Investor Sophistication and Capital Income Inequality*

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Abstract

What contributes to the growing income inequality across U.S. households? We develop an information-based general equilibrium model that links capital income derived from financial assets to a level of investor sophistication. Our model implies income inequality between sophisticated and unsophisticated investors that is growing in investors' aggregate and relative sophistication in the market. We show that our model is quantitatively consistent with the data from the U.S. market. In addition, we provide supporting evidence for our mechanism using a unique set of cross-sectional and time-series predictions on asset ownership and stock turnover.

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The rise in wealth and income inequality in the United States and worldwide has been one of the most hotly discussed topics over the last few decades in policy and academic circles.[1] A significant step towards understanding the patterns in the data is the vast empirical and theoretical literature on wage inequality, skill-biased technological change, and the polarization of the U.S. labor market.[2] Less understood thus far has been inequality in capital income generated in financial markets. An important component of total income, capital income is by far the most polarized part of household income in the United States, and it exhibits a strong upward trend in polarization.[3] A growing literature in economics and finance[4] analyzes household behavior in financial markets and especially its impact on financial returns. Some of the robust general trends in household behavior are a growing non-participation in high-return investments and a decline in trading activity. Anecdotal evidence suggests that an ever present and growing disparity in investor sophistication, or investor access to superior investment technologies, is partly responsible for these trends. An early articulation of this argument is Arrow (1987) and more recently Piketty (2014); however, micro-founded, quantitative treatments of such mechanisms are still missing.

In this paper, we provide a micro-founded mechanism for the return differential and show that, when embedded in a general equilibrium framework, it can go a long way in explaining the growth in capital income inequality, qualitatively and quantitatively. The main friction in the model is heterogeneity in investor sophistication. Intuitively, when information about financial assets is costly to process, individuals with different access to financial resources differ in terms of their capacity to acquire and process information. Wealthier investors gain access to better information, which allows them to earn a higher income on the assets they

[1] For a summary of the literature, see Piketty and Saez (2003); Atkinson, Piketty, and Saez (2011). A comprehensive discussion of the topic is also provided in the 2013 Summer issue of the Journal Economic Perspectives and in Piketty (2014).

[2] Representative contributions to this line of research include Katz and Autor (1999); Acemoglu (1999); Autor, Katz, and Kearney (2006, 2008); and Autor and Dorn (2013).

[3] Using the data from the Survey of Consumer Finances we document that approximately 25% of households actively participate in financial markets. Capital income accounts for approximately 15% of this group's total income, ranging from 40% to less than 1%. Between 1989 and 2010, the ratio of the capital income of the group in the 90th percentile of the wealth distribution relative to that of the median group increased from 29 to 57.

[4] Most recently represented by Calvet, Campbell, and Sodini (2007) and Chien, Cole, and Lustig (2011).

hold. As a result, their wealth diverges from that of less wealthy investors with relatively less information. In addition, the relatively unsophisticated investors perceive their information disadvantage through asset prices and allocate their investments away from the allocations of sophisticated investors, resulting in further divergence.

Our mechanism provides an explanation for the growing presence of sophisticated, institutional investors in risky asset classes, over the last 20-30 years (Gompers and Metrick (2001)). Specifically, the average institutional equity ownership has more than doubled over the last few decades, and it accounts now for more than 60% of the total stock ownership. Our hypothesis also fits well with a puzzling phenomenon of the last two decades of a growing retrenchment of retail investors from trading and stock market ownership in general (Stambaugh (2014)),[5] even though direct transaction costs, if anything, have fallen significantly. We document such avoidance of risky assets both for direct stock ownership and ownership of intermediated products, such as actively managed equity mutual funds. Specifically, we find that direct stock ownership has been falling steadily over the last 30 years, while flows into equity mutual funds coming from less sophisticated, retail investors began to decline and turn negative starting from the early 2000s, implying a drop in cumulative flows by 2012 by an astounding 70% of their 2000 levels.

To formalize our arguments and to assess their qualitative and quantitative match to the data, we build a noisy rational expectations equilibrium model with endogenous information acquisition and capacity constraints in the spirit of Van Nieuwerburgh and Veldkamp (2009, 2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013). We generalize this theoretical framework by allowing for meaningful heterogeneity both across assets and across investors. Specifically, we consider an economy with many risky assets and one riskless asset. The risky assets differ in terms of volatilities of their fundamental payoffs. A fraction of investors are endowed with high capacity for processing information and the remaining fraction have lower, yet positive capacity. Thus, everyone in the economy has the ability to

[5] We view the Stambaugh (2014) study as complementary to ours. It aims to explain the decreasing profit margins and activeness of active equity mutual funds using exogenously specified decline in individual investors' stock market participation. In contrast, our study endogenizes such decreasing participation as part of the mechanism which explains income inequality.

learn about assets payoffs, but to different degrees. Investors have mean-variance preferences with equal risk aversion coefficients and learn about assets payoffs from optimal private signals subject to an entropy constraint on information (Sims (2003)). Based on the observed assets characteristics, investors decide which assets to learn about, how much information to process about these assets, and how much wealth to invest.

In a departure from existing work, both the number of assets that are actively traded (i.e., learned about) in the market, and the mass of investors choosing to learn about each asset are determined endogenously. In equilibrium, learning exhibits *specialization*, *preference for volatility*, and *strategic substitutability*. First, each investor chooses to invest all her capacity into learning about one asset, while trading the other assets in the portfolio based only on her priors. Second, investors prefer to learn about assets with highly volatile payoffs, since that is where the gains from spending information capacity are the greatest, *ceteris paribus*. Third, the gains from learning about an asset decrease with the mass of investors already learning about it thus making investors prefer stocks that are relatively less explored. Therefore, despite the specialization at the investor level, the aggregate economy has an interior solution for the number of assets that are learned about, reflecting the strategic substitutability in learning.

We provide an analytical characterization of the model's predictions, which we then quantify in the parameterized model. First, in the cross-section of investors, sophisticated investors generate higher capital income relative to unsophisticated investors. This divergence is driven by three forces: (i) sophisticated investors have better information to identify profitable assets (average effect), (ii) sophisticated investors adjust their portfolios in real time (dynamic effect), and (iii) unsophisticated investors reduce their exposure to assets with large sophisticated ownership, due to the impact of sophisticated ownership on prices (general equilibrium effect). Second, heterogeneity in sophistication also affects trading intensity asymmetrically. Sophisticated investors frequently trade their assets while unsophisticated investors turn over their risky assets much less.

We next investigate the response of our outcome variables to shocks to sophistication. First, we show that sophisticated ownership increases with aggregate sophistication, which can be interpreted as general progress in information-processing technologies. This result holds even if we keep the relative sophistication of the two investor types constant. Intuitively, the more an investor knows, the easier it is for her to learn on the margin. This effect reinforces the general equilibrium effect that the same growth in capacity moves prices more when it is applied to the sophisticated investors' capacity than when it increases the capacity of the unsophisticated investors. This asymmetry leads to unsophisticated investors being priced out of the risky asset market.

Second, we show that in response to growth in aggregate sophistication, the mechanism implies a robust, unique way in which investors expand their risky portfolio holdings and change their portfolio composition. Specifically, we show that sophisticated investors keep moving down in the asset volatility dimension. At the same time, unsophisticated investors retrench from risky assets and hold safer assets.

Third, the symmetric expansion in capacity leads to lower expected market returns. These results play an important role in that they cut against plausible alternative explanations, such as the model with heterogeneous risk aversion or differences in trading costs.

To evaluate the quantitative fit of our theoretical predictions to the data, we calibrate the model using U.S. data spanning the period from 1989 to 2012. We parameterize the model using micro data on stocks and aggregate retail and institutional portfolios, which allows us to pin down details of the stochastic structure of assets payoffs. In our calibration, we set the parameters based on the first half of our sample period, and treat the second subperiod data moments as a test for the dynamic effect coming from progress in information technology. Specifically, in order to generate the dynamic predictions of the model, we introduce aggregate (not investor-specific) progress in information technology, which increases the average equity ownership rate of sophisticated investors from 23% (the data average for 1989-2000) to 43% (the data average for 2001-2012), while keeping the remaining parameters unchanged.

We show that the analytical predictions from the model are qualitatively and quantitatively borne out in the empirical evidence. First, sophisticated investors, on average, exhibit higher rates of returns that are approximately 2.7 percentage points per year higher in the model, compared to a 3 percentage point difference in the data. Hence, the information friction delivers return inequality qualitatively and quantitatively, which gives the model a chance to account for capital income polarization when we map it to the household data. As an additional unique feature of our mechanism, the model predicts that cross-sectional asset turnover is monotonically increasing with asset return volatility and with the ownership share of sophisticated investors, both results being consistent with the data. Second, we show that the dynamic predictions further confirm our economic mechanism. In response to symmetric growth in technology, sophisticated investors increase their ownership of equities by first entering the most volatile stocks and subsequently moving into stocks with medium and low volatility–a pattern we also document in the data. At the same time, sophisticated investors' entry into equity induces higher asset turnover, in magnitudes consistent with the data, both in the time series and in the cross-section of stocks.

In our main result, we use our parametrized model to explore the consequences of our mechanism for capital income inequality. We link information heterogeneity to past wealth and returns on financial assets. Intuitively, a high fixed cost and low marginal cost of access to information would endogenously lead to wealthier individuals obtaining better access to information, along the lines outlined in Arrow (1987). Here, we take this as a guiding principle in mapping the investors in our model into different wealth deciles in the Survey of Consumer Finances. Specifically, in the population of households who participate in asset markets, we use the ratio of average financial wealth of the 10% wealthiest investors relative to 50% poorest investors in 1989 as a proxy for *initial* relative investor sophistication, and we posit that the growth in financial wealth maps directly into a subsequent increase in investor sophistication. We then show that introducing this feedback generates *endogenous* evolution of capacity and capital income that can match capital income inequality growth in the data. The model implies an average inequality growth of 71% between 1989 and 2010, versus 83% in the data. Moreover, we can closely match the evolution of the growth rate over the entire sample period. These results further imply that from the perspective of our model, wealth is a good proxy for sophistication. This exercise can be viewed as a quantification of the economic mechanism proposed by Arrow (1987), in which financial wealth facilitates access to more sophisticated investment techniques, and begets even more wealth. This result is also consistent with evidence in Chapter 12 of Piketty (2014) in which he attributes differences in portfolio returns among University endowments to differences in their investment levels.

In addition to our quantitative analysis, we provide a discussion of general empirical regularities which qualitatively correspond to the analytical predictions of the model. We show that unsophisticated investors tend to hold an increasingly larger fraction of their wealth in safer, liquid assets. They also reduce their aggregate equity ownership. In the data, we observe a steady outflow of unsophisticated, retail money from risky assets, such as direct equity and equity mutual funds, while the flows from sophisticated investors into such assets are generally positive. Somewhat surprisingly, these outflows in the data continued until recently despite a large increase in the risky assets valuations.

This paper spans three strands of literature: (1) the literature on household finance; (2) the literature on rational inattention; and (3) the literature on income inequality. While some of our contributions are specific to each of the individual streams, our additional value added comes from the fact that we integrate the streams into one unified framework within our research context.

Our results relate to a wide spectrum of research in household finance and portfolio choice. The main ideas we develop build upon an empirical work on limited capital market participation (Mankiw and Zeldes (1991); Ameriks and Zeldes (2001)), growing institutional ownership (Gompers and Metrick (2001)), household trading decisions (Barber and Odean (2001), Campbell (2006), Calvet, Campbell, and Sodini (2009b, 2009a), Guiso and Sodini (2012)), and investor sophistication (Barber and Odean (2000, 2009), Calvet, Campbell, and Sodini (2007), Grinblatt, Keloharju, and Linnainmaa (2009)). While the majority of the studies attribute limited participation rates to either differences in stock market participation costs (Gomes and Michaelides (2005), Favilukis (2013)) or preferences, we relate the decisions to differences in sophistication across investors.

Another building block of our paper is the literature on rational inattention and endogenous information capacity that originates with the papers of Sims (1998, 2003, 2006). More germane to our application are models of costly information of Van Nieuwerburgh and

Veldkamp (2009, 2010), Mondria (2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013). The literature on endogenous information acquisition generally assumes that informed investors have homogenous information capacity, face a homogeneous set of risky assets, or learn about a single asset in the aggregate. In contrast, we study the implications of a model with heterogeneous agents in an environment with many heterogeneous assets, and we solve for the endogenous allocation of investor types across assets types. We show that the implications of such a model for portfolio decisions and asset prices are very different than those of the model with homogeneity. In addition, we study the quantitative implications of information frictions for income processes of investors and the equilibrium holdings of assets with different characteristics, such as volatility or turnover, all features which are absent in the present literature.

Our last building block constitutes the literature on income inequality that dates back to the seminal work by Kuznets and Jenks (1953) and has been subsequently advanced by the work of Piketty (2003), Piketty and Saez (2003), Alvaredo, Atkinson, Piketty, Saez, et al. (2013), Autor, Katz, and Kearney (2006), and Atkinson, Piketty, and Saez (2011). In contrast to our paper, a vast majority of that literature explain total income inequality looking at the income earned in labor market (e.g., Acemoglu (1999, 2002); Katz and Autor (1999); Autor, Katz, and Kearney (2006, 2008); and Autor and Dorn (2013)); and they do not consider explanations that relate to informational sophistication of investors.

The closest paper in spirit to ours is Arrow (1987) who also considers information differences as an explanation of income gap. However, his work does not consider heterogeneity across assets or investors and does not attempt a quantitative evaluation of the strength of the forces in general equilibrium. Both these elements are crucial for the results of our paper, and especially to establish the validity of our mechanism. Thanks to having a richer, equilibrium framework, we are able to parameterize the model and show that it comes very close to the data moments. Another work related to ours is Peress (2004) who examines the role that wealth and decreasing absolute risk aversion play in investors' acquisition of information and participation in risky assets. In contrast to that paper, we focus on micro foundations of how investors attain superior rates of return on equity. In addition, we model how different investors allocate their money across disaggregated risky asset classes. This allows us to test our information-based mechanism using micro-level data.

The rest of the paper proceeds as follows. In Section 1, we provide the general equilibrium framework to study behavior and income evolution of heterogeneously informed individuals. In Section 2, we derive analytical predictions, which we subsequently take to the data. In Section 3, we establish our main results and provide additional evidence in favor of our proposed mechanism. Section 4 concludes. All the proofs and derivations are in the Appendix.

1 Theoretical Framework

We build a noisy rational expectations model of portfolio choice with heterogeneous information a la Grossman and Stiglitz (1980), following the information choice models of Van Nieuwerburgh and Veldkamp (2009, 2010), and Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013). Our specification departs from this work by introducing heterogeneity in investors' capacity to process information. In our model, all investors have the ability to learn about asset payoffs, but to different degrees. This heterogeneity allows us to investigate how different levels of capacity dispersion affect returns and investment behavior, and it is critical to our results concerning the evolution of inequality over time. Our methodological contribution is to solve for the equilibrium allocation of information capacity across assets and investors. In our solution, both the number of assets that are being learned about and the mass of investors learning about each asset are determined endogenously. In contrast, previous work assumes that all investors with positive information capacity learn about the same asset(s). Since learning about an asset affects the holdings of that asset, the endogenous allocation of investor learning allows us derive rich asset-level predictions, and it is critical to our test of the information mechanism.

1.1 Setup

A continuum of atomless investors of mass one, indexed by j, solve a portfolio choice problem to maximize mean-variance utility over wealth W_j, given common risk aversion coefficient $\rho > 0$. The financial market consists of one risk-free asset, with price normalized to 1 and payoff r, and $n > 1$ risky assets, indexed by i, with prices p_i, and independent payoffs $z_i = \overline{z} + \varepsilon_i$, with $\varepsilon_i \sim \mathcal{N}\left(0, \sigma_i^2\right)$. The risk-free asset has unlimited supply, and each risky asset has fixed supply, $\overline{x}$. For each risky asset, non-optimizing "noise traders" trade for reasons orthogonal to prices and payoffs (e.g. liquidity, hedging, or life-cycle reasons), such that the net supply available to the optimizing investors is $x_i = \overline{x} + \nu_i$, with $\nu_i \sim \mathcal{N}\left(0, \sigma_x^2\right)$, independent of payoffs and across assets. For simplicity, we introduce heterogeneity only in the volatility of payoffs, although the model can easily accommodate heterogeneity in supply and in mean payoffs.

Prior to making the portfolio decision, each optimizing investor can choose to obtain information about some or all of the risky asset payoffs. Mass $\lambda \in (0, 1)$ of investors have high capacity for obtaining information, K_1, and are labeled *sophisticated*, and mass $1 - \lambda$ of investors have low capacity, K_2, and are labeled *unsophisticated*, with $0 < K_2 < K_1 < \infty$. Information is obtained in the form of endogenously designed signals subject to this capacity limit. The investor's signal choice is modeled following the rational inattention literature (Sims (2003)), using entropy reduction as a measure of the amount of information acquired. Signals are then used to update the beliefs that inform each investor's portfolio allocation.

Optimization occurs in two stages. In the first stage, investors solve their information acquisition problem: they choose the distribution of signals to receive in order to maximize expected utility, subject to their information capacity. In the second stage, given the signals they receive, investors update their beliefs about the payoffs and choose their portfolio holdings to maximize utility.

1.2 Portfolio Choice

We begin by solving the portfolio problem in the second stage, for a given signal structure. Each investor chooses portfolio holdings q_{ji} to solve

$$\max_{\{q_{ji}\}_{i=1}^{n}} U_j = E_j (W_j) - \frac{\rho}{2} V_j (W_j) \tag{1}$$

$$s.t. \;\; W_j = r \left(W_{0j} - \sum_{i=1}^{n} q_{ji} p_i \right) + \sum_{i=1}^{n} q_{ji} z_i, \tag{2}$$

where E_j and V_j denote the mean and variance conditional on investor j's information set, and W_{0j} is initial wealth. Optimal portfolio holdings are given by

$$q_{ji} = \frac{\widehat{\mu}_{ji} - r p_i}{\rho \widehat{\sigma}_{ji}^2}, \tag{3}$$

where $\widehat{\mu}_{ji}$ and $\widehat{\sigma}_{ji}^2$ are the mean and variance of investor j's posterior beliefs about the payoff z_i, conditional on the investor's information.

1.3 Information Choice

Each investor can receive a separate signal s_{ji} on each of the asset payoffs, z_i. Given the optimal portfolio holdings, the information problem in the first stage becomes choosing the distribution of these signals to maximize expected utility, $E_{0j} [U_j]$, subject to the information constraint, $I (z; s_j) \leq K_j$, where $I (z; s_j)$ denotes the Shannon (1948) mutual information, measuring the information that the vector of private signals conveys about the vector of payoffs. This information constraint imposes a limit on the amount of uncertainty reduction that the signals can achieve. Since perfect information requires infinite capacity, each investor faces some residual uncertainty about the realized payoffs.

For analytical tractability, we make the following assumption about the signal structure:

Assumption 1 *The signals s_{ji} are independent across assets.*

Assumption 1 implies that the total quantity of information obtained by an investor can be expressed as a sum of the quantities of information obtained for each asset.[6] The information constraint becomes $\sum_{i=1}^{n} I\left(z_i; s_{ji}\right) \leq K_j$, where $I\left(z_i; s_{ji}\right)$ measures the information conveyed by the private signal s_{ji} about the payoff of asset i.

Investors decompose each payoff into a lower-entropy signal component and a residual component that represents the information lost through this compression: $z_i = s_{ji} + \delta_{ji}$.[7] For tractability, we introduce the following additional assumption:

Assumption 2 *The signal s_{ji} is independent of the data loss δ_{ji}.*

Since z_i is normally distributed, Assumption 2 implies that s_{ji} and δ_{ji} are also normally distributed, by Cramer's Theorem: $s_{ji} \sim \mathcal{N}\left(\overline{z}, \sigma_{sji}^2\right)$ and $\delta_{ji} \sim \mathcal{N}\left(0, \sigma_{\delta ji}^2\right)$ with $\sigma_i^2 = \sigma_{sji}^2 + \sigma_{\delta ji}^2$.[8] Hence, posterior beliefs are also normally distributed random variables, independent across assets, with mean $\widehat{\mu}_{ji} = s_{ji}$ and variance $\widehat{\sigma}_{ji}^2 = \sigma_{\delta ji}^2$. A perfectly precise signal would be associated with no information loss, such that the investor's posterior uncertainty would be zero. Conversely, a signal that consumes no information capacity would be completely uninformative: it would discard all information about the realized payoff, returning only the mean payoff, $\overline{z}$, and leaving the investor's posterior uncertainty equal to the prior uncertainty.

Using this signal structure and the resulting distribution of expected excess returns, the investor's information problem becomes choosing the variance of posterior beliefs to solve

$$\max_{\left\{\widehat{\sigma}_{ji}^2\right\}_{i=1}^{n}} \sum_{i=1}^{n} G_i \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2}, \tag{4}$$

$$s.t. \quad \prod_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} \leq e^{2K_j}, \tag{5}$$

where G_i represents the expected utility gain from learning about asset i. This gain is a function of the distribution of expected excess returns. It is an equilibrium object, and it is common across investor types and taken as given by each investor.

[6] Assumption 1 is common in the literature. Allowing for potentially correlated signals requires a numerical approach, and is beyond the scope of this paper.

[7] We discuss the difference between these compressed signals and the signals with additive noise usually employed in the literature in Section **??**.

[8] In general, the optimal signal structure may require correlation between the signal and the data loss, but Assumption 2 maintains this analytical tractability.

Lemma 1 *The solution to the maximization problem (4)-(5) is a corner: each investor allocates her entire capacity to learning about a single asset from the set of assets with maximal expected utility gains. The posterior beliefs of an investor j, learning about asset $l_j \in \arg\max_i G_i$, are normally distributed, with mean and variance*

$$\widehat{\mu}_{ji} = \begin{cases} s_{ji} & \text{if } i = l_j \\ \overline{z} & \text{if } i \neq l_j \end{cases} \quad \text{and} \quad \widehat{\sigma}^2_{ji} = \begin{cases} e^{-2K_j}\sigma_i^2 & \text{if } i = l_j \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases} \tag{6}$$

The linear objective function and the convex constraint imply that each investor specializes, learning about a single asset. Which asset that is—namely which are the assets with the biggest gains from learning—will be determined in equilibrium. Conditional on the realized payoff, the signal is normally distributed random variables whose mean is a weighted sum of the prior and the shock, $E\left(s_{ji}|z_i\right) = \overline{z} + \left(1 - e^{-2K_j}\right)\varepsilon_i$, and whose variance is $V\left(s_{ji}|z_i\right) = \left(e^{-2K_j} - e^{-4K_j}\right)\sigma_i^2$. Hence, as capacity $K_j \to \infty$, the conditional signal approaches the realized payoff, z_i, and the conditional variance approaches zero. For $K_j = 0$, the signal is always equal to the prior mean, $\overline{z}$, and the conditional variance is once again zero.

1.4 Equilibrium Prices

Given the solution to each investor's portfolio and information problem, the market clearing condition pins down equilibrium prices as linear combinations of the shocks.

Lemma 2 *The price of asset i is given by $p_i = a_i + b_i\varepsilon_i - c_i\nu_i$, with*

$$a_i = \frac{1}{r}\left[\overline{z} - \frac{\rho\sigma_i^2\overline{x}}{(1+\phi m_i)}\right], \quad b_i = \frac{\phi m_i}{r\left(1+\phi m_i\right)}, \quad c_i = \frac{\rho\sigma_i^2}{r\left(1+\phi m_i\right)}, \tag{7}$$

where $\phi \equiv \lambda\left(e^{2K_1} - 1\right) + (1-\lambda)\left(e^{2K_2} - 1\right)$ is a measure of the aggregate capacity for processing information in the economy, and m_i is the mass of investors learning about asset i, with $\sum_{i=1}^n m_i = 1$.

The price of an asset reflects the asset's payoff and supply shocks, with relative weights that depend on the mass of investors learning about the asset. For each asset, the quantity ϕm_i is an important statistic: it is a measure of the capacity that the market allocates to learning about that asset in equilibrium. For assets in markets with no information capacity ($\phi = 0$) and for assets that are not learned about ($m_i = 0$), the price only reflects the noise shock ν_i. As the capacity allocated to an asset increases, the asset's price comoves more strongly with the underlying payoff (c_i decreases and b_i increases, though at a decreasing rate). In the limit, as $\phi \to \infty$, the price approaches the discounted payoff, z_i/r.

1.5 Equilibrium Learning

Using equilibrium prices, we derive the conditions that determine the assets that are learned about in equilibrium and the mass of investors learning about each asset. Without loss of generality, let assets be ordered such that $\sigma_i^2 > \sigma_{i+1}^2$ for all $i \in \{1, ..., n-1\}$.

Lemma 3 *The equilibrium gain from learning about asset i is*

$$G_i = \frac{1 + \rho^2 \xi_i}{(1 + \phi m_i)^2}, \tag{8}$$

where $\xi_i \equiv \sigma_i^2 \left(\sigma_x^2 + \overline{x}^2\right)$ summarizes the properties of asset i. Let k denote the number of assets with strictly positive learning mass in equilibrium. Then, the masses $\{m_i\}_{i=1}^n$ are uniquely pinned down by the following conditions:

$$G_i = G^*, \quad \forall i \in \{1, ..., k\}, \tag{9}$$

$$G_i < G^*, \forall i \in \{k+1, ..., n\}, \tag{10}$$

$$\sum_{i=1}^{k} m_i = 1. \tag{11}$$

The equilibrium allocation of active investors across assets, $\{m_i\}_{i=1}^n$, is pinned down by the $k-1$ indifference conditions implied by equation (9), combined with the condition that each investor learns about some asset, (11). Condition (10) ensures that no other asset yields a higher gain from learning.

Lemma 3 implies the following properties for the equilibrium gains:

$$\frac{\partial G_i}{\partial \sigma_i^2} > 0, \quad \frac{\partial G_i}{\partial m_i} \leq 0, \quad \frac{\partial G_i}{\partial \phi} \leq 0.$$

Hence, learning in the model exhibits preference for volatility (high σ_i^2) and strategic substitutability (low m_i). Furthermore, for the assets that are actively traded, the value of learning about an asset also falls with the aggregate amount of information in the market, *ceteris paribus,* since more capacity overall increases the comovement between prices and payoffs, thereby reducing excess returns.

For sufficiently low information capacity, all investors learn only about the most volatile asset: for $\phi < \phi_1$, $m_1 = 1$ and $m_i = 0$ for all $i > 1$, where

$$\phi_1 \equiv \sqrt{\frac{1 + \rho^2 \xi_1}{1 + \rho^2 \xi_2}} - 1. \tag{12}$$

Hence, we endogenize the single-asset learning assumed by Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013) as an optimal outcome for low enough information processing capacity. Above this threshold, we obtain an interior solution: as the overall capacity in the economy increases, investors expand their learning towards lower volatility assets. For sufficiently high information capacity, or alternatively, for low enough dispersion in asset volatilities, all assets are learned about, thus endogenizing the assumption employed in models with exogenous signals.[9] However, in the presence of asset heterogeneity, even if all assets are learned about, there is heterogeneity in the information capacity allocated to each asset: since the equilibrium gain is increasing in σ_i^2 and decreasing in m_i, the mass of investors learning about each asset is increasing in σ_i^2. In turn, this heterogeneity has implications for

[9] This result also implies that if the degree of dispersion in asset payoff volatilities varies, learning will also vary, with economies/periods with high dispersion being characterized by more concentrated learning, and economies/periods with low dispersion being characterized by more diversified learning (and hence portfolios).

holdings, returns and turnover in the cross section of assets.[10] In the quantitative analysis of Section 3, we parameterize the model to obtain an interior solution, in order to take these implications to the data, as a test of our mechanism.

In order to further characterize aggregate learning, we introduce the following notation:

Definition 1 *Let ϕ_k be a threshold for ϕ, such that for any $\phi < \phi_k$, at most k assets are actively traded (learned about) in equilibrium, while for $\phi \geq \phi_k$, at least $k+1$ assets are actively traded in equilibrium.*

Using this definition, Lemma 3 implies that the threshold values of aggregate information capacity are monotonic: $0 < \phi_1 < \phi_2 < ... < \phi_{n-1}$. The following lemma further characterizes aggregate learning in response to variation in the level and in the dispersion of investor capacities.

Lemma 4 *Let $\phi_{k-1} \leq \phi \leq \phi_k$, such that $k > 1$ assets are actively traded, and consider changes in investor capacity such that k' is the new number of actively traded assets (potentially different from k for large enough changes in capacity). Then, equilibrium masses $\{m_i\}_{i=1}^n$ satisfy the following conditions:*

(i) There exists a threshold asset $\bar{\imath} < k'$, such that m_i is decreasing in ϕ for all assets $i \leq \bar{\imath}$, and increasing in ϕ for all assets $\bar{\imath} < i \leq k'$.

(ii) The quantity (ϕm_i) is increasing in ϕ for all assets $i \in \{1, ..., k'\}$.

(iii) The quantity $m_i(e^{2K_j} - 1)$, $j \in \{1, 2\}$, is increasing in K_j at an increasing rate for all assets $i \in \{1, ..., k'\}$.

Lemma 4 shows that as the amount of aggregate capacity ϕ increases, the amount of capacity allocated to each asset (ϕm_i) strictly increases for all assets that are learned about (part (*ii*)), even though the mass m_i of investors learning about the most volatile assets decreases, so that investors shift to new assets to be learned about (part (*i*)). Furthermore, the amount of capacity allocated to each asset by each investor group $(m_i(e^{2K_j} - 1))$ also increases, but it increases by more for the group of sophisticated investors, who have higher

[10] If assets are homogeneous, then $m_i = 1/n$ $\forall i \in \{1, ..., n\}$.

capacity (part (*iii*)). In Section 2, we use these results to derive analytical predictions on the patterns of investment, in response to both an increase in the cross-sectional dispersion of capacity and a symmetric growth of capacity.

1.6 The Value of Prices

We have presented each investor's information acquisition problem in terms of a constraint on the information obtained through private signals alone, disregarding the information content of prices. Specifically, we have assumed that investors cannot condition on prices in the information acquisition stage, even though they can condition on prices when choosing asset holdings in the second stage (which is necessary for market clearing, given the exogenous supply). When some agents in the economy acquire information through private signals, prices become informative about asset payoffs, because they reflect the demand of these privately informed investors. In the literature on portfolio choice with exogenous signals, investors are often assumed to learn about payoffs not only from their private signals, but also from equilibrium prices, which aggregate the information of all investors in the market (e.g. Admati (1985)). Hence, we now ask: would our investors have an incentive to allocate capacity to learning from prices rather than only from their own private signals? We show that the answer to this question is no: if the information contained in prices is costly to process, then prices are an inferior source of information when the investor has the opportunity to observe endogenously designed signals on the payoffs themselves.

We consider the signal choice of an individual investor, taking the choices of all other investors as given by the solution in Sections 1.4 and 1.5. Processing information through either prices or private signals consumes the investor's capacity. Hence, whatever the source of information, the investor cannot acquire a total quantity beyond her capacity limit, K_j.

Lemma 5 *If prices consume capacity, then the capacity-constrained investor chooses to devote all her capacity to learning about payoffs through private signals on asset payoffs, rather than devoting any capacity to learning from prices.*

The investor strictly prefers learning through the private signal in the presence of strategic substitutability, which corresponds to the model in this paper.[11] Our proof is based on the same logic as that of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013), although it is derived for a different information structure and extended to include the case in which the information content of prices is not necessarily processed perfectly.

Lemma 5 implies that investors will not allocate any capacity to learning about the supply shock, ν_i, either: learning about the noise traders is not useful, *unless* that information is combined with information processed from equilibrium prices. It is only joint information on both variables that can inform investors about payoffs, which is ultimately what they seek to learn.

2 Analytical Predictions

In this section, we present a set of analytical results implied by our model. We first present the predictions for capital income inequality followed by a set of theoretical predictions that are specific to the information-based mechanism. These results allow us to compare the model's implications with evidence from stock-level micro data. We conclude with a discussion of how variations of the baseline framework affect equilibrium outcomes.

2.1 Capital Income Inequality

Let π_{ji} denote the average profit per capita for an investor of type $j \in \{1,2\}$, from trading asset i:

$$\pi_{1i} \equiv \frac{Q_{1i}\left(z_i - rp_i\right)}{\lambda} \quad \text{and} \quad \pi_{2i} \equiv \frac{Q_{2i}\left(z_i - rp_i\right)}{1-\lambda}, \tag{13}$$

where Q_{1i} and Q_{2i} are the aggregate holdings of asset i for sophisticated and unsophisticated investors, respectively, obtained by integrating holdings q_{ji} across investors of each type:

$$Q_{1i} = \lambda \left[\frac{\left(\overline{z}_i - rp_i\right) + m_i\left(e^{2K_1} - 1\right)\left(z_i - rp_i\right)}{\rho\sigma_i^2}\right], \tag{14}$$

[11] Conversely, it is well known from the literature on learning with strategic complementarities that in such settings agents overweigh the public signals at the expense of their private signals (e.g. Morris and Shin (2002)).

$$Q_{2i} = (1-\lambda)\left[\frac{(\overline{z}_i - rp_i) + m_i\left(e^{2K_2}-1\right)(z_i - rp_i)}{\rho\sigma_i^2}\right]. \tag{15}$$

Our first result is that heterogeneity in information capacity across investors drives capital income inequality as sophisticated investors generate higher income than unsophisticated ones. This is summarized in Proposition 1.

Proposition 1 *If* $K_1 > K_2$ *then* $\sum_i \pi_{1i} - \sum_i \pi_{2i} > 0$.

The informational advantage manifests itself in two ways. First, sophisticated investors achieve relatively higher profits by holding a different average portfolio (the average effect). Second, they also achieve relatively higher profits by obtaining larger gains from shock realizations that are profitable relative to expectations, and incurring smaller losses on unprofitable shock realizations (the dynamic effect). These two effects show up in the average level and in the adjustment of holdings in response to shocks, and are summarized in Proposition 2.

Proposition 2 *Let* $K_1 > K_2$ *and* $\phi_{k-1} \leq \phi < \phi_k$, *such that the first* $k > 1$ *assets are actively traded in equilibrium. The following statements hold:*

(i) $E\left\{\frac{Q_{1it}}{\lambda} - \frac{Q_{2it}}{(1-\lambda)}\right\} > 0$ *for* $i \in \{1, ..., k\}$.

(ii) If $E(z_i - rp_i) > E(z_l - rp_l)$, *then* $E\left\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}\right\} > E\left\{\frac{Q_{1l}}{\lambda} - \frac{Q_{2l}}{(1-\lambda)}\right\}$, *for any two assets* $i, l \in \{1, ..., k\}$,.

(iii) $\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}$ *is increasing in excess returns,* $z_i - rp_i$, *for* $i \in \{1, ..., k\}$.

Proposition 2 demonstrates that sophisticated investors choose higher average holdings of risky assets (part (*i*)), since the risky assets are the ones with the largest gains from learning, *ceteris paribus*. On average, they also tilt their portfolios towards profitable assets more than unsophisticated investors do (part (*ii*)). Moreover, for every realized state x_i, z_i, sophisticated investors are able to adjust their portfolios (contemporaneously) upwards if the shock implies high returns and downwards if the shock implies low returns (part (*iii*)). Hence, also dynamically, they are able to outperform unsophisticated investors by responding to shock realizations in a way that increases their profits.

The differential dynamic adjustment of investor portfolios implies differences in trading intensity, as shown in Proposition 3

Proposition 3 *Let $K_1 > K_2$ and $\phi_{k-1} \leq \phi < \phi_k$, such that the first $k > 1$ assets are actively traded in equilibrium. Let V_{ji} denote the average per capita volume of trade generated by investors of type $j \in \{1, 2\}$ in asset i. Then, $V_{1i} \geq V_{2i}$ with equality for $i \in \{k+1, ...n\}$.*

To see explicitly the impact on capital income inequality coming from the dynamic effect, we express the total capital income of an average sophisticated investor as[12]

$$\sum_{i=1}^{n} \pi_{1i} \equiv \sum_{i=1}^{n} \alpha_i \pi_{2i}, \tag{16}$$

where, by (14) and (15),

$$\alpha_i \equiv \frac{\pi_{1i}}{\pi_{2i}} = \frac{(\overline{z}_i - rp_i) + m_i(e^{2K_1} - 1)(z_i - rp_i)}{(\overline{z}_i - rp_i) + m_i(e^{2K_2} - 1)(z_i - rp_i)}, \quad \forall i. \tag{17}$$

That is, capital income of an average sophisticated investor can be expressed as a weighted sum of an average unsophisticated investor's capital income from each asset, but the weights depart from 1 whenever the asset is actively traded ($m_i > 0$).

To see the dynamic effect, consider how variation in the weights α_i drives income differences. For assets that are actively traded in equilibrium, they vary depending on the realization of the shocks z_i and x_i. There are two possible scenarios. First, $\pi_{2i} > 0$, which by (17) implies $\pi_{1i} > 0$ and $\alpha_i > 1$. Hence, sophisticated investors have a larger gain in their (positive) capital income from asset i. Second, $\pi_{2i} < 0$ and either (i) $\pi_{1i} < 0$ and $0 < \alpha_i < 1$, or (ii) $\pi_{1i} > 0$ and $\alpha_i < 0$. The first case implies that sophisticated investors put a smaller weight in their portfolio on the loss, while the second case means that the profit of sophisticated investors puts a negative weight on the loss. In both cases, sophisticated investors either incur a smaller loss or realize a bigger profit, state by state.

[12]Here, we are implicitly assuming that profits are never exactly zero. For such case, the arguments extend trivially.

These arguments lead to the following comparative result: *increases* in sophistication heterogeneity lead to a *growing* capital income polarization. Intuitively, greater dispersion in information capacity means that, relative to unsophisticated investors, sophisticated investors receive higher-quality signals about the fundamental shocks, and as a result, they respond more strongly to realized excess profits $z_i - rp_i$. This is the essence of Proposition 4.

Proposition 4 *Consider an increase in capacity dispersion of the form $K_1' = K_1 + \Delta_1 > K_1$, $K_2' = K_2 - \Delta_2 < K_2$, with Δ_1 and Δ_2 chosen such that total information capacity ϕ remains unchanged. Then, the ratio $\sum_i \pi_{1i} / \sum_i \pi_{2i}$ increases, that is, capital income becomes more polarized.*

The results show that heterogeneity in capacity generates heterogeneity in portfolios, which in result decreases the relative participation of unsophisticated investors. Below, we explore the intuitive reasons behind unsophisticated investors' retrenchment from risky assets in the presence of informationally superior, sophisticated investors.

Intuition Suppose that the realized state is such that in equilibrium $z_i - rp_i > 0$ for some asset i. Consider one of a set of *homogeneous* investors with capacity K_2 who learns about asset i and receives the mean signal for her type, $S_2 = \bar{z}_i e^{-2K_2} + z_i(1 - e^{-2K_2})$. Her allocation is then

$$q_{2i} = e^{2K_2} \left(\frac{S_2 - rp_i}{\rho \sigma_i^2} \right),$$

where $e^{-2K_2}\sigma_i^2$ is the variance of the her posterior beliefs.

Let the allocation of investors to learning about different assets, $\{m_i\}_{i=1}^n$, also be fixed at the equilibrium level, and exogenously increase the capacity of mass $\gamma < m_i$ of investors to $K_1 > K_2$ so that they become more sophisticated. These new sophisticated investors have average demand given by

$$q_{1i} = e^{2K_1} \left(\frac{S_1 - rp_i}{\rho \sigma_i^2} \right),$$

where the mean signal they receive is $S_1 = \bar{z}_i e^{-2K_1} + z_i(1 - e^{-2K_1})$.

There are two effects that lead to an increased relative participation of sophisticated investors in risky assets in this example: a partial equilibrium one and a general equilibrium one.

First, absent any price adjustment, the partial equilibrium effect is that the remaining unsophisticated investors do not change their demand q_{2i} for asset i. But the new sophisticated investors now demand more, because *(i)* $S_1 > S_2$ (we are considering a good state where $z_i > \bar{z}_i$), and *(ii)* their signal is more precise ($e^{-2K_1}\sigma_i^2 < e^{-2K_2}\sigma_i^2$). Hence, in partial equilibrium, we would observe growth in sophisticated investors' ownership. However, we would see no change in the strategies of unsophisticated investors.

Second, there is the general equilibrium effect working through price adjustment, which makes unsophisticated investors perceive an *informational disadvantage* in trading asset i after sophisticated investors enter. In particular, in accordance with the market clearing conditions, the price will adjust to the greater demand from the relatively more informed investors. Through that price adjustment, both types of investors will see their profits fall, but only unsophisticated ones will choose to reduce their holdings. Their signals are not of a high enough quality to sustain previous positions as the optimal choice. Through this general equilibrium effect, the entry of sophisticated investors spills over to an informational disadvantage for unsophisticated investors and causes their retrenchment from trading the asset.

2.2 Testing the Mechanism

In this section, we provide further analytical characterization of our model's predictions. These analytical results, together with the quantitative predictions from our parameterized model, serve as a test of the main mechanism of the model when compared with the same features in the data.

We start with the characterization of properties of the market return in response to growth in the overall level of information in the economy. As aggregate information increases, prices contain a growing amount of information about the fundamental shocks, and excess market return drops. This is summarized in Proposition 5.

Proposition 5 (Market Value) *Growth in total information processing capacity leads to*

(i) higher average prices, $\frac{d\overline{p}_i}{d\phi} \geq 0$*;*

(ii) lower average market excess returns, $dE\left(z_{it} - rp_{it}\right)/d\phi \leq 0$*.*

Next, in Proposition 6, we consider the effects of a pure increase in dispersion of sophistication, without changing the aggregate level of sophistication in the economy. Such polarization in capacities implies polarization in holdings.

Proposition 6 *Consider an increase in capacity dispersion of the form* $K_1' = K_1 + \Delta_1 > K_1$, $K_2' = K_2 + \Delta_2 < K_2$, *with* Δ_1 *and* Δ_2 *chosen such that total information capacity* ϕ *remains unchanged. Then, the average ownership difference* $E\left\{\sum_i \frac{Q_{1i}}{\lambda} - \sum_i \frac{Q_{2i}}{1-\lambda}\right\}$ *increases.*

Using Proposition 4, we can show that the aggregate *symmetric* growth in information technology, modeled as a common growth rate of both K_1 and K_2, leads to a growing retrenchment of unsophisticated investors and hence an increased ownership of risky assets by sophisticated (Proposition 7), as well as growing capital income polarization (Proposition 8).

Proposition 7 (Dynamic Ownership) *Consider symmetric information capacity, such that* $K_{1t} = K_t$ *and* $K_{2t} = K_t\gamma$, $\gamma \in (0,1)$, *and consider* $\phi_{k-1} < \phi < \phi_k$ *such that the first* $k > 1$ *assets are actively traded in equilibrium. In equilibrium, the average ownership share by sophisticated investors increases across all assets: for all i*

$$dE\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{1-\lambda}\}/dK > 0.$$

Proposition 8 (Capital Income Polarization) *Consider symmetric information capacity, such that* $K_{1t} = K_t$ *and* $K_{2t} = K_t\gamma$, $\gamma \in (0,1)$, *and consider* $\phi_{k-1} < \phi < \phi_k$ *such that the first* $k > 1$ *assets are actively traded in equilibrium. In equilibrium, the average capital income becomes more polarized:*

$$dE\{\sum_i \pi_{1i} / \sum_i \pi_{2i}\}/dK > 0.$$

3 Quantitative Results

In this section, we present the results corresponding to our analytical predictions. We first discuss the parametrization of the model using stock-level micro data, and show the quantitative impact of information frictions. Further, we present results that help us to identify our economic mechanism in the data. Then, we proceed to our main result: We use the predictions of the parameterized model to establish the link between differences in investors' sophistication – proxied by differences in their financial wealth – and capital income inequality, using a sample of individuals from the Survey of Consumer Finances. Finally, we proceed with the discussion of alternative mechanisms, and provide additional empirical evidence that supports our analytical predictions.

3.1 Parametrization

Our analytical design combines a portfolio framework with information frictions. Thus, in order to parameterize the model it is essential that we use data with a similar level of granularity. Our evidence is based on institutional portfolio holdings data from Thomson Reuters. These data contain a large sample of portfolios of publicly traded equity held by institutional investors and come from quarterly reports required by law and submitted by institutional investors to the Securities and Exchange Commission (SEC). While the official requirement for reporting is that the minimum asset size exceed $100 million, and thus not all investors are in the data, in reality, the data are comprehensive as more than 95% of all dollar investments are reported. Overall, thanks to their rich micro-level structure, the data allow us to directly test the predictions of our model for portfolio composition and its evolution for different asset classes.

To map the model to the data, we need to identify the heterogeneity in information capacity across investors. To this end, we define sophisticated investors as those classified as investment companies or independent advisors (types 3 and 4) in the Thomson data set. These investors include wealthy individuals, mutual funds, and hedge funds. Among all types, these two groups are known to be particularly active in their information production efforts; in turn, other groups, such as banks, insurance companies, or endowments and pensions are more passive by nature. Our definition of unsophisticated investors is other shareholders who are not part of Thomson data. These are individual (retail) investors.

To provide the empirical verification of the proposed investor classification, we show the evolution of cumulative returns of portfolios held by the two types of investors using data over the time period 1989-2012. We proceed in three steps. First, we obtain the market value of each stock held by all investors of a given type. Market value of each stock is the product of the number of combined shares held by a given investor type and the price per share of that stock, obtained from CRSP. Since the number of shares held by unsophisticated investors is not directly observable, we impute this value by taking the difference between the total number of shares available for trade and the number of shares held by *all* institutional investors. Second, we calculate the value shares of each stock in the aggregate portfolio by taking the ratio of market value of each stock relative to the total value of the portfolio of each type of investor. Third, we obtain the return on the aggregate portfolio by matching each asset share with their next month realized return and calculating the value-weighted aggregated return. We repeat this procedure separately for sophisticated and unsophisticated investors.

To compare portfolio performance of the two investor types, we calculate cumulative values of $1 invested by each group in January 1989 using time series of the aggregated monthly returns ending in December 2012. We present the two series in Figure 1.

Our results indicate that portfolio returns of sophisticated investors indeed systematically outperform those of unsophisticated investors. The value of $1 invested in January 1989 grows to $5.32 at the end of 2012 for sophisticated investors and only to $3.28 for unsophisticated investors.

We now proceed to the details of the parametrization of the model that we subsequently use to assess the validity of our economic mechanism. We use stock-level micro data and aggregated investors' equity shares, which allows us to test the model's predictions regarding portfolio allocations and asset turnover across assets and over time. We parameterize the model to match key moments of the data for the period 1989-2000. We think of this as the

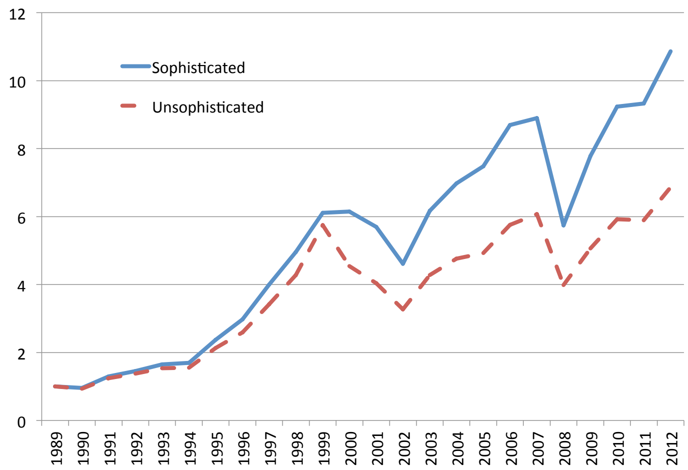

Figure 1: Cumulative Return in Equity Markets.

initial period in our model and treat it as a point of departure for our dynamic comparative statics exercises.

The key parameters of our model are the information capacity of each investor type (K_1 and K_2), the averages and volatilities of the fundamental shocks ($\bar{z}_i, \sigma_i$) and the supply shocks ($\bar{x}_i, \sigma_{xi}, , i = 1, ..., n$), the risk aversion parameter (ρ), and the fraction of sophisticated investors (λ).

For parsimony, we restrict some parameters and normalize the natural candidates. In particular, we normalize $\bar{x} = 5, \bar{z} = 10$ and restrict $\sigma_{xi} = \sigma_x$. To capture heterogeneity in assets returns, we set the lowest volatility $\sigma_n = 1$ and assume that volatility changes linearly across assets, which means that it can be parameterized by a single number, the slope of the line.[13] We pick the remaining parameters to match the following targets in the data (based on 1989-2000 averages): (i) aggregate equity ownership of sophisticated investors, equal to 23%; (ii) real risk-free interest rate, defined as the average nominal return on 3-month Treasury bills minus inflation rate, equal to 2.5%; (iii) average annualized stock market return in excess of the risk-free rate, equal to 11.9%; (iv) average monthly equity turnover, defined as the total monthly volume divided by the number of shares outstanding, equal to 9.7%; (v) the ratio of the 90th percentile to the median of the cross-sectional idiosyncratic volatility

[13]In particular, we set $\sigma_i = \sigma_n + \alpha(n - i)/n$ which, given our normalization of σ_n, leaves only α to be determined.

of stock returns, equal to 3.54. In addition, we arbitrarily set the fraction of assets about which agents learn to 50%.

To generate the dynamic predictions of our model, we assume a symmetric growth in information capacity of each investor type, in order to match the 2001-2012 average equity ownership rate of sophisticated investors, equal to 43%. The progress in information capacity required to achieve this target amounts to a roughly 9.5% capacity growth annually (for 11 years from the middle of the first sub period to the middle of the second sub period). We think of this approach as a way of modeling technological progress in investment technology which affects both types of investors in the same way—hence, the reported results are not driven by differential growth but come solely from the general equilibrium effects of our mechanism.

The above procedure leaves us with one key parameter left—the ratio of information capacity of sophisticated versus unsophisticated investors, K_1/K_2. We set this parameter to 10% in this section, and use Survey of Consumer Finances data to pin it down when we use the model to predict capital income polarization in the next section. The parameters and model fit are presented in Tables 1 and 2.

Table 1: Parameter Values

Parameter	Value
K_1, K_2, λ, n	0.577, 0.0577, 0.2, 10
$\bar{z}_i$, $\bar{x}_i$	10, 5
ρ	1.0865
σ_{xi}	0.41 for all assets i
$\{\sigma_i\}$, $i = 1, ..., 10$ assets	$\{1.5026, 1.4468, 1.3909, 1.3351, 1.2792, 1.2234, 1.1675,$ $1.1117, 1.0558, 1\}$
K_1, K_2 ex-post	1.57, 0.157

3.2 Quantifying the Information Friction

In this section, we explore the quantitative implications of our information frictions, and contrast them with the stock-level micro evidence. We first discuss findings related to returns

Table 2: Parametrization: Model Fit

Statistic	Data	Model
Market Return	11.9%	11.9%
Average Turnover	9.7%	9.7%
Sophisticated Investors' Ownership	23%	23%
Informed Trading	n.a.	50%

inequality. Next, we show that predictions specific to our economic mechanism are borne out in the data, which we view as independent tests of our model.

3.2.1 Returns Inequality

We report the results in Table 3. The parameterized model implies a 2.7 percentage point advantage (14% versus 11.3%) in average portfolio returns between sophisticated and unsophisticated investors, which accounts for 90% of the difference in the data for the 1989-2000 period (13.4% versus 10.4%). Thus, the model can account for a significant fraction of the empirical difference in returns across the two investor types. Given that our mechanism has an economically large implication for the difference in performance across agents with different information capacities, this suggests that a similarly large economic effect may also exist within the household sector. In particular, if sophistication can be approximated by financial wealth (as implied by a setting in Arrow (1987)), then our mechanism would imply a growing disparity in capital incomes across households. We explore the quantitative implications of this hypothesis in Section 3.3.

Decomposing the Return Differential As our analytical results suggest, sophisticated investors outperform unsophisticated investors for two reasons: (i) they are more exposed to risk because they hold a larger share of risky assets (compensation for risk); and (ii) they have informational advantage (compensation for skill).

Below, we decompose the returns of each investor type to shed light on the relative importance of these two effects. To this end, we first derive the conditional CAPM model in

Table 3: Market Averages by Subperiod: Data and Model

	1989-2000		2001-2012	
Statistic	Data	Model	Data	Model
Market Return	11.9%	11.9%	2.4%	3.5%
Sophisticated Investors' Return	13.4%	14%	2.9%	3.7%
Unsophisticated Investors' Return	10.4%	11.3%	1.6%	3.4%
Average Equity Turnover	9.7%	9.7%	16%	14%
Sophisticated Investors' Ownership	23%	23%	43%	43%

our setup, which gives the asset pricing equation

$$E_t(r_{it}) - r = \frac{cov_t(r_{it}, r_{mt})}{var_t(r_{mt})}(E_t(r_{mt}) - r), \tag{18}$$

so that the conditional market β_{it} of asset i is $\frac{cov_t(r_{it}, r_{mt})}{var_t(r_{mt})}$. In the above, r_{it} is the return on asset i in period t, p_{mt} is the price of the market portfolio ($p_m := \sum x_i p_i$), and $r_{mt} = \sum z_{it} x_{it}/p_{mt}$ is the market return.[14]

In order to evaluate returns on a portfolio of average investor type $j = S, U$, define portfolio weights of investor j at time t as

$$\omega_{ijt} = \frac{q_{ijt} p_{it}}{\sum_l q_{ljt} p_{lt}}.$$

The time t expected excess return on the portfolio of type j is

$$R_{jt} = \sum_i \omega_{ijt}(E_t(r_{it}) - r),$$

and the ex-ante expectation of that return is

$$\bar{R}_j = \alpha_j + \bar{\beta}_j(E(r_{mt}) - r)) \tag{19}$$

[14] Both the expectation and the covariance are conditional on the information available to the average investor in period t. For detailed derivation, see Appendix.

where $\bar{\beta}_j = \sum_i E\omega_{ijt} E\beta_{it}$, and

$$\alpha_j = \sum_i cov(\omega_{ijt}, E_t(r_{it})) + \sum_i E\omega_{ijt} cov(\beta_{it}, E_t(r_{mt})).$$

Quantitatively, the pure skill effect captured by α_j accounts for all of the return differential in the model. To see that, replace α_S with α_U to get the counterfactual return of the sophisticated investors if their skill effect was the same as the unsophisticated investors':

$$\hat{R}_S = \alpha_U + \bar{\beta}_S(E(r_{mt}) - r)).$$

According to $\hat{R}_S$, the sophisticated portfolio would generate an annualized return of 11.2% versus 11.3% of the unsophisticated portfolio[15] (determined by $\bar{R}_U$), which means actually more than 100% of the return differential is due to skill.

This exercise can also shed light on whether the skill compensation differential $\alpha_S > \alpha_U$ in the model comes from superior active response to shocks or just different portfolio weights across types *on average*. These two effects are captured by the first and second expression which determines alpha, respectively. Quantitatively, active portfolio adjustment in response to shocks is the dominant force determining alpha: It accounts for 84% of the sophisticated investors' α_S and 80% of the unsophisticated investors' α_U.

3.2.2 Testing the Mechanism

The results in the previous section demonstrate a significant impact of our information mechanism on the return differential, which will be the driving force behind accounting for the capital income polarization in the next section. Before we proceed to our analysis of capital income, however, we provide a set of quantitative predictions for the benchmark parametrization that allow us to provide additional support for our mechanism by comparing it to the corresponding data moments. These are robust predictions of our mechanism and are proven analytically in Section 2. Below, we show a good fit of these results not only

[15] α_1/α_2 is equal to 1.67 in the model.

qualitatively but also quantitatively.

Market Averages Technological progress in information capacity in the model implies large changes in average market returns, cross-sectional return differential, and turnover. We report these statistics generated by the model and observed in the data in Table 3.

The changes implied by the model not only match the changes in the data qualitatively, but they also come close quantitatively. Both the model and the data imply a decrease in market return and a decrease in the return differential of portfolios held by sophisticated and unsophisticated investors. Intuitively, in the model, lower market return is a result of an increase in quantity of information: The price reflects that and tracks much more closely the actual return z than it does in the initial parametrization with lower overall capacity (for additional intuition, see Proposition 5).

The model also predicts a sharp increase in average asset turnover, in magnitudes consistent with the data. As with the market return, this result is a direct implication of our mechanism and is not driven by changes in asset volatility. In fact, fundamental asset volatilities (σ_is) are held at the same level across the two sub-periods in the model. Intuitively, higher turnover in the model is driven by more informed trading by sophisticated investors, both due to their holding a larger share of the market as well as them receiving more precise signals about asset payoffs.

Expansion of Ownership Investors in our model prefer to learn about assets with higher volatility. In particular, upon increasing their information capacity, they first invest it in the most volatile asset until the benefits from a unit of information become equalized with those of the second-highest volatility asset, then third, and so forth (see Proposition 3). This process implies a particular way in which sophisticated investors expand their portfolio holdings as their capacity (through overall capacity) increases. Specifically, we should see that sophisticated investors exhibit the highest initial growth in ownership for the the highest-volatility assets, then lower-volatility assets, etc. This prediction is robustly borne out in the data, as exhibited in Figure 2, which shows the evolution of this growth in the model

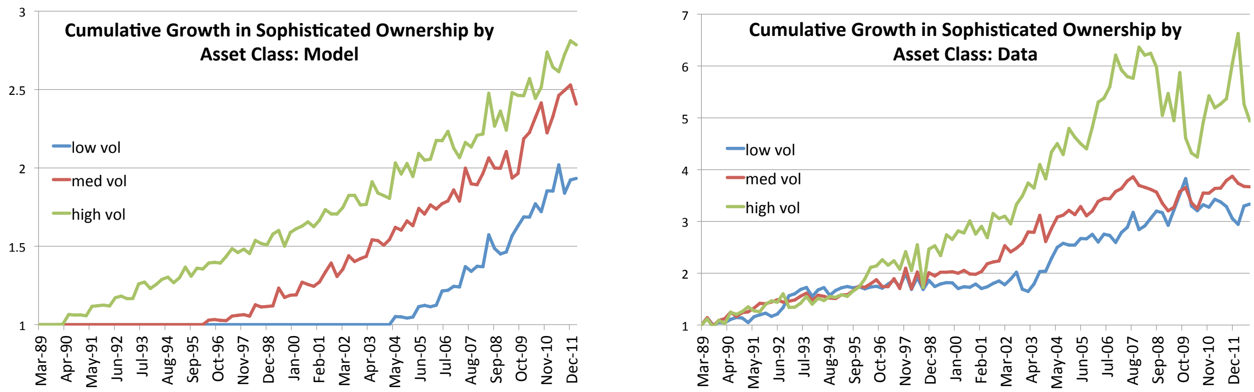

Figure 2: Cumulative Growth in Sophisticated Investors' Ownership: Data and Model

and in the data over the period 1989-2012.[16]

In Figure 3, we show *the change* in asset ownership by sophisticated investors over the periods 1989-2000 and 2001-2012, where assets are sorted by volatility of their returns. This cross-sectional change underlies the average ownership targets in the model of 23% in the initial period and 43% in the later period. Both the data and the model exhibit a hump-shaped profile of the increase and they are also very close quantitatively.

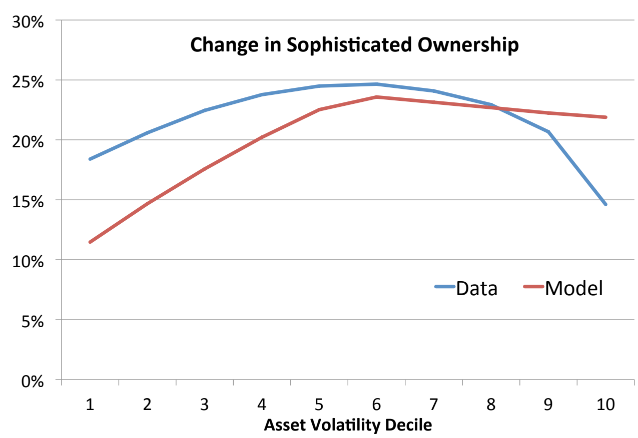

Figure 3: Absolute Change in Sophisticated Investors' Ownership

In conclusion, even though we parameterize the model to match the *aggregate* ownership levels of sophisticated investors in the pre- and post-2000 period, the model is also able to

[16]To generate this graph in the model, we increase aggregate capacity from zero to the level that matches 48% sophisticated ownership, which is the last point in the data.

explain quantitatively *how* ownership increases *across* asset volatility classes, both in terms of timing of the growth levels and in terms of the absolute magnitudes of the ownership changes.

Cross-sectional Turnover Our model implies cross-sectional variation in asset turnover, driven by differential investment of investors' information capacity. Intuitively, if an asset is more attractive and investors invest more in it, then there are more investors with precise signals about this asset's returns, and these investors want to act on such better information by taking larger and more volatile positions. Since the sophisticated investors receive more precise signals, and they have preference towards high-volatility assets, we should see a positive relationship between volatility and turnover. We report turnover in relation to return volatility in the model and in the data in Table 4.

Table 4: Turnover by Asset Volatility

Volatility quintile	1	2	3	4	5	Mean
1989-2000						
Data	5%	8.5%	10.5%	12.5%	11.5%	9.7%
Model	9%	9%	9.3%	9.9%	10.8%	9.7%
2001-2012						
Data	11%	14.6%	17%	18.4%	19.3%	16%
Model	12.5%	13.6%	14.2%	15%	15.4%	14%

The first two rows compare data and the model prediction for the initial parametrization to 1989-2000 data. Both data and model show increasing patterns in turnover as volatility goes up, which are quantitatively close to each other. In the next two rows, we compare data for the 2001-2012 period to results generated from the dynamic exercise in the model in which we increase overall capacity. The model implies an increase in average turnover compared to that in an earlier period and additionally matches the cross-sectional pattern of the increase. This effect is purely driven by our information friction, since the fundamental volatilities remain constant over time in this exercise.[17]

[17] Our model also implies a positive turnover-ownership relationship, which we further confirm in the data.

At first it may appear as though our results are at odds with evidence in Barber and Odean (2000) who find that trading intensity is negatively related to performance, while our findings would suggest that sophisticated investors achieve superior returns and they trade more. We argue that the two are not necessarily contradictory. First, we highlight the relationship between gross (portfolio) returns and turnover while their result is about net returns to emphasize the role of transaction costs. In fact, Barber and Odean (2000) do find that turnover is weakly positively related to gross returns. Second, their data set covers a subset of (*retail*) investors over the short period of 1991-1997. Hence, the capital income distribution is likely truncated from above. Our data set instead also includes most sophisticated investors and our theory predicts that these most sophisticated investors have disproportionately higher information advantage. Finally, we look at turnover at the stock level, while they look at turnover by investor type; these are conceptually different statistics, and in the presence of any investor-fixed effects, the investor-based results would be skewed.

3.3 Capital Income Inequality

In our analysis so far we have obtained a set of parameters crucial for understanding the workings of investors' opportunity set. Crucially, these parameters are calibrated in the economy with non-trivial heterogeneity across investors and assets. In this section, we use this parameterization of the stochastic environment of our economy implied by micro-evidence to shed light on the main question of our paper: the dynamics of capital income inequality. We perform this analysis on a group of U.S. households from the Survey of Consumer Finances (SCF). The SCF data set has been a standard testing set for questions related to households finance and thus is a reliable source for our purpose.

A critical element for our analysis involves the measurement of investor sophistication. Following the work of Arrow (1987) and Calvet, Campbell, and Sodini (2009b), we use wealth levels as proxies for investor sophistication. The idea is that wealthier individuals have access to better information production or processing technologies, which in the language of our model means they have greater information capacity. We investigate, through the lens of

This result is consistent with the empirical findings in Chordia, Roll, and Subrahmanyam (2011).

our model, how initial wealth levels can propagate information capacity differences through time.

Specifically, we map investor types in our model into households in different wealth groups in the SCF. We then explicitly map the ratio of wealth levels into initial ratio of information capacities and posit that the growth in information capacity is linearly related to the growth of the financial portfolio of each investor type. Hence, differences in rates of return endogenously propagate into different capacity levels in subsequent periods. The guiding principle of our exercise is the existence of a technology of obtaining capacity that is characterized by high fixed and low marginal costs, as explored in Arrow (1987). Below, we first proceed to describe the relevant groups of individuals in the SCF, and then present model results.

In order to map our investor types into household types in the SCF, we restrict our data to households who participate in capital markets. Specifically, we focus on households with non-zero investment in either stocks or non-money market mutual funds, or a brokerage account (about 30% of the population). We then consider two subsets of households: a group of 10% of households with the highest level of total wealth at each point in time (sophisticated investors) and a group of 50% of households with the lowest level of total wealth (unsophisticated investors).[18]

We proceed to set the initial ratio of investors' information capacity, K_1/K_2 in the model, to the 1989 ratio of average financial wealth in the top 10% and the bottom 50% of the total wealth distribution of our households. In the data, this ratio is equal to 29.92. We then pick the initial aggregate capacity level to match the excess return on the market portfolio, equal to 11.9% in the data.[19] We then assume that the growth of each investor type's capacity

[18] Income ratios are highly dispersed cross-sectionally, with sophisticated investors earning at the minimum 45 times more dollar income than unsophisticated ones. This dispersion also grows strongly over time up to 150 in 2004. Even though it subsequently diminishes slightly, it remains at a very high level of at least 100. In the data, we also find that the ratio in rates of returns for sophisticated vs. unsophisticated investors on average equals 1.7 and varies between 1.1 and 2.15 in the time series–which suggests that the capital income polarization is not driven mechanically by financial wealth differences.

[19] We also set the initial wealth in order to match the average initial (1989) return on wealth of 5%, consistent with the SCF in 1989. The parameterization procedure gives capacity levels equal to $K_1 = 0.694$ and $K_2 = 0.0231$.

is equal to her return on equity. We simulate the model for 21 years forward, which is the time span of our data set. As the outcome of the experiment, we obtain the endogenous capital income dispersion growth implied by our mechanism. The results of this exercise are presented in Table 5 and Figure 4.

Table 5: Capital Income Dispersion: Data and Model

	Data 1989-2010	Model
Capital Income Dispersion Growth	83%	71%

We obtain a 71% growth in capital income inequality (83% in the data), which is over 93% of the growth observed in the data. We conclude that our mechanism implies a strong role of wealth as a proxy for sophistication and growth in wealth as a proxy for growth in sophistication, especially in explaining capital income polarization observed in the data. As Figure 4 shows, the model matches well not only the overall growth but also the dynamics of the increase in capital income polarization.

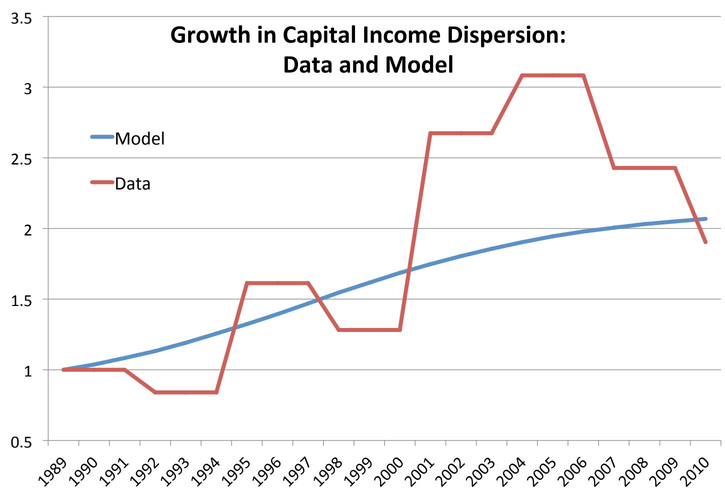

Figure 4: Cumulative Growth in Capital Income Dispersion

3.4 Additional Supporting Evidence

So far, we presented quantitative results supporting analytical predictions that are based on our parameterized model. Specifically, our theoretical predictions imply that differences in capital income can stem from two sources: heterogeneity in prices of investable assets and the differential exposure of investors to holding such assets. In this section, we provide additional evidence on each of these channels that offers support for our predictions qualitatively but cannot be assessed quantitatively.

Unsophisticated Investors' Retrenchment We show that cross-sectional differences in assets holdings of investors with different levels of sophistication are consistent with predictions of our model and thus contribute to capital income inequality and its growing polarization. Our main prediction is that unsophisticated investors should be more likely to invest in assets with lower expected values. In the quantitative tests of the model in Figure 2, we show that sophisticated investors allocate their wealth first into assets with highest level of volatility and subsequently into assets with lower levels of volatility. Now, we provide additional evidence which suggests similar investors' preferences.

Our first piece of evidence is based on SCF data regarding households' holdings in liquid wealth. The idea of this test is that unsophisticated investors should be more likely to invest in safe (liquid) assets. SCF provides detailed classification of wealth invested in such assets that include checking accounts, call accounts, money market accounts, coverdell accounts, and 529 educational state-sponsored plans. As before, in each period, we divide households into two groups: top 10% and bottom 50% of the wealth distribution. For each of the groups, we calculate the average ratio of liquid wealth to total financial wealth. Higher ratios would imply greater exposure to low-profit assets. We present the two time series in Figure 5.

We find evidence that strongly supports predictions of our model. First, the average ratio of liquid wealth for sophisticated investors, equal to 15.3%, is significantly lower than that for unsophisticated investors, which in our sample equals 25%. In addition, while the exposure to liquid assets by sophisticated investors is generally non-monotonic (u-shaped), similar investment for unsophisticated investors exhibits a strong positive time trend, especially in

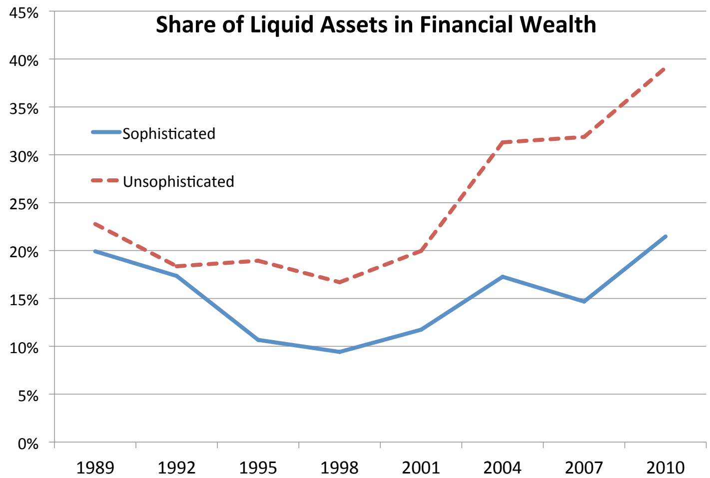

Figure 5: Share of Liquid Wealth in Financial Wealth: Survey of Consumer Finances.

the last 20 years: The average investment goes up from 16.7% in 1998 to 39% in 2010. This evidence strongly supports our economic mechanism in that differences in information capacity lead to retrenchment by unsophisticated investors from risky assets and relocation to safer assets.

We further confirm this claim using evidence on institutional holdings from Thomson Reuters. To this end, we calculate average (equal-weighted) equity ownership of sophisticated investors (mutual funds and hedge funds) and unsophisticated (retail) investors. We report the respective time series quarterly averages of the ownership over the period 1989-2012 in Figure 6.

The results paint a picture that is generally consistent with our model's predictions. Although the average ownership level of unsophisticated investors is higher in an unconditional sample and equals 61%, the time-series evidence clearly indicates a very strong pattern: The average equity ownership for unsophisticated investors goes down while that for sophisticated investors significantly goes up.[20] We argue that this evidence is consistent with the view that the observed expansion of relative financial wealth drives the expansion of information capacities. Realizing a positive shock to information capacity sophisticated investors enter

[20]The visible positive trend in active ownership has been documented before by Gompers and Metrick (2001) and is even stronger if one accounts for differences in market values across assets and the preference of sophisticated investors for large-cap stocks.

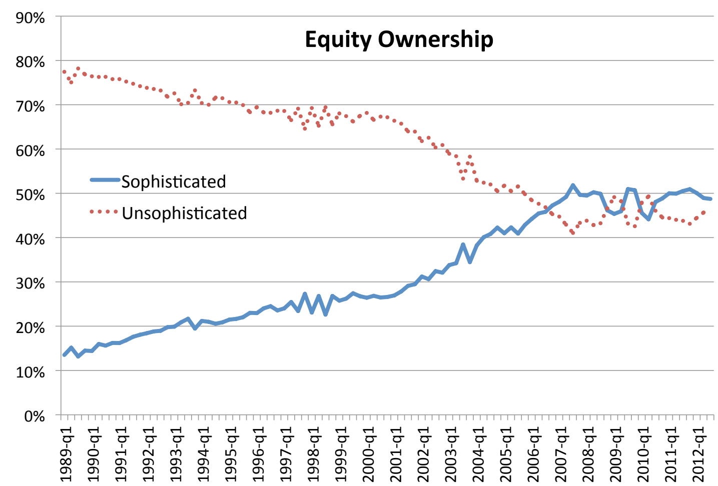

Figure 6: Equity Ownership by Sophistication Type.

the profitable equity market at the expense of unsophisticated investors who perceive the informational disadvantage in the market and as a result move away from equity. Notably, the retrenchment of unsophisticated investors from directly holding equity happened despite the overall strong performance of equity markets over the same time period. This suggests that investors do not simply respond to past trends in equity returns.

As a final auxiliary prediction we consider money flows into mutual funds. The idea is that equity mutual funds are more risky than non-equity funds. As such, unsophisticated investors should be less likely to invest in the former, especially if information capacity gets more polarized.

To test this prediction in the data we use mutual fund data from Morningstar. Morningstar classifies different funds into those serving institutional investors and individuals whose investment is at least \$100,000 (institutional funds) and those serving individual investors with investment value less than \$100,000 (retail funds). For the purpose of testing our predictions, we define sophisticated investors as those investing in institutional funds and unsophisticated investors as those investing in retail funds. Subsequently, we calculate cumulative aggregated dollar flows into equity and non-equity funds, separately for each investment type. Our data span the period 1989-2012. We present the results in Figure 7.

We find that the cumulative flows from sophisticated investors into equity and non-

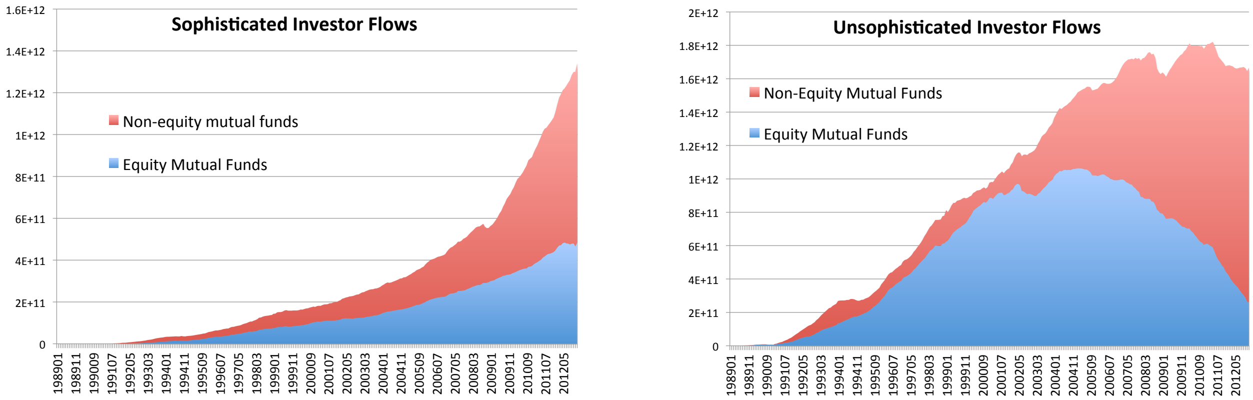

Figure 7: Cumulative Flows to Mutual Funds by Sophistication Type: Equity vs. Non-Equity

equity funds increase steadily over the whole sample period. In contrast, the flows from unsophisticated investors display a visibly different pattern. The flows into equity funds keep increasing until 2000 but subsequently decrease at a significant rate of more than 3 times by 2012. Moreover, the decrease in cumulative flows to equity mutual funds coincides with a significant increase in cumulative flows to non-equity funds. Overall, these findings support predictions of our model: Sophisticated investors have a large exposure to risky assets and subsequently add extra exposure to less risky assets, whereas unsophisticated investors leave riskiest assets and move into safer assets as they perceive higher information disadvantage.

One could point out that the increase in equity fund flows by unsophisticated investors observed in the early sample period is inconsistent with our model. However, we argue that this result could still be rationalized by contrasting it with the steady decrease in holdings of individual equity documented earlier. To the extent that individual equity holdings are more risky than diversified equity portfolios, such as mutual funds, this only means that in the earlier period unsophisticated investors reallocate their wealth from riskier to safer asset class.

Stock Selection Ability The second building block of our economic mechanism is the ability of sophisticated investors to better choose assets. Our quantitative evaluation maps the model prediction to the observed differences in performance between sophisticated and unsophisticated investors. Here, we provide an additional qualitative result in which we show that equity holdings of sophisticated investors are higher for stocks which realize higher returns.

To conduct this test, we obtain data on stock returns come from Center for Research on Security Prices (CRSP), and for each stock we calculate the market shares of sophisticated investors. Next, we estimate the regression model over the period January 1989-December 2012 with stock/month as a unit of observation. Our dependent variable is the share of sophisticated investors in month t and the independent variable is return corresponding to the stock in month t. Our regression model includes year-month fixed effects and standard errors are clustered at the stock level to account for the cross-sectional correlation in the data. We report the results of this estimation in Table 6.

Table 6: Future Returns Explain Sophisticated Investors' Ownership

Variable	Value	Standard Error
Future Return	0.048	0.00845
Constant	0.300	0.00007
Year-Month-Fixed Effects	Yes	
Number of Observations	1,525,787	

We find strong evidence that sophisticated investors tend to invest more in stocks that generate higher returns (which is consistent with our model's prediction summarized in Proposition 2). Hence, we conclude that sophisticated investors in our sample exhibit superior stock-selection ability. This finding corroborates that in a number of other studies that show the strong existence of stock-picking ability among sophisticated investors, such as actively managed equity mutual funds (e.g., Daniel, Grinblatt, Titman, and Wermers (1997), Cohen, Coval, and Pástor (2005), Kacperczyk, Sialm, and Zheng (2005), Kacperczyk and Seru (2007)). At the individual level, there is ample anecdotal evidence that shows superior investment ability of wealthy investors such as Warren Buffett or Carl Icahn.

Overall, our evidence is consistent with the premise of our economic mechanism that sophisticated investors are good at choosing assets and relocating their resources to the most profitable ones.

3.5 Discussion of Alternative Mechanisms

Our findings so far strongly suggest that heterogeneity in information capacity has the ability to explain cross-sectional and time-series patterns in capital income inequality, while simultaneously producing results that are consistent with other micro-level financial data. While the information friction constitutes a plausible economic mechanism, there may certainly exist other mechanisms which could contribute to the patterns observed in the data. To our knowledge, our paper is the first to embed the proposed friction in an equilibrium model and formally test its predictions vis a vis the data to provide an independent verification for the proposed mechanism. Our view is that any alternative mechanism put forward should be subject to the same verification procedure. While their formal modeling is beyond the scope of this paper, below we discuss some of the popular alternative explanations in light of micro-evidence.

Risk aversion differences It is possible that capital income inequality in the data is driven by differences in risk aversion among investors. In particular, if one group of investors is less risk averse they would hold a greater share of risky assets with higher expected returns and hence would have higher expected capital income. Such a setting would also encompass situations in which investors are exposed to different levels of volatility in areas outside capital markets, like labor income.

Within our mean-variance specification, growing differences in risk aversion would produce growing *aggregate* ownership in risky assets of less risk-averse investors, but it would not generate any difference in investor-specific rates of return on equity, or differential growth in ownership reported in Figure 2. The reason being that differences in risk aversion imply different exposures to *risky portfolio* but not different risky portfolio weights, and hence imply a uniform proportional retrenchment of the high risk aversion agents from equity. This result, supported additionally by regression results in Section 3.4, suggests that the excess market performance is driven by sophisticated investors explicitly picking different portfolio shares (as opposed to pure timing). Finally, differences in risk aversion across investors cannot explain the cross-sectional turnover profile of assets (in Table 4).[21]

We are not the first ones to point out that preference-based approaches to explaining household portfolio choice suffer from serious drawbacks. Analysis in Dumas (1989), as well as arguments in Chien, Cole, and Lustig (2011) suggest that differences in risk preferences cannot account for observed differences in rates of returns across agents with different degrees of sophistication.[22]

Saving rates Another explanation for the growing capital income polarization could be differences in savings rates among households. If less wealthy households have lower savings rates (which go towards financial wealth), then their financial wealth will grow slower than that of the more wealthy households and hence capital incomes of the two groups will diverge. We view this explanation as separate from our mechanism, since it implies no differences in rates of return on financial wealth, and works through pure size effects of the financial wealth. This directly contradicts our finding, presented in Figure 8, on differences in rates of return on financial wealth across wealth groups in the Survey of Consumer Finances. In particular, for each of the group identified by us as sophisticated and unsophisticated in Section 3.3 (top 10% and bottom 50% of wealth distribution), we compute a rough measure of rate of return by taking capital income relative to financial wealth for each group of households. Figure 8 plots the ratio of these rates of return in our data. Sophisticated investors earn on average a 70% higher rate of return on their financial wealth relative to unsophisticated investors, with the number being positive in all survey years. We conclude that there are large and persistent return differences between household groups, which are captured by

[21] Note that in a model with CRRA specification, portfolio weights will also be identical *across risky assets*, and hence even in that specification, rates of return on equity will be equalized across investor types.

[22] Using a different approach, Chien, Cole, and Lustig (2011) explore the role of exogenous heterogeneity in investment technologies in explaining wealth distribution. We build a micro-founded model of such heterogeneity, and focus on capital income (flows) and differences in rates of return.

our mechanism and by the nature of return construction are not driven by savings rates differences.

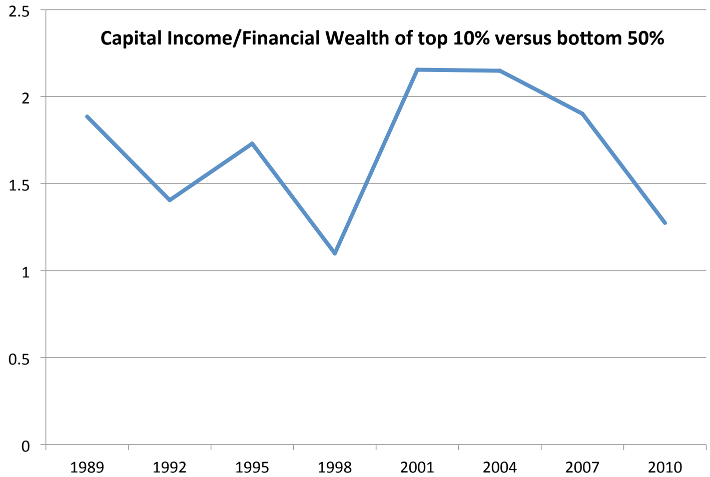

Figure 8: Return on Financial Wealth by Sophistication Type

Age-dependent investment policies Capital income polarization in our data also does not seem to be driven by age demographics. In principle, our measure of capital income includes capital gains and the endogenous household decision of when to realize them. This could depend on a variety of life-cycle factors which we will proxy by using investor age. Figure 9 plots the average age of sophisticated and unsophisticated investors in the SCF. As expected, the wealthy households are older on average. However, the figure reveals that there is no special time-series dynamics to the age difference that could possibly explain the observed capital income dynamics.

Further analysis also rules out the possibility that capital income differences are the outcome of differences in market returns over time combined with buy-and-hold, passive strategies of households in different age cohorts. Figure 10 plots, for each year, the past 15-year and 5-year cumulative return on holding the market (left panel)[23]. It simulates the cumulative return of a household which buys the market index at age 40 and sells at 44 or at 55 - a completely passive strategy. As we can see, the cumulative return on the passive

[23]The patterns we document are essentially the same for other choices of the two horizons, like 20-year and 10-year cumulative returns.

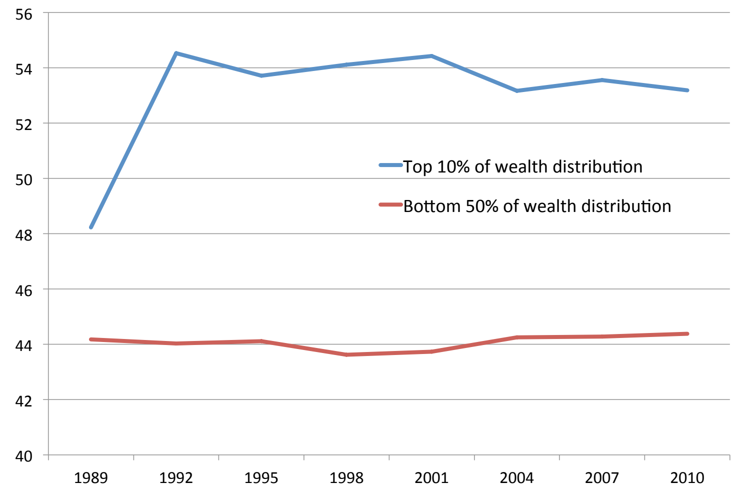

Figure 9: Age by Sophistication Type

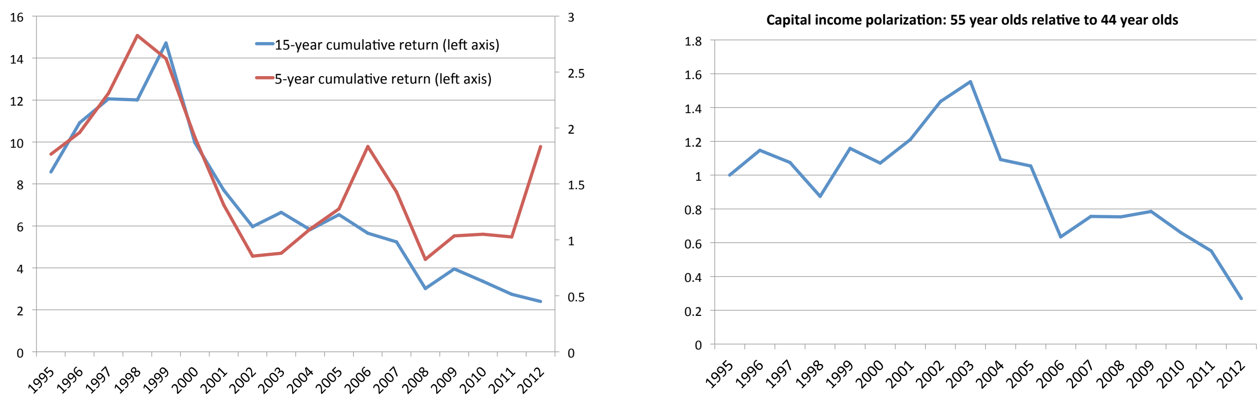

Figure 10: Cumulative market return on a 5-year and 15-year passive investment in market (left panel) and implied growth in polarization (right panel).

strategy actually exhibits a declining trend, more so for the 15-year strategy. This implies that if all investors used the passive strategy and the only difference would be how much of the capital gain is realized by each age group, we should observe the 44-year olds' capital income (driven by cumulative return over the last 5 years) to go up relative to the 55-year olds' (driven by returns over the last 15 years). This is presented in the right panel of Figure 10. Compared with the data in Figure 4, it is clear that the passive investment strategies imply a counterfactual evolution of capital income polarization.

Transaction costs Finally, we discuss the potential of differences in transaction costs across investors to account for the data. To the extent that less sophisticated investors face higher transaction costs in risky asset markets they would be willing to participate less, as argued in Gomes and Michaelides (2005) and others.

While this explanation might have some merit to explain cross-sectional patterns in the data, we believe it is less likely to explain the time-series results. In particular, we observe that more sophisticated households generate significantly greater gap in their incomes over time, which is hard to reconcile with the fact that there was not much change in the overall quantity of transaction costs, as reported in French (2008). In fact, if anything, growth in internet access and services made an access to more direct investing extremely easy and relatively less costly for the average citizen as opposed to just the few privileged ones.

4 Concluding Remarks

What contributes to the growing income inequality across households? This question has been of great economic and policy relevance for at least several decades starting with a seminal work by Kuznets. We approach this question from the perspective of capital income that is known to be highly unequally distributed across individuals. We propose a theoretical information-based framework that links capital income derived from financial assets to a level of investor sophistication. Our model implies the presence of income inequality between sophisticated and unsophisticated investors that is growing in the extent of total sophistication in the market and in relative sophistication across investors. Additional predictions on asset ownership, market returns, and turnover help us pin down the economic mechanism and rule out alternative explanations. The quantitative predictions of the model match qualitatively and quantitatively the observed data.

Although our empirical findings are strictly based on the U.S. market, our model should have similar implications for other financial markets. For example, qualitatively, we know that income inequality in emerging markets tends to be even larger than the one documented for the U.S. To the extent that financial sophistication in such markets is much more skewed one could rationalize within our framework the differences in capital incomes. Similarly, the U.S. market is considered to be the most advanced in terms of its total sophistication, which is possibly why we find a greater dispersion in capital income compared to other developed markets, such as those in Europe or Asia.

More generally, one could argue that although the overall growth of investment resources and competition across investors with different skill levels are generally considered as a positive aspect of a well-functioning financial market, our work suggests that one should assess any policy targeting overall information environment in financial markets as potentially exerting an offsetting and negative effect on socially relevant issues, such as distribution of income. We leave detailed evaluation of such policies for future research.

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Appendix

Model

Portfolio Choice We begin by solving investors' portfolio problem in the second stage, for a given signal structure. Each investor chooses portfolio holdings q_{ji} to solve

$$\max_{\{q_{ji}\}_{i=1}^{n}} U_j = E_j(W_j) - \frac{\rho}{2} V_j(W_j) \quad s.t. \;\; W_j = r\left(W_{0j} - \sum_{i=1}^{n} q_{ji} p_i\right) + \sum_{i=1}^{n} q_{ji} z_i,$$

where W_{0j} is initial wealth and E_j and V_j denote the mean and variance conditional on investor j's information set:

$$E_j(W_j) = E_j\left[rW_{0j} + \sum_{i=1}^{n} q_{ji}(z_i - rp_i)\right] = rW_{0j} + \sum_{i=1}^{n} q_{ji}\left[E_j(z_i) - rp_i\right],$$

$$V_j(W_j) = V_j\left[rW_{0j} + \sum_{i=1}^{n} q_{ji}(z_i - rp_i)\right] = \sum_{i=1}^{n} q_{ji}^2 V_j(z_i).$$

Let $\widehat{\mu}_{ji} \equiv E_j[z_i]$ and $\widehat{\sigma}_{ji}^2 \equiv V_j[z_i]$. The investor's portfolio problem is to maximize

$$U_j = rW_{0j} + \sum_{i=1}^{n} q_{ji}\left(\widehat{\mu}_{ji} - rp_i\right) - \frac{\rho}{2}\sum_{i=1}^{n} q_{ji}^2 \widehat{\sigma}_{ji}^2.$$

The first order conditions with respect to q_{ji} yield the optimal portfolio holdings, given by $q_{ji} = \frac{\widehat{\mu}_{ji} - rp_i}{\rho \widehat{\sigma}_{ji}^2}$.

Since W_{0j} does not affect the optimization, we normalize it to zero. The indirect utility function becomes

$$U_j = \frac{1}{2\rho}\sum_{i=1}^{n} \frac{\left(\widehat{\mu}_{ji} - rp_i\right)^2}{\widehat{\sigma}_{ji}^2}.$$

Posterior Beliefs The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that posterior beliefs are normally distributed, with mean and variance given by Bayesian updating,

$$\widehat{\mu}_{ji} = \overline{z} + \frac{Cov(s_{ji}, z_i)}{\sigma_{sji}^2}\left(s_{ji} - \overline{s}_{ji}\right) = \overline{z} + s_{ji} - \overline{z} = s_{ji},$$

$$\widehat{\sigma}_{ji}^2 = \sigma_i^2\left(1 - \frac{Cov^2(s_{ji}, z_i)}{\sigma_{sji}^2 \sigma_i^2}\right) = \sigma_i^2\left(1 - \frac{\sigma_{sji}^2}{\sigma_i^2}\right) = \sigma_{\delta ji}^2.$$

Conditional Distribution of Signals Conditional on the realized payoff, the signal is a normally distributed random variable, with mean and variance given by Bayesian updating,

$$E\left(s_{ji}|z_i\right) = \overline{s}_{ji} + \frac{Cov(s_{ji}, z_i)}{\sigma_i^2}\left(z_i - \overline{z}\right) = \overline{z} + \frac{\sigma_{sji}^2}{\sigma_i^2}\varepsilon_i = \begin{cases} \overline{z} + \left(1 - e^{-2K_j}\right)\varepsilon_i & \text{if } i = l_j \\ \overline{z} & \text{if } i \neq l_j, \end{cases}$$

$$V\left(s_{ji}|z_i\right) = \sigma_{sji}^2\left(1 - \frac{Cov^2(s_{ji}, z_i)}{\sigma_{sji}^2 \sigma_i^2}\right) = \sigma_{sji}^2\left(1 - \frac{\sigma_{sji}^2}{\sigma_i^2}\right) = \begin{cases} \left(1 - e^{-2K_j}\right)e^{-2K_j} & \text{if } i = l_j \\ 0 & \text{if } i \neq l_j. \end{cases}$$

Information Constraint Let $H(z)$ denote the entropy of z, and let $H(z|s_j)$ denote the conditional entropy of z given the vector of signals s_j. Then

$$I(z; s_j) \equiv H(z) - H(z|s_j) \overset{(1)}{=} \sum_{i=1}^n H(z_i) - H(z|s_j) \overset{(2)}{=} \sum_{i=1}^n H(z_i) - \sum_{i=1}^n H(z_i|z^{i-1}, s_j)$$

$$\overset{(1)}{=} \sum_{i=1}^n H(z_i) - \sum_{i=1}^n H(z_i|s_j) \overset{(3)}{=} \sum_{i=1}^n H(z_i) - \sum_{i=1}^n H(z_i|s_{ji}) = \sum_{i=1}^n I(z_i; s_{ji})$$

where (1) follows from the independence of the payoffs z_i; (2) follows from the chain rule for entropy, where $z^{i-1} = \{z_1, ..., z_{i-1}\}$; (3) follows from the independence of the signals s_{ji}.

Hence, under the assumption that the signals are independent, $I(z; s_j) = \sum_{i=1}^n I(z_i; s_{ji})$.

For each asset i, the entropy of $z_i \sim f(z_i) = \mathcal{N}(\overline{z}, \sigma_i^2)$ is

$$\begin{aligned} H(z_i) &\equiv \int f(z_i) \ln \frac{1}{f(z_i)} dz_i = \int f(z_i) \ln \left\{ \sqrt{2\pi\sigma_i^2} \exp\left[\frac{(z_i - \overline{z})^2}{2\sigma_i^2} \right] \right\} dz_i \\ &= \int f(z_i) \left[\frac{1}{2} \ln\left(2\pi\sigma_i^2\right) + \frac{(z_i - \overline{z})^2}{2\sigma_i^2} \right] dz_i \\ &= \frac{1}{2} \ln\left(2\pi\sigma_i^2\right) + \frac{1}{2\sigma_i^2} \int f(z_i)(z_i - \overline{z}_i)^2 dz_i = \frac{1}{2} \ln\left(2\pi e\sigma_i^2\right). \end{aligned}$$

The signal structure, $z_i = s_{ji} + \delta_{ji}$, implies that

$$\begin{aligned} I(z_i; s_{ji}) &= H(z_i) + H(s_{ji}) - H(z_i, s_{ji}) \\ &= \frac{1}{2} \log\left(2\pi e\sigma_i^2\right) + \frac{1}{2} \log\left(2\pi e\sigma_{sji}^2\right) - \frac{1}{2} \log\left[(2\pi e)^2 \left|\Sigma_{z_i s_{ji}}\right|\right] \\ &= \frac{1}{2} \log\left(\frac{\sigma_i^2 \sigma_{sji}^2}{\left|\Sigma_{z_i s_{ji}}\right|}\right) = \frac{1}{2} \log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right), \end{aligned}$$

where $\left|\Sigma_{z_i s_{ji}}\right| = \sigma_{sji}^2 \sigma_{\delta ji}^2$ is the determinant of the variance-covariance matrix of z_i and s_{ji}.

Across assets,

$$I(z; s_j) = \sum_{i=1}^n I(z_i; s_{ji}) = \tfrac{1}{2} \sum_{i=1}^n \log\left(\frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right) = \tfrac{1}{2} \log\left(\prod_{i=1}^n \frac{\sigma_i^2}{\sigma_{\delta ji}^2}\right).$$

The information constraint can be written as $\prod_{i=1}^n \frac{\sigma_i^2}{\sigma_{\delta ji}^2} \leq e^{2K_j}$.

Finally, since $\widehat{\sigma}_{ji}^2 = \sigma_{\delta ji}^2$, the information constraint becomes $\prod_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} \leq e^{2K_j}$.

Conversely, for an additive noise signal structure with $s_{ji} = z_i + \delta_{ji}$, $I(z_i; s_{ji}) = \frac{1}{2}\log\left(\frac{\sigma_i^2+\sigma_{\delta ji}^2}{\sigma_{\delta ji}^2}\right)$, and the information constraint would be $\prod_{i=1}^{n} \frac{\sigma_i^2+\sigma_{\delta ji}^2}{\sigma_{\delta ji}^2} \leq e^{2K_j}$.

Information Objective Expected utility is given by

$$E_{0j}[U_j] = \frac{1}{2\rho} E_{0j}\left[\sum_{i=1}^{n} \frac{(\widehat{\mu}_{ji} - rp_i)^2}{\widehat{\sigma}_{ji}^2}\right] = \frac{1}{2\rho}\sum_{i=1}^{n} \frac{E_{0j}\left[(\widehat{\mu}_{ji} - rp_i)^2\right]}{\widehat{\sigma}_{ji}^2} = \frac{1}{2\rho}\sum_{i=1}^{n}\left(\frac{\widehat{R}_{ji}^2 + \widehat{V}_{ji}}{\widehat{\sigma}_{ji}^2}\right),$$

where $\widehat{R}_{ji}$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, $\widehat{\mu}_{ji} - rp_i$. Conjecture (and later verify) that prices are normally distributed, $p_i \sim \mathcal{N}\left(\overline{p}_i, \sigma_{pi}^2\right)$.

$$\widehat{R}_{ji} \equiv E_{0j}(\widehat{\mu}_{ji} - rp_i) = \overline{z} - r\overline{p}_i,$$

$$\widehat{V}_{ji} \equiv V_{0j}(\widehat{\mu}_{ji} - rp_i) = Var(\widehat{\mu}_{ji}) + r^2\sigma_{pi}^2 - 2rCov(\widehat{\mu}_{ji}, p_i).$$

The signal structure implies that $Var(\widehat{\mu}_{ji}) = \sigma_{sji}^2$.

Following Admati (1985), conjecture (and later verify) that prices are $p_i = a_i + b_i\varepsilon_i - c_i\nu_i$, for some coefficients $a_i, b_i, c_i \geq 0$. We compute $Cov(\widehat{\mu}_{ji}, p_i)$ exploiting the fact that posterior beliefs and prices are conditionally independent given payoffs:

$$Cov(\widehat{\mu}_{ji}, p_i) = \frac{Cov(\widehat{\mu}_{ji}, z_i)Cov(z_i, p_i)}{\sigma_i^2}.$$

Since $Cov(z_i, p_i) = b_i\sigma_i^2$ and $Cov(\widehat{\mu}_{ji}, z_i) = \sigma_{sji}^2$, then $Cov(\widehat{\mu}_{ji}, p_i) = b_i\sigma_{sji}^2$. Then

$$\widehat{V}_{ji} = \sigma_{sji}^2 + r^2\sigma_{pi}^2 - 2rb_i\sigma_{sji}^2 = (1 - rb_i)^2\sigma_i^2 + r^2c_i^2\sigma_x^2 - (1 - 2rb_i)\widehat{\sigma}_{ji}^2.$$

Hence the distribution of expected excess returns is normal with mean and variance:

$$\widehat{R}_{ji} = \overline{z} - ra_i \quad \text{and} \quad \widehat{V}_{ji} = (1 - rb_i)^2\sigma_i^2 + r^2c_i^2\sigma_x^2 - (1 - 2rb_i)\widehat{\sigma}_{ji}^2.$$

Expected utility becomes

$$E_{0j}[U_j] = \frac{1}{2\rho}\sum_{i=1}^{n}\left[\frac{(\overline{z} - ra_i)^2 + (1 - rb_i)^2\sigma_i^2 + r^2c_i^2\sigma_x^2 - (1 - 2rb_i)\widehat{\sigma}_{ji}^2}{\widehat{\sigma}_{ji}^2}\right] = \frac{1}{2\rho}\sum_{i=1}^{n} G_i\frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} - \frac{1}{2\rho}\sum_{i=1}^{n}(1 - 2rb_i),$$

where $G_i \equiv (1 - rb_i)^2 + \frac{r^2c_i^2\sigma_x^2}{\sigma_i^2} + \frac{(\overline{z} - ra_i)^2}{\sigma_i^2}$, and where the second summation is independent of the investor's choices.

Hence, the investor's objective is to maximize $\sum_{i=1}^{n} G_i\frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2}$. Note that the gain G_i is the same across all investors and it is taken as given by each individual investor. It is determined in equilibrium, as a function of equilibrium prices.

Proof of Lemma 1 The linear objective function and the convex constraint imply a corner solution for the optimal allocation of attention for each investor: each investor allocates all capacity to learning about a single asset. For all other assets, the posterior variance is equal to the prior variance. Let l_j index the asset about which investor j learns. The information constraint becomes

$\prod_{i=1}^{n} \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} = \frac{\sigma_{l_j}^2}{\widehat{\sigma}_{jl_j}^2} = e^{2K_j}$, and hence the variance of the investor's beliefs is given by

$$\widehat{\sigma}_{ji}^2 = \begin{cases} e^{-2K_j}\sigma_i^2 & \text{if } i = l_j, \\ \sigma_i^2 & \text{if } i \neq l_j. \end{cases}$$

The mean of posterior beliefs is derived above.

The investor's problem becomes picking the asset l_j to maximize

$$\sum_{i=1}^{n} G_i \frac{\sigma_i^2}{\widehat{\sigma}_{ji}^2} = \left(e^{2K_j} - 1\right) G_{l_j} + \sum_{i=1}^{n} G_i.$$

Since $e^{2K_j} > 1$, the objective is maximized by allocating all capacity to the asset with the largest utility gain: $l_j \in \arg\max_i G_i$.

Proof of Lemma 2 The market clearing condition for each asset in state (z_i, x_i) is

$$\int_{M_{1i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_1}\rho\sigma_i^2}\right) dj + \int_{M_{2i}} \left(\frac{s_{ji} - rp_i}{e^{-2K_2}\rho\sigma_i^2}\right) dj + (1 - m_{1i} - m_{2i}) \left(\frac{\overline{z} - rp_i}{\rho\sigma_i^2}\right) = x_i,$$

where M_{1i} denotes the set of measure $m_{1i} \in [0, \lambda]$ of sophisticated investors who choose to learn about asset i, and M_{2i} denotes the set of measure $m_{2i} \in [0, 1-\lambda]$, of unsophisticated investors who choose to learn about asset i.

Using the conditional distribution of the signals, $\int_{M_{1i}} s_{ji} dj = m_{1i} \left[\overline{z} + \left(1 - e^{-2K_1}\right)\varepsilon_i\right]$ for the type-1 investors, and similarly for the type-2 investors.

The market clearing condition can be written as $\alpha_1 \overline{z} + \alpha_2 \varepsilon_i - x_i = \alpha_1 r p_i$, where

$$\alpha_1 \equiv \frac{1 + m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)}{\rho\sigma_i^2} \text{ and } \alpha_2 \equiv \frac{m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)}{\rho\sigma_i^2}.$$

We obtain identification of the coefficients in $p_i = a_i + b_i\varepsilon_i - c_i\nu_i$ as

$$a_i = \frac{1}{r}\left[\overline{z} - \frac{\overline{x}}{\alpha_1}\right],\ b_i = \frac{\alpha_2}{r\alpha_1}, \text{ and } c_i = \frac{1}{r\alpha_1}.$$

Let $\Phi_i \equiv m_{1i}\left(e^{2K_1} - 1\right) + m_{2i}\left(e^{2K_2} - 1\right)$ be a measure of the information capacity allocated to learning about asset i in equilibrium.

Further substitution yields

$$a_i = \frac{1}{r}\left(\overline{z} - \frac{\rho\sigma_i^2\overline{x}}{1+\Phi_i}\right), \;\; b_i = \frac{1}{r}\left(\frac{\Phi_i}{1+\Phi_i}\right), \quad c_i = \frac{1}{r}\left(\frac{\rho\sigma_i^2}{1+\Phi_i}\right).$$

Since the gain factor G_i for each i is the same across all investors, regardless of their type, we assume that the participation of sophisticated and unsophisticated investors in learning about a particular asset is proportional to their mass in the population: $m_{1i} = \lambda m_i$ and $m_{2i} = (1-\lambda)\, m_i$, where m_i is the total mass of investors learning about asset i. Hence $\Phi_i = \phi m_i$, where $\phi \equiv \lambda\left(e^{2K_1}-1\right) + (1-\lambda)\left(e^{2K_2}-1\right)$ is a measure of the total capacity for processing information available in the economy. Substitution yields

$$a_i = \frac{1}{r}\left(\overline{z} - \frac{\rho\sigma_i^2\overline{x}}{1+\phi m_i}\right), \;\; b_i = \frac{1}{r}\left(\frac{\phi m_i}{1+\phi m_i}\right), \;\; c_i = \frac{1}{r}\left(\frac{\rho\sigma_i^2}{1+\phi m_i}\right).$$

Proof of Lemma 3 Using equilibrium prices, equilibrium gains become

$$G_i \equiv (1-rb_i)^2 + \frac{r^2c_i^2\sigma_x^2}{\sigma_i^2} + \frac{(\overline{z}-ra_i)^2}{\sigma_i^2} = \left[1 - \frac{\phi m_i}{1+\phi m_i}\right]^2 + \left(\frac{1}{1+\phi m_i}\right)^2\rho^2\sigma_i^2\sigma_x^2 + \left(\frac{1}{1+\phi m_i}\right)^2\rho^2\sigma_i^2\overline{x}^2 = \frac{1+\rho^2\sigma_i^2\left(\sigma_x^2+\overline{x}^2\right)}{(1+\phi m_i)^2}.$$

Hence $G_i = \frac{1+\rho^2\xi_i}{(1+\phi m_i)^2}$, where $\xi_i \equiv \sigma_i^2\left(\sigma_x^2 + \overline{x}^2\right)$.

By Lemma 1, each investor allocates capacity to a single asset among the assets with the highest gain. First suppose that there is only one asset that is being learned about and let this asset be denoted by l: $m_l = 1$ and $m_i = 0$ for all other assets $i \neq l$. Then $G_l = \frac{1+\rho^2\xi_l}{(1+\phi)^2}$. Consider an individual investor's gain from learning about a second arbitrary asset $i \neq l$: at the current equilibrium, $G_i = 1 + \rho^2\xi_i$. The investor will switch to learning about asset i unless $G_l > G_i$, which is equivalent to $\frac{1+\rho^2\xi_l}{1+\rho^2\xi_i} > (1+\phi)^2$. Hence, learning about a single asset is sustained only if this inequality is satisfied. Since $1+\phi > 1$, the inequality holds only if $\xi_l > \xi_i$ for any $i \neq l$. We have assumed, without loss of generality, that assets in the economy are ordered such that, for all $i \in \{1, ..., n-1\}$, $\sigma_i^2 > \sigma_{i+1}^2$, which implies that $\xi_i > \xi_{i+1}$. Hence, $l = 1$: the asset learned about is the asset with the highest volatility. Moreover, since the left-hand side of the inequality is decreasing in ξ_i, the threshold for starting to learn about the second asset, taking shocks and risk aversion as given, is $\phi_1 \equiv \sqrt{\frac{1+\rho^2\xi_1}{1+\rho^2\xi_2}} - 1$. At this threshold market capacity, investors begin learning about the second asset, ξ_2.

Second, suppose that $\phi \geq \phi_1$, such that at least two assets are learned about. We show that gains must be equated among all assets with positive learning mass. Consider any two assets h, l, with strictly positive masses, $m_h, m_l > 0$, and suppose that they have unequal gains. WLOG, let $G_h < G_l$. Consider any investor learning about asset h. His utility is $\left(e^{2K_j}-1\right)G_h + \sum_{i=1}^n G_i < \left(e^{2K_j}-1\right)G_l + \sum_{i=1}^n G_i$. Hence, this investor will have an incentive to switch to learning about asset l instead. The gain is decreasing in the mass of investors learning about an asset. Hence, as investors switch from learning about asset h to asset l, m_h falls (hence G_h rises) and m_l rises (hence G_l falls). This process continues until in the new equilibrium, $G_h = G_l$, and each individual investor is just indifferent between learning about either asset. Note that this process also implies that the masses m_i are strictly decreasing across the assets that are learned about. Let the equilibrium gain for assets with positive mass be denoted by G^*.

Finally, the gains of all assets with zero learning mass must be strictly lower than G^*. Otherwise, an individual investor would once again have the incentive to deviate and learn about one of these assets.

Proof of Lemma 4 We begin by deriving expressions for the masses m_i. The necessary and sufficient conditions for determining $\{m_i\}_{i=1}^n$ in equilibrium are $\sum_{i=1}^k m_i = 1$ and $\frac{1+\phi m_i}{1+\phi m_1} = c_{i1}$, for any $i \in \{2, ..., k\}$, where $c_{i1} \equiv \sqrt{\frac{1+\rho^2 \xi_i}{1+\rho^2 \xi_1}} < 1$, and $m_i = 0$ for any $i \in \{k+1, ..., n\}$.

Recursively,

$$m_i = c_{i1} m_1 - \frac{1}{\phi}(1 - c_{i1}) \qquad \forall i \in \{2, ..., k\}.$$

Hence $1 = \sum_{i=1}^k m_i = m_1 + m_1 \sum_{i=2}^k c_{i1} - \frac{1}{\phi} \sum_{i=2}^k (1 - c_{i1})$, which results in a solution for m_1 given by

$$m_1 = \frac{1 + \frac{1}{\phi} \sum_{i=2}^k (1 - c_{i1})}{1 + \sum_{i=2}^k c_{i1}}.$$

We next prove the three claims in the lemma:

(i) For any $\phi > 0$, m_1 changes continuously with ϕ. Differentiating m_1 with respect to ϕ,

$\frac{dm_1}{d\phi} = -\frac{1}{\phi^2} \left[\frac{\sum_{i=2}^k (1 - c_{i1})}{1 + \sum_{i=2}^k c_{i1}} \right] < 0$, since $0 < c_{i1} < 1$ for all $i > 1$. Hence, m_1 is decreasing in ϕ.

Likewise, for each asset $i \in \{2, ..., k\}$, m_i changes continuously with ϕ. Differentiating m_i with respect to ϕ,

$\frac{dm_i}{d\phi} = c_{i1} \left(\frac{dm_1}{d\phi} \right) + \frac{1}{\phi^2}(1 - c_{i1})$. Substituting in the derivative of m_1 and rearranging we obtain

$\frac{dm_i}{d\phi} = \frac{1}{\phi^2} \left[1 - c_{i1} \left(\frac{k}{\sum_{j=1}^k c_{j1}} \right) \right]$, where we have used the fact that $c_{11} = 1$. Since $c_{i1} < 1$ and c_{i1} is strictly decreasing in i, then $\frac{dm_i}{d\phi}$ is increasing in i.

Next, consider the case of a local increase in ϕ to some $\phi' < \phi_k$, such that no new assets are learned about in equilibrium. Since $\Sigma_i m_i = 1$, there must be at least one asset $i > 1$ for which $\frac{dm_i}{d\phi} > 0$, and this asset defines the cutoff $\bar{i}$.

Finally, suppose that $k < n$, and consider the case of an increase in ϕ to some ϕ' with $\phi_k \leq \phi' < \phi_{k'}$, such that $k' > k$ assets are learned about in equilibrium (with $k' \leq n$). Let the equilibrium masses associated with aggregate capacity ϕ' be denoted by m_i' for $i \in \{1, ..., k'\}$. For the new assets, $m_i' > m_i = 0$ for all $i \in \{k+1, ..., k'\}$, hence the mass is increasing in ϕ. From above, $m_1' < m_1$. Since $\Sigma_i m_i = 1$, the new cutoff will be some $\bar{i}' > \bar{i}$.

(ii) First, consider the case of a local increase in ϕ to some $\phi' < \phi_k$, such that no new assets are learned about in equilibrium. For assets that are not learned about, $i > k$, $m_i = 0$ both before and after the capacity increase, hence $\frac{d(\phi m_i)}{d\phi} = 0$. For any two assets that are learned about, $i, l \leq k$, equating equilibrium gains implies $1 + \phi m_i = (1 + \phi m_l) c_{il}$, where $c_{il} \equiv \sqrt{\frac{1+\rho^2 \xi_i}{1+\rho^2 \xi_l}} > 0$. Totally differentiating, $\frac{d(\phi m_i)}{d\phi} = \frac{d[(1+\phi m_l)c_{il}]}{d\phi}$ yields

$$m_i + \phi \frac{dm_i}{d\phi} = \left(m_l + \phi \frac{dm_l}{d\phi}\right) c_{il}.$$

Suppose that there exists an asset $i \leq k$ such that $\frac{d(\phi m_i)}{d\phi} \leq 0$. Then for all other assets $l \leq k$, $l \neq i$, we must also have $\left(m_l + \phi \frac{dm_l}{d\phi}\right) c_{il} \leq 0$. Since $c_{il} > 0$, $m_l > 0$, and $\phi > 0$, then we must have that $\frac{dm_l}{d\phi} < 0$. But this contradicts (i): $\Sigma_i m_i = 1$, hence there must be at least one asset for which $\frac{dm_i}{d\phi} \geq 0$. Hence for all $i \leq k$, $\frac{d(\phi m_i)}{d\phi} > 0$.

Second, consider the case of an increase in ϕ to some ϕ' with $\phi_k \leq \phi' < \phi_{k'}$, such that $k' > k$ assets are learned about in equilibrium (with $k \leq n$). For assets which remain passively traded, $i > k'$, $m_i = m_i' = 0$; hence, there is no change in the aggregate capacity allocated to these assets. For the new assets that are actively traded, $i \in \{k+1, ..., k'\}$, $m_i' > m_i = 0$, hence, $\phi' m_i' > \phi m_i$. Finally, an asset i that was actively traded both before and after the increase, $i \leq k$, had, prior to the increase, a utility gain strictly larger than that of an asset l that was previously not learned about, which implies that $1 + \phi m_i < c_{il}$. After the increase, asset i has a utility gain equal to that of asset l: $1 + \phi' m_i' = (1 + \phi' m_l') c_{il}$. Substituting the inequality for c_{il} into the equality, we obtain $\phi' m_i' - \phi m_i > \phi' m_l' (1 + \phi m_i)$. Since the right-hand side is positive, it follows that $\phi' m_i' > \phi m_i$, which completes the proof.

(iii) Let $K_1 = K$ and $K_2 = \gamma K$, for some $\gamma \in (0,1)$, and consider the case of a local increase in capacity K such that no new assets are learned about in equilibrium. Let $m_{i\phi} \equiv \frac{dm_i}{d\phi}$. The derivatives we are interested in are

$$\frac{d[m_i(e^{2K}-1)]}{dK} = 2e^{2K} m_i + m_{i\phi}(e^{2K} - 1)\frac{d\phi}{dK}$$

$$\frac{d[m_i(e^{2K\gamma}-1)]}{dK} = 2\gamma e^{2K\gamma} m_i + m_{i\phi}(e^{2K\gamma} - 1)\frac{d\phi}{dK}$$

where $\frac{d\phi}{dK} = 2\lambda e^{2K} + 2\gamma(1-\lambda)e^{2K\gamma} > 0$.

First, consider the case in which $m_{i\phi} > 0$. Then, since $e^{2K} > e^{2K\gamma} > \gamma e^{2K\gamma}$,

$$\frac{d[m_i(e^{2K}-1)]}{dK} > \frac{d[m_i(e^{2K\gamma}-1)]}{dK} > 0.$$

Next, consider the case in which $m_{i\phi} < 0$. Factoring out $2e^{2K}$ yields

$$\begin{aligned}\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} &= 2e^{2K}\left\{m_i + m_{i\phi}\left(e^{2K}-1\right)\left[\lambda + (1-\lambda)\gamma e^{2K(\gamma-1)}\right]\right\} \\ &= 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + (1-\lambda)\gamma\left(e^{2K}-1\right)e^{2K(\gamma-1)}\right]\right\} \\ &= 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + (1-\lambda)\gamma\left(e^{2K\gamma}-e^{2K(\gamma-1)}\right)\right]\right\} \\ &> 2e^{2K}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + (1-\lambda)\left(e^{2K\gamma}-1\right)\right]\right\},\end{aligned}$$

where the inequality follows from $m_{i\phi} < 0$, $\gamma < 1$, $e^{2K} > 1$, and $e^{2K(\gamma-1)} < 1$. Using the definition of ϕ, we obtain

$$\frac{d\left[m_i\left(e^{2K}-1\right)\right]}{dK} > 2e^{2K}\left(m_i + \phi m_{i\phi}\right) = 2e^{2K}\left[\frac{d(\phi m_i)}{d\phi}\right] > 0,$$

where the last inequality follows from part (ii) above.

Similarly, also for the case in which $m_{i\phi} < 0$,

$$\begin{aligned}\frac{d\left[m_i\left(e^{2K\gamma}-1\right)\right]}{dK} &= 2\gamma e^{2K\gamma}m_i + m_{i\phi}\left(e^{2K\gamma}-1\right)\left[2\lambda e^{2K} + 2\gamma(1-\lambda)e^{2K\gamma}\right] \\ &= 2e^{2K\gamma}\left\{\gamma m_i + m_{i\phi}\left[\lambda\left(e^{2K}-e^{2K(1-\gamma)}\right) + \gamma(1-\lambda)\left(e^{2K\gamma}-1\right)\right]\right\} \\ &> 2\gamma e^{2K\gamma}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-e^{2K(1-\gamma)}\right) + (1-\lambda)\left(e^{2K\gamma}-1\right)\right]\right\} \\ &= 2\gamma e^{2K\gamma}\left\{m_i + m_{i\phi}\left[\lambda\left(e^{2K}-1\right) + \lambda\left(1-e^{2K(1-\gamma)}\right) + (1-\lambda)\left(e^{2K\gamma}-1\right)\right]\right\} \\ &= 2\gamma e^{2K\gamma}\left\{m_i + m_{i\phi}\phi + m_{i\phi}\lambda\left(1-e^{2K(1-\gamma)}\right)\right\}\end{aligned}$$

where the inequality follows from $\gamma < 1$, $m_{i\phi} < 0$, and the term in square brackets being positive. Using the definition of ϕ, we obtain

$$\frac{d\left[m_i\left(e^{2K\gamma}-1\right)\right]}{dK} > 2\gamma e^{2K\gamma}\left\{\left[\frac{d(\phi m_i)}{d\phi}\right] + \lambda m_{i\phi}\left(1-e^{2K(1-\gamma)}\right)\right\} > 0,$$

where the last inequality follows from part (ii) above and from $m_{i\phi} < 0$ and $1 < e^{2K(1-\gamma)}$.

Finally, note that

$$\lambda\left\{\frac{d[m_i(e^{2K}-1)]}{dK}\right\} + (1-\lambda)\left\{\frac{d[m_i(e^{2K\gamma}-1)]}{dK}\right\} = \left[\frac{d(\phi m_i)}{d\phi}\right]\left(\frac{d\phi}{dK}\right).$$

Plugging in $d\phi/dK$,

$$\lambda\left\{\frac{d[m_i(e^{2K}-1)]}{dK}\right\}+(1-\lambda)\left\{\frac{d[m_i(e^{2K\gamma}-1)]}{dK}\right\} = \lambda\left\{2e^{2K}\left[\frac{d(\phi m_i)}{d\phi}\right]\right\}+(1-\lambda)\left\{2\gamma e^{2K\gamma}\left[\frac{d(\phi m_i)}{d\phi}\right]\right\}.$$

Since the first term on the left-hand side is greater than the first term on the right-hand side, and since $2e^{2K} > 2\gamma e^{2K\gamma}$, it must be the case that the second element of this weighted average is smaller, which implies that

$\frac{d[m_i(e^{2K}-1)]}{dK} > \frac{d[m_i(e^{2K\gamma}-1)]}{dK}$, which concludes the proof.

Proof of Lemma 5 We consider the choice of an individual investor, taking the choices of all other investors as given, characterized by the solution in the main text.

Case A. First, we consider the case in which the investor treats the price as any other random variable that cannot be processed perfectly for free. Suppose that the investor allocates capacity to learning the price of asset i. This investor will observe a compressed representation of the price, s^p_{ji}, that is the result of the decomposition $p_i = s^p_{ji} + \varepsilon_{ji}$,with $s^p_{ji} \sim \mathcal{N}\left(\overline{p}_i, \sigma^2_{spji}\right), \varepsilon_{ji} \sim \mathcal{N}\left(0, \sigma^2_{\varepsilon ji}\right)$, and $\sigma^2_{pi} = \sigma^2_{spji} + \sigma^2_{\varepsilon ji}$. The amount of capacity consumed by the price signal is

$I\left(p_i; s^p_{ji}\right) = \frac{1}{2}\log\left(\frac{\sigma^2_{pi}}{\sigma^2_{\varepsilon ji}}\right).$

The quantity of information about *payoffs* that is conveyed by the price signal is

$I\left(z_i; s^p_{ji}\right) = H\left(z_i\right) + H\left(s^p_{ji}\right) - H\left(z_i, s^p_{ji}\right) = \frac{1}{2}\log\left(\frac{\sigma^2_i\sigma^2_{spji}}{\left|\Sigma_{z_i sp_{ji}}\right|}\right),$

where $\left|\Sigma_{z_i sp_{ji}}\right|$ is the determinant of the variance-covariance matrix of z_i and s^p_{ji}. Using the fact that z_i and s^p_{ji} are conditionally independent given prices,

$$Cov\left(z_i, s^p_{ji}\right) = \frac{Cov\left(z_i, p_i\right) Cov\left(p_i, s^p_{ji}\right)}{\sigma^2_{pi}}.$$

Using the solution for equilibrium prices, $Cov\left(z_i, p_i\right) = b_i\sigma^2_i$. Using the signal structure, $Cov\left(p_i, s^p_{ji}\right) = \sigma^2_{spji}$. Hence

$$Cov\left(z_i, s^p_{ji}\right) = \frac{b_i\sigma^2_i\sigma^2_{spji}}{\sigma^2_{pi}}.$$

The determinant becomes

$$\left|\Sigma_{z_i sp_{ji}}\right| = \sigma^2_i\sigma^2_{spji}\left(\frac{\sigma^2_{pi}\sigma^2_{pi} - b^2_i\sigma^2_i\sigma^2_{spji}}{\sigma^2_{pi}\sigma^2_{pi}}\right)$$

so that

$I\left(z_i; s^p_{ji}\right) = \frac{1}{2}\log\left(\frac{\sigma^2_{pi}}{c^2_i\sigma^2_{xi} + \frac{b^2_i\sigma^2_i}{\sigma^2_{pi}}\sigma^2_{\varepsilon ji}}\right).$

Next, we show that $I\left(z_i; s^p_{ji}\right) \leq I\left(p_i; s^p_{ji}\right)$. Suppose not. Then, in order for the reverse inequality to hold, it must be the case that

$c^2_i\sigma^2_{xi} < \left(1 - \frac{b^2_i\sigma^2_i}{\sigma^2_{pi}}\right)\sigma^2_{\varepsilon ji} \quad \Leftrightarrow \quad \sigma^2_{pi} < \sigma^2_{\varepsilon ji},$

which is a contradiction. Hence, $I\left(z_i; s^p_{ji}\right) \leq I\left(p_i; s^p_{ji}\right)$, with equality if and only if $\sigma^2_{pi} = \sigma^2_{\varepsilon ji}$, which occurs only if $I\left(p_i; s^p_{ji}\right) = 0$. Hence for any positive capacity dedicated to the price signal, the effective amount of information about the payoff is less than the capacity consumed in order to receive the signal.

Case B. Next, we consider the case in which the price itself is a perfectly observed signal that nonetheless consumes capacity. Suppose that the investor uses capacity to learn from p_i, and let posterior beliefs about z_i conditional on p_i be denoted by y_i. Then $y_i \sim \mathcal{N}\left(\overline{y}_i, \sigma^2_{yi}\right)$, with

$$\overline{y}_i = \sigma^2_{yi}\left[\tfrac{1}{\sigma^2_i}\overline{z}_i + \tfrac{b^2_i}{c^2_i\sigma^2_{xi}}z_i - \tfrac{b_i}{c_i\sigma^2_{xi}}\left(x_i - \overline{x}_i\right)\right]$$

$$\tfrac{1}{\sigma^2_{yi}} = \frac{1}{\frac{1}{\sigma^2_i} + \frac{b^2_i}{c^2_i\sigma^2_{xi}}}.$$

The information contained in the price of asset i is $I\left(z_i; p_i\right) = \frac{1}{2}\log\left(\frac{\sigma^2_i}{\sigma^2_{yi}}\right)$. Using the solution for equilibrium prices, this variance is given by

$$\sigma^2_{yi} = \frac{\sigma^2_i}{1+\left(\frac{\phi m_i}{\rho\sigma_i\sigma_{xi}}\right)^2}.$$

We next demonstrate that the investor's ex-ante expected utility is higher when allocating all her capacity to learning from private signals than when allocating at least a portion of her capacity to learning from prices, owing to strategic substitutability. The investor's objective is to maximize

$$\widetilde{E}_{1j}\left[U_{2j}\right] = \frac{1}{2\rho}\sum_{i=1}^{n}\left(\frac{\widetilde{V}_{ji} + \widetilde{R}^2_{ji}}{\widetilde{\sigma}^2_{ji}}\right), \tag{20}$$

subject to

$$\prod_{i=1}^{n}\left(\frac{\sigma^2_i}{\widetilde{\sigma}^2_{ji}}\right) \leq e^{2K_j}, \tag{21}$$

where $\widetilde{R}_{ji}$ and $\widetilde{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns, $\left(\widetilde{\mu}_{ji} - rp_i\right)$, $\widetilde{\mu}_{ji}$ and $\widetilde{\sigma}^2_{ji}$ denote the mean and variance of the investor's posterior beliefs about the payoff z_i, and the tilde indicates that these variables are computed under a signalling mechanism that allows for learning from prices.

Suppose that the investor uses capacity to learn from p_i, and let posterior beliefs about z_i conditional on p_i be denoted by y_i. Then, the investor designs a signal conditional on the information obtained from the price, $y_i = \widetilde{s}_{ji} + \widetilde{\delta}_{ji}$, where we maintain the same two independence assumptions that were used in setting up the private signal in the absence of learning from the price. Under this signal structure, the ex-ante mean is the same, regardless of whether the investor learns from p_i or not: $\widetilde{R}_{ji} = \overline{z}_i - r\overline{p}_i$. The ex-ante variance of expected excess returns is given by $\widetilde{V}_{ji} = Var_{1j}\left(\widetilde{\mu}_{ji}\right) + r^2\sigma^2_{pi} - 2rCov_{1j}\left(\widetilde{\mu}_{ji}, p_i\right)$. Using the formula for partial correlation and exploiting the fact that signals and prices are conditionally independent given beliefs,

$$Cov_{1j}\left(\widetilde{\mu}_{ji}, p_i\right) = \frac{Cov_{1j}\left(\widetilde{\mu}_{ji}, y_i\right) Cov_{1j}\left(y_i, p_i\right)}{\sigma_{yi}^2}.$$

Using the signal structure, $Cov_{1j}\left(\widetilde{\mu}_{ji}, y_i\right) = Var\left(\widetilde{s}_{ji}\right)$, $Var\left(\widetilde{s}_{ji}\right) = \sigma_{yi}^2 - \widetilde{\sigma}_{ji}^2$, and using equilibrium prices, $Cov_{1j}\left(y_i, p_i\right) = b_i \sigma_i^2$. Hence

$$Cov_{1j}\left(\widetilde{\mu}_{ji}, p_i\right) = b_i \sigma_i^2 - \frac{b_i \sigma_i^2 \widetilde{\sigma}_{ji}^2}{\sigma_{yi}^2}$$

Hence $\widetilde{V}_{ji} = \left(1 - 2rb_i\right)\sigma_i^2 + r^2\sigma_{pi}^2 - \left(\sigma_i^2 - \sigma_{yi}^2\right) - \left[1 - 2rb_i\left(\frac{\sigma_i^2}{\sigma_{yi}^2}\right)\right]\widetilde{\sigma}_{ji}^2$, if the investor learns from p_i.

Conversely, if the investor does not allocate any capacity to learning from prices, $V_{ji} = \left(1 - 2rb_i\right)\sigma_i^2 + r^2\sigma_{pi}^2 - \left(1 - 2rb_i\right)\widetilde{\sigma}_{ji}^2$, where we have used the fact that the information constraint implies that the investor's posterior variance, here denoted by $\widetilde{\sigma}_{ji}^2$, is the same in both cases. Both cases imply a corner solution, with the investor allocating all capacity to learning about a single asset. The remaining question is: will the investor allocate any capacity to learning from the price, or will she use all capacity on the private signal? It can be easily seen that for any positive level of capacity allocated to the price signal, $V_{ji} > \widetilde{V}_{ji}$. Hence, the investor's ex-ante utility is lower when she devotes any positive amount of capacity to learning from prices. Learning from prices increases the covariance between the investor's posterior beliefs and equilibrium prices, thereby reducing the investor's excess returns. This case is similar to that of Kacperczyk, Van Nieuwerburgh, and Veldkamp (2013), who show that prices are an inferior source of information in a portfolio choice model with an additive constraint on the sum of signal precisions.
Hence, regardless of the informativeness of prices relative to the investor's capacity, the investor is always better off learning through signals that provide information directly on the payoffs. In our framework prices lose their special role as publicly available signals.

Analytical Predictions

Proof of Proposition 1. Using equations (14)-(15), the difference in profits for asset i is given by

$$\pi_{1i} - \pi_{2i} = \frac{m_i\left(e^{2K_1} - e^{2K_2}\right)\left(z_i - rp_i\right)^2}{\rho\sigma_i^2} \geq 0.$$

This difference is zero if $m_i = 0$ or $K_1 = K_2$. For $K_1 > K_2$, it is strictly positive for assets that are learned about in equilibrium (i.e., if $m_i > 0$). Also, $K_1 > K_2 > 0$ implies $\phi > 0$. It follows that $m_i > 0$ for at least one i. ■

Proof of Proposition 2. Using equations (14)-(15), the ownership difference for asset i becomes

$$\frac{Q_{1i}}{\lambda}-\frac{Q_{2i}}{(1-\lambda)}=m_i\left(e^{2K_1}-e^{2K_2}\right)\left(\frac{z_i-rp_i}{\rho\sigma_i^2}\right).$$

(i) For $i>k$, $m_i=0$, and hence the ownership difference is equal to zero. For $i\leq k$, $m_i>0$, and the expected ownership differential is given by

$$E\left\{\frac{Q_{1i}}{\lambda}-\frac{Q_{2i}}{(1-\lambda)}\right\}=\frac{m_i\overline{x}_i\left(e^{2K_1}-e^{2K_2}\right)}{1+\phi m_i}, \tag{22}$$

where we have used the fact that expected excess returns are, by equations (**??**) and (7),

$$E\left(z_i-rp_i\right)=\frac{\rho\sigma_i^2\overline{x}_i}{1+\phi m_i}. \tag{23}$$

Since $K_1>K_2$ and $\overline{x}_i>0$, the result follows.
(ii) First, we show that if $E(z_i-rp_i)>E(z_l-rp_l)$, then $m_i>m_l$. Since $i,l<k$, their gain factors are equated, $G_i=G_l$. Using (23), and the fact that $\overline{x}_i=\overline{x}$ and $\sigma_{xi}=\sigma_x$ for all i, the gain factor of asset i can be written as

$$G_i=\frac{1+\rho^2\left(\sigma_x^2+\overline{x}^2\right)\sigma_i^2}{\rho^2\overline{x}^2\sigma_i^4}\left[E(z_i-rp_i)\right]^2,$$

and a corresponding expression holds for G_l. The inequality in excess returns implies that

$$\frac{1+\rho^2\left(\sigma_x^2+\overline{x}^2\right)\sigma_i^2}{\sigma_i^4}<\frac{1+\rho^2\left(\sigma_x^2+\overline{x}^2\right)\sigma_l^2}{\sigma_l^4},$$

which reduces to $\sigma_i^2>\sigma_l^2$. Proposition 3 implies that m_i is increasing in ξ_i, which, under the maintained assumptions that $\overline{x}_i$ and σ_{xi}^2 are equal across i, implies that m_i is increasing in σ_i^2. Hence, $m_i>m_l$.
Next, from the expression for the expected ownership differential in (22), the difference in expected relative ownership across the two assets is

$$E\left\{\frac{Q_{1i}}{\lambda}-\frac{Q_{2i}}{(1-\lambda)}\right\}-E\left\{\frac{Q_{1l}}{\lambda}-\frac{Q_{2l}}{(1-\lambda)}\right\}=\frac{\overline{x}\left(e^{2K_1}-e^{2K_2}\right)(m_i-m_l)}{(1+\phi m_i)(1+\phi m_l)}>0,$$

which completes the proof. ■

Proof of Proposition ??. Using equations (14)-(15), the state-by-state ownership difference for asset i becomes

$$\frac{Q_{1i}}{\lambda}-\frac{Q_{2i}}{(1-\lambda)}=m_i\left(e^{2K_1}-e^{2K_2}\right)\left(\frac{z_i-rp_i}{\rho\sigma_i^2}\right).$$

If $i\leq k$, the equilibrium level of $m_i>0$ is an ex-ante decision, and hence it is constant across realizations. The result follows. ■

Proof of Proposition 4. Our derivation keeps the aggregate information quantity ϕ constant, and hence the masses m_i unchanged, by equation (9), which in turn implies that prices also remain unchanged, by equations (**??**) and (7). By equations (13), (14), and (15), relative capital income is

$$\frac{\sum_i \pi_{1i}}{\sum_i \pi_{2i}} = \frac{\sum_i \{(\overline{z}_i - rp_i)(z_i - rp_i) + m_i(e^{2K_1} - 1)(z_i - rp_i)^2\}}{\sum_i \{(\overline{z}_i - rp_i)(z_i - rp_i) + m_i(e^{2K_2} - 1)(z_i - rp_i)^2\}}.$$

Since $K_1' > K_1$ and $K_2' < K_2$, each element of $\sum_i \pi_{1i}$ increases and each element of $\sum_i \pi_{2i}$ decreases. ■

Proof of Proposition 5. (i) From equations (**??**) and (7), the average equilibrium price of asset i can be expressed as

$$\overline{p}_i = \frac{1}{r}\left(\overline{z}_i - \frac{\rho\sigma_i^2 \overline{x}_i}{1 + \phi m_i}\right).$$

For $i > k$, $m_i = 0$, and $\overline{p}_i$ remains unchanged. For $i \leq k$, $m_i > 0$, and $\overline{p}_i$ is increasing in ϕm_i, which in turn is increasing in ϕ, per Proposition 4.
(ii) Equilibrium expected excess returns are

$$E\left(z_{it} - rp_{it}\right) = \frac{\rho\sigma_i^2 \overline{x}_i}{1 + \phi m_i}.$$

For $i > k$, $m_i = 0$, and expected excess returns remain unchanged. For $i \leq k$, $m_i > 0$, and the expected excess return of asset i is decreasing in ϕm_i, which in turn is increasing in ϕ, per Proposition 4. ■

Proof of Proposition 6. The average ownership difference is given by

$$E\left\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{(1-\lambda)}\right\} = \frac{m_i \overline{x}_i \left(e^{2K_1} - e^{2K_2}\right)}{1 + \phi m_i}.$$

For our designed deviation of information capacities, the aggregate information quantity ϕ constant, and hence the masses m_i are unchanged by equation (9). Polarization in e^{2K_1} versus e^{2K_2} gives the result. ■

Proof of Proposition 7. Using equations (14) and (15), the expected difference in asset ownership is given by

$$E\left\{\frac{Q_{1i}}{\lambda} - \frac{Q_{2i}}{1-\lambda}\right\} = \frac{1 + m_i\left(e^{2K_1} - 1\right)}{1 + \phi m_i}\overline{x}_i - \frac{1 + m_i\left(e^{2K_2} - 1\right)}{1 + \phi m_i}\overline{x}_i.$$

Since average quantities have to be equal to average supply $\bar{x}_i$, it is enough to show that the first element of the sum is increasing. It is given by

$$\frac{dE\{\frac{Q_{1i}}{\lambda}\}}{dK} = \frac{\frac{d[m_i(e^{2K}-1)]}{dK}(1+\phi m_i) - \frac{d\phi m_i}{d\phi}\frac{d\phi}{dK}m_i(e^{2K}-1)}{(1+\phi m_i)^2}\overline{x}_i$$

The sign of the expression is determined by the sign of

$$\begin{aligned} sign\left(\frac{dE\{\frac{Q_{1i}}{\lambda}\}}{dK}\right) &= sign\left(\frac{d[m_i(e^{2K}-1)]}{dK} - \frac{d\phi m_i}{d\phi}\frac{d\phi}{dK}m_i(e^{2K}-1)\frac{1}{1+\phi m_i}\right) \\ &= sign\left(\frac{d[m_i(e^{2K}-1)]}{dK} - \frac{d\phi m_i}{d\phi}(e^{2K}-1)\frac{2(\lambda e^{2K}+(1-\lambda)\gamma e^{2K\gamma})}{\lambda e^{2K}+(1-\lambda)e^{2K\gamma}}\right) \end{aligned}$$

In the proof of Proposition 4, we show that

$$\frac{d[m_i(e^{2K}-1)]}{dK} > 2e^{2K}\frac{d\,(\phi m_i)}{d\phi} > 0.$$

Using that expression, we obtain

$$sign\left(\frac{dE\{\frac{Q_{1i}}{\lambda}\}}{dK}\right) = sign\left(2e^{2K} - (2e^{2K}-2)\frac{\lambda e^{2K}+(1-\lambda)\gamma e^{2K\gamma}}{\lambda e^{2K}+(1-\lambda)e^{2K\gamma}}\right) > 0,$$

where the last inequality is guaranteed by $\frac{\lambda e^{2K}+(1-\lambda)\gamma e^{2K\gamma}}{\lambda e^{2K}+(1-\lambda)e^{2K\gamma}} < 1$. ■

Proof of Proposition 8. Using equations (13) and (14), the expected income from holding asset i for the sophisticated investors is given by:

$$E(\pi_{1i}) = \frac{m_i(e^{2K}-1)(\sigma_i^2+\rho^2\xi_i) - \phi m_i\sigma_i^2 + \rho^2\xi_i}{\rho(1+\phi m_i)^2}$$

and hence, the ratio of expected profits is

$$\frac{E\pi_{1i}}{E\pi_{2i}} = \frac{m_i(e^{2K}-1)(\sigma_i^2+\rho^2\xi_i) - \phi m_i\sigma_i^2 + \rho^2\xi_i}{m_i(e^{2K\gamma}-1)(\sigma_i^2+\rho^2\xi_i) - \phi m_i\sigma_i^2 + \rho^2\xi_i}$$

which can be written as

$$\frac{E\pi_{1i}}{E\pi_{2i}} = \frac{m_i(e^{2K}-1)\alpha - \phi m_i + \omega}{m_i(e^{2K\gamma}-1)\alpha - \phi m_i + \omega}$$

where

$$\alpha = 1 + \frac{\rho^2\xi_i}{\sigma_i^2} \text{ and } \omega = \alpha - 1.$$

Then consider the difference between old and new expected profit between two levels of overall capacity $K^* > K$, with K^* associated with the endogenous mass of investors m_i^* and

K with m_i:

$$\Delta \equiv \frac{m_i^*(e^{2K^*}-1)\alpha - \phi^* m_i^* + \omega}{m_i^*(e^{2K^*\gamma}-1)\alpha - \phi^* m_i^* + \omega} - \frac{m_i(e^{2K}-1)\alpha - \phi m_i + \omega}{m_i(e^{2K\gamma}-1)\alpha - \phi m_i + \omega}.$$

We will show that $\Delta > 0$, i.e.

$$\frac{m_i^*(e^{2K^*}-1)\alpha - \phi^* m_i^* + \omega}{m_i^*(e^{2K^*\gamma}-1)\alpha - \phi^* m_i^* + \omega} > \frac{m_i(e^{2K}-1)\alpha - \phi m_i + \omega}{m_i(e^{2K\gamma}-1)\alpha - \phi m_i + \omega}.$$

Suppose that expected profits for each investor are positive (which must be true for them to hold the asset), then the above is equivalent to

$$[m_i^*(e^{2K^*}-1)\alpha - \phi^* m_i^* + \omega][m_i(e^{2K\gamma}-1)\alpha - \phi m_i + \omega] > [m_i(e^{2K}-1)\alpha - \phi m_i + \omega][m_i^*(e^{2K^*\gamma}-1)\alpha - \phi^* m_i^* + \omega].$$

Multiplying through and rearranging,

$$\begin{aligned}
&\alpha\omega[m_i^*(e^{2K^*}-1) - m_i(e^{2K}-1) - (m_i^*(e^{2K^*\gamma}-1) - m_i(e^{2K\gamma}-1))] \\
&+ m_i^*(e^{2K^*}-1)\alpha m_i(e^{2K\gamma}-1)\alpha - m_i^*(e^{2K^*}-1)\alpha\phi m_i \\
&- \phi^* m_i^* m_i(e^{2K\gamma}-1)\alpha \\
&> \\
&+ m_i(e^{2K}-1)\alpha m_i^*(e^{2K^*\gamma}-1)\alpha - m_i(e^{2K}-1)\alpha\phi^* m_i^* \\
&- \phi m_i m_i^*(e^{2K^*\gamma}-1)\alpha
\end{aligned}$$

Since the first term in square brackets is positive by Proposition 4, for our result to hold it is enough to show that (factoring out $\alpha m_i^* m_i > 0$)

$$\begin{aligned}
&\alpha[(e^{2K^*}-1)(e^{2K\gamma}-1) - (e^{2K}-1)(e^{2K^*\gamma}-1)] - (e^{2K^*}-1)\phi - \phi^*(e^{2K\gamma}-1) \\
&> -(e^{2K}-1)\phi^* - \phi(e^{2K^*\gamma}-1)
\end{aligned}$$

which can be written as

$$\alpha[(e^{2K^*}-1)(e^{2K\gamma}-1) - (e^{2K}-1)(e^{2K^*\gamma}-1)] - [(e^{2K\gamma}-e^{2K})\phi^* + \phi(e^{2K^*}-e^{2K^*\gamma})] > 0$$

To obtain a closed-form expression for the second bracketed term, plug in the definition of

ϕ, to obtain

$$
\begin{aligned}
&(e^{2K\gamma}-e^{2K})[\lambda(e^{2K^*}-1)+(1-\lambda)(e^{2K^*\gamma}-1)]+(e^{2K^*}-e^{2K^*\gamma})[\lambda(e^{2K}-1)+(1-\lambda)(e^{2K\gamma}-1)] \\
&=(e^{2K\gamma}-1)\lambda(e^{2K^*}-1)+(e^{2K\gamma}-1)(1-\lambda)(e^{2K^*\gamma}-1) \\
&-(e^{2K}-1)\lambda(e^{2K^*}-1)-(e^{2K}-1)(1-\lambda)(e^{2K^*\gamma}-1) \\
&+(e^{2K^*}-1)\lambda(e^{2K}-1)+(e^{2K^*}-1)(1-\lambda)(e^{2K\gamma}-1) \\
&-(e^{2K^*\gamma}-1)\lambda(e^{2K}-1)-(e^{2K^*\gamma}-1)(1-\lambda)(e^{2K\gamma}-1) \\
&=(e^{2K^*}-1)(e^{2K\gamma}-1)-(e^{2K}-1)(e^{2K^*\gamma}-1)
\end{aligned}
$$

Hence, a sufficient condition for $\Delta > 0$ is

$$(\alpha-1)[(e^{2K^*}-1)(e^{2K\gamma}-1)-(e^{2K}-1)(e^{2K^*\gamma}-1)]>0 \tag{24}$$

Since $\alpha > 1$, it is enough to show that the term in square brackets is positive. To see that, define $f(K^*) = \left(e^{2K^*}-1\right)\left(e^{2K\gamma}-1\right)-\left(e^{2K}-1\right)\left(e^{2K^*\gamma}-1\right)$ and notice that $f(K) = 0$. Furthermore, also notice that $f'(K^* = K) = 0$ and $f'(K^*) = 0$ for all K^* if $\gamma \in \{0,1\}$, and that $f'(K^*)$ has a single maximum with respect to γ for each K^*, and that maximum is attained at $\gamma \in (0,1)$. To see that, calculate

$$
\begin{aligned}
f'_\gamma \equiv \frac{df'(K^*)}{d\gamma} &= 2\left(2Ke^{2K^*}e^{2K\gamma}-e^{2K^*\gamma}\left(e^{2K}-1\right)(1+2\gamma K)\right) \\
&> 2e^{2K^*\gamma}e^{2K}\left(2K+\left(\frac{1}{e^{2K}}-1\right)(1+2\gamma K)\right).
\end{aligned}
$$

Clearly, $f'_\gamma = 0$ for a single value of γ. Additionally, by the arguments in the proof of Proposition 4, we know that at $\gamma = 0$, $f'_\gamma = 0$. Hence, for any K^*, K, $f' = 0$ for $\gamma \in \{0,1\}$, f' is increasing in γ at $\gamma = 0$ and f' has a single maximum with respect to γ. It follows that for all γ between zero and one, $f'(K^*) > 0$, and hence equation (24) is satisfied. ■

Derivation of (18) and (19) We can derive the conditional CAPM equation in our model by using the market clearing condition (we omit the time notation for clarity of exposition):

$$\int \frac{s_{ij}}{\hat{\sigma}_{ij}^2}dj - rp_i \int \frac{1}{\hat{\sigma}_{ij}^2}dj = \rho x_i,$$

where $\hat{\sigma}_{ij}^2$ is equal to σ_i^2 if the investor is not learning, $e^{-2K_j}\sigma_i^2$ if investor type j is learning about asset i. s_{ij} are not correlated with $\hat{\sigma}_{ij}^2$, so we can express the market clearing as

$$\frac{1}{\Theta_i}(S_i - rp) = \rho x_i$$

where $s_{ij} = \bar{z}_i$ for investors who don't learn and

$$S_i = \int s_{ij} dj,$$

$$\frac{1}{\Theta_i} = \int \frac{1}{\hat{\sigma}_{ij}^2} dj.$$

That gives

$$rp_i = S_i - \rho \Theta_i x_i.$$

Define the market payoff: $\sum z_i x_i$. The (conditional) covariance of z_i with the market is a function of the average conditional variance of z_i:

$$cov(z_i, \sum x_i z_i) = x_i \Theta_i,$$

so the price of the asset is

$$p_i = \frac{1}{r}[S_i - \rho cov(z_i, \sum x_i z_i)],$$

Consider the weighted average $p_m := \sum x_i p_i$ as the price of the market portfolio, and $\sum z_i x_i / p_m$ the return on the market. Then the expected return is S_i/p_i, given by

$$E(r_i) - r = \rho p_m cov(r_i, r_m),$$

For the whole market,

$$E(r_m) - r = \rho p_m cov(r_m, r_m).$$

Substituting out $p_m \rho$ gives

$$E(r_i) - r = \frac{cov(r_i, r_m)}{var(r_m)}(E(r_m) - r)$$

So the market β_i of asset i is $\frac{cov(r_i, r_m)}{var(r_m)}$. By i.i.d. shocks assumption,

$$cov(r_i, r_m) = 1/(p_i p_m) cov(z_i, \sum x_i z_i) = \Theta_i x_i / (p_i p_m)$$

and

$$var(r_m) = p_m^2 var(\sum_i x_i z_i) = p_m^2 \sum x_i^2 \Theta_i.$$

Using the conditional CAPM, we can price the return on strategy $\{\omega_{ijt}\}_{i=1}^n$ at time t,

$$R_{jt} = \sum_i \omega_{it}(E_t(r_{it}) - r).$$

The unconditional expectation of that return is

$$E_0 \sum_i \omega_{it}(E_t(r_{it}) - r) = \sum_i cov_0(\omega_{it}, E_t(r_{it})) + \sum_i E_0 \omega_{it} E_0[E_t(r_{it}) - r]$$

$$= \sum_i cov_0(\omega_{it}, E_t(r_{it})) + \sum_i E_0 \omega_{it} E_0[\beta_{it}(E_t(r_{mt}) - r)],$$

where the covariance and the expectation are unconditional. We can thus express the expected return on the portfolio as

$$\bar{R}_j = \sum_i cov_0(\omega_{it}, E_t(r_{it})) + \sum_i E_0 \omega_{it} cov_0(\beta_{it}, E_t(r_{mt})) + \bar{\beta}(E_0(r_{mt}) - r)]$$

where $\bar{\beta} = \sum_i E_0 \omega_{it} E_0 \beta_{it}$.