Toward a theory of monopolistic competition*

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Abstract

We propose a general model of monopolistic competition, which encompasses existing models while being flexible enough to take into account new demand and competition features. Even though preferences need not be additive and/or homothetic, the market outcome is still driven by the sole variable elasticity of substitution. We impose elementary conditions on this function to guarantee empirically relevant properties of a free-entry equilibrium. Comparative statics with respect to market size and productivity shock are characterized through necessary and sufficient conditions. Furthermore, we show that the attention to the CES based on its normative implications was misguided: constant mark-ups, additivity and homotheticity are neither necessary nor sufficient for the market to deliver the optimum. In addition, monopolistic competition is shown to mimic oligopolistic competition under free entry. Finally, our approach can cope with heterogeneous firms and consumers, as well as with a multisector economy.

Keywords: monopolistic competition, general equilibrium, additive preferences, homothetic preferences

JEL classification: D43, L11, L13.

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1 Introduction

In a survey of the various attempts made in the 1970s and 1980s to integrate oligopolistic competition within the general equilibrium framework, Hart (1985) has convincingly argued that these contributions have failed to produce a consistent and workable model. Unintentionally, the absence of a general equilibrium model of oligopolistic competition has paved the way to the success of the CES model of monopolistic competition developed by Dixit and Stiglitz (1977). This model has been used in so many economic fields that a large number of scholars view it as *the* model of monopolistic competition (Brakman, and Heijdra, 2004). For example, Head and Mayer (2014) observe that the CES is "nearly ubiquitous" in the trade literature. However, owing to its extreme simplicity, this model dismisses several important effects that contradict basic findings in economic theory. To mention a few, unlike what the CES predicts, prices and firm sizes are affected by entry, market size and consumer income, while markups vary with costs. Recent empirical studies conducted at the firm level provide direct evidence for these findings.

In addition, tweaking the CES or using other specific models in the hope of obviating those difficulties prevents a direct confrontation between alternative specifications. We acknowledge that such a research strategy is motivated by tractability, but this is done at the risk of losing sight of the fragility of the results. For example, under the CES market prices do not depend on market size and individual income. By nesting quadratic preferences into a quasi-linear utility, Melitz and Ottaviano (2008) show that prices depend on market size, but suppress the per capita income effect. Prices depend on per capita income under the linear expenditure system in an open economy (Simonovska, 2015), but this effect disappears in a closed economy under additive preferences (Zhelobodko et al., 2012). Prices are independent of the number of competitors in the CES model of monopolistic competition, but not under oligopolistic competition (d'Aspremont et al., 1996). In sum, it seems fair to say that the theory of general equilibrium with imperfectly competitive markets is still in infancy and looks like a scattered field of incomplete and insufficiently related contributions in search of a more general approach. This is where we hope to contribute.

Our purpose is to build a general equilibrium model of monopolistic competition that has the following two desirable features. First, it encompasses existing models of monopolistic competition, such as the CES, quadratic, CARA, additive, and homothetic preferences. Second, it displays enough flexibility to take into account demand and competition attributes in a way that will allow us to determine under which conditions many findings are valid. To this end, we consider a setting in which preferences are unspecified and characterize them through necessary and sufficient conditions for various comparative static effects to hold. This should be useful to the applied economists in discriminating between the different specifications used in their setups.

By modeling monopolistic competition as a noncooperative game with a continuum of players, we are able to obviate at least two major problems. First, since each firm is negligible to the market, our setup captures Triffin's (1947) idea that a negligible cross-price elasticity between any two varieties is the main distinguishing feature of monopolistic competition. Second, because individual firms are unable to manipulate incomes and profits, firms do not have to make full general equilibrium calculations before choosing their profit-maximizing strategy. The approach followed in this paper thus concurs with Mas-Colell (1984, p. 19) for whom "the theory of monopolistic competition derives its theoretical importance not from being a realistic complication of the theory of perfect competition, but from being a simplified, tractable limit of oligopoly theory."

Our main findings may be summarized as follows. First, using the concept of Frechet-differentiability, which applies to the case where the unknowns are functions rather than vectors, we determine a general demand system that includes a wide range of special cases used in the literature. In particular, at any symmetric consumption profile, preferences are additive if and only if the marginal rate of substitution between any two varieties depends only upon their consumption level, while preferences are homothetic if and only if the marginal rate of substitution depends only upon the number of available varieties. Therefore, when preferences are neither additive nor homothetic, the demand for a variety must depend on its consumption level and the number of available varieties.

Second, even though the case of heterogeneous firms is studied in this paper, starting with symmetric firms allows us to insulate the impact of various types of preferences on the market outcome. In the words of Chamberlin (1933): "We therefore proceed under the heroic assumption that both demand and cost curves for all the "products" are uniform throughout the group." That said, we show that, at a symmetric free-entry equilibrium, *firms' markup is equal to the inverse of the equilibrium value of the elasticity of substitution*. As a consequence, the properties of an equilibrium depend on how the elasticity of substitution function behaves when the per variety consumption and the mass of firms vary. By imposing plausible conditions on this function and using simple analytical arguments, we are able to disentangle the various determinants of firms' behavior. In particular, although firms do not compete strategically, *our model mimics oligopolistic markets with free entry* and generates within a general equilibrium framework findings akin to those obtained in industrial organization.

Third, our setup is especially well suited to conduct detailed comparative static analyses in that we can determine the necessary and sufficient conditions for various thought experiments undertaken in the literature. The most common experiment is to study the impact of market size on the market outcome. What market size signifies is not always clear because it compounds two variables, i.e., the number of consumers and their willingness-to-pay for the product under consideration. The impact of population size and income level on prices, output and the number of firms need not be the same because these two parameters affect firms' demand in different ways. An increase in population or income raises demand, thereby fostering entry and lower prices. But an income hike also raises consumers' willingness-to-pay, which tends to push prices upward. The final impact is thus a priori ambiguous.

We show that a larger market results in a lower market price and bigger firms if and only if the elasticity of substitution responds more to a change in the mass of varieties than to a change in the per variety consumption. This is so in the likely case where the entry of new firms does not render varieties much more differentiated. Regarding the mass of varieties, it increases with the population size if varieties do not become too similar when their number rises. Thus, like most oligopoly models, *monopolistic competition exhibits the standard pro-competitive effects associated with market size and entry.* An increase in individual income generates similar, but not identical, effects to a population hike if and only if varieties become closer substitutes when their range widens. The CES is the only utility for which price and output are independent of both income and market size. Our model also allows us to study the impact of a productivity or trade liberalization shock on markups. When all firms face the same productivity hike, we show that *the nature of preferences determines the extent of the pass-through.* Specifically, there is incomplete pass-through if and only if the elasticity of substitution decreases with the per capita consumption. Note that, even with the same consumers and non-strategic firms, standard assumptions on preferences are not sufficient to rule out anti-competitive effects. For this, we need additional assumptions.

Fourth, ever since Chamberlin (1933), the question of whether the market under- or over-provides diversity is one of the most studied issues in the theory of imperfect competition. It is well known that the CES is the only additive utility for which the market achieved the optimum (Dixit and Stiglitz, 1978; Dhingra and Morrow, 2015). The conventional wisdom holds that the constant markup associated with the CES is necessary for this result to hold. Non-CES homothetic preferences typically generate markups that vary with the number of firms. Yet, we show that, for *any* homothetic utility, there exists a transformation of this utility that yields another homothetic utility, which generically differs from the CES, for which the market and optimal outcomes are the same. Therefore, we may conclude that the optimality of the market equilibrium is not driven by a constant markup. What is more, we show that homotheticity is not even required for this result to hold, as we provide an example of a non-homothetic utility where the optimum is reached at the market outcome. Therefore, the attention the CES has received regarding its normative implications was misguided: constant mark-ups, additivity or homotheticity, three properties verified by the CES, are neither necessary nor sufficient for the market to deliver the optimum.

Last, we discuss three extensions of the baseline model. The first one addresses the almost untouched issue of consumer heterogeneity in monopolistic competition. Although most models rely on the assumption of identical consumers, it should be clear consumers are heterogeneous in tastes and incomes. We show that the results discussed in the foregoing hold true when consumers are not "too" different.

The second extension allows for Melitz-like heterogeneous firms. When preferences are non-additive, the profit-maximizing strategy of a firm depends directly on the strategies chosen by all the other types' firms, which vastly increases the complexity of the problem. Despite of this, we are able to show that, regardless of the distribution of marginal costs, the elasticity of substitution across varieties produced by firms enjoying the same productivity level now depends on the number of entrants and the cutoff cost. Furthermore, our approach paves the way to the study of asymmetric preferences in that a model with heterogeneous firms supplying symmetric varieties is isomorphic to a model with homogeneous firms selling asymmetric varieties.

In the last extension, we consider a two-sector economy. The main additional difficulty stems from the fact that the sector-specific expenditures depend on the upper-tier utility. Under a fairly mild assumption on the marginal utility, we prove the existence of an equilibrium and show that many of our results hold true for the monopolistically competitive sector. This highlights the idea that our model can be used as a building block to embed monopolistic competition in full-fledged general equilibrium models coping with various economic issues.

Related literature. Different alternatives have been proposed to avoid the main pitfalls of the CES model. Behrens and Murata (2007) propose the CARA utility that captures price competition effects, while Zhelobodko et al. (2012) use general additive preferences to work with a variable elasticity of substitution, and thus variable markups. Vives (1999) and Ottaviano et al. (2002) show how the quadratic utility model obviates some of the difficulties associated with the CES model, while delivering a full analytical solution. Bilbiie et al. (2012) use general symmetric homothetic preferences in a real business cycle model. Last, pursuing a different, but related, objective, Mrázová and Neary (2013) study a class of demands which implies an invariant relationship between the demand elasticity and the curvature of demand schedule. Subsection 2.3 explains how this class fits in our general model.

In the next section, we describe the demand and supply sides of our setup. In particular, the primitive of the model being the elasticity of substitution function, we discuss how this function may vary with the per variety consumption and the mass of varieties. In Section 3, we prove the existence and uniqueness of a free-entry equilibrium and characterize its various properties when firms are symmetric. Section 4 discusses the optimality of the market outcome when preferences are homothetic. We study the case of heterogeneous consumers and firms in Section 5, as well as the case of a two-sector economy. Section 6 concludes.

2 The model and preliminary results

Consider an economy with L identical consumers, one sector and one production factor – labor, which is used as the numéraire. Each consumer is endowed with y efficiency units of labor. On the supply side, there is a continuum of firms producing each a horizontally differentiated good under increasing returns. Each firm supplies a single variety and each variety is supplied by a single firm.

2.1 Consumers

Let $\mathbb{N}$, an arbitrarily large number, be the mass of potential varieties (see below for a precise definition of "arbitrarily large"). As all potential varieties are not necessarily made available to consumers, we denote by $N \leq \mathbb{N}$ the endogenous mass of available varieties. Since we work with a continuum of varieties, the space of potential varieties is a functional space. Therefore, a *consumption profile* $\mathbf{x} \geq 0$ is a (Lebesgue-measurable) mapping from the space of potential varieties $[0, \mathbb{N}]$ to $\mathbb{R}_+$. Since a *market price profile* $\mathbf{p} \geq \mathbf{0}$ must belong to the dual of the space of consumption profiles (Bewley, 1972), we assume that both $\mathbf{x}$ and $\mathbf{p}$ belong to $L_2([0, \mathbb{N}])$. This implies that both $\mathbf{x}$ and $\mathbf{p}$ have a finite mean and variance; we denote by x_i (p_i) the consumption level (price) of variety i. The space L_2 may be viewed as the most natural infinite-dimensional extension of $\mathbb{R}^n$.

To start with, we give examples of preferences used in applications of monopolistic competition.

1. Additive preferences (Spence, 1976; Dixit and Stiglitz, 1977; Kühn and Vives, 1999). Preferences are *additive* over the set of varieties if

$$\mathcal{U}(\mathbf{x}) \equiv \int_0^{\mathbb{N}} u(x_i)\mathrm{d}i, \tag{1}$$

where u is differentiable, strictly increasing, strictly concave, and such that $u(0) = 0$. The CES, which has been used extensively in many fields and the CARA (Behrens and Murata, 2007) are special cases of (1).

2. Homothetic preferences. A tractable example of non-CES homothetic preferences used in the macroeconomic and trade literature is the *translog* (Bergin and Feenstra, 2009; Bilbiie et al., 2012; Feenstra and Weinstein, 2015). There is no closed-form expression for the translog utility function. Nevertheless, by appealing to the duality principle in consumption theory, these preferences can be described by the following expenditure function:

$$\ln E(\mathbf{p}) = \ln \mathcal{U}_0 + \frac{1}{\mathbb{N}} \int_0^{\mathbb{N}} \ln p_i \mathrm{d}i - \frac{\beta}{2\mathbb{N}} \left[\int_0^{\mathbb{N}} (\ln p_i)^2 \mathrm{d}i - \frac{1}{\mathbb{N}} \left(\int_0^{\mathbb{N}} \ln p_i \mathrm{d}i \right)^2 \right].$$

A broad class of homothetic preferences is given by what is known as *Kimball's flexible aggregator*, which has been introduced by Kimball (1995) as a production function used in the macroeconomic literature (Charie et al., 2000; Smets and Wouters, 2007). A utility functional $\mathcal{U}(\mathbf{x})$ is said to be described by Kimball's flexible aggregator if there exists a strictly increasing and strictly concave function $\theta(\cdot)$ such that $\mathcal{U}(\mathbf{x})$ satisfies

$$\int_0^{\mathbb{N}} \nu \left(\frac{x_i}{\mathcal{U}(\mathbf{x})} \right) \mathrm{d}i = 1 \tag{2}$$

for any consumption bundle $\mathbf{x}$. Whenever $\mathcal{U}(\mathbf{x})$ satisfies (2), it is single-valued, continuous, increasing, strictly quasi-concave, and linear homogeneous. This class of preferences has the same cardinality as the class of additive preferences because both are parameterized by the family of strictly increasing and concave functions (u and ν).[1]

[1] Unlike $u(\cdot)$ in (**??**), the function $\nu(\cdot)$ in (**??**) seems to have no clear economic meaning per se. However, the

3. **Quadratic preferences.** An example of preferences that are neither additive nor homothetic is the *quadratic* utility:

$$\mathcal{U}(\mathbf{x}) \equiv \alpha \int_0^{\mathbb{N}} x_i \mathrm{d}i - \frac{\beta}{2} \int_0^{\mathbb{N}} x_i^2 \, \mathrm{d}i - \frac{\gamma}{2} \int_0^{\mathbb{N}} \left(\int_0^{\mathbb{N}} x_i \, \mathrm{d}i \right) x_j \mathrm{d}j, \tag{3}$$

where α, β,and γ are positive constants (Dixit, 1979; Singh and Vives, 1984; Ottaviano et al., 2002; Melitz and Ottaviano, 2008).

This incomplete list of examples should be sufficient to show that the authors who use monopolistic competition appeal to a variety of models that display very different properties. It is unclear how these functional forms relate to each other and, more importantly, it is hard to assess the robustness of the theoretical predictions derived for specific demand systems and to match them to the empirical results obtained with other demand systems. This points to the need of a more general setup in which we can cast all these special cases and compare their properties. Having this in mind, we choose to describe individual preferences by a *general utility functional* $\mathcal{U}(\mathbf{x})$ defined over $L_2([0, \mathbb{N}])$.

In what follows, we make two assumptions about $\mathcal{U}$, which seem to be close to the "minimal" set of requirements for our model to be nonspecific while displaying the desirable features of existing models of monopolistic competition. First, for any N, the functional $\mathcal{U}$ is *symmetric* in the sense that any Lebesgue measure-preserving mapping from $[0, N]$ into itself does not change the value of $\mathcal{U}$. Intuitively, this means that renumbering varieties has no impact on the utility level.

Second, the utility function exhibits love for variety if, for any $N \leq \mathbb{N}$, a consumer strictly prefers to consume the whole range of varieties $[0, N]$ than any subinterval $[0, k]$ of $[0, N]$, that is,

$$\mathcal{U}\left(\frac{X}{k} I_{[0,k]}\right) < \mathcal{U}\left(\frac{X}{N} I_{[0,N]}\right), \tag{4}$$

where $X > 0$ is the consumer's total consumption of the differentiated good and I_A is the indicator of $A \sqsubseteq [0, N]$. We show in Appendix 1 in the Supplemental Material that consumers exhibit love for variety if $\mathcal{U}(\mathbf{x})$ is continuous and strictly quasi-concave. The convexity of preferences is often interpreted as a "taste for diversification" (Mas-Collel et al., 1995, p.44). The definition of "love for variety" is weaker than that of convex preferences because the former, unlike the latter, involves symmetric consumption only.

For any given N, the utility functional $\mathcal{U}$ is said to be Frechet-differentiable in $\mathbf{x} \in L_2([0, \mathcal{N}])$ when there exists a unique function $D(x_i, \mathbf{x})$ from $[0, N] \times L_2$ to $\mathbb{R}$ such that, for all $\mathbf{h} \in L_2$, the equality

$$\mathcal{U}(\mathbf{x} + \mathbf{h}) = \mathcal{U}(\mathbf{x}) + \int_0^N D(x_i, \mathbf{x}) \, h_i \, \mathrm{d}i + \circ \left(||\mathbf{h}||_2 \right) \tag{5}$$

elasticities of $\nu(\cdot)$ and $\nu'(\cdot)$ have a clear economic meaning (Kimball, 1995). Note that we fall back to the CES when $\nu(\cdot)$ is a power function.

holds, $||\cdot||_2$ being the L_2-norm (Dunford and Schwartz, 1988).[2] The function $D(x_i, \mathbf{x})$ is the *marginal utility of variety* i. That $D(x_i, \mathbf{x})$ does not depend directly on $i \in [0, N]$ follows from the symmetry of preferences. From now on, we focus on utility functionals that satisfy (5) for all $\mathbf{x} \geq 0$ and such that the marginal utility $D(x_i, \mathbf{x})$ is decreasing and differentiable with respect to the consumption x_i of variety i. Evidently, $D(x_i, \mathbf{x})$ (strictly) decreases with x_i if $\mathcal{U}$ is (strictly) concave. The reason for restricting ourselves to decreasing marginal utilities is that this property allows us to work directly with well-behaved demand functions.

Maximizing the utility functional $\mathcal{U}(\mathbf{x})$ subject to (i) the budget constraint

$$\int_0^N p_i x_i \mathrm{d}i = Y, \tag{6}$$

where Y is the consumer's income and (ii) the availability constraint

$$x_i \geq 0 \text{ for all } i \in [0, N] \quad \text{and} \quad x_i = 0 \text{ for all } i \in]N, \mathbb{N}]$$

yields the following inverse demand function for variety i:

$$p_i = \frac{D(x_i,\, \mathbf{x})}{\lambda} \quad \text{for all } i \in [0, N], \tag{7}$$

where λ is the Lagrange multiplier of the consumer's optimization problem. Expressing λ as a function of Y and $\mathbf{x}$, we obtain

$$\lambda(Y, \mathbf{x}) = \frac{\int_0^N x_i D(x_i, \mathbf{x})\, \mathrm{d}i}{Y}, \tag{8}$$

which is the marginal utility of income at the consumption profile $\mathbf{x}$ under income Y.

A large share of the literature focusing on additive or homothetic preferences, it is important to provide a characterization of the corresponding demands. First, Goldman and Uzawa (1964) show that preferences are additive if and only if the marginal rate of substitution between varieties i and j, $D(x_i,\, \mathbf{x})/D(x_j,\, \mathbf{x})$, depends only upon the consumptions x_i and x_j, while preferences are homothetic if and only if the marginal rate of substitution between varieties i and j depends only upon the consumption ratios $\mathbf{x}/x_i$ and $\mathbf{x}/x_j$ (for the proof see Appendix 2 in the Supplemental Material).

2.2 Firms: first- and second-order conditions

Let $\mathbf{\Omega}$ be the set of active firms. There are increasing returns at the firm level, but no scope economies that would induce a firm to produce several varieties. The continuum assumption

[2]The concept of Frechet-differentiability extends the standard concept of differentiability in a fairly natural way.

distinguishes monopolistic competition from other market structures in that it is the formal counterpart of the basic idea that a firm's action has no impact on the others. As a result, by being negligible to the market, each firm may choose its output (or price) while accurately treating market variables as given. However, for the market to be in equilibrium, firms must accurately guess what these variables will be.

Firms share the same fixed cost F and the same constant marginal cost c. In other words, to produce q_i units of its variety, firm $i \in \mathbf{\Omega}$ needs $F + cq_i$ efficiency units of labor. Hence, firm i's profit is given by

$$\pi(q_i) = (p_i - c)q_i - F. \tag{9}$$

Since consumers share the same preferences, the consumption of each variety is the same across consumers. Therefore, product market clearing implies $q_i = Lx_i$. Firm i maximizes (9) with respect to its output q_i subject to the inverse market demand function $p_i = LD/\lambda$, while the market outcome is given by a Nash equilibrium. The Nash equilibrium distribution of firms' actions is encapsulated in $\mathbf{x}$ and λ. In the CES case, this comes down to treating the price-index parametrically, while under additive preferences the only payoff-relevant market variable is λ.

Plugging $D(x_i, \mathbf{x})$ into (9), the program of firm i is given by

$$\max_{x_i} \pi_i(x_i, \mathbf{x}) \equiv \left[\frac{D(x_i, \mathbf{x})}{\lambda} - c\right] L\, x_i - F.$$

Setting

$$D_i' \equiv \frac{\partial D(x_i, \mathbf{x})}{\partial x_i} \qquad D_i'' \equiv \frac{\partial D^2(x_i, \mathbf{x})}{\partial x_i^2},$$

the first-order condition for profit-maximization are given by

$$D(x_i, \mathbf{x}) + x_i D_i' = [1 - \bar{\eta}(x_i, \mathbf{x})]\, D(x_i, \mathbf{x}) = \lambda c, \tag{10}$$

where

$$\bar{\eta}(x_i, \mathbf{x}) \equiv -\frac{x_i}{D(x_i, \mathbf{x})} \frac{\partial D(x_i, \mathbf{x})}{\partial x_i}$$

is the elasticity of the inverse demand for variety i. Since λ is endogenous, we seek necessary and sufficient conditions for a unique (interior or corner) profit-maximizer to exist regardless of the value of $\lambda c > 0$. The argument involves two steps.

Step 1. For (10) to have at least one positive solution regardless of $\lambda c > 0$, it is sufficient to assume that, for any $\mathbf{x}$, the following conditions hold:

$$\lim_{x_i \to 0} D = \infty \qquad \lim_{x_i \to \infty} D = 0. \tag{11}$$

Indeed, since $\bar{\eta}(0, \mathbf{x}) < 1$, (11) implies that $\lim_{x_i \to 0}(1 - \bar{\eta})D = \infty$. Similarly, since $0 <$

$(1-\bar{\eta})D < D$, it ensues from (11) that $\lim_{x_i \to \infty}(1-\bar{\eta})D = 0$. Because $(1-\bar{\eta})D$ is continuous, it follows from the intermediate value theorem that (10) has at least one positive solution. Note that (11) does not hold when D/λ displays a finite choke price. It is readily verified that (10) has at least one positive solution when the choke price exceeds λc.

Step 2. The first-order condition (10) is sufficient if the profit function π is strictly quasi-concave in x_i. If the maximizer of π is positive and finite, the profit function is strictly quasi-concave in x_i for any positive value of λc if and only if the second derivative of π is negative at any solution to the first-order condition. Since firm i treats λ parametrically, the second-order condition is given by

$$x_i D_i^{''} + 2D_i^{'} < 0. \tag{12}$$

This condition means that firm i's marginal revenue $(x_i D_i' + D)L/\lambda$ is strictly decreasing in x_i. It is satisfied when D is concave, linear or not "too" convex in x_i. Furthermore, (12) is also a necessary and sufficient condition for the function π to be strictly quasi-concave for all $\lambda c > 0$, for otherwise there would exist a value of λc such that the marginal revenue curve intersects the horizontal line λc more than once. Observe also that (12) means that the revenue function is strictly concave. Since the marginal cost is independent of x_i, this in turn implies that π_i is strictly concave in x_i. When firms are quantity-setters, the profit function π_i is strictly concave in x_i if this function is strictly quasi-concave in x_i (for the proof see Appendix 3 in the Supplemental Material). Therefore, the profit function π_i is strictly quasi-concave in x_i for all values of λc if and only if

(**A**) *firm i's marginal revenue decreases in* x_i.

Observe that (**A**) is equivalent to the well-known condition obtained by Caplin and Nalebuff (1991) for a firm's profits to be quasi-concave in its own price, which is stated as follows: the Marshallian demand $\mathcal{D}(p_i, \mathbf{p}, Y)$ for variety i is such that $1/\mathcal{D}$ is convex in p_i. Note, first, that $\mathcal{D}(p_i, \mathbf{p}, Y)$ is well defined in our framework. Indeed, because the consumer's budget set is convex and weakly compact in $L_2([0, \mathbb{N}])$, while $\mathcal{U}$ is continuous and strictly quasi-concave, there exists a unique utility-maximizing consumption profile $\mathbf{x}^*(\mathbf{p}, Y)$ (Dunford and Schwartz, 1988). Plugging $\mathbf{x}^*(\mathbf{p}, Y)$ into (7) – (8) and solving (7) for x_i yields

$$x_i = \mathcal{D}(p_i, \mathbf{p}, Y), \tag{13}$$

which is weakly decreasing in its own price.[3] Since the Caplin-Nalebuff condition is known to be the least stringent one for a firm's profit to be quasi-concave under price-setting firms, (**A**) is therefore the least demanding condition when firms compete in quantities.

[3] Since D is continuously decreasing in x_i, there exists at most one solution of (7) with respect to x_i. If there is a finite choke price ($D(0, \mathbf{x}^*)/\lambda < \infty$), there may be no solution. To encompass this case, the Marshallian demand should be formally defined by $\mathcal{D}(p_i, \mathbf{p}, y) \equiv \inf\{x_i \geq 0 \mid D(x_i, \mathbf{x}^*)/\lambda(y, \mathbf{x}^*) \leq p_i\}$.

A sufficient condition commonly used in the literature is as follows (Krugman, 1979; Vives, 1999):

(Abis) *the elasticity of the inverse demand* $\bar{\eta}(x_i, x)$ *increases in* x_i.

It is readily verified that **(Abis)** is equivalent to

$$-x_i \frac{D_i''}{D_i'} < 1 + \bar{\eta}(x_i, \mathbf{x}).$$

Since the expression (12) may be rewritten as follows:

$$-x_i \frac{D_i''}{D_i'} < 2.$$

while $\bar{\eta} < 1$ it must be that **(Abis)** implies **(A)**.

2.3 The elasticity of substitution

Definition. The elasticity of substitution plays a central role in the CES model of monopolistic competition. The common wisdom is that this concept is relevant in this case only. We show in this paper that this opinion is unwarranted in that the elasticity of substitution can be extended to cope with general preferences, while being the relevant primitive of a general model of monopolistic competition. This will allow us to show that, in our model, the comparative static results are driven by the demand side.

To achieve our goal, we use an infinite-dimensional version of the *elasticity of substitution function* given by Nadiri (1982, p. 442). Setting $D_i = D(x_i, \mathbf{x})$ for notational simplicity, the elasticity of substitution between varieties i and j for a given $\mathbf{x}$ is given by

$$\bar{\sigma}(x_i, x_j, \mathbf{x}) = -\frac{D_i D_j (x_i D_j + x_j D_i)}{x_i x_j \left[D_i' D_j^2 - \left(\frac{\partial D_i}{\partial x_j} + \frac{\partial D_j}{\partial x_i} \right) D_i D_j + D_j' D_i^2 \right]}.$$

Since $\mathbf{x}$ is defined up to a zero measure set, it must be that

$$\frac{\partial D_i(x_i, \mathbf{x})}{\partial x_j} = \frac{\partial D_j(x_j, \mathbf{x})}{\partial x_i} = 0$$

for all $j \neq i$, that is, the cross-price elasticity between any two varieties is negligible. Therefore, we obtain

$$\bar{\sigma}(x_i, x_j, \mathbf{x}) = -\frac{D_i D_j (x_i D_j + x_j D_i)}{x_i x_j \left(D_i' D_j^2 + D_j' D_i^2 \right)}.$$

Setting $x_i = x_j = x$ implies $D_i = D_j$, and thus we come to

$$\bar{\sigma}(x, \mathbf{x}) = \frac{1}{\bar{\eta}(x, \mathbf{x})}. \tag{14}$$

Evaluating $\bar{\sigma}$ at a symmetric consumption pattern, where $\mathbf{x} = xI_{[0,N]}$, yields

$$\sigma(x, N) \equiv \bar{\sigma}(x, xI_{[0,N]}).$$

Hence, regardless of the structure of preferences, at any symmetric consumption pattern *the elasticity of substitution depends only upon the individual consumption and the mass of varieties.* Hence, when firms are symmetric, the consumption vector $\mathbf{x}$ can be summarized with the number of varieties N and the consumption per variety x. In other words, the dimensionality of the problem is reduced to two variables. When firms are heterogeneous, the consumption pattern is no longer symmetric, but we will see in Section 5 how σ keeps its relevance.

Given the above considerations, we may consider the function $\sigma(x, N)$ as the *primitive* of the model. There are two more reasons for making this choice. First, we will see that what matters for the properties of the symmetric equilibrium is how $\sigma(x, N)$ varies with x and N. More precisely, we will show that the behavior of the market outcome can be characterized by necessary and sufficient conditions stating how σ vary with x and N. Rather than using the partial derivatives of σ, it will be more convenient to work with the elasticities $\mathcal{E}_x(\sigma)$ and $\mathcal{E}_N(\sigma)$. Specifically, the signs of these two expressions ($\mathcal{E}_x(\sigma) \gtrless 0$ and $\mathcal{E}_N(\sigma) \lesseqgtr 0$) and their relationship ($\mathcal{E}_x(\sigma) \gtrless \mathcal{E}_N(\sigma)$) will allow us to characterize completely the market outcome.

Second, since the elasticity of substitution is an inverse measure of the degree of product differentiation across varieties, we are able to appeal to the theory of product differentiation to choose the most plausible assumptions regarding the behavior of $\sigma(x, N)$ with respect to x and N and to check whether the resulting predictions are consistent with empirical evidence.

Remark. Our approach could be equivalently reformulated by considering the manifold (σ, $\mathcal{E}_x(\sigma)$, $\mathcal{E}_N(\sigma)$), which is parameterized by the variables x and N. Being generically a two-dimensional surface in $\mathbb{R}^3$, this manifold boils down to a one-dimensional locus in Mrázová and Neary (2013). The one-dimensional case encompasses a wide variety of demand systems, including those generated by additive preferences ($\mathcal{E}_N(\sigma) = 0$). We want to stress that the approach developed by Mrázová and Neary could equally be useful to cope with homothetic preferences ($\mathcal{E}_x(\sigma) = 0$).

2.3.1 Examples

To develop more insights about the behavior of σ as a function of x and N, we give below the elasticity of substitution for the different types of preferences discussed in the foregoing.

(i) When the utility is additive, we have:

$$\frac{1}{\sigma(x, N)} = r(x) \equiv -\frac{xu''(x)}{u'(x)}, \tag{15}$$

which means that σ depends only upon the per variety consumption when preferences are additive.

(ii) When preferences are homothetic, $D(x, \mathbf{x})$ evaluated at a symmetric consumption profile depends solely on the mass N of available varieties. Setting

$$\varphi(N) \equiv \eta(1, N)$$

and using (14) yields

$$\frac{1}{\sigma(x, N)} = \varphi(N). \tag{16}$$

For example, under translog preferences, we have $\varphi(N) = 1/(1 + \beta N)$.

Since the CES is additive, the elasticity of substitution is independent of N. Furthermore, since the CES is also homothetic, it must be that

$$r(x) = \varphi(N) = \frac{1}{\sigma}.$$

It is, therefore, no surprise that the constant σ is the only demand parameter that drives the market outcome under CES preferences.

Using (15) and (16), it is readily verified that $\mathcal{E}_N(\sigma) = 0$ if and only if preferences are additive, while $\mathcal{E}_x(\sigma) = 0$ if and only if preferences are homothetic. The CES satisfies $\mathcal{E}_N(\sigma) = \mathcal{E}_x(\sigma) = 0$.

2.3.2 How does $\sigma(x, N)$ vary with x and N?

While our framework allows for various patterns of σ, it should be clear that they are not equally plausible. This is why most applications of monopolistic competition focus on subclasses of utilities to cope with particular effects. For instance, Bilbiie et al. (2012) use the translog expenditure function to capture the pro-competitive impact of entry on markups. Indeed, $\sigma(x, N) = 1 + \beta N$ increases with the number of varieties. On the same grounds, working with additive preferences Krugman (1979) assumes "without apology" that $\sigma(x, N) = 1/r(x)$ decreases with individual consumption x. This suggests the following conditions:

$$\mathcal{E}_x(\sigma) \leq 0 \leq \mathcal{E}_N(\sigma). \tag{17}$$

However, translating these conditions into our more general framework is not straightforward. First, x refers here to the consumption of *all* varieties. Yet, when preferences are additive σ does not depend on the whole consumption profile $\mathbf{x}$. Second, the pro-competitive effects associated with entry are not equivalent to assuming (17) because x and N are tied together at the equilibrium. Consequently, the answer to the question raised in the title of this subsection is a priori unclear. Nevertheless, we can offer two insights.

First, we have seen that the love of variety is defined when the total individual consumption Nx is constant. Under the same assumption, for entry to trigger more competition, consumers must perceive varieties as closer substitutes when their number increases, which is in accordance with the common wisdom in industrial organization (Tirole, 1988, ch.7). In this event, it is readily verified that the following two relationships must hold:

$$\frac{\mathrm{d}x}{x} = -\frac{\mathrm{d}N}{N}$$
$$\frac{\mathrm{d}\sigma}{\sigma} = \frac{\partial\sigma}{\partial N}\frac{N}{\sigma}\frac{\mathrm{d}N}{N} + \frac{\partial\sigma}{\partial x}\frac{x}{\sigma}\frac{\mathrm{d}x}{x}.$$

Plugging the first expression into the second, we obtain

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}N}\right|_{Nx=\mathrm{const}} = \frac{\sigma}{N}\left[\mathcal{E}_N(\sigma) - \mathcal{E}_x(\sigma)\right].$$

Therefore, the elasticity of substitution weakly increases with N if and only if

$$\mathcal{E}_x(\sigma) \leq \mathcal{E}_N(\sigma) \tag{18}$$

holds for all $x > 0$ and $N > 0$. The condition (18) is appealing because it boils down to $\mathcal{E}_x(\sigma) < 0$ and $\mathcal{E}_N(\sigma) = 0$ for additive preferences, whereas $\mathcal{E}_x(\sigma) = 0$ and $\mathcal{E}_N(\sigma) > 0$ for homothetic preferences. What is more, (18) is less stringent than either of these conditions. Therefore, we see it as a natural generalization that generates pro-competitive effects of entry for preferences that need not be additive or homothetic. Recent empirical evidence pointing to such effects can be found in Feenstra and Weinstein (2015).

Second, as will be seen in 3.2.3, the variation of σ with respect to x captures how much firms' price react to a common change in their marginal cost (e.g. the exchange rate). Specifically, the pass-through is smaller than or equal to 100 percent if and only if $\mathcal{E}_x(\sigma) \leq 0$ holds. Although several contributions seem to back up a pass-trough smaller than 1 (Berman et al., 2011; De Loecker et al., 2014), it also fails to be unambiguous, as shown by Martin (2013) and Weyl and Fabinger (2012). Therefore, assuming $\mathcal{E}_x(\sigma) \leq 0$ appears to be too restrictive. Based on these considerations, *we find it reasonable to consider* (18) *as our most-preferred assumption.*

3 Market equilibrium

3.1 Existence and uniqueness of a symmetric free-entry equilibrium

We first determine prices, outputs and profits when the mass of firms is fixed, and then find N by using the zero-profit condition. When N is exogenously given, the market equilibrium is given by the functions $\bar{\mathbf{q}}(N)$, $\bar{\mathbf{p}}(N)$ and $\bar{\mathbf{x}}(N)$ defined on $[0, N]$, which satisfy the following four conditions: (i) no firm i can increase its profit by changing its output, (ii) each consumer maximizes her utility subject to her budget constraint, (iii) the product market clearing condition

$$\bar{q}_i = L\bar{x}_i \qquad \text{for all } i \in [0, N]$$

and (iv) the labor market balance

$$c\int_0^N q_i \mathrm{d}i + NF = yL \tag{19}$$

hold, where we have assumed that each consumer is endowed with y efficiency units of labor. The study of market equilibria where the number of firms is exogenous is to be viewed as an intermediate step toward monopolistic competition, where the number of firms is pinned down by free entry and exit.

Since we focus here on symmetric free-entry equilibria, we find it reasonable to study symmetric market equilibria, which means that the functions $\bar{\mathbf{q}}(N)$, $\bar{\mathbf{p}}(N)$ and $\bar{\mathbf{x}}(N)$ become the scalars $\bar{q}(N)$, $\bar{p}(N)$ and $\bar{x}(N)$. For this, consumers must have the same income, which holds when profits are uniformly distributed across consumers. In this case, the budget constraint (6) must be replaced by the following expression:

$$\int_0^N p_i x_i \mathrm{d}i = Y \equiv y + \frac{1}{L}\int_0^N \pi_i \mathrm{d}i. \tag{20}$$

where the unit wage has been normalized to 1. Being negligible to the market, each firm accurately treats Y as a given.

Each firm facing the same demand, the function $\pi(x_i, \mathbf{x})$ is the same for all i. In addition, (**A**) implies that $\pi(x_i, \mathbf{x})$ has a unique maximizer for any $\mathbf{x}$. As a result, the market equilibrium must be symmetric. Using $\pi_i \equiv (p_i - c)Lx_i - F$, (20) boils down to the labor market balance (19) yields the only candidate symmetric equilibrium for the per variety consumption:

$$\bar{x}(N) = \frac{y}{cN} - \frac{F}{cL}. \tag{21}$$

Therefore, $\bar{x}(N)$ is positive if and only if $N \leq Ly/F$. The product market clearing condition implies that the candidate equilibrium output is

$$\bar{q}(N) = \frac{yL}{cN} - \frac{F}{c}. \tag{22}$$

Plugging (22) into the profit maximization condition (24) shows that there is a unique candidate equilibrium price given by

$$\bar{p}(N) = c\frac{\sigma\left(\bar{x}(N), N\right)}{\sigma\left(\bar{x}(N), N\right) - 1}. \tag{23}$$

Clearly, if $N > Ly/F$, there exists no interior equilibrium. Accordingly, we have the following result: *If* (**A**) *holds and* $N \leq Ly/F$*, then there exists a unique market equilibrium. Furthermore, this equilibrium is symmetric.*

Rewriting the equilibrium conditions (10) along the diagonal yields

$$\bar{m}(N) \equiv \frac{\bar{p}(N) - c}{\bar{p}(N)} = \frac{1}{\sigma(\bar{x}(N), N)}. \tag{24}$$

This expression shows that, for any given N, *the equilibrium markup* $\bar{m}(N)$ *varies inversely with the elasticity of substitution.* The intuition is easy to grasp. We know from industrial organization that product differentiation relaxes competition. When the elasticity of substitution is lower, varieties are poorer substitutes, thereby endowing firms with more market power. It is, therefore, no surprise that firms have a higher markup when σ is lower. It also follows from (24) that the way σ varies with x and N shapes the properties of market outcome. In particular, this demonstrates that assuming a constant elasticity of substitution amounts to adding very strong restraints on the way the market works.

Combining (21) and (23), the equilibrium operating profits earned by a firm when there are N firms are given by

$$\bar{\pi}(N) = \frac{c}{\sigma\left(\bar{x}(N), N\right) - 1} L\bar{x}(N). \tag{25}$$

It is legitimate to ask how $\bar{\pi}(N)$ varies with the mass of firms. There is no straightforward answer to this question. However, the expression (25) suffices to show how the market outcome reacts to the entry of new firms depends on how the elasticity of substitution varies with x and N. This confirms why static comparative statics may yield ambiguous results in different setups.

We now pin down the equilibrium value of N by using the zero-profit condition. Therefore, a consumer's income is equal to her sole labor income: $Y = y$. A *symmetric free-entry equilibrium* (SFE) is described by the vector (q^*, p^*, x^*, N^*), where N^* solves the zero-profit condition

$$\pi^*(N) = F, \tag{26}$$

while $q^* = \bar{q}(N^*)$, $p^* = \bar{p}(N^*)$ and $x^* = \bar{x}(N^*)$. The Walras Law implies that the budget constraint $N^* p^* x^* = y$ is satisfied. Without loss of generality, we restrict ourselves to the domain of parameters for which $N^* < Ly/F$.

Combining (24) and (26), we obtain a single equilibrium condition given by

$$\bar{m}(N) = \frac{NF}{Ly}. \tag{27}$$

When preferences are non-homothetic, (21) and (23) show that L/F and y enter the function $\bar{m}(N)$ as two distinct parameters. This implies that L and y have a different impact on the equilibrium markup, while a hike in L is equivalent to a drop in F. However, when preferences are homothetic, it is well known that the effects of L and y on the equilibrium are the same.

For the condition (26) to have a unique solution N^* for *all* values of $F > 0$, it is necessary and sufficient that $\bar{\pi}(N)$ strictly decreases with N. Differentiating (25) with respect to N, we obtain

$$\begin{aligned}\bar{\pi}'(N) &= \bar{x}'(N)\,\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{cLx}{\sigma\left(x, yL/(cLx+F)\right)-1}\right]\Bigg|_{x=\bar{x}(N)} \\ &= -\frac{y}{cN^2}\left(\sigma - 1 - x\frac{\partial\sigma}{\partial x} + \frac{cLx}{cLx+F}\frac{yL}{cLx+F}\frac{\partial\sigma}{\partial N}\right)\Bigg|_{x=\bar{x}(N)}.\end{aligned}$$

Using (21) and (26), we find that the second term in the right-hand side of this expression is positive if and only if

$$\mathcal{E}_x(\sigma(x,N)) < \frac{\sigma(x,N)-1}{\sigma(x,N)}\left[1 + \mathcal{E}_N(\sigma(x,N))\right]. \tag{28}$$

Therefore, $\bar{\pi}'(N) < 0$ for all N if and only if (28) holds. This implies the following proposition.

Proposition 1. *Assume* (**A**). *Then, there exists a free-entry equilibrium for all* $c > 0$ *and* $F > 0$ *if and only if* (28) *holds for all* $x > 0$ *and* $N > 0$. *Furthermore, this equilibrium is unique, stable and symmetric.*

Thus, we may safely conclude that the set of assumptions required to bring into play monopolistic competition must include (28). This condition allows one to work with preferences that display a great deal of flexibility. Indeed, σ may decrease or increase with x and/or N. Evidently, (28) is satisfied when the folk wisdom conditions (17) hold. More generally, the conditions (18) and (28) define a range of possibilities which is broader than the one defined by (17).

Under additive preferences, (28) amounts to assuming that $\mathcal{E}_x(\sigma) < (\sigma-1)/\sigma$, which means that σ cannot increase "too fast" with x. In this case, as shown by (27), there exists a unique SFE while the markup function $\bar{m}(N)$ increases with N provided that the slope of m is smaller than F/Ly. In other words, a market mimicking anti-competitive effects need not preclude the existence and uniqueness of a SFE (Zhelobodko et al., 2012). When preferences are homothetic, (28) holds if and only if $\mathcal{E}_N(\sigma)$ exceeds -1, which means that varieties cannot become too differentiated when their number increases.

Local conditions. It is legitimate to ask what Proposition 1 becomes when (28) does not hold for all $x > 0$ and $N > 0$. In this case, there may exist several stable SFEs, so that Propositions 2-4 discussed below hold true for small shocks at any stable SFE. Of course, when there is multiplicity of equilibria, different patterns may arise at different equilibria because the functions $\mathcal{E}_x(\sigma)$ and $\mathcal{E}_N(\sigma)$ need not behave in the same way at each stable equilibrium.

3.2 Comparative statics

In this subsection, we study the impact of a shock in the GDP on the SFE. A higher total income may stem from a larger population L, a higher per capita income y, or both. Next, we will discuss the impact of firm's productivity. To achieve our goal, it proves to be convenient to work with the markup as the endogenous variable. Setting $m \equiv FN/(Ly)$, we may rewrite the equilibrium condition (27) in terms of m only:

$$m\sigma\left(\frac{F}{cL}\frac{1-m}{m}, \frac{Ly}{F}m\right) = F. \tag{29}$$

Note that (29) involves the four structural parameters of the economy: L, y, c and F. Furthermore, it is readily verified that the left-hand side of (29) increases with m if and only if (28) holds. Therefore, to study the impact of a specific parameter, we only have to find out how the corresponding curve is shifted. In our comparative static analysis, we will refrain from following an encyclopedic approach in which all cases are systematically explored.

3.2.1 The impact of population size

Let us first consider the impact of L on the market price p^*. Differentiating (29) with respect to L, we find that the right-hand side of (29) is shifted upwards under an increase in L if and only if (18) holds. As a consequence, the equilibrium markup m^*, whence the equilibrium price p^*, decreases with L. This is in accordance with Handbury and Weinstein (2015) who observe that the price level for food products falls with city size. Second, the zero-profit condition implies that L always shifts p^* and q^* in opposite directions. Therefore, firm sizes are larger in bigger markets, as suggested by the empirical evidence provided by Manning (2010).

How does N^* change with L? Differentiating (25) with respect to L, we have

$$\left.\frac{\partial\bar{\pi}}{\partial L}\right|_{N=N^*} = \frac{cx}{\sigma(x,N)-1} + \frac{\partial\bar{x}(N)}{\partial L}\frac{\partial}{\partial x}\left.\left(\frac{cLx}{\sigma(x,N)-1}\right)\right|_{x=x^*,N=N^*}. \tag{30}$$

Substituting F for $\bar{\pi}(N^*)$ and simplifying, we obtain

$$\left.\frac{\partial\bar{\pi}}{\partial L}\right|_{N=N^*} = \left.\left[\frac{cx\sigma}{(\sigma-1)^3}\left(\sigma-1-\mathcal{E}_x(\sigma)\right)\right]\right|_{x=x^*,N=N^*}.$$

Since the first term in the right-hand side of this expression is positive, (30) is positive if and only if the following condition holds:

$$\mathcal{E}_x(\sigma) < \sigma - 1. \tag{31}$$

In this case, a population growth triggers the entry of new firms. Otherwise, the mass of varieties falls with the population size. Indeed, when $\mathcal{E}_x(\sigma)$ exceeds $\sigma - 1$, increasing the individual consumption makes varieties much closer substitutes, which intensifies competition. Under such circumstances, the benefits associated with diversity are low, implying that consumers value more the volumes they consume. This in turn leads a certain number of firms to get out of business. Furthermore, when the mass of firms increases with L, the labor market balance condition implies that N^* rises less than proportionally because q^* also increases with L. Observe also that (28) implies (31) when preferences are additive, while (31) holds true under homothetic preferences because $\mathcal{E}_x(\sigma) = 0$.

The following proposition comprises a summary.

Proposition 2. *If $\mathcal{E}_x(\sigma)$ is smaller than $\mathcal{E}_N(\sigma)$ at the SFE, then a higher population size results in a lower markup and larger firms. Furthermore, the mass of varieties increases with L if and only if* (31) *holds in equilibrium.*

What happens when $\mathcal{E}_x(\sigma) > \mathcal{E}_N(\sigma)$ at the SFE? In this event, a higher population size results in a higher markup, smaller firms, a more than proportional rise in the mass of varieties, and a lower per variety consumption. In other words, a larger market would generate anti-competitive effects, which do not seem very plausible.

3.2.2 The impact of individual income

We now come to the impact of the per capita income on the SFE. One expects a positive shock on y to trigger the entry of new firms because more labor is available for production. However, consumers have a higher willingness-to-pay for the incumbent varieties and can afford to buy each of them in a larger volume. Therefore, the impact of y on the SFE is a priori ambiguous.

Differentiating (29) with respect to y, we see that the left-hand side of (29) is shifted downwards by an increase in y if and only if $\mathcal{E}_N(\sigma) > 0$. In this event, the equilibrium markup decreases with y. To check the impact of y on N^*, we differentiate (25) with respect to y and get

$$\left.\frac{\partial \bar{\pi}(N)}{\partial y}\right|_{N=N^*} = \left.\left[\frac{\partial \bar{x}(N)}{\partial y}\frac{\partial}{\partial x}\left(\frac{cLx}{\sigma\left(x, N\right) - 1}\right)\right]\right|_{x=x^*, N=N^*}.$$

After simplification, this yields

$$\left.\frac{\partial \bar{\pi}(N)}{\partial y}\right|_{N=N^*} = \left.\frac{L}{N}\frac{\sigma - 1 - \sigma\mathcal{E}_x(\sigma)}{(\sigma - 1)^2}\right|_{x=x^*, N=N^*}.$$

Hence, $\partial\bar{\pi}(N^*)/\partial y > 0$ if and only if the following condition holds:

$$\mathcal{E}_x(\sigma) < \frac{\sigma - 1}{\sigma}, \tag{32}$$

a condition more stringent than (31). Thus, if $\mathcal{E}_N(\sigma) > 0$, then (32) implies (28). As a consequence, we have:

Proposition 3. *If $\mathcal{E}_N(\sigma) > 0$ at the SFE, then a higher per capita income results in a lower* *markup and bigger firms. Furthermore, the mass of varieties increases with y if and only if* (32) *holds in equilibrium.*

Thus, when entry renders varieties less differentiated, the mass of varieties need not rise with income. This is because the decline in prices is too strong for more firms to operate at a larger scale.

3.2.3 The impact of firm productivity

Firms' productivity (trade barriers) is typically measured by marginal costs (trade costs). To uncover the impact on the market outcome of a productivity shock common to all firms, we conduct a comparative static analysis of the SFE with respect to c and show that *the nature of preferences determines the extent of the pass-through.* The left-hand side of (29) is shifted downwards under a decrease in c if and only if

$$\mathcal{E}_x(\sigma) < 0 \tag{33}$$

holds. In this case, both the equilibrium markup m^* and the equilibrium mass of firms $N^* = (yL/F) \cdot m^*$ increases with c. In other words, when $\mathcal{E}_x(\sigma) < 0$, the pass-through is smaller than 1. This is because varieties becomes more differentiated, which relaxes competition.

It must be kept in mind that the price change occurs through the following three channels. First, when facing a change in its own marginal cost, a firm changes its price more or less proportionally by balancing the impact of the cost change on its markup and market share. Second, since all firms face the same cost change, they all change their prices, which affects the toughness of competition and, thereby, the prices set by the incumbents. Third, as firms change their pricing behavior, the number of firms in the market changes, changing the markup of the active firms. Under homothetic preferences, the markup remains the same regardless of the productivity shock, implying that the pass-through is equal to 1. Indeed, we have seen that the markup function $m(\cdot)$ depends only upon N, and thus (27) does not involve c as a parameter.

The impact of technological shocks on firms' size leads to ambiguous conclusions. For example, under quadratic preferences, q^* may increase and, then, decreases in response to a steadily drop in c.

The following proposition comprises a summary.

Proposition 4. *If firms face a drop in their marginal production cost,* (i) *the market price decreases and* (ii) *the markup and number of firms increase if and only if* (33) *holds at the SFE.*

This proposition has an important implication. If the data suggest a pass-through smaller than 1, then it must be that $\mathcal{E}_x(\sigma) < 0$. In this case, (31) must hold while (28) is satisfied when $\mathcal{E}_N(\sigma)$ exceeds -1, thereby a bigger or richer economy is more competitive and more diversified than a smaller or poorer one.

Remark. When $\mathcal{E}_x(\sigma) > 0$, the pass-through exceeds 1, so that p^* decreases more than proportionally with c. As noticed in 2.3.2, though rather uncommon, a pass-through larger than 1 cannot be ruled out a priori.

3.2.4 Summary

Let us make a pause and recall our main results. We have found a necessary and sufficient condition for the existence and uniqueness of a SFE (Proposition 1) and provided a complete characterization of the effect of a market size or productivity shock (Propositions 2 to 4). Given that (17) implies (28), (18) and (32), we may conclude as follows: *if* (17) *holds, a unique SFE exists* (Proposition 1), *a larger market or a higher income leads to lower markups, bigger firms and a larger number of varieties* (Propositions 2 and 3), *and the pass-through rate is smaller than* 1 (Proposition 4). However, Propositions 1-4 still hold under conditions more general than (17). These conditions define the shaded area in Figure 1, in which (17) is described by the fourth quadrant.

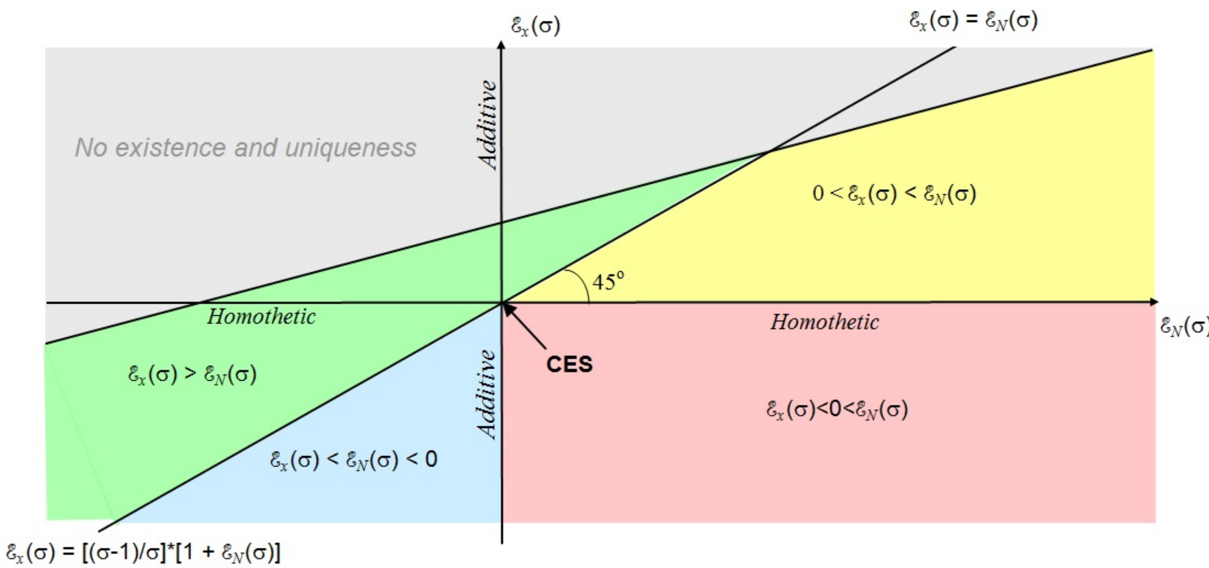

Fig. 1. The space of preferences

3.3 Monopolistic versus oligopolistic competition

It should be clear that Propositions 4-6 have the same nature as results obtained in similar comparative analyses conducted in oligopoly theory (Vives, 1999). They may also replicate less standard anti-competitive effects under some specific conditions (Chen and Riordan, 2008).

The markup (24) stems directly from preferences through the sole elasticity of substitution because we focus on monopolistic competition. However, in symmetric oligopoly models the markup emerges as the outcome of the interplay between preferences *and* strategic interactions. Nevertheless, at least to a certain extent, both settings can be reconciled.

To illustrate, consider the case of an integer number N of quantity-setting firms and of an arbitrary utility $U(x_1, ..., x_N)$. The inverse demands are given by

$$p_i = \frac{U_i}{\lambda} \qquad \lambda = \frac{1}{Y}\sum_{j=1}^{N} x_j U_j.$$

Assume that firms do not manipulate consumers' income through profit redistribution. Firm i's profit-maximization condition is then given by

$$\frac{p_i - c}{p_i} = -\frac{x_i U_{ii}}{U_i} + \mathcal{E}_{x_i}(\lambda) = -\frac{x_i U_{ii}}{U_i} + \frac{x_i U_i + \sum_{j=1}^{N} x_i x_j U_{ij}}{\sum_{j=1}^{N} x_j U_j}. \tag{34}$$

Set

$$r_{\mathrm{o}}(x, N) \equiv -\frac{x U_{ii}(x, ..., x)}{U_i(x, ..., x)} \qquad r_{\mathrm{c}}(x, N) \equiv \frac{x U_{ij}(x, ..., x)}{U_i(x, ..., x)} \quad \text{for } j \neq i.$$

The symmetry of preferences implies that $r_{\mathrm{o}}(x, N)$ and $r_{\mathrm{c}}(x, N)$ are independent of i and j.

Combining (34) with the symmetry condition $x_i = x$, we obtain the markup condition:

$$\frac{p - c}{p} = \frac{1}{N} + \left(1 - \frac{1}{N}\right)\left[r_{\mathrm{o}}(x, N) + r_{\mathrm{c}}(x, N)\right]. \tag{35}$$

The elasticity of substitution s_{ij} between varieties i and j is given by (see Nadiri, 1982, p. 442)

$$s_{ij} = -\frac{U_i U_j (x_i U_j + x_j U_i)}{x_i x_j \left[U_{ii} U_j^2 - 2U_{ij} U_i U_j + U_{jj} U_i^2\right]}. \tag{36}$$

When the consumption pattern is symmetric, (36) boils down to

$$s(x, N) = \frac{1}{r_{\mathrm{o}}(x, N) + r_{\mathrm{c}}(x, N)}. \tag{37}$$

Combining (35) with (37), we get

$$\frac{p - c}{p} = \frac{1}{N} + \left(1 - \frac{1}{N}\right)\frac{1}{s(x, N)}. \tag{38}$$

Unlike the profit-maximization condition (38), product and labor market balance, as well as the zero-profit condition, do not depend on strategic considerations. Since

$$\frac{p - c}{p} = \frac{1}{\sigma(x, N)} \tag{39}$$

under monopolistic competition, comparing (39) with (38) shows that *the set of Cournot symmetric free-entry equilibria is the same as the set of equilibria obtained under monopolistic competition if and only if* $\sigma(x, N)$ *is given by*

$$\frac{1}{\sigma(x,N)} = \frac{1}{N} + \left(1 - \frac{1}{N}\right)\frac{1}{s(x,N)}.$$

As a consequence, by choosing appropriately the elasticity of substitution as a function of x and N, monopolistic competition is able to replicate not only the direction of comparative static effects generated in symmetric oligopoly models with free entry, but also their magnitude. Hence, we find it to say that *monopolistic competition under non-separable preferences mimics oligopolistic competition.* As a consequence, Propositions 2-4 hold true under oligopolistic competition under free entry and symmetric preferences.

4 When is the free-entry equilibrium socially optimal?

In this section, our aim is to delve deeper into a variety of issues discussed in the literature on optimum product diversity. Since working with general preferences renders the formal analysis more complex without adding anything important to our results, we consider the case homothetic preferences. Without loss of generality, we assume that U is homogeneous of degree one in $\mathbf{x}$. In the case of symmetric consumption profiles $\mathbf{x} = xI_{[0,N]}$, we have

$$U\left(xI_{[0,N]}\right) \equiv \phi(N,x) = x\phi(N,1).$$

Setting $\psi(N) \equiv \phi(N,1)/N$ and $X \equiv xN$, we obtain

$$\phi(X,N) = X\psi(N). \tag{40}$$

Hence, at a symmetric consumption pattern $x_i = x$, homothetic preferences are *separable* in the total consumption X and the mass N of varieties. Preferences exhibit a love for variety if and only if $\psi(N)$ increases with N.

4.1 The social optimum

The planner aims to solve the following optimization problem:

$$\max U(\mathbf{x}) \qquad \text{s.t.} \quad L = c\int_0^N q_i \mathrm{d}i + NF \quad \text{and} \quad q_i = Lx_i.$$

Using symmetry, the socially optimal outcome is given by the solution to

$$\max_{X,N} X\psi(N) \qquad \text{s.t.} \quad L = cLX + NF. \tag{41}$$

subject to

(i) the labor balance condition

$$c\int_0^N q_i + fN = L,$$

(ii) and the availability constraints:

$$x_i = 0 \text{ for all } i \in]N, \mathbb{N}].$$

It is reasonable to assume that X and N are substitutes. This is so if and only if $1/\psi(N)$ is convex. It is readily verified that this condition is equivalent to the inequality:

$$\frac{\psi''(N)}{\psi'(N)} < 2\frac{\psi'(N)}{\psi(N)}. \tag{42}$$

The following result provides a characterization of the optimum (for the proof see Appendix 4 in the Supplemental Material).

Proposition 5. *Assume consumers are variety-lovers while X and N are substitutes. Then, there exists a unique social optimum. Furthermore, the optimum involves a positive range of varieties if and only if*

$$\frac{L}{F} > \lim_{N\to 0}\frac{\psi(N)}{\psi'(N)}. \tag{43}$$

Otherwise, the optimum is given by the corner solution

$$X^O = \frac{1}{c} \qquad N^O = 0. \tag{44}$$

Observe that the optimum may involve the supply of a single variety even when consumers are variety-lovers. Indeed, the labor balance constraint may be rewritten as follows:

$$X = \frac{1}{c} - \frac{F}{cL}N.$$

Therefore, the unique solution of the social planner's problem is the corner solution given by (44) if and only if the slope $F/(cL)$ exceeds the slope of the indifference curve at $(0, 1/c)$ in the plane (N, X). Put differently, the marginal rate of substitution between X and N is too small for more than one variety to be produced.

4.2 Is there over- or under-provision of diversity?

It is well known that the comparison of the social optimum and market outcome often leads to ambiguous conclusions (Spence, 1976). We illustrate this difficulty in the case of homothetic preferences.

4.2.1 When do the equilibrium and optimum coincide under homothetic preferences?

The ratio of the first-order conditions of (41) is given by

$$X \frac{\psi'(N)}{\psi(N)} = \frac{F}{cL}. \tag{45}$$

Using $X \equiv xN$, we may rewrite (45) as follows:

$$\mathcal{E}_{\psi}(N) \equiv N \frac{\psi'(N)}{\psi(N)} = \frac{F}{cLx}. \tag{46}$$

As for the market equilibrium condition (27), it may be reformulated as follows:

$$\frac{m(N)}{1 - m(N)} = \frac{F}{cLx}. \tag{47}$$

Comparing (46) and (47) shows that the social optimum and the market equilibrium are identical if and only if

$$\mathcal{E}_{\psi}(N) = \frac{m(N)}{1 - m(N)}, \tag{48}$$

while it is readily verified that there is excess (resp., insufficient) variety if and only if the right-hand side term of (48) is larger (resp., smaller) than the left-hand side term.

Clearly, the condition (48) is unlikely to be satisfied unless some strong restrictions are imposed on preferences. The general belief is that this condition holds only for the CES. Yet, we find it natural to ask whether there are other homothetic preferences for which the SFE is optimal. In the next proposition, we show that there exists a mapping from the set of homothetic preferences into itself such that the SFE and the optimum coincide for an infinite set of homothetic preferences, which includes the CES (see Appendix A for a proof). However, as shown by Example 1, there are homothetic preferences that do not satisfy this property, even when markups are constant along the diagonal. Moreover, as shown by Example 1 below, there exist homothetic preferences that do not satisfy this property, even when markups are constant along the diagonal. As a consequence, working with a subset of homothetic preferences may generate versatile welfare properties, which means that care is needed when drawing policy recommendations based on models that use homothetic preferences and monopolistic competition. As a consequence, working with a subset of homothetic preferences may generate welfare properties that do not hold for others, which means that care is needed when drawing policy recommendations based on models that use homothetic preferences and monopolistic competition.

Proposition 6. *For any homothetic utility $U(\mathbf{x})$, there exists an homothetic utility $V(\mathbf{x})$ that is generically non-CES such that the market equilibrium and optimum coincide for the homothetic utility given by $[U(\mathbf{x})V(\mathbf{x})]^{1/2}$.*

Example 1. The equivalence does not hold for all homothetic preferences, even when the markup is constant along the diagonal.

To see it, consider the following class of generalized CES preferences:

$$\mathcal{U}(\mathbf{x}) = \mathbb{E}\left[\ln\left(\int_0^{\mathbb{N}} x_i^\rho \mathrm{d}i\right)\right], \tag{49}$$

where $0 < \rho < 1$ is distributed according to the probability cumulative distribution $H(\rho)$ over $[0, 1]$. When this distribution is degenerate, (49) is equivalent to the standard CES. The idea behind (49) is that consumers are unsure about the degree of differentiation across varieties.

It is readily verified that the elasticity of substitution $\bar{\sigma}(x, \mathbf{x})$ is now given by

$$\bar{\sigma}(x, \mathbf{x}) = \frac{\mathbb{E}\left(\rho \frac{x^{\rho-1}}{\int_0^N x_j^\rho \mathrm{d}j}\right)}{\mathbb{E}\left[(\rho - \rho^2)\frac{x^{\rho-1}}{\int_0^N x_j^\rho \mathrm{d}j}\right]}, \tag{50}$$

which is variable, implying that (49) is non-CES. Regardless of the shape of the distribution $H(\rho)$, (49) describes a strictly convex symmetric preference over $L_2([0, \mathbb{N}])$.[4] Therefore, as in the foregoing, it is legitimate to focus on symmetric outcomes.

Evaluating $\bar{\sigma}(x, \mathbf{x})$ at a symmetric outcome $\mathbf{x} = xI_{[0,N]}$ yields

$$\sigma(x, N) = \frac{\mathbb{E}(\rho)}{\mathbb{E}(\rho) - \mathbb{E}(\rho^2)} = \frac{\mathbb{E}(\rho)}{\mathbb{E}(\rho) - [\mathbb{E}(\rho)]^2 - \mathrm{Var}(\rho)}, \tag{51}$$

the value of which depends on the distribution H. Hence, at any symmetric outcome, everything work as if preferences were CES with a constant elasticity of substitution given by (51). Following the line of Appendix B, the SFE can be shown to be optimal if and only if

$$\mathrm{Var}(\rho) = 0.$$

This amounts to assuming that the distribution of ρ is degenerate. If not, the SFE is *not* optimal.

While Example 1 shows that a constant elasticity of substitution is not a necessary condition for the optimality of the equilibrium, Proposition 6 relies on homotheticity, a property shared by the CES. It is therefore legitimate to ask whether homotheticity is a necessary condition for the SFE to be optimal. The example below shows that it is not.

Example 2. The equivalence may hold for non-homothetic preferences displaying a variable markup. This is shown for the following class of non-homothetic preferences (see Appendix B):

$$\mathcal{U}(\mathbf{x}) = \mathbb{E}\left[\int_0^{\mathbb{N}} (\ln x_i^r + 1)^{\frac{\rho}{r}} \mathrm{d}i\right], \tag{52}$$

[4] Symmetry holds because any Lebesgue-measure preserving mapping of $[0, \mathbb{N}]$ into inself preserves the value of $\ln\left(\int_0^{\mathbb{N}} x_i^\rho \mathrm{d}i\right)$ for any $\rho \in]0, 1[$.

where $r \equiv \text{Var}(\rho)/\mathbb{E}(\rho)$. Observe that the constant r is positive and smaller than 1 because ρ is distributed over the interval $[0, 1]$. When the distribution is degenerate, (52) boils down to the CES with the elasticity of substitution equal to $1/(1-\rho)$.

To sum up, a constant markup is neither a necessary nor a sufficient condition for the market equilibrium to be optimal.

Remark. Examples 1 and 2 shed further light on σ as the primitive of the model (see Section 2.3). We know that standard CES preferences imply that σ is constant everywhere, hence along the diagonal. Example 1 shows that there exist non-CES symmetric preferences for which σ is constant along the diagonal. Regarding now Example 2, (B.1) in Appendix B implies

$$\sigma(x, N) = \frac{r \ln x + 1}{r \ln x + 1 - \mathbb{E}(\rho)}.$$

Therefore, even though (52) is homothetic and not additive, along the diagonal σ depends solely on x, as in the case of additive preferences.

4.2.2 The optimal shifter

Another way to approach the diversity issue is to proceed along the line suggested by Dixit and Stiglitz (1974) who argued that the mass of varieties could enter the utility functional as a specific argument. Given $\phi(X, N)$, it is natural to map this function into another homothetic preference $\mathbb{A}(N)\phi(X, N)$, where $\mathbb{A}(N)$ is a *shifter* that depends on N only. Observe that the utility $\mathbb{A}(N)U(\mathbf{x})$ is homothetic and generates the same equilibrium outcome as $U(\mathbf{x})$, for the elasticity of substitution $\sigma(N)$ is unaffected by introducing the shifter $\mathbb{A}(N)$.

An example of shifter used in the literature is given by the augmented-CES:

$$\mathcal{U}(\mathbf{x}, N) \equiv N^{\nu} \left(\int_0^{\mathbb{N}} x_i^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\sigma/(\sigma-1)}. \tag{53}$$

In Benassy (1996), ν is a positive constant that captures the consumer benefit of a larger number of varieties. The idea is to separate the love-for-variety effect from the competition effect generated by the degree of product differentiation, which is inversely measured by the elasticity of substitution σ. Blanchard and Giavazzi (2003) takes the opposite stance by assuming σ in (53) to increase with N and setting $\nu = -1/[\sigma(N) - 1]$. Under this specification, increasing the number of varieties does not raise consumer welfare but intensifies competition among firms.

To determine the shifter $\mathbb{A}(N)$ that guarantees optimal product diversity, we observe that (48) is to be rewritten as follows in the case of $\mathbb{A}(N)\phi(X, N)$:

$$\mathcal{E}_{\mathbb{A}}(N) + \mathcal{E}_{\psi}(N) = \frac{m(N)}{1 - m(N)}. \tag{54}$$

We then show in Appendix C that *there always exists a shifter* $A(N)$ *such that* (54) *holds for all*

N if and only if $U(x)$ is replaced with $A(N)U(x)$. What the shifter does is to align the optimum to the equilibrium, which remains the same.

Furthermore, it is readily verified that there is excess (insufficient) variety if and only if the right-hand side term of (54) is larger (smaller) than the left-hand side term. We can even go one step further. Indeed, if we use the shifter $N^{\nu}\mathbb{A}(N)$, there is growing under-provision of varieties when the difference $\nu - 1/(\sigma(N^*) - 1) < 0$ falls, but growing over-provision when $\nu - 1/(\sigma(N^*) - 1) > 0$ rises. Therefore, for any positive or negative number Δ there exists a shifter such that $N^* - N^o = \Delta$. In other words, by taking a power transformation of $N^{\nu}\phi(N, x)$, we can render the discrepancy between the equilibrium and the optimum arbitrarily large, or arbitrarily small, by changing the value of ν.

In sum, *by choosing the appropriate shifter, the gap between the market equilibrium and the social optimum can be made equal to any arbitrary positive or negative constant.*

5 Extensions

In this section, we extend our baseline model to the cases of heterogeneous agents. Needless to say, assuming identical consumers and symmetric firms is very restrictive. We first discuss the case of heterogeneous consumers, which is almost untouched in the literature. We then discuss the case of heterogeneous firms where the literature is huge. We conclude this section by discussing the case of a multi-sector economy.

5.1 Heterogeneous consumers

Accounting for consumer heterogeneity is not easy but doable. Assume that consumers have different labor incomes $y \in \mathbb{R}_+$ and tastes parametrized by $\theta \in \Theta \subset \mathbb{R}^n$. For example, θ may describe the ideal variety of the consumers whose type is θ. Denoting by $\mathcal{D}(p_i, \mathbf{p}; y, \theta)$ the Marshallian demand for variety i of a (y, θ)-type consumer, the aggregate demand faced by firm i is given by

$$\Delta(p_i, \mathbf{p}) \equiv L \int_{\mathbb{R}_+ \times \Theta} \mathcal{D}(p_i, \mathbf{p}; y, \theta) \mathrm{d}G(y, \theta), \tag{55}$$

where G is a continuous joint probability distribution of income y and taste θ.

Ever since the Sonnenschein-Mantel-Debreu theorem (Mas-Colell et al., 1995, ch.17), it is well known that the aggregate demand function need not inherit the properties of the individual demand functions. However, for each variety i the aggregate demand $\Delta(p_i, \mathbf{p})$ is decreasing in p_i regardless of the income-taste distribution G. Other properties of $\mathcal{D}$ crucially depend on the relationship between income and taste. Indeed, since firm i's profit is given by $\pi(p_i, \mathbf{p}) = (p_i - c)\Delta(p_i, \mathbf{p}) - F$, the first-order condition for a symmetric equilibrium becomes

$$p\left[1-\frac{1}{\varepsilon(p,N)}\right]=c, \tag{56}$$

where $\varepsilon(p,N)$ is the price elasticity of $\Delta(p,\mathbf{p})$ evaluated at the symmetric outcome. If $\varepsilon(p,N)$ is an increasing function of p and N, most of the results derived in the above sections hold true. Indeed, integrating consumers' budget constraints across $\mathbb{R}_+\times\Theta$ and applying the zero-profit condition yields the markup.

$$m(N)=\frac{NF}{LY} \qquad \text{where} \quad Y\equiv\int_{\mathbb{R}_+\times\Theta} y\mathrm{d}G(y,\theta). \tag{57}$$

Note that (57) differs from (27) only in one respect: the individual income y is replaced with the mean income Y, which is independent of L. Consequently, if $\varepsilon(p,N)$ decreases both with p and N, a population hike or a productivity shock affects the SFE as in the baseline model (see Propositions 2 and 4). In contrast, the impact of an increase in Y is ambiguous because it depends on how θ and y are related.

There is no reason to expect the aggregate demand to exhibit an increasing price elasticity even when the individual demands satisfy this property. To highlight the nature of this difficulty, we show in Appendix 5 in the Supplemental Material that

$$\begin{gathered}\frac{\partial\varepsilon(p,N)}{\partial p}=\int_{\mathbb{R}_+\times\Theta}\frac{\partial\varepsilon(p,N;\,y,\theta)}{\partial p}s(p,N;\,y,\theta)\mathrm{d}G(y,\theta)-\\ -\frac{1}{p}\int_{\mathbb{R}_+\times\Theta}\left[\varepsilon(p,N;\,y,\theta)-\varepsilon(p,N)\right]^2 s(p,N;\,y,\theta)\mathrm{d}G(y,\theta),\end{gathered} \tag{58}$$

where $\varepsilon(p,N;\,y,\theta)$ is the price elasticity of $\mathcal{D}(p_i,\mathbf{p};y,\theta)$ evaluated at a symmetric price outcome ($p_i=p_j=p$), while $s(p,N;\,y,\theta)$ stands for the share of demand of the (y,θ)-type consumers in the aggregate demand, evaluated at the same symmetric price outcome:

$$s(p,N;y,\theta)\equiv\left.\frac{\mathcal{D}(p,\mathbf{p};\,y,\theta)}{\Delta(p,\mathbf{p})}\right|_{\mathbf{p}=pI_{[0,N]}}. \tag{59}$$

The expression (58) shows that the effect of heterogeneity in tastes and income generally differ. In particular, consumers with different incomes and identical tastes have different willingness-to-pay for the same variety, which increases the second term in (58). In contrast, when consumers have the same income but differ in their ideal variety, one may expect the second term in (58) to be close to zero when the market provides those varieties. Because the second term of (58) is negative, the market demand may exhibit decreasing price elasticity even when individual demands display increasing price elasticities. Nevertheless, (58) has an important implication.

Proposition 7. *If individual demand elasticities are increasing and their variance is not too large, then the elasticity of the aggregate demand is increasing.*

In this case, all the properties of Section 4 hold true. This is because the elasticity of the aggregate demand satisfies (**Abis**) when the individual demands also satisfy this property while consumers are not too different. However, when consumers are very dissimilar, like in the Sonnenschein-Mantel-Debreu theorem, the aggregate demand may exhibit undesirable properties.

5.2 Heterogeneous firms

It is natural to ask whether the approach developed in this paper can cope with Melitz-like heterogeneous firms? In this event, the consumption pattern ceases to be symmetric, making the problem infinitely dimensional. Yet, all firms of a given type will supply the same output. As shown below, making the elasticity of substitution type-specific will allow us to use σ for studying heterogeneous firms at the cost of one only additional dimension, i.e. the firm's type.

In what follows, we consider the one-period framework used by Melitz and Ottaviano (2008), the mass of potential firms being given by $\mathbb{N}$. Prior to entry, risk-neutral firms face uncertainty about their marginal cost while entry requires a sunk cost F_e. Once this cost is paid, firms observe their marginal cost drawn randomly from the continuous probability distribution $\Gamma(c)$ defined over $\mathbb{R}_+$, with a density $\gamma(c)$. After observing its type c, each entrant decides to produce or not, given that an active firm must incur a fixed production cost F. Under such circumstances, the mass of entrants, N_e, typically exceeds the mass of operating firms, N. Even though varieties are differentiated from the consumer's point of view, firms sharing the same marginal cost c behave in the same way and earn the same profit at equilibrium. As a consequence, we may refer to any variety/firm by its c-type only.

The equilibrium conditions are as follows:

(i) the profit-maximization condition for c-type firms:

$$\max_{x_c} \Pi_c(x_c, \mathbf{x}) \equiv \left[\frac{D\left(x_c, \mathbf{x}\right)}{\lambda} - c \right] L x_c - F; \tag{60}$$

(ii) the zero-profit condition for the cutoff firm:

$$(p_{\bar{c}} - \hat{c}) q_{\hat{c}} = F,$$

where $\hat{c}$ is the cutoff cost. At the equilibrium, firms are sorted out by decreasing order of productivity, which implies that the mass of active firms is equal to $N \equiv N_e \Gamma(\hat{c})$;

(iii) the product market clearing condition:

$$q_c = L x_c$$

for all $c \in [0, \hat{c}]$;

(iv) the labor market clearing condition:

$$N_eF_e + \int_0^{\hat{c}} (F + cq_c)\mathrm{d}\Gamma(c) = yL;$$

(v) firms enter the market until their expected profits net of the entry cost F_e are zero:

$$\int_0^{\hat{c}} \Pi_c(x_c, \mathbf{x})\mathrm{d}\Gamma(c) = F_e. \tag{61}$$

Since the distribution Γ is given, the profit-maximization condition implies that the equilibrium consumption profile is entirely determined by the set of active firms, which is fully described by $\hat{c}$ and N_e. In other words, a variety supplied by an active firm can be viewed as a point in the set

$$\Omega \equiv \{(c, \nu) \in \mathbb{R}^2_+ \mid c \leq \hat{c}; \nu \leq N_e\gamma(c)\}.$$

In the case of homogeneous firms, the variable N is sufficient to describe the set of active firms, so that $\Omega = [0, N]$.

It follows from (60) that there is perfect sorting across firms' types at any equilibrium, and thus firms with a higher productivity earn higher profits. For any c_i and c_j, it also follows from (60) that

$$\frac{D(x_{c_i}, \mathbf{x})\left[1 - \bar{\eta}(x_{c_i}, \mathbf{x})\right]}{D(x_{c_j}, \mathbf{x})\left[1 - \bar{\eta}(x_{c_j}, \mathbf{x})\right]} = \frac{c_i}{c_j}. \tag{62}$$

The condition (**A**) of Section 2.2 implies that, for any given $\mathbf{x}$, a firm's marginal revenue $D(x, \mathbf{x})\left[1 - \bar{\eta}(x, \mathbf{x})\right]$ decreases with x regardless of its marginal cost. Therefore, it ensues from (62) that $x_i > x_j$ if and only if $c_i < c_j$. In other words, more efficient firms produce more than less efficient firms. Furthermore, since $p_i = D(x, \mathbf{x})/\lambda$ and D decreases in x for any given $\mathbf{x}$, more efficient firms sell at lower prices than less efficient firms. As for the markups, (62) yields

$$\frac{p_i/c_i}{p_j/c_j} = \frac{1 - \bar{\eta}(x_{c_j}, \mathbf{x})}{1 - \bar{\eta}(x_{c_i}, \mathbf{x})}.$$

Consequently, more efficient firms enjoy higher markups – as in De Loecker and Warzynski (2012) – if and only if $\bar{\eta}(x, \mathbf{x})$ increases with x, i.e., (**Abis**) holds. Therefore, if (**A**) holds more efficient firms produce larger outputs and charge lower prices than less efficient firms. In addition, *more efficient firms have higher markups if and only if* (**Abis**) *holds.*

Very much as in 3.1 where N is treated parametrically, we assume for the moment that $\hat{c}$ and N_e are given, and consider the game in which the corresponding active firms compete in quantities. Because we work with general preferences, the quantity game cannot be solved pointwise. Indeed, the profit-maximizing output of a c-type firm depends on what the other types of firms do. We show in Appendix 6 of the Supplemental Material that, for any $\hat{c}$ and N_e, there exists an equilibrium $\bar{\mathbf{x}}(\bar{c}, N_e)$ of the quantity game. Observe that the counterpart of $\bar{\mathbf{x}}(\hat{c}, N_e)$ in the case of symmetric firms is $\bar{x}(N)$ given by (21). Furthermore, because all the c-type firms sell at the same price that depends on c, the consumption of a variety is c-specific, which makes c-specific the consumption of the corresponding c-type varieties.

The operating profits of a c-type firm made at an equilibrium $\mathbf{x}^*(\bar{c}, N_e)$ of the quantity game are as follows:

$$\bar{\pi}_c(\hat{c}, N_e) \equiv \max_{x_c} \left[\frac{D\left(x_c, \bar{\mathbf{x}}(\hat{c}, N_e)\right)}{\lambda\left(\bar{\mathbf{x}}(\hat{c}, N_e)\right)} - c \right] L x_c,$$

which is the counter-part of $\bar{\pi}(N)$ in the case of heterogeneous firms. Note that the perfect sorting of firms implies that $\bar{\pi}_c(\hat{c}, N_e)$ decreases with c.

A free-entry equilibrium with heterogeneous firms is defined by a pair $(\hat{c}^*, N_e^*)$ that satisfies the zero-expected-profit condition for each firm:

$$\int_0^{\hat{c}} [\bar{\pi}_c(\hat{c}, N_e) - F] \mathrm{d}\Gamma(c) = F_e, \tag{63}$$

as well as the cutoff condition

$$\bar{\pi}_c(\hat{c}, N_e) = F. \tag{64}$$

Thus, regardless of the nature of preferences and the distribution of marginal costs, the heterogeneity of firms amounts to replacing the variable N by the two variables $\hat{c}$ and N_e because $N = \Gamma(\hat{c})N_e$ when $\bar{x}(N)$ is replaced by $\bar{\mathbf{x}}(\hat{c}, N_e)$. As a consequence, *the complexity of the problem increases from one to two dimensions.*

Dividing (63) by (64) yields the following new equilibrium condition:

$$\int_0^{\hat{c}} \left[\frac{\bar{\pi}_c(\hat{c}, N_e)}{\bar{\pi}_{\hat{c}}(\hat{c}, N_e)} - 1 \right] \mathrm{d}\Gamma(c) = \frac{F_e}{F}. \tag{65}$$

5.2.1 Making the elasticity of substitution type-specific

When firms are symmetric, we have seen that the sign of $\mathcal{E}_N(\sigma)$ plays a critical role in comparative statics. Since firms of a given type are symmetric, the same holds here. The difference is that the mass of active firms is now determined by the two endogenous variables $\hat{c}$ and N_e. As a consequence, understanding how the mass of active firms responds to a population hike requires studying the way the left-hand side of (65) varies with $\hat{c}$ and N_e. Let $\sigma_c(\hat{c}, N_e)$ be the equilibrium value of the elasticity of substitution between any two varieties supplied by c-type firms:

$$\sigma_c(\hat{c}, N_e) \equiv \bar{\sigma}[\bar{x}_c(\hat{c}, N_e), \bar{\mathbf{x}}(\hat{c}, N_e)].$$

In this case, we may rewrite $\bar{\pi}_c(\hat{c}, N_e)$ as follows:

$$\bar{\pi}_c(\hat{c}, N_e) = \frac{c}{\sigma_c(\hat{c}, N_e) - 1} L\bar{x}_c(\hat{c}, N_e), \tag{66}$$

which is the counter-part of (25). Hence, by making σ type-specific, we are able to use the elasticity of substitution for studying heterogeneous firms at the cost of one additional dimension, i.e. the firm's type c.

Using the envelope theorem and the profit-maximization condition (60), we obtain:

$$\mathcal{E}_c\left(\bar{\pi}_c(\hat{c}, N_e)\right) = 1 - \sigma_c(\hat{c}, N_e). \tag{67}$$

Combining this with (66) allows us to rewrite the equilibrium conditions (64) and (65) as follows:

$$\frac{\hat{c}}{\sigma_{\hat{c}}(\hat{c}, N_e) - 1} L\bar{x}_{\hat{c}}(\hat{c}, N_e) = F, \tag{68}$$

and

$$\int_0^{\hat{c}} \left[\exp\left(\int_c^{\hat{c}} \frac{\sigma_z(\hat{c}, N_e) - 1}{z} \mathrm{d}z\right) - 1\right] \mathrm{d}\Gamma(c) = \frac{F_e}{F}. \tag{69}$$

Let $\hat{c} = g(N_e)$ be the locus of solutions to (68) and $\hat{c} = h(N_e)$ the locus of solutions to (69).[5] A free-entry equilibrium $(\hat{c}^*, N_e^*)$ is an intersection point of the two loci $\hat{c} = g(N_e)$ and $\hat{c} = h(N_e)$ in the $(N_e, \hat{c})$-plane, and thus the properties of the equilibrium $(\hat{c}^*, N_e^*)$ depend only upon the slopes of these two curves, which in turn are determined by the behavior of $\sigma_c(\hat{c}, N_e)$. In particular, if $\sigma_c(\hat{c}, N_e)$ increases with $\hat{c}$, the left-hand side of (65) increases with $\hat{c}$. Intuitively, when $\hat{c}$ increases, the mass of firms rises as less efficient firms stay in business, which intensifies competition and lowers markups. In this case, the selection process is tougher. This is not the end of the story, however. Indeed, the competitiveness of the market also depends on how N_e affects the degree of differentiation across varieties.

5.2.2 Properties of the free-entry equilibrium

The elasticity of substitution being the keystone of our approach, it is legitimate to ask whether imposing some simple conditions on σ_c (similar to those used in Section 3) can tell us something about the slope of $g(N_e)$. The left-hand side of (69) increases with N_e if and only if $\sigma_c(\hat{c}, N_e)$ increases in N_e. This amounts to assuming that, for any given cutoff $\hat{c}$, the relative impact of entry on the low-productivity firms (i.e., the small firms) is larger than the impact on the high-productivity firms. As implied by (67), $\mathcal{E}_c\left(\bar{\pi}_c(\hat{c}, N_e)\right)$ decreases in N_e if and only if $\sigma_c(\hat{c}, N_e)$

[5] We give below sufficient conditions for the left-hand side of (69) to be monotone in $\hat{c}$ and N_e, two conditions that guarantee that the locus $\hat{c} = h(N_e)$ is well defined.

increases in N_e. This leads us to impose an additional condition that implies that firms face a more competitive market when the number of active firms rises.

(**B**) *The equilibrium profit of each firm's type decreases in* $\hat{c}$ *and* N_e.

The intuition behind this assumption is easy to grasp: a larger number of entrants or a higher cutoff leads to lower profits, for the mass of active firms rises. When firms are symmetric, the equilibrium operating profits depend only upon the number N of active firms (see (25)). Thus, (**B**) amounts to assuming that these profits decrease with N. Using Zhelobodko et al. (2012), it is readily verified that any additive preference satisfying (**A**) also satisfies (**B**).

As implied by (**B**), $g(N_e)$ is downward-sloping in the $(N_e, \hat{c})$-plane. Furthermore, it is shifted upward when L rises. As for $h(N_e)$, it is independent of L but its slope is a priori undetermined. Three cases may arise. First, if the locus $h(N_e)$ is upward-sloping, there exists a unique free-entry equilibrium, and this equilibrium is stable. Furthermore, both N_e^* and $\hat{c}^*$ increase with L (see Figure 1a). Second, under the CES preferences, $h(N_e)$ is horizontal, which implies that N_e^* rises with L while $\hat{c}^*$ remains constant.

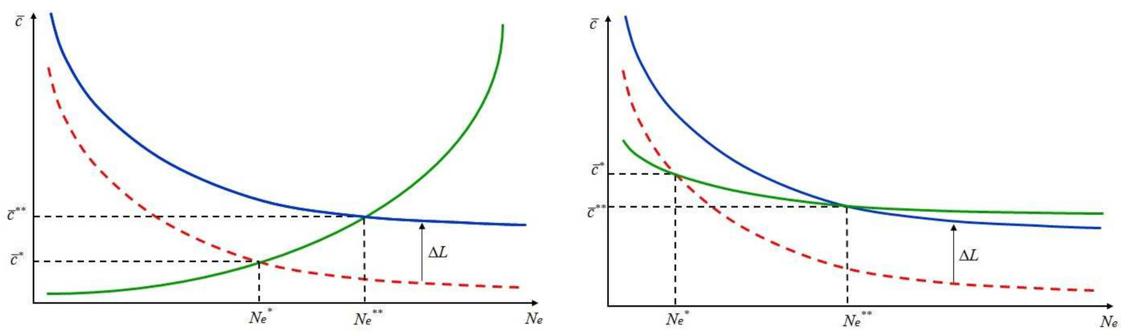

Fig. 2. Cutoff and market size

Last, when $h(N_e)$ is downward-sloping, two subcases must be distinguished. In the first one, $h(N_e)$ is less steep than $g(N_e)$. As a consequence, there still exists a unique free-entry equilibrium. This equilibrium is stable and such that N_e^* increases with L, but $\hat{c}^*$ now decreases with L (see Figure 1b). In the second subcase, $h(N_e)$ is steeper than $g(N_e)$, which implies that the equilibrium is unstable because $h(N_e)$ intersects $g(N_e)$ from below. In what follows, we focus only upon stable equilibria.

In sum, we end up with the following property:

Proposition 8. *Assume* (**B**). *Then, the equilibrium mass of entrants always increases with* L.

When $\sigma_c(\hat{c}, N_e)$ increases both with $\hat{c}$ and N_e, the locus $h(N_e)$ is downward-sloping. Indeed, when N_e rises, so does the left-hand side of (69). Hence, since $\sigma_c(\hat{c}, N_e)$ also increases with $\hat{c}$, it must be that $\hat{c}$ decreases for (69) to hold. As a consequence, we have:

Proposition 9. *Assume* (**B**). *If* $\sigma_c(\hat{c}, N_e)$ *increases with* $\hat{c}$ *and* N_e, *then the equilibrium cutoff decreases with* L. *If* $\sigma_c(\hat{c}, N_e)$ *increases with* $\hat{c}$ *and decreases with* N_e, *then* $\hat{c}^*$ *increases with* L.

Given $\hat{c}$, we know that the number of active firms N is proportional to the number of entrants N_e. Therefore, assuming that $\sigma_c(\hat{c}, N_e)$ increases with N_e may be considered as the counterpart of

(18), which is one of our most preferred assumptions in the case of symmetric firms. Indeed, as shown in Section 2.3, (18) can be reformulated as follows: $\sigma(\bar{x}(N), N)$ increases with N. In this case, the pro-competitive effect generated by entry exacerbates the selection effect across firms. In response to a hike in L, the two effects combine to induce the exit of the least efficient active firms. This echoes Melitz and Ottaviano (2008) who show that a trade liberalization shock gives rise to a similar effect under quadratic preferences. In the present setup, the impact of population size on the number of entrants remains unambiguous. In contrast, the cutoff cost behavior depends on how the elasticity of substitution $\sigma_c(\hat{c}, N_e)$ varies with N_e. In other words, even for plausible preferences generating pro-competitive effects, predictions regarding the direction of the firms' selection are inherently fragile.[6]

Note that what we said in Section 3.2 about local versus global conditions equally applies here. Indeed, when $\sigma_c(\hat{c}, N_e)$ increases with $\hat{c}$ and N_e in an neighborhood of the equilibrium, the above argument can be repeated to show that the equilibrium cutoff decreases with L for small changes in L. Note also that the result on complete pass-through under homothetic preferences shown in 3.2.3 still holds when firms are heterogeneous (for a proof see in Appendix 7 in the Supplemental Material).

Heterogeneous firms or asymmetric preferences. The assumption of symmetric preferences puts a strong structure on substitution between variety pairs. Without affecting the nature of our results, this assumption can be relaxed to capture a more realistic substitution pattern. Indeed, we have seen that varieties sharing the same marginal cost c may be viewed as symmetric, whereas varieties produced by c_i-type and c_j-type firms are asymmetric when c_i and c_j obey different substitution patterns. As a consequence, *a model with heterogeneous firms supplying symmetric varieties is isomorphic to a model with symmetric firms selling varieties whose degree of differentiation varies with their type c.*

5.3 Two-sector economy

Following Dixit and Stiglitz (1977), we consider a two-sector economy involving a differentiated good supplied under increasing returns and monopolistic competition, and a homogeneous good - or a Hicksian composite good - supplied under constant returns and perfect competition. Both goods are normal. Labor is the only production factor and is perfectly mobile between sectors. Consumers share the same preferences given by $U(\mathcal{U}(\mathbf{x}), x_0)$ where the functional $\mathcal{U}$ $(\mathbf{x})$ satisfies the properties stated in Section 2, while x_0 is the consumption of the homogeneous good. The upper-tier utility U is strictly quasi-concave, continuously differentiable, strictly increasing in each

[6]Results are ambiguous when $\sigma_c(\hat{c}, N_e)$ decreases with $\hat{c}$. In this case, the left-hand side of (69) may be non-monotone in $\hat{c}$. As a result, the mapping $h(N_e)$ may cease to be single-valued, which potentially leads to the existence of multiple equilibria. However, note that at any specific equilibrium, the behavior of $\hat{c}$ with respect to L depends solely on whether $h(N_e)$ is locally upward-sloping or downward-sloping.

argument, and such that the demand for the differentiated product is always positive.[7]

Choosing the unit of the homogeneous good for the marginal productivity of labor to be equal to 1, the equilibrium price of the homogeneous good is equal to 1. Since profits are zero at the SFE, the budget constraint is given by

$$\int_0^N p_i x_i \mathrm{d}i + x_0 = E + x_0 = y, \tag{70}$$

where the expenditure E on the differentiated good is endogenous because competition across firms affects the relative price of this good.

Using the first-order condition for utility maximization yields

$$p_i = \frac{U_1'(\mathcal{U}\ (\mathbf{x}), x_0)}{U_2'(\mathcal{U}\ (\mathbf{x}), x_0)} D(x_i, \mathbf{x}).$$

Let p be the price of the differentiated good. Along the diagonal $x_i = x$, the above condition becomes

$$p = S(\phi(x, N), x_0) D(x, xI_{[0,N]}), \tag{71}$$

where S is the marginal rate of substitution between the differentiated and homogeneous goods:

$$S(\phi, x_0) \equiv \frac{U_1'(\varphi(x, N), x_0)}{U_2'(\varphi(x, N), x_0)}$$

and $\varphi(x, N) \equiv \mathcal{U}\left(xI_{[0,N]}\right)$.

The quasi-concavity of the upper-tier utility U implies that the marginal rate of substitution decreases with $\varphi(x, N)$ and increases with x_0. Therefore, for any given (p, x, N), (71) has a unique solution $\bar{x}_0(p, x, N)$, which is the income-consumption curve. The two goods being normal, this curve is upward sloping in the plane (x, x_0).

For any given $x_i = x$, the love for variety implies that the utility level increases with the number of varieties. However, it is reasonable to suppose that the marginal utility D of an additional variety decreases. To be precise, we assume that

(C) *for all $x > 0$, the marginal utility D weakly decreases with the number of varieties.*

Observe that **(C)** holds for additive and quadratic preferences. Since $\varphi(x, N)$ increases in N, S decreases. As D weakly decreases in N, it must be that x_0 increases for the condition (71) to be satisfied. In other words, $\bar{x}_0(p, x, N)$ increases in N.

We now determine the relationship between x and m by using the zero-profit condition, as we did it above. Since by definition $m \equiv (p - c)/p$, for any given p the zero-profit and product market clearing conditions yield the per variety consumption as a function of m only:

[7]Our results hold true if the choke price is finite but sufficiently high.

$$x = \frac{F}{cL}\frac{1-m}{m}. \tag{72}$$

Plugging (72) and $p = c/(1-m)$ into $\bar{x}_0$, we may rewrite $\bar{x}_0(p, x, N)$ as a function of m and N only:

$$\hat{x}_0(m, N) \equiv \bar{x}_0\left(\frac{c}{1-m}, \frac{F}{cL}\frac{1-m}{m}, N\right).$$

Plugging (72) and $p = c/(1-m)$ into the budget constraint (70) and solving for N, we obtain the income y at which consumers choose the quantity $\hat{x}_0(m, N)$ of the homogeneous good:

$$N = \frac{Lm}{F}\left[y - \hat{x}_0(m, N)\right]. \tag{73}$$

Since $\bar{x}_0$ and $\hat{x}_0$ vary with N identically, $\hat{x}_0$ also increases in N. Therefore, (73) has a unique solution $\hat{N}(m)$ for any $m \in [0, 1]$.

Moreover, (73) implies that $\partial\hat{N}/\partial y > 0$, while $\partial\hat{N}/\partial L > 0$ because the income-consumption curve is upward slopping. In other words, if the price of the differentiated product is exogenously given, an increase in population size or individual income leads to a wider range of varieties.

Since $\hat{N}(m)$ is the number of varieties in the two-sector economy, the equilibrium condition (29) must be replaced with the following expression:

$$m\sigma\left[\frac{F}{cL}\frac{1-m}{m}, \hat{N}(m)\right] = 1. \tag{74}$$

The left-hand side $m\sigma$ of (74) equals zero for $m = 0$ and exceeds 1 when $m = 1$. Hence, by the intermediate value theorem, the set of SFEs is non-empty. Moreover, it has an infimum and a supremum, which are both SFEs because the left-hand side of (74) is continuous. In what follows, we denote the corresponding markups by $m_{\inf}$ and $m_{\sup}$; if the SFE is unique, $m_{\inf} = m_{\sup}$. Therefore, the left-hand side of (74) must increase with m in some neighborhood of $m_{\inf}$, for otherwise there would be an equilibrium to the left of $m_{\inf}$, a contradiction. Similarly, the left-hand side of (74) increases with m in some neighborhood of $m_{\sup}$.

Since $\partial\hat{N}/\partial y > 0$, (74) implies that an increase in y shifts the locus $m\sigma$ upward if and only if $\mathcal{E}_N(\sigma) > 0$, so that the equilibrium markups $m_{\inf}$ and $m_{\sup}$ decrease in y. Consider now an increase in population size. Since $\partial\hat{N}/\partial L > 0$, (74) implies that an increase in L shifts the locus $m\sigma$ upward if both $\mathcal{E}_x(\sigma) < 0$ and $\mathcal{E}_N(\sigma) > 0$ hold. In this event, the equilibrium markups $m_{\inf}$ and $m_{\sup}$ decrease in L.

Summarizing our results, we come to a proposition.

Proposition 10. *Assume* **(C)**. *Then, the set of SFEs is non-empty. Furthermore,* (i) *an increase in individual income leads to a lower markup and bigger firms at the infimum and supremum SFEs if and only if $\mathcal{E}_N(\sigma) > 0$ and* (ii) *an increase in population size yields a lower markup and bigger firms at the infimum and supremum SFEs if $\mathcal{E}_x(\sigma) < 0$ and $\mathcal{E}_N(\sigma) > 0$.*

This extends to a two-sector economy what Propositions 2 and 3 state in the case of a one-sector economy. Proposition 10 also shows that the elasticity of substitution keeps its relevance for studying monopolistic competition in a multisector economy. In contrast, studying how N^* changes with L or y is a harder problem because the equilibrium number of varieties depends on the elasticity of substitution between the differentiated and homogeneous goods.

6 Concluding remarks

We have shown that monopolistic competition can be considered as the marriage between oligopoly theory and the negligibility hypothesis, thus confirming Mas-Colell's (1984) intuition. Using the concept of elasticity of substitution, we have also provided a complete characterization of the market outcome and of the comparative statics implications in terms prices, firm size, and mass of firms/varieties. Somewhat ironically, the concept of elasticity of substitution, which has vastly contributed to the success of the CES model of monopolistic competition, thus keeps its relevance in the case of general preferences, both for symmetric and heterogeneous firms. The fundamental difference is that the elasticity of substitution ceases to be constant and now varies with the key-variables of the setting under study. We take leverage on this to make clear-cut predictions about the impact of market size and productivity shocks on the market outcome.

Furthermore, we have singled out our most preferred set of assumptions and given a disarmingly simple sufficient condition for the standard comparative statics effects to hold true. But we have also shown that relaxing these assumptions does not jeopardize the tractability of the model. Future empirical studies should shed light on the plausibility of the assumptions discussed in this paper by checking their respective implications. It would be unreasonable, however, to expect a single set of conditions to be universally valid.

We would be the last to say that monopolistic competition is able to replicate the rich array of findings obtained in industrial organization. However, it is our contention that models such as those presented in this paper may help avoiding several of the limitations imposed by the partial equilibrium analyses of oligopoly theory. Although we acknowledge that monopolistic competition is the limit of oligopolistic equilibria, we want to stress that monopolistic competition may be used in different settings as a substitute for oligopoly models when these ones appear to be unworkable.

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Appendix

A. Proof of Propositon 6.

(i). We show the existence of $V(\mathbf{x})$. The marginal utility of a variety $i \in [0, N]$ is given by

$$\mathcal{U}'(x_i, \mathbf{x}) = \frac{1}{2} D_U(x_i, \mathbf{x}) \left[\frac{V(\mathbf{x})}{U(\mathbf{x})}\right]^{1/2} + \frac{1}{2} D_V(x_i, \mathbf{x}) \left[\frac{U(\mathbf{x})}{V(\mathbf{x})}\right]^{1/2}, \tag{A.1}$$

where D_U (D_V) is the marginal utility of U (V). Computing the elasticity $\bar{\eta}(x_i, \mathbf{x})$ of the inverse demand and using

$$\bar{\sigma}(x, \mathbf{x}) = \frac{1}{\bar{\eta}(x_i, \mathbf{x})},$$

where $\bar{\sigma}(x, \mathbf{x})$ is the elasticity of substitution between varieties i and j at $x_i = x_j = x$, we get

$$\bar{\sigma}(x, \mathbf{x}) = \frac{\frac{D_U(x,\mathbf{x})}{U(\mathbf{x})} + \frac{D_V(x,\mathbf{x})}{V(\mathbf{x})}}{\frac{D_U(x,\mathbf{x})}{U(\mathbf{x})}\bar{\eta}_U(x, \mathbf{x}) + \frac{D_V(x,\mathbf{x})}{V(\mathbf{x})}\bar{\eta}_V(x, \mathbf{x})},$$

where $\bar{\eta}_U$ ($\bar{\eta}_V$) is elasticity of D_U (D_V) at x. Evaluating $\bar{\sigma}(x, \mathbf{x})$ at a symmetric consumption pattern $x = xI_{[0,N]}$ with N available varieties yields

$$\frac{1}{\sigma(N)} = \frac{1}{2}\left[\frac{1}{\sigma_U(N)} + \frac{1}{\sigma_V(N)}\right], \tag{A.2}$$

where σ_U (σ_V) is the elasticity of substitution associated with U (V). Furthermore, it is readily verified that the function $\psi(N)$ associated with (A.2) is given by

$$\psi(N) = [\psi_U(N)\psi_V(N)]^{1/2}. \tag{A.3}$$

Plugging (A.2) and (A.3) in (48), the optimum and equilibrium are identical if and only if

$$\frac{2\sigma_U(N) + 2\sigma_V(N)}{2\sigma_U(N)\sigma_V(N) - \sigma_U(N) - \sigma_V(N)} = \mathcal{E}_{\psi_U}(N) + \mathcal{E}_{\psi_V}(N), \tag{A.4}$$

Since we assume $U(\mathbf{x})$ to be given, $\sigma_U(N)$ and $\mathcal{E}_{\psi_U}(N)$ are both given functions of N.

We now determine $V(\mathbf{x})$ by using the class of preferences described by the Kimball's flexible aggregator (2): there exists an increasing and convex function $\nu(\cdot)$ such that for any consumption pattern $\mathbf{x}$ we have

$$\int_0^{\mathcal{N}} \nu\left(\frac{x_i}{V}\right) \mathrm{d}i = 1, \tag{A.5}$$

where $V = V(\mathbf{x})$. Evaluating (A.5) at a symmetric pattern $\mathbf{x} = xI_{[0,N]}$ implies that

$$\psi_V(N) = \frac{1}{N\nu^{-1}(1/N)}. \tag{A.6}$$

Setting

$$z = \nu^{-1}(1/N), \tag{A.7}$$

(A.6) becomes

$$\psi_V = \frac{\nu(z)}{z}.$$

Hence,

$$\mathcal{E}_{\psi_V}(N) = \frac{1}{\mathcal{E}_\nu(z)} - 1. \tag{A.8}$$

It is be readily verified that

$$\frac{1}{\sigma_V(N)} = r_\nu(z) \equiv -\frac{z\nu''(z)}{\nu'(z)}. \tag{A.9}$$

Using (A.6), (A.8) and (A.9) shows that (A.4) becomes a non-linear second-order differential equation in $\nu(z)$ where z is given by (A.7):

$$\nu''(z) = -\frac{\nu'(z)}{z}\frac{\left[2\sigma_U\left(\frac{1}{\nu(z)}\right)-1\right]\left[\mathcal{E}_{\psi_U}\left(\frac{1}{\nu(z)}\right)+\frac{\nu(z)-z\nu'(z)}{z\nu'(z)}\right]-2}{\left[\mathcal{E}_{\psi_U}\left(\frac{1}{\nu(z)}\right)+\frac{\nu(z)-z\nu'(z)}{z\nu'(z)}+2\right]\sigma_U\left(\frac{1}{\nu(z)}\right)}. \tag{A.10}$$

The Picard-Lindelöf theorem implies that (A.10) has a solution when $\mathcal{E}_{\psi_U}$ and σ_U are well-behaved functions.

(ii) We show that $U^{1/2}V^{1/2}$ is generically described by non-CES preferences. The argument goes by contradiction. Assume that $U^{1/2}V^{1/2}$ is a CES utility whose elasticity of substitution is σ. It follows from (A.1) that

$$\frac{2}{\sigma} = \frac{1}{\sigma_U(N)} + \frac{1}{\sigma_V(N)},$$

or, equivalently,

$$\frac{2}{\sigma} = \frac{1}{\sigma_U[1/\nu(z)]} - \frac{\nu'(z)}{\nu''(z)z}. \tag{A.11}$$

Combining (A.10) and (A.11) shows that $U^{1/2}\cdot V^{1/2}$ is CES only if

$$-\frac{\nu''(z)z}{\nu'(z)} = \frac{2}{\sigma} - \frac{1}{\sigma_U[1/\nu(z)]} = \frac{\left[2\sigma_U\left(\frac{1}{\nu(z)}\right)-1\right]\left[\mathcal{E}_{\psi_U}\left(\frac{1}{\nu(z)}\right)+\frac{\nu(z)-z\nu'(z)}{z\nu'(z)}\right]-2}{\left[\mathcal{E}_{\psi_U}\left(\frac{1}{\nu(z)}\right)+\frac{\nu(z)-z\nu'(z)}{z\nu'(z)}+2\right]\sigma_U\left(\frac{1}{\nu(z)}\right)}.$$

Therefore,

$$\nu'(z) = \frac{\nu(z)}{z}\left[\frac{\sigma+1}{\sigma-1} - \mathcal{E}_{\psi_U}\left(\frac{1}{\nu(z)}\right)\right]. \tag{A.12}$$

Observe that (A.10) is a second-order differential equation whose space of solutions is generically a two-dimensional manifold, for the solution is pinned down by fixing the values of two arbitrary integration constants. In contrast, (A.12) is a first-order differential equation, which has a unique solution up to one integration constant. Therefore, to guarantee that $U^{1/2}\cdot V^{1/2}$ is a non-CES preference, it is sufficient to choose $\nu(z)$ that satisfies (A.10) but not (A.12). Q.E.D.

B. Consider (52) and set:

$$u(x;\rho) \equiv (\ln x_i^r + 1)^{\frac{\rho}{r}},$$

$$\epsilon_u(x;\rho) \equiv \frac{u_x(x;\rho)x}{u(x;\rho)}, \qquad r_u(x;\rho) \equiv -\frac{u_{xx}(x;\rho)x}{u_x(x;\rho)}.$$

Following the line of Appendix A and using (52), the elasticity of substitution $\bar{\sigma}(x, \mathbf{x})$ between varieties i and j when $x_i = x_j = x$ is given by

$$\bar{\sigma}(x, \mathbf{x}) = \frac{\mathbb{E}\left[\frac{u'(x;\rho)}{\int_0^N u(x,\rho)\mathrm{d}j}\right]}{\mathbb{E}\left[\frac{u'(x;\rho)}{\int_0^N u(x;\rho)\mathrm{d}j} r_u(x;\rho)\right]}.$$

Evaluating $\bar{\sigma}(x, \mathbf{x})$ at a symmetric consumption pattern $x = xI_{[0,N]}$ with N available varieties yields

$$\sigma(x, N) = \frac{\mathbb{E}\left[\epsilon_u(x;\rho)\right]}{\mathbb{E}\left[\epsilon_u(x;\rho) r_u(x;\rho)\right]} = \frac{r\ln x + 1}{r \ln x + 1 - \mathbb{E}(\rho)}. \tag{B.1}$$

Combining (B.1) with the SFE conditions (19), (24) and (26), we find that the equilibrium individual consumption level must solve

$$\frac{\mathbb{E}\left[\epsilon_u(x;\rho)\left(1 - r_u(x;\rho)\right)\right]}{\mathbb{E}\left[\epsilon_u(x;\rho)\right]} = \frac{cLx}{cLx + F}. \tag{B.2}$$

As for the optimality condition, it is readily verified to be

$$\mathbb{E}\left[\epsilon_u(x;\rho)\right] = \frac{cLx}{cLx + F}. \tag{B.3}$$

Computing $\epsilon_u(x;\rho)$ and $r_u(x;\rho)$, and setting $r \equiv \mathrm{Var}(\rho)/\mathbb{E}(\rho)$, it can be shown that the left-hand sides of (B.2) and (B.3) are such that

$$\mathbb{E}\left[\epsilon_u(x;\rho)\right] = \frac{\mathbb{E}(\rho)}{r\ln x + 1} = \frac{\mathbb{E}\left[\epsilon_u(x,\rho)\left(1 - r_u(x,\rho)\right)\right]}{\mathbb{E}\left[\epsilon_u(x,\rho)\right]}.$$

Thus, the equilibrium condition (B.2) coincides with the optimality condition (B.3). Q.E.D.

C. Using (54), it is readily verified that $\mathbb{A}(N)$ must be the solution to

$$\frac{\mathrm{d}A}{\mathrm{d}N} = \left[\frac{m(N)}{1 - m(N)} - \frac{N}{\psi(N)}\frac{\mathrm{d}\psi}{\mathrm{d}N}\right]\frac{A(N)}{N},$$

that is, a linear differential equation in N that has a unique solution up to a positive constant.