be bound by a short-sale ban, as indicated by Figure 3(a). However, skewness is less affected when more traders are informed because the short-sale prohibition binds less frequently, which leads to a smaller increase in the degree of pessimism. For the same reason, the effect of a short-sale prohibition on negative skewness is less substantial when information acquisition is endogenous, as indicated by Figure 13(b). For most values of λ, the impact of a short-sale ban on skewness is small when information acquisition is endogenous. Overall, it appears as though a short-sale prohibition slightly increases the potential for crashes.

6 Conclusion

I evaluate the effects of a short-sale ban using a rational expectations equilibrium framework. Although a short-sale ban is seldom binding, a prohibition against short sales affects many equilibrium attributes because it alters uninformed traders' beliefs. Relative to a market in which short sales are permitted, I find that prohibiting short sales reduces market efficiency but increases liquidity. Additionally, the aggregate welfare of informed and uninformed traders falls, even though more traders choose to become informed when short sales are forbidden. Furthermore, barring short sales appears to have little effect on the potential for market crashes.

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Appendix

Proof of Lemma 1. First, (8) can be rewritten as

$$v = k - \frac{\lambda(x-p)}{\gamma\sigma^2}. \tag{A.1}$$

Then, the conditional density of x given k is derived by using a convolution of x and v:

$$\frac{1}{2\pi\varepsilon\eta} e^{-x^2/2\sigma^2} e^{-v^2/2\eta^2} \tag{A.2}$$

$$= \frac{1}{2\pi\varepsilon\eta} e^{-x^2/2\sigma^2} \exp\left[-\frac{(\lambda(p-x)+\gamma\sigma^2 k)^2}{2\gamma^2\sigma^4\eta^2}\right] \tag{A.3}$$

$$\propto \exp\left[-\frac{\theta^2}{2\gamma^2\sigma^4\eta^2\varepsilon^2}\left(x - \frac{\lambda\varepsilon^2(\lambda p + \gamma\sigma^2 k)}{\theta^2}\right)^2\right], \tag{A.4}$$

where (A.3) follows from substituting (A.1). Thus, $\tilde{x}$ is conditionally normally distributed with mean given by μ and standard deviation given by Σ. The respective lower and upper truncation points, α and β, are derived by substituting $v = 1$ and $v = 0$ into (A.1) and solving for x. The conditional expectation and variance of $\tilde{x}$ are obtained using well-known properties of truncated normal random variables (see, e.g., Greene, 1993, p. 685). □

Proof of Lemma 2. Because the distribution of the signal, $\hat{k}$, observed by uninformed traders depends on whether informed traders are short-sale constrained, there are two regions of integration with respect to x defined by whether informed traders are short-sale constrained. If informed traders are short-sale constrained, then by definition they hold zero shares in equilibrium. Together with (26), this implies that $x \leq \hat{p}$ when informed traders are short-sale constrained. Thus, the first integration region is right truncated at $\hat{p}$. If informed traders are not short-sale constrained, then uninformed traders infer that informed traders hold a quantity of shares between zero and $\hat{k}$ because $v \geq 0$. This implies that $\hat{p} < x < \hat{p} + \gamma\sigma^2\hat{k}/\lambda$ when informed traders are not short-sale constrained. Thus, the respective lower and upper truncation points for the second integration region are $\hat{p}$ and $\hat{\beta}$.

The conditional expectation of x given $\hat{k}$ is derived using a convolution of x and v. If $x \leq \hat{p}$, then it follows immediately from (28) that $v = \hat{k}$. Conversely, if $x > \hat{p}$, then (28) indicates that

$$v = \hat{k} - \frac{\lambda(x-\hat{p})}{\gamma\sigma^2}. \tag{A.5}$$

Then, the conditional expectation of x is

$$\frac{\frac{1}{2\pi\varepsilon\eta}\int_{-\infty}^{\hat{p}} xe^{-x^2/2\sigma^2}e^{-v^2/2\eta^2}\,\mathrm{d}x + \frac{1}{2\pi\varepsilon\eta}\int_{\hat{p}}^{\hat{\beta}} xe^{-x^2/2\sigma^2}e^{-v^2/2\eta^2}\,\mathrm{d}x}{\frac{1}{2\pi\varepsilon\eta}\int_{-\infty}^{\hat{p}} e^{-x^2/2\sigma^2}e^{-v^2/2\eta^2}\,\mathrm{d}x + \frac{1}{2\pi\varepsilon\eta}\int_{\hat{p}}^{\hat{\beta}} e^{-x^2/2\sigma^2}e^{-v^2/2\eta^2}\,\mathrm{d}x} \tag{A.6}$$

$$= \frac{\frac{1}{2\pi\varepsilon\eta}\int_{-\infty}^{\hat{p}} xe^{-x^2/2\sigma^2}e^{-\hat{k}^2/2\eta^2}\,\mathrm{d}x + \frac{1}{2\pi\varepsilon\eta}\int_{\hat{p}}^{\hat{\beta}} xe^{-x^2/2\sigma^2}\exp\big[-\frac{(\lambda(\hat{p}-x)+\gamma\sigma^2\hat{k})^2}{2\gamma^2\sigma^4\eta^2}\big]\,\mathrm{d}x}{\frac{1}{2\pi\varepsilon\eta}\int_{-\infty}^{\hat{p}} e^{-x^2/2\sigma^2}e^{-\hat{k}^2/2\eta^2}\,\mathrm{d}x + \frac{1}{2\pi\varepsilon\eta}\int_{\hat{p}}^{\hat{\beta}} e^{-x^2/2\sigma^2}\exp\big[-\frac{(\lambda(\hat{p}-x)+\gamma\sigma^2\hat{k})^2}{2\gamma^2\sigma^4\eta^2}\big]\,\mathrm{d}x} \tag{A.7}$$

$$= \frac{\frac{\varepsilon}{\eta}\phi\big(\frac{\hat{p}}{\varepsilon}\big)\phi\big(\frac{\hat{k}}{\eta}\big) + \frac{\gamma\sigma^2\varepsilon}{\theta^2}\big[\frac{1}{\theta}\phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\lambda^2\hat{\beta}\varepsilon + \gamma\sigma^2\eta\big(\phi\big(\frac{\hat{p}}{\varepsilon}\big)\phi\big(\frac{\hat{k}}{\eta}\big) - \frac{1}{\sqrt{2\pi}}\phi\big(\frac{\hat{\beta}}{\varepsilon}\big)\big)\big]}{\frac{1}{\eta}\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big) + \frac{1}{\theta}\phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2}, \tag{A.8}$$

where (A.7) follows from substituting $v = \hat{k}$ into the first term and (A.5) into the second term in both the numerator and the denominator. Algebra yields (29).

The conditional variance of x given $\hat{k}$ is also derived using a convolution of x and v. The integration bounds are the same as above. It follows that the conditional variance of x is

$$\frac{\frac{1}{2\pi\varepsilon\eta}\int_{-\infty}^{\hat{p}}(x-\mathbb{E}[x|\hat{k}])^2 e^{-x^2/2\sigma^2}e^{-v^2/2\eta^2}\,\mathrm{d}x+\frac{1}{2\pi\varepsilon\eta}\int_{\hat{p}}^{\hat{\beta}}(x-\mathbb{E}[x|\hat{k}])^2 e^{-x^2/2\sigma^2}e^{-v^2/2\eta^2}\,\mathrm{d}x}{\frac{1}{2\pi\varepsilon\eta}\int_{-\infty}^{\hat{p}}e^{-x^2/2\sigma^2}e^{-v^2/2\eta^2}\,\mathrm{d}x+\frac{1}{2\pi\varepsilon\eta}\int_{\hat{p}}^{\hat{\beta}}e^{-x^2/2\sigma^2}e^{-v^2/2\eta^2}\,\mathrm{d}x} \tag{A.9}$$

$$= \frac{\frac{1}{2\pi\varepsilon\eta}\int_{-\infty}^{\hat{p}}(x-\mathbb{E}[x|\hat{k}])^2 e^{-x^2/2\sigma^2}e^{-\hat{k}^2/2\eta^2}\,\mathrm{d}x}{\frac{1}{2\pi\varepsilon\eta}\int_{-\infty}^{\hat{p}}e^{-x^2/2\sigma^2}e^{-\hat{k}^2/2\eta^2}\,\mathrm{d}x+\frac{1}{2\pi\varepsilon\eta}\int_{\hat{p}}^{\hat{\beta}}e^{-x^2/2\sigma^2}\exp\big[-\frac{(\lambda(\hat{p}-x)+\gamma\sigma^2\hat{k})^2}{2\gamma^2\sigma^4\eta^2}\big]\,\mathrm{d}x}$$

$$+\frac{\frac{1}{2\pi\varepsilon\eta}\int_{\hat{p}}^{\hat{\beta}}(x-\mathbb{E}[x|\hat{k}])^2 e^{-x^2/2\sigma^2}\exp\big[-\frac{(\lambda(\hat{p}-x)+\gamma\sigma^2\hat{k})^2}{2\gamma^2\sigma^4\eta^2}\big]\,\mathrm{d}x}{\frac{1}{2\pi\varepsilon\eta}\int_{-\infty}^{\hat{p}}e^{-x^2/2\sigma^2}e^{-\hat{k}^2/2\eta^2}\,\mathrm{d}x+\frac{1}{2\pi\varepsilon\eta}\int_{\hat{p}}^{\hat{\beta}}e^{-x^2/2\sigma^2}\exp\big[-\frac{(\lambda(\hat{p}-x)+\gamma\sigma^2\hat{k})^2}{2\gamma^2\sigma^4\eta^2}\big]\,\mathrm{d}x} \tag{A.10}$$

$$= \frac{\frac{\varepsilon}{\eta}\phi\big(\frac{\hat{p}}{\varepsilon}\big)\phi\big(\frac{\hat{k}}{\eta}\big)\big(2\mathbb{E}[x|\hat{k}]-\hat{p}\big)+\frac{1}{\eta}\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)(\mathbb{E}[x|\hat{k}]^2+\varepsilon^2)}{\frac{1}{\eta}\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)+\frac{1}{\theta}\phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2}$$

$$+\frac{\frac{1}{\theta}\phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\big[\big(\mathbb{E}[x|\hat{k}]-\frac{\lambda^2\varepsilon^2\hat{\beta}}{\theta^2}\big)^2+\frac{\gamma^2\sigma^4\eta^2\varepsilon^2}{\theta^2}\big]\hat{\Omega}\gamma\sigma^2}{\frac{1}{\eta}\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)+\frac{1}{\theta}\phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2}$$

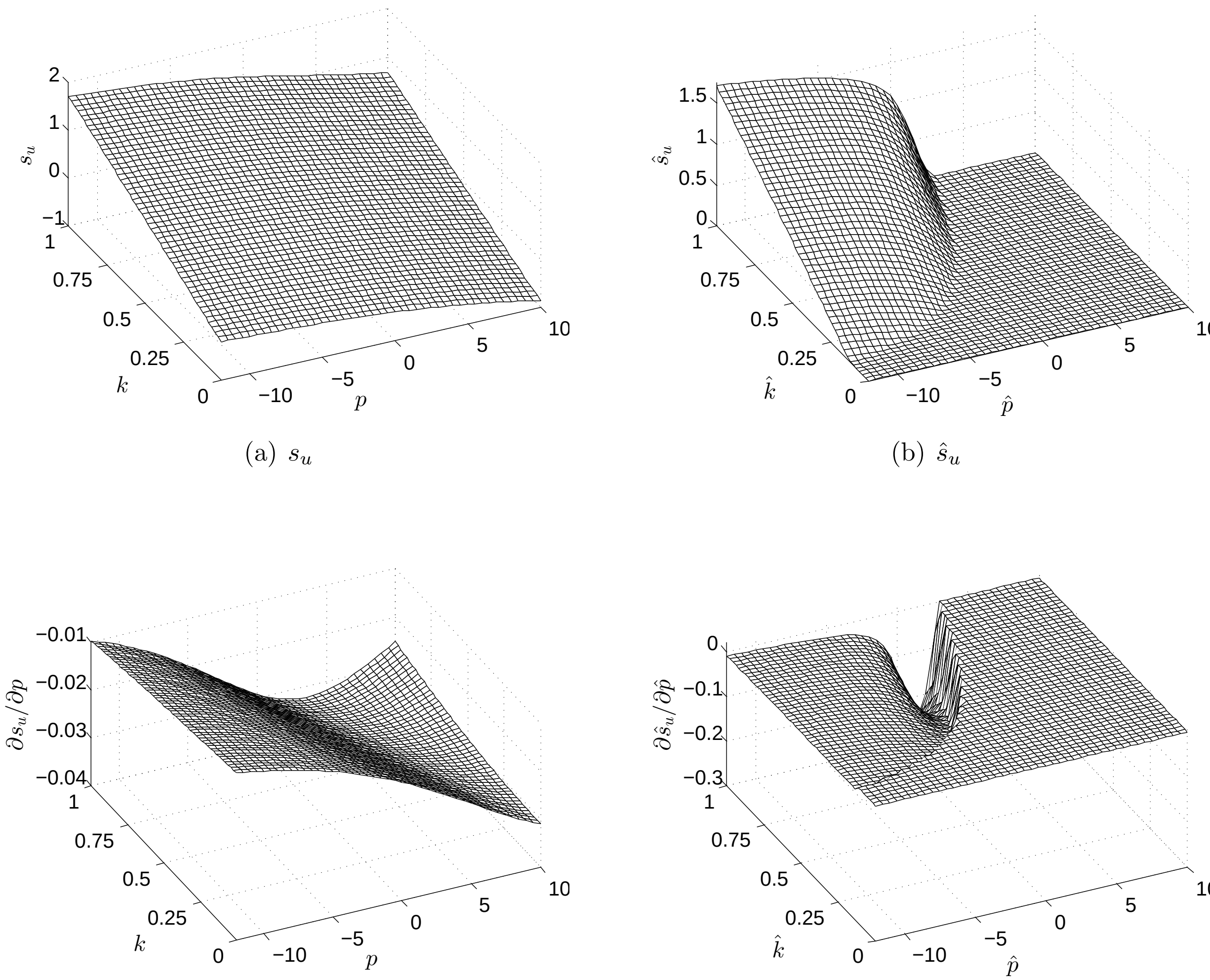
$$+\frac{\frac{2}{\theta^2}\big[\frac{1}{\sqrt{2\pi}}\phi\big(\frac{\hat{\beta}}{\varepsilon}\big)-\phi\big(\frac{\hat{p}}{\varepsilon}\big)\phi\big(\frac{\hat{k}}{\eta}\big)\big]\gamma^2\sigma^4\varepsilon\eta\mathbb{E}[x|\hat{k}]}{\frac{1}{\eta}\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)+\frac{1}{\theta}\phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2}$$

$$+\frac{\frac{1}{\theta^4}\big[\phi\big(\frac{\hat{p}}{\varepsilon}\big)\phi\big(\frac{\hat{k}}{\eta}\big)(\theta^2\hat{p}+\lambda^2\varepsilon^2\hat{\beta})-\frac{1}{\sqrt{2\pi}}\phi\big(\frac{\hat{\beta}}{\varepsilon}\big)\hat{\beta}(\lambda^2\varepsilon^2+\theta^2)\big]\gamma^2\sigma^4\varepsilon\eta}{\frac{1}{\eta}\phi\big(\frac{\hat{k}}{\eta}\big)\Phi\big(\frac{\hat{p}}{\varepsilon}\big)+\frac{1}{\theta}\phi\big(\frac{\lambda\hat{\beta}}{\theta}\big)\hat{\Omega}\gamma\sigma^2}, \tag{A.11}$$

where (A.10) follows from substituting $v=\hat{k}$ and (A.5) into the first and second terms, respectively, in both the numerator and denominator. Algebra yields (30). □

Table I: Parameter Values.

Variable	Symbol	#1	#2	#3
Variance of $\tilde{x}$	ε^2	3	5	3
Variance of $\tilde{y}$	σ^2	1	1	1
Variance of $\tilde{v}$	η^2	0.09	0.09	0.04
Risk aversion coefficient	γ	1	1	1
Stock endowment	w_s	1	1	1
Bond endowment	w_b	0	0	0



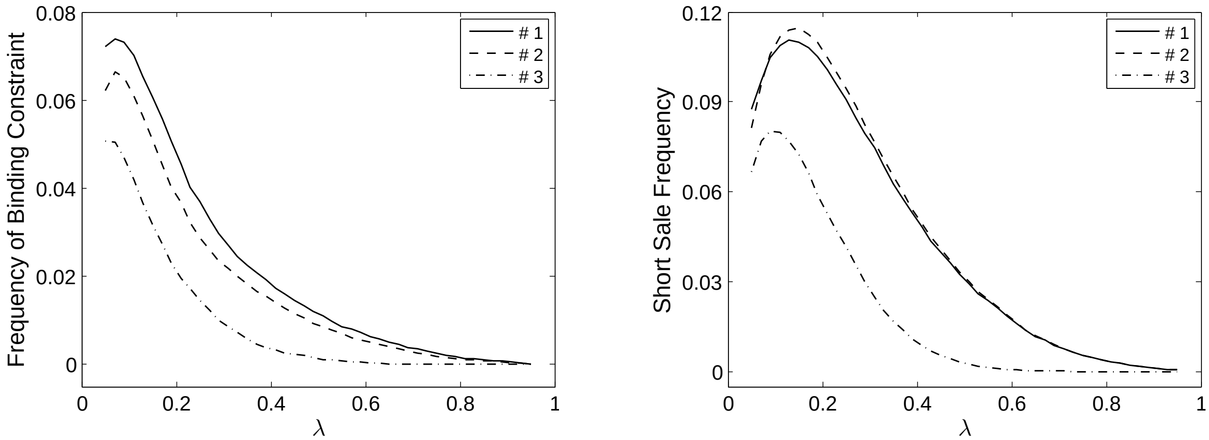
(a) s_u

(b) $\hat{s}_u$

(c) $\partial s_u/\partial p$

(d) $\partial \hat{s}_u/\partial \hat{p}$

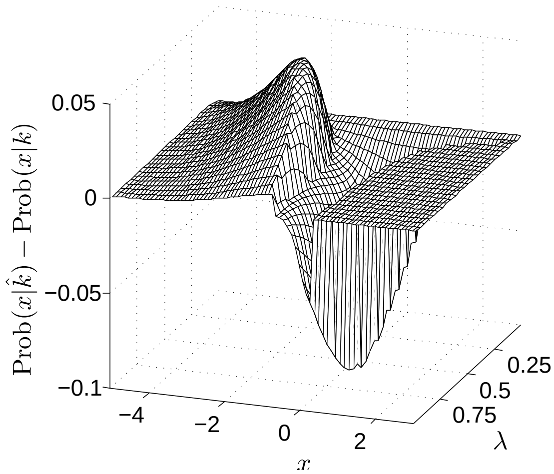
Figure 1: Demand Functions. (a) An uninformed trader's demand when short sales are permitted is plotted. (b) An uninformed trader's demand when short sales are prohibited is plotted. (c) The partial derivative of an uninformed trader's demand with respect to the stock price when short sales are permitted is plotted. (d) The partial derivative of an uninformed trader's demand with respect to the stock price when short sales are prohibited is plotted.



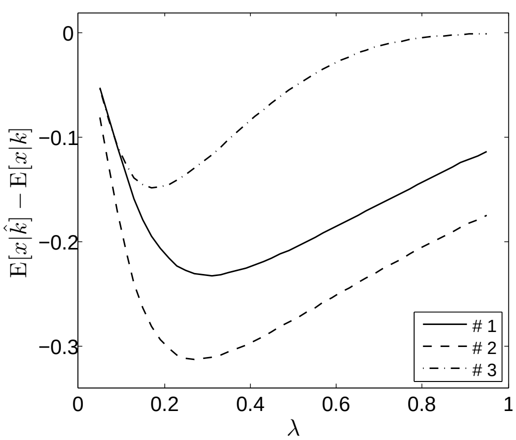
(a) Binding Constraint

(b) Short Sales

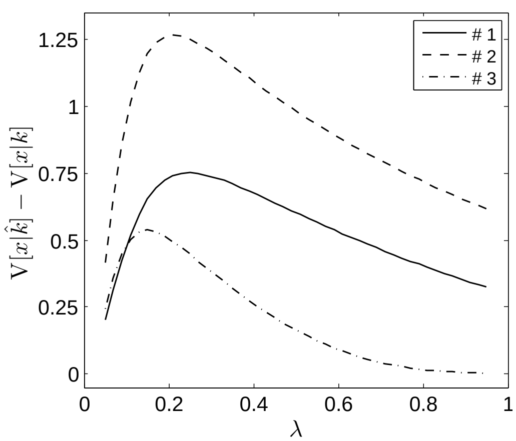
Figure 2: Frequency of Binding Constraint and Short Sales. (a) The frequency with which a short-sale prohibition is binding for informed traders is plotted. (b) The frequency with which informed traders hold a short position in the absence of a short-sale prohibition is plotted.



(a) Conditional Distribution



(b) Conditional Mean



(c) Conditional Variance

Figure 3: Conditional Distribution and Moments. (a) The difference between the conditional distribution of x given $\hat{k}$ and the conditional distribution of x given k is plotted. (b) The difference between the average conditional expectation of x with and without a short-sale prohibition is plotted. (c) The difference between the average conditional variance of x with and without a short-sale prohibition is plotted.

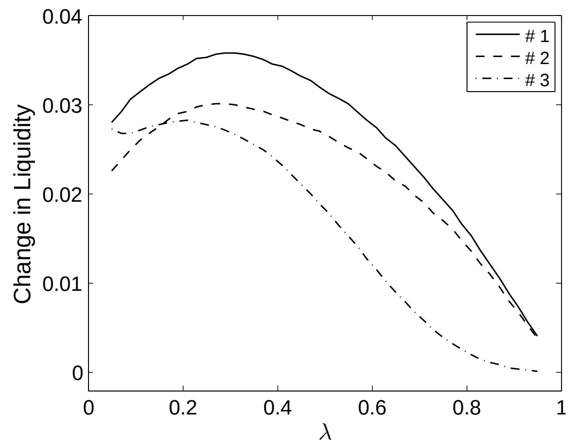
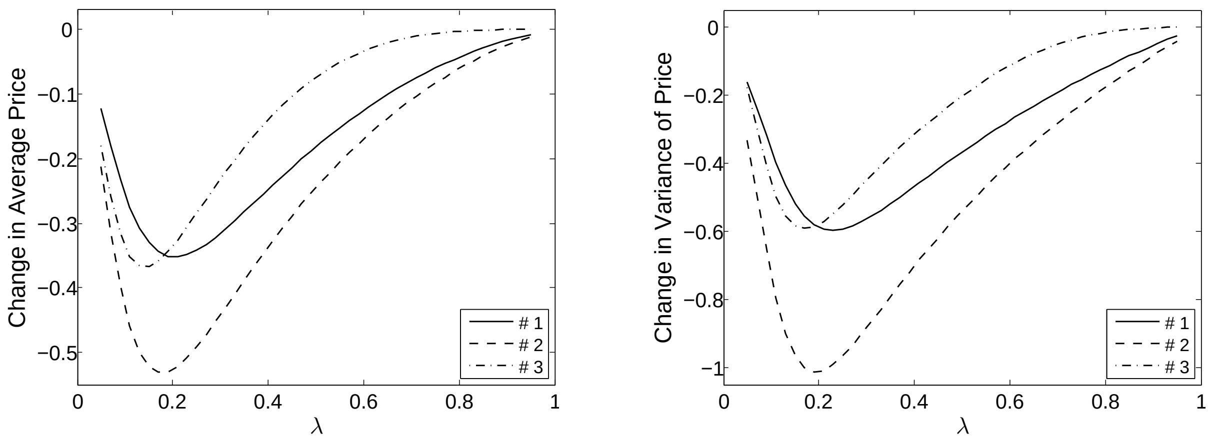


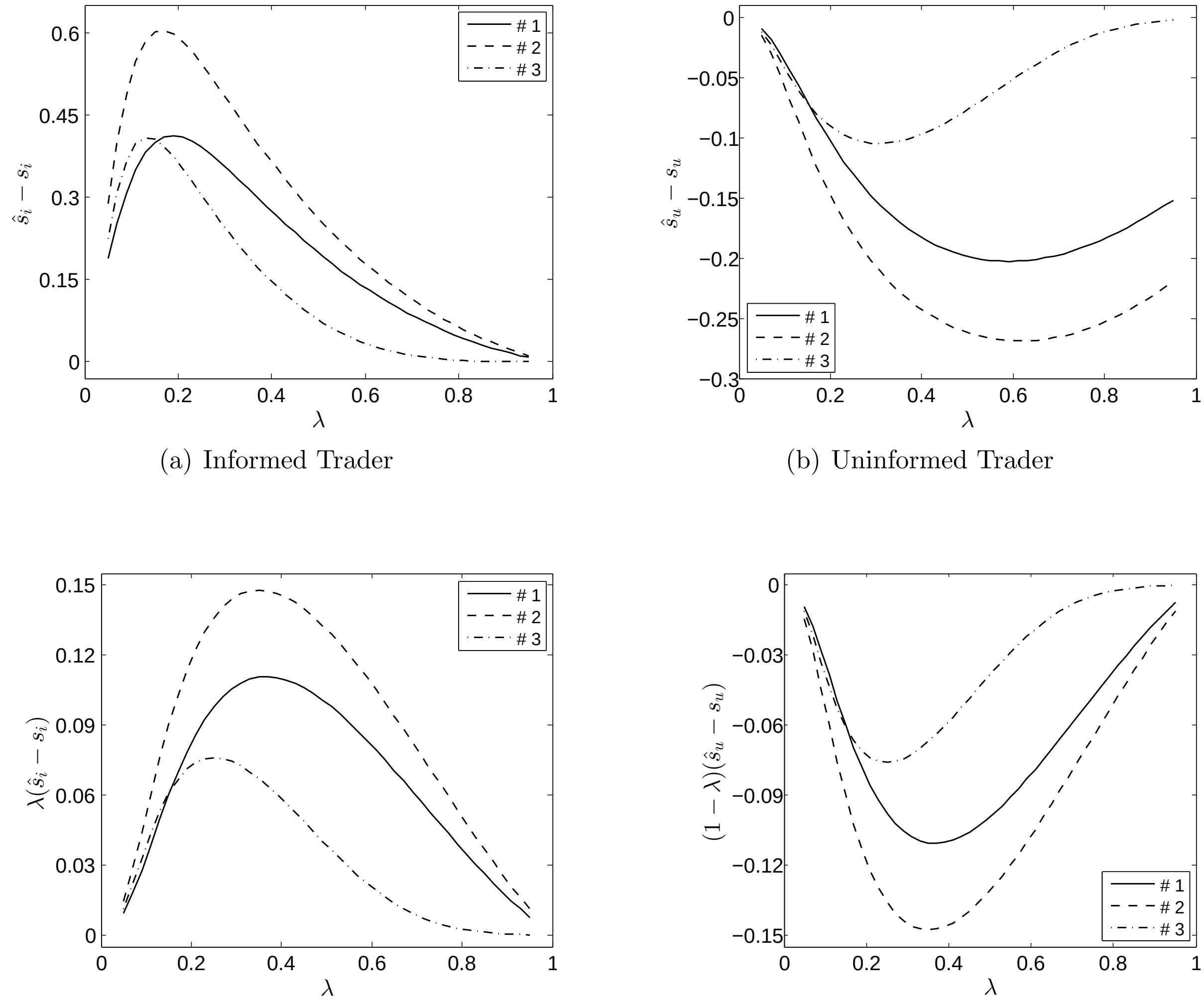
Figure 4: Liquidity. The difference between market liquidity with and without a short-sale prohibition in plotted.



(a) Mean

(b) Variance

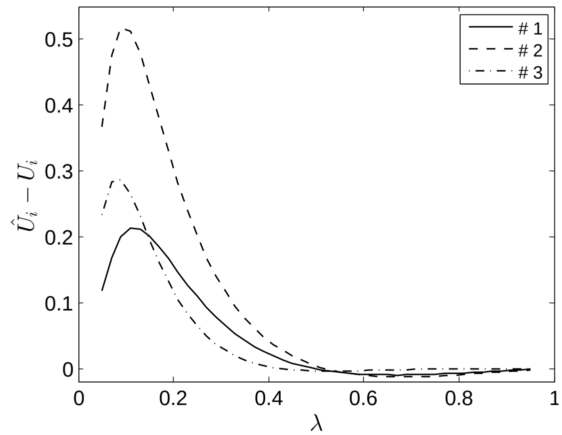
Figure 5: Stock Price. (a) The difference between the average stock price with and without a short-sale prohibition is plotted. (b) The difference between the variance of the stock price with and without a short-sale prohibition is plotted.



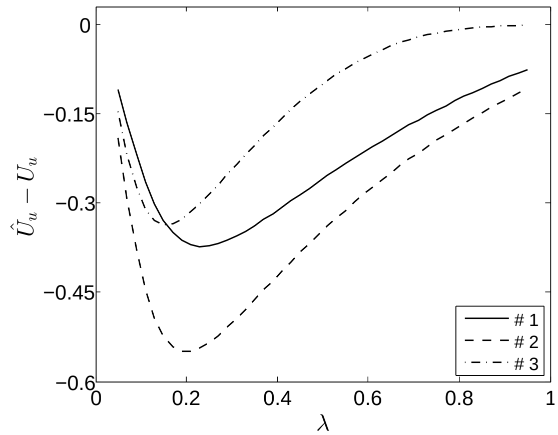
(a) Informed Trader (b) Uninformed Trader

(c) Informed Traders (d) Uninformed Traders

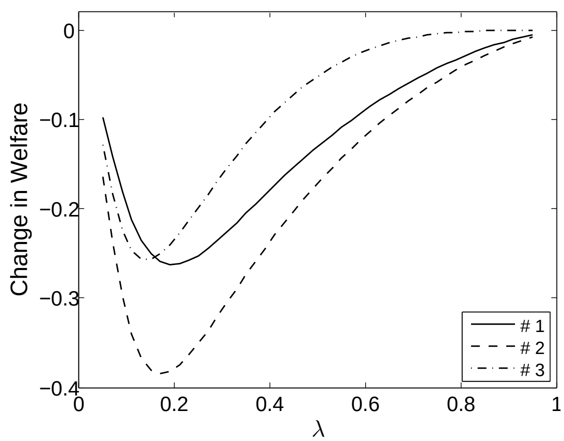
Figure 6: Stock Allocations. (a) The difference between an informed trader's average stock allocation with and without a short-sale prohibition is plotted. (b) The difference between an uninformed trader's average stock allocation with and without a short-sale prohibition is plotted. (c) The difference between the average amount of stock held by all informed traders with and without a short-sale prohibition is plotted. (d) The difference between the average amount of stock held by all uninformed traders with and without a short-sale prohibition is plotted.



(a) Informed Trader

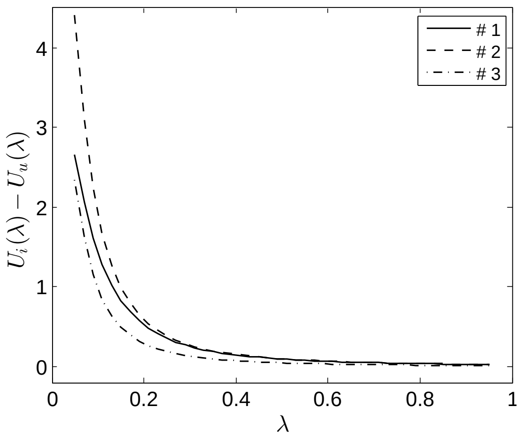


(b) Uninformed Trader

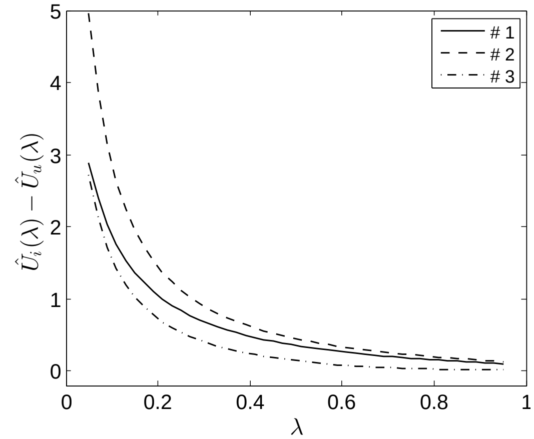


(c) Aggregate Welfare

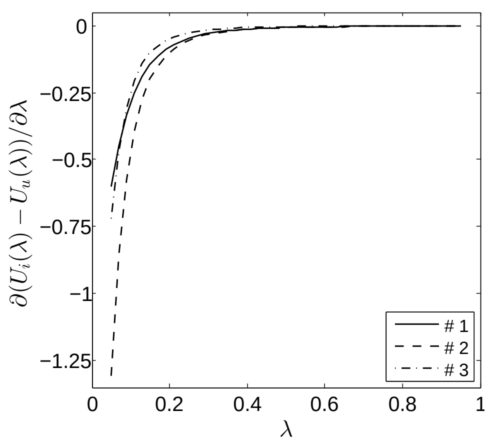
Figure 7: Welfare. (a) The difference between an informed trader's welfare with and without a short-sale prohibition is plotted. (b) The difference between an uninformed trader's welfare with and without a short-sale prohibition is plotted. (c) The difference between the aggregate welfare of informed and uninformed traders with and without a short-sale prohibition is plotted.



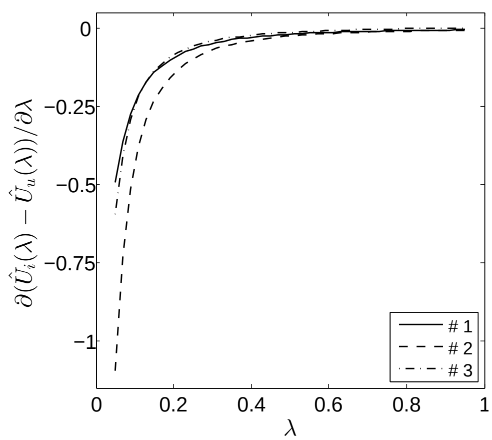
(a) Short Sales Permitted



(b) Short Sales Prohibited



(c) Short Sales Permitted



(d) Short Sales Prohibited

Figure 8: Welfare Differences. (a) The difference between an informed and an uninformed trader's expected utility when short sales are permitted is plotted. (b) The difference between an informed and uninformed trader's expected utility when short sales are prohibited is plotted. (c) The partial derivative of the difference between an informed and an uninformed trader's expected utility with respect to the mass of informed traders when short sales are permitted is plotted. (d) The partial derivative of the difference between an informed and an uninformed trader's expected utility with respect to the mass of informed traders when short sales are prohibited is plotted.

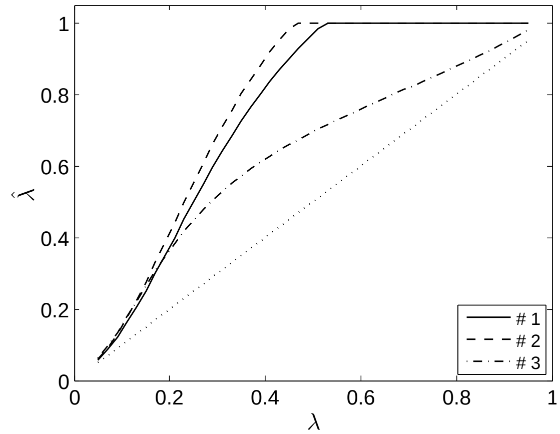
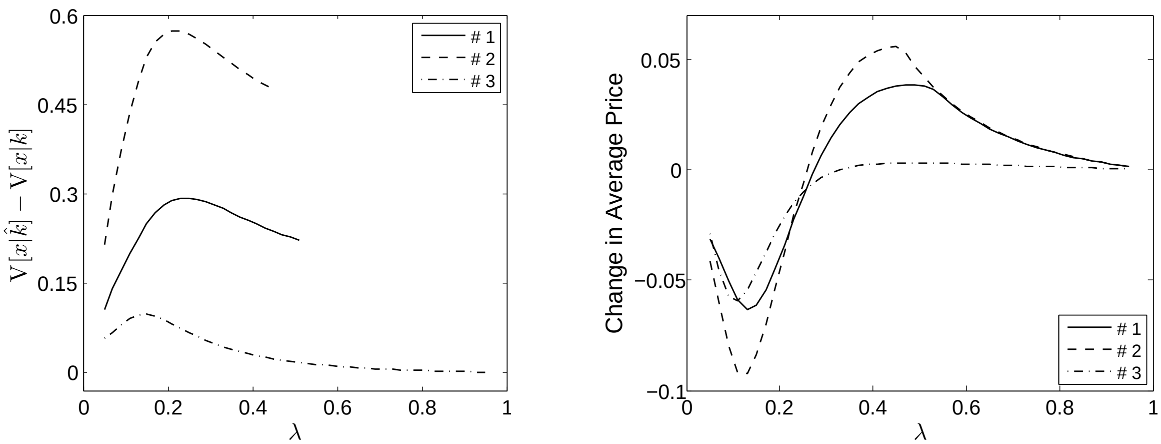


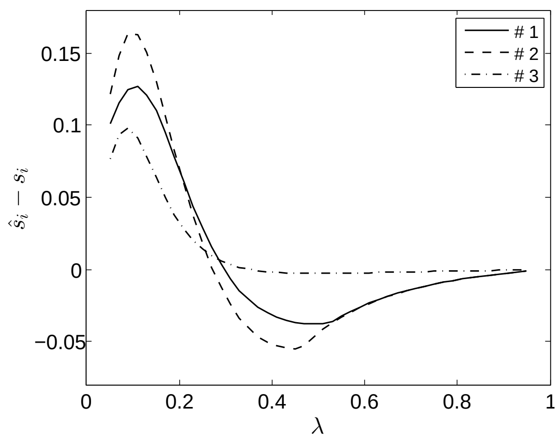
Figure 9: Fraction of Informed Traders. The mass of informed traders when short sales are prohibited is plotted. The dotted line denotes the mass of informed traders when short sales are permitted.



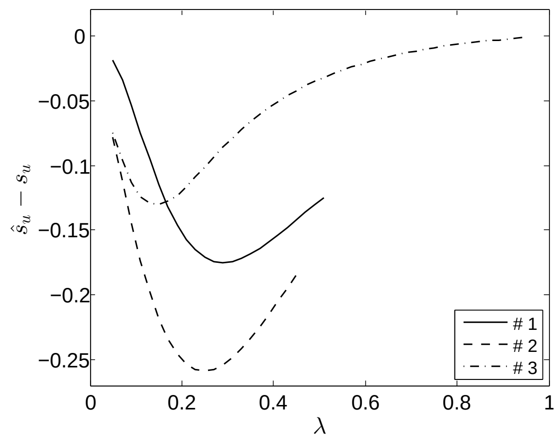
(a) Conditional Variance

(b) Average Stock Price

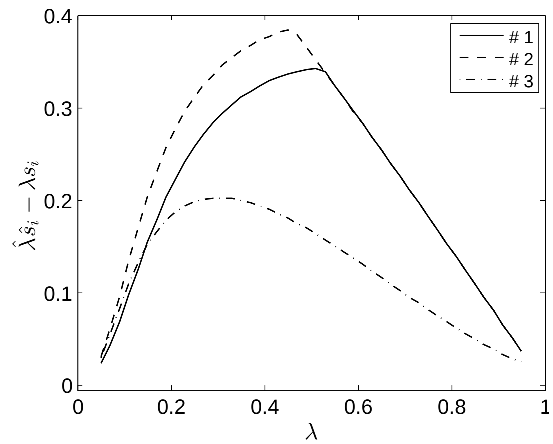
Figure 10: Conditional Variance and Stock Price. (a) The difference between the average conditional variance of x with and without a short-sale prohibition when information acquisition is endogenous is plotted. (b) The difference between the average stock price with and without a short-sale prohibition when information acquisition is endogenous is plotted.



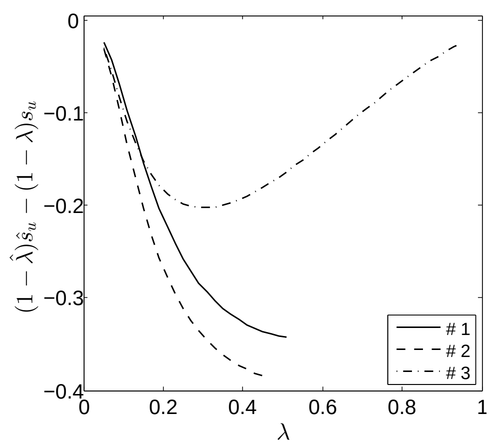
(a) Informed Trader



(b) Uninformed Trader

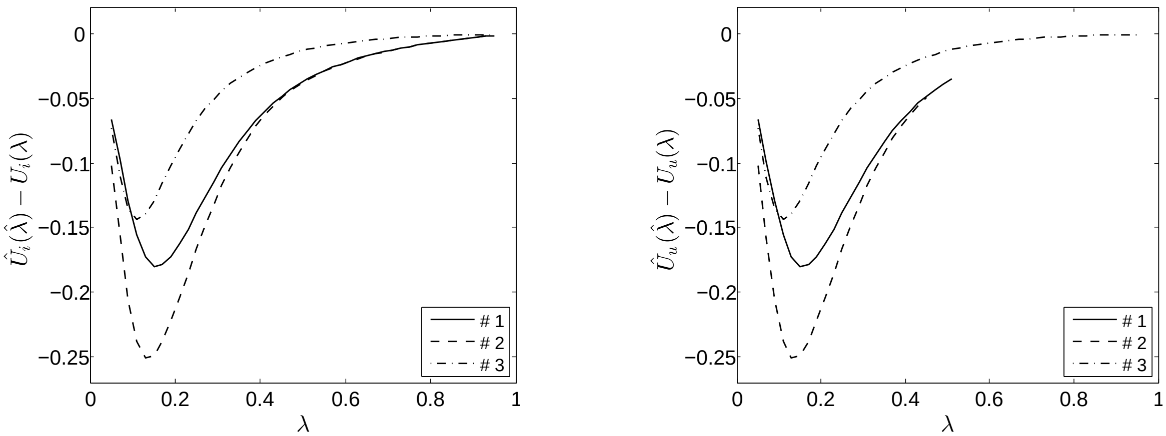


(c) Informed Traders



(d) Uninformed Traders

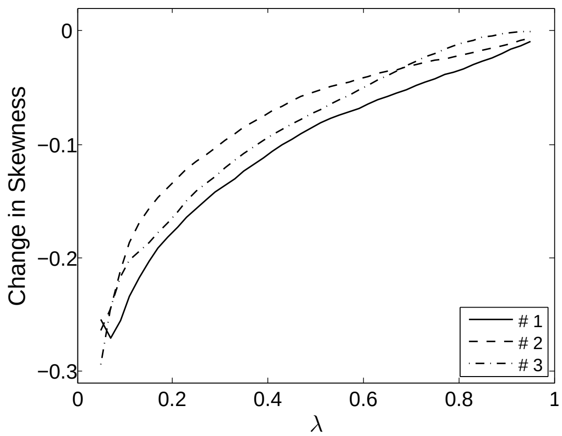
Figure 11: Stock Allocations. (a) The difference between an informed trader's average stock allocation with and without a short-sale prohibition when information acquisition is endogenous is plotted. (b) The difference between an uninformed trader's average stock allocation with and without a short-sale prohibition when information acquisition is endogenous is plotted. (c) The difference between the average amount of stock held by all informed traders with and without a short-sale prohibition when information acquisition is endogenous is plotted. (d) The difference between the average amount of stock held by all uninformed traders with and without a short-sale prohibition when information acquisition is endogenous is plotted.



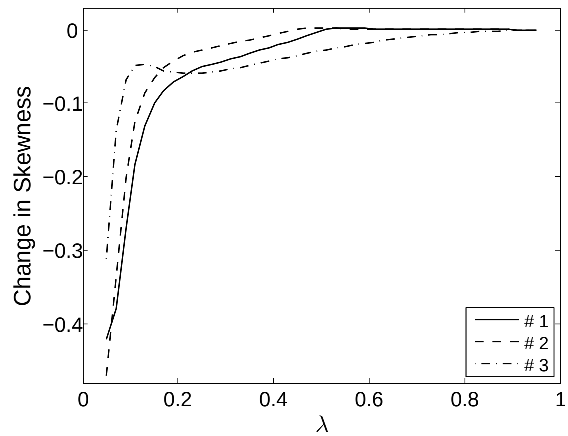
(a) Informed Trader

(b) Uninformed Trader

Figure 12: Welfare. (a) The difference between an informed trader's welfare with and without a short-sale prohibition when information acquisition is endogenous is plotted. (b) The difference between an uninformed trader's welfare with and without a short-sale prohibition when information acquisition is endogenous is plotted.



(a) Exogenous Information Acquisition



(b) Endogenous Information Acquisition

Figure 13: Skewness. (a) The difference between the price skewness with and without a short-sale prohibition when there is a fixed mass of informed traders is plotted. (b) The difference between the price skewness with and without a short-sale prohibition when information acquisition is endogenous is plotted.