The Marginal Voter's Curse*

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Abstract

An intuitive explanation for voter abstention is that a voter is uncertain which policy or candidate to vote for, and so defers to the rest of the electorate to make the decision. In majoritarian elections this has been formally modelled as a strategic response to the swing voter's curse, which arises because the rare event of a pivotal vote conveys substantial information. In electoral systems other than majority rule, however, the standard pivotal voting calculus may not apply. This paper analyzes one such system, namely proportional representation, where additional votes continue to push the policy outcome in one direction or the other. A new strategic incentive for abstention arises in that case, to avoid the "marginal voter's curse" of pushing the policy outcome in the wrong direction. Intuitively, conditioning on the rare event of a pivotal vote might seem to have a greater impact on behavior, but the marginal voter's curse actually presents a larger disincentive for voting than the swing voter's curse. This and other predictions of the model are confirmed empirically by a series of laboratory experiments.

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1 Introduction

An intuitive explanation for voter abstention is that a citizen is unsure which policy or candidate he should vote for, and anticipates that his peers will make a better choice on his behalf than he can make for himself. This could also explain why citizens often cast incomplete ballots, voting in some races but abstaining in others, even after voting costs are sunk. It is also consistent with extensive empirical evidence that abstention and partial ballots are most likely from citizens who (by various measures) lack political information or expertise.[1] In an influential paper, Feddersen and Pesendorfer (1996) demonstrate formally that uninformed citizens have a strategic incentive to abstain from voting, to avoid the *swing voter's curse* of overturning an informed decision: when others are voting informatively, the better of two alternatives is more likely to win by one vote than lose by one vote, so one additional vote for the inferior alternative is more likely to be pivotal than one vote for the superior alternative. A rational voter restricts attention to these pivotal events, and concludes that if his own vote is pivotal, it is likely mistaken.

Intuitively, the logic of delegating one's decision to those who know better seems not to depend strongly on the institutional details of the electoral system. Empirically, Sobbrio and Navarra (2010) indeed document that uninformed citizens abstain from voting in a variety of electoral systems. The pivotal voting calculus that generates the swing voter's curse, however, is specific to the particular institution of Majority Rule.[2] This paper therefore explores the incentives for participation in the alternative electoral system of Proportional Representation (PR, hereafter), which is increasingly popular, for example in Europe and Latin America. Under an ideal PR system, a party that receives 37% of the votes in a legislative election receives 37% of the seats in the legislature. Since every vote slightly shifts the vote shares that the two parties receive, every vote slightly adjusts the composition of the legislature. If the policy outcome ultimately results from bargaining within the legislature then shifting the composition of the legislature translates into shifting the location of the final policy outcome. In that sense, *every* vote is pivotal in a PR system.[3]

[1] See the references in McMurray (2012). In particular, Lassen (2005) and Banerjee et al. (2010) provide evidence from a natural experiment and a field experiment, respectively, that information has a causal impact on voter participation.

[2] McMurray (2014) also demonstrates an incentive to abstain when the winning candidate in a majoritarian system interprets her margin of victory as a signal from voters of the magnitude of an optimal policy, and responds optimally. That mechanism relies on candidates overreacting to uninformed votes, however, and so plays no role in the model below, where policy outcomes are mechanically tied to voting.

[3] As a practical matter, of course, legislatures have limited numbers of seats, so PR systems

A citizen will only value another's judgment if they share a common interest. Accordingly, the model below adopts the basic common-value setup of Feddersen and Pesendorfer (1996): there are two policy extremes, 0 and 1, associated with parties A and B, respectively. Nature designates one of these as superior, and *independent* voters prefer policies that are as close as possible to this optimum. There may also be *partisan* voters who prefer one of the two extremes regardless of Nature's choice. As an illustrative example, consider the allocation of public money, either to education or to national defense. Many voters will wish to allocate public funds to whatever program will truly make the greatest contribution to social welfare, but this depends on many unknown variables, such as the true labor market returns to schooling and the true threat of foreign invasion. Thus, voters with different opinions on these more primitive questions may favor different spending allocations, even though they ultimately share the same objective. Other voters may prefer one or the other type of spending, regardless of the consequences for social welfare. The departure from Feddersen and Pesendorfer (1996) is in replacing majority rule payoffs with PR payoffs. Majority rule produces the extreme policy x favored by the party who receives more votes: $x = 0$ if $a > b$ and $x = 1$ if $a < b$. Under PR, the policy outcome is instead a convex combination $x = \frac{a}{a+b}(0) + \frac{b}{a+b}(1) = \frac{b}{a+b}$ of the two extremes, with weights determined by the two parties' vote shares. Continuing the example above, $x = \frac{2}{3}$ could be interpreted as devoting two thirds of available funding to military endeavors and one third of the funding to education.

Under majority rule, a voter's expected utility is determined simply by the relative probabilities of $x = 0$ and $x = 1$, and the benefit of voting is simply proportional to the probability of changing this outcome. The source of the swing voter's curse is the fact that, when voting is informative, an additional vote for the inferior party is more likely to be pivotal than an additional vote for the superior party. Under PR, this pivot probability is not relevant: instead, what matters is the magnitude of the policy change $x_{a,b+1} - x_{a,b} = \frac{b+1}{a+b+1} - \frac{b}{a+b}$, say from voting B. This magnitude is largest, however, when the existing vote share for party B is small. When voting is informative, this is most likely when the optimal policy is 0. In that case, an additional B vote pushes the policy outcome in the wrong direction. Thus, whereas majority rule produces a swing voter's curse because a mistaken vote is more likely to be pivotal than a vote for the side that is truly superior, PR generates a *marginal*

cannot match the vote shares precisely. It is not therefore the case that *every* vote is pivotal, per se, but it is still the case that there are many pivotal events (i.e. as many as there are seats in the legislature), in contrast with standard majoritarian models. The "ideal" PR system considered below can be viewed as an approximation of this, where the additional smoothness serves to simplify the analysis.

voter's curse because a mistaken vote is likely to have a larger policy impact than a vote for the superior side. In either system, then, voters have a strategic incentive to abstain.

It is not immediately obvious whether the marginal voter's curse or the swing voter's curse should be stronger. In a majority rule system, the utility consequences of a mistaken vote can be dramatic, but only with miniscule probability; in a PR system, the consequences of mistakes are small, but unavoidable. It turns out, however, that the comparative static is unambiguous, at least in large elections: no matter how large the share of partisan voters, the marginal voter's curse is actually stronger than the swing voter's curse. The essential intuition for this is that, under majority, the damage caused by a mistaken vote can be corrected by a single vote for the optimal policy. Under PR, by contrast, each vote dilutes the impact of subsequent votes, so that multiple votes of support are required for undoing the mal-effects of a single mistake. As a simple illustration of this, suppose that the better of two alternatives received 3 out of 5 votes, or a 60% vote share. One additional vote for the opposite party would reduce this vote share to 50% (i.e. 3 out of 6), and an additional vote of support would bring it back up, but only to 57% (i.e. 4 out of 7). In other words, it is not sufficient to give lots of votes to the superior side; the electorate must also give as few votes as possible to the inferior side, so that the better side not only wins, but wins by a large margin. If there are no partisans, in fact, turnout under PR is negligible in large elections, in contrast with majority rule (McMurray, 2013), because everyone defers to the miniscule fraction of the electorate who are most nearly infallible.

In either electoral system, turnout is highest when the electorate is most partisan, because citizens who lack expertise worry less about canceling the votes of better-informed independents when the share of independents is small. For a given level of partisanship, the precise level of voter participation depends on the distribution of expertise. In general, however, improving voter expertise has an ambiguous effect on voter participation, because a citizen's incentive to vote is increasing in his own information but decreasing in the information of his peers. For any combination of parameters and in either electoral system, the logic of McLennan (1998) ensures that socially optimal behavior can occur in equilibrium. Welfare is higher under majority rule, where in large elections, the policy outcome converges almost surely to whichever side is truly optimal for society. This is not the case under PR: if 20% of the electorate are partisan (i.e. 10% on each side), for example, then even by voting unanimously, the best independents can hope for is a 90% vote share.

We test our theoretical results in the laboratory. We implement a 2x3 between subjects design and vary both the voting rule and the share of partisans in the electorate. While behavior is far from the point predictions, most comparative statics are in line with the model. First, poorly informed subjects abstain significantly more than well informed subjects, with the latter almost never abstaining. Second, abstention of poorly informed subjects decrease with the share of partisans in both voting systems. And third, abstention rates among poorly informed subjects are weakly higher in PR treatments.

The comparison of electoral systems, and proportional representation in particular, has been a subject of growing interest in recent literature, including both theoretical studies (e.g. Herrera, Morelli and Nunnari, 2015; Herrera, Morelli and Palfrey, 2014; Kartal, 2014a; Faravelli and Sanchez-Pages, 2014; Matakos, Troumpounis and Xefteris, 2014; and Faravelli, Man and Walsch, 2015) and laboratory experiments (e.g. Herrera, Morelli, and Palfrey, 2014; and Kartal, 2014b). This literature focuses on private values and costly voting, however, whereas the emphasis on the present paper is strategic abstention for informational reasons. Literature on strategic abstention also includes both theoretical studies (e.g. Feddersen and Pesendorfer, 1996; Krishna and Morgan, 2011, 2012; McMurray, 2013) and laboratory experiments (e.g. Battaglini, Morton and Palfrey, 2008, 2010; and Morton and Tyran, 2011), but these focus exclusively on majority rule. To our knowledge, the present paper is the first to analyze strategic abstention under proportional representation or to compare majority rule and PR in a common-value environment.

2 The Model

2.1 Model Description

An electorate consists of N citizens where, following Myerson (1998), N is finite but unknown, and follows a Poisson distribution with mean n. Together, these citizens must choose a policy x from the interval $[0, 1]$. Parties A and B are associated with the policy positions 0 and 1 on the left and right, respectively, and each citizen chooses an action in $\{A, B, 0\}$, which can be interpreted respectively as a vote for party A, a vote or for party B, or abstention from voting. Let a and b denote the numbers of A and B votes. In a Proportional Representation (PR) electoral system, the final policy outcome x is a convex combination of the two parties' policy positions, with weights given by the parties' vote shares $\lambda_A = \frac{a}{a+b}$ and $\lambda_B = \frac{b}{a+b}$. This reduces simply to

$$x(a, b) = 0\lambda_A + 1\lambda_B = \lambda_B = \frac{b}{a+b}. \tag{1}$$

Section 3.2 contrasts this with the case of majority rule, in which x is simply the policy position associated with the party that receives the largest number of votes (i.e. 0 or 1), breaking ties if necessary by a coin toss.

Some citizens are A or B *partisans*, and prefer policies as close as possible to the positions of the two parties: $u_A(x) = 1 - x$ and $u_B(x) = x$. The rest of the electorate are *non-partisan*, or *independent*, and have preferences that depend on an unknown state of the world $\omega = \{\alpha, \beta\}$ with uniform prior $\Pr(\alpha) = \Pr(\beta) = \frac{1}{2}$. For these citizens, policy 0 is optimal if $\omega = \alpha$ but policy 1 is optimal if $\omega = \beta$, and the utility of other policies merely depends on the distance from the optimum:

$$u(x|\alpha) = 1 - x \qquad\qquad u(x|\beta) = x. \tag{2}$$

At the beginning of the game, each citizen is independently designated as an A partisan with probability p, as a B partisan with probability p, and as an independent with probability $I = 1 - 2p$.

The optimal policy cannot be observed directly, but non-partisans observe private signals $s_i \in \{s_\alpha, s_\beta\}$ that are informative of the true state of the world.[4] These signals are of heterogeneous quality, reflecting the fact that citizens differ in their expertise on the issue at hand. Specifically, each citizens is endowed with information quality $q_i \in [0, 1]$, drawn independently from a common distribution F which, for simplicity, is continuous and has full support. Conditional on ω and on $q_i = q$, signals are then drawn independently from the following distribution,

$$\Pr(s_\alpha|\alpha, q) = \Pr(s_\beta|\beta, q) = \frac{1}{2}(1 + q).$$

With this specification, q specifies the correlation coefficient between s and ω. In particular, citizens with $q = 0$ are totally uninformed and citizens with $q = 1$ are perfectly informed about the state of the world. More generally, by Bayes' rule, posterior beliefs are given by

$$\phi_\alpha(q, s) = \Pr(\alpha|s, q) = \begin{cases} \frac{1}{2}(1 + q) & \text{if } s = s_\alpha \\ \frac{1}{2}(1 - q) & \text{if } s = s_\beta \end{cases} \tag{3}$$

and $\phi_\beta(q, s) = 1 - \phi_\alpha(q, s)$.

Partisan citizens have a dominant strategy to vote for the party they favor, so the only strategic choice is that of independents, who must choose an action for every realization of q_i and s_i. Let $\sigma : [0, 1] \times \{s_\alpha, s_\beta\} \to \Delta\{A, B, 0\}$ denote the mixed strategy of such a citizen, and let Σ denote the set of strategies. Abusing notation

[4] Partisans could receive signals as well, of course, but would ignore them.

slightly, denote a pure strategy simply as $\sigma(q,s) = j$ for any $j \in \{A, B, 0\}$. The ultimate policy outcome depends on the realizations of N and ω and the partisan type of each voter, together with the voting strategy and the realization of private information (q_i, s_i) for each independent. The equilibrium concept used is Bayesian Nash equilibrium. With the assumption of Poisson population uncertainty, such equilibria are necessarily *symmetric*, meaning that citizens with the same type and same private information take the same action, and are thus fully characterized by a single voting strategy $\sigma^* \in \Sigma$ for independents.[5]

2.2 Definitions

Depending on the voting strategy, parties may receive votes from non-partisan citizens with particular realizations of private information, in addition to partisan support. Integrating over the space of signals, therefore, the total probability $v_j(\omega)$ with which a citizen votes for party $j \in \{A, B\}$ in state $\omega \in \{\alpha, \beta\}$ can be written as

$$v_j(\omega) = p + I \int_0^1 \left(\sum_{s=s_\alpha, s_\beta} \sigma(j|q,s) \Pr(s|\omega, q) \right) dF(q). \tag{4}$$

This expression can also be interpreted as the expected vote share of party j. By the decomposition property of Poisson random variables (Myerson 1998), the numbers N_A and N_B of A and B votes are independent Poisson random variables with means $R_\omega \equiv n v_A(\omega)$ and $S_\omega \equiv n v_B(\omega)$. Thus, the probability of a particular electoral outcome $N_A = a$ and $N_B = b$ is given by

$$\Pr(a, b|\omega) = \left(\frac{e^{-R_\omega} R_\omega^a}{a!} \right) \left(\frac{e^{-S_\omega} S_\omega^b}{b!} \right). \tag{5}$$

By the environmental equivalence property of Poisson games (Myerson 1998), an individual from within the game reinterprets N_A and N_B as the numbers of A and B votes cast by his peers; by voting himself, he can add one to either total. If he abstains, the policy outcome will be chosen by his peers, and the individual will receive expected utility

$$Eu(0|q,s) = \sum_{\omega=\alpha,\beta} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} u[x(a,b)|\omega] \Pr(a,b|\omega) \phi_\omega, \tag{6}$$

[5] In games of Poisson population uncertainty, the finite set of citizens who actually play the game can be viewed as a random draw from an infinite set of *potential* citizens, for whom strategies could be defined (see Myerson 1998). The distribution of opponent behavior is therefore the same for any two individuals within the game (unlike a game between a finite set of players), implying that a best response for one citizen is a best response for all.

where $x(a,b)$ depends on the electoral system. Note that this expectation depends on private information only through the posteriors ϕ_ω. Similarly, an individual can obtain either of the following by voting for A or B, respectively.

$$Eu(A|q,s) = \sum_{\omega=\alpha,\beta} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} u[x(a+1,b)|\omega] \Pr(a,b|\omega)\phi_\omega \tag{7}$$

$$Eu(B|q,s) = \sum_{\omega=\alpha,\beta} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} u[x(a,b+1)|\omega] \Pr(a,b|\omega)\phi_\omega \tag{8}$$

and also that these expectations depend implicitly (through $\Pr(a,b|\omega)$) on the strategy σ adopted by a citizen's peers. The individual's best response $\sigma^{br}(\sigma)$ is to take the pure action $j \in \{A,B,0\}$ that maximizes $Eu(j|q,s)$, and a Bayesian Nash equilibrium $\sigma^* \in \Sigma$ is a fixed point of $\sigma : \Sigma \to \Sigma$.

Both for PR and for majority rule, the analysis below emphasizes the importance of a *posterior threshold strategy* $\sigma_{\bar{\phi}_\alpha\bar{\phi}_\beta} \in \Sigma$, which is characterized by posterior thresholds $\bar{\phi}_\alpha + \bar{\phi}_\beta \in [1,2]$ such that

$$\sigma_{\bar{\phi}_\alpha\bar{\phi}_\beta}(q,s) = \begin{cases} A & \text{if } \phi_\alpha \geq \bar{\phi}_\alpha \\ B & \text{if } \phi_\beta \geq \bar{\phi}_\beta \\ 0 & \text{else} \end{cases}.$$

When following a posterior threshold strategy, a citizen votes A if he is sufficiently confident that $\omega = \alpha$ and votes B if he is sufficiently confident that $\omega = \beta$, abstaining with some probability if $\bar{\phi}_\alpha + \bar{\phi}_\beta > 1$. Also of interest is a *quality threshold strategy* $\sigma_{\bar{q}} \in \Sigma$, defined such that

$$\sigma_{\bar{q}}(q,s) = \begin{cases} A & \text{if } q \geq \bar{q} \text{ and } s = s_\alpha \\ B & \text{if } q \geq \bar{q} \text{ and } s = s_\beta \\ 0 & \text{else} \end{cases}.$$

That is, a citizen votes if his expertise exceeds $\bar{q}$ and abstains otherwise, and conditional on voting, he simply votes for the party whose position seems superior, on the basis of his private signal s.[6]

Under a quality threshold strategy $\sigma_{\bar{q}}$, (4) reduces to the following, so that the probabilities $v_+ \equiv v_A(\alpha) = v_B(\beta)$ and $v_- \equiv v_A(\beta) = v_B(\alpha)$ of voting for the right

[6] Given the formulation in (3), a quality threshold strategy can also be interpreted as a posterior threshold strategy for which the posterior thresholds $\bar{\phi}_\alpha = \bar{\phi}_\beta = \bar{\phi}$ coincide. In that case, $\bar{q} = 2\left(\bar{\phi} - \frac{1}{2}\right)$.

or wrong alternative, respectively, are independent of the state.

$$v_+(\bar{q}) = p + I\int_{\bar{q}}^1 \frac{1}{2}(1+q)\,dF(q) = p + \frac{1}{2}I[1-F(\bar{q})][1+m(\bar{q})] \quad (9)$$

$$v_-(\bar{q}) = p + I\int_{\bar{q}}^1 \frac{1}{2}(1-q)\,dF(q) = p + \frac{1}{2}I[1-F(\bar{q})][1-m(\bar{q})], \quad (10)$$

where $m(\bar{q}) = E(q|q > \bar{q})$ is the average expertise among citizens who actually vote. From these, we can three write expressions useful later on, namely: expected turnout $T \equiv v_+ + v_-$, the expected winning margin $W \equiv v_+ - v_-$, and the likelihood ratio $L \equiv \frac{v_+}{v_-}$ of right votes to wrong votes:

$$\begin{aligned} T(\bar{q}) &= 2p + I[1-F(\bar{q})] \quad (11)\\ W(\bar{q}) &= I[1-F(\bar{q})]\,m(\bar{q})\\ L(\bar{q}) &= \frac{K+[1-F(\bar{q})][1+m(\bar{q})]}{K+[1-F(\bar{q})][1-m(\bar{q})]} \end{aligned}$$

where $K \equiv \frac{2p}{I}$is the ratio of partisans to non-partisans in the population.

As $\bar{q}$ increases, voting is limited to an increasingly elite group of the most expert citizens. $F(\bar{q})$ and $m(\bar{q})$ therefore increase and, accordingly, $T(\bar{q})$ and $W(\bar{q})$ decrease.

3 Equilibrium Analysis

3.1 Proportional Representation

Under proportional representation, $x(a,b) = \lambda_B = \frac{b}{a+b}$ as defined as in (1), so utility reduces to

$$\begin{aligned} u(x|\alpha) &= 1 - x = \lambda_A = \frac{a}{a+b}\\ u(x|\beta) &= x = \lambda_B = \frac{b}{a+b}, \end{aligned}$$

and expected utility reduces from (6) through (8) to the following.

$$\begin{aligned} E[u(0|q,s)] &= \phi_\alpha E_{a,b}\left(\tfrac{a}{a+b}|\alpha\right) + \phi_\beta E_{a,b}\left(\tfrac{b}{a+b}|\beta\right)\\ E[u(A|q,s)] &= \phi_\alpha E_{a,b}\left(\tfrac{a+1}{a+b+1}|\alpha\right) + \phi_\beta E_{a,b}\left(\tfrac{b}{a+b+1}|\beta\right)\\ E[u(B|q,s)] &= \phi_\alpha E_{a,b}\left(\tfrac{a}{a+b+1}|\alpha\right) + \phi_\beta E_{a,b}\left(\tfrac{b+1}{a+b+1}|\beta\right). \end{aligned}$$

Relative to abstaining, the expected benefit $\Delta_{0j} E[u(x)|q,s] = E[u(j|q,s)] - E[u(0|q,s)]$ of voting for candidate $j \in \{A, B\}$ depends merely on the expected changes $E(\Delta_{0j}\lambda_A|\omega)$ and $E(\Delta_{0j}\lambda_B|\omega)$ that this induces in the vote shares of each party, as follows.

$$\begin{aligned}
\Delta_{0A} E[u(x)|q,s] &= \phi_\alpha E_{a,b}\left(\tfrac{a+1}{a+b+1} - \tfrac{a}{a+b}|\alpha\right) + \phi_\beta E_{a,b}\left(\tfrac{b}{a+b+1} - \tfrac{b}{a+b}|\beta\right) \quad (12)\\
&= \phi_\alpha E(\Delta_{0A}\lambda_A|\alpha) + \phi_\beta E(\Delta_{0A}\lambda_B|\beta)\\
\Delta_{0B} E[u(x)|q,s] &= \phi_\alpha E_{a,b}\left(\tfrac{a}{a+b+1} - \tfrac{a}{a+b}|\alpha\right) + \phi_\beta E_{a,b}\left(\tfrac{b+1}{a+b+1} - \tfrac{b}{a+b}|\beta\right) \quad (13)\\
&= \phi_\alpha \Delta_{0B} E(\lambda_A|\alpha) + \phi_\beta \Delta_{0B} E(\lambda_B|\beta).
\end{aligned}$$

The first term in (12) is positive, and reflects the benefit of increasing A's vote share when $\omega = \alpha$. The second term is negative, reflecting the disutility of decreasing B's vote share when $\omega = \beta$. Accordingly, the benefit of voting A is increasing in ϕ_α and decreasing in ϕ_β. Symmetrically, (13) is increasing in ϕ_β and decreasing in ϕ_α. The consequence of this is that a citizen who is sufficiently confident that $\omega = \alpha$ wishes to vote A, while a citizen who is sufficiently confident that $\omega = \beta$ wishes to vote B. In other words, as Proposition 1 now states, the best response to any voting strategy can be characterized as a posterior threshold strategy. In particular, the best response to a posterior threshold strategy is another posterior threshold strategy, so standard fixed point arguments on the pair of posterior thresholds guarantee the existence of an equilibrium. In fact, the symmetry of the model is such that these equilibrium thresholds can be symmetric, in which case equilibrium voting reduces simply to a quality threshold strategy $\sigma_{\bar{q}^*}$.

Proposition 1 *If $\sigma^{br} \in \Sigma$ is a best response to $\sigma \in \Sigma$ then σ^{br} is a posterior threshold strategy. Moreover, there exists a quality threshold $\bar{q}^* \in [0,1]$ such that the quality threshold strategy $\sigma_{\bar{q}^*}$ is a Bayesian Nash equilibrium.*

As noted in Section 2.2, a consequence of a quality threshold strategy is that $v_A(\alpha) = v_B(\beta)$ and $v_A(\beta) = v_B(\alpha)$, so that N_A has the same distribution in state α that N_B has in state β, and vice versa. This implies that $\Pr(a, b|\omega) = \Pr(b, a|\tilde{\omega})$, where $\tilde{\omega}$ is the opposite state from ω, and therefore that $E(\Delta_{0A}\lambda_A|\omega) = E(\Delta_{0B}\lambda_B|\omega)$ and $E(\Delta_{0A}\lambda_B|\omega) = E(\Delta_{0B}\lambda_A|\tilde{\omega})$. Since $\phi_\alpha(q, s_\alpha) = \phi_\beta(q, s_\beta) = \frac{1}{2}(1+q)$ and $\phi_\alpha(q, s_\beta) = \phi_\beta(q, s_\alpha) = \frac{1}{2}(1-q)$, this further implies that the benefit $\Delta_{0A} E[u(x)|q, s_\alpha]$ of voting A in response to a signal s_α is the same as the benefit $\Delta_{0B} E[u(x)|q, s_\beta]$ of voting B in response to a signal s_β. In particular, since $E(\Delta_{0j}\lambda_A|\omega) + E(\Delta_{0j}\lambda_B|\omega) = 0$ for any $j \in \{A, B\}$, (12) and (13) reduce to

$$\Delta_{0A} E[u(x)|q, s_\alpha] = \Delta_{0B} E[u(x)|q, s_\beta] = \frac{1+q}{2} E(\Delta_{0A}\lambda_A|\alpha) + \frac{1-q}{2} E(\Delta_{0A}\lambda_B|\beta), \quad (14)$$

which is positive if and only if q exceeds the threshold $\bar{q}^{br}$, defined as follows:

$$\bar{q}^{br} = \frac{E\left(\Delta_{0A}\lambda_A|\beta\right) - E\left(\Delta_{0A}\lambda_A|\alpha\right)}{E\left(\Delta_{0A}\lambda_A|\alpha\right) + E\left(\Delta_{0A}\lambda_A|\beta\right)}. \tag{15}$$

If $\bar{q} = 0$ then everyone votes. Intuitively, this may seem likely to be the case in equilibrium, because voting is costless, and because every citizen has a private signal that is informative regarding the optimal policy. The standard pivotal voting calculus could generate a swing voter's curse, as in Feddersen and Pesendorfer (1996), but that calculus is irrelevant here, because A positive fraction of the electorate abstain , but that is because of the pivotal voting calculus, which is irrelevant here, where every vote influences the policy outcome. As Theorem 1 now states, however, a strictly positive fraction of the electorate necessarily abstain in equilibrium.

Theorem 1 (Marginal Voter's Curse) *If a quality threshold strategy σ_{q^*} is a Bayesian Nash equilibrium then $q^* > 0$.*

The logic of Theorem 1 is simple. First, note that the impact $\frac{a+1}{a+b+1} - \frac{a}{a+b}$ of one additional A vote on the vote share of party A is largest when a is small and b is large. When his peers are voting informatively, however, this is most likely to occur when $\omega = \beta$, in which case the additional A vote reduces utility. Similarly, one additional B vote likely has the greatest impact when $\omega = \alpha$. In other words, a vote for the inferior party is likely to exert greater influence on the policy outcome than a vote for the superior party. By voting, then, a totally uninformed citizen would suffer from a *marginal voter's curse*. To avoid this, such a citizen abstains in equilibrium, and the only citizens who vote are those who are sufficiently confident that they have correctly identified the true state of the world.

3.2 Majority Rule

An interesting question is how turnout in a PR system compares, theoretically, with turnout under majority rule. Accordingly, this section derives the latter.[7] The assumptions of this model are the same as those made above, except that the policy outcome x is now 0 whenever A votes outnumber B votes, and 1 otherwise (breaking a tie, if necessary, by a fair coin toss). For non-partisans, therefore, expected utility can be written as

$$\begin{aligned} Eu\left(x|\alpha\right) &= 1 \times \Pr\left(x=0|\alpha\right) + 0 \times \Pr\left(x=1|\alpha\right) = \Pr\left(x=0|\alpha\right) \\ Eu\left(x|\beta\right) &= 0 \times \Pr\left(x=0|\beta\right) + 1 \times \Pr\left(x=1|\beta\right) = \Pr\left(x=1|\beta\right), \end{aligned}$$

[7]This generalizes the derivation in McMurray (2013) to include partisan voters.

where the right-hand-side probabilities depend on a citizen's own voting decision. In particular, if a citizen abstains then

$$\Pr(x=0|\omega)=\Pr(a>b|\omega)+\frac{1}{2}\Pr(a=b|\omega).$$

By voting for A, he can change this to

$$\Pr(x=0|\omega)=\Pr(a>b+1|\omega)+\frac{1}{2}\Pr(a=b+1|\omega).$$

The difference between these two expressions is the probability with which a single additional A vote is *pivotal* (event piv_A), reversing the outcome of the election:

$$\Delta_{0A}\Pr(x=0|\omega)=\frac{1}{2}\Pr(a=b|\omega)+\frac{1}{2}\Pr(a=b+1|\omega)\equiv\Pr(piv_A|\omega).$$

In terms of this pivot probability, the expected benefit $\Delta_{0A}Eu(x|q,s)$ to a citizen with $q_i=q$ and $s_i=s$ (and therefore posteriors ϕ_α and ϕ_β) of voting A instead of abstaining is simply

$$\begin{aligned}\Delta_{0A}E[u(x)|q,s] &= \phi_\alpha\Delta_{0A}\Pr(x=0|\alpha)-\phi_\beta\Delta_{0A}\Pr(x=1|\beta)\\ &= \phi_\alpha\Pr(piv_A|\alpha)-\phi_\beta\Pr(piv_A|\beta),\end{aligned}\tag{16}$$

which is positive if and only if ϕ_α exceeds

$$\bar{\phi}_\alpha^{br}=\frac{\Pr(piv_A|\beta)}{\Pr(piv_A|\alpha)+\Pr(piv_A|\beta)}.$$

Similarly, a vote for B is pivotal with probability

$$\Delta_{0B}\Pr(x=1|\omega)=\frac{1}{2}\Pr(a=b|\omega)+\frac{1}{2}\Pr(a+1=b|\omega)\equiv\Pr(piv_B|\omega)$$

and thus provides expected benefit

$$\Delta_{0B}E[u(x)|q,s]=-\phi_\alpha\Pr(piv_B|\alpha)+\phi_\beta\Pr(piv_B|\beta)\tag{17}$$

which is positive if and only if ϕ_β exceeds

$$\bar{\phi}_\beta^{br}=\frac{\Pr(piv_B|\alpha)}{\Pr(piv_B|\alpha)+\Pr(piv_B|\beta)}.$$

By logic identical to Proposition 1, Proposition 2 characterizes the best response to any voting strategy as a posterior threshold strategy, and states that an equilibrium strategy exists. Given the symmetry of the model, the posterior thresholds may coincide, thus reducing to a quality threshold strategy. The proof is quite similar to those above, and is thus omitted.

Proposition 2 *If $\sigma^{br} \in \Sigma$ is a best response to $\sigma \in \Sigma$ then σ^{br} is a posterior threshold strategy. Moreover, there exists a quality threshold $\bar{q}^* \in [0,1]$ such that the quality threshold strategy $\sigma_{\bar{q}^*}$ is a Bayesian Nash equilibrium.*

When voting follows a quality threshold strategy, it is symmetric with respect to ω, as in Section 3.1. Since posteriors $\phi_\alpha(q, s_\alpha) = \phi_\beta(q, s_\beta) = \frac{1}{2}(1+q)$ and $\phi_\alpha(q, s_\beta) = \phi_\beta(q, s_\alpha) = \frac{1}{2}(1-q)$ are symmetric as well, (16) and (17) reduce to

$$\Delta_{0A} E[u(x)|q, s_\alpha] = \Delta_{0B} E[u(x)|q, s_\beta] = \frac{1+q}{2} \Pr(piv_A|\alpha) - \frac{1-q}{2} \Pr(piv_A|\beta),$$

which is positive if and only if q exceeds the threshold $\bar{q}$, defined by

$$\bar{q} = \frac{\Pr(piv_A|\beta) - \Pr(piv_A|\alpha)}{\Pr(piv_A|\alpha) + \Pr(piv_A|\beta)}. \tag{18}$$

If citizens follow a quality threshold strategy then, when $\omega = \alpha$, they are more likely to vote for candidate A than candidate B. Accordingly, candidate A is more likely to be ahead by one vote than behind by one vote. This implies, however, that an additional vote for candidate B is more likely to be pivotal than an additional vote for candidate A. Similarly, when $\omega = \beta$, a vote for B is less likely to be pivotal than a vote for A. These observations, together with the symmetry of quality threshold strategies, imply that (18) is positive, as Proposition 2 now states. Thus, as in Feddersen and Pesendorfer (1996), relatively uninformed citizens abstain in equilibrium, to avoid the *swing voter's curse* of overturning an informed electoral decision.

Theorem 2 (Swing Voter's Curse) *If a quality threshold strategy $\sigma_{\bar{q}^*}$ is a Bayesian Nash equilibrium then $\bar{q}^* > 0$.*

3.3 Welfare

Drawing on the equilibrium analysis above, this section analyzes voter welfare. Ex ante, the probabilities of being an A partisan and a B partisan are the same, and between these groups, the election outcome is zero-sum. It is therefore uncontroversial to interpret the expected utility of an independent citizen as an ex-ante measure of social welfare. Since every citizen receives an informative private signal but only some report their signals in equilibrium, valuable information is lost. Intuitively, this may seem to justify efforts to increase voter participation, say by punishing non-voters with stigma or fines. To the contrary, however, Proposition 3 states that equilibrium abstention actually improves welfare.

Proposition 3 *Whether the electoral system is majority rule or PR, if σ^{**} is the socially optimal voting strategy then σ^{**} also constitutes a Bayesian Nash equilibrium, and specifies that a positive fraction $v_0^{**} > 0$ of the electorate abstain.*

The proof of Proposition 3 relies on the logic of McLennan (1998), that in a common-value environment such as this, whatever is socially optimal is also individually optimal. The necessity of abstention then follows from Theorem 1 for PR and from Proposition 2 for majority rule. To see how it can be welfare improving to throw away signals, note that citizens actually have not one but two pieces of private information: their signal realizing s_i and their expertise q_i. In an ideal electoral system, all signals would be utilized, but would be weighted according to their underlying expertise. In a standard electoral system (whether majority rule or PR), votes are instead weighted equally. Abstention gives citizens a crude mechanism for transferring weight from the lowest quality signals to those that reflect better expertise.

4 Asymptotic Results

The results above apply for a population of any finite size n. Sections 4.1 and 4.2 now derive the limit of equilibrium behavior as n grows large, for PR and majoritarian elections, respectively. This is relevant because electorates do tend to be large, and also simplifies the analysis to facilitate a direct comparison of the two electoral systems, which is the topic of Section 4.3.

4.1 Proportional Representation

For PR, asymptotic results are made possible by Lemma 1, which offers an algebraic simplification of the formulas obtained previously, in terms of the expected numbers R_ω and S_ω of votes for A and B in state ω.

Lemma 1 *For any n and for any quality threshold strategy σ, the following identity holds:*

$$\begin{aligned} E\left(\Delta_{0A}\lambda_A|\omega\right) &= -E\left(\Delta_{0A}\lambda_B|\omega\right) = E\left(\Delta_{0B}\lambda_B|\tilde{\omega}\right) = -E\left(\Delta_{0B}\lambda_A|\tilde{\omega}\right) \\ &= \frac{S_\omega + \frac{{R_\omega}^2 - {S_\omega}^2 - S_\omega}{2} e^{-(R_\omega + S_\omega)}}{\left(R_\omega + S_\omega\right)^2}. \qquad (19) \end{aligned}$$

As noted above, an equilibrium quality threshold $\bar{q}_n^*$ for a particular n must solve $\Delta_{0A} E\left[u\left(x\right)|q, s_\alpha\right] = \Delta_{0B} E\left[u\left(x\right)|q, s_\beta\right] = 0$. Combining (14) and (19), this is equivalent to the following.

$$
\begin{aligned}
0 &= \frac{1+\bar{q}}{2}\left(\frac{S_\alpha + \frac{R_\alpha^2 - S_\alpha^2 - S_\alpha}{2} e^{-(R_\alpha+S_\alpha)}}{\left(R_\alpha+S_\alpha\right)^2}\right) - \frac{1-\bar{q}}{2}\left(\frac{S_\beta - \frac{R_\beta^2 - S_\beta^2 - S_\beta}{2} e^{-\left(R_\beta+S_\beta\right)}}{\left(R_\beta+S_\beta\right)^2}\right) \\
&= \left(\begin{array}{c} \frac{1+\bar{q}}{n(v_+ + v_-)^2}\left(v_- + \frac{n\left(v_+^2 - v_-^2\right) - v_-}{2} e^{-n(v_+ + v_-)}\right) \\ -\frac{1-\bar{q}}{n(v_+ + v_-)^2}\left(v_+ - \frac{n\left(v_+^2 - v_-^2\right) - v_-}{2} e^{-n(v_+ + v_-)}\right) \end{array}\right)
\end{aligned}
$$

or, more compactly,

$$
\frac{1+\bar{q}}{1-\bar{q}} = \frac{v_+ - \frac{nTW - v_-}{2} e^{-nT}}{v_- + \frac{nTW - v_-}{2} e^{-nT}}. \tag{20}
$$

Equation (20) must be satisfied for a given population size parameter n, but as n grows large, the right-hand side converges simply to the ratio $\frac{v_+}{v_-} = L\left(\bar{q}\right)$.[8] Thus, the limit $q^P = \lim_{n\to\infty} \bar{q}_n^*$ of any sequence of equilibrium quality threshold must solve the simpler equation,

$$
L\left(\bar{q}\right) = \frac{1+\bar{q}}{1-\bar{q}}. \tag{21}
$$

Proposition 4 shows that such a solution to (20) exists, and is unique. Uniqueness in the limit does not imply a unique equilibrium in any game with finite size parameter n, but if there are multiple equilibrium participation thresholds then the implication of Proposition 4 is that these all converge to each other in the limit, at a level that is determined entirely by the level p of partisans and the distribution F of expertise. Note that expected turnout (11) strictly decreases in $\bar{q}$, so that a unique limiting quality threshold q^P implies a unique level of turnout, as well. The expected margin of victory $v_+\left(q^P\right) - v_-\left(q^P\right)$ is uniquely determined by q^P as well. In large elections, of course, actual turnout and actual victory margins converge to their expectations.

Proposition 4 also derives the comparative static implications of (21). Intuitively, it might seem that the marginal voter's curse should attenuate as n grows large, because the damage caused by one mistaken vote shrinks, so citizens should be less worried about making mistakes. If this intuition were accurate then abstention would decline as the electorate grows large, and turnout would tend toward 100% in

[8] This convergence is not trivial, but is demonstrated formally in the proof of Proposition 4.

the limit. Contrary to this intuition, however, the first part of Proposition 4 states that q^P is strictly positive, meaning that a positive fraction of the electorate always abstains. In fact, if there are no partisans then q^P equals one, meaning that turnout actually tends to 0% in the limit. This is because the policy outcome is a weighted average of the two extremes, with weights corresponding to vote shares. Citizens wish to vote as unanimously as possible in favor of the superior side, and this is accomplished by limiting participation to those who are the least likely to err.

Proposition 4 *In the PR system, there exists a unique solution q^P to (21), which exhibits the following properties:*
(i) $q^P > 0$ for any partisan share p. If $p = 0$ then $q^P = 1$.
(ii) q^P decreases strictly with p.
(iii) q^P increases with increases in the distribution F of expertise that satisfy the monotone likelihood ratio property.

The result that equilibrium outcomes depend uniquely on the distribution of expertise can be illustrated by solving the limiting equilibrium condition (21) for $K = \frac{2p}{1-2p}$, which is simply a monotonic transformation of the partisan share. That is, q^P must solve

$$\frac{1 - F(\bar{q})}{\bar{q}} [m(\bar{q}) - \bar{q}] = K, \tag{22}$$

so for any distribution F of expertise and participation threshold q^P, it is simple to determine the fraction of partisans for which q^P is the limiting equilibrium participation threshold. For a uniform distribution, for instance, $F(\bar{q}) = \bar{q}$ and $m(\bar{q}) = \frac{1+\bar{q}}{2}$, so (22) reduces to

$$q^P = (K + 1) - \sqrt{K(K + 2)}. \tag{23}$$

Using the uniform distribution, Figure 1 plots the left- and right-hand sides of (21) for various levels p of partisanship. Evidently, the left-hand side of (21) is maximized precisely at the intersection of the two. Indeed, this must always be the case: it is straightforward to show that (21) coincides exactly with the first-order condition for maximizing $L(\bar{q})$. The intuition for Proposition 4 is related to the intuition for this phenomenon. To see this, first note that the left-hand side of (21) represents the likelihood ratio of a correct vote to an incorrect vote, for an average voter, chosen at random. The objective of independent voters is precisely to make this ratio as large as possible, so that the policy outcome will be as close as possible to whatever is truly optimal. Similarly, the right-hand side of (21) is the ratio of the likelihood ratio of a correct vote to an incorrect vote for the marginal independent voter, whose expertise is right at the participation threshold. Equating

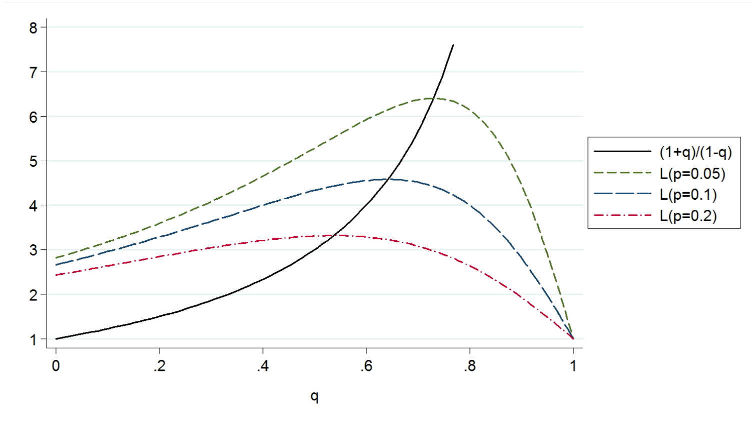

Figure 1: Left and right-hand side of equation (21) when the distribution F of expertise is uniform for various levels of partisans.

the average and marginal likelihood ratios serves to maximize the average, just as equating the average and marginal costs of a firm's production serves to minimize the average: if the marginal voter's likelihood ratio is not as good as the average voter's then increasing the participation threshold removes votes of below-average quality, thus improving the average; if the marginal voter's likelihood ratio is better than the average voter's then raising the participation threshold removes votes of above-average quality, thus making things worse.

The second part of Proposition 4 states that the marginal voter's curse is most severe when there are fewer partisans. Since partisans always vote, this implies that turnout is lower with fewer partisans, as well. With no partisans at all, average quality always exceeds marginal quality, because the marginal voter is precisely the one with the lowest expertise—except at the very top of the domain of expertise; thus, the equilibrium quality threshold rises all the way to $q^P = 1$. From an independent's perspective, however, adding partisans adds noise to the electoral process. For any participation threshold $\bar{q}$, therefore, a higher level of partisanship reduces the average accuracy of a vote, as Figure 1 makes clear. The accuracy of the marginal voter is unchanged, however, and strictly improves with $\bar{q}$, implying that the solution q^P to (21) is lower, as stated in the second part of Proposition 4. Since partisans always vote, this has the clear effect of raising turnout.

The last part of Proposition 4 states that improving the distribution of expertise has the effect of raising the limiting participation threshold q^P. The intuition for this merely complements that of increasing partisanship: improving expertise improves the correct-to-incorrect vote ratio for any participation threshold $\bar{q}$, so the solution to (21) is higher than before. In the case of partisanship, lowering q^P unambiguously raises voter participation. For changes in the distribution of expertise, however, the impact on voter participation is ambiguous: on one hand, raising citizens above the participation threshold increases turnout by transforming non-voters into voters, but on the other hand, raising the participation threshold lowers turnout, by transforming voters into non-voters.

Proposition 5 delineates situations in which a first-order stochastic dominance shift in F (which is weaker than a shift that satisfies MLRP) has an unambiguous impact on turnout: (1) improving nonvoters' information has no effect on turnout, if it does not lift them above the participation threshold; (2) improving the expertise of voters who are already above the participation threshold raises the margin of victory of the superior alternative, thereby strengthening the marginal voter's curse so that the participation threshold rises, and turnout falls; (3) moderate improvements in nonvoter's information increase turnout twice, first by pushing these non-voters above the participation threshold and then, since this lowers the margin of victory and weakens the marginal voter's curse, the participation threshold rises further, so that turnout increases still further.

Proposition 5 *If F and G both have log-concave densities f and g and G first-order stochastically dominates F then*

(i) If $G(q) = F(q)$ for all $q \geq q_F^P$ then $q_G^P = q_F^P$.

(ii) If $G(q) = F(q)$ for all $q \leq q_F^P$ then $q_G^P > q_F^P$.

(iii) If $G(q) = F(q)$ for all $q \geq m_F\left(q_F^P\right)$ and $G\left(q_F^P\right) < F\left(q_F^P\right)$ then $q_G^P < q_F^P$.

4.2 Majority Rule

Like Section 4.1, this section considers how the equilibrium threshold changes as n grows large. Here, however, the focus is on majority rule. Myerson (2000) provides a useful preliminary result, which is that pivot probabilities in large elections can be closely approximated as follows.

$$\begin{aligned}
\Pr\left(piv_A|\alpha\right) &\approx \frac{e^{-n\left[\sqrt{v_+}-\sqrt{v_-}\right]^2}}{4n\sqrt{\pi\sqrt{v_+v_-}}}\frac{\sqrt{v_+}+\sqrt{v_-}}{\sqrt{v_+}} \\
\Pr\left(piv_A|\beta\right) &\approx \frac{e^{-n\left[\sqrt{v_+}-\sqrt{v_-}\right]^2}}{4n\sqrt{\pi\sqrt{v_+v_-}}}\frac{\sqrt{v_+}+\sqrt{v_-}}{\sqrt{v_-}},
\end{aligned}$$

implying that the limit $q^M = \lim_{n\to\infty} \bar{q}_n^*$ of any sequence of equilibrium quality thresholds must solve the following approximation of (18).[9]

$$\bar{q} = \frac{\sqrt{v_+} - \sqrt{v_-}}{\sqrt{v_+} + \sqrt{v_-}}$$

or, equivalently,

$$L = \frac{v_+}{v_-} = \left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2. \tag{24}$$

Proposition 6 now states that a solution to (24) exists. If the distribution F of expertise is well-behaved, this solution is unique.[10] As in PR, uniqueness in the limit implies that if multiple equilibria exist then they all converge to the same behavior, and uniquely determine expected voter turnout and the expected margin of error, as well, where actual turnout and margins of error converge to their expectations. Uniqueness also facilitates the derivation of comparative statics, which according to Proposition 6 match those of PR: a higher partisan share leads to a lower q^M, and a better-informed electorate leads to a higher q^M.

Proposition 6 *In majority rule, there exists a solution q^M to (24), which lies strictly between* 0 *and* 1*. Moreover, if F has a log-concave density then q^M is unique, and satisfies the following additional properties:*
(i) q^M strictly decreases in p
(ii) q^M increases with increases in the distribution F of expertise that satisfy the monotone likelihood ratio property.

As above, the result that equilibrium outcomes depend uniquely on the distribution of expertise can be illustrated by solving the limiting equilibrium condition (24) for K.

$$\frac{1-F(\bar{q})}{\bar{q}}\left(\frac{1+(\bar{q})^2}{2}m(\bar{q}) - \bar{q}\right) = K. \tag{25}$$

[9] This approximation actually requires that the number of votes tend to infinity, not just the number of citizens, but this is guaranteed by the result below that $q^M < 1$ no matter what fraction of the electorate is partisan.

[10] Bagnoli and Bergstrom (2005) show that log-concavity is satisfied by many of the most standard density functions, but log-concavity is actually stronger than necessary for uniqueness: one can easily construct examples that exhibit unique equilibria but are not log-concave. The important thing, as the proof the proposition indicates, is that raising the participation threshold $\bar{q}$ should not increase the average expertise $m(\bar{q})$ of citizens above the threshold too quickly. This could be violated, for example, if the distribution of expertise included atoms, or "spikes" of probability.

For a given distribution F of expertise, it is then straightforward for any $\bar{q}$ to find the level of K (and therefore the level of p) such that $q^M = \bar{q}$ is the limiting equilibrium quality threshold.

In stating that q^M is strictly positive, the first part of Proposition 6 implies that, for any level of partisanship, a positive fraction of the electorate abstain from voting, no matter how large the electorate grows. This was also true of PR, as stated in Proposition 4. Under PR, a positive fraction of the electorate also continues voting, no matter how large the electorate grows, except in the case of $p = 0$, for which $q^P = 1$. In contrast, Proposition 6 states that q^M is strictly less than one for any p (including zero), a difference emphasized further in Section 4.3.

The logic for the result that q^M decreases in p is analogous to the corresponding result for q^P: when the fraction of partisans is low, an uninformed independent worries that he will cancel the vote of a better-informed independent, but when the fraction of partisans is high, it is more likely that he is canceling the vote of a partisan; in the former case he wishes to abstain, but in the latter case he wishes to vote. Mathematically, an increase in p lowers the average vote quality for any participation threshold, and therefore the correct-to-incorrect vote ratio, which is the left-hand side of (24). Since $L(\bar{q})$ increases in $\bar{q}$, this implies a solution that is lower than before. Similar logic underlies the last part of Proposition 6, because improving the distribution of expertise raises $L(\bar{q})$ for any $\bar{q}$, and the solution to (24) is higher than before. As in PR, however, Proposition 7 states that changes in the distribution of information have ambiguous consequences for voter turnout: (1) if the information of non-voters improves, but not by enough to push them above the participation threshold, then participation does not change; (2) if the information of voters improves then the marginal voter's curse becomes stronger, and turnout declines; (3) if non-voters are lifted only slightly above the participation threshold then turnout increases, both because non-voters now vote and because the marginal voter's curse is not as strong. The proof is essentially identical to that of Proposition 5, and is thus omitted.[11]

Proposition 7 *If F and G both have log-concave densities f and g and G first-order stochastically dominates F then*

(i) If $G(q) = F(q)$ for all $q \geq q_F^M$ then $q_G^M = q_F^M$.

(ii) If $G(q) = F(q)$ for all $q \leq q_F^M$ then $q_G^M > q_F^M$.

(iii) If $G(q) = F(q)$ for all $q \geq m_F\left(q_F^M\right)$ and $G\left(q_F^M\right) < F\left(q_F^M\right)$ then $q_G^M < q_F^M$.

[11] This generalizes Proposition 3 of McMurray (2013), which treats only the case of $p = 0$.

4.3 Comparison

Sections 4.1 and 4.2 emphasize the similarities between the comparative static implications of the marginal voter's curse for PR and the swing voter's curse for majoritarian electoral systems. Maintaining a focus on large electorates, this section now compares the levels of equilibrium voter participation under the two regimes (assuming a log-concave density of expertise, so that equilibrium behavior under either electoral system is unique). Such a comparison is surprisingly unambiguous, because of the strong similarity between the limiting equilibrium conditions (21) and (24) for PR and majority rule.

Intuitively, it might seem that conditioning on the event a pivotal vote should have a much greater impact on behavior than conditioning on the marginal impact of a nudge in one direction or the other—especially in large elections, where a pivotal vote is so extremely rare, and where the magnitude of the nudge is vanishingly small. If so, abstention should be much higher—and turnout much lower—under majority rule than under PR. As Theorem 3 now states, however, the opposite is true: q^P exceeds q^M, meaning that voter participation is lower.

Theorem 3 *If f is log-concave then $q^P > q^M > 0$.*

In stating that $q^P > q^M$, Theorem 3 leaves open the possibility that the two thresholds are quite close to one another, so that the difference is negligible. For specific distributions, this is straightforward to investigate. Suppose, for example, that F is uniform and that partisans comprise one third of the electorate (i.e. $p = \frac{1}{6}$, and therefore $K = \frac{1}{2}$). From (23), this implies that $q^P \approx 0.38$, implying approximately 75% turnout in large PR elections (i.e. 62% turnout among independents and 100% turnout among partisans). From (25), $q^M \approx 0.19$, implying approximately 87% turnout in large majoritarian elections (i.e. 81% turnout among independents and 100% turnout among partisans). Similar computations can be made for any level of partisanship, and corresponding turnout levels are displayed in Figure 2. Evidently, there is a substantial gap between q^P and q^M for all but the highest levels of partisanship.

The gap between turnout under majority rule and PR is most notable when there are no partisans. In that case, as Section 4.1 explains, turnout under PR tends toward 0%, because of strategic unraveling: citizens with below-average expertise abstain, so as to not bring down the average vote quality, but then the average among those who are still voting is higher, and citizens below this average abstain, and so on. Since the marginal citizen is always below average, this unraveling continues until only the infinitessimal fraction of the most expert citizens remain.

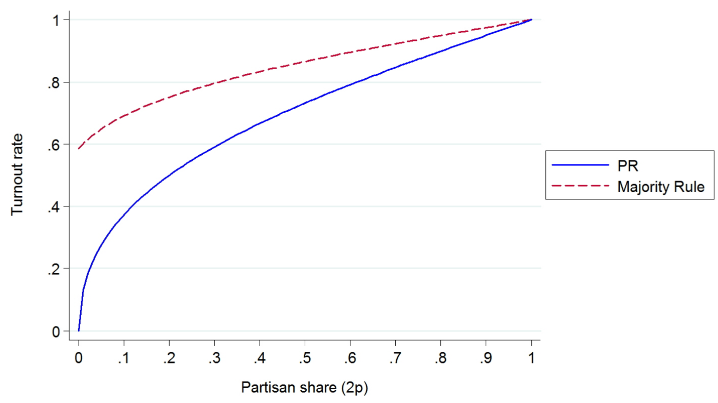

Figure 2: Turnout among independent voters as a function of the partisan share $(2p)$ and the voting rule.

Intuitively, it might seem that turnout should unravel under majority rule, just as it does under PR, because regardless of the electoral system, the marginal citizen always has less expertise than the average citizen, and so should eventually abstain, to get out of the way. To the contrary, however, Proposition 6 states that a substantial fraction of the electorate continue to vote, no matter how large the electorate grows. As McMurray (2013) explains, this reflects a trade-off between the quantity of information and the quality of information: holding the number of voters fixed, electoral outcomes are better when the expertise behind those votes is higher. Holding expertise fixed, however, increasing the number of votes also improves election accuracy, just as in the classic "jury theorem" of Condorcet (1785). For a citizen with below-average expertise, voting has the opposite effects of decreasing the average quality of information while increasing the quantity of information. These balance in the limit so that turnout remains substantial. In PR, by contrast, the quality of information matters but the quantity of information does not: fixing the distribution of expertise, there is no advantage to having more votes. Winning an election by 50 to 40 is the same as winning by 500 to 400, for example, because the policy outcomes $\frac{50}{90} = \frac{500}{900}$ are the same. Thus, quality considerations dominate, and turnout unravels in that case.

An alternative intuition for the discrepancy between turnout levels under majority rule and PR makes use of the optimality arguments above. As Section 4.1 notes, the limiting equilibrium condition (21) coincides with the first-order condition for maximizing $L(\bar{q}) = \frac{v_+}{v_-}$—or equivalently, for maximizing $\frac{v_+}{v_+ + v_-}$. The latter is simply

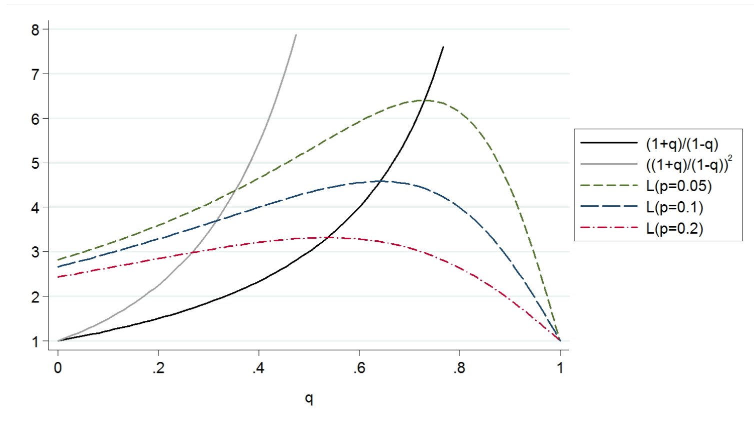

Figure 3: Left and right-hand sides of equations (21) and (24) with a uniform distribution F of expertise, for various levels of partisanship.

a formula for expected utility in large elections (i.e. where actual vote shares have converged to expectations), as can be seen from (1) and (2). In other words, $L(\bar{q})$ can be viewed as a monotonic transformation of voters' objective function (in large elections), and the quality threshold adjusts to the level q^P that maximizes this objective. The condition (24) for majority rule equates $L(\bar{q})$ to $\left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2$ instead of to $\frac{1+\bar{q}}{1-\bar{q}}$. Since the latter maximizes $L(\bar{q})$, the former does not, as Figure 3 illustrates for a uniform distribution. The figure also illustrates how $\left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2 > \frac{1+\bar{q}}{1-\bar{q}}$ guarantees that $q^M < q^P$, which is the crux of the proof of Theorem 3.

That q^P maximizes the objective function for PR in large elections begs the question of whether q^M maximizes the objective function for majoritarian elections. Indeed, this turns out to be the case—a feature that seems not to have been noted in existing literature on majority rule. Under majority rule, expected utility is given by the probability that $N_+ > N_-$ (plus half the probability of a tie, which becomes vanishingly small in large elections), where N_+ is the number of votes that match the state of the world and N_- is the number of votes that do not. Since each voter is more likely to cast a correct vote than an incorrect vote, the expected number nv_+ of good votes exceeds the expected number nv_- of mistakes. As Figure 4 illustrates, large deviations from this expectation are less likely than small deviations; if N_- does exceed N_+, the most likely margin of victory is a single vote.

As Myerson (2000) discusses, deviations from the expected election outcome become exponentially less likely as n grows large. The magnitude of a set of events

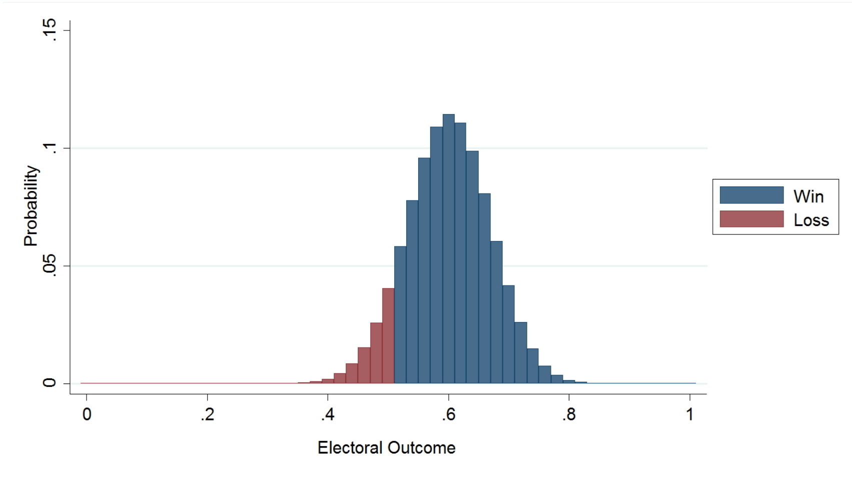

Figure 4: When the expected election outcome is a win, the most likely instance of a loss coincides with the event of a pivotal vote.

therefore converges to the magnitude of the most likely event within that set. In the case of election outcomes, this means that the magnitude of the conglomerate event of N_- exceeding N_+ in large elections is the same as the magnitude of exceeding N_+ by exactly one vote, which is the same as the magnitude of an exact tie. Specifically, Myerson (2000) gives this magnitude as $-\left(\sqrt{v_+}-\sqrt{v_-}\right)^2$, which is a monotonic function of $\sqrt{v_+}-\sqrt{v_-}$. The first-order condition for minimizing the latter is none other than the limiting equilibrium condition (24) for q^M. Intuitively, what matters is not only that the expected vote share v_+ exceeds v_-, but also that the variances of N_+ and N_- are small relative to their expectations, so that accidents in which $N_- > N_+$ do not occur.[12] The standard deviation of a Poisson random variable is the square root of its mean; thus, $\sqrt{nv_+}$ and $\sqrt{nv_-}$ represent the expected numbers of correct and incorrect votes, measured in standard deviations instead of in numbers of votes, and $\sqrt{v_+}-\sqrt{v_-}$ is proportional to the difference.

The results that PR and majority rule each maximize their respective objective functions, but generate different levels of voter participation, begs the question of which system is better for social welfare. To answer this, first define x_n^M and x_n^P to be equilibrium policy outcomes under majority rule and PR, respectively, and let $u^M = \lim_{n\to\infty} E\left[u\left(x_n^M\right)\right]$ and $u^P = \lim_{n\to\infty} E\left[u\left(x_n^P\right)\right]$ denote the limits of expected utility under either regime. Intuitively, the result of Proposition 3 that equilibrium

[12]These two considerations correspond to the considerations of quality and quantity, discussed above.

abstention improves welfare, together with the result of Theorem 3 that participation in large elections is higher under a majoritarian regime than under PR, might seem to suggest that welfare should be higher under PR than under majority rule. However, the opposite actually turns out to be true, as Proposition 8 now states: in large elections, welfare is higher under majority rule than under PR, especially for high levels of partisanship.

Proposition 8 *$u^M = 1$ for all $p < 1$. u^P decreases in p, from 1 for $p = 0$ to $\frac{1}{2}$ for $p = 1$.*

The comparison here of welfare has little to do with the comparison of turnout from Theorem 3. What drives the result is that, under majority rule, A partisans and B partisans negate one another's influence, so that the majority decision is determined entirely by the behavior of independent voters, no matter how small this group is. In a large election, a majority of these almost surely identify the true state of the world. If there are no partisans, then PR delivers the same outcome in the limit, as abstention is limited to an increasingly elite fraction of voters, the election outcome tends toward unanimity, and the policy outcome converges to the desired extreme. A positive mass of partisan votes for either side, however, bounds the policy outcome away from 0 and 1, implying some utility loss, which is increasing in p. Actually, even with no partisans, PR would be inferior to majority voting if the domain of the distribution of expertise were bounded below one, so that even the most elite citizens were incapable of agreeing unanimously on the state of the world.

5 Experiment

The model presented above is difficult to test with observational data, due to the endogeneity problems inherent in much of the empirical literature on turnout. To avoid this problem, we generate new data through a series of laboratory experiments. Implementation in the laboratory poses a number of challenges. First, financial and space constraints limit the number of participants, making it difficult to test asymptotic results for large elections. Second, technical features of the model such as the Poisson population uncertainty and the continuum of possible types, while elegant and convenient for theoretical derivations, are difficult to explain to experimental subjects. To circumvent these challenges we implement a simplified version of the model above, and derive equilibrium predictions computationally. As explained below, the simple version of the game features the main comparative statics of the model above.

5.1 Experimental Design and Procedures

Subjects interacted for 40 periods. The instructions in each period were identical. In each period, subjects interacted in groups of six. At the beginning of each round, the color of a triangle was chosen randomly to be either blue or red with equal probability. Subjects were not told the color of the triangle, but were told that their goal would be work together as a group to guess the color of the triangle. Independently, each would observe one ball (a *signal*) drawn randomly from an urn with 20 blue and red balls. With 40% probability, a participant would be designated as a *high* type (H), and 19 of the 20 balls in the urn would be the same color as the triangle. With 60% probability, a participant would be designated as a *low* type (L), in which case only 13 of the 20 balls would be the same color as the triangle. Individual were told their own types, but did not know the types of the other five members of their group.

After observing their signals, each subject had to take one of three actions: vote Blue, vote Red, or abstain from voting. Regardless of which action they chose, however, they were told that their action choice might be replaced at random, by the choice of a computer: with 10% probability, their vote choice was changed to Abstain.[13] With probability p the voting choice was replaced with a Blue vote, and with probability p it was replaced with a Red vote. Replacements of votes were determined independently across subjects. The partisanship parameter p was one of the treatment variables of the experiment. We considered three different values: $p = 0\%$, $p = 12.5\%$ and $p = 25\%$. The second treatment variable, the *voting rule*, determined subjects' payoff as a function of the group votes. In the Majority Rule (M) treatments, subjects each received payoffs of 100 points if the number of votes for the color of the triangle exceeded the number of votes for the other color, 50 points in case of a tie and 0 points otherwise. In the case of Proportional Representation (P) treatments, subjects each received as payoff in points the percentage of non-abstention votes that had the same color as the triangle—or, if everyone abstained, a payoff equal to 50 points.

For each of the two voting rules and each of the three levels of partisanship, Table 1 lists the equilibrium abstention rates $\sigma^*_{0,H}$ and $\sigma^*_{0,L}$ for high- and low-type individuals. If they vote at all, both types of citizens should vote in accordance with their private signals. High-type individuals should always vote, but the equilibrium

[13] This form of population uncertainty follows Feddersen and Pesendorfer (1996). With a known number of voters, the swing voter's would depend heavily on whether that number is even or odd. If it is odd, for example, there is always an equilibrium with full participation, because a vote is then pivotal only if the rest of the electorate is evenly split. In that case, a citizen infers no information beyond his or her own signal, and therefore has a strict incentive to vote.

Treatment	Voting Rule	% Partisans	$\sigma^*_{0,H}$	$\sigma^*_{0,L}$
M0	Majority Rule	0	0%	100%
M25	Majority Rule	25	0%	0%
M50	Majority Rule	50	0%	0%
P0	Proportional Representation	0	0%	100%
P25	Proportional Representation	25	0%	100%
P50	Proportional Representation	50	0%	0%

Table 1: Equilibrium abstention rates for high and low types, for all treatments.

strategy of low-type voters varies by treatment. Under majority rule, they should abstain when $p = 0$ but vote whenever p is positive. Under Proportional Representation, low-type individuals should abstain unless p is at its highest level, so that a vote is 50% likely to be replaced by a partisan computer. We summarize these predictions in the following hypotheses:

Hypothesis 1 *High types should vote (weakly) more than low types*

Hypothesis 2 *The frequency of abstention of high types should not change with the number of partisans or with the voting rule.*

Hypothesis 3 *Under either voting rule, the frequency of abstention of low types decreases with the number of partisans.*

Hypothesis 4 *The frequency of abstention of low types voters is weakly lower under majority rule than under PR.*

Experiments were conducted at the Experimental Economics Laboratory at the University of Valencia (LINEEX) in November 2014. We ran one session for each treatment, with 60 subjects each. No subject participated in more than one session. Students interacted through computer terminals, and the experiment was programmed and conducted with the software z-Tree (Fischbacher 2007). All experimental sessions were organized along the same procedure: subjects received detailed written instructions (see Appendix A3), which an instructor read aloud. Before starting the experiment, students were asked to answer a questionnaire to check their full understanding of the experimental design. Right after that, subjects played one of the treatments for 40 periods and random matching. Matching occurred within matching groups of 12 subjects, which generated 5 independent groups in each session. At the end of each round, each subject was given the information about the color of the triangle, their original and their final vote, and the total numbers of Blue votes, Red votes, and abstentions in their group (though they could not tell whether these were the intended votes of the other participants, or computer overrides). In P treatments, they also observed the percentage of votes that matched the color of the triangle; in M treatments, they instead were told whether the color of the Triangle received more, equal, or fewer votes than the other color. To determine payment at the end of the experiment, the computer randomly selected five periods and participants earned the total of the amount earned in these periods. Points were converted to euros at the rate of 0.025€. In total, subjects earned an average of 14.21€, including a show-up fee of 4 Euros. Each experimental session lasted approximately an hour.

5.2 Experimental Results

Figure 5 displays empirical abstention rates for all treatments, for voters of high and low types. The figure shows several interesting patterns that we formalize and test with the regression presented in Table 2. Table 2 displays the results of a random effects GLS regression of the frequency of abstention on dummies for each possible combination of voter type, level of partisanship, and voting rule (except for the combination of high types and 0% partisanship under majority rule, which is the reference category).[14]

Let's focus first on the behavior of high types. According to the theoretical predictions, these voters should never abstain. Empirically, abstention is indeed extremely low in all treatments, ranging from 0.3% only to 3.2%. Overall, we cannot reject the null hypothesis that the frequency of abstention across high-type voters is constant across all treatments ($\chi^2_4 = 3.28$, $p = 0.512$), in line with Hypothesis 2.

Figure 5 indicates a stark contrast in behavior across voter types: while the frequency of abstention of high-type voters is 1.73%, this frequency is 33.53% across low types. We find indeed a significant difference at under every single treatment.[15] This finding is line with Hypothesis 1: better informed voters tend to participate more in elections. See, however, that in contrast with the theoretical predictions, we find a strict significance in all treatments. In treatments where the percentage of partisans is 50%, for instance, all types of voters should always vote and therefore we

[14] The regressions lacks temporal variables and therefore ignores potential evolution in behavior throughout the experiment in order to ease interpretation of the coefficients. The results are robust to introducing temporal variables.

[15] $\chi^2_1 = 45.77$, $p < 0.001$ in $M0$, $\chi^2_1 = 82.46$, $p < 0.001$ in $M25$, $\chi^2_1 = 32.90$, $p < 0.001$ in $M50$, $\chi^2_1 = 15.17$, $p < 0.001$ in $M0$, $\chi^2_1 = 90.53$, $p < 0.001$ in $M25$, and $\chi^2_1 = 18.36$, $p < 0.001$ in $M50$.

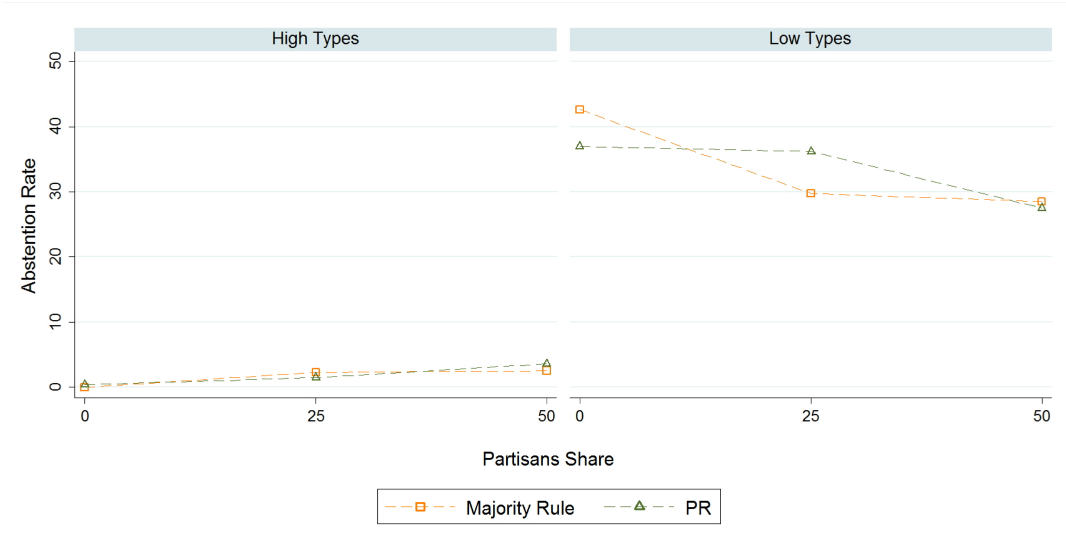

Figure 5: Observed abstention for each treatment, by voter type.

shouldn't such strict difference. This brings us to study behavior of low-type voters more closely.

Recall from Table 1 that we have corner solutions in all treatments. That is, low types should either all abstain or all vote. Figure 5 indicates that average frequencies are more moderate. However, this is not surprising as the model abstracts from certain aspects that do influence behavior. A perhaps more insightful question is whether the theory captures the comparative statics with respect to the voting mechanisms and to the level of partisans in the electorate. Let's attack these questions in turn. Figure 5 suggests that, in line with the predictions, the level of abstention decreases with the percentage of partisans in the electorate. Indeed, the percentages of abstention are 42.6%, 29.7% and 28.5% in treatments M0, M25, and M50, respectively, and 37.0%, 36.2% and 27.5% in treatments P0, P25, and P50. Moreover, the biggest drop coincides with this one predicted by theory. In the case of M treatments we can indeed reject the null hypothesis that the level of abstention is not constant across different levels of partisans in line with Hypothesis 3 ($\chi^2_2 = 58.75$, $p < 0.001$). Instead, we can't reject the null hypothesis in the case of P treatments ($\chi^2_1 = 2.17$, $p = 0.338$).

Do we observe significant differences across voting mechanisms? According to theory we should only observe significant differences the level of partisans is 25%, and in this case we should observe that abstention is substantially higher under PR. Although the magnitude of the difference is off, this is exactly what we find in the

Variable	Coef.	Std. Err.	z	P>z	95% C.I.
$High \times M25$	0.021	0.012	1.81	0.071	$[-0.002, 0.045]$
$High \times M50$	0.019	0.006	3.26	0.001	$[0.008, 0.031]$
$High \times P0$	0.008	0.006	1.47	0.142	$[-0.003, 0.020]$
$High \times P25$	0.011	0.011	1.04	0.298	$[-0.010, 0.032]$
$High \times P50$	0.032	0.022	1.45	0.147	$[-0.011, 0.075]$
$Low \times M0$	0.420	0.062	6.77	0.000	$[0.299, 0.542]$
$Low \times M25$	0.293	0.038	7.63	0.000	$[0.218, 0.368]$
$Low \times M50$	0.284	0.050	5.69	0.000	$[0.186, 0.382]$
$Low \times P0$	0.361	0.093	3.89	0.000	$[0.179, 0.543]$
$Low \times P25$	0.359	0.040	9.08	0.000	$[0.281, 0.436]$
$Low \times P50$	0.272	0.061	4.5	0.000	$[0.154, 0.391]$
Constant	0.003	0.006	0.54	0.588	$[-0.009, 0.015]$
Observations	14,400		σ_u	.209	ρ .313
Subjects	360		σ_e	.309	

Table 2: Random effects GLS regression of the probability of abstention on a constant and a number of dummies indicating the interaction between voter type, voting rule, and level of partisanship.

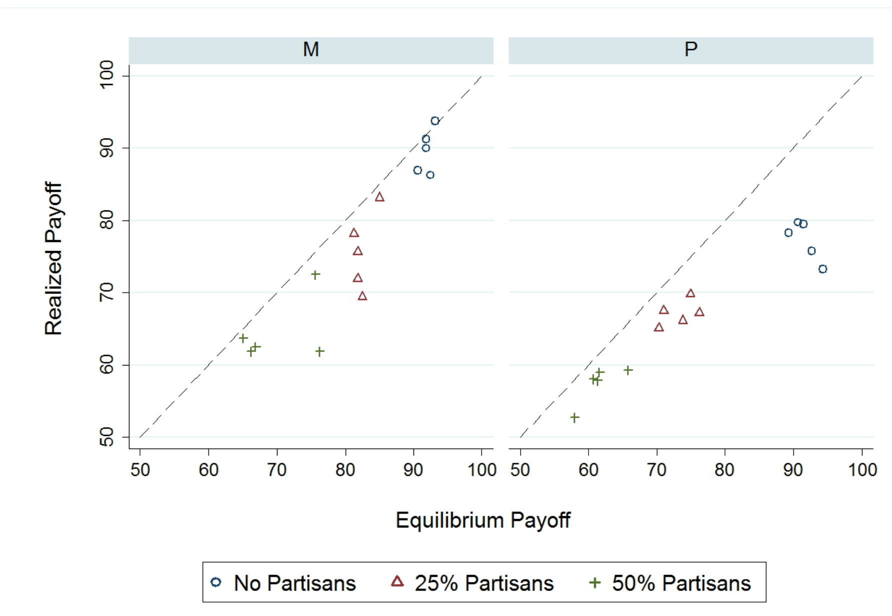

Figure 6: Realized payoff vs equilibrium payoff in each independent group.

data: we find no significant differences when the level of partisans is 0% or 50% ($\chi_1^2 = 0.43$, $p = 0.514$ and $\chi_1^2 = 0.04$, $p = 0.844$ respectively), but a statistically significant difference in favor of PR when the level of partisans is 25% ($\chi_1^2 = 16.76$, $p < 0.001$).

So far we have been focusing solely on abstention. According to theory, if they didn't abstain they should always vote their signal. This is not what we always observe. Overall, they deviate from voting their signal 11.6% of the time: 4.5% across high-type voters and 16.5% across low-type voters.[16] Additionally, these frequencies seems to increase with the level of partisans. This is consistent with models of quantal response equilibrium, where mistakes are less prevalent when the payoff difference across actions is smaller.[17]

Finally, let's turn to welfare. Figure 6 displays the realized average payoff in each independent group vis-à-vis the prediction for the realized draws. Actual payoffs are lower than would have been obtained by following the equilibrium strategy, but not by much: on average, realized payoffs were 91.7% as those that would have been obtained by following the equilibrium strategies. In other words, deviations from equilibrium reduced payoffs by only 8.3%. There is some heterogeneity across voting rules: realized payoffs under majority rule were 93.7% as high as equilibrium payoffs would have been, while payoffs under PR were 89.7% as high as equilibrium payoffs. A clear outlier among all the treatments is $P0$, where overparticipation of low types reduced payoff substantially, to 84.3% of the equilibrium levels. This is not too surprising given the mechanical effect of the partisans, however; realized payoffs clearly decrease with the level of partisanship. A similar regression to the one in Table 2 where the independent variable is payoff and the random effect is on each group shows that payoffs were higher in P0 than P25, higher in P25 than P50, higher in M0 than M25, and higher in M25 than M50, by amounts that are statistically significant at the 1% level in all cases.

6 Conclusion

The electoral system of Proportional Representation has become increasingly popular in recent decades, but its properties remain less well understood than those of majority rule. In particular, the swing voter's curse identified by Feddersen and

[16] This anomaly has been found systematically in experimental studies on information aggregation. See, for instance, Bouton et al. (2014) or Bouton et al. (2015).

[17] See Guarnaschelli, McKelvey and Palfrey (2000) and Holt and Goeree (2005) for applications of QRE to voting.

Pesendorfer (1996) has been extremely influential in understanding voter participation in majoritarian systems, but existing literature has not provided an analogous result for PR. This paper fills that gap by demonstrating a marginal voter's curse, which gives citizens a strategic incentive to abstain in PR systems, just as under majority rule. This suggests that the incentive to abstain in deference to those with better information is not highly sensitive to the electoral system, but results more intrinsically from the common-values assumption, together with the inevitable heterogeneity of expertise.

While it is not obvious, ex ante, which electoral system should provide greater incentive for voter turnout, the analysis above finds that equilibrium turnout is unambiguously higher under majority rule. This is in contrast with Herrera, Morelli, and Palfrey (2014), who analyze a private-value model with costly voting, and find that turnout can be higher in either system. To observers who view high levels of voter participation as intrinsically desirable, this may make PR unattractive. The welfare analysis above does not place value on participation, per se, and in fact demonstrates that some abstention is desirable, in the sense that the resulting policy outcome better matches the unknown state of the world than it would if everyone were to have voted. Nevertheless, the mechanism that leads to lower turnout under PR is shown to lower welfare directly: namely, proportional representation makes the influence of partisan voters more difficult to negate.

Several aspects of the analysis above would benefit from future extension. For example, the superiority of majority rule results at least partly from the assumption that the state of the world is binary, so that the optimal policy ultimately lies at one of the two extremes; in situations where the optimal policy is likely to lie between the two extremes, proportional rule may be advantageous. Another useful extension would be to consider an imbalance between partisan groups. Feddersen and Pesendorfer (1996) show that, when partisan groups are imbalanced, uninformed independents no longer abstain, instead voting to neutralize the partisan bias. Turnout is likely to increase under PR for a similar reason, but by even more, as negating the influence of one partisan requires more than one independent vote. Lastly, of course, is the issue of voting costs, which may have differential effects under the two electoral systems.

7 Appendix

Proof of Proposition 1. In response to σ, a citizen prefers to vote A if $\Delta_{0A} E\left[u\left(x\right)|q,s\right]$ exceeds the maximum of zero and $\Delta_{0B} E\left[u\left(x\right)|q,s\right]$. Since (12) is increasing in ϕ_α and (13) is decreasing in ϕ_α, these inequalities are satisfied for any ϕ_α above some threshold $\bar{\phi}_\alpha^{br}$. By symmetric reasoning, a citizen prefers to vote B if ϕ_β exceeds a threshold $\bar{\phi}_\beta^{br}$. Accordingly, these thresholds characterize a posterior threshold strategy that is the unique best response to σ.

If $\sigma_{\bar{q}}$ is a quality threshold strategy with quality threshold $\bar{q}$ then, as noted in Section 2.2, $v_A(\alpha) = v_B(\beta)$ and $v_A(\beta) = v_B(\alpha)$. This implies that symmetric electoral outcomes are equally probable. That is, N_A has the same distribution in state α that N_B has in state β, and vice versa, so that $\Pr(a, b|\alpha) = \Pr(b, a|\beta)$. In that case, it is straightforward to show that $\Delta_{0A} E[u(x)|q, s_\alpha] = \Delta_{0B} E[u(x)|q, s_\beta]$ and $\Delta_{0A} E[u(x)|q, s_\beta] = \Delta_{0B} E[u(x)|q, s_\alpha]$, implying that $\bar{\phi}_\alpha^{br} = \bar{\phi}_\beta^{br}$. Thus, the best response to $\sigma_{\bar{q}}$ is another quality threshold strategy $\sigma_{\bar{q}^{br}}$. In this way, $\bar{q}^{br}(\bar{q})$ can be interpreted as a continuous function from the compact set $[0, 1]$ of quality thresholds into itself. A fixed point $\bar{q}^*$exists by Brouwer's theorem, and the corresponding quality threshold strategy $\sigma_{\bar{q}^*}$ constitutes its own best response. ■

Proof of Theorem 1. The logic of this proof is to show that the best response to the quality threshold strategy σ_0 with full participation is a quality threshold strategy $\sigma_{\bar{q}^{br}(0)}$ with a best-response quality threshold $\bar{q}^{br}(0) > 0$, implying less-than-full participation. To see this, first note that if citizens follow σ_0 then (4) reduces so that $v_A(\alpha) = v_B(\beta) > v_A(\beta) = v_B(\alpha)$. Therefore, the distribution of N_A when $\omega = \alpha$ first-order stochastically dominates the distribution of N_A when $\omega = \beta$, and the distribution of N_B when $\omega = \beta$ first-order stochastically dominates the distribution of N_A when $\omega = \alpha$. The difference $\Delta_{0A}\lambda_A = \frac{a+1}{a+b+1} - \frac{a}{a+b}$ is decreasing in a and increasing in b, however, which implies that the distribution of $\Delta_{0A}\lambda_A$ when $\omega = \beta$ first-order stochastically dominates the distribution of $\Delta_{0A}\lambda_A$ when $\omega = \alpha$. Thus, $E(\Delta_{0A}\lambda_A|\beta) > E(\Delta_{0A}\lambda_A|\alpha)$, implying that (15) is strictly positive. ■

Proof of Proposition 2. When voters follow a quality threshold strategy,

$$
\begin{aligned}
\Pr(piv_A|\beta) - \Pr(piv_A|\alpha) &= \frac{1}{2}\left[\Pr(a+1=b|\beta) - \Pr(a+1=b|\alpha)\right] \\
&= \frac{1}{2}\left[\Pr(a=b+1|\alpha) - \Pr(a+1=b|\alpha)\right] \\
&= \frac{1}{2}\sum_{k=0}^{\infty}\frac{e^{-nv_A(\alpha)-nv_B(\alpha)}n^{2k+1}}{k!\,(k+1)!}\left[v_A^{k+1}(\alpha)\,v_B^k(\alpha) - v_A^k(\alpha)\,v_B^{k+1}(\alpha)\right] \\
&> 0,
\end{aligned}
$$

where the second equality follows from the symmetry of the strategy and the inequality follows because $v_A(\alpha) > v_B(\alpha)$. Thus, the best-response quality threshold (18) is strictly positive, implying that an equilibrium quality threshold must be positive as well. ∎

Proof of Proposition 3. McLennan (1998) points out that, in a common interest game such as this, any strategy σ^{**} is socially optimal is also individually rational, and thus constitutes an equilibrium. This logic applies whether the electoral system is PR or Majority Rule. That a positive fraction $v_0^{**} > 0$ of the electorate necessarily abstain in equilibrium follows from Theorem 1 for PR and from Proposition 2. ∎

Proof of Lemma 1. From (5) we have

$$\begin{aligned} E\left(\frac{a+1}{a+b+1}|\omega\right) &= e^{-R_\omega - S_\omega} \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\frac{R_\omega^a}{a!}\right)\left(\frac{S_\omega^b}{b!}\right)\left(\frac{a+1}{a+b+1}\right) \\ E\left(\frac{a}{a+b}|\omega\right) &= e^{-R_\omega - S_\omega} \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\frac{R_\omega^a}{a!}\right)\left(\frac{S_\omega^b}{b!}\right)\left(\frac{a}{a+b}\right). \end{aligned}$$

Omitting the ω subscripts, this can be reduced as follows.

$$\begin{aligned} \sum_{b=0}^{\infty}\frac{S^b}{b!}\frac{a}{a+b} &= \frac{a}{S^a}\sum_{b=0}^{\infty}\int_0^S \frac{d}{dr}\left(\frac{1}{b!}\frac{r^{a+b}}{a+b}\right)dr = \\ &= \frac{a}{S^a}\int_0^S \sum_{b=0}^{\infty}\left(\frac{1}{b!}r^{a+b-1}\right)dr = \begin{cases} \frac{a}{S^a}\int_0^S r^{a-1}e^r dr & \text{for } a \geq 1 \\ 1/2 & \text{for } a = 0 \end{cases} \end{aligned}$$

and

$$\sum_{b=0}^{\infty}\frac{S^b}{b!}\frac{a+1}{a+b+1} = \frac{a+1}{S^{a+1}}\int_0^S r^a e^r dr.$$

By inverting the series and integral operators again in the series over a, we have equal to

$$
\begin{aligned}
& E\left(\frac{a+1}{a+b+1}|\omega\right)-E\left(\frac{a}{a+b}|\omega\right) \\
= \;& e^{-R-S}\left(\left(\sum_{a=0}^{\infty}\frac{R^a}{a!}\left(\frac{a+1}{S^{a+1}}\int_0^S r^a e^r dr\right)\right)-\left(\sum_{a=1}^{\infty}\frac{R^a}{a!}\left(\frac{a}{S^a}\int_0^S r^{a-1}e^r dr\right)+\frac{1}{2}\right)\right) \\
= \;& e^{-R-S}\left(\int_0^S \frac{1}{S}\left(\sum_{a=0}^{\infty}\frac{\left(\frac{R}{S}r\right)^a}{a!}+\sum_{a=1}^{\infty}\frac{\left(\frac{R}{S}r\right)^a}{(a-1)!}\right)e^r dr-\left(\int_0^S \frac{R}{S}\sum_{a=1}^{\infty}\frac{\left(\frac{R}{S}r\right)^{a-1}}{(a-1)!}e^r dr+\frac{1}{2}\right)\right) \\
= \;& e^{-R-S}\left(\int_0^S \frac{1}{S}\left(e^{\frac{R}{S}r}+\frac{R}{S}re^{\frac{R}{S}r}\right)e^r dr-\left(\int_0^S \frac{R}{S}e^{\frac{R}{S}r}e^r dr+\frac{1}{2}\right)\right) \\
= \;& e^{-R-S}\frac{1}{S^2}\int_0^S e^{\left(1+\frac{R}{S}\right)r}\left(S+Rr\right)\,dr-\left(R\frac{1-e^{-(R+S)}}{R+S}+\frac{e^{-(R+S)}}{2}\right) \\
= \;& \frac{S}{(R+S)^2}+\frac{e^{-(R+S)}}{(R+S)^2}\frac{R^2-S^2-S}{2}
\end{aligned}
$$

In sum,

$$
E\left(\Delta_{0A}\lambda_A|\omega\right)=\frac{S_\omega+\frac{R_\omega{}^2-S_\omega{}^2-S_\omega}{2}e^{-(R_\omega+S_\omega)}}{\left(R_\omega+S_\omega\right)^2}.
$$

That $E\left(\Delta_{0B}\lambda_B|\tilde{\omega}\right)=E\left(\Delta_{0A}\lambda_A|\omega\right)$ follows from the symmetry of a quality threshold strategy, and since $\lambda_A+\lambda_B=1$, $\Delta_{0j}\lambda_A+\Delta_{0j}\lambda_B=0$ for $j\in\{A,B\}$. ■

Proof of Proposition 4. The equilibrium condition (20) is continuous in n (and in v_+, v_-, T, and W, which also vary continuously with n) so a sequence $\bar{q}_n^*$ of solutions to (20) must converge to a solution of the limit of (20). That is, letting $l\equiv\lim_{n\to\infty}\frac{nTW-v_-}{2}e^{-nT}$, q^P must solve

$$
\frac{1+\bar{q}}{1-\bar{q}}=\frac{v_+-l}{v_-+l}
$$

or, equivalently,

$$
\bar{q}=\frac{v_+-v_--2l}{v_++v_-}.
$$

Suppose we have $l>0$, then q^P satisfies

$$
\bar{q}<\frac{v_+-v_-}{v_++v_-}\leq 1,
$$

implying a positive turnout rate $T > 0$, and therefore that $nT \to +\infty$. In that case, $l = \lim_{n\to\infty} \frac{nTW - v_-}{2} e^{-nT} = 0$, yielding a contradiction.

Studying the sign of the derivative of

$$L(\bar{q}) = \frac{K + [1 - F(\bar{q})][1 + m(\bar{q})]}{K + [1 - F(\bar{q})][1 - m(\bar{q})]}$$

we obtain for any K the condition:

$$L'(\bar{q}) \gtrless 0 \quad \iff \quad L(\bar{q}) \gtrless \frac{1 + \bar{q}}{1 - \bar{q}}$$

For any $K > 0$ we have $L(0) > 1$ and $L(1) = 1$, moreover the function $\frac{1+\bar{q}}{1-\bar{q}}$ is increasing from 1 to infinity. Thus, q^P must be positive, and by the condition above $L(\bar{q})$ is increasing initially, must eventually cross $\frac{1+\bar{q}}{1-\bar{q}}$ and is decreasing from there after. Hence $L(\bar{q})$ has a unique maximum which solves $L(\bar{q}) = \frac{1+\bar{q}}{1-\bar{q}}$ which therefore coincides with the unique solution q^P. Moreover, q^P decreases in K as the function $L(\bar{q})$ decreases in K for all $\bar{q}$.

If $K = 0$ then $L(\bar{q}) = \frac{1+m(\bar{q})}{1-m(\bar{q})}$ which is always increasing, hence we must have $L(\bar{q}) > \frac{1+\bar{q}}{1-\bar{q}}$ for all $\bar{q} \in [0, 1)$, hence: $q^P = 1$ solves $L(\bar{q}) = \frac{1+\bar{q}}{1-\bar{q}} = +\infty$.

As for stochastic dominance, we first want to show that any MLR shift of the distribution increases $L(\bar{q})$for any $K \geq 0$ and any $\bar{q} \in [0, 1)$. By 9 and 10 we can write:

$$L(\bar{q}) = \frac{K + \int_{\bar{q}}^1 (1 + q) f(q)\, dq}{K + \int_{\bar{q}}^1 (1 - q) f(q)\, dq} = \frac{\int_0^1 \alpha(q) f(q)\, dq}{\int_0^1 \beta(q) f(q)\, dq}$$

$$\alpha(q) := K + I_{\bar{q}}(1 + q), \qquad \beta(q) := K + I_{\bar{q}}(1 - q)$$

where $I_{\bar{q}}$ is the indicator function of the interval $[\bar{q}, 1]$ and therefore $\alpha(q)$ and $\beta(q)$ are both non-negative and respectively non-decreasing and non-increasing functions of q for all K.

We need to show that for any distribution g such that $\frac{g(q)}{f(q)}$ is increasing, we have:

$$\frac{\int_0^1 \alpha(q) f(q)\, dq}{\int_0^1 \beta(q) f(q)\, dq} < \frac{\int_0^1 \alpha(q) g(q)\, dq}{\int_0^1 \beta(q) g(q)\, dq}$$

We can write the above inequality as:

$$\int_0^1 \int_0^1 \alpha(q) \beta(q') f(q) g(q')\, dq dq' < \int_0^1 \int_0^1 \alpha(q) \beta(q') f(q') g(q)\, dq dq'$$

hence we need to show that the following function Γ is negative, namely:

$$\Gamma := \int_0^1 \int_0^1 \alpha(q)\,\beta(q')\left[f(q)\,g(q') - f(q')\,g(q)\right] dq dq' < 0$$

Splitting the domain of integration in two symmetric parts we have:

$$\Gamma = \left(\begin{array}{c} \iint\limits_{q>q'} \alpha(q)\,\beta(q')\left[f(q)\,g(q') - f(q')\,g(q)\right] dq dq' + \\ \iint\limits_{q'>q} \alpha(q)\,\beta(q')\left[f(q)\,g(q') - f(q')\,g(q)\right] dq dq' \end{array} \right)$$

Renaming the variables (q as q') in the second double integral, and collecting similar terms, we have:

$$\begin{aligned} \Gamma &= \iint\limits_{q>q'} \left[\alpha(q)\,\beta(q') - \alpha(q')\,\beta(q)\right]\left[f(q)\,g(q') - f(q')\,g(q)\right] dq dq' \\ &= \iint\limits_{q>q'} \beta(q)\,\beta(q')\left[\frac{\alpha(q)}{\beta(q)} - \frac{\alpha(q')}{\beta(q')}\right] f(q)\,f(q')\left[\frac{g(q')}{f(q')} - \frac{g(q)}{f(q)}\right] dq dq' \end{aligned}$$

For any $q > q'$ and the properties listed above the first square brackets is positive while the second is negative.

In sum, for any $K \geq 0$ and any $\bar{q} \in [0, 1)$ a strict MLR shift increases $L(\bar{q})$. Hence if q^P solves

$$q^P : L(\bar{q}) = \frac{1 + \bar{q}}{1 - \bar{q}}$$

given that the RHS is increasing in $\bar{q}$ and unchanged after a MLR shift of F, the new solution after such a shift must be larger than q^P. ■

Proof of Proposition 5. As a preliminary step, note that the following expression is equivalent to the conditional mean $m(\bar{q}) = E(q|q \geq \bar{q})$, as can be seen by integrating by parts.

$$m(\bar{q}) = \bar{q} + \int_{\bar{q}}^1 \frac{1 - F(q)}{1 - F(\bar{q})} dq. \tag{26}$$

As a second preliminary step, rewrite the limiting equilibrium conditions (21) and (24) in terms of G as follows:

$$\frac{1-G(\bar{q})}{\bar{q}}\left[m_G(\bar{q})-\bar{q}\right] = K \tag{27}$$

$$\frac{1-G(\bar{q})}{\bar{q}}\left(\frac{1+\bar{q}^2}{2}m_G(\bar{q})-\bar{q}\right) = K. \tag{28}$$

1. If $G(q)=F(q)$ for all $q\geq q_F^P$ then from (26) it is clear that $m_G\left(q_F^P\right)=m_F\left(q_F^P\right)$. If q_F^P solves (21), therefore, then it solves (27) as well.

2. If $G(q)=F(q)$ for all $q\leq q_F^P$ but G first-order stochastically dominates F then from (26) it is clear that $m_G\left(q_F^P\right)>m_F\left(q_F^P\right)$. Therefore,

$$\frac{1-G\left(q_F^P\right)}{q_F^P}\left[m_G\left(q_F^P\right)-q_F^P\right]>\frac{1-F\left(q_F^P\right)}{q_F^P}\left[m_F\left(q_F^P\right)-q_F^P\right]=K. \tag{29}$$

By the log-concavity of g, the left-hand side of (27) is decreasing in $\bar{q}$ (as shown in the proof of Proposition 4), so the unique solution q_G^P must exceed q_F^P.

3. If $G(q)=F(q)$ for all $q\geq m_F\left(q_F^P\right)$ and $G\left(q_F^P\right)<F\left(q_F^P\right)$ then

$$\begin{aligned}
\int_{q_F^P}^1 qg(q)\,dq &= \int_{q_F^P}^1 qf(q)\,dq+\int_{q_F^P}^{m_F\left(q_F^P\right)} q\left[g(q)-f(q)\right]dq \\
&< \int_{q_F^P}^1 qf(q)\,dq+\int_{q_F^P}^{m_F\left(q_F^P\right)} m_F\left(q_F^P\right)\left[g(q)-f(q)\right]dq \\
&= m_F\left(q_F^P\right)\left\{\int_{q_F^P}^1 f(q)\,dq+\int_{q_F^P}^{m_F\left(q_F^P\right)}\left[g(q)-f(q)\right]dq\right\} \\
&= m_F\left(q_F^P\right)\left[1-G\left(q_F^P\right)\right],
\end{aligned}$$

so $m_G\left(q_F^P\right)=\frac{\int_{q_F^P}^1 qg(q)dq}{1-G\left(q_F^P\right)}<m_F\left(q_F^P\right)$. Thus, the inequality in (29) is reversed. By the log-concavity of g, the left-hand side of (27) is decreasing in $\bar{q}$ (as shown in the proof of Proposition 4), so the unique solution q_G^P must be less than q_F^P. ■

Proof of Proposition 6. In the expression (24) we can isolate K, yielding:

$$\frac{1-F(\bar{q})}{\bar{q}}\left(\frac{1+\bar{q}^2}{2}m(\bar{q})-\bar{q}\right)=K. \tag{30}$$

To prove existence, first note that the left-hand side of (30) approaches zero as $\bar{q}$ approaches 1. Thus, $q^M = 1$ solves (30) if and only if $K = 0$. Moreover, if $\lim_{q \to 1} \frac{f'(q)}{f(q)} < \infty$ then $0 < q^M < 1$. In that case, however, the condition that $\lim_{q \to 1} \frac{f'(q)}{f(q)} < \infty$ implies that no sequence of equilibrium quality thresholds can converge to one, by Theorem 3 of McMurray (2013).[18] As $\bar{q}$ tends to zero, the left-hand side of (30) tends to infinity. Thus, for any K, there exists a solution $0 < q^M < 1$ by the Intermediate Value Theorem and continuity of the left-hand side of (30).

As for uniqueness, for $K = 0$, Theorem 4 of McMurray (2013) states that the solution to (30) is unique on the open interval $(0, 1)$.[19] For $\bar{q}$ below that solution, therefore, the left-hand side of (30) is positive (tending to infinity as $\bar{q}$ tends to zero), while for $\bar{q}$ above that solution, it is negative. When it is positive, inspection of (30) reveals that we must have

$$m(\bar{q}) > \frac{2\bar{q}}{1 + \bar{q}^2}. \tag{31}$$

In that case, the left-hand side of (30)is also decreasing in $\bar{q}$. To see this, differentiate the left-hand side of (30) to obtain

$$-f(\bar{q}) \left(\frac{1 + \bar{q}^2}{2} \frac{m(\bar{q})}{\bar{q}} - 1 \right) + \frac{1 - F(\bar{q})}{2} \left(-\frac{1 + q^2}{q^2} m(\bar{q}) + \frac{1 + q^2}{q} m'(\bar{q}) \right).$$

By (31), the first term in this expression is negative. The second term is negative if and only if

$$m'(\bar{q}) < \frac{m(\bar{q})}{\bar{q}},$$

which is a condition equivalent to a decreasing $m(\bar{q}) - \bar{q} = E(q - \bar{q} | q \geq \bar{q})$, sometimes called the *mean residual lifetime* of a machine: a machine "ages" with time if its mean residual lifetime decreases. According to Lemma 2 of Bagnoli and Bergstrom (2005), a necessary and sufficient condition for the latter is that f is log-concave. Thus, under log-concavity the left-hand side of (30) is decreasing whenever it is positive, implying a unique solution for any $K > 0$.

The latter also implies that, if K increases, then q^M decreases under log-concavity.

The argument for MLR shifts is analogous to the one obtained for the PR system.

■

[18] As that paper notes, this condition merely rules out electorates that are arbitrarily close to being perfectly informed, and is sufficient for the result but not necessary.

[19] $\bar{q} = 1$ constitutes another solution but cannot be the limit of a sequence of equilibrium thresholds.

Proof of Theorem 3. From the characterization results, we know that $q^M \in (0,1)$ is the (possibly not unique) solution to:

$$q^M : L(\bar{q}) = \left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2$$

while $q^P \in (0,1]$ is the unique solution to:

$$q^P : L(\bar{q}) = \left(\frac{1+\bar{q}}{1-\bar{q}}\right)$$

Thus, we have

$$\begin{aligned}
\bar{q} &= q^P \in (0,1] \implies L(\bar{q}) = \left(\frac{1+\bar{q}}{1-\bar{q}}\right) < \left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2 \\
\bar{q} &= 0 \implies L(\bar{q}) > 1 = \left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2
\end{aligned}$$

Hence the continuous functions $L(\bar{q})$ and $\left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2$ must cross at some value $q^M \in (0, q^P)$. ■

Proof of Proposition 8. As n grows large, equilibrium strategies converge pointwise to σ_{q^M} under majority rule and to σ_{q^P} under PR. In either regime, actual vote shares converge in probability to expected vote shares. That is, $\frac{N_+}{N} \to_p \frac{nv_+}{n} = v_+$ and $\frac{N_-}{N} \to_p \frac{nv_-}{n} = v_-$.

For $\bar{q} = q^M$, v_+ exceeds v_-, implying that $\Pr(N_+ > N_-)$ converges almost surely to one, and utility converges almost surely $u^M = \lim_{n\to\infty} \Pr(N_+ > N_-) + \frac{1}{2} \lim_{n\to\infty} \Pr(N_+ = N_-) = 1 + 0$. This logic is valid for any $p < 1$. (For $p = 1$, $\Pr(N_+ > N_-) = \Pr(N_+ < N_-)$ for any n, implying that $u^M = \frac{1}{2} + 0$).

With Proportional Representation, utility is given by $\frac{N_+}{N_+ + N_-}$, which converges in probability to $u^P = \frac{v_+}{v_+ + v_-} = \frac{L(q^P)}{L(q^P)+1}$. This decreases in p since it increases in L, which decreases in p. As explained in Section 4.1, $\lim_{\bar{q}\to q^P} L(\bar{q}) = \infty$ for $p = 0$, implying that $u^P = 1$. From (9) and (10) it is also clear that $L(\bar{q}) = \frac{1}{2}$ when $p = 1$ for any $\bar{q}$, implying that $u^P = \frac{1}{2}$ in that case. ■

References

[1] Bagnoli, Mark, and Ted Bergstrom. 2005. "Log-Concave Probability and Its Applications." *Economic Theory*, 26(2): 445-469.

[2] Banerjee, Abhijit V., Selvan Kumar, Rohini Pande, and Felix Su. 2011. "Do Informed Voters Make Better Choices? Experimental Evidence from Urban India." Manuscript, Harvard University.

[3] Battaglini, Marco, Rebecca B. Morton, and Thomas R. Palfrey. 2010. "The Swing Voter's Curse in the Laboratory." *Review of Economic Studies*, 77(1): 61-89.

[4] Battaglini, Marco, Rebecca B. Morton, and Thomas R. Palfrey. 2008. "Information aggregation and strategic abstention in large laboratory elections." *The American Economic Review P&P*, 98(2): 194–200.

[5] Bouton, Laurent, Micael Castanheira and Aniol Llorente-Saguer. 2014. "Divided Majority and Information Aggregation: Theory and Experiment." Manuscript, Max Planck Institute for Collective Goods.

[6] Bouton, Laurent, Aniol Llorente-Saguer and Frédéric Malherbe. 2015. "Veto, Abstention and Framing: An Experiment on Majority with Veto Power." Manuscript, Queen Mary Unviersity of London.

[7] Condorcet, Marquis de. 1785. *Essai sur l'application de l'analyse a la probabilite' des decisions rendues a la pluralite' des voix [Essay on the Application of Analysis to the Probability of Majority Decisions].* Paris: De l'imprimerie royale. Trans. Iain McLean and Fiona Hewitt. 1994.

[8] Faravelli, Marco, Priscilla Man and Randall Walsh. 2015. "Mandate and Paternalism: A Theory of Large Elections", Working Paper, University of Queensland.

[9] Faravelli, Marco and Santiago Sanchez-Pages. 2014. "(Don't) Make My Vote Count." *Journal of Theoretical Politics,* forthcoming.

[10] Feddersen, Timothy J. and Wolfgang Pesendorfer. 1996. "The Swing Voter's Curse." *The American Economic Review*, 86(3): 408-424.

[11] Fischbacher, Urs. 2007. "z-Tree - Zurich Toolbox for Readymade Economic Experiments." *Experimental Economics*, 10(2): 171-178.

[12] Goeree, Jacob and Charles Holt. 2005. "An Explanation of Anomalous Behavior in Models of Political Participation." *American Political Science Review*, 99(2): 201-213.

[13] Guarnaschelli, Serena, Richard McKelvey and Thomas R. Palfrey. 2000. "An Experimental Study of Jury Decision Rules" *American Political Science Review*, 94(2): 407-423.

[14] Herrera, Helios, Massimo Morelli, and Salvatore Nunnari. 2015. "Turnout across Democracies." *The American Journal of Political Science*, forthcoming.

[15] Herrera, Helios, Massimo Morelli, and Thomas Palfrey. 2014. "Turnout and Power Sharing." *The Economic Journal*, forthcoming.

[16] Kartal, Melis. 2014a. "A Comparative Welfare Analysis of Electoral Systems with Endogenous Turnout," *The Economic Journal*, forthcoming.

[17] Kartal, Melis. 2014b. "Laboratory elections with endogenous turnout: proportional representation versus majoritarian rule." *Experimental Economics*, forthcoming.

[18] Krishna Vijay and John Morgan. 2011."Overcoming Ideological Bias in Elections." *Journal of Political Economy*, 119(2): 183-211.

[19] Krishna Vijay and John Morgan. 2012. "Voluntary Voting: Costs and Benefits." *Journal of Economic Theory*, 147(6): 2083–2123.

[20] Lassen, David. 2005. "The Effect of Information on Voter Turnout: Evidence from a Natural Experiment." *American Journal of Political Science*, 49(1): 103-118.

[21] Matakos, Konstantinos, Orestis Troumpounis and Dimitrios Xefteris. 2014. "Turnout and polarization under alternative electoral systems." Manuscript, Lancaster University.

[22] McLennan, Andrew. 1998. "Consequences of the Condorcet Jury theorem for Beneficial Information Aggregation by Rational Agents." *American Political Science Review*, 92(2): 413-418.

[23] McMurray, Joseph C. 2013. "Aggregating Information by Voting: The Wisdom of the Experts versus the Wisdom of the Masses." *The Review of Economic Studies*, 80(1): 277-312.

[24] McMurray, Joseph C. 2012. "The Paradox of Information and Voter Turnout." Manuscript, Brigham Young University.

[25] McMurray, Joseph C. 2014. "Mandates, Multiple Candidates, and the Signaling Voter's Curse." Manuscript, Brigham Young University.

[26] Morton, Rebecca B., and Jean-Robert Tyran. 2011. "Let the experts decide? Asymmetric information, abstention, and coordination in standing committees." *Games and Economic Behavior*, 72(2): 485-509.

[27] Myerson, Roger. 1998. "Population Uncertainty and Poisson Games." *International Journal of Game Theory*, 27(3): 375-392.

[28] Sobbrio, Francesco and Pietro Navarra. 2010. "Electoral Participation and Communicative Voting in Europe." *European Journal of Political Economy*, 26(2): 185-207.