

# PERFORMANCE PAY, TRADE AND INEQUALITY

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## Abstract

This paper introduces moral hazard into a standard general equilibrium model with heterogeneous firms, to study the impact of trade liberalization on wage inequality between identical workers. Trade liberalization operates on two margins of inequality, generating between- and within-firm wage dispersion. While the former channel has been studied in recent papers, the latter is novel in the literature. In the model, within-firm wage dispersion increases in firm productivity as a result of differential intensity in optimal performance-pay compensation across firms. International trade liberalization triggers labor reallocations towards high productivity firms that result in higher within-firm inequality.

## 1 Introduction

Our understanding of the impact of international trade on wage inequality has evolved substantially in the last twenty years. In the early 1990's, most economists dismissed the role of trade as a driving force behind the steep increases in wage inequality that had been observed in many countries around the world since the late 1970's. Standard factor proportions theory was not easily reconcilable with increasing inequality in developing countries, the absence of significant reallocations of labor across industries and evidence showing that standard human capital variables like education and experience could account for only minor shares of the level and growth of inequality in both developed and developing countries.<sup>1</sup>

In recent years, however, a new generation of trade models has caught up to these empirical challenges by shifting its focus from industries to firms, as the basic units of analysis. This research agenda has been fueled by numerous studies documenting a set of stylized facts regarding heterogeneity in firm-level outcomes within industries, including systematic differences between exporting and non-exporting firms. Recent trade theories place particular emphasis on the finding that more productive firms pay higher average wages, even after

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<sup>1</sup>See Katz and Autor (1999) and Goldberg and Pavcnik (2007) for evidence on developed and developing countries, respectively.

controlling for worker characteristics, including education, occupation and industry.<sup>2</sup> In a recent study using Brazilian data, Helpman et al. (2012) report that 38% of the variance of log wages within sector-occupation cells in 1990 can be accounted for by the variation in wage premia across firms. These facts are compatible with models of firm heterogeneity that feature search frictions and bargaining (Davidson et al. (2008), Helpman et al. (2010), Coşar et al. (2011)), efficiency wages (Verhoogen (2008), Davis and Harrigan (2011)), and fair wage constraints (Egger and Kreickemeier (2009), Amiti and Davis (2011)), in which ex-ante identical workers receive higher wages in more productive firms and wages are systematically related to the export status of the firm. However, with the exception of Verhoogen (2008) (discussed below), workers employed in the same firm receive identical wages in these models.

This literature therefore cannot elucidate an equally sizable component of residual wage inequality (34%) reported in Helpman et al. (2012), namely, within-firm wage dispersion. This evidence is corroborated in recent empirical studies in the United States and several European countries, collected in Lazear and Shaw (2008). Overall, they report that within-firm wage variation ranges from 60 to 80 percent of the total wage dispersion in each of those countries. In a study of Mexican plants, Frías et al. (2012) find that an exogenous increase in the incentive to export, triggered by the peso devaluation in 1994, resulted in higher within-plant wage dispersion.

The purpose of this paper is to develop a theoretical framework to study this important and relatively unexplored dimension of wage inequality, emphasizing its links to international trade. To do so, I extend a standard two-country, general equilibrium model with heterogeneous firms (Melitz (2003)), by adding two key ingredients. First, moral hazard, which generates within-firm wage dispersion between identical workers as firms pay for performance in order to align the incentives of employees with their best interests. In particular, I study a sequential production process during which workers stochastically make mistakes that are detrimental to product quality. Workers can reduce the frequency of their mistakes by exerting costly effort at each production stage. Firms, in turn, monitor the performance of their workers, observing outcomes (mistakes/successes) but not inputs (effort choices). Importantly, as the frequency of tasks increases, individual performance converges to a Brownian process. This feature of the model allows for a simple characterization of optimal contracts that builds on the seminal work of Holmstrom and Milgrom (1987).

Second, I introduce cross-firm differences in compensation policies by allowing for complementarity between firm productivity and the performance of workers in generating product quality. Each firm designs a set of optimal contracts, providing incentives that implement desired effort levels. Because high productivity firms have a comparative advantage in generating quality, they find it optimal to offer higher-powered incentives.<sup>3</sup> As a result, in equilibrium, wages are relatively more dispersed in more productive firms.

Heterogeneity in performance-pay contracts across firms generates implications for (residual) wage inequality that remain unexplored in the trade literature.<sup>4</sup> To illustrate these,

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<sup>2</sup>Evidence of size and exporter wage premia is reported in Bernard and Jensen (1995), Amiti and Davis (2011) and Helpman et al. (2012) for US, Indonesian and Brazilian firms, respectively.

<sup>3</sup>This pattern is consistent with firm-level evidence in Bloom and Van Reenen (2007), who report a positive correlation between the extent to which firms reward performance and total sales in the United States, France, Germany, and the United Kingdom.

<sup>4</sup>Workers are identical, except for their ex-post income. Thus, from an empirical perspective, wage variation generated by the model should be understood as residual (or within-group) inequality (i.e. wage variation across workers of identical observable characteristics such as education, gender, experience, etc).

consider the variance of (log) wages in any one of the two countries in the model, denoted  $Var(w)$ , which can be decomposed as

$$Var(w) = Var[E(w|\theta)] + E[Var(w|\theta)],$$

where  $\theta$  indexes the set of active firms in a given equilibrium.  $E(w|\theta)$  and  $Var(w|\theta)$  denote the mean and variance of wages across workers employed in firms with productivity  $\theta$ , respectively. In turn,  $Var[\cdot]$  and  $E[\cdot]$  integrate over the distribution of workers across firms. The total wage variance is the sum of (i) the variance of average wages across firms (*between-firm inequality*) and (ii) the average of within-firm wage variances (*within-firm inequality*). As mentioned, recent theoretical studies link trade liberalization to residual wage inequality through mechanisms that operate exclusively on the between-firm component of wage inequality, in which firms of different sizes pay different wages to identical workers but there is no wage dispersion inside firms. The model developed in this paper is, to the best of my knowledge, the first to link trade and residual wage inequality through both channels.

More specifically, the key features and implications of the model regarding the effect of international trade on wage inequality can be summarized as follows:

(a) Performance pay generates wage dispersion within firms. By punishing or rewarding employees according to their performance, high-powered incentives amplify the effect of the idiosyncratic component of performance on wages. This implies  $Var(w|\theta) > 0$  in every firm  $\theta$ .

(b) Different firms design different performance-pay contracts, generating cross-firm variation in the first and second moments of firm-level wage distributions. In particular, more productive firms offer higher-powered incentives and hence  $Var(w|\theta)$  increases in productivity. Moreover, because equilibrium in the labor market requires workers to be indifferent between employment in any firm, high productivity firms also offer higher expected wages to compensate for higher effort levels. This generates variation in  $E(w|\theta)$  across firms, which translates into positive between-firm inequality.

(c) In equilibria featuring selection of more productive firms into exporting, international trade liberalization (i.e. a lower variable trade cost) alters the distribution of workers across firms by triggering general equilibrium reallocations of labor towards high productivity firms. Specifically, when firm productivity is Pareto distributed, I show that the distribution of workers across firms in a post-liberalization equilibrium first-order stochastically dominates the corresponding pre-liberalization equilibrium distribution. In combination with (b), this leads to monotonic increases in within-firm inequality. As in Helpman et al. (2010), however, the effects on between-firm inequality are non-monotonic and difficult to characterize analytically without further assumptions.

The mechanism advanced in this paper is both distinct from, and complementary to, the work of Verhoogen (2008). In the latter, an exchange-rate devaluation increases firm-level wage variances (among identical workers) in exporting firms, as they upgrade quality by paying higher efficiency wages to workers employed in the export production line. In Verhoogen (2008), however, effort-wage schedules are exogenous and a characterization of equilibrium changes in the distribution of workers across firms is not provided. This prevents a general equilibrium analysis of the impact of trade on within-firm inequality, which is the main goal of this paper.<sup>5</sup>

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<sup>5</sup>The importance of characterizing equilibrium changes in the distribution of workers across firms in response to trade liberalization cannot be overstated. To illustrate this in a stark way, note that within-firm

Importantly, the results in this paper do not require trade-induced effects on firm-level wage distributions. In fact, in the model, there is no quality upgrading or downgrading associated to exporting and hence  $E(w|\theta)$  and  $Var(w|\theta)$  do not change in response to trade liberalization. Heterogeneity in performance-pay contracts ensures that reductions in variable trade costs will still impact the overall variance of wages, purely through labor reallocations. Naturally, this mechanism will, in turn, be amplified by increases in the firm-level variances driven by quality upgrading.<sup>6</sup>

There are a number of studies in which within-firm wage dispersion is driven by workforce composition, such as Bustos (2011), Harrigan and Reshef (2011), Monte (2011), Burstein and Vogel (2012) and Caliendo and Rossi-Hansberg (2012).<sup>7</sup> In these models, workers are heterogeneous due to differences in ability or human capital, thus they can explain variation in skill premia, as opposed to wage dispersion between identical workers.<sup>8</sup> In addition, wages are determined in competitive labor markets and thus do not contain either firm- or match-specific components.

Several key features of the model are consistent with different pieces of empirical evidence. The emphasis on performance pay is motivated by evidence that its prevalence has grown considerably in the last 30 years in the United States. Lemieux et al. (2009) report that, by the late 1990s, performance-pay jobs accounted for as much as 45% of the jobs of male workers and show that this trend can account for a significant share of the growth in wage inequality in the U.S.<sup>9</sup> Cross-firm differences in performance-pay policies are, in turn, consistent with evidence from the managerial economics literature. Bloom and Van Reenen (2007) report that large firms tend to rely on incentive pay more intensively than smaller firms. Moreover, the empirical results in Kugler and Verhoogen (2012) support the assumption that larger firms have a comparative advantage in producing high-quality goods. Finally, evidence that trade liberalization induces market share reallocations towards high productivity firms is provided by Pavcnik (2002) and Trefler (2004), for Chile and Canada, respectively.

The outline of the paper is the following. The next section introduces the theoretical framework, sequentially describing the timing of events, market structure, the production

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inequality in an economy could *decrease* even in a situation in which firm-level wage variances *increase in every firm*. In principle, this could occur if trade liberalization induced labor reallocations towards firms with initially low firm-level wage variances.

<sup>6</sup>Footnote 22 discusses how quality upgrading can be introduced in the model, along the lines of Verhoogen (2008).

<sup>7</sup>In Yeaple (2005) and Sampson (2012), differences in workforce composition generate only between-firm wage inequality, since firms hire workers of a single type.

<sup>8</sup>To the extent that firms observe skills that are hidden to the econometrician, these models are also compatible with residual wage inequality. This, however, does not imply that they are readily applicable to study the effect of trade liberalization on residual wage dispersion. To do so, this class of models would have to be augmented with a theory of what is and what is not observable to the econometrician and, crucially, explain how the latter component varies across firms. Even then, a disadvantage of this approach is that its applicability would depend on the quality/detail of the specific dataset at hand. The results in this paper broadly apply to residual wage inequality, since the latter originates from idiosyncratic variation in workers' performance that is unobservable to firms and thus (presumably) to the econometrician, regardless of the dataset.

<sup>9</sup>In particular, using data from the PSID, Lemieux et al. (2009) show that the fraction of U.S. male workers on performance-pay jobs (i.e. workers earning piece rates, commissions, or bonuses) increased from about 30 percent in the late 1970s to over 40 percent in the late 1990s. They also show that wages are less equally distributed on performance-pay than non performance-pay jobs and conclude that the growth of performance-pay has contributed to about 25 percent of the increase in the variance of log wages between the late 1970s and the early 1990s.

process and the convergence of individual performance to a Brownian process, and individual preferences. Section 3 studies firms' optimal performance-pay contracts and profit maximization, embedding the moral hazard problem in a monopolistic competition model with heterogeneous firms. Section 4 analyzes the general equilibrium of the model, under free entry and trade balance conditions. Section 5 studies how trade liberalization affects the distribution of firm productivity, how labor is reallocated across firms and the implications of the theory for wage inequality between- and within-firms. The final section discusses extensions and topics for future versions of this paper. The Appendix (coming soon) contains the proofs of the main results.

## 2 Model

There are two countries, Home and Foreign. To focus squarely on within-industry residual wage dispersion, I assume that each country is populated by identical workers that consume a single differentiated good. In addition, both countries are identical in terms of market structure and technological access, although the size of their labor forces may differ. I focus on the description of the Home economy and use an asterisk to denote foreign variables.

Firms are heterogeneous in productivity and endogenously choose the quantity and quality of output, and market(s) to serve in the presence of international trade costs. The main departure from the literature is the existence of moral hazard in the production process. Firms respond by tying compensation to individual performance, generating between- and within-firm wage inequality.

### 2.1 Setup

The timing of events in the model combines elements of Melitz (2003) and Holmstrom and Milgrom (1987). A competitive fringe of risk neutral firms may potentially enter the differentiated sector. Upon incurring a sunk entry cost of  $f_e > 0$  units of the numeraire, a firm observes its productivity  $\theta$ , independently drawn from a distribution  $G_\theta(\theta)$ , with  $\theta > 0$ . Firms then decide whether to exit, produce solely for the domestic market, or produce for both the domestic and export markets. A successful entrant becomes a monopolistic producer of a single variety of good  $X$ . Production requires a fixed cost of  $f_d > 0$  units of the numeraire. In addition, exporting involves a fixed cost of  $f_x > 0$  units of the numeraire and an iceberg variable trade cost, such that  $\tau > 1$  units of a variety must be exported for one unit to arrive in the foreign market. Since all firms with the same productivity behave symmetrically in equilibrium, I index firms and varieties by  $\theta$  from now onward.

The quantity and quality of output depend on both the mass and effort of workers allocated to a sequential production process with stochastic performance. In the presence of moral hazard, each firm hires a mass of workers and designs performance-pay contracts to implement desired effort sequences. Workers can accept or reject contracts prior to starting production. In the former case, workers choose effort at each stage of the production process, having observed their personal histories of realized performance in previous tasks. At the end of the production process, contracts are executed and consumption takes place.

As in standard monopolistic competition models, the equilibrium features free entry and balanced trade. In addition, because workers are homogeneous, equilibrium in the labor market requires that every contract offered by any firm should be individually rational,

yielding the same expected utility, denoted  $\bar{u} > 0$ . The latter is endogenously determined by either a labor market clearing condition in the single-sector model or by an indifference condition if an outside sector is added to the model.<sup>10</sup>

## 2.2 Sequential Production with Stochastic Performance

Every firm has access to the same technology, which requires each worker to perform a sequence of  $T$  symmetric tasks, indexed by  $\tau = 1, \dots, T$ . Each task spans an interval of time of length  $\Delta \equiv 1/T$ . Worker  $i$  chooses a sequence  $\{\mu_{i\tau}^\Delta\}_{\tau=1}^T$  of possibly history-dependent effort levels for each task  $\tau$ , where  $\mu_{i\tau}^\Delta \geq \mu_{\min} > 0$ .<sup>11</sup> This choice generates a stochastic sequence of worker-specific performance outcomes  $\{z_{i\tau}\}_{\tau=1}^T$ , where  $z_{i\tau}$  is equal to 1 if worker  $i$  successfully completes task  $\tau$  and equal to  $-1$  in the event of a mistake, for  $\tau = 1, \dots, T$ . For a fixed  $\Delta$ , the probability of success in any task  $\tau$ , denoted  $\pi_{i\tau}^\Delta$ , is given by

$$\pi_{i\tau}^\Delta \equiv P(z_{i\tau} = 1 | z_{i1}, \dots, z_{i\tau-1}) = \frac{1}{2} + b(\mu_{i\tau}^\Delta) \frac{\Delta^{1/2}}{2},$$

where  $b(\cdot)$  is continuous and bounded. Conditional on effort, the expected performance of worker  $i$  in task  $\tau$  is  $E_\tau(z_{i\tau}) = b(\mu_{i\tau}^\Delta) \Delta^{1/2}$  and thus it is also natural to assume that  $b(\cdot)$  is increasing. Note that, conditional on effort choices,  $z_{i\tau}$  is independent of  $z_{i'\tau'}$  for any two tasks  $\tau$  and  $\tau'$  and any two workers  $i$  and  $i'$  (unless, of course,  $\tau = \tau'$  and  $i = i'$ ). The randomness of a task's outcome captures unmodeled determinants of a worker's performance such as idiosyncratic skills, match-effects and variation in the quality of inputs used in the production process.

Let  $Z_{i\tau}^\Delta$  denote the cumulative performance of worker  $i$  up to task  $\tau$ , i.e.  $Z_{i\tau}^\Delta = \Delta^{1/2} \sum_{\tau'=1}^{\tau} z_{i\tau'}$ . Equivalently,  $-Z_{i\tau}^\Delta$  is the number of mistakes in excess of successes of worker  $i$  up to task  $\tau$ , i.e. the *net* number of mistakes. To characterize the convergence of the path of cumulative performance as the duration of tasks  $\Delta$  approaches zero, I embed the discrete process  $\{Z_{i\tau}^\Delta\}_{\tau=1}^T$  in continuous time by linearly interpolating between the points  $(0, 0)$ ,  $(\Delta, Z_{i1}^\Delta)$ ,  $(2\Delta, Z_{i2}^\Delta), \dots, (1, Z_{iT}^\Delta)$ . In other words, I construct a function  $Z_i^\Delta(t)$  satisfying

$$Z_i^\Delta(t) = \left(1 - \frac{t}{\Delta} + \left\lfloor \frac{t}{\Delta} \right\rfloor\right) Z_{i\lfloor t/\Delta \rfloor}^\Delta + \left(\frac{t}{\Delta} - \left\lfloor \frac{t}{\Delta} \right\rfloor\right) Z_{i\lfloor (t+1)/\Delta \rfloor}^\Delta,$$

for  $t \in [0, 1]$  and the initial condition  $Z_{i0}^\Delta = 0$ , where  $\lfloor x \rfloor$  is the integer part of  $x \in \mathbb{R}$ . Thus  $Z_i^\Delta(t)$  is a random element of the space of continuous real functions,  $C[0, 1]$ . Analogously, let  $\mu_i^\Delta(t)$  denote the linear interpolation of  $\{\mu_{i\tau}^\Delta\}_{\tau=1}^T$ . Endowing  $C[0, 1]$  with the uniform metric, I obtain the following result.

**Lemma 1** *In the sequential production process over the unit time interval with task duration  $\Delta = 1/T$ , consider a sequence of effort choices  $\{\mu_{i\tau}^\Delta\}_{\tau=1}^T$  and the corresponding process of*

<sup>10</sup>Contracts yielding a lower expected utility than this outside option would fail to attract workers. Exceeding  $\bar{u}$  would not be profit-maximizing. Positive wages and cost of effort imply that, in equilibrium,  $\bar{u} > 0$  (see Corollary 3).

<sup>11</sup> $\mu_{\min}$  is the lowest feasible effort level for any worker. For the purpose of describing technology, it suffices to take the sequence of effort levels as given. Optimal effort choices are analyzed in Section 3.1 in the continuous-time limit of this production process.

cumulative performance  $\{Z_{i\tau}^\Delta\}_{\tau=1}^T$  for worker  $i$ . Suppose that  $\mu_i^\Delta(t) \rightarrow \mu_i(t)$  a.s. as  $\Delta \rightarrow 0$ , for  $t \in [0, 1]$ . If  $b(\cdot)$  is continuous and bounded then, as  $\Delta \rightarrow 0$ ,  $Z_i^\Delta(t)$  converges in distribution to a stochastic process  $Z_i(t)$ , such that:

$$Z_i(t) = \int_0^t b(\mu_i(t')) dt' + \varepsilon_i(t),$$

for  $t \in [0, 1]$ , where  $\varepsilon_i(t)$  is a standard Brownian motion.

The crux of this result is showing that deviations of cumulative performance  $\{Z_{i\tau}^\Delta\}_{\tau=1}^T$  from its expected value follow a martingale process. Convergence to a standard Brownian motion is then a direct application of standard results from functional limit theory for martingales (Hall and Heyde (1980)). The assumptions of Bernoulli task outcomes and unidimensional effort are not essential.<sup>12</sup>

Lemma 1 states that, when task duration approaches zero, the path of cumulative performance of worker  $i$  converges to a Brownian process whose drift is determined by the worker's effort choices. The remainder of the paper concentrates on this limiting case, due to computational ease of optimal contracts. The value of Lemma 1 is to provide an economically relevant interpretation of this continuous-time environment as the limit of the sequential production model introduced above.

This technology determines the quantity and quality of output. Physical output of each variety ( $y$ ) depends on firm productivity ( $\theta$ ) and the mass of workers ( $h$ ) allocated to this process:

$$y(\theta) = \theta h. \quad (1)$$

On the other hand, product quality ( $q$ ) depends on the performance of these workers. In particular, I assume that quality is described by a function  $q(\theta, N)$  that depends both on firm productivity and on the average net number of mistakes in the production process, denoted  $N$ . In order to derive analytical solutions I assume the following CES specification for product quality:

$$q(\theta, N) = \left[ \theta^\lambda + \left( \frac{1}{N} \right)^\lambda \right]^{\frac{1}{\lambda}}. \quad (2)$$

Recall that  $-Z_{i\tau}^\Delta$  is the net number of mistakes of worker  $i$  in the production process with task length  $\Delta$ . Therefore, in the continuous-time limit,  $N = -h^{-1} \int_0^h Z_i(1) di$ . In turn, the parameter  $\lambda$  in equation (2) controls the degree of complementarity between productivity and average performance. I assume  $\lambda < 0$ , which implies that quality is strictly log-submodular in  $\theta$  and  $N$ . Intuitively, this property means that fewer mistakes lead to higher marginal quality in high productivity firms.<sup>13</sup> In other words, under  $\lambda < 0$ , high productivity firms have a comparative advantage in producing higher-quality output.

Two additional remarks are in order. First, conditional on effort,  $N$  is (almost surely) a constant. Because the  $Z_i$ 's are independent across workers, the strong law of large numbers (SLLN) implies that the firm fully diversifies the impact of idiosyncratic individual performance,  $\varepsilon_i(1)$ , on product quality.<sup>14</sup> An implication is that equilibrium firm-level variables

<sup>12</sup>See Hellwig and Schmidt (2002) for a generalization of these assumptions in the context of a principal-agent model.

<sup>13</sup>Formally, because  $q(\theta, N)$  is differentiable, strict log-submodularity is equivalent to a negative cross-partial derivative of  $\log q(\theta, N)$ . Kugler and Verhoogen (2012) provide evidence consistent with this assumption.

<sup>14</sup>This property also relies on the assumption that firms hire a continuum of workers.

such as quality, employment, output and prices are deterministic with probability one. Second, for quality to be well-defined under log-submodularity in equation (2), it is necessary that  $N > 0$ . In light of the previous remark, suffices to assume:

$$b(\mu_i) = -\mu_i^{-b}/b, \quad (3)$$

where  $b > 0$ . Note that (3) is increasing and concave in effort, and satisfies the requirements of Lemma 1 since  $b(\cdot) \subset [b(\mu_{\min}), 0]$ .

To illustrate the properties of (2) under assumptions (3) and  $\lambda < 0$ , consider the special case in which every worker exerts a constant effort  $\mu = \mu_i(t)$ , for  $t \in [0, 1]$  and  $i \in [0, h]$ . From Lemma 1,  $N = -b(\mu) > 0$  and thus the firm almost surely achieves a positive quality  $q(\theta, \mu^{-b}/b)$ , which is increasing, concave and log-supermodular in productivity and effort.

## 2.3 Demand

Home is populated by a continuum of identical risk-neutral workers of mass  $L$ . The preferences of any worker  $i$  depend on the consumption of a differentiated product  $X_i$  and on the sequence of effort  $\mu_i \equiv \{\mu_i(t); t \in [0, 1]\}$  exerted during the production process:

$$U(X_i, \mu_i) = \frac{X_i}{\exp\left(\int_0^1 \ln k(\mu_i(t)) dt\right)}, \quad (4)$$

where  $k(x) \equiv x^\delta$ ,  $\delta > 1$  is the instantaneous cost-of-effort function.  $X_i$  indexes the consumption of a continuum of horizontally and vertically differentiated varieties, defined as

$$X_i \equiv \left[ \int_{j \in J} (q(j)x_i(j))^{\frac{\nu-1}{\nu}} dj \right]^{\frac{\nu}{\nu-1}},$$

where  $j$  indexes varieties,  $J$  is the set of varieties available in the market,  $x_i(j)$  and  $q(j)$  denote the consumption and quality of variety  $j$ , respectively, and  $\nu > 1$  is the elasticity of substitution across varieties. The quality-adjusted price index dual to  $X_i$  is denoted by  $P$ .<sup>15</sup>

For a worker earning a wage  $w_i$ , the familiar two-stage budgeting solution yields  $PX_i = w_i$  and individual demand  $x_i(j) = w_i q(j)^{\nu-1} p(j)^{-\nu} / P^{1-\nu}$ . Other than for final consumption, the differentiated product  $X$  is also demanded by firms as they set up production and export activities (fixed costs). These activities are assumed to use the output of each variety in the same way as is demanded by final consumers. Denoting total expenditure on the differentiated good by  $E$ , the aggregate demand for variety  $j$ , denoted  $x(j)$ , can then be written as

$$x(j) = q(j)^{\nu-1} \frac{p(j)^{-\nu}}{P^{1-\nu}} E.$$

The revenue of producer  $j$ , denoted  $r(j)$ , is equal to the aggregate expenditure on variety  $j$ . Therefore,

$$r(j) = p(j)x(j) = Aq(j)^\rho x(j)^\rho, \quad (5)$$

where  $A \equiv P^{1-\nu} E^{\frac{1}{\nu}}$ ,  $\rho \equiv (\nu - 1)/\nu$  and  $0 < \rho < 1$ .

For expositional purposes, it is convenient to simplify notation by setting the aggregate consumption index in Home to be the numeraire ( $P = 1$ ) in order to express utility solely as a function of income and effort choices, i.e.  $U(w_i, \mu_i) = U(X_i, \mu_i)$ .

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<sup>15</sup>Specifically,  $P \equiv \left[ \int_{j \in J} \left( \frac{p(j)}{q(j)} \right)^{1-\nu} dj \right]^{\frac{1}{1-\nu}}$ .



### 3 The Firm's Problem

This section studies the problem of firm  $\theta$  in two steps. The first step takes firm employment and output quality as given, while seeking to characterize the optimal contracts that the firm designs in order to attain the targeted quality at minimum cost. The second step sets up the profit maximization problem, in which the firm determines employment, quality and whether to export given demand in the domestic and foreign markets.

#### 3.1 Optimal Performance-pay Contracts

The cost of a given output quality  $q_0 = q(\theta, N_0)$  is determined by the cost of providing incentives such that the average net number of mistakes in the production process is  $N_0$ . A *performance-pay contract* for any worker  $i$  is an arbitrary function  $w_i = w_i(Z_i^1)$ , stipulating the wage of worker  $i$  based on the realized path of individual performance  $Z_i^1$ , i.e.  $Z_i^1 \equiv \{Z_i(t); t \in [0, 1]\}$ .<sup>16</sup> As anticipated, workers accept or reject contracts prior to starting production at time  $t = 0$ , select effort in each task  $t$  having observed  $\{Z_i(t'); t' \in [0, t]\}$  and receive wages upon completion of all tasks at time  $t = 1$ . In order to attain  $q_0$ , a firm employing  $h$  workers designs a set of contracts and effort sequences  $\{w_i, \mu_i; i \in [0, h]\}$  that minimize expected total compensation subject to: (i) inducing fewer than  $N_0$  mistakes, (ii) the stochastic processes of individual performance, (iii) incentive compatibility constraints and (iv) participation constraints:

$$\begin{aligned} \min_{\{w_i, \mu_i; i \in [0, h]\}} & \int_0^h E[w_i(Z_i^1)] di \\ \text{s.t.} \quad & (i) \ N_0 \geq -h^{-1} \int_0^h Z_i(1) di \\ & (ii) \ Z_i(t) = \int_0^t b(\mu_i(t')) dt' + \varepsilon_i(t), \text{ for } i \in [0, h] \\ & (iii) \ \mu_i \in \arg \max_{\hat{\mu}_i} E[U(w_i, \hat{\mu}_i)], \text{ for } i \in [0, h] \\ & (iv) \ E[U(w_i, \mu_i)] \geq \bar{u}, \text{ for } i \in [0, h] \end{aligned} \tag{6}$$

The following proposition characterizes the solution to this problem for the case in which  $N_0$  is low enough to motivate the firm to implement effort levels greater than  $\mu_{\min}$ .<sup>17</sup> The assumed functional forms for  $k(\cdot)$  and  $b(\cdot)$  are not necessary to establish parts (a) and (b) although they are used in (c) and (d) and to guarantee the existence of a solution.

**Proposition 2 (Cost-minimizing contracts)** *Suppose that  $N_0 < -b(\mu_{\min})$ . Then there exists a global minimizer in problem (6), denoted  $\{w_i^*, \mu_i^*; i \in [0, h]\}$ , such that:*

- (a) *Effort:*  $\mu_i^*(t) = \mu \equiv (bN_0)^{-1/b}$ , for all  $t \in [0, 1]$  and  $i \in [0, h]$
- (b) *Contracts:*  $\log(w_i^*) = \alpha + \beta Z_i(1)$ ,  $\alpha, \beta \in \mathbb{R}$ , for all  $i \in [0, h]$ , where

<sup>16</sup> Although potentially relevant to study within-firm wage variation, this paper does not deal with any form of group-based compensation schemes. The emphasis on individual incentives can be motivated empirically. Lazear and Shaw (2007) report that the share of large US firms in which more than 20 percent of their workforce is subject to some form of individual incentives, like a performance bonus, has grown from 38 percent in 1987 to 67 percent in 1999. The comparable share of firms using any form of 'gain-sharing' or group-based incentives was 7 percent in 1987 and 24 percent in 1999.

<sup>17</sup> The opposite case is uninteresting, since the firm can simply satisfy (i) by offering a constant wage that ensures participation, trivializing the moral hazard problem.

(c) *Piece rate*:  $\beta = \delta\mu^b$

(d) *Fixed compensation*:  $\alpha = \ln(\bar{u}\mu^\delta) + \delta/b - \mu^{2b}\delta^2/2$

**Proof.** Appendix. ■

The solution to problem (6) has several important features. First, the optimal contract for worker  $i$  is a log-linear function of  $i$ 's cumulative performance at time  $t = 1$  and implements a constant effort in each task of the production process. The model thus inherits the simple structure of contracts in Holmstrom and Milgrom (1987). As in that paper, tasks (time periods) are technologically independent and consumption takes place after production, eliminating any scope for improved statistical inference and for consumption smoothing throughout the production process. A conceptually significant departure relative to Holmstrom and Milgrom (1987) is the specification of the objective functions of firms and workers. In particular, the firm's cost minimization problem (6) arises naturally in the context of the broader profit maximization problem studied in the next section. Moreover, the utility function (4) plays a key role in ensuring that wages are positive for all realizations of individual performance  $Z_i^1$ .<sup>18</sup> Because wages fuel the demand side of the model, this is an essential property for embedding the moral hazard problem in general equilibrium.

Second, the firm's cost minimizing strategy is to offer identical contracts to its  $h$  employees. In principle, the firm could offer different contracts to different workers. However, this is not cost-effective. The symmetry of optimal effort levels -part (a)- follows from the convexity of the effort cost function  $k(\cdot)$ . Intuitively, convexity implies that the cost of compensating a worker for a higher-than-average effort exceeds the cost reduction of inducing another worker to exert a lower-than-average effort level.

Third, under the assumed functional forms for  $k(\cdot)$  and  $b(\cdot)$ , parts (c) and (d) of Proposition (2) provide an analytical expression for the optimal contract. Incentive compatibility requires the intensity of performance pay (proxied by  $\beta$ ) to increase in effort, which is consistent with numerous empirical studies documenting performance gains from performance pay.<sup>19</sup> The firm adjusts the fixed component of compensation  $\alpha$  to ensure that the participation constraint is satisfied with equality.

Importantly, Proposition (2) has implications for the distribution of wages within the firm, as summarized in the following corollary.

**Corollary 3 (Conditional firm-level wages)** *Suppose that the firm implements a constant effort  $\mu$  such that  $\mu_i(t) = \mu$  for all  $t \in [0, 1]$  and  $i \in [0, h]$ . Then:*

(a) *The (random) wage of worker  $i$  is*

$$w_i^* = \bar{u}\mu^\delta e^{\delta\mu^b[\varepsilon_i(1) - \delta\mu^b/2]},$$

*for all  $i \in [0, h]$ .*

(b) *The conditional mean and variance of firm-level wages are, respectively,*

$$\begin{aligned} E[w_i^*|\mu] &= \bar{u}\mu^\delta, \\ \text{Var}[w_i^*|\mu] &= \left(\bar{u}\mu^\delta e^{\delta\mu^b}\right)^2. \end{aligned}$$

<sup>18</sup>In Holmstrom and Milgrom (1987), firms and workers have negative exponential (CARA) objective functions defined over cumulative performance at  $t = 1$  and compensation, respectively. Moreover, effort costs are measured in monetary units. The optimal contract is a linear function of a normally distributed random variable and thus the support of the wage distribution is  $\mathbb{R}$ .

<sup>19</sup>See, for example, Parent (1999), Lazear (2000) and references cited in Lazear and Shaw (2007).

(c) The average wage that implements  $\mu$ , denoted  $\omega(\mu) \equiv h^{-1} \int_0^h w_i^* di$ , converges almost surely to  $E[w_i^*|\mu]$ .

**Proof.** Appendix. ■

Parameter restrictions  $\delta > 1$  and  $b > 0$  guarantee that the conditional mean and variance of firm-level wages increase in effort  $\mu$ . It is straightforward to check that the standard deviation of log wages,  $\delta\mu^b$ , also increases in effort. The next section endogeneizes the choice of effort. Together with Corollary (3), they provide a mapping between wages and firm productivity that can be used to analyze wage variation between and within firms and the implications of international trade for wage inequality.

Part (c) of Corollary (3) follows from the SLLN. As with output quality, the firm effectively diversifies the impact of idiosyncratic performance on the average wage paid to its employees. In the next section, we take the approximation to be exact,  $\omega(\mu) = \bar{u}\mu^\delta$ , and treat average wages as deterministic in the firm's profit maximization problem.  $\delta > 1$  guarantees that  $\omega(\mu)$  is increasing and convex in  $\mu$ .

### 3.2 Profit Maximization

The formulation of the profit maximization problem can be simplified by using three properties of the model. First, the linearity of the production function (1) implies that, given a choice of product quality  $q_0$ , the marginal cost of physical output  $y$  is constant and equal to  $\omega(\mu_0)/\theta$  for a firm with productivity  $\theta$ , where  $\mu_0$  is implicitly defined by  $q_0 = q(\theta, b(\mu_0))$ . If firms can price- and quality-discriminate between domestic and foreign buyers, then the profit maximization problem of firm  $\theta$  is additively separable in the profits of domestic and foreign markets.<sup>20</sup> As usual, conditional on the entry decision, the CES demand structure ensures that the firm finds it profitable to serve domestic consumers. In turn, the firm exports if and only if the gross profit from foreign sales exceeds the fixed cost of exporting.

Second, the optimal choices of product quality in the domestic and foreign markets, denoted  $q_d(\theta)$  and  $q_x(\theta)$ , respectively, are identical, i.e.  $q_d(\theta) = q_x(\theta)$ .<sup>21</sup> There is no product quality upgrading or downgrading associated to exporting in this model.<sup>22</sup>

<sup>20</sup> Allowing for quality discrimination, the firm can in principle choose to supply different product qualities in the home and foreign markets. If so, workers allocated to different 'production lines' will earn different expected wages. Note that, in equilibrium, workers are indifferent between employment in either production line because every contract generates the same expected utility.

<sup>21</sup> To see this, it is convenient to introduce the 'quality cost function'  $c(\theta, q) \equiv \omega(\mu(\theta, q))$ , where  $\mu(\theta, q)$  is implicitly defined by  $q = q(\theta, b(\mu))$ . Intuitively, the firm can increase revenue in a given market by either expanding output or quality. Optimality requires that choices of output and quality in each market satisfy the equality of relative marginal revenue and relative marginal cost. From (5), the marginal revenue of output divided by the marginal revenue of quality is  $q_m/y_m$  in market  $m = \{H, F\}$ . In turn, the marginal cost of output divided by the marginal cost of quality is given by  $c(\theta, q_m) / (c_q(\theta, q_m) y_m)$ . Note that variable trade costs increase the marginal costs of output and quality proportionally in market  $F$ , thus they do not distort the relative marginal cost of output across markets. Therefore,  $c_q(\theta, q_m) = c(\theta, q_m) / q_m$ , for  $m = \{H, F\}$ . Since  $c(\theta, q)$  is strictly convex in  $q$ , for  $q \geq 0$ ,  $q_m$  is unique and thus  $q_d(\theta) = q_x(\theta)$ . Geometrically, the marginal and average costs of quality intersect at  $q_m$ . Therefore,  $q_m$  minimizes the average cost of quality in firm  $\theta$ .

<sup>22</sup> Quality upgrading induced by exporting can be easily introduced to the model by assuming that foreign consumers trade off quality and quantity differently than domestic consumers (see Verhoogen (2008)). For example, letting  $X_i^* = \left[ \int_{j \in J^*} (q^*(j)^\chi x_i^*(j))^{\frac{\nu-1}{\nu}} dj \right]^{\frac{\nu}{\nu-1}}$ , and  $\chi > 1$ . Alternatively, if  $\chi < 1$  would lead exporters to downgrade quality. This suggests that export destinations matter, as they may amplify or dampen the link between trade and inequality advanced in this paper. This extension is left for future versions of the paper.

Third, for an exporting firm, the optimal allocation of total output, denoted  $y(\theta)$ , between the domestic and foreign markets, denoted  $y_d(\theta)$  and  $y_x(\theta)$ , respectively, satisfies the standard condition of equal marginal revenues in the two markets. From (5), this requires  $[y_x(\theta)/y_d(\theta)]^{1-\rho} = \tau^{-\rho}(A^*/A)$ , which implies that firm revenue can be written as a function of total output and product quality:

$$r(\theta) \equiv r_d(\theta) + r_x(\theta) = Aq(\theta)^\rho y(\theta)^\rho \Upsilon(\theta)^{1-\rho}. \quad (7)$$

The variable  $\Upsilon(\theta)$  is a measure of foreign market access of firm  $\theta$  that decreases in the variable trade cost  $\tau$ . As in Helpman et al. (2010),  $\Upsilon(\theta) \equiv 1 + I_x(\theta) [\tau^{-\rho}(A^*/A)]^{1/(1-\rho)}$ , where the indicator  $I_x(\theta)$  equals 1 if firm  $\theta$  exports and 0 otherwise. Implicit in (7), domestic producers always serve the domestic market. This is guaranteed by the CES demand structure.

The firm's problem can thus be formulated as choices of total output  $y$ , team effort  $\mu$  and export decision  $I_x$  that solve

$$\Pi(\theta) \equiv \max_{\substack{y \geq 0, \mu \geq \mu_{\min}, \\ I_x \in [0,1]}} \left\{ Aq(\theta, b(\mu))^{\frac{\rho}{\lambda}} y^\rho \left[ 1 + I_x \tau^{-\frac{\rho}{1-\rho}} \left( \frac{A^*}{A} \right)^{\frac{1}{1-\rho}} \right]^{1-\rho} - \frac{\omega(\mu)}{\theta} y - f_d - I_x f_x \right\}.$$

The existence of a fixed production cost implies that there is a zero-profit cutoff  $\theta_d$  such that firms drawing a productivity  $\theta < \theta_d$  exit without producing. Similarly, the existence of a fixed exporting cost implies that there is an exporting cutoff  $\theta_x$  such that firms drawing a productivity  $\theta < \theta_x$  do not find it profitable to serve the export market. For consistency with a large empirical literature that finds evidence of self-selection of the more efficient firms into the export market, I focus on values of trade costs for which  $\theta_{\min} < \theta_d < \theta_x$ .<sup>23</sup> This implies that the firm market access variable can be written as

$$\Upsilon(\theta) \begin{cases} \Upsilon_x & \text{if } \theta \geq \theta_x, \\ 1 & \text{if } \theta < \theta_x, \end{cases}$$

where  $\Upsilon_x \equiv 1 + \tau^{-\frac{\rho}{1-\rho}} (A^*/A)^{\frac{1}{1-\rho}} > 1$ .

For a given choice of quality, the first-order condition for total output requires that the marginal revenue of output be equal to the constant marginal production cost. With CES demand, this implies that the variable cost equals a constant fraction of firm revenue, as in equation (8) below. Similarly, the optimal choice of team effort weighs the marginal revenue generated by improved quality against the marginal increase in compensation. In an interior solution,  $\mu(\theta) > \mu_{\min}$ , dividing the first-order conditions for output and quality yields an equality of relative marginal revenues and relative marginal costs. As equation (9) shows, this implies that the optimal choice of team effort is attained when the percentage increase in quality induced by a marginal increase in effort is equal to the percentage increase in compensation. As a result,

$$\rho r(\theta) = \frac{\omega(\mu(\theta))}{\theta} y(\theta), \quad (8)$$

$$\frac{q_c(\theta, b(\mu(\theta))) b'(\mu(\theta))}{q(\theta, b(\mu(\theta)))} = \frac{\omega'(\mu(\theta))}{\omega(\mu(\theta))}. \quad (9)$$

---

<sup>23</sup>In general, the exact condition that yields  $\theta_d < \theta_x$  depends on both trade costs and relative demand shifters (see equation (21)). In the case of symmetric countries, the condition is  $(f_x/f_d)\tau^{\frac{\rho}{1-\rho}} > 1$ , as in Melitz (2003).

Note that equation (9) depends on a single unknown,  $\mu(\theta)$ . The assumed functional forms for product quality and team performance allow a closed-form solution for optimal team effort. Because product quality is log-supermodular in productivity and team performance, only firms with productivity above a cutoff  $\theta_\mu$  find it profitable to induce a team effort higher than the minimum  $\mu_{\min}$ . I assume that  $\theta_m$  is sufficiently high, so that  $\theta_d \geq \theta_\mu$  in equilibrium.<sup>24</sup> Therefore:

$$\mu(\theta) = \mu_{\min} \kappa_\mu \theta^{1/b}, \quad (10)$$

where  $\theta_\mu = (\kappa_\mu)^{-b}$ . Constants  $\kappa_\mu$ ,  $\kappa_q$ ,  $\kappa_\omega$ ,  $\kappa_r$ ,  $\kappa_y$  (introduced below) are positive and defined in the Appendix.

Team effort determines optimal product quality, denoted (with a slight abuse of notation)  $q(\theta) \equiv q(\theta, b(\mu(\theta)))$ , and the average wage  $\omega(\theta)$  according to Corollary (3):

$$\begin{aligned} q(\theta) &= \kappa_q \theta, \\ \omega(\theta) &= \kappa_\omega \bar{u} \theta^{\delta/b}. \end{aligned}$$

Product quality is proportional to productivity. High productivity firms thus pay higher average wages to compensate their employees for the disutility of effort associated to the production of quality.

From the first-order condition for output (8), the expression for firm revenue (7) and the solution for the average wage  $\omega(\theta)$ , I solve for revenue and total output as functions of the demand shifters  $A$  and the reservation utility  $\bar{u}$ . Total employment,  $h(\theta)$ , follows from the production function (1). Therefore:

$$r(\theta) = \kappa_r \Upsilon(\theta) (A \bar{u}^{-\rho})^{1/(1-\rho)} \theta^\Gamma, \quad (11)$$

$$y(\theta) = \kappa_y \Upsilon(\theta) (A \bar{u}^{-1})^{1/(1-\rho)} \theta^{\Gamma-(1-\rho)}, \quad (12)$$

$$h(\theta) = \kappa_y \Upsilon(\theta) (A \bar{u}^{-1})^{1/(1-\rho)} \theta^{\Gamma-2}, \quad (13)$$

where  $\Gamma \equiv (2 - \delta/b)/(1 - \rho)$ . The condition  $\delta/b < 2\rho$  ensures that  $\Gamma > 2$ , so that revenue, output and employment increase in productivity. Note that, as usual in models with a fixed exporting cost and selection into export markets, firm revenue, output and employment increase discontinuously at the exporting cutoff as the marginal exporter incurs  $f_x$ . This is not the case for quality, team effort and average wage, since there is no motif for quality upgrading (or downgrading) associated to exporting in this model (see footnote in page 11).

Finally, the first-order condition (8) also implies that firm profits can be written as a function of revenue and the fixed costs,

$$\Pi(\theta) = (1 - \rho)r(\theta) - f_d - I_x(\theta)f_x. \quad (14)$$

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<sup>24</sup>In the case  $\theta_d < \theta_\mu$ , product quality and thus firm revenue cease to be power functions of firm productivity when  $\theta_d \leq \theta < \theta_\mu$ , which precludes a closed-form analysis of the general equilibrium. However, the model has a similar structure to the case developed in the main text. For example, quality, output and revenue increase in firm productivity. The main difference is that optimal contracts do not vary across firms with productivity  $\theta \in [\theta_d, \theta_\mu]$ . Therefore, firms in this range pay the same average wage (i.e. no between-firm inequality) and exhibit the same degree of within-firm wage dispersion.

## 4 Equilibrium

The general equilibrium of the model shares a common structure with the extensive literature that builds on Melitz (2003). This section explains how to compute the remaining endogenous variables in the model. Further details can be found in the Appendix.

The zero-profit cutoff  $\theta_d$  is the productivity level that leaves firms indifferent between exiting and producing for the domestic market. In turn, the exporting cutoff  $\theta_x$  leaves firms indifferent between exporting and producing exclusively for the domestic market. From the expressions for revenue (11) and profits (14), these two conditions require

$$\kappa_r(1 - \rho) (A\bar{u}^{-\rho})^{1/(1-\rho)} \theta_d^\Gamma = f_d \quad (15)$$

and

$$\kappa_r(1 - \rho) (\Upsilon_x - 1) (A\bar{u}^{-\rho})^{1/(1-\rho)} \theta_x^\Gamma = f_x, \quad (16)$$

respectively.

Free entry implies that the expected profits of successful entrants should equal the sunk entry cost; that is,  $\int_{\theta_d}^{\infty} \Pi(\theta) dG_\theta(\theta) = f_e$ . Using the Pareto productivity assumption, the expression linking revenue to firm productivity (11) and the conditions characterizing the productivity cutoffs (15) and (16), the free entry condition can be written as

$$\frac{f_d}{(z/\Gamma - 1)} \left( \frac{\theta_{\min}}{\theta_d} \right)^z \left[ 1 + \left( \frac{f_x}{f_d} \right) \left( \frac{\theta_d}{\theta_x} \right)^z \right] = f_e. \quad (17)$$

For future reference, note that the ratio of productivity cutoffs  $\theta_x/\theta_d$  is inversely related to the domestic cutoff  $\theta_d$ . Since equation (17) does not depend directly on the transport cost, it follows that changes in  $\tau$  induce  $\theta_d$  and  $\theta_x/\theta_d$  to change in opposite directions.

Equations (15), (16), (17) and their Foreign counterparts can be used to solve for the productivity cutoffs and demand shifters in Home and Foreign ( $\theta_d, \theta_x, \theta_d^*, \theta_x^*, A, A^*$ ) as functions of the reservation utilities  $\bar{u}$  and  $\bar{u}^*$ . The demand shifters, in turn, determine firm market access variables  $\Upsilon(\theta)$  and  $\Upsilon^*(\theta)$ .

The mass of firms and expenditure in Home and Foreign are determined by imposing market clearing and trade balance. First, note that total expenditure is proportional to the mass of firms in each country. This follows from the market clearing condition (5), which implies that aggregate expenditure on domestic varieties equals total revenues of domestic firms. In Home, this is written as

$$E = M \int_{\theta_d}^{\infty} r(\theta) dG_\theta(\theta), \quad (18)$$

where  $M$  denotes the mass of firms in Home. A similar equation applies in Foreign, linking  $E^*$  and  $M^*$ . Trade balance requires the equality of export sales of domestic and foreign firms. This is formally stated as

$$M \frac{\Upsilon_x - 1}{\Upsilon_x} \int_{\theta_x}^{\infty} r(\theta) dG_\theta(\theta) = M^* \frac{\Upsilon_x^* - 1}{\Upsilon_x^*} \int_{\theta_x^*}^{\infty} r^*(\theta) dG_\theta(\theta), \quad (19)$$

after using  $r_x(\theta) = r(\theta) (\Upsilon_x - 1) / \Upsilon_x$  for  $\theta \geq \theta_x$  and an analogous expression for export sales of foreign firms. Next, the definition of the demand shifter  $A$  and the choice of numeraire

( $P = 1$ ) determine the expenditure in Home,  $E = A^\nu$ . Equation (18), its counterpart in Foreign and the trade balance condition (19) can then be used to solve for  $M$ ,  $M^*$  and  $E^*$ . The price index in Foreign follows from  $A^* = (P^*)^{1-\nu} (E^*)^{\frac{1}{\nu}}$ .

Finally, the reservation utilities  $\bar{u}$  and  $\bar{u}^*$  are pinned-down by imposing labor market clearing in each country. In Home, this requires equating labor supply,  $L$ , and labor demand,  $M \int_{\theta_d}^{\infty} h(\theta) dG_\theta(\theta)$ . Substituting for firm employment using expression (13) and solving for  $\bar{u}$  yields

$$\bar{u} = A \left[ \frac{M}{L} \kappa_y \int_{\theta_d}^{\infty} \Upsilon(\theta) \theta^{\Gamma-1-s} dG_\theta(\theta) \right]^{1-\rho}. \quad (20)$$

In the same way, labor market clearing in Foreign yields  $\bar{u}^*$ .

## 5 Trade Liberalization, Selection and Inequality

This section begins by analyzing the impact of trade liberalization, modeled as a fall in the transport cost  $\tau$ , on firm selection and labor reallocations across firms. This sets the stage for the analysis of wage inequality. Throughout, this section assumes that productivity follows a Pareto distribution  $G_\theta(\theta) = 1 - (\theta_{\min}/\theta)^z$  for  $\theta \geq \theta_{\min}$  and  $z > 1$ . The Pareto distribution is not only tractable, but together with other assumptions in the model, implies a Pareto firm-size distribution which typically provides a reasonable approximation to observed data (Axtell (2001)).

### 5.1 Firm Selection

There is substantial empirical evidence that episodes of trade liberalization shape the equilibrium distribution of firm productivity by inducing low productivity firms to exit and some firms to start exporting.<sup>25</sup> In the model, these findings are consistent with equilibria in which lower variable trade costs result in higher domestic cutoffs  $\theta_d$ . Through the free-entry condition (17), a higher  $\theta_d$  implies a lower ratio of productivity cutoffs  $\theta_x/\theta_d$ , increasing the fraction of exporting firms.<sup>26</sup> I will refer to the class of equilibria satisfying this property as equilibria in which trade liberalization leads to firm selection.

**Definition 4** *An equilibrium exhibits **firm selection in response to trade liberalization** if a marginal fall in the transport cost increases the domestic cutoff  $\theta_d$ .*

Besides its empirical relevance, this class of equilibria is of interest because, as I show below, the impact of trade liberalization on wage inequality can be sharply characterized in this setting. Under what conditions does trade liberalization lead to firm selection? As in other models in the trade literature, it is difficult to derive a necessary and sufficient condition under which this property holds. However, firm selection can be ensured in special cases that enhance the tractability of the equilibrium.

To see this, divide equation (16) by (15) to obtain

$$(\Upsilon_x - 1) \left( \frac{\theta_x}{\theta_d} \right)^\Gamma = \frac{f_x}{f_d}, \quad (21)$$

<sup>25</sup>See, for example, Pavcnik (2002), Treffer (2004) and Bustos (2011).

<sup>26</sup>With Pareto-distributed productivity, it is straightforward to verify that the fraction of exporting firms, given by  $[1 - G_\theta(\theta_x)] / [1 - G_\theta(\theta_d)]$ , is a strictly increasing function of the ratio of productivity cutoffs  $\theta_x/\theta_d$ .

and recall the definition of the market access measure,  $\Upsilon_x \equiv 1 + \tau^{-\frac{\rho}{1-\rho}} (A^*/A)^{\frac{1}{1-\rho}}$ . Since free entry implies that  $\theta_x/\theta_d$  and  $\theta_d$  are inversely related, expression (21) implies that the equilibrium exhibits firm selection if and only if a fall in variable trade costs translates into higher market access  $\Upsilon_x$ . This is evidently the case when countries are symmetric and thus  $A^*/A = 1$ , as stated in part (a) of Proposition (5) below. More generally, it is necessary and sufficient that the direct effect of  $\tau$  on  $\Upsilon_x$  (i.e. holding relative demand  $A^*/A$  constant) is not overturned by the equilibrium response of  $A^*/A$ . Part (b) of Proposition (5) gives a sufficient condition limiting the elasticity of relative demand  $A^*/A$  with respect to  $\tau$ .

**Proposition 5** *An equilibrium exhibits firm selection in response to trade liberalization if one of the following conditions hold:*

- (a) *Countries are of equal size, i.e.  $L = L^*$ .*
- (b) *The reservation utilities  $\bar{u}$  and  $\bar{u}^*$  are exogenously determined and satisfy*

$$T(f_x/f_d, \tau)^{-1} < \frac{\bar{u}^*}{\bar{u}} < T(f_x/f_d, \tau),$$

where  $T(f_x/f_d, \tau) \equiv \frac{(f_x/f_d)^{1-2(1-\rho)z/\Gamma} + \tau^{2\rho z/\Gamma}}{2(f_x/f_d)^{1-(1-\rho)z/\Gamma} \tau^{\rho z/\Gamma}}$ .

**Proof.** Appendix. ■

To the best of my knowledge, variants of the Melitz (2003) model which analytically characterize the effect of variable trade costs on the domestic cutoff typically rely on at least one of these two conditions.<sup>27</sup> These ensure that the equilibrium displays a block structure that allows the productivity cutoffs and demand shifters to be determined solely by equations (15), (16) and (17) in each country.

Condition (a) states that trade liberalization always induces firm selection in the case of symmetric countries. Asymmetry is allowed under condition (b), which is usually introduced in the literature by assuming the existence of a homogeneous good that is produced in every country under perfect competition and constant returns to labor. In this case, expected wages (and thus the reservation utility) are proportional to labor productivity in the homogeneous sector.

Bounds on the admissible degree of asymmetry, however, are defined by  $T(f_x/f_d, \tau)$ . The appendix shows (i)  $T(f_x/f_d, \tau) > 1$  for finite values of  $\tau$  and  $f_x \geq f_d$  and (ii)  $T$  is increasing in both arguments.<sup>28</sup> These bounds are necessary because of the existence of a home market effect in the model.<sup>29</sup> Intuitively, when countries are asymmetric, a fall in transport costs induces the differentiated product industry to concentrate disproportionately in the country with the larger domestic market, i.e. the country with a higher reservation utility. If the demand asymmetry is sufficiently high, this effect may overturn the direct effect of transport costs on firm selection, thereby reducing the domestic cutoff  $\theta_d$ . This

<sup>27</sup>For example, countries are symmetric in Melitz (2003). Helpman et al. (2010) derive closed-form solutions for  $\theta_d$  only under symmetry or with an outside sector. Their analysis focuses on how changes in the fraction of exporting firms shape inequality. An exception is Demidova and Rodriguez-Clare (2011), who show that unilateral trade liberalization induces firm selection in the context of a small open economy variant of Melitz (2003).

<sup>28</sup> $f_x \geq f_d$  is a standard assumption that ensures only the most productive firms export in equilibrium, in line with the extensive evidence of selection into exporting.

<sup>29</sup>Home market effects are a standard feature in models of monopolistic competition with costly trade, dating back to Krugman (1980). See Helpman and Krugman (1985), chapter 10, for an example in a model with both differentiated and homogeneous sectors.



effect becomes stronger with lower transport costs, which explains why the admissible degree of asymmetry is increasing in  $\tau$ .

## 5.2 Labor Reallocations Across Firms

Firm selection in response to trade liberalization leads to shifts in the distribution of firm productivity that trigger reallocations of labor towards high productivity firms. This section formalizes this argument by first deriving the distribution of employment across firms and then establishing how it is affected by trade liberalization. Since optimal compensation policies differ across firms, labor reallocations have implications for the equilibrium distribution of wages in the economy which are studied in the next section.

The distribution of employment across firms, denoted  $G_h(\theta)$ , measures the fraction of workers employed in firms with productivity below  $\theta$ ,

$$G_h(\theta) = \frac{\int_{\theta_d}^{\theta} h(\theta') dG_{\theta}(\theta')}{\int_{\theta_d}^{\infty} h(\theta') dG_{\theta}(\theta')}.$$

Provided that firm productivity is not too dispersed (i.e.  $z$  is large enough), the integral in the denominator of this expression will converge. In this case, it is possible to use the solution for firm employment (13) and the Pareto productivity assumption to obtain

$$G_h(\theta) = \begin{cases} 1 - \frac{(\theta/\theta_d)^{\Lambda} + (\Upsilon_x - 1)(\theta_x/\theta_d)^{\Lambda}}{(\Upsilon_x - 1)(\theta_x/\theta_d)^{\Lambda} + 1} & \text{if } \theta_d \leq \theta \leq \theta_x, \\ 1 - \frac{\Upsilon_x(\theta/\theta_d)^{\Lambda}}{(\Upsilon_x - 1)(\theta_x/\theta_d)^{\Lambda} + 1} & \text{if } \theta_x \leq \theta, \end{cases}$$

where  $\Lambda \equiv \Gamma - 1 - s - z$  and  $z > 2 + s + \Gamma$ .

An important property of the model is that the distribution of employment across firms is fully determined by the productivity cutoffs and three parameters,  $\Lambda$ ,  $\Gamma$  and  $f_x/f_d$ . To check this, note that equation (21) implies that market access  $\Upsilon_x$  can be written as  $\Upsilon_x = 1 + (f_x/f_d)(\theta_x/\theta_d)^{-\Gamma}$ . This property allows me to characterize changes in the distribution of employment in terms of changes in the productivity cutoffs across equilibria. To do this, let subscripts 0 and 1 denote outcomes corresponding to two equilibria of the model.

**Proposition 6** *Consider any two equilibria indexed by 0 and 1 such that:*

- (i)  $\theta_{d,0} < \theta_{d,1}$ ,
- (ii)  $\frac{\theta_{x,0}}{\theta_{d,0}} > \frac{\theta_{x,1}}{\theta_{d,1}}$ ,
- (iii) *parameters  $\Lambda$ ,  $\Gamma$  and  $f_x/f_d$  are the same in both equilibria.*

*Then the distribution of employment across firms in equilibrium 1 first-order stochastically dominates the distribution of employment across firms in equilibrium 0. That is, for all  $\theta$ ,*

$$G_{h,1}(\theta) \leq G_{h,0}(\theta), \text{ with strict inequality for some } \theta.$$

**Proof.** Appendix. ■

This result allows a comparison of employment distributions across equilibria in which cutoffs satisfy conditions (i) and (ii). A special case of interest is the class of equilibria that exhibit firm selection as a response to trade liberalization. In any such equilibrium, a fall in variable trade costs induces low productivity firms to exit and results in a higher proportion of exporting firms, in line with conditions (i) and (ii) of Proposition 6. This yields the following result.

**Corollary 7** *Consider any equilibrium that exhibits firm selection as a response to trade liberalization. Then the employment distribution that follows a trade liberalization first-order stochastically dominates the initial employment distribution.*

Corollary (7) provides a sharp characterization of labor reallocations towards high productivity firms following trade liberalization. In the next section, we exploit this result to study the impact of trade liberalization on wage inequality.

### 5.3 Wage Inequality

There are two sources of heterogeneity in individual wages, a firm-specific component  $\theta$  and a worker-specific component  $\varepsilon_i = \varepsilon_i(1)$ . The distribution of wages in the economy (and thus measures of wage inequality) will therefore depend on the underlying distributions of firm productivity  $\theta$  and idiosyncratic performance  $\varepsilon$ .

To formalize this point, combine the firm's optimal choice of effort (10) with parts (a) and (b) of Corollary (3), to obtain the wage of worker  $i$  employed in firm  $\theta$ ,

$$w(\theta, \varepsilon_i) = \bar{u}\eta_0\theta^{\delta/b} \frac{e^{\eta_1\varepsilon_i\theta}}{E_\theta[e^{\eta_1\varepsilon\theta}]}. \quad (22)$$

where  $\eta_0 \equiv (\mu_{\min}\kappa_\mu)^\delta$  and  $\eta_1 \equiv \delta(\mu_{\min}\kappa_\mu)^b$  are positive constants. Next, let  $\int_{\underline{\varepsilon}}^{\varepsilon(\theta,w)} dG_\varepsilon(\varepsilon)$  denote the fraction of employees in firm  $\theta$  with wages lower than  $w$ , i.e.  $\varepsilon(\theta, w)$  satisfies  $w = w(\theta, \varepsilon(\theta, w))$ . Then the wage distribution, denoted  $G_w(w)$ , is given by

$$G_w(w) = \int_{\theta_d}^{\infty} \int_{\underline{\varepsilon}}^{\varepsilon(\theta,w)} dG_\varepsilon(\varepsilon) dG_h(\theta). \quad (23)$$

The distribution of wages is therefore a mixture of the distributions of  $\theta$  and  $\varepsilon$ .

To study wage inequality in this model, I focus on a specific inequality measure constructed from (23), the variance of log wages.<sup>30</sup> Besides its widespread application in empirical studies of wage inequality,<sup>31</sup> this approach yields analytical results for within-firm inequality that are robust arbitrary distributional assumptions for  $\varepsilon$ . In addition, unlike other popular measures of inequality such as the Gini coefficient and the 90-10 wage gap, the variance can be decomposed into between- and within-firm components. As discussed in the Introduction, this property allows me to highlight different channels through which international trade can have an impact on wage inequality. At the end of the section, I verify the robustness of the results using an alternative measure of inequality, the mean log deviation.

In the model, different firms select different compensation policies to reward their employees. This implies that within-firm wage distributions differ across firms and thus inequality measures will crucially depend on the equilibrium allocation of workers across firms. The variance of log wages depends on the employment distribution and just the mean and variance

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<sup>30</sup>The logarithmic transformation ensures that this measure of inequality is invariant to proportional shifts in the wage distribution, e.g. changes in the reservation utility  $\bar{u}$  in equation (22). That is, if Home's wage distribution in an initial equilibrium 0 is simply a scaled-up version of that in another equilibrium 1, then the variance of log wages is the same in both equilibria.

<sup>31</sup>For example, among recent empirical studies, Lemieux (2006), Helpman et al. (2012) and Card et al. (2012) use variance decompositions of log wages to analyze changes in inequality in the US, Brazil and Germany, respectively.

of the within-firm log wage distributions, denoted  $w_M(\theta)$  and  $w_V(\theta)$ , respectively. Letting  $\tilde{w}(\theta, \varepsilon_i) = \log w(\theta, \varepsilon_i)$  and using the expression for individual wages (22) yields

$$w_M(\theta) = E_\theta [\tilde{w}(\theta, \varepsilon_i)] = \kappa_M + \frac{\delta}{b} \log \theta - \log E_\theta [e^{\eta_1 \varepsilon \theta}] , \quad (24)$$

$$w_V(\theta) = Var_\theta [\tilde{w}(\theta, \varepsilon_i)] = (\eta_1 \theta)^2 , \quad (25)$$

where  $\kappa_M$  is a constant term. Given the equilibrium employment distribution,  $G_h(\theta)$ , these expressions can be integrated across firms to obtain the standard decomposition of the total variance of log wages into between and within-firm components,

$$Var(\tilde{w}(\theta, \varepsilon_i)) = Var[w_M(\theta)] + E[w_V(\theta)] .$$

The between-firm component,  $Var[w_M(\theta)]$ , is equal to the variance of average log wages across firms. The within-firm component,  $E[w_V(\theta)]$ , is equal to the average within-firm variance. Henceforth, I will refer to this second component as the *residual* variance of log wages. This allows me to avoid confusion with the within-firm variances  $w_V(\theta)$  and also to highlight implications of the analysis in this section for the empirical assessment of the impact of trade on inequality. In empirical studies such as Helpman et al. (2012), the between-firm component is the estimated variance of the firm-fixed effects in a regression of individual wages that also controls for observable worker characteristics. The within-firm component is the variance of the regression residuals.

As in the previous related literature, wage inequality across ex-ante identical workers in the model is partly driven by cross-firm variation in average wages, i.e. between-firm inequality. Earlier models have shown that this variation can be generated by search frictions, efficiency wages or fair wage considerations,<sup>32</sup> while in this model firms compensate their workers for exerting different effort levels.

Unlike other models in the literature, however, part of the wage variation arises from differences in within-firm variances across firms. As long as worker performance is only a noisy signal of effort, firms deal with the moral hazard problem by paying for performance, which results in within-firm wage dispersion. Moreover, within-firm inequality varies across firms since high productivity firms offer higher-powered incentives that magnify the variance of idiosyncratic performance between their employees. Note that  $w_V(\theta)$  increases in firm productivity even when the variance of idiosyncratic performance is identical in every firm. Cross-firm variation in inequality is a necessary ingredient for trade liberalization to have an impact on inequality through the within-firm component. When combined with the labor reallocations towards high productivity firms that result from trade liberalization, this mechanism generates increasing residual wage inequality.

Next, I show that if the initial equilibrium exhibits firm selection in response to trade liberalization, then the change in the residual variance is necessarily positive. The change in the between-firm variance, however, cannot be signed without imposing more structure on the distribution of the idiosyncratic component of individual performance  $\varepsilon$ .

Formally, let subscripts 0 and 1 denote outcomes corresponding to equilibria before and after trade liberalization, respectively. Consider first the change in the residual variance,

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<sup>32</sup>See the discussion in the Introduction.

which can be written as

$$\begin{aligned}\Delta E[w_V(\theta)] &= \int_{\theta_{\min}}^{\infty} w_V(\theta) [dG_{h,1}(\theta) - dG_{h,0}(\theta)], \\ &= \int_{\theta_{\min}}^{\infty} w'_V(\theta) [G_{h,0}(\theta) - G_{h,1}(\theta)] d\theta, \\ &> 0.\end{aligned}$$

The first line uses the fact that, in any equilibrium of the model, the within-firm variance depends only on firm-productivity. The second line follows after integrating by parts. From equation (25), the within-firm variance increases in  $\theta$ , thus  $w'_V(\theta) > 0$ . Moreover, if the initial equilibrium exhibits firm selection in response to trade liberalization, then  $G_{h,0}(\theta) \geq G_{h,1}(\theta)$  for all  $\theta$ , with strict inequality for some  $\theta$ . Intuitively, trade liberalization generates labor reallocations towards high inequality firms, and this results in an unambiguous increase in the residual variance of log wages.

In turn, the change in the between-firm variance is given by

$$\begin{aligned}\Delta Var[w_M(\theta)] &= \int_{\theta_{\min}}^{\infty} [w_M(\theta)]^2 [dG_{h,1}(\theta) - dG_{h,0}(\theta)] - \Delta(\bar{w}^2), \\ &= 2 \int_{\theta_{\min}}^{\infty} w'_M(\theta) w_M(\theta) [G_{h,0}(\theta) - G_{h,1}(\theta)] d\theta - \Delta(\bar{w}^2),\end{aligned}$$

where  $\Delta(\bar{w}^2) \equiv (\bar{w}_1)^2 - (\bar{w}_0)^2$  and  $\bar{w}_q \equiv \int_{\theta_{\min}}^{\infty} w_M(\theta) dG_{h,q}(\theta)$  is the mean log wage in equilibrium  $q = \{0, 1\}$ . As in the analysis of the residual variance, the second line is obtained after integrating by parts. However, the change in  $Var[w_M(\theta)]$  cannot, in general, be signed. First, note from (24) that the mean log wage is not necessarily increasing in firm productivity.<sup>33</sup> Furthermore, labor reallocations towards high productivity firms also imply a rise in the mean log wage,  $\bar{w}_1 > \bar{w}_0$ , that tends to reduce the between-firm variance in the aftermath of trade liberalization. I summarize these results in the following Proposition.

**Proposition 8** *Trade liberalization leads to an increase in the residual variance of log wages if and only if the equilibrium exhibits firm selection as a response to trade liberalization. The change in the between-firm variance of log wages cannot, in general, be signed.*

Although the variance of log wages is a popular measure for inequality comparisons in applied work, it may conflict with the Lorenz criterion (Foster and Ok (1999)).<sup>34</sup> The latter, however, incorporates some principles that are generally regarded as fundamental to the theory of inequality measurement.<sup>35</sup> For this reason, I close this section by analyzing the impact of trade liberalization using a Lorenz-consistent measure, the mean log deviation (MLD). This index, introduced by Theil (1967), belongs to the class of generalized entropy measures and, as such, it can be decomposed into between and within components.<sup>36</sup> The

<sup>33</sup> Actually, it is possible to construct examples in which, when productivity is high enough, the mean log wage decreases in  $\theta$ .

<sup>34</sup> The Lorenz criterion states that a distribution  $F$  is more unequal than distribution  $F'$  if and only if the Lorenz curve of  $F$  lies below the Lorenz curve of  $F'$  everywhere in the domain.

<sup>35</sup> Atkinson (1970) showed that this criterion is equivalent to second-order stochastic dominance when the two distributions have equal mean.

<sup>36</sup> Generalized entropy measures have several desirable properties. Cowell (2011), chapter 3, shows that an inequality measure belongs to this class if and only if it simultaneously satisfies the weak principle of transfers, decomposability, scale independence and the population principle.

definition and decomposition of the MLD are given by

$$\begin{aligned} MLD &\equiv E \left[ \log \left( \frac{\omega(\theta)}{w(\theta, \varepsilon_i)} \right) \right] \\ &= \int_{\theta_{\min}}^{\infty} \log \left( \frac{\bar{w}}{\omega(\theta)} \right) dG_h(\theta) + \int_{\theta_{\min}}^{\infty} E_{\theta} \left[ \log \left( \frac{\omega(\theta)}{w(\theta, \varepsilon_i)} \right) \right] dG_h(\theta) \end{aligned}$$

The second equality states that the MLD of wages can be decomposed into the MLD of mean wages across firms (between-firm MLD) and the average MLD of wages within firms (residual MLD). The impact of trade liberalization on the MLD index can be evaluated using the expression for individual wages (22) and Corollary (7). The results are qualitatively identical to those obtained for the variance of log wages.

**Proposition 9** *Trade liberalization leads to an increase in the residual MLD of wages if and only if the equilibrium exhibits firm selection as a response to trade liberalization. The change in the between-firm MLD of wages cannot, in general, be signed.*

**Proof.** Appendix. ■

## 6 Concluding Remarks

Evidence from firm-level studies consistently show that wage dispersion within firms is a major component of wage inequality in many countries. This paper is, to the best of my knowledge, the first in the literature to develop a theoretical framework to study the determinants of within-firm wage dispersion, its variation across firms and links to changes in international trade costs. Moreover, in light of the magnitude and growth in residual wage dispersion, the focus is on modeling within-firm wage inequality between identical workers.

Although the hypothesis that quality depends on employee performance appears to be a natural assumption, I do not regard it as an essential part of the mechanism linking trade liberalization to wage inequality. An interesting topic for future work is to think about alternative settings that would lead high productivity firms to offer higher-powered incentives.

A common feature in related studies in the literature that is absent in this framework are exporter wage premia. In the model, conditional on productivity, exporting does not induce firms to pay higher wages. As mentioned, however, this feature can be easily incorporated into the model by assuming that foreign buyers have a relatively higher preference for quality than domestic consumers. This extension would also generate higher within-firm inequality in exporting firms, conditional on productivity, which is consistent with the empirical evidence reported in Frías et al. (2012). Importantly, the analysis shows that exporter wage premia are not necessary for international trade to have an impact on within-firm wage inequality. Introducing exporter wage premia would reinforce the main results of the paper.

There are a number of additional topics worth exploring in future versions of this draft. One of them is the impact of trade liberalization on ex-post welfare. On one hand, lower trade costs lead to lower consumption prices and higher expected wages. However, labor reallocations towards high productivity firms can potentially hurt unlucky workers who, despite high effort levels, end up receiving very low wages due to poor ex-post performance.

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# A Appendix

Coming soon.