

Information Acquisition and Strategic Sequencing in Bilateral Trading: Is Ignorance Bliss?*

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Abstract

This paper examines optimal sequencing of complementary deals with privately known values. It is shown that an informed buyer begins with the high value seller to minimize future holdup. Together, the buyer's sequencing and the sellers' pricing response determine the value of information to the buyer: it is negative for moderate complements and positive for strong complements. That is, for moderate complements the buyer would optimally sequence uninformed even with no information cost, while for strong complements she would seek unlikely deals. By mitigating the holdup, informed sequencing increases trade and thus welfare. Evidence on land assembly supports our findings.

JEL Classifications: C70, D80, L23.

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1 Introduction

Acquiring complementary goods and services often entails dealing with independent sellers. Examples include: a real estate developer buying adjacent parcels from different landowners; an employer recruiting a team of employees; a lobbyist securing bipartisan support; and a vaccine manufacturer obtaining required antigens from patent holders. In many cases, the buyer needs to deal with the sellers one-by-one – perhaps, convening multiple sellers is infeasible, or the sellers fear leaking business plans. Given the complementarity between them,

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careful sequencing of the sellers should therefore be an important bargaining tool for the buyer.¹ Complicating the buyer's strategy, however, is her potential uncertainty about each deal's worth. In this paper, we explore optimal informed sequencing and its value to the buyer. Our main observation is that when sequencing the sellers, ignorance may be bliss for the buyer even though it may reduce trade.

Our base model features one buyer and two sellers of complementary goods. The buyer's joint valuation is commonly known while her stand-alone valuations are private and initially unknown.² The buyer can discover all her valuations at a cost prior to meeting with the sellers. In each meeting, the seller offers a confidential price, which the buyer pays upon acceptance. The buyer's meeting sequence as well as her purchase history are public – perhaps, due to the visibility of such transactions.

Our analysis reveals that equilibrium prices trend upward: ignoring past payments, each seller charges the marginal value of his good, which, given the complementarity, rises. To counter the price surge and improve her bargaining position against future holdup, an informed buyer begins with the high value seller. Together, the buyer's sequencing and the sellers' pricing response determine the value of information for the buyer. For moderate complements, we show that the value of information is *negative*; in particular, the price increase by the leading seller outweighs the benefit of informed sequencing. Hence, even with *costless* information, the buyer would optimally commit to being uninformed (or ignorant) so the sellers would not “read” into her sequence. She might achieve such commitment by: overloading herself with other tasks (Aghion and Tirole, 1997); delegating the sequencing to an uninformed third party; or letting the sellers self-sequence.³ For strong complements, the value of information is positive since the pricing of the leading seller is now favorable to an informed buyer, implying that the buyer would optimally become informed even though she

¹For an interesting discussion and further applications of sequencing in bilateral trading, see Sebenius (1996) and Wheeler (2005).

²In particular, the buyer's stand-alone valuations are assumed to be more uncertain than her joint valuation. For instance, a developer may be less sure about the success of a smaller shopping mall built on a single parcel; a lobbyist may be more worried about a passage of legislation through only one-party endorsement; or a vaccine manufacturer may be more uncertain about the effectiveness of the vaccine that uses only a subset of the required antigens.

³For instance, an employer can assign scheduling of job interviews to an (uninformed) administrative assistant or ask job candidates to pick an interview slot from available ones. In some applications, the buyer's sheer concern for “fairness” may also commit her to random (or uninformed) sequencing, as is the case for judicial recruitments (Greenstein and Sampson 2004, ch.7).

is unlikely to acquire a single item.^{4,5}

In many applications, the buyer may fail to follow her optimal information strategy because it is unobservable to the sellers.⁶ With unobservability, the buyer is unable to influence prices; thus, she seeks information too much for moderate complements (when the pricing effect is negative) and too little for strong complements (when the pricing effect is positive). The suboptimal information acquisition clearly hurts the buyer but it may improve welfare. Note that for complements, (social) efficiency requires a joint purchase, which exposes the buyer to a holdup. By strategic sequencing, an informed buyer is able to mitigate this problem and, in turn, is more likely to purchase the bundle than the uninformed, implying social value to informed sequencing.

For robustness, we consider several extensions pertaining to the bargaining protocol and information structure. Most notably, we show that the buyer may prefer sequential procurement to an auction, in which the sellers make simultaneous price offers. The reason is that while eliminating the holdup problem, the auction encourages both sellers to target the buyer's extra surplus from complementarity. We also show that strategic sequencing substitutes other sources of bargaining power: it is less valuable to a buyer who is likely to make the offers.

There is some evidence in favor of our findings. In land assembly, Fu et al. (2002), Cunningham (2013), and Brooks and Lutz (2013) all estimate a significant premium to assembled parcels. In particular, Cunningham (2013) finds that "parcels toward the center of the development may command a larger premium than those at the edge, suggesting that developers retain or are perceived to retain some design flexibility."⁷ Similarly, Fu et al. (2002) "identify patterns in the sequencing of acquisition among heterogeneous owners that reflect the trade-off of the opportunity cost of not assembling the preferred set of sites vs. exposure to greater hold-out risk." Given strong complementarity in land assembly, our model also predicts an informed sequence from high to low value parcel. In employee recruiting, however, interview order seems unimportant. Kelsky (2015), a career consultant, writes: "[Academic]

⁴To increase demand, the leading seller offers a discount to alleviate the buyer's holdup and on average, does so more for an informed buyer.

⁵The value of information is trivially zero for weak complements (as would be the case for unrelated goods) in our model and thus not the focus of our discussion here.

⁶It is conceivable that a realtor privately researches alternative uses of land parcels; an employer secretly studies resumes (CVs) before setting up job interviews; or a lobbyist privately investigates the long-term political significance of a democratic vs. republican support.

⁷Notable architectural redesigns due to holdouts include Macy's and Rockefeller Center in New York.

Departments come up with a list of workable dates, and then contact candidates more or less randomly...In all of my years on faculty search committees, I never saw a particular position in the order of campus visits yield better or worse outcomes for a candidate.” Consistently, Willihnganz and Meyers (1993) find that interview order had no effect on employment in a large utility company. Nonetheless, job candidates’ interest in the order is consistent with the potential unobservability of who actually sets up interviews.

Related Literature. Our paper relates to a growing literature on optimal negotiation sequence. With two exceptions discussed below, this literature assumes commonly-known valuations, so information acquisition is a nonissue. Marx and Shaffer (2007) show that with contingent price contracts, the buyer strictly prefers to negotiate first with the weaker seller to extract rents from the stronger seller. Xiao (2014) finds the same ordering in a complementary-goods setting with noncontingent cash offers. Li (2010) studies an infinite-horizon bargaining model of complementary goods and establishes that any ordering is sustainable in equilibrium.⁸ A similar indeterminacy is proved by Moresi et al. (2008) in a fairly general model of bilateral negotiations.⁹ Our paper is also related to Noe and Wang (2004) and Krasteva and Yildirim (2012a) who note that the buyer is (weakly) better off conducting negotiations confidentially.

Our paper is closest to Chatterjee and Kim (2005) and Krasteva and Yildirim (2012b). Chatterjee and Kim examine a bargaining model in which the buyer values one item twice as much as the other but the exact valuations are her private information. These authors do not study the value of information, which is at the heart of our investigation. Krasteva and Yildirim explore a similar setting to this paper except that they rule out ex ante information acquisition. Here we consider complementary settings where information cost is not too high and sequencing is purely informational.¹⁰ Nevertheless, we find that even with costless information, the buyer might choose to stay uninformed.

The strategic value of being uninformed has also been indicated in other contexts. For instance, Carrillo and Mariotti (2000) argue that a decision-maker with time-inconsistent preferences may choose to remain ignorant of the state to control future consumption. In

⁸See also Horn and Wolinsky (1988) and Cai (2000) who assume a fixed order of negotiations.

⁹In a labor union-multiple firms framework, Marshall and Merlo (2004) examine “pattern bargaining” where the buyer offers in the second negotiation the contract that is agreed upon in the first negotiation. In their case with non-pattern sequential negotiations, the buyer does not, however, care about the order. See also Banerji (2002).

¹⁰The sequencing in Krasteva and Yildirim (2012b) is driven by the ex ante heterogeneity of the sellers’ bargaining powers.

a principal-agent framework, Riordan (1990), Cremer (1995), Dewatripont and Maskin (1995) and Taylor and Yildirim (2011), among others, show that an uninformed principal may better motivate an agent while Kessler (1998) makes a similar point for the agent who may stay ignorant to obtain a more favorable contract. Perhaps, in this vein, papers closest in spirit to ours are those that incorporate signaling. Among them, Kaya (2010) examines a repeated contracting model without commitment and finds that the principal may delay information acquisition to avoid costly signaling through contracts. In a duopoly setting with role choice, Mailath (1993) and Daughety and Reinganum (1994) show that the choice of production period (as well as production level) may have signaling value and dampen incentives to acquire information. The issue of signaling in our setting is very different from these models, and the value of information critically depends on the prior belief in a non-monotonic way.

The rest of the paper is organized as follows. The next section sets up the base model, followed by the equilibrium characterization with exogenous information in Section 3. Section 4 endogenizes information. We explore several robustness issues in Section 5 and the case of substitutes in Section 6. Section 7 concludes. The proofs of formal results are relegated to an appendix.

2 Base model

A risk-neutral buyer (b) aims to purchase two complementary goods such as adjacent land parcels from two risk-neutral sellers (s_i , $i = 1, 2$). It is commonly known that the buyer's joint value is 1, while her stand-alone value for good i , v_i , is an independent draw from a nondegenerate Bernoulli distribution where $\Pr\{v_i = 0\} = q \in (0, 1)$ and $\Pr\{v_i = \frac{1}{2}\} = 1 - q$. We say that as q increases, goods become stronger complements for the buyer. In particular, with probability q^2 goods are believed to be perfect complements. The outside option of each player is normalized to 0.

The buyer meets with the sellers only once and in the sequence of her choice: $s_1 \rightarrow s_2$ or $s_2 \rightarrow s_1$. Refer to Figure 1. Prior to the meetings, the buyer can privately discover both v_1 and v_2 by paying a fixed cost $c > 0$.¹¹ In each meeting, the buyer receives a confidential price offer p_i and if previously uninformed, privately learns her stand-alone value v_i at no extra cost – perhaps, through free consultation. The offer is “exploding” in that it compels a

¹¹We rule out $c = 0$ in the analysis to avoid a trivial equilibrium multiplicity when the value of information is exactly zero, though some of our key results will hold even for $c = 0$.

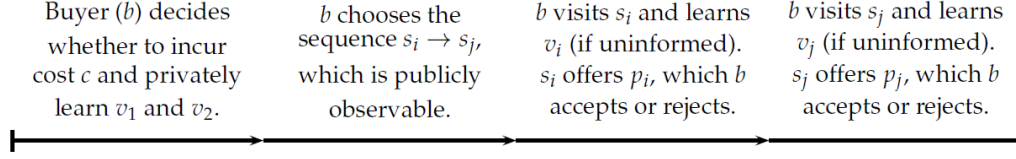


Figure 1: Timing and Information Structure

purchasing decision without visiting the next seller. We assume that the buyer's sequence as well as purchase history are public. Our solution concept is perfect Bayesian equilibrium.

Note that under complements, a joint sale is (socially) efficient. We break indifferences in favor of efficiency (i.e., buying and selling more units) unless it is uniquely determined in equilibrium.

Discussion of the model. Our base model is designed to identify sequencing as the only source of signaling and bargaining power for the buyer.¹² As such, we assume that the sellers are on the short side of the market and make the price offers. For instance, there may be many realtors competing to acquire the adjacent land parcels or many employers trying to recruit among scarce talents. Exploding offers are ubiquitous in labor and real estate markets (e.g., Niederle and Roth, 2009; Armstrong and Zhou, 2011; Lippman and Mamer, 2012). We also assume that the sellers are ex ante identical so that sequencing is trivial if the buyer is uninformed. The fact that the sequence and purchase history are public can be justified by the publicity surrounding the buyer-seller meetings or their timing. Finally, we restrict attention to one-time bilateral interactions. This greatly simplifies the analysis and is reasonable if the buyer has a limited time to undertake the project or an employer is in urgent need of filling vacancies. In Section 5, we check the robustness of our model.

We begin our analysis with exogenous information and then examine information acquisition. Without loss of generality, we re-label the sellers so that seller 1 refers to the first or leading seller in the sequence unless stated otherwise.

3 Informed vs. uninformed sequencing

Suppose that it is commonly known whether the buyer sequences informed (I) or uninformed (U). Given ex ante identical sellers, sequencing is inconsequential for an uninformed

¹²It is readily verified that if stand-alone values were also commonly known, then the buyer would receive the same payoff of 0 regardless of the sequence.

buyer. For an informed buyer, let $\theta_1(v_i, v_{-i})$ be the probability that the first (-place) seller has stand-alone value v_i . To ease the analysis, we assume that equal sellers are treated equally: $\theta_1(v, v) = \frac{1}{2}$, which reduces sequencing decision to choosing $\theta_1(\frac{1}{2}, 0)$. Let $h \in \{0, 1\}$ indicate the buyer's purchase history and $(p_1^z, p_2^z(h))$ denote the corresponding pair of prices where $z = I, U$. Our first result shows that under weak complements, equilibrium prices do not respond to informed sequencing.

Lemma 1 Suppose $q \leq \frac{1}{2}$. In equilibrium, **(a)** $(p_1^z, p_2^z(h)) = (\frac{1}{2}, \frac{1}{2})$ for all z and h , and **(b)** the buyer purchases the bundle with certainty.

If goods were independent, i.e., $q = 0$, each seller would post his monopoly price of $\frac{1}{2}$, inducing a joint purchase irrespective of the buyer's information. Lemma 1 implies that the same applies to weak complements, $q \leq \frac{1}{2}$. Lemma 1 is, however, uninteresting for our purposes as it trivially rules out information acquisition. For $q > \frac{1}{2}$, Proposition 1 characterizes the equilibrium in which prices do respond to the buyer's information and thus the focus of our ensuing analysis.¹³

Proposition 1 Suppose $q > \frac{1}{2}$. In equilibrium, $p_2^U(h = 0) = p_2^I(h = 0) = \frac{1}{2}$. Moreover, for

(a) uninformed buyer:

$$p_1^U = \begin{cases} \frac{1-q}{2} & \text{with prob. } \frac{1-q}{q} \\ \frac{1}{2} & \text{with prob. } \frac{2q-1}{q} \end{cases} \quad \text{and } p_2^U(h = 1) = \begin{cases} \frac{1}{2} & \text{with prob. } 1-q \\ 1 & \text{with prob. } q \end{cases};$$

(b) informed buyer: $p_1^I = p_2^I(h = 1) = \frac{1}{2}$ and $\theta_1(\frac{1}{2}, 0) > \frac{1}{2}$ for $q \leq \frac{1}{\sqrt{2}}$; and

$$p_1^I = \begin{cases} \frac{1-q^2}{2} & \text{with prob. } \frac{1-q^2}{q^2} \\ \frac{1}{2} & \text{with prob. } \frac{2q^2-1}{q^2} \end{cases} \quad \text{and } p_2^I(h = 1) = \begin{cases} \frac{1}{2} & \text{with prob. } 1-q^2 \\ 1 & \text{with prob. } q^2 \end{cases},$$

and $\theta_1(\frac{1}{2}, 0) = 1$ for $q > \frac{1}{\sqrt{2}}$.

(c) Demand: A buyer with $v_1 = 0$ accepts only the low p_1^z but all $p_2^z(h = 1)$ whereas a buyer with $v_1 = \frac{1}{2}$ accepts all p_1^z but only the low $p_2^z(h = 1)$.

¹³For $q > \frac{1}{2}$, there is also a trivial equilibrium such that $(p_1^U, p_2^U(h)) = (p_1^I, p_2^I(h))$. The following is one: regardless of her information, the buyer picks a "favorite" seller to visit first and the sellers offer their uninformed prices, with an off-equilibrium belief that switching the sequence would mean a high-value first seller with certainty and engender the price pair $(\frac{1}{2}, \frac{1}{2})$, leaving no surplus to the buyer.

To understand Proposition 1, notice that with sunk payments, a key strategic concern for the buyer is being held up by the second seller. Consider an uninformed buyer. Upon observing a prior purchase, the second seller optimally charges the buyer's marginal value from the bundle, which is $\frac{1}{2}$ or 1. He must strictly mix between these prices; otherwise, a sure price of 1 would strictly discourage a low value buyer from acquiring the first good and lead the second seller to reduce his price to $\frac{1}{2}$, whereas a sure price of $\frac{1}{2}$ would guarantee the sale of the first good and encourage the second seller to raise his price to 1 given that the prior strictly favors a low value buyer, $q > \frac{1}{2}$. Not surprisingly, seller 2 mixes according to the prior on the first good and thus stochastically increases his price with the probability of a low value buyer, q . Note that a low value buyer demands the first good in the hope of paying less than the full surplus for the second. In particular, in equilibrium, such a buyer expects to pay $\frac{1+q}{2}$ for the second good and is therefore willing to pay $\frac{1-q}{2}$ for the first, which is exactly what seller 1 might offer. Seller 1 might, however, also offer a high price of $\frac{1}{2}$ to target a high value buyer. Seller 1's mixing between these two prices accommodates that of 2's by keeping his posterior "unbiased" at $\frac{1}{2}$. As q increases, seller 1 drops his discount price, $\frac{1-q}{2}$, to (partially) subsidize a low value buyer for the subsequent holdup, but interestingly he also drops the frequency, $\frac{1-q}{q}$, of this enticing offer so that his subsidy is not captured by seller 2.¹⁴ The uninformed prices in part (a) also explain the equilibrium demand in part (c): a low value buyer purchases the first good only at the discount price, upon which she proceeds to purchase the second with certainty, while the opposite is true for a high value buyer.

Note that because each seller prices at the buyer's marginal value, uninformed prices trend upward: the first seller charges no higher and the second seller charges no less than the stand-alone value. Hence, an informed buyer is more likely to sequence the sellers from high to low value. If this sequencing is strict, namely $\theta_1(\frac{1}{2}, 0) = 1$, then the informed buyer has low value for the first good only in the case of perfect complements, occurring with probability q^2 . Substituting this posterior for the prior q in the uninformed prices yields informed prices in part (b) so long as $q > \frac{1}{\sqrt{2}}$. That is, an informed buyer begins with the high value seller if goods are strong complements. For moderate complements, $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$, the informed buyer might mix over the sequence although due to rising prices, she is still strictly more likely to begin with the high value seller, $\theta_1(\frac{1}{2}, 0) \in (\frac{1}{2}, 1]$. Such mixing over the sequence requires equal prices, which can only be at $\frac{1}{2}$.

¹⁴Indeed, with probability $\frac{1-q}{q}q = 1 - q$, a low value buyer acquires both goods but ends up with a loss of $\frac{1-q}{2}$, illustrating the holdup problem.

Inspecting Proposition 1, we can determine the buyer's payoff and identify the two key effects of being informed: sequencing and pricing. Recall that the first seller offers the discount price to entice a low value buyer, leaving her with no expected surplus. This means that despite a joint purchase, a low value buyer incurs a loss if she receives a high price from the second seller. Such holdup does not apply to a high value buyer because she can opt to purchase only the first good. Corollary 1 records this useful observation about the payoffs.

Corollary 1 *A low value buyer of the first good ($v_1 = 0$) obtains an expected payoff of 0 while a high value buyer ($v_1 = \frac{1}{2}$) obtains a positive expected payoff equal to her expected payoff from the first purchase.*

From Corollary 1 and Proposition 1, the expected payoff of an uninformed buyer is found to be

$$\begin{aligned} B^U(q) &= (1-q) \frac{1-q}{q} \left(\frac{1}{2} - \frac{1-q}{2} \right) \\ &= \frac{(1-q)^2}{2} \text{ if } q > \frac{1}{2}, \end{aligned} \quad (1)$$

where $1-q$ is the probability that $v_1 = \frac{1}{2}$ and $\frac{1-q}{q}$ is the probability of the discount price, $\frac{1-q}{2}$, by the first seller. For strong complements, the expected payoff of an informed buyer is analogously found by replacing $1-q$ in (1) with $1-q^2$ – the probability that $v_1 = \frac{1}{2}$ under strategic sequencing. For moderate complements, the expected informed payoff is zero since the first seller targets the high value buyer; hence,

$$B^I(q) = \begin{cases} \frac{(1-q^2)^2}{2} & \text{if } q > \frac{1}{\sqrt{2}} \\ 0 & \text{if } \frac{1}{2} < q \leq \frac{1}{\sqrt{2}}. \end{cases} \quad (2)$$

To identify the two effects of being informed, we also compute a counterfactual payoff for the buyer in which she sequences informed but the sellers are “nonstrategic” in that they keep their uninformed prices. Substituting the probability $1-q^2$ for $1-q$ in the first term of (1), we find the expected informed payoff with nonstrategic sellers:

$$\bar{B}^I(q) = \frac{(1-q^2)(1-q)}{2} \text{ if } q > \frac{1}{2}. \quad (3)$$

Evidently, $\bar{B}^I(q) > B^U(q)$, implying a *positive sequencing effect* of being informed: given uninformed prices, the buyer strictly benefits from the ability to match a high value good

with a low price seller. Moreover, $B^I(q) < \bar{B}^I(q)$ for $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$, and $B^I(q) > \bar{B}^I(q)$ for $q > \frac{1}{\sqrt{2}}$; so the *pricing effect* of being informed is *negative* for moderate complements and *positive* for strong complements. As indicated by Corollary 1, the direction of the pricing effect depends on the first seller. Note from Proposition 1 that the first seller offers an expected price of $\frac{q}{2}$ to an uninformed buyer while he offers a higher price of $\frac{1}{2}$ for moderate complements and a lower expected price of $\frac{q^2}{2}$ for strong complements to an informed buyer. Intuitively, informed sequencing increases the probability that the first seller faces a high value buyer. For moderate complements, this probability is significant enough that the second seller chooses a low price, ruling out a holdup and in turn, inducing aggressive pricing by the first seller. For strong complements, the probability of a high value buyer is less significant and thus the second seller also puts weight on the full – surplus extracting – price of 1, leading the first seller to decrease his average price for a low value buyer. An interesting implication of the pricing effect is that for strong complements, an informed buyer prefers *strategic* sellers who read into her sequencing to those who do not while for moderate complements, she prefers nonstrategic sellers.

From Corollary 1, it is clear that an informed buyer sequences to reduce the risk of holdup by the last seller.¹⁵ We therefore predict that an informed buyer is more likely to purchase the bundle than the uninformed. To confirm, we calculate from Proposition 1 that an uninformed buyer purchases the bundle with probability

$$q \frac{1-q}{q} + (1-q)(1-q) = (1-q)(2-q),$$

whereas an informed buyer purchases the bundle with certainty for moderate complements and with probability $(1-q^2)(2-q^2)$ for strong complements. Since $q^2 < q$, we have

Corollary 2 *An informed buyer is strictly more likely to purchase the bundle than an uninformed buyer.*

Note that both informed and uninformed buyers are less likely to purchase the bundle of stronger complements, i.e., a greater q , due to the increased chance of holdup. Nevertheless, given the complementarity, the buyer should be less inclined to purchase a single good.

Corollary 3 *An informed buyer is strictly more likely to purchase the bundle than a single good. The same is true for an uninformed buyer if and only if $q < \frac{3}{4}$.*

¹⁵This strategy is consistent with the evidence on land assembly alluded to in the introduction.

Corollary 3 follows because unable to sequence optimally, the uninformed buyer guards against the holdup by acquiring only one unit when the holdup is sufficiently likely. Together with Corollary 2, this result points to the social value of information. The value of information to the buyer, however, depends on the sequencing and pricing effects, as we study next.

4 Information acquisition

Before examining information acquisition when it is unobservable to the sellers, we establish two benchmarks, one in which the buyer can publicly commit to visiting the sellers informed or uninformed, and the other in which a social planner dictates such commitment.

Optimal information acquisition. By definition, the buyer's value of information is the difference between her informed and uninformed payoffs: $\Delta(q) \equiv B^I(q) - B^U(q)$. Using (1) and (2), we have

$$\Delta(q) = \begin{cases} \frac{(1-q)^2(q^2+2q)}{2} & \text{if } q > \frac{1}{\sqrt{2}} \\ -\frac{(1-q)^2}{2} & \text{if } \frac{1}{2} < q \leq \frac{1}{\sqrt{2}}. \end{cases} \quad (4)$$

Eq.(4) implies that for moderate complements, the buyer is strictly worse off being informed! As discussed above, informed sequencing causes the first seller to set the high price in this case, leaving no surplus to the buyer. Put differently, for moderate complements, the negative pricing effect of being informed dominates the positive sequencing effect. For strong complements, both effects are positive and so is the value of information, which the buyer weighs against the cost of information, c .

Proposition 2 *If goods are strong complements, $q > \frac{1}{\sqrt{2}}$, and the information cost is low enough, $c < \Delta(q)$, then the buyer optimally acquires information. If, on the other hand, goods are moderate complements, $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$, she optimally stays uninformed.*

Hence, the buyer prefers informed sequencing if and only if goods are strong complements and the information cost is low. Otherwise, even with no information cost, the buyer prefers to sequence uninformed. The buyer can credibly remain uninformed by: (1) significantly raising her own cost, perhaps through overloading with multiple tasks (Aghion and Tirole, 1997); (2) delegating her sequencing decision to an uninformed third party; or (3) letting the sellers self-sequence.

Note that if the sellers were nonstrategic, the value of information would be positive for all $q > \frac{1}{2}$. To see this, we subtract (1) from (3):

$$\bar{\Delta}(q) \equiv \frac{(1-q)^2 q}{2}. \quad (5)$$

Interestingly, $\bar{\Delta}(q) < \Delta(q)$ for $q > \frac{1}{\sqrt{2}}$. That is, for strong complements, the buyer has a greater incentive to be informed when the sellers are strategic and read into her sequence, which simply follows from the positive pricing effect identified above.

Since informed sequencing increases the probability of a joint sale, the buyer's optimal information strategy is unlikely to be (socially) efficient, which we demonstrate next.

Efficient information acquisition. Suppose that a social planner who maximizes the expected welfare can publicly instruct the buyer whether or not to acquire information. Consider an uninformed buyer. From Proposition 1, the expected welfare defined as the expected total surplus is computed to be

$$\begin{aligned} W^U(q) &= q \left[\frac{1-q}{q}(1) + \frac{2q-1}{q}(1-q)\left(\frac{1}{2}\right) \right] + (1-q) \left[q\left(\frac{1}{2}\right) + (1-q)(1) \right] \\ &= \frac{1}{2}(1-q)(3+q). \end{aligned}$$

Similarly, the expected welfare under an informed buyer is $W^I(q) = \frac{1}{2}(1-q^2)(4-q^2)$ if $q > \frac{1}{\sqrt{2}}$, and $W^I(q) = 1$ if $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$ since in the latter case, the bundle is purchased with certainty. Hence, the social value of information is $\Delta^W(q) \equiv W^I(q) - W^U(q)$ or

$$\Delta^W(q) = \begin{cases} \Delta(q) + \frac{1-q^2}{2} & \text{if } q > \frac{1}{\sqrt{2}} \\ \frac{q^2+2q-1}{2} & \text{if } \frac{1}{2} < q \leq \frac{1}{\sqrt{2}}. \end{cases} \quad (6)$$

Comparing (6) with (4), we readily conclude:

Proposition 3 *The social value of information is positive and exceeds its private value to buyer; i.e., $\Delta^W(q) > 0$ and $\Delta^W(q) > \Delta(q)$. Hence, the buyer's optimal information acquisition is less than efficient.*

Given complementarity, welfare is maximized by a joint sale and informed sequencing helps with this objective. A joint sale, however, increases the risk of holdup; to minimize it, the buyer seeks information less often than is efficient.

Armed with these benchmarks, we now turn to the base model in which information acquisition is unobservable to the sellers and thus the buyer cannot commit to being informed or uninformed.

Equilibrium information acquisition. To fix ideas, consider the case of moderate complements for which the buyer would commit to sequencing uninformed. If the sellers believed this to be the buyer's strategy, they would offer their uninformed prices, yielding a positive value of information, $\bar{\Delta}(q)$. In the case of strong complements, the (commitment) value of information, $\Delta(q)$, is positive so information acquisition is likely when unobservable, too. It is, however, less than optimal as Proposition 4 shows. In its statement, let ϕ^* be the buyer's equilibrium probability of being informed.

Proposition 4 *When unobservable to the sellers, the buyer acquires information more (resp. less) frequently than optimal for moderate (resp. strong) complements. Formally, if $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$, then $\phi^* > 0$ for $c < \bar{\Delta}(q)$, and if $q > \frac{1}{\sqrt{2}}$, then $\phi^* < 1$ for $c \in ((1+q)\bar{\Delta}(q), \Delta(q))$.*

The reason behind the suboptimal information acquisition is that when it is unobservable, the buyer cannot control the pricing effect of being informed. As identified in Section 3, the pricing effect is negative for moderate complements and ignoring this, the buyer relies too much on informed sequencing while the opposite holds for strong complements under which the pricing effect is positive.

It is intuitive that by restricting her ability to commit, the unobservability of information acquisition cannot make the buyer better off than her optimal strategy. It may, however, strictly increase the welfare by encouraging informed sequencing for moderate complements. As mentioned above, although the buyer would want to sequence the sellers of moderate complements uninformed, this is not credible. She would not sequence them informed either because the value of information, $\Delta(q)$, is negative in this region, establishing strict mixing in equilibrium for $c < \bar{\Delta}(q)$. The unobservability may also lower the welfare: for strong complements, the buyer acquires information even less frequently than efficient.

5 Robustness

In this section, we consider five robustness issues regarding the bargaining protocol and information structure.

5.1 Sequential procurement vs. auction

Up to now, we have assumed sequential procurement of goods and services. This is natural if, as with job interviews, the buyer has a capacity or privacy concern to deal with both sellers. Absent such concerns, the buyer could alternatively hold an auction in which she receives simultaneous price offers from the sellers and decides which good(s) to acquire after being informed of all prices and valuations. The obvious advantage of an auction over sequential procurement is that the buyer avoids the holdup problem and will incur no ex post loss. Its potential disadvantage is that having no sequence, both sellers are likely to target the buyer's extra surplus from complementarity. Therefore, in the auction, the sellers are expected to coordinate prices to avoid exceeding the buyer's joint valuation, but this makes them less generous in their price discounts. Lemma 2 confirms this conjecture.

Lemma 2 *In the auction, there is a symmetric-price equilibrium, $p_i^A = p_{-i}^A = \frac{1}{2}$, for all q . For $q \geq \frac{\sqrt{5}-1}{2}$, there is also a continuum of asymmetric-price equilibria: $p_i^A \in \left[\frac{1-q}{2}, 1 - \frac{1}{2q}\right]$ and $p_{-i}^A = 1 - p_i^A$.*

The multiplicity of equilibria is not surprising because the sellers play a simultaneous game of price coordination in the auction. In equilibrium, prices sum to the joint valuation of 1, with each being no lower than the discount price, $\frac{1-q}{2}$, offered by the leading seller under uninformed sequencing (see Proposition 1(a)). Note that the buyer purchases the bundle in the symmetric equilibrium but enjoys no surplus – i.e., $B^A(q) = 0$ whereas in the asymmetric equilibria, she receives a positive expected payoff, $B^A(q) \in \left[\frac{(1-q)^2}{2q}, \frac{(1-q)q}{2}\right]$, as she may realize a high value on the lower price item. To understand the buyer's choice between sequential procurement and auction, consider uninformed sequencing. This comparison is the most meaningful because the auction is strategically equivalent to uninformed sequencing except that price offers are “nonexploding” – i.e., all purchases are decided after visiting both sellers.¹⁶ Using (1), it is evident that the buyer will choose sequential procurement if she anticipates symmetric pricing in the auction and Lemma 2 indicates that such an equilibrium always exists. On the other hand, the buyer may also choose an auction if she anticipates asymmetric pricing. Proposition 5 records these observations.

Proposition 5 *For all q , there is an equilibrium in which an uninformed buyer chooses sequential*

¹⁶Moreover, in either setting, the buyer reveals no price information interim and eventually learns all her valuations.

procurement over an auction. This equilibrium is unique if $q \in \left(\frac{1}{2}, \frac{\sqrt{5}-1}{2}\right)$; otherwise, there is also an equilibrium in which she holds an auction.

Hence, the buyer may adopt sequential procurement as assumed in the base model. While the multiplicity of equilibria in the auction prevents a clear prediction of this choice for all q , it is worth noting that the symmetric-price equilibrium maximizes the sellers' joint payoff and is therefore likely to be their "focal point".

Based on the strategic equivalence alluded to above, an alternative interpretation of Proposition 5 is that the buyer may prefer exploding offers to nonexploding offers under sequential procurement. Again, while the former expose the buyer to a holdup, they also compel the first seller to significantly cut price to entice the initial purchase.

5.2 Correlated values

In the base model, we have also assumed that stand-alone values, v_i , are independent. But, in many applications, they may be (positively) correlated.¹⁷ For instance, a developer who is unable to acquire adjacent land parcels for a shopping mall may appraise each similarly for a smaller project. Here we show that correlation reduces the incentive for informed sequencing. To this end, consider the following joint distribution of stand-alone values:

$\Pr(v_1, v_2)$	0	$\frac{1}{2}$
0	$q^2 + rq(1 - q)$	$(1 - r)q(1 - q)$
$\frac{1}{2}$	$(1 - r)q(1 - q)$	$(1 - q)^2 + rq(1 - q)$

where $r \in [0, 1]$ denotes the correlation coefficient, with $r = 0$ and 1 referring to the base model and (ex ante) homogenous goods, respectively.

The equilibrium characterization with correlation closely mimics Proposition 1 (see Proposition A2). In particular, the expected uninformed payoff in (1) remains intact since, as in the base model, the buyer's equilibrium payoff depends on the first deal. The expected informed payoff in (2) is, however, slightly modified by replacing the posterior q^2 with $\Pr(0, 0)$, which implies

$$B^{C,I}(q; r) = \begin{cases} \frac{[1 - \Pr(0, 0)]^2}{2} & \text{if } q > \bar{q}(r) \\ 0 & \text{if } \frac{1}{2} < q \leq \bar{q}(r) \end{cases} \quad (7)$$

where $\bar{q}(r) \geq \frac{1}{2}$ uniquely solves $\Pr(0, 0) = \frac{1}{2}$ such that $\bar{q}'(r) < 0$, $\bar{q}(0) = \frac{1}{\sqrt{2}}$, and $\bar{q}(1) = \frac{1}{2}$. By subtracting (1) from (7), we obtain the value of information under correlation:

¹⁷The argument for negatively correlated goods is symmetric.

$$\Delta^C(q; r) = \begin{cases} \frac{[1 - \Pr(0,0)]^2 - (1-q)^2}{2} & \text{if } q > \bar{q}(r) \\ -\frac{(1-q)^2}{2} & \text{if } \frac{1}{2} < q \leq \bar{q}(r). \end{cases} \quad (8)$$

As expected, $\Delta^C(q; 0) = \Delta(q)$. Moreover, $\Delta^C(q; 1) = 0$. This makes sense because when goods are homogeneous, the buyer's ability to match a high value good with a low price seller under informed sequencing is inconsequential. More generally, informed sequencing becomes less consequential when goods are more correlated and thus ex ante less heterogeneous: formally, $\Delta^C(q; r)$ is strictly decreasing in r for $q > \bar{q}(r)$. It is, however, worth noting that since $\Pr(0, 0)$ is increasing in r , $\bar{q}'(r) < 0$; that is, correlation reduces the incentive to remain uninformed by increasing the likelihood of perfect complements.

5.3 Partial information

In the base model, the buyer can discover both valuations ex ante by paying a fixed cost; that is, information is all-or-nothing. If the marginal cost of information is significant, however, the buyer may choose to learn only one valuation. We argue that the buyer is unlikely to gain from such "partial" information. Suppose that prior to meeting with the sellers, the buyer privately discovers only v_i . If she approaches seller i second, then she engenders the uninformed equilibrium described in Proposition 1 and obtains a positive expected payoff for $q > \frac{1}{2}$. If, instead, the buyer approaches seller i first, she receives an expected payoff of 0 irrespective of v_i . For a low value buyer, this follows from Corollary 1. For a high value buyer, this follows because seller i would infer the buyer's valuation from sequencing and charge a sure price of $\frac{1}{2}$, leaving no surplus to the buyer. We therefore obtain Proposition 6.

Proposition 6 *Suppose $q > \frac{1}{2}$ and that the buyer is privately informed of v_i only. Then, she optimally sequences seller i second and receives her uninformed payoff in (1).*

Proposition 6 justifies our focus on all-or-nothing information. Intuitively, the buyer cannot exploit partial information as it leaks through her sequencing; to avoid this, the buyer begins with the seller of the uncertain good, effectively committing to behaving uninformed. This contrasts with a fully informed buyer whose sequencing leaves a significant probability that the first seller has a low value item.

5.4 Seller's vs. buyer's market

To identify (informed) sequencing as a source of bargaining power for the buyer, we have also assumed that sellers make the price offers – i.e., each operates in a seller's market. We predict that the buyer will value sequencing less if she expects a buyer's market. To confirm, let $m_i \in \{s_i, b\}$ denote the state of market i , which favors either seller i or the buyer as the price-setter. We assume that sellers already know their respective market conditions but the buyer needs to find out.¹⁸ Specifically, the buyer is assumed to learn m_1 and m_2 at an interim stage between information acquisition and meeting with the sellers.¹⁹ Letting $\Pr(m_1, m_2)$ be the joint probability distribution over the states of the markets, the following proposition shows that the buyer discounts the value of information by the likelihood of facing a seller's market in each meeting.

Proposition 7 *In the setting just described, the value of information to the buyer is $\bar{\Delta}^m(q) = \Pr(s_1, s_2)\Delta(q)$.*

Intuitively, if both markets turn in the buyer's favor, there is no value to informed sequencing because the buyer, informed or uninformed, offers 0 to each seller and secures the highest payoff of 1. The equivalence of informed and uninformed payoffs is also true if only one market turns in the buyer's favor. The reason is that an uninformed buyer optimally begins with the buyer's market in order to improve her outside option in the seller's market. Such sequencing, however, provides her with the same amount of information as informed sequencing, engendering the same reservation payoff and price offer in the seller's market.²⁰ Hence, the buyer cares about informed sequencing to the extent that she anticipates sellers' markets. Put differently, the buyer views strategic sequencing as a substitute to other sources of bargaining power – it is most valuable for the buyer with the least bargaining power.

5.5 Uncertain joint value

In the base model, we have also maintained that the buyer's joint value is commonly known. This fits well applications where the buyer has a large winning project: e.g., a retail chain opening a large enough store that ensures monopolizing a local market; a vaccine company

¹⁸In the real estate market, the buyer can discover the market condition from the stock of listings or expert opinions while in the labor market, the employer can ascertain it from the initial screening of candidates, anticipating that peer institutions receive similar applications.

¹⁹Though convenient, this timing of events is not crucial. Our conclusion in Proposition 7 would not change if the buyer learned m_1 and m_2 before her information decision.

²⁰Recall that an uninformed buyer learns her valuations as she meets with the sellers.

acquiring all the necessary antigens to guarantee an effective vaccine; and a lobbyist seeking bipartisan support that secures the favorable legislation. In other applications, the buyer's joint value may be uncertain – at least initially. For instance, an academic department may be unsure of the synergy level between faculty candidates. Here we show that consistent with the base model, the buyer has an incentive to learn her joint value to avoid future holdup. To distinguish it from the information incentive due to sequencing, suppose that stand-alone values are equal and commonly known, $v_1 = v_2 = v \in [0, \frac{1}{2}]$. The joint value, however, is uncertain: $V = 1$ or $\bar{V} > 1$ where $\Pr\{V = \bar{V}\} = \alpha \in (0, 1)$. As in the base model, the buyer can privately discover V at a cost prior to meeting with the sellers or wait until she meets with both (so the second purchase is always informed). Proposition 8 characterizes the value of information in this setting.

Proposition 8 *Consider the setting with an uncertain joint value as described above. In equilibrium, the buyer's uninformed payoff is $B^{J,U}(\alpha) = 0$ whereas her informed payoff and thus her value of information is*

$$\Delta^J(\alpha) = B^{J,I}(\alpha) = \begin{cases} \alpha(\bar{V} - 1) & \text{if } \alpha \leq \frac{v}{v+\bar{V}-1} \\ \frac{v}{1-v}\alpha(\bar{V} - 1) & \text{if } \frac{v}{v+\bar{V}-1} < \alpha \leq \frac{1-v}{\bar{V}-v} \\ 0 & \text{if } \alpha > \frac{1-v}{\bar{V}-v} \end{cases} .$$

An uninformed buyer receives no expected surplus because the first seller offers the highest price acceptable in expectation. This means that an uninformed buyer may realize a loss after the second purchase if her joint value turns out to be low. To minimize such holdup, the buyer therefore has an incentive to approach the sellers informed. An informed purchase from the first seller, however, leads the second seller to be more optimistic about a high joint value and raise price, fully extracting the buyer's surplus when a high joint value is sufficiently likely, i.e., $\alpha > \frac{1-v}{\bar{V}-v}$. Similar to the base model, Proposition 8 indicates that a negative pricing effect may completely outweigh the benefit of informed purchases. Moreover, $\Delta^J(\alpha)$ and $\Delta(q)$ together imply that the buyer is likely to discover her stand-alone values for strong complements and her joint value for moderate complements.

6 Substitutes

Sequential procurement and thus the issues of information acquisition and sequencing can also be pertinent to substitutes – e.g., parcels at rival locations and job candidates with comparable skills. We, however, argue that with substitutes, sequential procurement is undesirable for the buyer as it forecloses competition between the sellers; instead, the buyer is likely to hold an auction with simultaneous price offers. To make the point, let the buyer’s joint value be 1 (as in the base model) but her stand-alone values be independently distributed such that $\Pr\{v_i = 1\} = q_u$ and $\Pr\{v_i = \frac{1}{2}\} = 1 - q_u$. Clearly, with probability q_u^2 , goods are perfect substitutes whereas with probability $(1 - q_u)^2$, they are independent. We assume $q_u > \frac{1}{2}$ so that perfect substitutes are more likely.²¹

Proposition 9 *Consider substitute goods with $q_u > \frac{1}{2}$. Then, an uninformed buyer strictly prefers auction to sequential procurement.*

Proposition 9 is easily understood for (almost) perfect substitutes, $q_u \approx 1$. Unsurprisingly, the auction engenders the most competitive prices of 0 and in turn, the highest expected payoff of 1 for the buyer. In contrast, sequential procurement results in the monopoly prices of 1 and yields the lowest payoff of 0. The latter follows because with no previous purchase, the last seller sets his monopoly price and anticipating this, so does the first seller, leaving no surplus to the buyer. The buyer continues to receive monopoly prices under sequential procurement for imperfect substitutes, $q_u > \frac{1}{2}$, but due to competition, lower prices are likely in the auction. In particular, the proof of Proposition 9 establishes that there is no pure strategy equilibrium in the auction: the sellers trade off pricing for perfect substitutes and pricing for independent units.

7 Conclusion

In this paper, we have explored the optimal sequencing of complementary negotiations with privately known values. Our analysis has produced three main observations. First, an informed buyer begins with the high value seller to mitigate future holdup. Second, because of the sellers’ pricing response, the buyer may be strictly worse off with informed sequencing; that is, ignorance may be bliss. And third, the buyer underinvests in information from

²¹Again, the comparison is for an uninformed buyer because as mentioned in Section 5.1, the auction is strategically equivalent to uninformed sequencing except that price offers are nonexploding.

a social stand point: informed sequencing increases the likelihood of an efficient (joint) purchase but also the risk of holdup. As mentioned in the Introduction, the empirical evidence on land assembly corroborates our first observation in that real estate developers are estimated to assemble land parcels with a flexible design in mind to dissuade possible holdouts. On the other hand, the evidence on labor market supports our second observation in that job candidates are often advised by career consultants against reading into the interview sequence. Nonetheless, job candidates' interest in the sequence is consistent with their potential uncertainty about who actually schedules interviews.

Appendix A

As in the text, we re-label the sellers so that the sequence is $s_1 \rightarrow s_2$ unless stated otherwise. For future reference, Proposition A1 characterizes the equilibrium with the following information structure: the buyer privately knows $z \in \{I, U\}$ but the sellers commonly believe that $\Pr\{z = I\} = \phi \in [0, 1]$. Conditional on this information structure, let $q_1(\phi) = \Pr\{v_1 = 0|\phi\}$ be the posterior belief that s_1 is of low value.

Proposition A1. *In equilibrium, $p_2(h = 0) = \frac{1}{2}$ and*

(a) *if $q_1(\phi) < \frac{1}{2}$, then $p_1 = p_2(h = 1) = \frac{1}{2}$ and the buyer purchases the bundle with certainty;*

(b) *if $q_1(\phi) = \frac{1}{2}$, then $p_1 = \frac{\beta}{2}$ and $p_2(h = 1) = \begin{cases} \frac{1}{2} & \text{with prob. } \beta \\ 1 & \text{with prob. } 1 - \beta \end{cases}$,*

where $\beta \geq \frac{1}{2}$. The buyer purchases from s_1 with certainty and s_2 only if $v_1 = 0$ or $p_2(h = 1) = \frac{1}{2}$;

(c) *if $q_1(\phi) > \frac{1}{2}$, then*

$$p_1 = \begin{cases} \frac{1-q_1(\phi)}{2} & \text{with prob. } \frac{1-q_1(\phi)}{q_1(\phi)} \\ \frac{1}{2} & \text{with prob. } \frac{2q_1(\phi)-1}{q_1(\phi)} \end{cases} \quad \text{and } p_2(h = 1) = \begin{cases} \frac{1}{2} & \text{with prob. } 1 - q_1(\phi) \\ 1 & \text{with prob. } q_1(\phi) \end{cases}.$$

Moreover, a buyer with $v_1 = 0$ accepts only low p_1 but all $p_2(h = 1)$ whereas a buyer with $v_1 = \frac{1}{2}$ accepts all p_1 but only the low $p_2(h = 1)$.

Proof. Consider pricing by s_2 . Clearly $p_2(h = 0) = \frac{1}{2}$ since s_2 realizes a positive payoff only if $v_2 = \frac{1}{2}$. Let $h = 1$. Then a buyer with $v_1 = 0$ accepts any offer $p_2(h = 1) \leq 1$ whereas a buyer with $v_1 = \frac{1}{2}$ accepts only $p_2(h = 1) \leq \frac{1}{2}$. Thus, s_2 's optimal price is

$$p_2(h=1) = \begin{cases} \frac{1}{2} & \text{if } \hat{q}_1(\phi, 1) \leq \frac{1}{2} \\ 1 & \text{if } \hat{q}_1(\phi, 1) \geq \frac{1}{2}, \end{cases} \quad (\text{A-1})$$

where $\hat{q}_1(\phi, h) = \Pr\{v_1 = 0 | \phi, h\}$ is the posterior conditional on the buyer's information and purchase history.

Anticipating $p_2(h=1)$, a buyer with v_1 is willing to pay s_1 up to $\bar{p}_1(v_1)$ such that

$$\max\{1 - \bar{p}_1(v_1) - p_2(h=1), v_1 - \bar{p}_1(v_1)\} = 0,$$

or simplifying,

$$\bar{p}_1(v_1) = \max\{1 - p_2(h=1), v_1\}. \quad (\text{A-2})$$

Next we show that $\hat{q}_1(\phi, 1) \leq \frac{1}{2}$ in equilibrium. Suppose, to the contrary, that $\hat{q}_1(\phi, 1) > \frac{1}{2}$. Then $p_2(h=1) = 1$, $\bar{p}_1(v_1=0) = 0$, and $\bar{p}_1(v_1=\frac{1}{2}) = \frac{1}{2}$. But this would imply $p_1 = \frac{1}{2}$ and in turn $\hat{q}_1(\phi, 1) = 0$ – a contradiction. We exhaust two possibilities for $\hat{q}_1(\phi, 1)$.

$\hat{q}_1(\phi, 1) < \frac{1}{2}$: Then $p_2(h=1) = \frac{1}{2}$ from (A-1), and $\bar{p}_1(v_1=0) = \bar{p}_1(v_1=\frac{1}{2}) = \frac{1}{2}$ from (A-2). This implies $\hat{q}_1(\phi, h) = q_1(\phi)$ and thus $q_1(\phi) < \frac{1}{2}$, which reveals that the buyer purchases the bundle with certainty, proving part (a).

$\hat{q}_1(\phi, 1) = \frac{1}{2}$: By (A-1), s_2 is indifferent between the prices $\frac{1}{2}$ and 1. Suppose s_2 offers $\frac{1}{2}$ with probability β . Then, by (A-2), $\bar{p}_1(v_1=\frac{1}{2}) = \frac{1}{2}$ and $\bar{p}_1(v_1=0) = \frac{\beta}{2}$. Let s_1 mix between the prices $\frac{1}{2}$ and $\frac{\beta}{2}$ by offering the latter with probability $\gamma \in [0, 1]$. Evidently, the buyer always accepts $\frac{\beta}{2}$ whereas only the buyer with $v_1 = \frac{1}{2}$ accepts $\frac{1}{2}$. Using Bayes' rule, we therefore have $\hat{q}_1(\phi, 1) = \frac{\gamma q_1(\phi)}{\gamma q_1(\phi) + 1 - q_1(\phi)}$, which, given $\hat{q}_1(\phi, 1) = \frac{1}{2}$, implies $\gamma = \frac{1 - q_1(\phi)}{q_1(\phi)}$. For $q_1(\phi) = \frac{1}{2}$, $\gamma = 1$ or $p_1 = \frac{\beta}{2}$. By the buyer's optimal purchasing decision, this means $\frac{\beta}{2}(1) \geq \frac{1}{2}(1 - q_1(\phi))$ or equivalently $\beta \geq \frac{1}{2}$, resulting in the equilibrium multiplicity in part (b). Finally, for $q_1(\phi) > \frac{1}{2}$, $\gamma \in (0, 1)$. Such strict mixing by s_1 requires $\frac{\beta}{2} = \frac{1 - q_1(\phi)}{2}$ or $\beta = 1 - q_1(\phi)$, proving part (c). ■

Proof of Lemma 1. Suppose $q \leq \frac{1}{2}$. For $z = U$, $\phi = 0$ and $q_1(0) = q$. For $z = I$ or $\phi = 1$, it must be that $q_1(1) \leq \frac{1}{2}$; otherwise, if $q_1(1) > \frac{1}{2}$, equilibrium prices in Proposition A1 would imply an informed sequence strictly from high to low value – i.e., $\theta_1(\frac{1}{2}, 0) = 1$, and in turn, $q_1(1) = q^2 < \frac{1}{2}$ – a contradiction. Given that $q_1(0) \leq \frac{1}{2}$ and $q_1(1) \leq \frac{1}{2}$, Proposition A1 further reveals that $(p_1^z, p_2^z(h)) = (\frac{1}{2}, \frac{1}{2})$ for $z = I, U$ and $h = 0, 1$, inducing a joint purchase, where the sellers' indifference at $q_1(\phi) = \frac{1}{2}$ is broken in favor of efficient pricing. ■

Proof of Proposition 1. Suppose $q > \frac{1}{2}$. For $z = U$, parts (a) and (c) are immediate from Proposition A1 since $q_1(0) = q$. Next, consider $z = I$. If $q > \frac{1}{\sqrt{2}}$, the proof of Lemma 1 has

established that $\theta_1(\frac{1}{2}, 0) = 1$ and $q_1(1) = q^2 > \frac{1}{2}$. Therefore, $q_1(1) \leq \frac{1}{2}$ if $q \in (\frac{1}{2}, \frac{1}{\sqrt{2}}]$, where sellers break indifference in favor of efficient pricing at $q_1(1) = \frac{1}{2}$. This implies $\theta_1(\frac{1}{2}, 0) > \frac{1}{2}$ given that by Bayesian updating,

$$\begin{aligned} q_1(1) &= \frac{q^2 \frac{1}{2} + q(1-q)[1 - \theta_1(\frac{1}{2}, 0)]}{\frac{1}{2}} \\ &= q^2 + 2q(1-q)[1 - \theta_1(\frac{1}{2}, 0)]. \end{aligned} \quad (\text{A-3})$$

Applying Proposition A1, we obtain parts (b) and (c) for $z = I$. ■

Proof of Corollary 1. Obvious from Proposition 1. ■

Proof of Corollary 2. Directly follows from the probability of a joint purchase found in the text. ■

Proof of Corollary 3. From Proposition 1, an informed buyer obtains the bundle of moderate complements with probability 1. She obtains a single unit of strong complements if and only if she has at least one high value and receives a high second price, whose probability is $(1 - q^2)q^2$ and strictly less than $(1 - q^2)(2 - q^2)$. The probability of a single purchase by an uninformed buyer is

$$(1 - q)q + q \frac{2q - 1}{q} (1 - q) = (1 - q)(3q - 1),$$

where the first term is the probability that $v_1 = \frac{1}{2}$ and $p_2^U(h = 1) = 1$, while the second term is the probability of rejecting $p_1^U = \frac{1}{2}$ due to $v_1 = 0$ and accepting $p_2^U(h = 0)$ due to $v_2 = \frac{1}{2}$. Since the probability of a joint purchase is $(1 - q)(2 - q)$, the buyer is more likely to purchase both if and only if $q < \frac{3}{4}$. ■

Proof of Proposition 2. Directly follows from (4). ■

Proof of Proposition 3. Directly follows from (4) and (6). ■

Proof of Proposition 4. Consider first $q > \frac{1}{\sqrt{2}}$. Then $\theta_1(\frac{1}{2}, 0) = 1$ and $q_1(1) = q^2$ (by (A-3)), which imply $q_1(\phi) = \phi q_1(1) + (1 - \phi)q \geq \phi q^2 + (1 - \phi)q > \frac{1}{2}$. Moreover, the value of information under unobservable acquisition is $\hat{\Delta}(\phi) = q(1 - q) \frac{1 - q_1(\phi)}{2}$. Comparing with (4), $\hat{\Delta}(\phi) \leq \Delta(q)$ for all ϕ . To determine when $\phi^* < 1$, note that for $c < \Delta(q)$, it is optimal for the buyer to acquire information. With unobservability, however, $\phi^* = 1$ requires $c \leq \hat{\Delta}(1) = q(1 - q) \frac{1 - q^2}{2} = (1 + q)\bar{\Delta}(q)$. Therefore, $\phi^* < 1$ for $(1 + q)\bar{\Delta}(q) < c < \Delta(q)$, as claimed. Next, consider $\frac{1}{2} < q \leq \frac{1}{\sqrt{2}}$. For $\phi^* = 0$, $q_1(0) = q$ and $\hat{\Delta}(0) = q(1 - q) \frac{1 - q}{2} = \bar{\Delta}(q)$. Hence, $\phi^* > 0$ for $c < \bar{\Delta}(q)$, as desired. ■

Proof of Lemma 2. In an auction, the sellers play a simultaneous-move pricing game and thus the equilibrium occurs at the intersection of their best responses. Consider seller i 's best response $P_i(p_{-i})$ to price p_{-i} by the other seller. Note that if $p_{-i} \leq \frac{1}{2}$, then $p_i = 1 - p_{-i}$ generates a sale for s_i only if $v_{-i} = 0$, while $p_i = \frac{1}{2}$ guarantees a sale. Comparing s_i 's resulting payoffs, $(1 - p_{-i})q$ and $\frac{1}{2}$, it follows that $P_i(p_{-i}) = 1 - p_{-i}$ if $p_{-i} \leq 1 - \frac{1}{2q}$, and $P_i(p_{-i}) = \frac{1}{2}$ if $1 - \frac{1}{2q} \leq p_{-i} \leq \frac{1}{2}$. If, on the other hand, $p_{-i} > \frac{1}{2}$, then since $v_{-i} \leq \frac{1}{2}$, seller s_{-i} realizes a sale only if the buyer acquires the bundle. Given this, the price $p_i = 1 - p_{-i}$ ensures a sale for s_i whereas $p_i = \frac{1}{2}$ is accepted only if $v_i = \frac{1}{2}$, leading to the respective payoffs: $1 - p_{-i}$ and $(1 - q)\frac{1}{2}$. From here, $P_i(p_{-i}) = 1 - p_{-i}$ if $\frac{1}{2} < p_{-i} \leq \frac{1+q}{2}$, and $P_i(p_{-i}) = \frac{1}{2}$ if $p_{-i} \geq \frac{1+q}{2}$. To sum up,

$$P_i(p_{-i}) = \begin{cases} 1 - p_{-i} & \text{if } 0 \leq p_{-i} \leq 1 - \frac{1}{2q} \\ \frac{1}{2} & \text{if } 1 - \frac{1}{2q} \leq p_{-i} \leq \frac{1}{2} \\ 1 - p_{-i} & \text{if } \frac{1}{2} \leq p_{-i} \leq \frac{1+q}{2} \\ \frac{1}{2} & \text{if } p_{-i} \geq \frac{1+q}{2}. \end{cases} \quad (\text{A-4})$$

In equilibrium, $P_i(P_{-i}(p_i^A)) = p_i^A$ for all i , which, given (A-4), implies that $p_i^A + p_{-i}^A = 1$. Clearly, $p_i^A = p_{-i}^A = \frac{1}{2}$ satisfies this condition for all q . Moreover, the only asymmetric prices that satisfy this condition are: $p_i^A \in [\frac{1-q}{2}, 1 - \frac{1}{2q}]$ and $p_{-i}^A = 1 - p_i^A \in [\frac{1}{2q}, \frac{1+q}{2}]$. The interval for p_i^A is nonempty if and only if $q \geq \frac{\sqrt{5}-1}{2}$. ■

Proof of Proposition 5. By Lemma 2, $p_i^A = p_{-i}^A = \frac{1}{2}$ is an equilibrium for all q when the buyer holds an auction, resulting in the expected payoff $B^A = 0$. Under sequential procurement with an uninformed buyer, $B^U = 0$ for $q \leq \frac{1}{2}$ (by Lemma 1) and $B^U = \frac{(1-q)^2}{2}$ for $q > \frac{1}{2}$ (by (1)). Therefore, there is an equilibrium, in which the buyer chooses sequential procurement with an off-equilibrium belief that symmetric pricing would occur under the auction. For $q \in (\frac{1}{2}, \frac{\sqrt{5}-1}{2})$, $B^A = 0$ is the unique equilibrium payoff and $B^U > B^A$, indicating a unique equilibrium with sequential procurement. For $q \geq \frac{\sqrt{5}-1}{2}$, given the equilibrium pricing in Lemma 2, $B^A \in [\frac{(1-q)^2}{2q}, \frac{(1-q)q}{2}] \cup \{0\}$. Note that $B^U < \frac{(1-q)^2}{2q}$, implying that in this region there is also an equilibrium in which the buyer chooses to hold an auction and the sellers charge asymmetric prices $p_i^A \in [\frac{1-q}{2}, 1 - \frac{1}{2q}]$ and $p_{-i}^A = 1 - p_i^A$. ■

Proposition A2. (Informed prices with correlation) As defined in Section 5.2, let $\bar{q}(r)$ be the unique solution to $\Pr(0,0) = \frac{1}{2}$, where $\Pr(0,0) = q^2 + rq(1-q)$. In equilibrium, $p_1^I = p_2^I(h = 1) = \frac{1}{2}$ for $q \leq \bar{q}(r)$ with $\theta_1(\frac{1}{2}, 0) > \frac{1}{2}$ for $q > \frac{1}{2}$; and

$$p_1^I = \begin{cases} \frac{1-\Pr(0,0)}{2} & \text{with prob. } \frac{1-\Pr(0,0)}{\Pr(0,0)} \\ \frac{1}{2} & \text{with prob. } \frac{2\Pr(0,0)-1}{\Pr(0,0)} \end{cases} \quad \text{and } p_2^I(h=1) = \begin{cases} \frac{1}{2} & \text{with prob. } 1 - \Pr(0,0) \\ 1 & \text{with prob. } \Pr(0,0) \end{cases}$$

and $\theta_1(\frac{1}{2}, 0) = 1$ for $q > \bar{q}(r)$.

Proof. Using the joint distribution $\Pr(v_1, v_2)$ in Section 5.2, the posterior belief in (A-3) generalizes to:

$$\begin{aligned} q_1(1) &= \frac{\Pr(0,0) \times \frac{1}{2} + \Pr(\frac{1}{2}, 0) [1 - \theta_1(\frac{1}{2}, 0)]}{\frac{1}{2}} \\ &= \Pr(0,0) + 2\Pr(\frac{1}{2}, 0) \left[1 - \theta_1(\frac{1}{2}, 0) \right]. \end{aligned}$$

By Proposition A1, if $q_1(1) > \frac{1}{2}$, then $\theta_1(\frac{1}{2}, 0) = 1$ and $q_1(1) = \Pr(0,0)$. Therefore $q_1(1) > \frac{1}{2}$ if and only if $\Pr(0,0) > \frac{1}{2}$, or equivalently $q > \bar{q}(r)$. On the other hand, for $q \leq \bar{q}(r)$, $q_1(1) \leq \frac{1}{2}$, which requires $\theta_1(\frac{1}{2}, 0) > \frac{1}{2}$. Equilibrium prices follow from Proposition A1. ■

Proof of Proposition 6. Suppose $q > \frac{1}{2}$ and that the buyer is privately informed of v_i only. If s_i is second in the sequence, the buyer receives the uninformed payoff in (1), $B^U(q) > 0$, because she is uninformed of v_{-i} and the second seller's pricing depends only on the prior q in this case. Suppose, instead, that s_i is first and let $\tilde{q}_1 = \Pr\{v_i = 0 | s_i \text{ is first}\}$. If $v_i = 0$, the buyer receives an expected payoff of 0 because, by Proposition A1, for $\tilde{q}_1 \leq \frac{1}{2}$, each seller charges $\frac{1}{2}$ whereas for $\tilde{q}_1 > \frac{1}{2}$, s_i sets his low price to leave no expected surplus. Hence, a buyer with $v_i = 0$ strictly prefers to sequence s_i second and obtain $B^U(q) > 0$. This implies $\tilde{q}_1 = 0$ and by Proposition A1, an expected payoff of 0 for the buyer when approaching s_i first. Therefore, in equilibrium, s_i is sequenced second, yielding the buyer her uninformed payoff in (1). ■

Proof of Proposition 7. Let $m = (m_1, m_2)$. By definition, the buyer's expected value of information is

$$\bar{\Delta}^m(q) = \Pr(b, b)\Delta^{(b,b)}(q) + \sum_{i=1}^2 \Pr(s_i, b)\Delta^{(s_i,b)}(q) + \Pr(s_1, s_2)\Delta^{(s_1,s_2)}(q).$$

If $m_1 = m_2 = b$, the buyer optimally offers 0 to each seller, implying $B^I(q) = B^U(q) = 1$ and in turn, $\Delta^{(b,b)}(q) = 0$. If, on the other hand, $m_i = s_i$ for $i = 1, 2$, the setting reduces to our base model, implying $\Delta^{(s_1,s_2)}(q) = \Delta(q)$ where $\Delta(q)$ is as stated in (4). It therefore remains

to prove that if $m_i = s_i$ and $m_{-i} = b$, then $\Delta^{(s_i, b)}(q) = 0$. Suppose $m = (s_i, b)$. We consider uninformed and informed buyers in turn.

Uninformed buyer: If the sequence is $s_{-i} \rightarrow s_i$, the buyer always purchases from s_{-i} (at price 0) and thus the optimal price by s_i is given by (A-1) where $\phi = 0$ and $\hat{q}_1(0, 1) = q$. This means that the buyer's uninformed payoff is: $B^U(s_{-i} \rightarrow s_i) = \begin{cases} \frac{1}{2} & \text{if } q \leq \frac{1}{2} \\ \frac{1-q}{2} & \text{if } q > \frac{1}{2} \end{cases}$. If, however, the sequence is $s_i \rightarrow s_{-i}$, the buyer's expected payoff from rejecting s_i 's offer is $\frac{1-q}{2}$, which is simply the expected payoff from acquiring good i only. Therefore, the highest acceptable price by s_i in the first period satisfies $\max\{1 - \bar{p}_1, v_{-i} - \bar{p}_1\} = \frac{1-q}{2}$, revealing $p_1 = \frac{1+q}{2}$ and an expected payoff: $B^U(s_i \rightarrow s_{-i}) = \frac{1-q}{2}$. Comparing the two payoffs, $B^U = B^U(s_{-i} \rightarrow s_i)$.

Informed buyer: If the sequence is $s_{-i} \rightarrow s_i$, the optimal price by s_i is given by (A-1) where $\phi = 1$ and $\hat{q}_1(1, 1) = q_1(1)$ since the buyer always purchases from s_{-i} . If the sequence is $s_i \rightarrow s_{-i}$, the highest price acceptable to the buyer in the first meeting satisfies $\max\{1 - \bar{p}_1, v_{-i} - \bar{p}_1\} = v_i$. Therefore, the optimal price by s_i is

$$p_1 = \begin{cases} \frac{1}{2} & \text{if } q_2(1) \leq \frac{1}{2} \\ 1 & \text{if } q_2(1) \geq \frac{1}{2} \end{cases} \quad (\text{A-5})$$

where $q_2(1) = \Pr\{v_{-i} = 0 | s_{-i} \text{ is second}\}$. The buyer with $v_{-i} = 0$ accepts p_1 for sure whereas a buyer with $v_{-i} = \frac{1}{2}$ accepts only the low p_1 . Let $\hat{\theta}_k(v_{-i}) = \Pr\{s_{-i} \text{ is } k\text{th} | v_{-i}\}$ and $q_k(1) = \Pr\{v_{-i} = 0 | s_{-i} \text{ is } k\text{th}\}$. Then, by Bayes' rule,

$$q_k(1) = \frac{q\hat{\theta}_k(0)}{q\hat{\theta}_k(0) + (1-q)\hat{\theta}_k(\frac{1}{2})}. \quad (\text{A-6})$$

We show that there is no equilibrium in which $B^I \neq B^U$. If, in equilibrium, $\hat{\theta}_k(0) = \hat{\theta}_k(\frac{1}{2}) = 1$ for some $k = 1, 2$, then $q_k(1) = q$ and, by (A-1) and (A-5), $B^I = B^U$. Suppose $\hat{\theta}_k(0) \in (0, 1)$. Then $q_k(1)$ is uniquely pinned down for $k = 1, 2$ using (A-6). We consider three cases for $q_k(1)$.

- $q_k(1) \leq \frac{1}{2}$ for $k = 1, 2$: Since $\hat{\theta}_{-k}(v_{-i}) = 1 - \hat{\theta}_k(v_{-i})$, by (A-6), such an equilibrium belief requires $2q - 1 \leq q\hat{\theta}_k(0) - (1-q)\hat{\theta}_k(\frac{1}{2}) \leq 0$ and in turn, $q \leq \frac{1}{2}$. From (A-1) and (A-5), we therefore have that the informed and uninformed prices by s_i is $\frac{1}{2}$, resulting in $B^I = B^U$.
- $q_k(1) > \frac{1}{2}$ for $k = 1, 2$: By (A-6), this requires $q > \frac{1}{2}$, which, by (A-1) and (A-5), induces the informed and uninformed prices of 1 by s_i . Therefore, $B^I = B^U$.

- $q_k(1) \leq \frac{1}{2} < q_{-k}(1)$: By (A-1) and (A-5), approaching s_{-i} in k th place results in a price of $\frac{1}{2}$ by s_i , while approaching s_{-i} in $-k$ th place results in a price of 1 by s_i . Therefore, a buyer with $v_{-i} = 0$ has a strict preference to approach s_{-i} k th, i.e. $\hat{\theta}_k(0) = 1$. By (A-6), however, this implies that $q_{-k}(1) = 0 < q_k(1)$, contradicting the existence of an equilibrium with $q_k(1) \leq \frac{1}{2} < q_{-k}(1)$.

Consequently, there is no equilibrium with $B^I \neq B^U$. An equilibrium with $B^I = B^U$ obtains by setting $\hat{\theta}_1(0) = \hat{\theta}_1(\frac{1}{2}) \in (0, 1)$, resulting in $q_k(1) = q$ for $k = 1, 2$, which, by (A-1) and (A-5), yields identical informed and uninformed pricing by s_i . ■

Proof of Proposition 8. Suppose that $v_1 = v_2 = v \in [0, \frac{1}{2}]$ and the joint value is $V = 1$ or $\bar{V} > 1$ where $\Pr\{V = \bar{V}\} = \alpha \in (0, 1)$. Let $\hat{\alpha}(h) = \Pr(V = \bar{V}|h)$. Then, the optimal price by the second seller upon observing a prior purchase is

$$p_2(h = 1) = \begin{cases} 1 - v & \text{if } \hat{\alpha}(1) \leq \frac{1-v}{\bar{V}-v} \\ \bar{V} - v & \text{if } \hat{\alpha}(1) \geq \frac{1-v}{\bar{V}-v} \end{cases}. \quad (\text{A-7})$$

Without a prior purchase, the second seller trivially offers $p_2^z(h = 0) = v$. The pricing by the first seller depends on whether the buyer is informed or uninformed.

Uninformed buyer: Let \bar{p}_1 be the first seller's maximum price acceptable to the buyer. Denoting by $E[\cdot]$ the usual expectation operator, \bar{p}_1 satisfies: $\max\{E[V] - \bar{p}_1 - E[p_2(h = 1)], v - \bar{p}_1\} = 0$, or

$$\bar{p}_1 = \max\{E[V] - E[p_2(h = 1)], v\}.$$

Since any higher price is rejected for sure, $p_1^U = \bar{p}_1$ and by Bayes' rule, $\hat{\alpha}(1) = \alpha$. Therefore, for $\alpha \leq \frac{1-v}{\bar{V}-1}$, $p_1^U = \alpha(\bar{V} - 1) + v$ and $p_2^U(h = 1) = 1 - v$ while for $\alpha > \frac{1-v}{\bar{V}-1}$, $p_1^U = v$ and $p_2^U(h = 1) = \bar{V} - v$. The resulting expected payoff for the buyer is $B^{I,U}(v) = 0$.

Informed buyer: In this case, \bar{p}_1 satisfies

$$\bar{p}_1 = \max\{V - E[p_2(h = 1)], v\}.$$

We consider three possibilities for $\hat{\alpha}(1)$.

- $\hat{\alpha}(1) < \frac{1-v}{\bar{V}-v}$: Then, $p_2(h = 1) = 1 - v$ by (A-7), implying that $\bar{p}_1^L = v$ is accepted for sure, whereas $\bar{p}_1^H = (\bar{V} - 1) + v$ is accepted only if $V = \bar{V}$. Therefore, for $\alpha \leq \frac{v}{v+\bar{V}-1} \left(< \frac{1-v}{\bar{V}-v} \right)$, the first seller optimally sets $p_1 = v$ (breaking the indifference at $\alpha = \frac{v}{v+\bar{V}-1}$ in favor of efficiency), which reveals $\hat{\alpha}(1) = \alpha$. As a result, for $\alpha \leq \frac{v}{v+\bar{V}-1}$, the

price pair $p_1^I = v$ and $p_2^I(h = 1) = 1 - v$ constitute an equilibrium, resulting in the payoff: $B^{I,I}(\alpha) = \alpha(\bar{V} - 1)$. For $\alpha \in \left(\frac{v}{v+\bar{V}-1}, \frac{1-v}{\bar{V}-v}\right)$, the first seller sets $p_1 = (\bar{V} - 1) + v$, which implies $\hat{\alpha}(1) = 1$ and a profitable deviation for the second seller to $p_2(h = 1) = \bar{V} - v$. Hence, $\hat{\alpha}(1) < \frac{1-v}{\bar{V}-v}$ only if $\alpha \leq \frac{v}{v+\bar{V}-1}$ resulting in $\Delta^I(\alpha) = B^{I,I}(\alpha)$.

- $\hat{\alpha}(1) = \frac{1-v}{\bar{V}-v}$: Then, the second seller is indifferent between $\bar{V} - v$ and $1 - v$. Suppose that he offers $1 - v$ with probability σ . Then, $E[p_2(h = 1)] = \bar{V} - v - \sigma(\bar{V} - 1)$, implying that $\bar{p}_1^L = v$ is accepted for sure by the buyer while $\bar{p}_1^H = v + \sigma(\bar{V} - 1)$ is accepted only if $V = \bar{V}$. The first seller is indifferent between \bar{p}_1^L and \bar{p}_1^H if $\sigma = \frac{(1-\alpha)v}{\alpha(\bar{V}-1)}$, in which case the first seller's mixing $\beta = \Pr(p_1 = v) = \frac{\alpha(\bar{V}-1)}{(1-\alpha)(1-v)} \leq 1$ engenders an equilibrium belief $\hat{\alpha}(1) = \frac{\alpha}{\beta + (1-\beta)\alpha} = \frac{1-v}{\bar{V}-v}$. Then, $\bar{p}_1^H = \frac{v}{\alpha}$. Note that $\sigma \leq 1$ for $\alpha \geq \frac{v}{v+\bar{V}-1}$ and $\beta \leq 1$ for $\alpha \leq \frac{1-v}{\bar{V}-v}$. Therefore, the price pair

$$p_1^I = \begin{cases} v & \text{with prob. } \beta \\ \frac{v}{\alpha} & \text{with prob. } 1 - \beta \end{cases} \quad \text{and} \quad p_2^I(h = 1) = \begin{cases} 1 - v & \text{with prob. } \sigma \\ \bar{V} - v & \text{with prob. } 1 - \sigma \end{cases}$$

is an equilibrium for $\alpha \in \left(\frac{v}{v+\bar{V}-1}, \frac{1-v}{\bar{V}-v}\right]$. In such an equilibrium, $\Delta^I(\alpha) = B^{I,I}(v) = \frac{v}{1-v}\alpha(\bar{V} - 1)$. Note that for $v = 0$, $\sigma = 0$ and in turn, $p_2^I(h = 1) = \bar{V}$. Then, $\bar{p}_1 = 0$. Using the efficient tie-breaking rule, the first seller offers $p_1^I = 0$. Let $\eta(V)$ denote the probability that the buyer accepts the first offer given V . Then, by Bayes' rule, $\hat{\alpha}(1) = \frac{\eta(\bar{V})\alpha}{\eta(\bar{V})\alpha + \eta(1)(1-\alpha)}$. Since efficiency is maximized for $\eta(\bar{V}) = 1$, $\hat{\alpha}(1) = \frac{1}{\bar{V}}$, and $\eta(1) = \frac{\alpha(\bar{V}-1)}{(1-\alpha)}$. Therefore, for $v = 0$, the price pair $p_1^I = 0$ and $p_2^I(h = 1) = \bar{V}$ is supported by $\eta(1) = \frac{\alpha(\bar{V}-1)}{(1-\alpha)(1-v)}$ and $\hat{\alpha}(1) = \frac{1-v}{\bar{V}-v}$. The buyer's payoff is $B^{I,I}(v = 0) = 0 = \Delta^I(\alpha)$.

- $\hat{\alpha}(1) > \frac{1-v}{\bar{V}-v}$: Then, $p_2(h = 1) = \bar{V} - v$ and $p_1^I = \bar{p}_1 = v$. The first price is always accepted by the buyer, implying that $\hat{\alpha}(1) = \alpha > \frac{1-v}{\bar{V}-v}$ and $B^{I,I} = 0 = \Delta^I(\alpha)$. ■

Proof of Proposition 9. Let $q_u > \frac{1}{2}$. Consider sequential procurement with uninformed buyer. If s_2 observes no prior purchase, he offers $p_2(h = 0) = 1$ since it is accepted with probability q_u , resulting in a payoff of q_u , whereas the alternative price of $\frac{1}{2}$ is accepted with certainty, resulting in a payoff of $\frac{1}{2}$. If s_2 observes a prior purchase, he offers $p_2(h = 1) = \frac{1}{2}$ since the buyer's marginal value for his good is $\frac{1}{2}$ or 0. Anticipating such pricing, the highest price, \bar{p}_1 , acceptable to the buyer in the first meeting satisfies: $\max \{1 - p_2(h = 1) - \bar{p}_1, v_1 - \bar{p}_1\} \geq 0$, or simplifying

$$\bar{p}_1 \leq \max \{1 - p_2(h = 1), v_1\}.$$

This implies $p_1 = 1$ since $p_1 = 1$ is accepted with probability q_u and $p_1 = \frac{1}{2}$ is accepted for sure. Given the equilibrium prices, sequential procurement yields a payoff of 0 to the buyer. To prove that the auction yields a positive payoff, it suffices to show that in equilibrium, the sellers choose prices lower than 1 with a positive probability. The following two claims make this point.

Claim 1 *In the auction, there is no pure strategy equilibrium.*

Proof of Claim 1. As in the standard Bertrand competition, $p_1 = p_2 = p > 0$ cannot arise in equilibrium because with probability q_u^2 , goods are perfect substitutes and a slightly lower price would guarantee a sale in this realization. Without loss of generality, suppose $p_1 < p_2$. If $\frac{1}{2} < p_1 < p_2$, then s_1 receives an expected profit $\pi_1 = [q_u(1 - q_u) + q_u^2] p_1$, implying a profitable deviation to $\tilde{p}_1 = \frac{p_2 + p_1}{2}$. The same profitable deviation also exists if $p_1 < p_2 \leq \frac{1}{2}$, because in this case, p_1 is accepted unless $v_2 = 1$ and $v_1 = \frac{1}{2}$, resulting in $\pi_1 = [1 - q_u(1 - q_u)] p_1$. Finally, if $p_1 \leq \frac{1}{2} < p_2$, the sellers' expected profits are

$$(\pi_1, \pi_2) = \begin{cases} (p_1, 0) & \text{if } p_1 < p_2 + \frac{1}{2} \\ ([1 - (1 - \sigma_1)q_u(1 - q_u)]p_1, q_u(1 - q_u)(1 - \sigma_1)p_2) & \text{if } p_1 = p_2 + \frac{1}{2} \\ ([1 - q_u(1 - q_u)]p_1, q_u(1 - q_u)p_2) & \text{if } p_1 > p_2 + \frac{1}{2} \end{cases}$$

where $\sigma_1 \in [0, 1]$ is an arbitrary tie-breaking rule when the buyer is indifferent. For $p_1 < p_2 + \frac{1}{2}$, s_2 clearly has a strict incentive to lower his price. The same is true for $p_1 = p_2 + \frac{1}{2}$, in which case the buyer is indifferent. For $p_1 > p_2 + \frac{1}{2}$, s_2 would deviate to $\tilde{p}_2 = \frac{p_2 + p_1 - \frac{1}{2}}{2}$. In sum, there is no pure strategy equilibrium. ■

Claim 2 *In the auction, the following c.d.f. constitutes a symmetric mixed strategy equilibrium:*

$$F(p) = \frac{1}{q_u^2} \left[1 - q_u(1 - q_u) - \frac{1 - q_u}{2p} \right] \text{ for } p \in \left[\frac{1 - q_u}{2(1 - q_u(1 - q_u))}, \frac{1}{2} \right].$$

Proof of Claim 2. Consider a symmetric mixed strategy equilibrium with a continuous support $\underline{p} < \bar{p} \leq \frac{1}{2}$ and no mass points. Then,

$$\pi(p) = [F(p)(1 - q_u) + (1 - F(p))(1 - q_u(1 - q_u))] p = \bar{\pi},$$

where $\bar{\pi}$ is the indifference profit across $p \in [\underline{p}, \bar{p}]$. Note that $\pi(\bar{p}) = (1 - q_u)\bar{p}$ is increasing in \bar{p} , implying a profitable deviation to $p \in (\bar{p}, \frac{1}{2}]$. Therefore, $\bar{p} = \frac{1}{2}$. Re-writing,

$$F(p) = \frac{1}{q_u^2} \left[1 - q_u(1 - q_u) - \frac{\bar{\pi}}{p} \right].$$

Since $F(\frac{1}{2}) = 1$, $\bar{\pi} = \frac{1-q_u}{2}$. Given this and the fact that $F(\underline{p}) = 0$, we find that $\underline{p} = \frac{1-q_u}{2(1-q_u(1-q_u))}$.

Thus,

$$F(p) = \frac{1}{q_u^2} \left[1 - q_u(1 - q_u) - \frac{1 - q_u}{2p} \right] \text{ for } p \in \left[\frac{1 - q_u}{2(1 - q_u(1 - q_u))}, \frac{1}{2} \right],$$

as claimed. It remains to show that there is no unilateral deviation incentive to $p \notin \left[\underline{p}, \frac{1}{2} \right]$. Without loss of generality, consider a deviation by s_1 . Clearly, $p_1 < \underline{p}$ is not profitable, because $\pi(p_1) = (1 - q_u(1 - q_u))p_1 < (1 - q_u(1 - q_u))\underline{p} = \pi(\underline{p})$. Next consider a deviation to $p_1 > \frac{1}{2}$. Since $p_1 > 1$ is rejected with probability 1, we restrict attention to $p_1 \in (\frac{1}{2}, 1]$. In this case, s_1 realizes a sale only if $v_1 = 1$, $v_2 = \frac{1}{2}$, and $1 - p_1 > \frac{1}{2} - p_2$, or equivalently $p_2 > p_1 - \frac{1}{2}$. We exhaust two cases:

- $p_1 - \frac{1}{2} \leq \underline{p}$: Then, $\pi(p_1) = q_u(1 - q_u)p_1$. Since this deviation profit is increasing in p_1 , the maximum deviation profit in this region is

$$\pi(\underline{p} + \frac{1}{2}) = q_u(1 - q_u) \left(\frac{1}{2} + \frac{1 - q_u}{2(1 - q_u(1 - q_u))} \right) < \frac{1 - q_u}{2} = \bar{\pi}.$$

Therefore, there is no incentive to deviate to $p_1 \in (\frac{1}{2}, \frac{1}{2} + \underline{p}]$.

- $\frac{1}{2} + \underline{p} < p_1 \leq 1$: Then, s_1 's probability of a sale is $q_u(1 - q_u) \Pr(p_2 > p_1 - \frac{1}{2})$ and his deviation profit is

$$\begin{aligned} \pi(p_1) &= q_u(1 - q_u) \left[1 - \frac{1}{q_u^2} \left(1 - q_u + q_u^2 - \frac{1 - q_u}{2(p_1 - \frac{1}{2})} \right) \right] p_1 \\ &= \frac{(1 - q_u)^2}{q_u} \left[\frac{1}{2(\tilde{p}_i - \frac{1}{2})} - 1 \right] p_1. \end{aligned}$$

Simple algebra shows that $\pi(p_1) < \frac{1 - q_u}{2} = \bar{\pi}$. ■

Together Claims 1 and 2 prove Proposition 9. ■

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