Search, Asset Liquidity and Business Cycles

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Abstract

This paper presents a real business cycle model with search frictions in the asset market, where equity liquidity is endogenously determined. I use the model to study how labor input, asset liquidity and asset prices fluctuate in response to productivity and liquidity shocks, and how, in turn, these fluctuations magnify the impact of productivity shocks on economic activity. A household's investment is restricted by its holdings of liquid assets in the model. When a negative productivity shock hits the economy, entry into the asset market declines. This effect discourages investment because of the reduced liquidity of assets, thereby amplifying the decrease in total output. After calibrating the model to match key features of the U.S. data, I find that productivity and liquidity shocks are equally important in explaining business cycles, which is contrary to findings in some recent studies. In addition, I show that productivity shocks can generate pro-cyclical movements of labor input, asset prices and liquidity, which is not the case for liquidity shocks. The model also generates significant positive correlations between productivity and output,between productivity and labor input and between productivity and asset liquidity, in agreement with my empirical observations.

JEL classification: E32, G12, G1

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1 Introduction

In this paper, I construct a real business cycle model with search frictions in the asset market to study how asset liquidity and prices fluctuate in response to productivity shocks and liquidity shocks, and how, in turn, these fluctuations magnify the impact of productivity shocks on economic activity. In the model, asset liquidity depends on the matching probability of a seller with an appropriate buyer in the asset market. In this way, an exogenous shock can generate a response in asset liquidity by changing the tightness of the asset market. After calibrating the model to U.S. data, I find that a negative productivity shock generates positive co-movements between asset liquidity and asset prices and that these effects act to magnify the impact on economic activity. The implied movements in output, investment and labor input are qualitatively consistent with empirical evidence.

Recently, many economists have argued that shocks to financial markets account for a large proportion of fluctuations in output and asset prices. In particular, a negative shock to asset liquidity reduces asset prices and stability in asset markets, which leads to declines in output and investment. Kiyotaki and Moore (2008) formulate this idea using a monetary model with liquidity constraints in which a firm's investment capability is restricted by the holding of liquid assets. A negative liquidity shock reduces capital investment, and thus reduces labor input and aggregate production by decreasing the marginal productivity of labor.

The liquidity shock hypothesis has become popular in macroeconomic models. Several authors have argued that there are good reasons to expect that financial shocks to asset liquidity are much more important than productivity shocks in accounting for fluctuations, such as Christiano, Motto and Rostagno (2008), Jermann and Quadrini (2009), Del Negro, Eggertsson, Ferrero and Kiyotaki (2011), Ajello (2011), *etc.* However, Shi (2012) shows that there is a fundamental problem with this argument — asset prices are counter-cyclical. He demonstrates that a negative financial shock to asset liquidity always generates an increase in the asset price. This counterfactual movement of asset price is generated because a negative liquidity shock reduces asset supply, and thus drives up the asset price. To resolve the contradiction, Shi suggests the inclusion of productivity shocks, in addition to liquidity shocks, are a good candidate to be included for investigating business cycles, because productivity shocks lower the asset price by reducing the asset demand. This approach leads to a more general question about the role of productivity in economic activity: What is the quantitative importance of productivity shocks, relative to liquidity shocks, in driving business cycles?

Many researchers believe the contribution of productivity shocks to business cycle fluctuations is very small because they assume that the variations in the structure of the financial market are purely driven by financial shocks. Changes in the financial market structure explain a large fraction of business cycles, as do financial shocks. This inference is true only if the assumption these researchers make about the structure of the financial market is true. However, if the changes in the structure of the financial market can be caused by shocks other than financial shocks, are financial shocks still important in explaining business cycles? More specifically, if the other shocks are productivity shocks, can they replace or even dominate the role of financial shocks in business cycles?

In light of the above literature, I construct a dynamic stochastic general equilibrium model in which asset liquidity is endogenously determined to study the contribution of productivity shocks to business cycle fluctuations. This framework is based on Kiyotaki and Moore (2008) and Shi (2012). A representative household consists of a large number of members who trade in the frictional asset market. A cost is incurred when a buyer enters the asset market. Buyers make tradeoffs between the expected trading surplus and the entry cost when choosing whether to participate in the asset market.[1] The greater number of buyers entering the market, the higher the probability that sellers will sell their assets in each period. At the same time, however, more entry costs have to be paid by buyers. Therefore, a shock that changes the trading surplus in the asset market will affect market tightness (*i.e.,* buyer to seller ratios) and asset liquidity.

The theory suggests that productivity shocks change asset liquidity and generate a large variation in GDP. In the model, a decrease in productivity reduces the trading surplus and the measure of buyers in the asset market, which in turn lowers the probability with which a seller can liquidate his/her assets. This lower level of asset liquidity reduces sellers' investment capability, generating a negative effect on the future production of consumption goods and labor supply, which implies stronger positive correlations between productivity and labor input and between productivity and total output. A model that does not include this indirect effect acting through asset liquidity will underestimate the impact of productivity shocks on business cycles.

In my empirical analysis, I estimate a structural vector auto-regressive model using U.S. quarterly data from 1980q1 to 2011q3. I find the following characteristics of U.S. business cycles. First, labor input, asset prices and asset liquidity are pro-cyclical. Figure 2 illustrates the dynamic paths of productivity, output, labor input, asset prices and the

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[1]See Mankiw and Zeldes (1991), and Haliassos and Bertaut (1995) for empirical evidence of limited participation of households in the asset markets.

bid-ask spread. It is clear that output, asset prices and labor input co-move positively. The bid-ask spread is negatively related to asset liquidity. Thus, asset liquidity is also pro-cyclical. Second, productivity and labor input are positively correlated. Moreover, productivity, asset liquidity and output are also positively related. Third, the combination of productivity and liquidity shocks accounts for approximately 50% of GDP volatility. Finally, productivity shocks are as important as liquidity shocks in driving business cycles.

Some of these observations have been documented previously. For example, basic real business cycle models, such as those of Barrio and King (1984), Prescott (1986) and King and Rebelo (1999), account well for co-movements among productivity, labor input and output; however, thees models are silent about the asset market. Jermann (1998), Campbell (1999), and Bernanke and Gertler (2000) do a good job in accounting for asset prices, but they did not explore the importance of shocks that originate from financial markets in business cycles.

The literature on liquidity shocks shows that financial shocks to asset liquidity are important in business cycles. My analysis confirms that the sum of productivity and liquidity shocks accounts for approximately 50% of the variations in GDP. However, instead of showing that liquidity shocks are the main driving force, I find that productivity shocks are as important as liquidity shocks in explaining business cycles. In addition, my model also documents that labor input is pro-cyclical in response to productivity shocks, which cannot be shown in Kiyotaki and Moore (2008) under reasonable parameter choices. However, my model still cannot solve the puzzle of the counter-cyclical movement of asset prices. Although negative productivity shocks reduce asset prices by reducing asset demand, they were unable compensate for the positive effect generated by asset liquidity during the most recent financial crisis.

The analysis and basic structure of the model are closely related to those of Kiyotaki and Moore (2008) and Shi (2012), who also study short-term dynamics driven by productivity and liquidity shocks in a model in which entrepreneur's investment capability is restricted by holdings of liquid assets. They do not, however, consider the impact of productivity on asset liquidity. Their models have significant success in accounting for the fluctuations of aggregate production and labor input with joint shocks to both productivity and asset liquidity. However, they underestimate the impact of productivity shocks on aggregate variables because they fail to include the indirect effect of productivity shocks acting through asset liquidity.

Ajello (2010) applies "Financing Gap" to measure financial shocks and finds that financial sector shocks account for significant amount of output and investment volatility.

Gertler and Kiyotaki (2012) develop a model that features both balance-sheet and financial accelerator effects to study banking instability. Again they do not consider the impact of productivity to the financial structure. Jermann and Quadrini (2009) suggest that imposing an intertemporal correlation between asset liquidity and productivity, a sizable adjustment costs to investment can make a pro-cyclical movement of asset prices with liquidity shocks. However, they do not provide a micro-foundation for this intertemporal correlation, which is the major contribution of this paper.

The model structure is also close to Cui and Radde (2013). They also introduce search frictions into the financial market. Their models have significant success in accounting for the fluctuations of liquidity share and asset prices. However, they do not consider the effects of an individual trader's behavior on the matching rate because the trading probability in the asset market is fixed in their paper. In addition, they do not calibrate liquidity shocks.

This paper is related to the literature on limited participation in the asset market. Mankiw and Zeldes (1991), and Haliassos and Bertaut (1995) present evidence of limited participation of households in the asset markets. Vissing-Jorgensen (2002) and Attanasio and Vissing-Jorgensen (2003) provide empirical evidence that limited asset market participation is important for estimating the elasticity of intertemporal substitution on U.S. data. Attanasio, Banks and Tanner (2002) find support for this argument in U.K. data. My approach differs from these papers in that I develop a process of entry decision to endogenize the participation rate of households in the asset markets.

This paper is also related to a large literature on the study of markets with search frictions. Kiyotaki and Wright (1989), Hosios (1990), Shi (1995, 1999), Lagos and Wright (2005), present a search environment with random match and price bargaining. However, none of these papers study asset prices and liquidity. Lagos (2010) develop an asset pricing model with search frictions. He shows assets are valued for their liquidity in an environment that all agents participate asset markets. Lagos (2011) proposes extensions of the asset pricing model to an environment with money to study monetary policy. In contrast to them, limited asset markets participation is an important element to study the impact of productivity shocks on asset liquidity in this paper.

The rest of the paper is organized as following: the environment of the model is described in section 2. Equilibrium is discussed in section 3. The empirical results are listed in section 4. Calibration and simulation are discussed in section 5. In section 6, I discuss the importance of search frictions in this model. My concluding remarks are offered in section 7.

2 The Model

2.1 Environment

Time is discrete and infinite, and is indexed by t. There are a continuum of households in the economy with measure one. Each household consists of a unit measure of members, divided into two groups according to an independent draw from a binomial distribution each period. With probability $\pi \in (0, 1)$ a member is an entrepreneur, and with probability $1 - \pi \in (0, 1)$ the member is a worker. Members' realizations are independent through time. Entrepreneurs have the ability to make new investments but cannot provide labor, while workers can provide labor but do not have investment opportunities. All members, who belong to the same household, share consumption and disutility from labor supply.

At the beginning of a period, households distribute their asset endowments before their members realize their types. Hence, an equal endowment is assigned to each member with instructions on its choices. A member's decisions follow the instructions which he/she receives from his/her household. Entrepreneurs sell equities and make investment into new projects. Workers provide labor, buy consumption goods, and also buy equities if they get matched with "brokers". Workers' disutility of labor is incurred at the end of the period. In each period, brokers collect equities from sellers and sell them to buyers in the frictional equity market. To simplify the exposition, I suppose households act as brokers. As a result, any unsold equity accrues to households at the end of the period.

At date t, a household's preferences are described by

$$E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t) - h((1 - \pi)l_t^w) - e_t(1 - \pi)c^f\} \tag{1}$$

where c denotes the total consumption of a household, which is a summation of the consumption of workers, $(1 - \pi)c^w$, and the consumption of entrepreneurs, πc^e. The superscripts w and e denote respectively workers and entrepreneurs. Entrepreneurs must divide their endowments between consumption and capital investment. In cases where capital investment is more profitable, the consumption of entrepreneurs is zero, and total consumption, c, equals the consumption of workers, $(1 - \pi)c^w$. The value of l_t^w denotes labor supply per worker. The functions $u(c)$ and $h(l)$ are twice continuously differentiable and have the following properties: $u'(c) > 0$, $u''(c) < 0$, $\forall c$; $h'(l) > 0$, $h''(l) > 0$, $\forall l$; $u'(0) = \infty$, $u'(\infty) = 0$, $h(0) = 0$, $h(\infty) = \infty$, $h'(0) = 0$, $h'(\infty) = \infty$. The last term, $e_t(1 - \pi)c^f$, is the total participation cost of the asset market, where e_t is the fraction of workers that

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participate in the asset market, and c^f is the fixed participation cost per worker in terms of utility. This cost includes all expenses incurred in undertaking trade in the asset market, such as costs of learning about the assets, effort costs, commissions, *etc.* Most of the expenses depend on time and effort spent, rather than money spent. Therefore, the entry cost is considered as a utility cost. It is assumed to be fixed for simplicity. The participation cost is incurred whenever a worker enters the asset market. Paying the cost does not mean that the worker gets to trade for sure. A worker's trading probability depends on the matching protocol in the asset market. The protocol will be discussed in more detail later. Households discount future at rate $\beta \in (0, 1)$.

The economy is also populated by a large number of firms who behave competitively. A firm operates a technology that uses labor and capital to produce consumption goods. Capital depreciates at rate $(1 - \sigma)$ after production, where $\sigma \in (0, 1)$. A firm's production function in period t is given by

$$y_t = A_t F(k_t^d, l_t^d), \tag{2}$$

where y_t denotes consumption goods produced in period t, A_t is the productivity level in period t, which follows a Markov process and k_t^d and l_t^d are firm's demands for capital and labor in period t.

In each period, a labor market, a capital market, a consumption goods market and an equity market operate. Firms participate in the labor market, the capital market and the consumption goods market. Entrepreneurs participate in the capital market and the equity market. The consumption goods market remains open throughout period. If entrepreneurs have some extra funds after investing, they can buy consumption goods. In contrast to entrepreneurs, workers participate in the labor market, the capital market and the consumption goods market. If they participate in the equity market, they will pay the fixed entry cost.

The labor, capital and consumption goods markets are Walrasion markets. Taking the wage rate w and the capital rental rate r as given, each firm chooses labor and capital input, each worker chooses labor and capital supply and each entrepreneur supplies capital only. In each period, firms sell consumption goods to workers and entrepreneurs. Consumption goods are the numeraire and they are non-storable across periods.

Search is random in the equity market. Sellers and buyers interact in the market with anonymous bilateral matching. The matching probability is determined by the matching function, $M(B_t, S_t)$. The values of B_t and S_t respectively represent the measures of buyers and sellers in the market in period t. Let ϕ_t^s and ϕ_t^b denote respectively the seller's

probability of selling and the buyer's probability of buying in the equity market. It follows that

$$\phi_t^s = \frac{M(B_t, S_t)}{S_t} = \phi^s(m_t) \tag{3}$$

$$\phi_t^b = \frac{M(B_t, S_t)}{B_t} = \frac{\phi^s(m_t)}{m_t}, \tag{4}$$

where $m_t = B_t/S_t$ is the tightness of the equity market. I assume sellers make a take-it-or-leave-it offer (x_t, q_t) to matched buyers. The value of x_t is the amount of consumption goods which the seller requires for one unit of the equity and q_t is the amount of equity which the seller offers the trading partner. For buyers in a desirable match, they take the offer (X_t, Q_t) as given and decide whether to accept it.

Entrepreneurs are granted a special ability which allows them to make new investments. One unit of investment costs one unit of consumption goods, and it takes one period for the investment to generate installed capital. Entrepreneurs need to raise funds to make new investments. Two possible ways are available to them: They can lend and sell their asset endowment; they can issue new equity in the asset market. Entrepreneurs' ability to raise funds is subject to two restrictions. The first constraint is the liquidity constraint, which states that only a fraction $\theta \in (0, 1)$ of total new investments can be financed by issuing new equity.[2]

$$s_{t+1}^e = i_t^e + \sigma s_t - q_t \geq (1 - \theta)i_t, \tag{5}$$

where s_t and s_{t+1}^e represent respectively entrepreneur's equity holding before and after trading with a broker in period t. Because the household distributes assets evenly to all members, each of them holds s_t. The first equality in (5) states that the entrepreneur's equity holding at the end of period t, s_{t+1}^e, equals the sum of new investment, i_t^e, and old capital after depreciation, σs_t, minus equities sold in the market, where σ is the capital survival rate.

The inequality in (5) states that an entrepreneur has to hold at least a fraction $(1-\theta)$ of new investments. To make the liquidity constraint effective, I suppose that funds cannot be reallocated between household members until all markets close. Otherwise, workers would shift more assets to matched entrepreneurs until (5) does not bind. The liquidity constraint can be rewritten as

$$\theta i_t^e \geq q_t - \sigma s_t,$$

[2]it is exactly same as the one modeled in Kiyotaki and Moore (2008) and Shi (2012).

where the right-hand side is the amount of new equity issued. This has to be lower than the upper limit on the investment that can be financed by issuing equity. This restriction prevents entrepreneurs from issuing more equity than they can buy back in the following period. The fraction $(1 - \theta)$ can also be interpreted as the down payment requirement for a new investment.

The entrepreneur's ability to raise funds is also restricted by the liquidity of equities. The equity market is frictional. If an entrepreneur cannot raise enough funds, he/she will lose the investment opportunity for the current period. To avoid this, entrepreneurs sell their asset endowment and new equity to brokers at a discounted price, which reflects the risk of not being able to sell them (both existing assets and new issuing equities) immediately. The discounted price depends on brokers' matching probabilities in the equity market. A broker offers a higher price to entrepreneurs only if he/she has a higher probability to resell the equities. After buying equities from entrepreneurs, brokers freely enter the equity market as sellers, whereas workers who intend to trade in the equity market need to pay a fixed participation cost c^f.

I take θ as a constant. But asset liquidity ϕ^s is endogenous. A change in the value of ϕ is considered as "a change in asset liquidity". Asset liquidity fluctuates with changes in the market tightness. A shock to the real sector, which affects households' willingness to trade in the equity market, changes asset liquidity and largely affects the stability of the equity market. Note that this is absent in models where asset liquidity is exogenous.

2.2   A household's problem

A household's state variables are its equity holdings s_t and the aggregate variables (K, A), where A is the realization of current productivity and K is the aggregate capital stock. In each period, a household chooses the investment i_t, consumption c_t, labor supply l_t, and trading decisions (e_t, x_t, q_t) to maximize value function $v_t(s_t; A_t, K_t)$. After suppressing the time subscript t and using $+1$ to denote variables in $t + 1$, I formulate the household's Bellman equation as

$$v(s; K, A) = \max_{(q,l,e,s_{+1})} u(c) - h(l) - e(1 - \pi)c^f + \beta E v(s_{+1}; K_{+1}, A_{+1}) \tag{6}$$

subject to

$$rs + \omega l^w - \frac{\pi \phi^s}{1 - \pi} QX = c^w \tag{7}$$

$$rs + x^d q^d = i^e + c^e \tag{8}$$

$$q + \sigma s \leq \theta i^e \tag{9}$$

$$\frac{\Omega}{U'(C)} \geq X \tag{10}$$

$$c^w \geq 0 \quad s_{+1} \geq 0 \quad q \geq 0 \quad l \geq 0 \quad X \geq 0 \tag{11}$$

$$e \in [0, 1]. \tag{12}$$

Equation (7) is a worker's total resource constraint. Here l^w and c^w are labor supply and consumption per worker. Workers need to buy consumption, c^w, and equities, Q. Only the matched workers purchase equities. A worker's probability of getting a match is $\pi \phi^s / (1 - \pi)$. The funds include capital rental income, $r_t s_t$, and wage income, ωl^w. The measure of labor is $(1 - \pi)$, so the total labor supply is $l = (1 - \pi) l^w$.

Equation (8) is an entrepreneur's resource constraint. Entrepreneurs get resources not only from renting capital, but also from selling equities. Resources are used to finance new investment, i^e and consumption, c^e. The value of q^d is the amount of equity a worker sells to a broker, and x^d is the discounted price the broker pays to the worker. For simplicity, I assume a broker can only purchase assets from one entrepreneur and a household can freely hire as many brokers as he/she want. Thus the measure of brokers is the same as the measure of entrepreneurs, and the trading quantity, q, satisfies $q = q^d$. With free entry of brokers, the following condition must hold:

$$x^d = \phi^s X. \tag{13}$$

The discounted price may be much lower than the equity price, X, because of the assumption on the unsold equities. If unsold equities are returned to entrepreneurs or unsold equities can generate some profits to brokers in the future, the value of the discounted price varies a lot. In this paper, the asset price refers to the value of X. To simplify the exposition, I refer to both entrepreneurs and borks as "sellers". A seller trades q units of equities at a unit price X with a buyer with a probability ϕ^s.

The price X that a broker posts to the matched worker must give a non-negative surplus to the worker. A worker's benefit of trade equals the expected marginal value of capital in the next period, discounted to the current period, Ω_t. The cost associated with the trade

is the forgone utility of consumption in the current period. Hence, the worker's surplus is given by

$$\Omega - XU'((1-\pi)C^w). \tag{14}$$

For a trade to take place, the asset price X, needs to satisfy

$$XU'((1-\pi)C^w) \le \Omega. \tag{15}$$

Rearranging it yields (10).

A household's capital holding at the beginning of period $t+1$ equals the summation of the old capital after depreciation, new investments and the net trade in the equity market:

$$s_{+1} = \sigma s + \pi i^e + \pi \phi^s Q - \pi \phi^s q. \tag{16}$$

Using s_{+1} in (16) to replace i^e in (8) yields

$$\pi r s + \pi x^d q = s_{+1} - \sigma s + \pi \phi^s q - \pi \phi^s Q + \pi c^e. \tag{17}$$

Combining the liquidity constraint, (9), and the budget constraint of the entrepreneur yields

$$(r + \frac{\sigma}{\theta})s + (x^d - \frac{1}{\theta})q \ge c^e. \tag{18}$$

Equations (8) and (9) can be replaced by (17) and (18) in the household's problem. Let the multipliers of (17) and (18) be λ^e and $\lambda^m \beta E v_{s_{+1}} \pi \phi^s$, the optimal choices must satisfy the following first-order conditions:

$$\beta E v_{s_{+1}} = \lambda^e \tag{19}$$

$$\pi u'(c) - \lambda^e \pi - \lambda^m \beta E v_{s_{+1}} \pi \phi^s \le 0, \text{ and } c^e \ge 0 \tag{20}$$

$$w u'(c) = h'((1-\pi)l^w) \tag{21}$$

$$\lambda^e \pi \phi^s (X - 1) + \lambda^m \beta E v_{s_{+1}} \pi \phi^s (\phi^s X - \frac{1}{\theta}) \le 0, \text{ and } q \ge 0 \tag{22}$$

$$\begin{cases} (\lambda^e \pi \frac{\partial \phi^s}{\partial e} + \lambda^m \beta E v_{s_{+1}} \pi \phi^s \frac{\partial \phi^s}{\partial e})Xq - \pi u'(c)QX\frac{\partial \phi^s}{\partial e} - (1-\pi)c^f > 0, & e = 1 \\ (\lambda^e \pi \frac{\partial \phi^s}{\partial e} + \lambda^m \beta E v_{s_{+1}} \pi \phi^s \frac{\partial \phi^s}{\partial e})Xq - \pi u'(c)QX\frac{\partial \phi^s}{\partial e} - (1-\pi)c^f = 0, & 0 < e < 1 \\ (\lambda^e \pi \frac{\partial \phi^s}{\partial e} + \lambda^m \beta E v_{s_{+1}} \pi \phi^s \frac{\partial \phi^s}{\partial e})Xq - \pi u'(c)QX\frac{\partial \phi^s}{\partial e} - (1-\pi)c^f < 0, & e = 0 \end{cases} \tag{23}$$

and the envelope condition:

$$v_s = u'(c)(1 - \pi)r + \lambda^e(\pi r + \sigma) + \lambda^m \beta E v_{s_{+1}} \pi \phi^s (r + \frac{\sigma}{\theta}). \tag{24}$$

Using (19) to replace the shadow price λ^e in (22) yields

$$\lambda^m \beta E v_{s_{+1}} (1 - \theta \phi^s X) \geq \beta E v_{s_{+1}} \theta (X - 1). \tag{25}$$

The cost of an investment is one and a fraction θ of new investments can be financed by issuing new equities. The equity price is x. The left hand side of (25) equals the down payment on the investment multiplied by the marginal value of equity holdings, and hence represents the cost of the investment in terms of utility. The equity trading surplus is $(X - 1)$. Since only a fraction θ of the new investment will be financed through equity, the trading benefit of one unit of investment in terms of utility can be expressed as $E v_{s_{+1}} \theta (X - 1)$. Entrepreneurs are willing to trade equities only if the benefit is enough to compensate the cost. For the liquidity constraint to bend, the shadow price λ^m must be positive. Clearly from (25), a positive λ^m requires that the asset price X be greater than 1 and less than $1/(\theta \phi^s)$. I will focus on the case where the liquidity constraint binds.

Lemma 1 *If the liquidity constraint is binding, the entrepreneur's consumption, $c^e = 0$ and the leverage ratio satisfies*

$$\frac{q_t}{s_t} = \frac{r + \sigma/\theta}{1/\theta - x^d}. \tag{26}$$

When the liquidity constraint is binding, the equity price is greater than one. Since sellers have full market power, the price X actually reflects the household's marginal rate of intertemporal substitution between future consumption and current consumption. If X is greater than 1, the marginal benefit from future consumption is larger than the marginal benefit from current consumption. Therefore, entrepreneurs will allocate no resources to current consumption, and, instead, will hold assets for future consumptions. The detailed proof is provided in appendix A.

This ratio, q_t/s_t, can also be interpreted as the leverage rate of the equity market. It depends on the down payment, θ, and the fundamental value of the asset, (x^d, r). As the price or the expected payoff of the asset increases, entrepreneurs can issue more equities based on the asset. There is no leverage restriction if the discount price reaches $\frac{1}{\theta}$ or higher. A higher price level returns more profits to sellers. If the price is high enough, the trading

surplus is enough to pay the down payment of a new investment. In this case, sellers can make as many investments and trade equities as they want.

Condition (23) reflects a worker's benefit and cost in the equity market. In a trade, a worker has to give up some current consumption to trade equity holdings in the following period. The trading benefit to a worker in terms of utility is $Xq\lambda^e$. So the first term in (23) represents the worker's marginal benefit of trading. The second term of (23) is the expected cost of the consumption which needs to be given up if the trade happens. $(1-\pi)c^f$ is the fixed participation cost. The entry rate is an interior solution only if the benefit and the cost are equal in the interval where $e \in (0,1)$. Using (19) to replace λ^e in (23) yields

$$\pi X \frac{\partial \phi^s}{\partial e}(\beta E v_{s+1} q + \lambda^m \beta E v_{s+1} \phi^s q - u'(c)Q) \begin{cases} > (1-\pi)c^f \text{ , and } e = 1. \\ = (1-\pi)c^f \text{ , and } 0 < e < 1. \\ < (1-\pi)c^f \text{ , and } e = 0. \end{cases} \quad (27)$$

Combining the optimality condition (19), the optimal asset price condition (10), and the envelope condition (24), yields the asset-pricing equation:

$$X = \beta\{\frac{u'(c_{+1})}{u'(c)}[X_{+1}\sigma + [(1-\pi)r_{+1} + X_{+1}\pi r_{+1}] + \lambda^m_{+1}X_{+1}\pi \phi^s_{+1}(r_{+1} + \frac{\sigma}{\theta})]\}. \quad (28)$$

The first two parts on the right hand side are the expected value of equities after depreciation and the expected rental income. The last term can be rewritten as

$$\lambda^m_{+1}X_{+1}\pi \phi^s_{+1}(r_{+1} + \frac{\sigma}{\theta}) = \lambda^m_{+1}X_{+1}\pi \phi^s_{+1}\frac{q_{+1}}{s_{+1}}(\frac{1}{\theta} - \phi^s X_{+1}) = X_{+1}\pi \phi^s_{+1}\frac{q_{+1}}{s_{+1}}(X_{+1} - 1). \quad (29)$$

The first equality is from the equation for the leverage ratio. If the liquidity constraint does not bind, λ^m is 0. Otherwise, the equation for the leverage ratio holds with equality, and c^e is zero. Therefore, the first equation in (29) always holds with equality. The second equality is from (25), when the liquidity constraint is binding. The trading surplus is $(X_{+1} - 1)$. Thus, the last term in (28) represents the expected trading surplus when one extra unit of equity is held. If the liquidity constraint is not binding, the last term of (29) is zero. Multiplying the sum of these three components by the household's discount factor, $\beta u'(c_{+1})/u'(c)$, I obtain the discounted value of future dividends, which has to be equal to the equity price X.

2.3   A firm's problem

In each period, firms choose capital and labor supply to minimize costs, subject to the production function. Output prices adjust so that profits are zero. It follows that:

$$r_t = A_t F_1'(k_t^d, l_t^d) \tag{30}$$

$$w_t = A_t F_2'(k_t^d, l_t^d). \tag{31}$$

3   Stationary search equilibrium

I focus on a stationary equilibrium where households are symmetric and make identical decisions. The time index is dropped and $+1$ is used to denote variables in $t+1$.

Definition 1 *A symmetric search equilibrium consists of price functions* (x, X, r, ω), *a household's policy functions* $(i, q, Q, l, c^w, c^e, e, s_{+1})$, *the demand factors of final goods producers,* (k^d, l^d), *and the law of motion of the aggregate capital stock, such that the following requirements are satisfied:*

(i) *Given price functions and the aggregate state* (K, A), *a household's value and policy functions are solved from the household's optimization problem.*

(ii) *Optimal conditions of firms,* $r = AF_1'(k^d, l^d)$, *and* $\omega = AF_2'(k^d, l^d)$ *hold.*

(iii) *All markets clear:*

$$\begin{aligned} goods: &\quad (1-\pi)c^w + \pi c^e + \pi i^e = AF(k^d, l^d) &\tag{32}\\ labor: &\quad l^d = (1-\pi)l^w = l &\tag{33}\\ capital: &\quad k^d = K = s &\tag{34}\\ equity: &\quad s_{+1} = \sigma s + \pi i^e. &\tag{35} \end{aligned}$$

(iv) *Aggregate capital holding* K_{+1} *satisfies the law of motion of capital:*

$$K_{+1} = \sigma K + \pi i^e. \tag{36}$$

(v) *In the frictional asset market, the trading price satisfies:*

$$X = \frac{\beta E v_{s_{+1}}}{u'(c)}.$$

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The trading offer posted by an seller has to be consistent with the offer accepted by the matched worker.

$$x = X, \quad and \ q = Q. \tag{37}$$

(vi) Symmetry condition: All households are identical. They make symmetric decisions in each period.

Although the quantitative results of this paper are based on a more general utility function, to understand the workings of the model it is useful to obtain some analytical results with linear utility, given by:

$$u(c) = u_0 c,$$

where u_0 is a parameter. The disutility of labor and the production functions have the following standard forms:

$$h(l) = l^\eta \quad \text{and } F(K, L) = K^\alpha L^{1-\alpha}.$$
$$\eta > 1 \quad \text{and } 0 < \alpha < 1.$$

Proposition 1 *There exists an equilibrium with $\lambda^m > 0$ and $e = 1$, if*

$$\left(\frac{1}{\theta} - \bar{\phi}^s\right)(1 - \sigma) > \frac{\pi}{\beta\theta} \tag{38}$$

and

$$
\begin{aligned}
c^f \leq \ & \frac{\pi}{1-\pi} X(K^1 : A) \underline{q} u_0 [X(K^1 : A) - 1 \\
& + \frac{\theta(X(K^1 : A) - 1)}{1 - \theta\bar{\phi}^s X(K^1 : A)} X(K^1 : A) \bar{\phi}^s] \phi_0 (\frac{1-\pi}{\pi})^\gamma \gamma,
\end{aligned}
\tag{39}
$$

where K^1 is the unique equilibrium capital holdings at $e = 1$ and the value of $\underline{q}$ and $\bar{\phi}^s$ are defined in Appendix B. In this equilibrium, the interest rate, the asset price and capital holdings are increasing in productivity,

Capital depreciation, $1 - \sigma$, is constant. The capital investment, which equals $D(K)$,[3]is strictly decrease in K at $e = 1$. Figure 1 depicts the steady state capital holding, K^*, as the intersection of the "break-even" level of investment and the actual capital investment. This solution represents corner equilibrium in the sense that all workers participate in the equity

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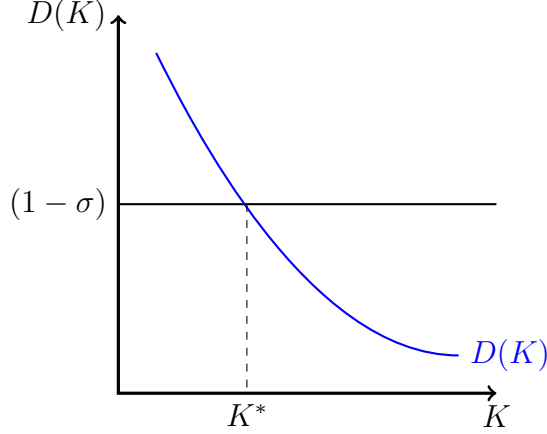
[3]It is defined in Appendix B

Figure 1: Steady State Equilibrium

market because the participation cost is low. Workers always a make positive expected profit from entering the equity market. Condition (38) requires that θ and σ cannot be too large. As the value of θ increases, the down payment required for an investment decreases. This process will stimulate new investments and equity trades in the equity market. Therefore, the trading benefit of an extra worker entering the market decreases relative to the fixed entry cost. If θ is large enough, the benefit of trade in the equity market is insufficient to cover the cost. Hence, the equilibrium entry rate e^* will deviate from 1. Similarly, a higher value of σ increases the entrepreneur's asset holding from the previous period, which in turn increases his investment capacity and equity trade. The upper limit on c^f is also necessary to obtain this result. If c^f is too high, the expected benefit of entering the equity market is insufficient to cover the large participation cost. Some workers will leave the equity market to avoid the entry cost. Therefore, the entry rate, $e = 1$, is no longer a steady state value. The detailed proof is provided in Appendix B.

Lemma 2 *In a steady state with a binding liquidity constraint, the equity price, X^*, and the interest rate, r^*, are positively related.*

The superscript $*$ indicates the value in steady state. Suppose the liquidity constraint is binding, asset prices can be rewritten as

$$X^* = F(r^*) = \frac{\beta}{1-\beta} r^* - \frac{\beta}{1-\beta}(1-\sigma). \tag{40}$$

The right hand side is the life-time capital return which is the difference between the capital rental income and the depreciation cost. As the steady state interest rate increases, the

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expected return to capital increases. Hence, buyers are willing to pay a higher price for equity, and vice versa. The proof of this lemma is discussed in Appendix A.

Proposition 2 *There is at least one equilibrium in which liquidity constraints are binding and the participation rate is strictly greater than zero and less than one, if condition (38) holds and if c^f is sufficiently large.*

Corollary 1 *In such a steady state equilibrium: (1) Suppose (K^2, e^2) is one pair of the equilibrium capital holding and entry rate, (K^2, e^2) are always lower than $(K^1, 1)$. (2) A permanent decrease in productivity results in decreases in the equilibrium values of traded equity, capital holding, investment and output and increases in the interest rate. The asset price also rises, and the movement of consumption is ambiguous.*

Proposition 2 characterizes an equilibrium in which the entry rate decision e is less than 1. Condition (38) is required to ensure that liquidity constraints are binding. The solution pair (K^2, e^2) is smaller than the solution from the case where the entry rate is equal to 1. Intuitively, if the entry rate is lower, the matching probability of a seller is also lower, and hence the trading prices brokers offer to entrepreneurs. Therefore, the total funds available for new investments are reduced. In a steady state, new investments have to be sufficient to cover the depreciation lost. The depreciation rate is constant, which means capital holdings are lower whenever the entry rate is smaller. A decrease in productivity reduces capital productivity. Hence, capital holdings, investments, output and equity trades decrease. Asset prices and the interest rate increase because the decrease in capital holdings increases the marginal product of capital. A negative TFP shock generates two effects on consumption. One is the wealth effect, which reduces consumption. A household uses its funds to buy consumption goods and invest in capital. A decrease in productivity reduces the marginal product of capital and the marginal profit of capital investment. Hence, households switch their funds from investment to consumption. This is the substitution effect. These two effects drive consumption in opposite directions. Thus, the movement of consumption is ambiguous.

4   Data

In this section, I estimate a structural vector autoregression (SVAR) for productivity (a), asset liquidity (s), labor input (h) and GDP per capita (y), using U.S. quarterly data covering the period from 1980q1 to 2011q3. Labor input is defined as the total hours worked

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by non-farm employees from Bureau of Labor Statistics. Asset liquidity is defined as the ratio of the bid-ask spread to the average asset price. If the asset is more illiquid, sellers pay a higher premium to attract more buyers and increase the probability of selling assets immediately. Therefore, the spread increases. The spread cost greatly depends on the time taken to complete a transaction. Therefore, this cost is an appropriate measurement of liquidity cost. With a one percent decrease in asset liquidity, I expect that the trading premium offered by the seller will also increase by approximately 1%. Therefore, I assume that the spread and asset liquidity are perfectly negatively correlated with equal variances. The bid-ask spreads are observed from COMPUSTAT and are available at an individual firm level. Hence, I take an average of the spreads from all firms in each quarter and use the average as the bid-ask spread in this paper.

Capital is defined as the sum of the capital from the previous period, the net fixed investment, changes in private inventory and nominal holding gains or losses.[4] Previous capital holdings, the net fixed investment and changes in private inventory are available on a quarterly basis. However, there are no quarterly data of nominal holding gains or losses. In each year, the nominal holding gains or losses account for a very small part of capital holdings, which is approximately 5%. However, if I ignore these data, capital holdings will be nearly halved after 30 years because capital is accumulated from period to period. Therefore, to obtain a more accurate measurement of capital, I evenly distribute the nominal holding gains or losses to each quarter within a year. After capturing capital, I back out total factor productivity (TFP) directly from the production function and define it as productivity in this paper.

Figure 2 illustrates the dynamic paths of productivity, output, labor input, asset prices and the spread. To present the relationships clearly, I have multiplied deviations of GDP, labor input and productivity by 10. Output and labor input clearly move up and down together with nearly the same amplitudes. Asset prices and the spread negatively co-move, especially after the year 2001, which implies that asset prices and liquidity should positively co-move. Productivity is pro-cyclical and seems to have lead business cycles in the last decade.

4.1 Short-Run VAR and Impulse Responses

In this section, I build an SVAR model for the mechanism described in the model. The structure is based on Sims (1986) and Bernanke (1986). The model is based on causality

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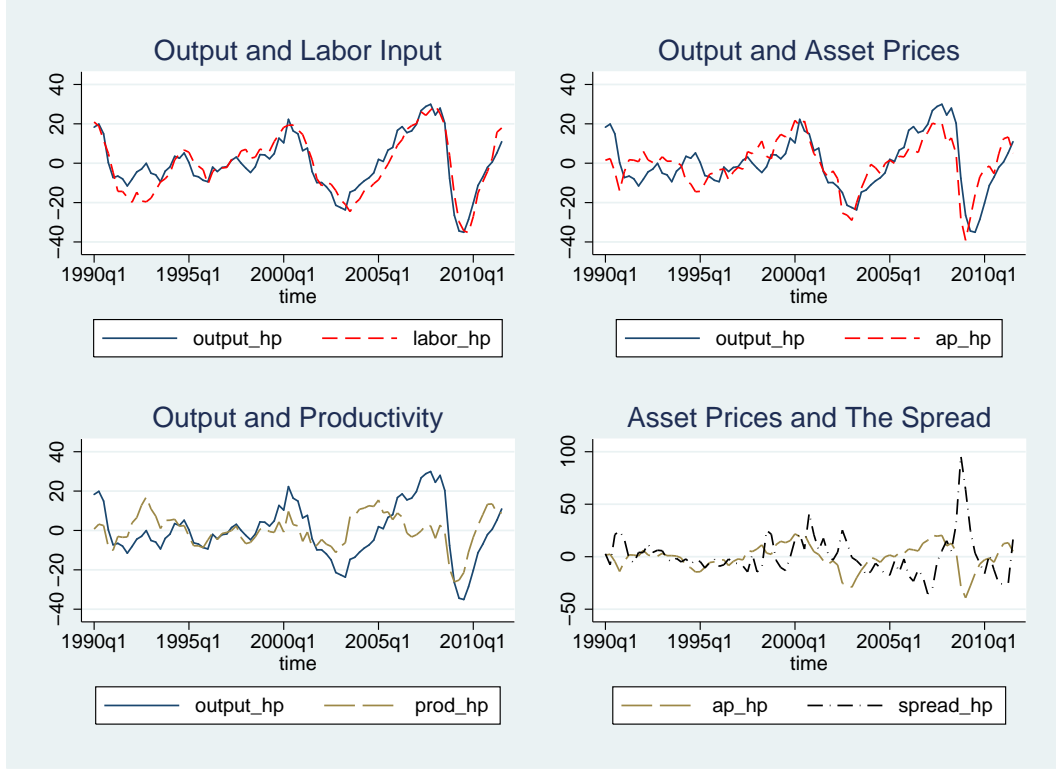[4]This is the definition used in NIPA tables.

Figure 2: *HP Detrended: Deviation of GDP, Employment, Productivity, Asset Prices and Spread from Trend*

relations from economic theory, and it includes the contemporaneous effects among the variables. This model includes four variables: GDP per capita, y, labor input, n, the ask-bid spread, s, and productivity, a. The causality relations are based on assumptions in the subsequent model section. Productivity shocks are ordered first. Productivity has a contemporaneous effect on asset market tightness and, hence, asset liquidity and the bid-ask spread. However, there is no potential contemporaneous impact of liquidity on the measure of productivity. Productivity and asset liquidity, then, combine to affect labor input and output.

Augmented Dickey-Fuller (ADF) unit root tests are applied to the levels, the first differenced and HP-detrended data. The results are reported in Table 1. The test fails to reject the null of a unit root in the levels, but it does reject the null when applied to the first differenced and HP-detrended data.[5]

The estimated correlations are reported in table 2. GDP is positively correlated with productivity and labor input, while it is negatively correlated with asset spread and pos-

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[5]Note: The test is based on an ADF test with four lags. The 1% significance critical value is -3.503, and the 5% significance critical value is -2.889

Table 1: Unit Root Tests.

Level	Results	First Diff	results	HP	results
y	-2.39	dy	-4.35	hp_y	-4.76
h	-1.60	dh	-4.19	hp_h	-4.83
a	-2.42	da	-4.43	hp_ah	-4.75
s	-2.91	ds	-5.63	hp_s	-4.23

itively correlated with asset liquidity. As in Prescott (1989) and Christiano, Eichenbaum and Vigfusson (2003, 2004), labor input and productivity are positively correlated. The magnitude of the correlation between the spread and productivity is unclear in the first-differenced case and is approximately 7%. If I use the HP-filter to detrend the data, the negative correlation increases significantly to 0.36.

Table 2: Unconditional Correlations

	First Differenced					HP-Detrended			
	da	ds	dh	dy		hp_a	hp_s	hp_h	hp_y
da	1.00				hp_a	1.00			
ds	-0.07	1.00			hp_s	-0.36	1.00		
dh	0.25	0.06	1.00		hp_h	0.25	-0.04	1.00	
dy	0.83	-0.02	0.66	1.00	hp_y	0.58	-0.15	0.85	1.00

The model can be written in matrix format as

$$AY = BX + E,$$

where A and Y are the $K \times K$ matrix, including all contemporary coefficients and contemporary variables. K is the number of variables. B is a $K \times (LK + 1)$ matrix containing coefficients of all past variables. X is a $(LK + 1)$ vector including a constant and all past variables. E is a matrix of unconditional errors.

4.1.1 The basic model without asset liquidity

In the basic model, asset liquidity is removed.

$$
\begin{aligned}
hp\_a &= \beta_1 lag(G) + e_x \\
hp\_h + a_{21} hp\_a &= \beta_2 lag(G) + e_n \\
hp\_y + a_{31} hp\_a + a_{32} hp\_h &= \beta_3 lag(G) + e_y.
\end{aligned}
$$

The matrix A is a 3×3 matrix including all contemporary coefficients. The variables are HP-detrended. The estimated coefficients are listed in (41).

$$
A = \begin{vmatrix} 1 & & \\ -0.07 & 1 & \\ (0.08) & & \\ -0.99 & -0.64 & 1 \\ (0.00) & (0.01) & \end{vmatrix} \; . \tag{41}
$$

The values of a_{21} are negative but not significant, which implies that the contemporary effect of productivity on labor input is weakly negative. This result is consistent with the previous finding in standard RBC models. The values of a_{31} and a_{32} are negative and significant, implying that productivity and labor input have a positive contemporary effect on output.

The decompositions of the variance of GDP are shown in Table 3. Productivity explains only approximately 25% of the GDP variances, and labor demand shocks[6] explain more than 65% of the total variances. The last 10% is explained by undefined shocks. Without asset liquidity, demand shocks are the major driving force of business cycles. In the next section, I will discuss how liquidity shocks change the pattern of the variance decompositions.

Table 3: model without asset liquidity

Response	GDP		
Impulse	Prod	Emp	GDP
Decomposition	25.6%	65.1%	9.3%

4.1.2 The model with asset liquidity

In this section, I use the bid-ask spread to represent asset liquidity and include it in the SVAR model. As in the theory discussed subsequently, I assume that productivity generates a contemporaneous effect on the bid-ask spread, but not the reverse. In addition, productivity and asset liquidity affect labor input and total output contemporaneously. The time interval $(1981q2 - 2011q3)$ is used. I assume that the shocks are orthogonal. The

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[6]They include all non-productivity shocks that change labor input, such as preference shocks, monetary shocks, *etc.*

estimated matrix A is listed in equation (42)

$$A_{80} = \begin{vmatrix} 1 & & & \\ 5.10 & 1 & & \\ (3.09) & & & \\ -0.01 & 0.005 & 1 & \\ (0.09) & (0.002) & & \\ -0.99 & -0.00 & -0.64 & 1 \\ (0.01) & (-0.00) & (0.01) & \end{vmatrix}. \tag{42}$$

The values in parentheses represent the standard deviations. The positive value of $a_{21} = 5.10$ implies that productivity has a negative effect on the spread and, hence, a positive effect on asset liquidity. The coefficient $a_{31} = -0.01$. Thus, the productivity and labor input are still positively related. The coefficients a_{41} and a_{43} are strictly negative, which means both productivity and demand shock increase he total output immediately. The coefficients a_{32} and a_{42} are approximately 0. Therefore, asset liquidity does not have a large contemporary effect on labor input and total output.

The results of the variance decomposition are shown in Table 4. Because asset liquidity and the spread are perfectly negatively correlated, the decomposition results of asset liquidity should be identical with the results of the spread. To make the comparison with the model results more clear, I report the decomposition of the spread as the decomposition of asset liquidity. Columns $2 - 5$ are the results of the variance decomposition of

Table 4: A Model with Asset Liquidity

Response	Asset Liquidity				GDP			
Impulse	Prod	Liq	Emp	GDP	Prod	Liq	Emp	GDP
Decomposition	3.5%	79.3%	12.1%	5.1%	21.5%	28.8%	47.3%	2.4%

asset liquidity. Columns $6 - 9$ record the results of the variance decomposition of the total output. The majority of the variance of asset liquidity is explained by itself. Productivity explains 3.5% of the variance. The variance of GDP is explained primarily by three components: productivity (21.5%), asset liquidity (28.8%), and labor input (47.3%). The sum of productivity and asset liquidity accounts for approximately 50% of the GDP volatility. Productivity shocks are as important as liquidity shocks in explaining business cycles. The variance accounted for by labor input shocks fall significantly to 46%.

Having shown that productivity shocks are important in explaining business cycles, I attempt to use technology innovations to explain productivity shocks In Appendix F. I

find technology shocks are also important in explaining business cycles. I also find that technology shocks generate pro-cyclical movements of labor input in most cases, which are consistent with the results of Christiano, Eichenbaum and Vigfusson (2004) [7]

5 Calibration and Simulation

In this section, I calibrate the model and study the nature of the fluctuations in asset prices, liquidity, investments and output with productivity shocks. The following function forms are used:

$$
\begin{aligned}
u(c) &= u_0 \frac{(c)^{1-\rho} - 1}{1 - \rho} \\
h(l) &= h_0 l^\eta, \quad \eta > 1 \\
F(K, L) &= K^\alpha L^{1-\alpha} \\
M(B, S) &= \phi_0 B^\gamma S^{1-\gamma}(1 - \mu) \\
\phi^s &= \phi_0 \left(\frac{e(1 - \pi)}{\pi}\right)^\gamma.
\end{aligned}
$$

The matching function is s Cobb-Douglas type function. A productivity shock changes buyers' willingness to trade and the market tightness and thus indirectly affects asset liquidity. There are also exogenous shocks that change asset liquidity directly, such as shocks on middlemen's balance sheets, shocks on middlemen's future expectations and shocks on fixed investment costs.[8][9] The value of μ is used to capture the exogenous shocks on asset liquidity. A shock to μ is named a liquidity shock rather than a matching efficiency shock in order to making a consistent definition with the data., although, a shock to μ and a shock to the matching efficiency ϕ_0 are equivalent.[10] A decrease in μ reduces matching probability and thus reduces asset liquidity. Assume that the quarterly TFP and liquidity processes are given by

$$
\begin{aligned}
\log A_{t+1} &= (1 - \delta_A) \log A^* + \delta_A \log A_t + \varepsilon_{A,t+1} & (43) \\
\mu_{t+1} &= (1 - \delta_\mu)\mu^* + \delta_\mu \mu_t + \varepsilon_{\mu,t+1}. & (44)
\end{aligned}
$$

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[7] For detail explanations, see Appendix F.

[8] Refer to Kiyotaki and Moore (2008), Negro, Eggertsson, Ferrero and Kiyotaki (2011) for further discussions about the shocks on liquidity.

[9] These shocks are different from productivity shocks. Productivity shocks are realized at the beginning of each period. When households make entry decisions in each period, productivity is observable. In contrast, an exogenous shock to asset liquidity is realized after workers enter asset markets.

[10] liquidity shocks include all non-productivity shocks that can generate fluctuations of asset liquidity.

where δ_A and δ_μ are the persistence of the TFP and the liquidity shocks, respectively. The value of A^* is the steady state TFP, which is normalized to 1. The steady state value of μ^* is normalized to 0. The values of ε_A and ε_μ represent the error terms of the two shocks. The variance-covariance matrix of the error terms is given by

$$cov = \begin{bmatrix} var(\varepsilon_A) & cov(\varepsilon_A, \varepsilon_\mu) \\ cov(\varepsilon_\mu, \varepsilon_A) & var(\varepsilon_\mu) \end{bmatrix}. \tag{45}$$

The model is calibrated at a quarterly frequency. I partition the model parameters into two groups: a general group and a special group. The general group includes $(\beta, \alpha, \rho, \eta, \sigma, u_0, \delta_A, var(\varepsilon_A))$, which are standard parameters in the business cycle literature. The special group includes $(\pi, \phi_0, c_f, \gamma, \delta_\mu, var(\varepsilon_\mu), cov(\varepsilon_A, \varepsilon_\mu))$.

The values of the discount factor, β; the capital share, α; and the relative risk aversion, ρ, are set equal to 0.992, 0.36 and 2, respectively. The curvature of disutility of labor, η, equals to 1.5, is determined from the target where the labor elasticity is 2. The annual investment to capital ratio, $4(1 - \sigma)$, is set equal to 0.066. This ratio is used to solve the depreciation rate σ. Given that the annual capital to output ratio is 2.92, I can solve for the steady state capital rental rate, r; wage rate, w; and the ratio of capital to labor, $\frac{K}{l}$. The target for total hours worked is 0.2, which is used to determine the steady state values of capital and output. Normalizing h_0 to 1, the value of u_0 can be solved from

$$u_0 = \frac{h_0 \eta l^{\eta - 1}}{c^\rho w}.$$

I obtain the steady state value of the broker's matching rate, ϕ^s, from the capital's law of motion.[11] Given the value of ϕ^s, I can get a relationship between the power of the matching function γ and the constant ϕ_0 from the matching function. The estimated range of the fraction of firms that adjust their capital holding in a quarter is between 0.2 and 0.4.[12] In this paper, I set the value equal to 0.24 as in Shi (2012). Hence, the quarterly value, π, equals 0.06. The participation rate in the asset market is approximately 50%. The entry cost of the equity market, c^f, is determined by the free entry condition.

The variance and the persistence of the TFP shock are observed from the data. However, the matching function and the distribution of liquidity shocks cannot be observed from the data. In the absence of direct observations, I jointly calibrate the power of the

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[11]I actually use two targets, k/y and hours of work, to determine two parameters, u_0 and ϕ_0, simultaneously.

[12]These values are from Doms and Dunne (1998) and Cooper, Haltiwanger and Power (1999).

<table>Table 5: Parameters</table>

Parameters	Value(general)	target
β: discount factor	0.992	exogenously chosen
ρ: relative risk aversion	2	exogenously chosen
u_0: constant in entrep. utility	0.109	ratio of capital to annual output=2.92
h_0: constant in labor disutility	1	normalization
η: curvature in labor disutility	1.5	labor supply elasticity=2
α: capital share	0.36	exogenously chosen
σ: survival rate of capital	0.984	annual investment/capital=0.066
A^*: steady-state TFP	1	normalization
δ_a: persistence of TFP	0.96	persistence of TFP
θ: fraction of new equity	0.135	equals the price discount rate
$var(\varepsilon_A)$: variance of TFP	0.367×10^{-4}	observable from data
π: fraction of entrepreneurs	0.06	fraction of investing firms=0.24
c^f: entry cost of workers	0.02	worker's entry rate e=0.5
γ: power of the matching function	0.315	relative volatility $\sigma_{\phi^s}/\sigma_y = 20$
ϕ_0: constant in the matching fun.	0.07	target labor supply L=0.2
δ_μ: persistence of the liq.	0.53	$corr(A, \phi) = 0.36$
$var(\varepsilon_\mu)$: variance of the liq.	0.02	$corr(A, L) = 0.25$
$cov(\varepsilon_\mu, \varepsilon_A)$: covariance of (TFP liq.)	5.6×10^{-4}	$corr(A, Y) = 0.6$

matching function, γ, , the constant, ϕ_0, the variance and the persistence of the liquidity shocks, $(var(\varepsilon_\mu), \delta_\mu)$, and the covariance between TFP and the liquidity shocks, $cov(\varepsilon_A, \varepsilon_\mu)$ to match five targets: the broker's matching rate, ϕ^s; the relative volatility of asset liquidity, σ_{ϕ^s}/σ_y; the correlation between TFP and asset liquidity, $corr(A, \phi^s)$; the correlation between TFP and labor input, $corr(A, l)$; and the correlation between TFP and output, $corr(A, Y)$.

All parameters are listed in Table 5. The value of δ_A is strictly greater than the value of δ_μ, which means that productivity is more persistent. The covariance between the two shocks is positive, which implies that a negative productivity shock will more likely accompany a negative liquidity shock.

The steady state values implied by this calibration are listed in Table 6. Asset liquidity is strictly less than 1, which means that brokers need time to liquidate their equity holdings. In this paper, the probability that a broker matches a desired buyer and executes a trade is 13.5%, which is similar to the result of Negro, Eggertsson, Ferrero and Kiyotaki (2010).[13] The asset price, X, is strictly greater than 1, which implies that the expected benefit of

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[13]In Negro, Eggertsson, Ferrero and Kiyotaki (2010), the liquidity of assets equals 15%, which is solved from the target where the relative quantity of liquid and illiquid assets is approximately 14%. In Shi (2012), the liquidity is solved from the target where the annual return on liquid assets is approximately 2%, which gives $\phi^s = 27\%$.

holding the asset is greater than the cost. Therefore, buyers have incentive to trade and hold the asset.

Table 6: Steady State Values

Variables	Steady State Values
Y: total output	0.8
K: total capital supply	9.31
L: total labor supply	0.2
C: total consumption	0.64
I: total investment	0.15
c^w: consumption per worker	0.68
i: investment per entrep.	2.56
l^w: labor supply per worker	0.21
e: worker's entry rate	0.50
X: asset price	1.78
q: asset trade per match	9.50
ω: wage rate	2.55
r: capital rental rate	0.03
ϕ^s: asset liquidity	0.13
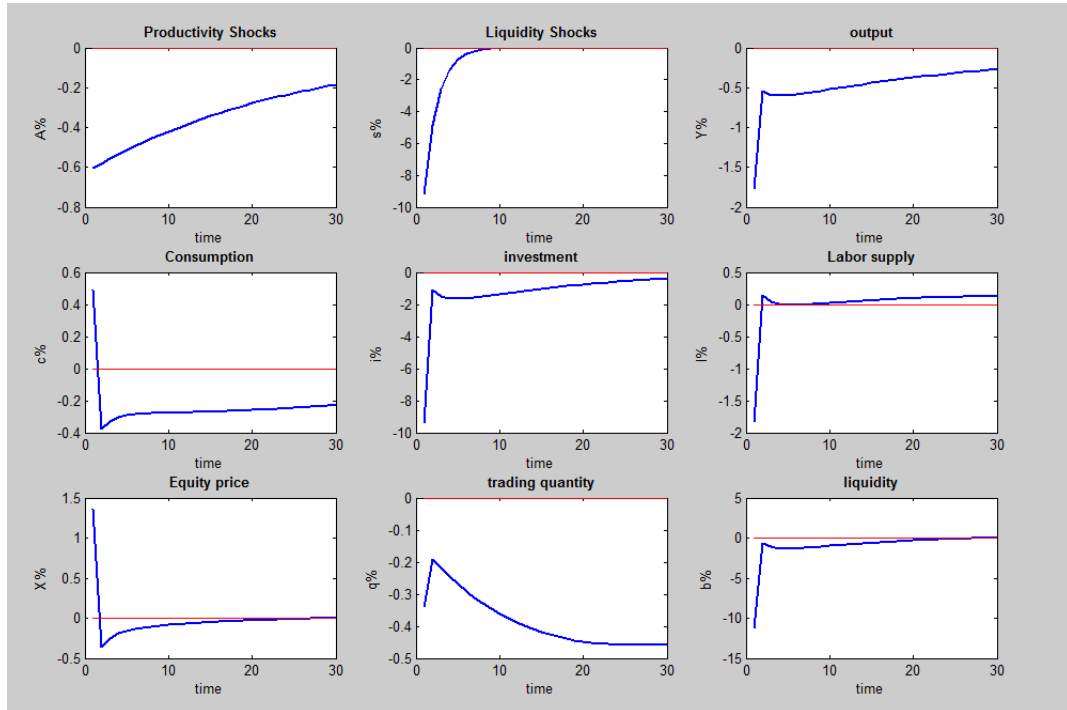
Figure 3: *Impulse Responses to Productivity and Liquidity Shocks*

The implied impulse response functions following joint shocks, namely, simultaneously

shocks to both productivity and the liquidity, are depicted in Figure 3.[14] The magnitude of the productivity shock is one standard deviation, and the liquidity shock is imposed according to the covariance between two shocks. In this economy, a negative shock to productivity reduces worker's labor income. Fewer workers enter the equity market due to a relatively expensive entry cost. The tightness of the equity market decreases, reducing both asset liquidity and prices. Entrepreneurs use initial asset holdings and funds obtained from brokers to finance new investments. The lower the asset price, the tighter the entrepreneur's resource constraint. Thus, investments made by entrepreneurs decrease, and consequently, total output and consumption decrease. At the same time, a negative matching efficiency shock hits the economy, which further reduces asset liquidity and, hence, further reduces total investment and output.

The asset price increases immediately because of a large deviation of the liquidity shocks. The degeneration of the liquidity shocks is much faster than that of the productivity shocks. Therefore, the asset price falls sharply and is lower than the steady state level for a long period.

6 Importance of Search Frictions

The importance of frictions depends on whether the frictions can help significantly improve the calibration results. Figure 4 provides a time series representation of the evolution of quarterly output growth from $1981q1$ to $2011q3$. The solid black line represents the observed data series. The blue dashed line represents the output fluctuations the empirical model identifies as being driven by productivity shocks. The red dashed-dotted line represents the fluctuations driven by liquidity shocks. Liquidity shocks are indicated as the only financial shocks in the paper. The aggregate of productivity and liquidity shocks explains approximately 50% of the output variations. Although a number of other researchers have found this result, they have also noted that liquidity shocks are the main driving force of business cycles, which is contradicted by the result in Figure 4, where the red line does not show more features similar to the black line than the blue line. In contrast, I find that productivity shocks are as important as liquidity shocks in business cycles.

In this section, I explore the reasons search frictions in the asset market are important in explaining business cycles. I find that market tightness in the asset market is the key

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[14]At the given parameter set, the Blanchard-Kahn condition holds. The number of eigenvalues is greater than one and equals the number of forward-looking variables. Thus, the steady state is unique. I also simulate the model 527 times. The probability of the entry decision e, which hits the upper and lower bounds, is 0. Thus, the occasionally binding scenario rarely happens in the model and can be ignored.
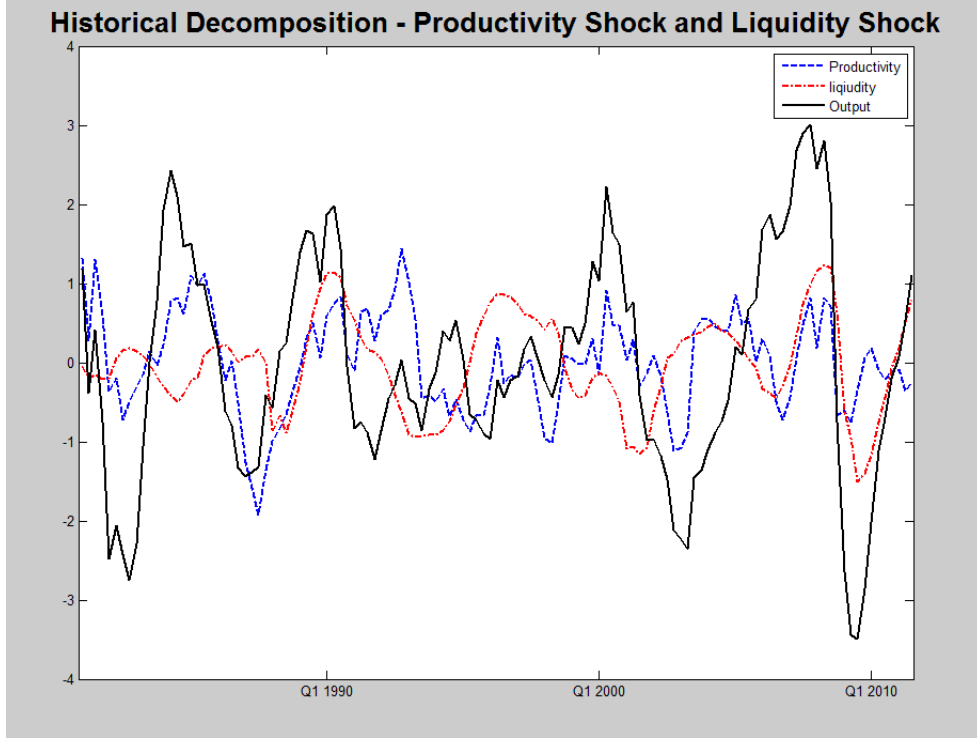
Figure 4: *Historical Decomposition*

element in linking productivity to the financial market.

6.1   Asset Liquidity

Asset prices are pro-cyclical. Shi (2012) argues that " *liquidity shocks cannot be the main driving force of business cycles because a negative liquidity shock leads asset price to rise rather than fall, and hence suggests that a negative liquidity shock must be accompanied by other concurrent shocks, such as productivity shocks, which reduce the demand for asset sufficiently.*" In my model, the asset market tightness links productivity to asset liquidity. A negative productivity shock reduces capital productivity and the measure of buyers in the asset market and, hence, lowers asset liquidity. One can question whether productivity shocks are still strong enough to lead pro-cyclical movements of the asset price when they generate persistent fluctuations of asset liquidity.

An unexpected decrease in productivity reduces investments and total output by reducing the marginal productivity of capital. These reductions decrease a household's consumption and drive up the marginal utility of consumption. Meanwhile, lower levels of investment and asset liquidity make the liquidity constraint (Eqn. (18)) tighter in the subsequent period and, hence, increase the expected marginal benefit of asset holding, Ω.

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According to (10), the asset price falls if the increase in the marginal utility of consumption dominates the increase in the expected marginal benefit of asset holding.

To illustrate this possibility, I compute the response of the equilibrium output to productivity shocks only. The six panels in Figure 5 depict the impulse response function for productivity, asset liquidity, consumption, investment, expected future utility of asset holdings and the asset price. As explained above, the negative shock to productivity decreases asset liquidity, consumption and investment. A decrease in asset liquidity tightens the liquidity constraint, which drives up the expected utility of asset holdings. The last panel shows that the asset price falls, which means the productivity shock does generate a significant decrease in the demand of the asset, even though it also generates a concurrent decrease in asset liquidity. Thus, productivity shocks can still play a visible role in explaining the pro-cyclical movement of asset price. Figure 3 further confirms this result by showing that the asset price still falls with joint shocks to both productivity and asset liquidity.
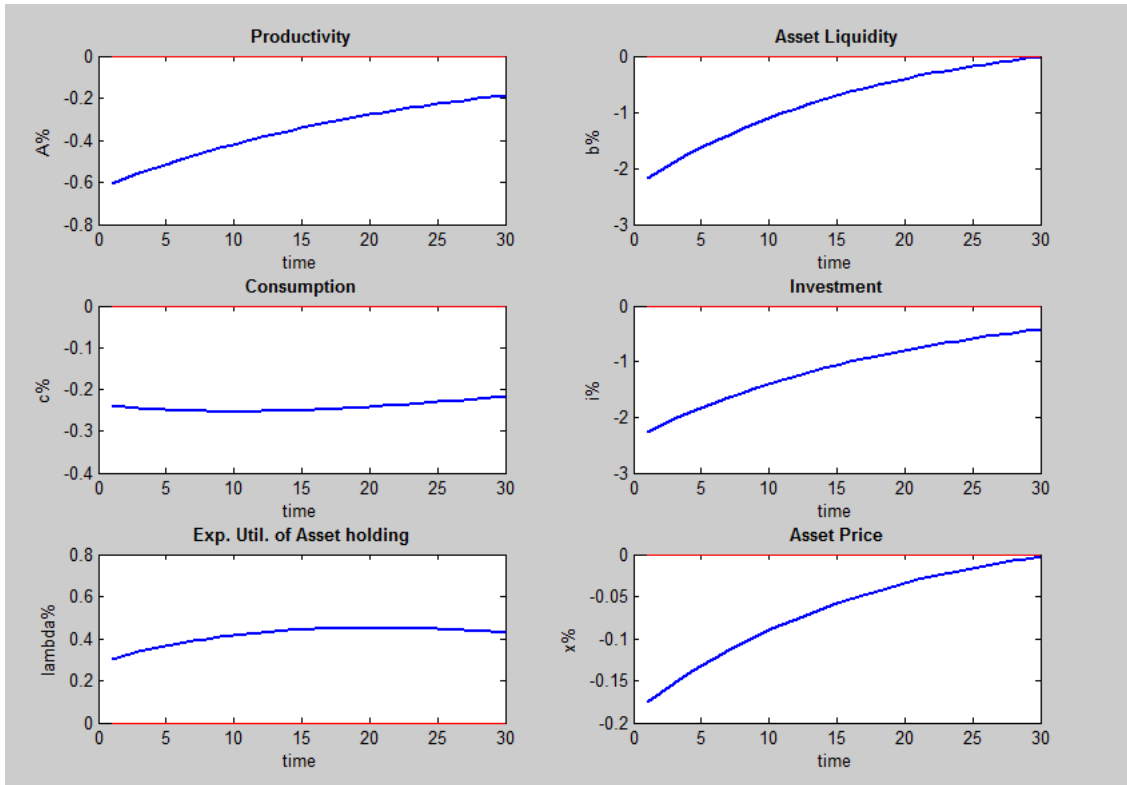
Figure 5: *Impulse Responses to a One Standard Deviation Productivity Shock*

6.2 Employment

Another notable feature of business cycles is the positive co-movement between GDP and labor input. Any shock that fails to generate this feature could not be considered a primary driving force of business cycles. In this section, I explain whether a productivity shock can generate this feature in my model. This issue needs to be discussed because the liquidity constraint reduces the substitution effect of productivity shocks on labor input. If the reduction is strong enough, then productivity shocks push labor input to move in the opposite direction.

Labor input, l, satisfies (21). Using the marginal product of labor to replace the wage rate w and taking a log form yields

$$(\eta - 1 + \alpha)\log(l) = \log(A) + \alpha\log(K) - \rho\log(c). \tag{46}$$

The constant term is suppressed. The value of η is greater than 1. Thus, labor input decreases in c and increases in K. A negative productivity shock reduces total income and consumption and, hence, increases the marginal utility of consumption, leading to an increase in labor supply. This process is the wealth effect on labor input. However, the productivity shock reduces the marginal productivity of capital. Thus households prefer to reallocate more funds to consumption. This reallocation leads to a decrease in labor productivity and the wage payment, and thus, the labor supply falls. This process is referred to as the substitution effect.

The substitution effect of labor input depends on the change in the expected marginal benefit of the capital holding. According to (24), the expected marginal benefit can be expressed as

$$v_{s+1} = u'(c_{+1})(1 - \pi)r_{+1} + \lambda^e_{+1}(\pi r_{+1} + \sigma) + \lambda^m_{+1}\beta E v_{s+2}\pi\phi^s_{+1}(r_{+1} + \frac{\sigma}{\theta}). \tag{47}$$

The first two terms are the marginal benefits of rental and selling incomes, respectively, which are exactly the same as those in the model without the liquidity constraint. The last term represents the benefit from relaxing the liquidity constraint (called the tightness premium), existing only in the model with liquidity constraints. As discussed above, a lower level of productivity tightens the liquidity constraint and, hence, increases the tightness premium. This increase raises the expected marginal benefit of capital holdings and, hence, reduces the substitution effect on labor input. If the reduction is large enough, then the wealth effect dominates the substitution effect. Therefore, the labor input rises.

Search frictions are important in moderating the unexpected reduction in the substitution effect. A negative productivity shock reduces the measure of buyers in the equity market because of the higher entry cost, which reduces entrepreneurs' investment capacity and capital holdings. This effect is reflected as a reduction of the seller's matching probability, ϕ_{+1}^s. According to (47), a lower value of ϕ_{+1}^s reduces the tightness premium and thus mitigates the unexpected reduction in the substitution effect.
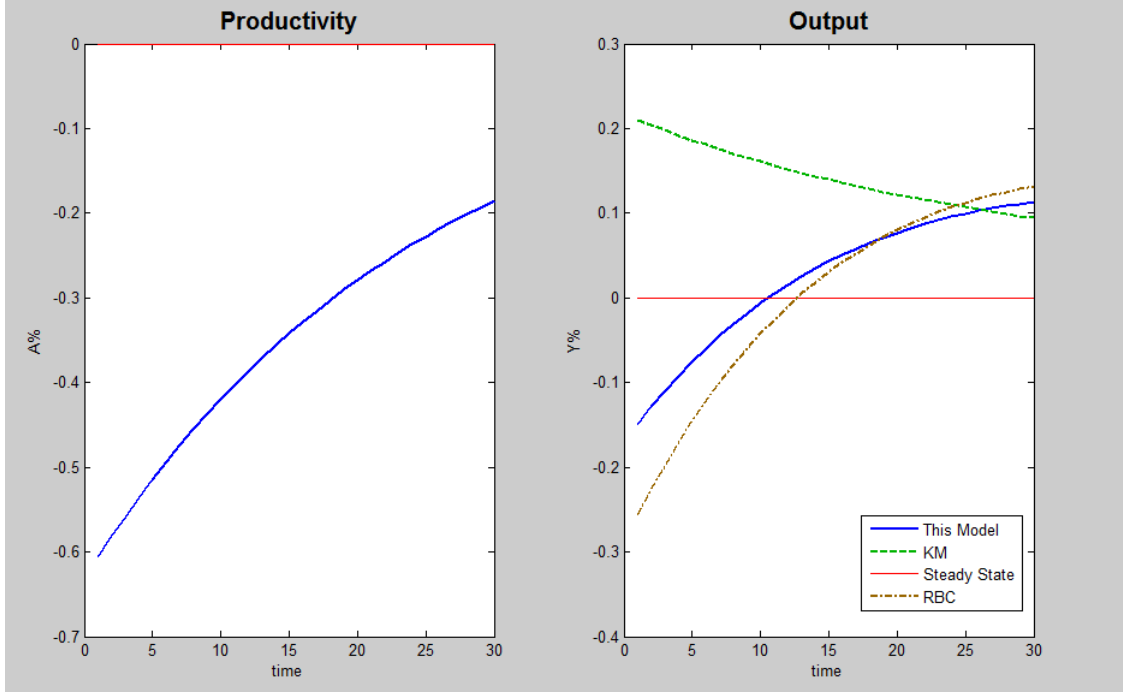
Figure 6: *Impulse Responses of Labor Input*

Figure 6 illustrates a comparison of the effects of one standard deviation of productivity shocks on labor inputs in different models. The solid blue line displays the dynamic path of labor input from the search model. The green dashed line represents the impulse response of labor inputs from Kiyotaki and Moore (2008),[15] and the brown dashed-dotted line

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[15] Although the model generates a positive dynamic path of labor input in Kiyotaki and Moore (2008), the model can generate a negative dynamic path with different parameter values. Equation (21) can be rewritten as

$$h'((1-\pi)l^w) = \frac{\partial F(K,L)}{\partial L}u'(c). \tag{48}$$

Thus, labor input depends on three functions: the disutility of labor supply, the production function and the utility function. The strict convexity of the disutility function implies that the left hand side of (48) strictly increases in labor supply, l^w. The production function is a general Cobb-Douglas function, and the parameter values are standard. Thus, the only feasible way to alter the moving direction of labor input is by changing the format of the utility function of consumption, particularly the relative risk aversion. For instance, if $\rho = 1$, labor input is pro-cyclical. In my paper, the relative risk aversion equals 2 because this value clarifies the comparison of the results. In addition, the assumption that households are risk averse is widely used in the related literature.

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represents the results from Prescott (1989) (A standard RBC model). Clearly, the liquidity constraint and search frictions in the asset market have a substantial effect on labor input, which can be traced to the effect of a decrease in productivity on the substitution effect of labor input. In Kiyotaki and Moore (2008), the liquidity constraint reduces the substitution effect of productivity on labor input significantly. Thus, the wealth effect dominates the substitution effect and increases labor input. In my model, search frictions in the asset market discourage buyers' entry, which reduces market tightness and, hence, reduces the tightness premium and mitigates the reduction of the substitution effect. Consequently, labor input decreases.

Figure 7 illustrates a comparison of impulse responses to productivity shocks and to the matching efficiency shocks in the search model. The three panels on the left report the impulse responses of labor input and the asset price to productivity shocks, and the three panels on the right report the impulse responses to efficiency shocks. It is clear that an efficiency shock generates a pro-cyclical movement of labor input, but it pushes asset prices counter-cyclically, which is counterfactual. This result implies that direct shocks to asset liquidity cannot be an independent source of business cycles. In contrast to liquidity shocks, productivity shocks with search frictions can generate positive co-movement between asset liquidity and asset prices as well as the pro-cyclical movement of labor input. Therefore, productivity shocks alone can be considered a source of business cycles.

6.3 Cyclical Properties

Figure 8 offers a comparison of the dynamic paths between the models with and without search frictions. Two exogenous shocks, namely, productivity and liquidity shocks, are introduced in Section 5. The blue solid line represents the impulse responses from the model with search frictions. The resaleability of assets is endogenously determined by market tightness and restricts the liquidity of both the new and existing assets. The green dashed line represents the dynamic results from Kiyotaki and Moore (2008). The resaleability is fixed and restricts the liquidity of the existing asset only. To enable an appropriate comparison, I compute other non-search model in which the resaleability constraint is imposed on both old and new assets. The dynamic results are represented by brown dashed-dotted lines in Figure 8. As the figure shows, the impulse responses are similar, even though different models are applied. This consistency suggests that search frictions do not significantly affect the joint impact of productivity and liquidity shocks on economic activity. This result is also consistent with my empirical result that although different
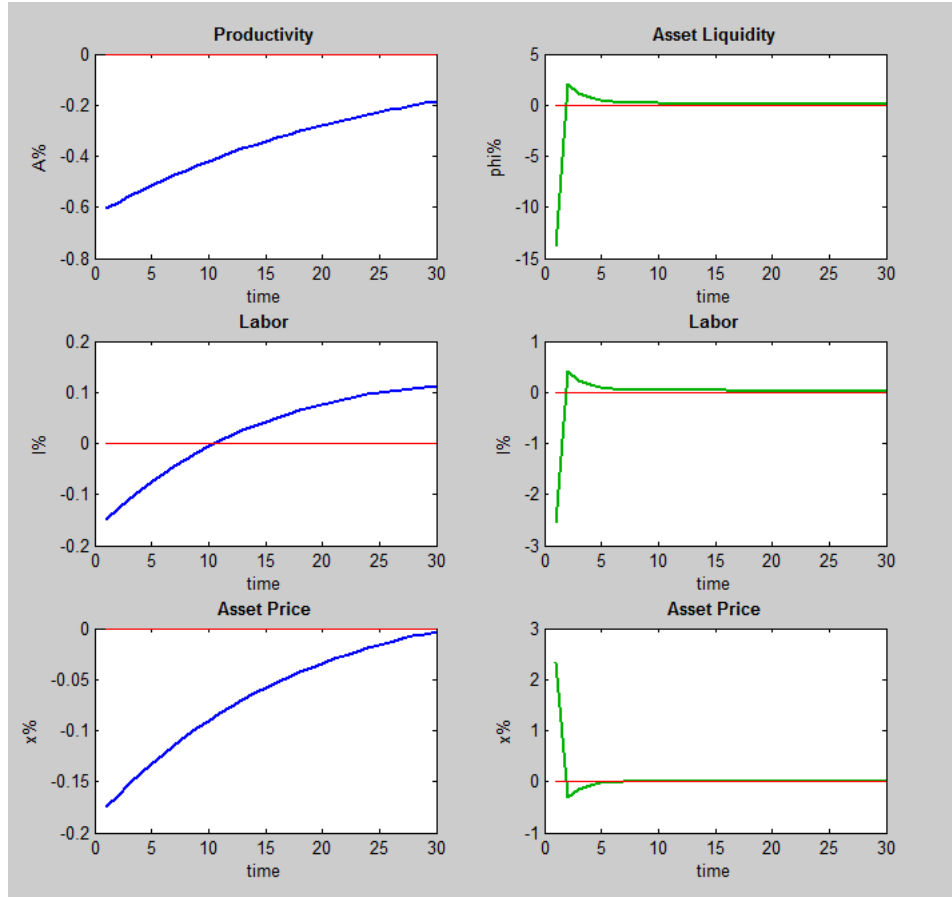
Figure 7: *Impulse Responses: a Productivity Shock and a Liquidity Shock*

assumptions have been used, the aggregate effect of productivity and asset liquidity shocks always accounts for approximately 50% of the variations in GDP. However, this result does not mean that search frictions cannot help clarify the components of business cycles. In this section, I document the importance of search frictions in matching certain cyclical properties.

My estimated model with search frictions explains well the behaviors of asset liquidity, total output and labor input. Table 7 reports the correlations and relative volatilities among some key business cycle variables for different models. In the table, the first column reports the results from my empirical analysis, the second column reports the results from the search model, and the third column lists the results from the model without search frictions in the asset market. The results from Kiyotaki and Moore (2008) are reported in the fourth column.

Search frictions visibly increase the correlations of productivity with some of the key business cycle variables. The search model generates correlations between productivity
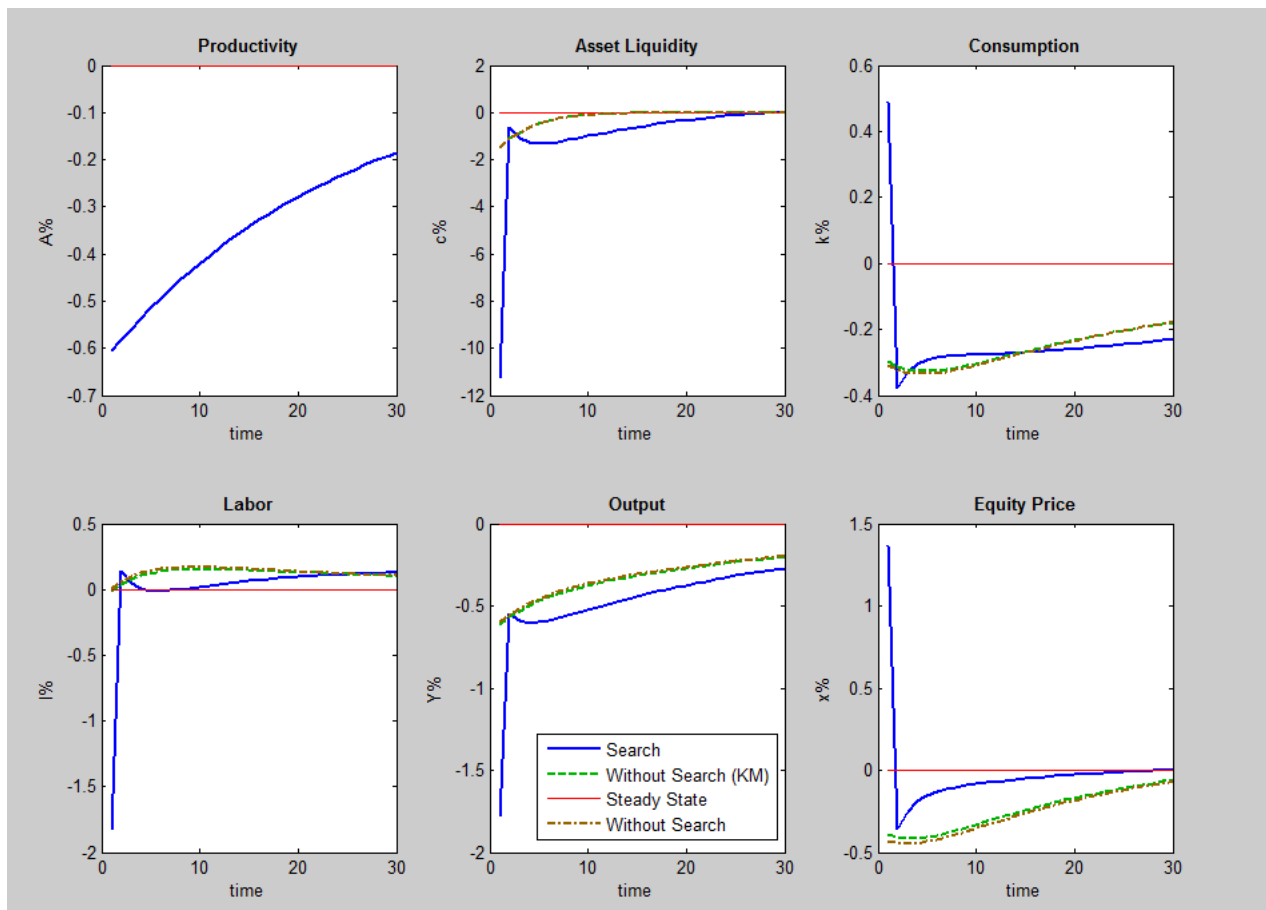
Figure 8: *A Comparison among Models with and without Search Frictions: Impulse Responses to One Standard Deviation Productivity Shocks and Accompanied Liquidity Shocks*

and asset liquidity and between productivity and labor input, $corr(A, \phi^s)$ and $corr(A, L)$, which are identical to the empirical results. In contrast, the correlations generated by the model without search frictions are much lower. The search model is also able to generate positive correlations between productivity and output and between labor input and output, $corr(A, L)$ and $corr(L, Y)$ that are nearly identical to those observed in the data. The search model also replicates relatively well both asset liquidity volatility and labor input volatility, although the volatility of productivity is somewhat understated. Thus, search frictions seem to be necessary for an accurate assessment of the importance of productivity shocks in business cycles.

Having shown that the estimated search model fits the data quite well, I use the model to address a related question of how the productivity shocks have contributed to the variations in GDP in the U.S.. Figures 4 and 9 provide a visual representation. I simulate each model (with and without search frictions) for 527 periods. In each period, two shocks are randomly

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Table 7: Correlations and Volatilities

Correlation[a][b]	US data	Search Model	Without Search	Without Search (KM)
$corr(A, \phi^s)$[#]	0.36	0.36(0.14)	0.05(0.15)	0.08(0.15)
$corr(A, L)$[#]	0.25	0.25(0.11)	$-0.04(0.15)$	$-0.004(0.15)$
$corr(A, Y)$[#]	0.6	0.6(0.14)	0.32(0.13)	0.3(0.13)
$corr(L, Y)$	0.85	0.93(0.01)	0.93(0.03)	0.93(0.03)
Volatilities				
σ_{ϕ^s}/σ_A[#]	20	20(0.07)	26(3.75)	27(3.75)
σ_A/σ_Y	0.62	0.37(0.01)	0.35(0.06)	0.37(0.06)
σ_L/σ_Y	1.1	1.33(0.01)	1.51(0.09)	1.49(0.09)

[a]Numbers in parentheses are standard deviations over 1000 simulations

[b]I simulate the model 1000 times. In each simulation, I randomly select 527 draws from the distributions of shocks. The length of the artificial time series equals that of the data: 127. The artificial time series are detrended using an HP-filter with $\lambda = 1600$.

drawn from the distributions discussed in Section 5.2. Then, I compute the variations in GDP and illustrate them in Figure 9. The first panel shows the result from the model with search frictions, and the decomposition results from the non-search models are displayed in panels 2 and 3. The blue areas are the variations in GDP explained by the productivity shocks, and the red areas are the variations explained by the efficiency shocks. As Figure 9 shows, the contribution of productivity shocks to variations in GDP is notable in the search model which is qualitatively similar to the result of the empirical model illustrated in Figure 4. In contrast, the productivity shocks seem irrelevant to the variations in GDP in the non-search models.

Table 8 presents the numerical results of the variance decomposition. The first panel reports the accumulated decomposition of the variance of GDP, and the second panel reports the accumulated decomposition of the variance of asset liquidity. The first row shows the results from the data.[16] [17] The second row reports the results of the search model, and the third and the forth rows report the results of the models without search frictions. In the search model, the productivity shocks account for approximately 58% of the variations in GDP, which is higher than what is observed in the data. However, these results are much better than the results from the model without search frictions. Moreover,

[16]Because only two shocks are studied in each model, to enable a proper comparison, I redefine the variables as the components explained by productivity and liquidity shocks only when I compute the results in the first row.

[17]I assume that liquidity variations, which are not explained by productivity shocks, are generated by the matching efficiency shocks. This approach is consistent with my theorem result. Therefore, the variations in GDP, which are led by the standard error of asset liquidity in the empirical model, are considered to be the contribution of the matching efficiency to GDP fluctuations.
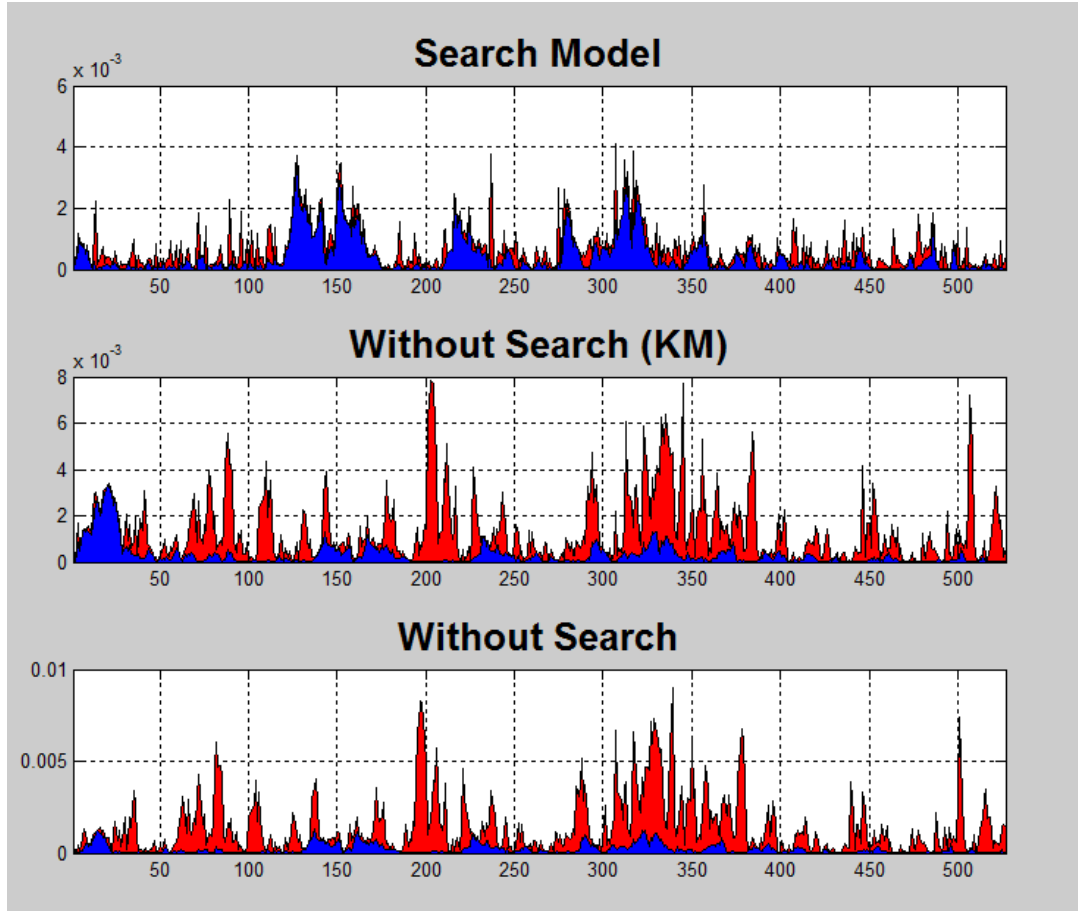
Figure 9: *Simulated GDP Decomposition*

in contrast to the models without search frictions, the search model is able to account for the contribution of productivity shocks on asset liquidity, although this contribution is somewhat overstated.

Table 8: Decompositions of GDP and Asset Liquidity

Shocks	GDP		Asset Liquidity	
	Productivity	Matching eff.	Productivity	Matching eff.
Data	0.48	0.52	0.03	0.97
Search Model	0.58	0.42	0.07	0.93
Without Search	0.14	0.86	0	1
Without Search (KM)	0.22	0.78	0	1

6.4 Why are the Indirect Effects of Productivity Shocks on GDP so Important?

As the results show in Table 8, the contribution of productivity shocks on GDP significantly increases to 58%. In addition, only 7% of the movement in liquidity is driven by the endogenous response to productivity shocks. One can question whether the indirect effects of productivity shocks through asset liquidity are still important in boosting the contribution of productivity shocks on GDP? I discuss this question further in this section.

The production function implies that the effect of a shock on GDP depends on its contributions to the fluctuations of capital investment and labor input. In the model without search frictions, an unexpected decrease in productivity reduces labor input by reducing the marginal productivity of labor. However, asset liquidity is fixed. Thus, the capital investment changes very little. The restriction is imposed that labor input cannot undergo a large decrease that leads to insufficient funds to clear the capital market. However, an unexpected decrease in asset liquidity reduces capital investment by reducing entrepreneur's investment capacity. In response to the large decline in capital investment, the labor input also decreases significantly. Therefore, liquidity shocks dominate productivity shocks in explaining business cycles. In contrast to the model without search friction, an unexpected productivity shock reduces the buyer's willingness to trade by reducing the marginal productivity of capital in the search model, thereby reducing the entry rate. These reductions explain approximately 7% of the variance in asset liquidity and only approximately 7% of the variances in capital investment and labor input. By inference, productivity shocks may explain an additional 7% of the variations in GDP. Why are the effects of productivity shocks on GDP so large, as shown in Table 8? In fact, the exogenous liquidity shocks to matching efficiency have a lesser effect on capital and labor than productivity shocks. An unexpected decrease in matching efficiency reduces entrepreneurs' investment capability without reducing the marginal productivity of capital. Thus, more buyers will enter the asset market to increase capital investment. Figure 10 compares the entry rates from a model with a productivity shock and a model with a matching efficiency shock. The blue solid line represents the entry rates driven by a productivity shock, and the green dashed line represents those driven by a matching efficiency shock. The matching efficiency shock raises the entry rate. This positive effect on entry rate reduces the impact of the matching efficiency shocks on capital and labor. In contrast, an unexpected negative productivity shock reduces the marginal productivity of capital and labor, which, in turn, decreases asset liquidity by reducing the measure of buyers in the asset market. Therefore, although

both shocks reduce asset liquidity, they act through different channels. A productivity shock reduces buyers' entry rate and, hence, further decreases capital investment and labor input. Thus, the indirect effects of productivity shocks acting through asset liquidity are still important in explaining business cycles, even though productivity shocks can only explain approximate 7% of the variation in asset liquidity.
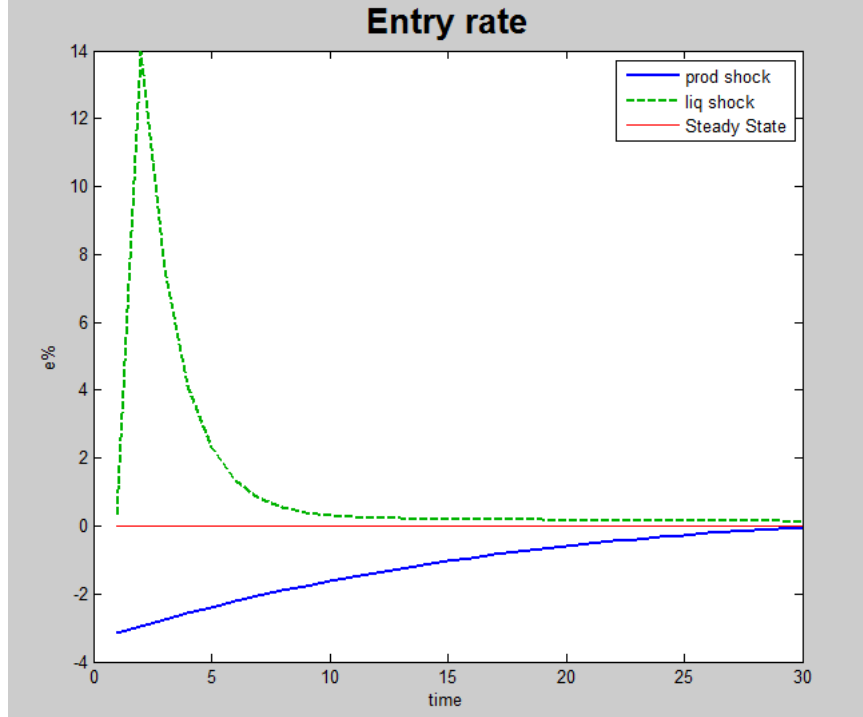

Figure 10: *Entry Rate of the Asset Market*

In summary, it is clear that the search model with productivity shocks can generate procyclical movements of labor input and asset prices. Moreover, imposing search frictions in the asset market is necessary for precisely estimating the importance of productivity shocks in business cycles. In contrast to the model with search frictions, the non-search model understates the correlations of productivity with some key business cycle variables and the volatility of productivity and, hence, severely underestimates the contribution of productivity shocks to business cycles.

6.5 Simulating the Financial Crisis: the Impact on Output, Asset Prices and Labor Input

Having shown that the estimated search model explains well the behaviors of total output, asset liquidity and asset prices under general conditions, another open question

is whether this model can capture certain specific facts in the current financial crisis. Particularly pertinent is the question of whether the search model can still generate pro-cyclical movements of asset prices and labor input given huge declines on asset liquidity and total productivity. The top two panels of Figure 11 depict productivity and liquidity shocks from the U.S. quarterly data in the period $2008q1 - 2011q3$. The bottom three panels show the responses of output, asset prices and labor input to productivity and the matching efficiency shocks.[18] It is clear that both total output and labor input decrease with negative joint shocks on productivity and the matching efficiency. However, the asset price is counter-cyclical in all cases. Thus, the counterfactual movement of asset prices is still a puzzle.

Comparing the dynamic paths of asset prices from the models with and without search frictions, we can see that the search model always generates smaller counter-cyclical movements of asset prices than the non-search model. In the search model, a negative productivity shock reduces asset demand by decreasing the marginal product of capital, and thus reduces the asset price. At the same time, a lower marginal product of capital also reduces buyers' incentives to trade in the asset market, and thus decreases their participation rate in the market. This lower participation rate reduces asset demand and, hence, further reduces the asset price. In contrast, the decrease on asset demand generated by a productivity shock acting through the buyer's participation rate does not exist in the model without search frictions. Therefore, although my model does not solve the puzzle of the counter-cyclical movement of asset prices in the current financial crisis, it points in the right direction toward solving the puzzle.

6.6   Robustness

Four alternative parameter sets[19] are evaluated to test the robustness of my results. Figures 12 - 19 provide visual representations. In Figures 12 - 15, two shocks, the productivity and matching efficiency shocks, are imposed simultaneously in the model. The imposed responses generated only by productivity shocks are illustrated in Figures 16-19. In each case, I find the following: productivity shocks generate large effects on asset liquid-

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[18]The Taylor approximation has been used to generate the dynamic paths. This approximation is accurate only if the shocks are in the neighborhood of the steady state. Clearly, both shocks are far from the steady state during the crisis. However, to my knowledge, for dynamic estimations with sequential shocks, this is most likely the best approach.

[19]In the first case, I test the power γ in the matching function. In the second case, I test the constant ϕ_0 in the matching function. In the third case, I test the fraction of entrepreneur π. In the last case, I test the relative risk aversion ρ. The widely used values of ρ are in the range $[1, 2]$. Thus, I choose $\rho = 1$ and $\rho = 1.5$ as two alternative cases.
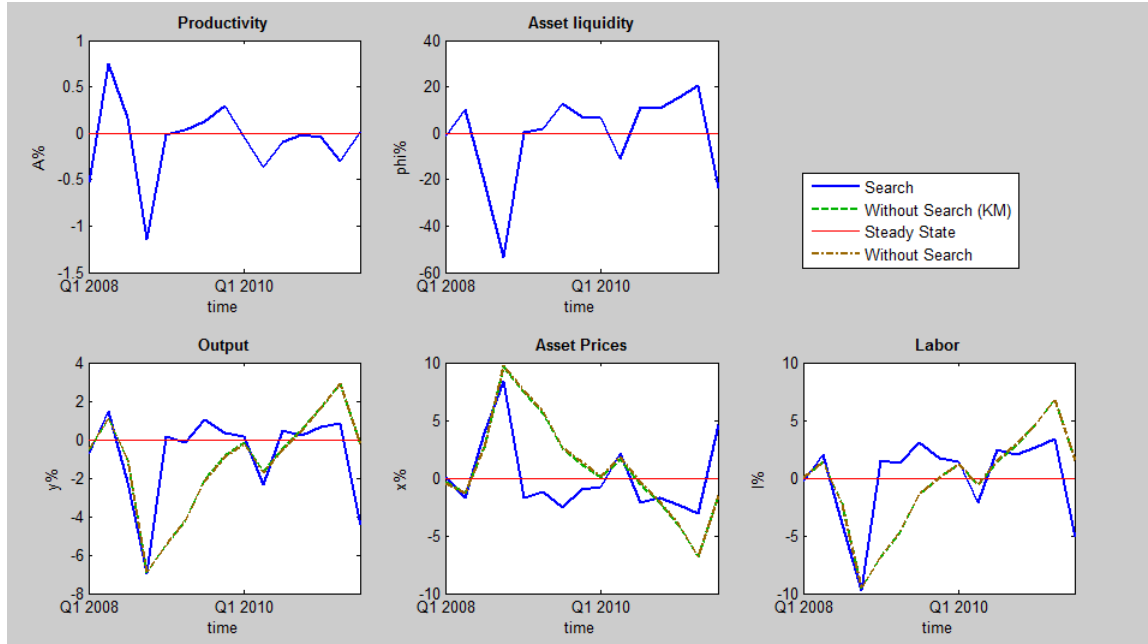
Figure 11: *Responses of Output, Asset Prices and Labor Input*

ity; asset liquidity and the asset price are positively co-moved with a negative productivity shock; the asset price and labor input decrease when productivity falls. These results are qualitatively consistent with the results from the basic model. In this sense, my results are robust.

7   Conclusion

I have constructed a model with search frictions in the asset market to study the importance of productivity shocks in business cycles. Two key features of the model are the following: (1) search frictions in equity market cause the asset price to depend on the market tightness; (2) only part of the population participates in the asset market. The asset market participation rate depends on the expected trading benefit in the market.

I calibrate the model and show that is does well in quantitatively matching some properties of the U.S. data. First, productivity shocks are as important as liquidity shocks in business cycles. Second, productivity shocks generate a pro-cyclical movement of labor input and also generate a positive co-movement between asset price and liquidity, which is not the case for liquidity shocks. However, the price effect generated by productivity shocks is not strong enough to solve the puzzle of the counterfactual movement of asset prices. Third, the model generates significant positive correlations between productivity

and output, between productivity and labor input and between productivity and asset liquidity, which are in line with my empirical findings.

My model could be used to study a number of other issues associated with business cycles. In particular, one could allow for money in the model and assess the effects of various monetary policies, such as policies that can increase asset liquidity by allowing the government to buy illiquid assets from private firms. These policy interventions increase the amount of liquid assets circulating in the economy, stimulate firms' investments, and boost the asset prices. The higher the asset prices, the lower the incentives private buyers will have to trade assets. Therefore, the effects of the monetary policies may not be so optimistic because they are mitigated by reducing the number of private buyers.

Alternatively, one could allow for heterogeneity in asset demand and study individual decisions and the wealth distribution. Thus, the model can also be used to study the effects of productivity and liquidity shocks on the wealth distribution and Gini coefficient.
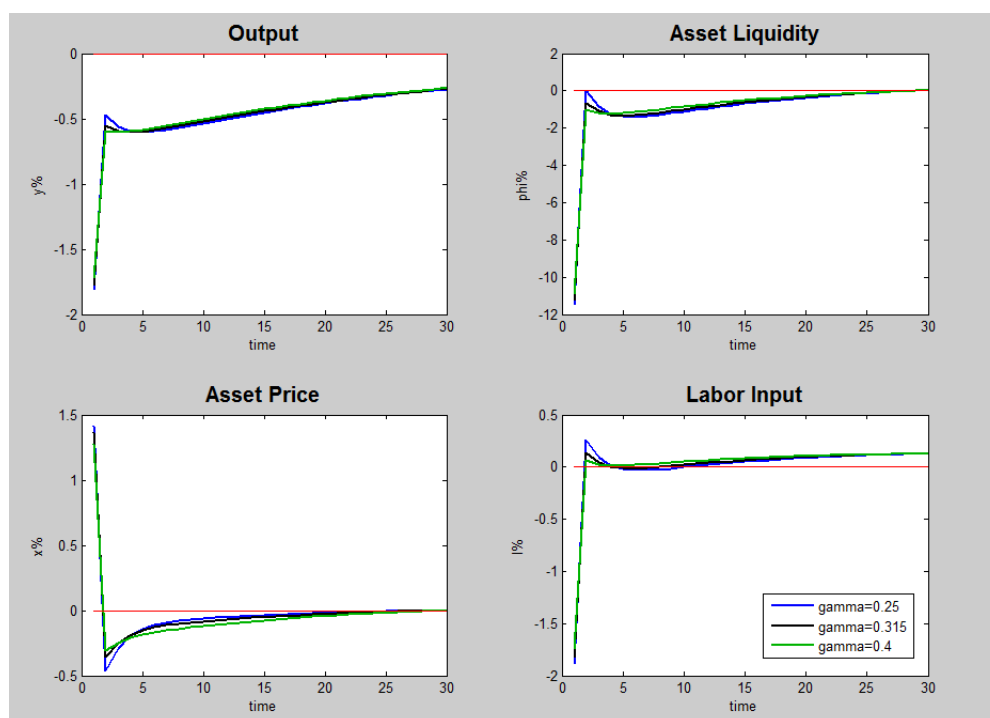

Figure 12: *Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different γ*
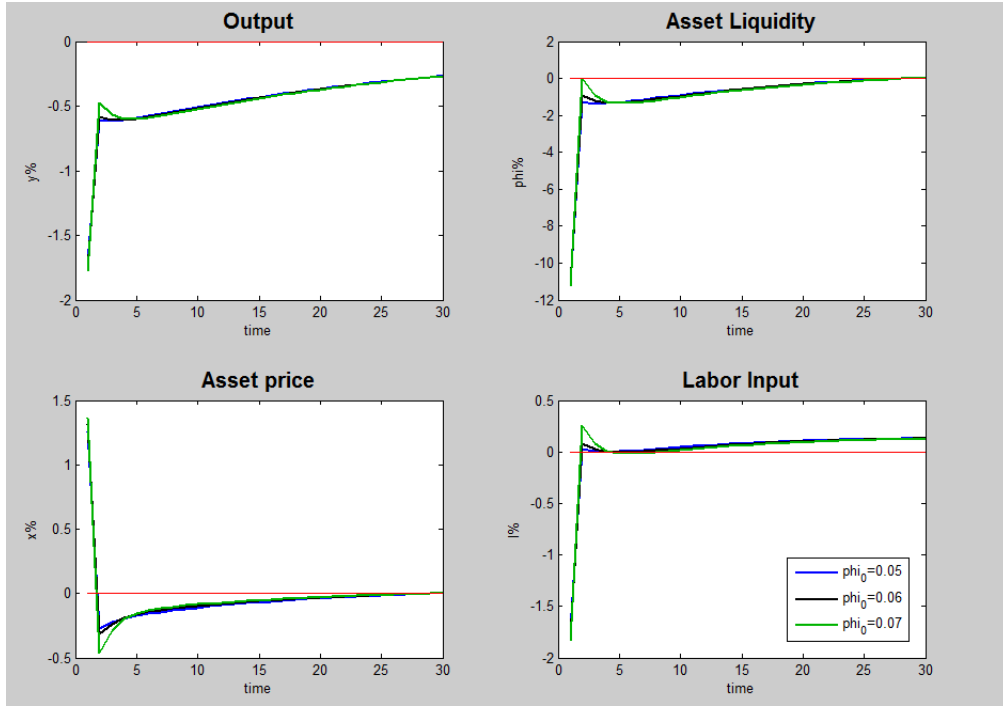
Figure 13: *Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different ϕ_0*
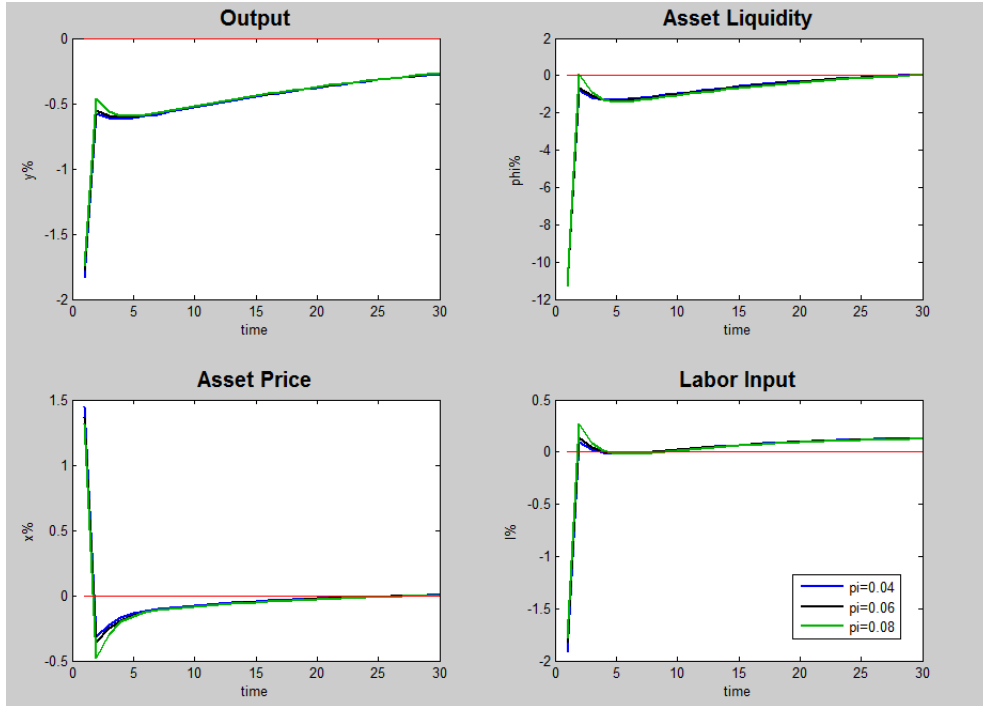

Figure 14: *Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different π*
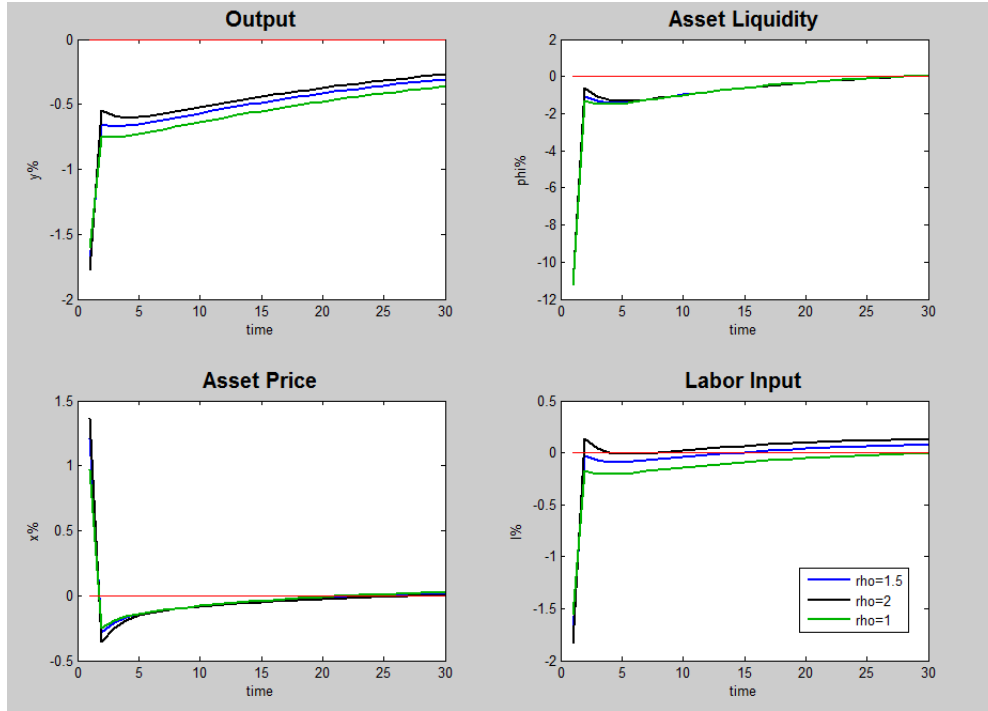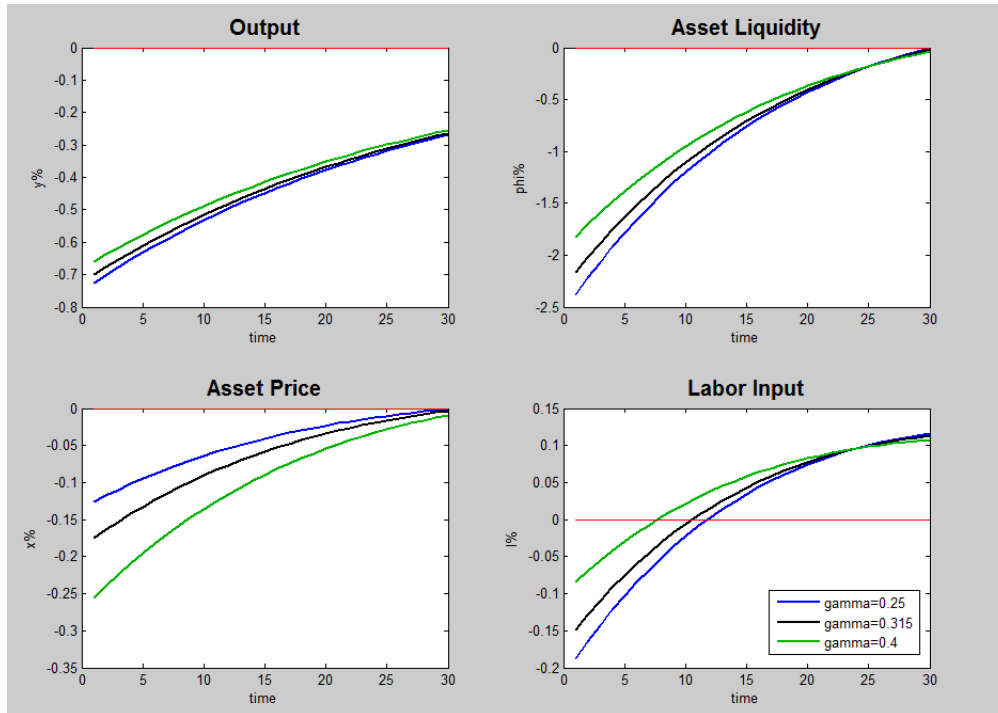
Figure 15: *Impose Responses to One Standard Deviation of Productivity Shocks and Accompanied Shocks to the Matching Efficiency with Different ρ*

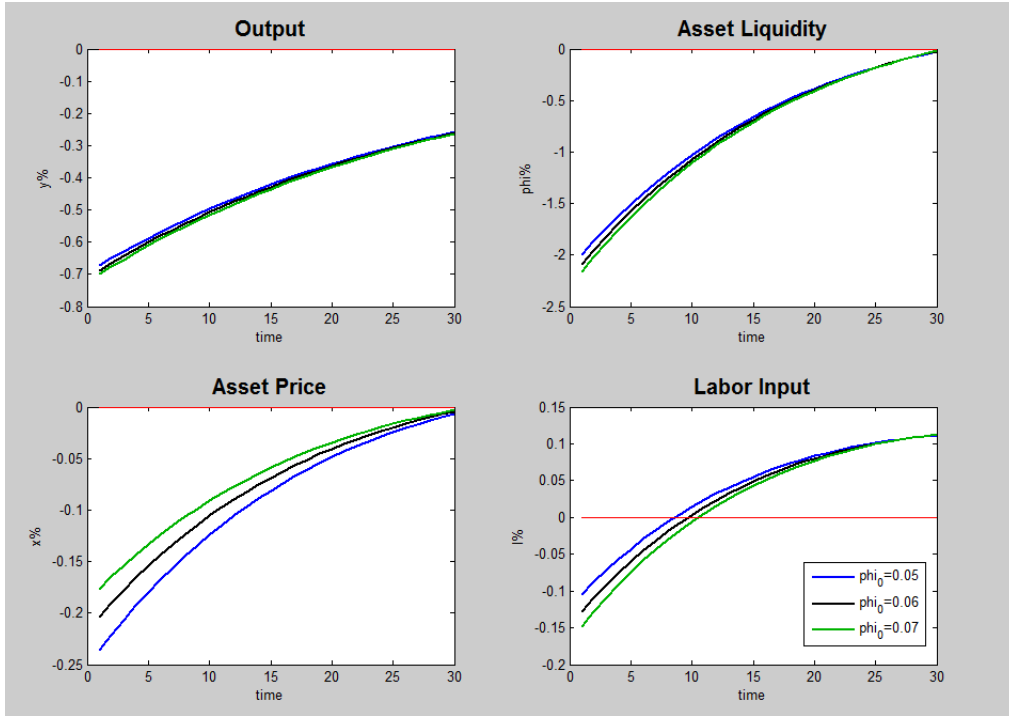Figure 16: *Impose Responses to One Standard Deviation of Productivity Shocks with different γ*

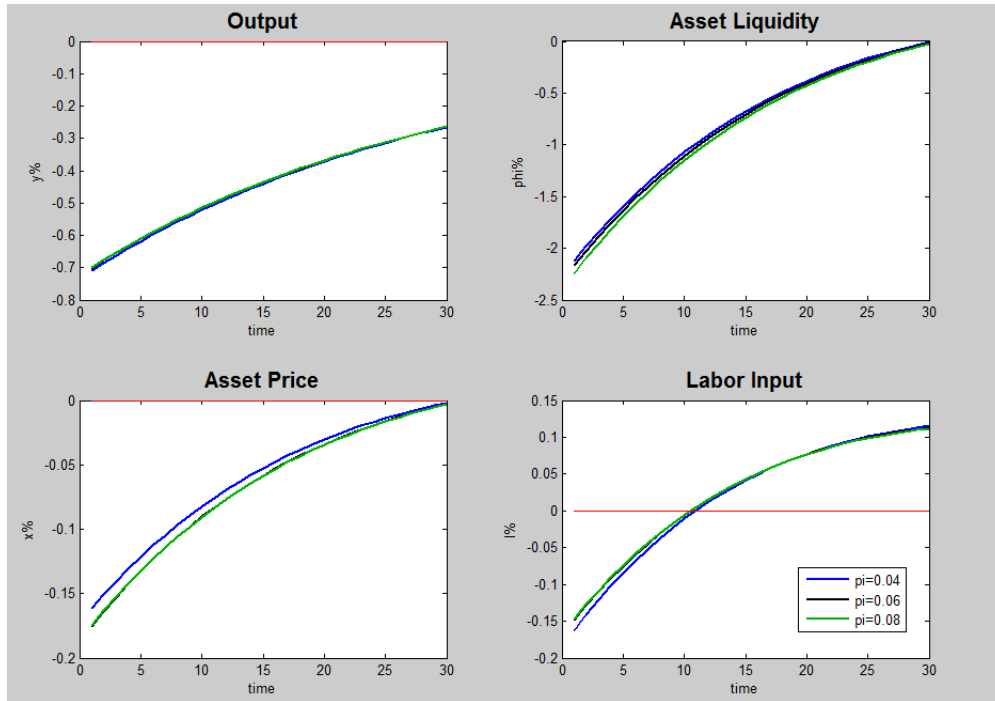Figure 17: *Impose Responses to One Standard Deviation of Productivity Shocks with different ϕ_0*

Figure 18: *Impose Responses to One Standard Deviation of Productivity Shocks with different π*

Figure 19: *Impose Responses to One Standard Deviation of Productivity Shocks with different ρ*
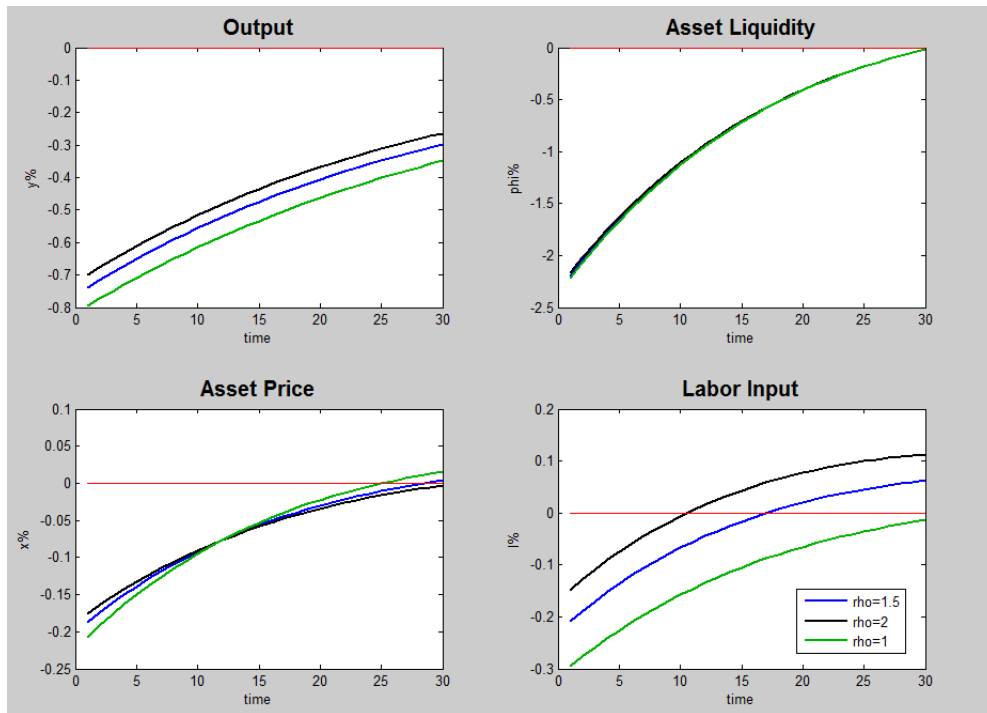
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Appendix:

A: Proof of Lemmas 1 and 2

Lemma 1:

According to (20), c^e is 0 if the following condition holds

$$\pi u'(c) < \lambda^e \pi + \lambda^m \beta E v_{s_{+1}} \pi \phi^s. \tag{49}$$

From (10) and (19), I obtain

$$\pi u'(c) = \pi \frac{\lambda^e}{X},$$

where π is less than 1. If X is greater than 1, $\pi u'(c)$ is always less than $\lambda^e \pi$. Because the last term of (7) is nonnegative, condition (7) always holds. In the household problem section, I have shown that the asset price X is greater than i if the liquidity constraint is binding. Therefore, if the liquidity constraint is binding, the entrepreneurs' consumption c^e is zero. The binding condition is only a sufficient condition of this result, not a necessary condition.

Lemma 2:

Replacing i in the capital market clearing condition with the entrepreneurs' budget constraint yields

$$k^* = \sigma k^* + \pi r^* k^* + \pi \phi^s X^* q^*. \tag{50}$$

The superscript $*$ indicates the value at the steady state. The liquidity constraint is binding. Thus, from (26), the equity trade, q, can be rewritten as

$$q^* = \frac{(r^* + \frac{\sigma}{\theta})k^*}{\frac{1}{\theta} - \phi^s X^*} > 0. \tag{51}$$

Using (51) to replace q in (50) yields

$$(1 - \sigma)(\frac{1}{\theta} - \phi^s X^*) = \pi r^* (\frac{1}{\theta} - \phi^s X^*) + \pi \phi^{s*} X^* (r^* + \frac{\sigma}{\theta}). \tag{52}$$

Equation (25) holds with equality because q^* is positive. Using (25) to replace λ^m in asset

price gives

$$
\begin{aligned}
(\frac{1}{\theta} - \phi^s X^*)X^* &= \beta\{(\frac{1}{\theta} - \phi^s X^*)r^* + (\frac{1}{\theta} - \phi^s X^*)X^*\sigma \\
&+ (X^* - 1)[\pi r^*(\frac{1}{\theta} - \phi^s X^*) + \pi\phi^{s*}X^*(r^* + \frac{\sigma}{\theta})]\}.
\end{aligned}
\tag{53}
$$

Combining (52) and (53) yields the asset price function at the steady state as

$$
X^* = F(r^*) = \frac{\beta}{1-\beta}r^* - \frac{\beta}{1-\beta}(1-\sigma).
\tag{54}
$$

The discount factor β is positive and less than 1. Therefore, the asset price X and capital rate r are positively correlated.

B: Proof of proposition 1

There are three steps to prove proposition 1. In step 1, I show there is an equilibrium capital holding when $e = 1$. Next, I show that the equilibrium asset price is within the domain $(1, \frac{1}{\theta})$ in step 2. In the last step, I show that the condition $\pi X q u'(c)(X + \lambda^m X \phi^s - 1)\frac{d\phi^s}{de}|_{e\in[0,1]} \geq (1-\pi)c^f$ holds. The left hand side is the marginal benefit of entering the market. The right hand side is the marginal cost. To achieve an equilibrium with $e = 1$, the marginal benefit must be no less than the marginal cost for all e in the domain $[0,1]$. Otherwise, fewer workers will enter the equity market.

Step 1: From (21), I obtain the labor supply as

$$
L = (\frac{u_0 A(1-\alpha)}{\eta})^{\frac{1}{\eta-1+\alpha}} K^{\frac{\alpha}{\eta-1+\alpha}}.
$$

Define BB as:
$$
BB = (\frac{u_0 A(1-\alpha)}{\eta})^{\frac{1}{\eta-1+\alpha}}.
$$

The capital rate and equity price x can be rewritten as

$$
r = r(K:A) = A\alpha(\frac{L}{K})^{1-\alpha} = A\alpha BB^{1-\alpha} K^{\frac{(1-\alpha)(1-\eta)}{\eta-1+\alpha}}
\tag{55}
$$

$$
X = X(K:A) = \frac{\beta}{1-\beta}A\alpha BB^{1-\alpha} K^{\frac{(1-\alpha)(1-\eta)}{\eta-1+\alpha}} - \frac{\beta}{1-\beta}(1-\sigma),
\tag{56}
$$

where η is strictly greater than 1. Given any productivity level, r and X decrease in K.

In a steady state, capital holdings are identical across the period. Suppose that K^1 is the steady state capital holding. Canceling K^1 on both sides of the capital law of motion and replacing q by (26) yields

$$(1 - \sigma) = \pi r(K^1 : A) + \pi \phi^s X(K^1 : A) \frac{r(K^1 : A) + \frac{\sigma}{\theta}}{1/\theta - \phi^s X(K^1 : A)}. \tag{57}$$

RHS(LHS) is denoted as the right (left) hand side of equation (57). LHS is a constant. $r(K : A)$ and $x(K^1 : A)$ decrease in K, and $\bar{\phi}^s$ is a constant with $e = 1$. Because $K \to 0 \Rightarrow RHS \to \infty$; $K \to \infty \Rightarrow RHS \to 0$, K^1 is the unique solution that satisfies (57). All the results are based on the assumptions that the liquidity constraint is binding and the entry rate is 1.

Step 2: In this step, I show that the asset price $X(K^1 : A)$ satisfies $1 < X(K^1 : A) < \frac{1}{\theta}$. Suppose $K = \underline{K}$ when $X(K : A) = \frac{1}{\theta \phi^s}$. This relation implies that $RHS(\underline{K}) = \infty$. RHS is a decreasing function of K, which means $\underline{K} < K^1$. Suppose $K = \bar{K}$ when $X(K : A) = 1$. From (55), I obtain

$$r(\bar{K} : A) = \frac{1}{\beta} - \sigma.$$

The difference between $LHS(\bar{K})$ and $RHS(\bar{K})$ is

$$\underbrace{(\frac{1}{\theta} - \bar{\phi}^s)(1 - \sigma) - \frac{\pi}{\beta\theta}}_{\text{part 1}} + \underbrace{\sigma\pi\frac{1}{\theta}(1 - \bar{\phi}^s)}_{\text{part 2}}.$$

Part 2 is positive. Condition (38) is required to show that part 1 is also positive. π is the fraction of entrepreneurs and should be a very small number. Thus, (38) is most likely to be held. Because RHS(K) is a decreasing function of K, $\bar{K}$ is strictly greater than K^1. The steady state capital holding, K^1, is in the domain $(\underline{K}, \bar{K})$. The price function $X(K : A)$ decreases in K, which implies that $X(K^1 : A)$ is greater than 1 and less than $\frac{1}{\theta\phi^s}$. Thus, the liquidity constraint is binding at the point $K = K^1$. $\bar{K}$ and $\underline{K}$ are solved by setting the value of (56) to 1 and $\frac{1}{\theta\phi^s}$, respectively.

$$\bar{K} = [\frac{\frac{1-\beta}{\beta} + 1 - \sigma}{A\alpha BB^{1-\alpha}}]^{\frac{\eta-1+\alpha}{(1-\alpha)(1-\eta)}} \tag{58}$$

$$\underline{K} = [\frac{\frac{1}{\theta b}\frac{1-\beta}{\beta} + 1 - \sigma}{A\alpha BB^{1-\alpha}}]^{\frac{\eta-1+\alpha}{(1-\alpha)(1-\eta)}}. \tag{59}$$

Because $\frac{1}{\theta\phi^s}$ is greater than 1 and $1 - \eta$ is negative, $\bar{K}$ is greater than $\underline{K}$. The domain $(\underline{K}, \bar{K})$ is not an empty set.

Step 3: Finally, I show that $e = 1$ is an optimal solution, which requires that the marginal benefit of entering the market be no less than the entry cost for all e in the domain $[0, 1]$. According to the condition (57), a lower level of e is associated with a smaller number of K. Capital rate and equity price decrease in K and, hence, decrease in e. The marginal benefit of entering the market at $e = 1$ is

$$\frac{\pi}{1 - \pi}X(K^1 : A)q(K^1 : A)u_0[X(K^1 : A) - 1 + \frac{\theta(X(K^1 : A) - 1)}{1 - \theta\bar{\phi}^s X(K^1 : A)}X(K^1 : A)\bar{\phi}^s]\frac{\partial\phi^s}{\partial e}|_{e=1}. \tag{60}$$

Both the capital rate and asset price decrease when the entry rate decreases. The lowest value of equilibrium q is greater than the value of $\underline{q}$. $\underline{q}$ is defined as

$$\underline{q} = \frac{r(\bar{K}) + \frac{\sigma}{\theta}}{\frac{1}{\theta}}\underline{K},$$

which is solved from (51) when r, K and $\phi^s X$ are at the lowest levels. Defining c^{f1} as

$$c^{f1} = \frac{\pi}{1 - \pi}X(K^1 : A)\underline{q}u_0[X(K^1 : A) - 1 + \frac{\theta(X(K^1 : A) - 1)}{1 - \theta\bar{\phi}^s X(K^1 : A)}X(K^1 : A)\bar{\phi}^s]\frac{\partial\phi^s}{\partial e}. \tag{61}$$

The value of c^{f1} is strictly lower than the smallest marginal benefit of entering the market. Therefore, if the marginal cost, c^f, is no more than c^{f1}, the optimal entry rate decision is 1. Because $X(K^1 : A) - 1$ and $\underline{q}$ are positive, c^{f1} is strictly greater than 0. Thus there always exists a positive c^f that is no more than c^{f1}. Consumption, investment and trade decisions can be solved from first order conditions and market clearing conditions.

Equations (55) and (56) imply that the capital rate and asset prices are increasing in productivity. As shown in Figure 20 a negative productivity shock shifts $D(x)$ to the left, which leads to a decrease in the steady state capital holdings. Therefore, the capital holdings also increase in productivity.

C: Proof of proposition 2

Two steps are needed to prove the existence of the equilibrium. In the first step, I suppose

there is an equilibrium such that the liquidity constraint is binding. I also solve a feasible range of c^f. Within the range, I can always find at least one equilibrium. In the second step, I show that the liquidity constraint is binding at this equilibrium. This proof requires that I show that the equilibrium price X should be greater than 1 and lower than $\frac{1}{\theta b}$.

Step 1: Suppose the liquidity constraint binds. According to the capital market clearing condition (57), the capital holding can be rewritten as

$$K = f(e),$$

where $f(e)$ is a continuous function in the domain $e \in [0, 1]$. According to (51), (55) and (56), the quantity of trade, capital rate and asset prices can also be modeled as a continuous function of e in its domain. The marginal benefit of entering the market is

$$\frac{1}{1-\pi}\pi X(e)q(e)u_0[X(e) + \frac{\theta(X(e)-1)}{1-\theta\phi^s(e)X(e)}X(e)\phi^s(e) - 1]\phi_0\gamma(\frac{1-\pi}{\pi})^\gamma(e^2)^{\gamma-1}. \quad (62)$$

The marginal benefit is a product of continuous functions, and hence, it is also a continuous function of e in the domain $e \in [0, 1]$. When e equals zero, the marginal benefit equals infinity. Suppose c^{f2} satisfies

$$\begin{aligned} c^{f2} &= \frac{1}{1-\pi}\pi X(K^1)q(K^1)u_0[X(K^1) \\ &+ \frac{\theta(X(K^1)-1)}{1-\theta\bar{\phi}^s X(K^1)}X(K^1)\bar{\phi}^s - 1]\phi_0\gamma(\frac{1-\pi}{\pi})^\gamma(e)^{\gamma-1}|_{e=1} > 0. \end{aligned} \quad (63)$$

The right hand side is the marginal benefit when the entry decision, e, equals one. Because X is strictly greater than 1, c^{f2} is positive. If the entry cost, c^f, is larger than c^{f2}, according to the fixed point theorem, there is at least one intersection between the marginal benefit of entering the market and the marginal cost. If the intersection is unique, it is the unique solution of this model. If the number of the intersection is greater than one, multiple solutions may exist.

Step 2: Suppose (e^2, K^2) is one of the intersections I obtain from step 1. In this step, I show that the liquidity constraint is binding at (e^2, K^2). This step requires that the equilibrium price X^2 be greater than 1 and less than $\frac{1}{\theta}$. The procedures are the same as in the proof of proposition 1. Given $e = e^2$, $\phi^s(e)$ is a constant, and RHS is a strictly decreasing function of K. At $\underline{K}$, the price is $\frac{1}{\theta}$, and RHS is equal to infinity. At $\bar{K}$, the price is 1, and RHS is

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greater than LHS if the condition

$$(\frac{1}{\theta} - \phi^s)(1 - \sigma) > \frac{\pi}{\beta\theta} \tag{64}$$

is satisfied. In this case, the broker's matching rate, ϕ^s, is not a constant but depends on the entry rate e. Hence, condition (64) changes as the value of e changes. When e equals one, the right hand side of equation (64) reaches its lowest level. Using $\bar{\phi}^s$ to replace ϕ^s in (38), I obtain (38). If condition (38) holds, RHS is greater than LHS for all e in the domain $[0, 1]$. Therefore, there is at least one K between $\underline{K}$ and $\bar{K}$ where RHS is equal to LHS. The equilibrium asset price is a decreasing function of K. Therefore, X^2 is in a domain $(1, \frac{1}{\theta})$. K^2 is one of the K values that makes RHS equal to LHS. Substituting K^2 and e^2 into (62), I obtain a unique positive c^{f2} where (K^2, e^2, x^2) are equilibrium values of the capital holding, entry rate and asset price.

K^1 is the capital holding solved from (57) at $e = 1$. Rearranging (57), I obtain

$$\phi^s(e) = \frac{\frac{1}{\theta}[1 - \sigma - \pi r(K)]}{\frac{\sigma}{\theta}\pi + (1 - \sigma)}. \tag{65}$$

A lower level of entry rate e reduces the value of $\phi^s(e)$. Restoring an equilibrium of (65) requires a lower level of capital holding K. Hence, a lower level of the equilibrium entry rate is always associated with a lower level of capital holding.

Equations (55) and (56) imply that $r(A : K)$ and $X(A : K)$ decrease as A decreases for any level of K. In (51), q is a function of r and X, which also decrease as A decreases for any level of K. Equation (27) can be rewritten as

$$\frac{\partial \phi^s}{\partial e} = (\frac{1 - \pi}{\pi})^\gamma e^{\gamma - 1} = \frac{(1 - \pi)c^f}{\pi X q u_0[X + \frac{\theta(X - 1)}{1 - \theta\phi^s X}X\phi^s - 1]}. \tag{66}$$

The value of γ is less than 1. Thus, given K, e and ϕ^s increase in A. Assuming

$$F(A : K) = \pi r(A : K) + \pi\phi^s(A : K)X(A : K)\frac{r(A : K) + \frac{\sigma}{\theta}}{1/\theta - \phi^s(A : K)X(A : K)},$$

where

$$(1 - \sigma) = D(A : K)$$

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in the steady state. The left hand side is a constant. The right hand side shifts down as A decreases. Therefore, the equilibrium capital holding K^2 decreases, as shown in Figure 20. where K^* is the equilibrium capital holding at A^*. When TFP decreases, the new
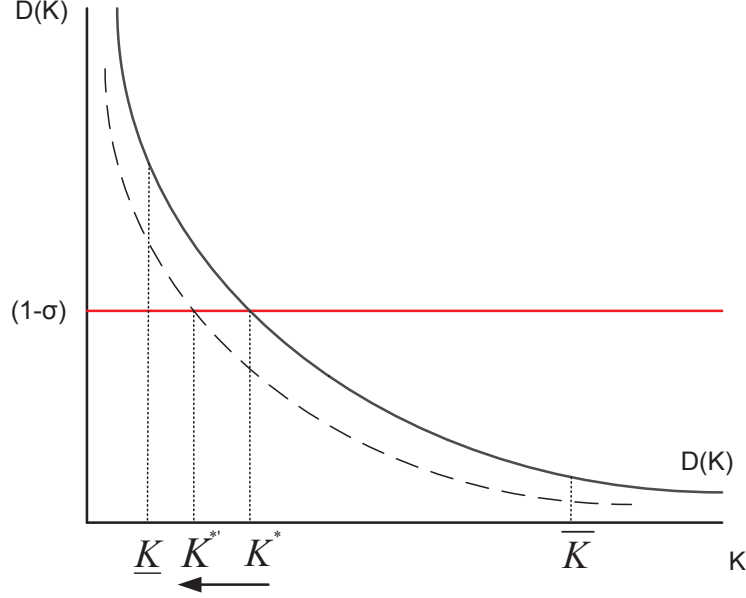

Figure 20: *Steady state value of K*

equilibrium of capital holding decreases to K^{**}, as do the entry and matching rates. In Lemma 2, the equilibrium values of X and r are positively co-moved as explained above. Suppose both X and r decrease with a negative productivity shock. In equilibrium, we have $D(A^{**} : K^{**}) = 1 - \sigma$. The decrease of X and r requires an increase of the matching rate ϕ^s to restore the equilibrium, which is a contradiction. Similarly, I can also show that X and r cannot be constant as A decreases. Thus, the only possible case is that both X and r increase with a negative productivity shock. A decrease in e increases the value of $\frac{\partial \phi^s}{\partial e}$. To maintain the equality of condition (66), the value of Xq also decreases with a negative productivity shock. The labor equation in Appendix B implies that the labor supply decreases as a negative shock on A, as does total output Y. The investment can be solved from the resource constraint of the entrepreneur:

$$i^e(A) = \frac{1}{\pi}[r(A)K(A) + \phi^s(A)X(A)q(A)].$$

Both $r(A)K(A)$ and $\phi^s(A)X(A)q(A)$ decrease in a negative productivity shock. Thus, the

equilibrium investment also increases in A. According to the market clearing condition of the goods market, I obtain

$$c(A) = Y(A) - \pi i^e(A).$$

Both $Y(A)$ and $i(A)$ increase in A, and the movement of $c(A)$ is ambiguous.

D: The model without search friction

A representative household's Bellman equation is:

$$v(s : K, A) = \max_{(l,i,s_{+1})} u(c) - h(l) + \beta Ev(s_{+1}; K_{+1}, A_{+1}) \tag{67}$$

subject to

$$rs + x(i^e + \sigma s - s_{+1}^e) \geq i^e + c^e \tag{68}$$

$$rs + \omega l^w + x(\sigma s - s_{+1}^w) \geq c^w \tag{69}$$

$$s_{+1}^e \geq (1 - \theta)i^e + (1 - b)\sigma s \tag{70}$$

$$c^w \geq 0 \quad s_{+1} \geq 0 \quad q \geq 0 \quad l \geq 0 \quad x \geq 0 \quad i \geq 0 \tag{71}$$

$$c = (1 - \pi)c^w, \quad l = (1 - \pi)l^w, \quad s_{+1} = \pi s_{+1}^e + (1 - \pi)s_{+1}^w, \tag{72}$$

where s_{+1}^e and s_{+1}^w are the entrepreneurs' and workers' asset holdings at the end of the period. Adding (68) $\times \pi$ and (69) $\times (1 - \pi)$ and replacing s_{+1}^e in (68) by (70) gives

$$rs + \omega l + (x - 1)\pi i^e + x(\sigma s - s_{+1}) \geq c \tag{73}$$

$$(r + b\sigma x)s - (1 - \theta x)i^e \geq c^e. \tag{74}$$

Supposing the multipliers of (73) and (74) are λ^{ee} and $\lambda^{mm}\pi u'(c)$, the FOCs for (c, s_{+1}, l, i^e) are

$$u'(c) - \lambda^{mm}u'(c) \leq \lambda^{ee}, \quad c^e \geq 0 \tag{75}$$

$$u'(c) \leq \lambda^{ee}, \quad c^w \geq 0 \tag{76}$$

$$\beta Ev_{s_{+1}} \leq \lambda^{ee}x \quad , \text{ and } s_{+1} \geq 0 \tag{77}$$

$$\omega u'((1 - \pi)c^w) \leq h'((1 - \pi)l^w) \quad , \text{ and } l \geq 0 \tag{78}$$

$$(x - 1) \leq (1 - \theta x)\lambda^{mm} \quad , \text{ and } i \geq 0. \tag{79}$$

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The envelope condition is

$$v_s = u'(c)(r + x\sigma) + \lambda^{mm}\pi u'(c)(r + b\sigma x). \tag{80}$$

Combining (77) and (80), I obtain the asset price as

$$x = \beta E\{\frac{u'(c_{+1})}{u'(c)}[r_{+1} + \sigma x_{+1} + \lambda^{mm}_{+1}\pi(r_{+1} + b\sigma x_{+1})]\}. \tag{81}$$

Supposing the liquidity constraint is binding, asset price x is greater than 1 and less than $1/\theta$. Equations (75) and (76) imply that the entrepreneurs' consumption c^e is 0. Steady state equations are listed below:

$$
\begin{aligned}
i^{e*} &= \frac{(1-\sigma)K^*}{\pi} \\
\lambda^{mm*} &= \frac{(x^*-1)}{1-\theta x^*} \\
x^* &= \beta[r^* + x^*\sigma + \pi\lambda^{mm*}(r^* + b\sigma x^*)] \\
\omega^* &= \frac{h'(l)}{u'(c)} \\
\omega^* &= (1-\alpha)A(\frac{K}{l})^\alpha \\
r^* &= \alpha A(\frac{l}{K})^{1-\alpha} \\
c^* &= AK^\alpha l^{1-\alpha} - (1-\sigma)K^* \\
(r^* + b\sigma x^*)K^* &= (1-\theta x^*)i^{e*}.
\end{aligned}
$$