Debt dilution and sovereign default risk*

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Abstract

We measure the effects of debt dilution on sovereign default risk and show how these effects can be mitigated with debt contracts promising borrowing-contingent payments. First, we calibrate a baseline model à la Eaton and Gersovitz (1981) with endogenous debt duration to mimic the public debt level, average duration of public debt, and the interest rate spread in Spain. In this model, the value of long-term bonds can be diluted. Second, we present a model in which sovereign bonds contain a covenant promising that after each time the government borrows, it pays to the holder of each long-term bond issued in previous periods the difference between the long-term bond price and the counterfactual price that would have been observed in the absence of current-period borrowing. This covenant eliminates debt dilution by disentangling the value long-term debt from future borrowing decisions. We quantify the effects of dilution by comparing the simulations of the model with and without this covenant. We find that dilution accounts for 78% of the default risk in the baseline economy. Similar default risk reductions can be obtained with borrowing-contingent payments that depend only on observed long-term bond prices. Using borrowing-contingent payments is welfare enhancing because it reduces the frequency of default episodes.

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Keywords: Sovereign Default, Debt Dilution, Debt Covenant, Long-term Debt, Endogenous Borrowing Constraints.

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1 Introduction

The sovereign debt crisis that started to unfold in Greece in 2010 and that spread out to other European nations in the following years led to costly fiscal consolidation packages and to disruptions in financial markets. The social costs of these events revived discussions about policies that could mitigate the likelihood and the negative effects of future debt crisis episodes.[1] But many of these discussions are not new and have been present following previous waves of sovereign defaults in developing countries.[2] This paper contributes to these discussions by presenting a measure of the effects of the debt dilution caused by governments' borrowing decisions and by showing how these effects can be mitigated with debt contracts that feature borrowing-contingent payments. If governments could commit to preserve the value of current debt issuances by limiting their future borrowing behavior and, therefore, by reducing the likelihood of a future sovereign default, they would be able to issue debt at a higher price. Sovereign debt dilution may become a problem when governments lack the ability to make such commitment.

Participants in various credit markets have made efforts to mitigate the dilution problem, which is suggestive of the relevance assigned to this issue. Corporate debt contracts often include covenants intended to limit debt dilution (Asquith et al., 2005, Smith and Warner, 1979, Rodgers, 1965, and Carey et al., 1993 discuss the use and effectiveness of corporate debt covenants) A seniority structure is common in corporate debt and collateralized loans to households— if existing debt is senior to new issuances, this may mitigate the dilution problem (Bizer and DeMarzo, 1992). In contrast, sovereign bonds typically do not present differences in legal seniority but include a pari passu clause and a negative pledge clause that prohibits future issuances of collateralized debt.[3] These clauses are intended to avoid making new debt senior to previously

[1]See, for instance, European Council (2012) and Claessens et al. (2012).

[2]For instance, Bolton and Skeel (2005), and Krueger and Hagan (2005) propose institutional modifications to sovereign debt markets. These proposals were motivated by the sovereign bond defaults of the late 1990s and early 2000s (see Sturzenegger and Zettelmeyer, 2006 for a review of these episodes). Eichengreen and Lindert (1989) discuss the sovereign debt defaults of the 1980s, which differ from the most recent episodes in terms of the sovereign debt instruments and creditor concentration. While the most recent defaults were declared on sovereign bonds held by dispersed investors around the world, the defaults of the 1980s were declared on syndicated bank loans owned by a relatively small number of banks in advanced economies. Defaults on sovereign bonds were common prior to 1940 (see Eichengreen and Lindert, 1989 and Fishlow, 1985 for a historical study of sovereign defaults).

[3]Sturzenegger and Zettelmeyer (2006) explain how in Ecuador's 2000 sovereign debt restructuring, the gov-

issued debt, but do not make existing debt senior to debt that will be issued in the future. The weaker protection against sovereign debt dilution may be due in part to the weak enforcement of sovereign debt claims.[4] Overall, it seems clear that existing sovereign debt contracts do not eliminate the risk of debt dilution.

The possibility of sovereign debt dilution has also received considerable attention in both academic and policy discussions. Several studies describe the benefits of eliminating debt dilution. For instance, Bizer and DeMarzo (1992) show how dilution may lead to equilibria with higher debt levels and higher interest rates implied by higher default probabilities. It has also been argued that dilution may lead to excessive issuance of short-term debt (Kletzer, 1984), or of debt that is hard to restructure after a default (Bolton and Jeanne, 2009), which in turn could increase the likelihood and/or severity of sovereign debt crises. Bolton and Skeel (2005) argue for the importance of being able to grant seniority to debt issued while the country is negotiating with holders of debt in default, as observed in corporate bankruptcy procedures. Borensztein et al. (2004) suggest changes in national and international laws that may facilitate the introduction of debt contracts that provide some protection against debt dilution.[5] While these studies suggest that debt dilution may be an important source of inefficiencies in debt markets, they do not quantify the effects of dilution.

We measure the effects of debt dilution using a default framework à la Eaton and Gersovitz

ernment exchanged defaulted bonds for new bonds that included a clause specifying that if a default occurred within 10 years following the restructuring agreement, the government would extend new bonds to the holders of the restructured debt. Sturzenegger and Zettelmeyer (2006) argue that the "effect of this was to offer a (limited) protection of bond holders against the dilution of their claims by new debt holders in the event of default." However, the inclusion of such debt covenants is much more an exception than a rule. It has also been argued that loans from institutions such as the International Monetary Fund or the World Bank receive de facto seniority over loans from private agents (see, for example, Saravia, 2010).

[4]This weak enforcement has lead to several proposals to induce more orderly sovereign debt restructurings (Bolton and Skeel, 2005; Borensztein et al., 2004; G-10, 2002; IMF, 2003; Krueger and Hagan, 2005; Paulus, 2002). This weak enforcement has been magnified by a more dispersed distribution of creditors over the last decades. Resorting to a syndicate to concentrate the credit supply available to governments could mitigate the dilution problem as long as the syndicate forced governments to internalize the effect of current borrowing on the value of outstanding debt. However, such an arrangement would be effective only if the government could credibly commit to borrow only from the syndicate. Besides, there may be adverse effects of too much concentration in the supply of funds. Wright (2005) shows that a higher concentration of creditors can help in attaining a constrained efficient allocation but shifts welfare gains away from the debtor country.

[5]Detragiache (1994), Eaton and Fernandez (1995), Niepelt (2008), Sachs and Cohen (1982), Tirole (2002), and UN (2004) also discuss inefficiencies raised by debt dilution.

(1981).[6] Formally, we analyze a small open economy that receives a stochastic endowment stream of a single tradable good. At the beginning of each period, when the government is not in default, it decides whether to default on its debt. While in default, the government suffers an endowment loss and cannot borrow. Each period, a government in default may be offered the opportunity of exiting the default. In order to exit the default, the government must pay a fraction of the debt in default. As in Arellano and Ramanarayanan (2012), we assume the government borrows by issuing both short-term and long-term non-contingent bonds, which implies that the debt duration is endogenous. Bonds are priced by foreign investors with preferences as in Piazzesi and Schneider (2007) and Rudebusch and Swanson (2012). We allow for shocks to the (exogenous) consumption process of foreign investors, which introduces time variation in the interest rates of default-free bonds.

We propose a new approach for the study of the effects of debt dilution. First, we modify the baseline model by assuming that long-term sovereign bonds include the following covenant: each time the government borrows, it has to compensate the holders of long-term debt issued in previous periods by paying the difference between the long-term bond price observed in the market and the counterfactual price that would have been observed in the absence of current-period borrowing. This borrowing-contingent covenant disentangles long-term bond prices from future borrowing behavior and thus, eliminates dilution caused by borrowing decisions. We measure the effects of dilution by comparing simulations of the baseline model (with dilution) with the ones of the modified model (without dilution). We impose discipline to our quantitative exercise by calibrating the baseline model to match data from Spain, an economy facing default risk.

We find that, if the sovereign eliminates debt dilution, the number of defaults per 100 years decreases from 2.8 (with dilution) to 0.6 (without dilution). That is, dilution accounts for 78% of the default risk in the simulations of the baseline model. We find that eliminating dilution

[6] This framework has been used in many recent studies. See, for instance, Aguiar and Gopinath (2006), Arellano (2008), Mendoza and Yue (2012), Boz (2011), Cuadra et al. (2010), D'Erasmo and Mendoza (2013), Durdu et al. (2013), Lizarazo (2013, 2012), and Yue (2010). These models share blueprints with the models used in studies of household bankruptcy—see, for example, Athreya et al. (2007), Chatterjee et al. (2007), Li and Sarte (2006), Livshits et al. (2008), and Sanchez (2010).

amounts to an ex-ante welfare increase equivalent to 0.4% of consumption. Thus, our exercise is indicative of the quantitative importance of dilution and supports the view that debt dilution should not be ignored in discussions of sovereign debt management and the international financial architecture (e.g., Borensztein et al., 2004).

Reducing the default frequency improves welfare because while all defaults are optimal conditional on the aggregate state at the time of the default, they need not be ex-ante optimal. In particular, eliminating dilution allows the government to choose a lower default risk. With dilution, default risk is high even if the government chooses very low debt levels that are not optimal in the current period and that would almost certainly not trigger a default in the following period. This is the case because future governments would optimally increase the debt level. Thus, as long as a government cannot control the choices of future governments, it cannot choose low default risk. Issuing long-term debt with borrowing-contingent payments allows the government to lower borrowing levels chosen by future governments.

The bond covenant that eliminates dilution also allows the government to lower its exposure to rollover risk.[7] In our benchmark economy, the government shortens the duration of its debt portfolio to mitigate the dilution problem at the expense of increasing rollover risk (see also Arellano and Ramanarayanan, 2012; Hatchondo and Martinez, 2013). With the debt covenant that eliminates dilution, the government finds it optimal to lower its exposure to rollover risk by increasing the duration of its debt portfolio. The average debt duration in the simulations is almost two years higher in the model without dilution than in the benchmark.

The second contribution of this paper is to show that gains from mitigating dilution could be obtained with debt covenants that are easier to implement. The covenant that eliminates dilution may be difficult to implement in practice. This is because the payments mandated by this covenant requires knowledge of the counterfactual price at which long-term bonds would trade in the absence of current-period borrowing. While that price can be easily computed in our simulations, it may be difficult to determine in practice. We show that most gains from

[7] Our model features endogenous rollover risk, i.e., future governments face the risk that the cost of borrowing is high (or low) at the time they have to pay maturing debt obligations. The high cost of borrowing may be either because the investors anticipate a higher default probability (for a given debt level) or because the default-free interest rate increases.

eliminating dilution can be obtained with a simple covenant that depends only on the bond price of long-term bonds. If borrowing-contingent payments depend on a constant reference price (instead of the price at which long-term bonds would trade in the absence of debt dilution), the default frequency can be reduced to 0.5 defaults every 100 years (compared to 0.6 in the economy without dilution). We also find that a borrowing-contingent covenant that promises to pay a predetermined share of current borrowing proceeds to the holder of each long-term bond issued in previous periods is less effective in bringing down the default frequency.

The debt covenants studied in this paper resemble covenants commonly used in debt markets. For instance, Chamon and Mauro (2006) show that 26 percent of government debt in emerging economies was indexed to a domestic interest rate in 2001 (an additional 7 percent was indexed to inflation). The debt covenants studied in this paper index debt payments to an interest rate that reflects default risk. In corporate debt contracts, covenants often transfer resources from debtors to creditors when credit quality deteriorates. Asquith et al. (2005) document the use and effects of such "interest-increasing performance pricing".[8] They find that interest rates are lower for debt contracts that feature these covenants, which is consistent with our results.

It should be emphasized that our findings are not based on the assumption that the government cannot default on covenant payments in the same way that it can default on other debt payments. Sovereign debt contracts often contain an acceleration clause and a cross-default clause (IMF, 2002 and Choi et al., 2011). The first clause allows creditors to accelerate all future payments owed to them if one of a set of pre-defined events of default takes place. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that in practice, when the government chooses to default on a payment it chooses to default on all its debt. The implementation of borrowing-contingent payments only requires that defaulting on borrowing-contingent payments would trigger acceleration and cross-default clauses and, therefore, a default on all government debt.

[8]Performance pricing loans explicitly make the interest on the loan a function of the borrower's current credit rating, leverage ratios, or other measures of financial performance.

1.1 Related literature

The most common modeling approach for the study of debt dilution is to focus on the effect of seniority clauses.[9] However, it is well known that seniority does not fully eliminate debt dilution if new borrowing increases the default probability (Bizer and DeMarzo, 1992). Therefore, in general, one cannot measure accurately the effects of dilution by comparing equilibria with and without seniority. Furthermore, seniority clauses may not be a practical instrument to curb debt dilution given that the weak enforcement of sovereign debt claims could be an obstacle to implementing a meaningful seniority structure: Governments typically exit defaults by offering a debt exchange that must be accepted by a sufficiently high fraction of bond holders. This limits the degree of discrimination that can be implemented with seniority clauses (holders of junior debt may oppose to participate in the exchange). In contrast, the enforcement we assume on the payments imposed by borrowing-contingent covenants is not stronger than the enforcement assumed on any other debt payment obligation.

A second approach that has been followed to study debt dilution is to compare equilibria obtained with long-term and with one-period bonds. This is in environments with an exogenous debt duration.[10] However, this approach is ill-suited to isolate the effects of debt dilution. Chatterjee and Eyigungor (2012) show that a model with only long-term debt in which creditors get fully compensated for any increase or decrease in the value of their debt claims is isomorphic to a model with only one-period bonds. Importantly, these compensation payments depend on the entire change in bond prices, not only to the fraction of the price change that is caused by borrowing decisions. This means that a model with one-period debt does more than just eliminate debt dilution. In addition, an environment in which the government issues only one-

[9]For instance, Bi (2006) analyzes a model with one-quarter and two-quarter bonds and studies the effects of making earlier issuances senior to new issuances. Chatterjee and Eyigungor (2013) studies the effect of seniority in a quantitative model of sovereign default. Using Argentina as a case study, they find that the default frequency decreases from 8.5 to 5.1 defaults every 100 years once seniority is introduced.

[10]Intertemporal debt dilution only appears with long-term bonds. With one-period bonds, when the government decides its current issuance level, the outstanding debt level is zero (either because the government honored its debt obligations at the beginning of the period or because it defaulted on them). Thus, the government cannot dilute the value of debt issued in previous periods. Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009) show that in a sovereign default framework, equilibrium default risk is significantly higher with long-term bonds than with one-period bonds.

period debt may feature a lower default frequency not only because the government issues debt that cannot be diluted away but also because the government may choose to carry lower debt levels in order to mitigate the higher rollover risk of having only one-period bonds. It should be noted in our benchmark model, the government can choose a debt portfolio with only one-period bonds but it does not find it optimal to do so.[11]

The debt covenants discussed in this paper amount to a sovereign-risk indexation of debt to mitigate default risk. Thus, these covenants resemble the price indexation of sovereign debt advocated by Calvo (1988) to mitigate the repudiation of sovereign debt through inflation. However, in Calvo's (1988) model, price indexation is used to eliminate bad equilibria in a multiple equilibria framework. In contrast, we study a framework in which the equilibrium with dilution is always bad (is conductive to high default risk) and indexation to sovereign risk moves the economy to a better equilibrium. Gains from sovereign debt indexation appear in our framework even without multiple equilibria. The gains originate because of time-inconsistency problem that arises because we allow the government to issue long-term debt.

The borrowing-contingent payments discusses in this paper also resemble taxes used in previous studies for eliminating overborrowing by private debtors (see Bianchi, 2011, and the references therein). These studies feature a pecuniary externality: individual borrowers do not internalize how their actions affect the probability of an aggregate crisis. Taxing private borrowing reduces the frequency of crises. In this paper, the borrowing by future governments increases the probability of a future default. Borrowing-contingent payments "tax" that borrowing and therefore reduce default risk.

The rest of the article proceeds as follows. Section 2 introduces the model. Section 3 discusses the calibration. Section 4 presents the results. Section 5 concludes and discusses possible extensions of our analysis.

[11]Hatchondo and Martinez (2013) argue that there may be a time inconsistency problem when governments choose the duration of sovereign debt: Government would like to commit to a duration that is significantly shorter than the one they choose when decisions are made sequentially.

2 The model

We first discuss the baseline model with debt dilution and later introduce the borrowing-contingent payments that allow us to quantify the role of debt dilution. The baseline model captures the interaction between foreign lenders and a small sovereign borrower with limited commitment. It extends the canonical Eaton and Gersovitz (1981) model in three dimensions: i) the average duration of sovereign debt is endogenous, ii) bond holders are risk averse and are subject to shocks, and iii) the recovery rate of debt in default is positive.

2.1 The baseline environment

Local endowment and preferences There is a single tradable good. The domestic economy receives a stochastic endowment stream y_t of this good, where y_t follows a Markov process.

The government's objective is to maximize the present expected discounted value of future utility flows of the representative agent in the economy, namely

$$E\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t\right)\right],$$

where E denotes the expectation operator, β denotes the subjective discount factor, and the utility function is assumed to display a constant coefficient of relative risk aversion denoted by γ. That is,

$$u\left(c\right)=\frac{c^{(1-\gamma)}-1}{1-\gamma}.$$

Asset space As in Arellano and Ramanarayanan (2012), we assume that the government can issue a short-term bond and a long-term bond. Short-term debt takes the form of a one-period bond. Long-term debt takes the form of a perpetuity with decaying coupon obligations. As in Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012), we assume that a long-term bond issued in period t entails a promise to pay one unit of the good in period $t+1$ and $(1-\delta)^{s-1}$ units in period $t+s$, with $s \geq 2$. The advantage of this payment structure is that it enables us to condense all future payment obligations derived from past long-term debt issuances into a one-dimensional state variable: the magnitude of long-term coupon obligations that mature in the current period.

Each period, the government makes two decisions. First, it decides whether to default. Second, it rebalances its debt portfolio. This implies that debt duration is endogenous, which is an important feature of the model given that the government's ability to dilute debt depends on the debt duration.

Bond holders We assume that the kernel that prices bonds issued by the domestic government is similar to the one that has been used in recent studies that account for the price behavior of U.S. government bonds. Following Piazzesi and Schneider (2007), we assume that bond holders consumption growth rate (denoted by g^*) follows an AR(1) process, namely

$$log(g_t^*) = (1-\rho^*){\mu_g}^* + \rho^* log(g_{t-1}^*) + \varepsilon_t^*, \tag{1}$$

where ${\mu_g}^*$ denotes the mean consumption growth, $|\rho^*| < 1$, and $\varepsilon_t^* \sim N\left(0, \sigma_{\epsilon^*}^2\right)$.

As in Piazzesi and Schneider (2007) and Rudebusch and Swanson (2012), we assume that bond holders preferences can be described by the recursive utility model proposed by Epstein and Zin (1989) and Weil (1989), which allows for a constant coefficient of relative risk aversion that can differ from the reciprocal of the intertemporal elasticity of substitution. Bond holders preferences are thus described by

$$V^*\left(c^*, g^*\right) = {c^*}^{1-\beta^*} E\left(V^*\left(c^{*\prime}, g^{*\prime}\right)^{1-\gamma^*} \mid g^*\right)^{\frac{\beta^*}{1-\gamma^*}},$$

where c^* denotes bond holders' consumption, β^* denotes their discount factor, and γ^* denotes their coefficient of relative risk aversion. This preference specification assumes a unitary elasticity of intertemporal substitution.

Given that preferences are homothetic, the function V^* depends linearly on c^*, i.e., $V^*\left(c^*, g^*\right) = c^* \tilde{V}^*\left(g^*\right)$, with

$$\tilde{V}^*\left(g^*\right) = {g^*}^{\frac{\rho^*\beta^*}{1-\rho^*\beta^*}} \times {\mu_{g^*}}^{\frac{(1-\rho^*)\beta^*}{(1-\beta^*)(1-\rho^*\beta^*)}} \times E\left(exp(\varepsilon^*)^{\frac{1-\gamma^*}{1-\rho^*\beta^*}}\right)^{\frac{\beta^*}{(1-\beta^*)(1-\gamma^*)}}.$$

This simplifies bond holders' stochastic discount factor, which can be expressed as

$$M\left(g^{*}, g^{*\prime}\right) = \beta^{*} \frac{g^{*\prime -\gamma^{*}} \tilde{V}^{*}(g^{*\prime})^{1-\gamma^{*}}}{E\left(\left(g^{*\prime}\tilde{V}^{*}(g^{*\prime})\right)^{1-\gamma^{*}} g^{*}\right)},$$

where $M\left(g^{*}, g^{*\prime}\right)$ denotes the value that bond holders assign to a payment of 1 unit of the good when their next-period consumption growth rate is $g^{*\prime}$ and their current consumption growth rate is g^{*}.

This pricing kernel assumes that i) the debt issued by the domestic government represents a small fraction of bond holders wealth and, thus, default decisions or variations in the market value of that debt do not affect bond holders consumption, and ii) domestic and foreign shocks are uncorrelated. The advantage of this pricing kernel is that it enables us to incorporate empirically plausible movements in the short-term risk-free interest rate and the term premium, which affect the choice of debt duration.[12]

Defaults We assume that when the government defaults, it does so on all current and future debt obligations. This is consistent with the behavior of defaulting governments in reality. As mentioned in the introduction, sovereign debt contracts often contain acceleration and cross-default clauses. These clauses imply that after a default event, future debt obligations become current.[13]

In order to sustain positive debt levels, we assume that sovereign defaults are ex-post socially costly. Once the government declares a default, it remains in default for a stochastic number of periods. While the government is in default, it cannot issue debt and domestic aggregate income is reduced by $\phi\left(y\right)$. As in Arellano (2008) and Chatterjee and Eyigungor (2012), we assume that it is proportionally more costly to default in good times ($\phi(y)/y$ is increasing in

[12]It should be noted that while Piazzesi and Schneider (2007) and Rudebusch and Swanson (2012) assume this preference specification to study the nominal yields of U.S. government debt, our environment abstracts from variation in the aggregate price level.

[13]The type of acceleration clauses depend on the details of each bond contract and on the jurisdiction under which the bond was issued. For instance, in some cases it is necessary that creditors holding a minimum percentage of the value of the bond issue request their debt to be accelerated for their future claims to become due and payable. In other cases, no such qualified majority is needed (see IMF, 2002 and Choi et al., 2011 for recent trends in the use of cross-default and acceleration clauses).

y). They show that this property is important in accounting for the dynamics of the interest rate spread in sovereign debt.[14] Mendoza and Yue (2012) show that this property of the cost of defaulting arises endogenously in a setup in which defaults affect the ability of local firms to acquire a foreign intermediate input good. Borensztein and Panizza (2009) surveys previous work about the costs of defaults and also present their own estimations. They find statistical evidence suggesting that output may fall following a sovereign default but the effect is short lived and does not last for more than one year.[15] They also encounter evidence that other costs of defaulting seem to be more long-lived. They find evidence of reputational costs (lower credit rating and higher borrowing cost after a default) and of disruptions in international trade.

We assume that the state of default ends with a time-invariant probability ξ. Once the default ends, we assume that the government offers a debt exchange to its creditors and that creditors accept the offer. Each bond holder receives a number α of new bonds per bond in default. We assume that the maturity structure of the exchanged debt is the same as the on of the debt in default. The government regains access to debt markets by paying its new debt obligations in the period of the exchange. Alternatively, the government can default on the new debt obligations, which triggers a new default period. This structure captures in a simple fashion the positive recovery rate of debt in default observed in the data (see Cruces and Trebesch, 2013 and Benjamin and Wright, 2008).

2.2 Recursive formulation of the baseline environment

The government cannot commit to future default and borrowing decisions. Thus, one may interpret this environment as a game in which the government making the default and borrowing decisions in period t is a player who takes as given the default and borrowing strategies of other

[14] Aguiar and Gopinath (2006) discusses that a cost of defaulting that is more sensitive to aggregate income shocks reduces the sensitivity of the interest rate spread on debt level. This reduces the marginal cost of debt issuances and induces the government to issue debt at higher spread levels.

[15] Borensztein and Panizza (2009) finds that the output growth rate tend to be 2.6 percent lower one year after the default but no statistically significant effect is found for longer lags. However, Borensztein and Panizza (2009) warn that their estimations do not fully control for endogeneity biases and, thus, it is not clear what fraction of the measured effect on output is caused by defaults per se. In addition, they discuss that output falls prior to the default, which could be attributed to adverse effects caused by the anticipation of the default (see also Levy Yeyati and Panizza (2011)).

players (governments) who will decide after t. We focus on Markov Perfect Equilibria. That is, we assume that in each period, the government's equilibrium default and borrowing strategies depend only on payoff-relevant state variables.

Continuation values given future borrowing and defaulting rules Let $\hat{d}$ denote the defaulting rule followed by future governments, $\hat{b}_S$ the borrowing rule for short-term bonds followed by future governments, $\hat{b}_L$ the borrowing rule for long-term bonds followed by future governments (the number of long-term coupon obligations that become due in the next-period), and $\hat{V}$ the continuation value at the beginning of next-period when future governments follow the decision rules $(\hat{d}, \hat{b}_S, \hat{b}_L)$, namely:

$$\hat{V}\left(b_S, b_L, y, g^*\right) = \hat{d}\left(b_S, b_L, y, g^*\right) \hat{V}^D\left(b_S, b_L, y, g^*\right) + (1 - \hat{d}\left(b_S, b_L, y, g^*\right))\hat{V}^R\left(b_S, b_L, y, g^*\right) \quad (2)$$

where b_S denotes the one-period bonds that mature in the current period, b_L denotes the long-term coupon obligations that mature in the current period, and $\hat{V}^R$ denotes the continuation value at the beginning of next-period when the next-period government repays its debt, borrows in the next period following the decision rules $(\hat{b}_S, \hat{b}_L)$, and then follows the decision rules $(\hat{d}, \hat{b}_S, \hat{b}_L)$ in every subsequent future period. That is:

$$\hat{V}^R(b_S, b_L, y, g^*) = u\left(c\right) + \beta E_{y', g^{*\prime}}\left[\hat{V}(b_S{}', b_L{}', y', g^{*\prime}) \mid y, g^*\right], \quad (3)$$

subject to

$$c = y - b_L - b_S + q_L(b_S{}', b_L{}', y, g^*)\left(b_L{}' - (1-\delta)b_L\right) + q_S(b_S{}', b_L{}', y, g^*)b_S{}'$$

where $b_S{}' = \hat{b}_S(b_S, b_L, y, g^*)$, $b_L{}' = \hat{b}_L(b_S, b_L, y, g^*)$, q_L denotes the bond price function for long-term debt, and q_S denotes the bond price function for short-term debt.

The function $\hat{V}^D$ denotes the continuation value at the beginning of next-period when the next-period government defaults on its debt, and then follows the decision rules $(\hat{d}, \hat{b}_S, \hat{b}_L)$ in every future period (in which it is not in default). Formally,

$$\hat{V}^D(b_S, b_L, y, g^*) = u\left(c\right) + \beta E_{y', g^{*\prime}} \left[(1-\xi)\hat{V}^D({b_S}', {b_L}', y', g^{*\prime}) + \xi \hat{V}(\alpha {b_S}', \alpha {b_L}', y', g^{*\prime}) \mid y, g^* \right], \tag{4}$$

subject to

$c = y - \phi(y)$,

${b_S}' = b_S(1+r)$, and

${b_L}' = b_L(1+r)$,

where r denotes the average short-term interest rate.[16] The functional equation (4) assumes that debt in default accumulates missed interest rate payments. This prevents a reduction in the present value of debt in default because of the delay in exiting from a default.

Current optimal decisions given future borrowing and defaulting rules In the current period, the government optimally chooses whether to default and how much debt to issue (if it does not default). Let V^R denote the optimal continuation value at the beginning of the current period when the government repays its current debt. The function V^R is determined by:

$$V^R(b_S, b_L, y, g^*) = \max_{{b_S}', {b_L}' \geq 0} \left\{ u\left(c\right) + \beta E_{y', g^{*\prime}} \left[\hat{V}({b_S}', {b_L}', y', g^{*\prime}) \mid y, g^* \right] \right\}, \tag{5}$$

subject to

$c = y - b_L - b_S + q_L({b_S}', {b_L}', y, g^*)\left({b_L}' - (1-\delta)b_L\right) + q_S({b_S}', {b_L}', y, g^*){b_S}'$, and

$q_L({b_S}', {b_L}', y, g^*) \geq \underline{q} \quad$ if ${b_L}' > (1-\delta)b_L$ and ${b_S}' > 0$.

Given that being in default is costly, for any income realization there is a portfolio with positive debt at which the government prefers to repay than to default. This means that the minimum present value of total debt payments expected from the government is positive (and

[16]The short-term interest rate satisfies:

$$\frac{1}{1+r} = E_{g^*}\left[E_{g^{*\prime}}\left[M\left(g^*, g^{*\prime}\right) \mid g^*\right]\right].$$

state-dependent) even after the government announces a default. This feature of the model combined with the presence of long-term debt enables the government to engineer a consumption boom by choosing a large issuance volume and declaring a default in the following period. This amounts to increasing the de-facto seniority of investors who purchase debt issued in the current period and diluting the value of previously issued long-term bonds.[17] The government may not always find it optimal to dilute existing debt by choosing large issuance volumes and then declaring a default. However, given that we do not observe that extreme behavior in sovereign debt markets and that allowing for that extreme behavior in the model may magnify the effects of debt dilution, we eliminate the option to fully dilute existing long-term debt by introducing a lower bound on the long-term bond price at which the government can issue debt. We report below that the lower bound chosen is rarely binding.

Let V^D denote the optimal continuation value at the beginning of the current period when the government defaults on its current debt. Given that no further action is taken by the government in a default period, $V^D = \hat{V}^D$.

Bond prices Bond holders are rational and anticipate future borrowing and defaulting rules when they compute the maximum value at which they are willing to buy the bonds issued by

[17]It may assist the intuition to consider the following environment: i) foreign investors are risk-neutral and the risk-free interest rate is zero, ii) domestic income (y) follows an i.i.d. process with $E(y) > 0$, iii) the government issues only a long-term bond, iv) the government settles the default by extending a total payment of φy in the default period, and v) the compensation received by each bond holder is proportional to the number of bonds owned by the bond holder. The budget constraint for the government reads as:

$$c = y - b_L + q(b'_L)\left(b'_L - (1-\delta)b_L\right).$$

If the default probability is 1 for sufficiently a large debt level, then

$$\lim_{b'_L \to \infty} q(b'_L) = \lim_{b'_L \to \infty} \frac{\varphi E(y)}{b'_L} = 0.$$

Even though the bond price is close to zero when the government issues a large amount of debt, the resources obtained by the current government are close to $\lim_{b'_L \to \infty} q(b'_L)b'_L = \varphi E(y)$. In this case, the market value of previously issued long-term debt is close to zero.

In our model, the government could also fully dilute outstanding long-term debt by choosing an issuance volume large enough that the share of currently issued debt in the total debt stock would be close to 1. Unlike in the simple example presented in this footnote, in our model the cost of increasing the dilution of outstanding long-term debt is increasing: the government will choose to remain longer in default. Thus, it is less clear whether there may be state combinations at which the government will fully dilute outstanding long-term debt.

the domestic government. The value of one-period bonds depends only on the defaulting rule because that rule determines i) in which states the government repays its one-period debt, and ii) in which states the government repays the one-period debt issued at the exit of a default, which is relevant for the cases in which the government defaults in the next period. Formally,

$$q_S({b_S}', {b_L}', y, g^*) = E_{y',g^{*\prime}} \left[M\left(g^*, g^{*\prime}\right) \left(1 - d' + d' q_S^{D'}\right) \mid y, g^* \right], \tag{6}$$

where $d' = \hat{d}\left({b_S}', {b_L}', y', g^{*\prime}\right)$ denotes the next-period default decision and $q_S^{D'} = q_S^D\left({b_S}', {b_L}', y', g^{*\prime}\right)$ denotes the next-period value of one-period bonds in default.

The value of a one-period bond in default is determined as follows:

$$q_S^D\left({b_S}', {b_L}', y, g^*\right) = E_{y',g^{*\prime}} \left[M\left(g^*, g^{*\prime}\right)(1+r)\left((1-\xi) q_S^{D'} + \xi\alpha\left(1 - d' + d' q_S^{DD\prime}\right)\right) \mid y, g^* \right]. \tag{7}$$

The government remains in default with probability $1 - \xi$. The continuation value in that case is represented by $q_S^{D'} = q_S^D\left((1+r){b_S}', (1+r){b_L}', y', g^{*\prime}\right)$. Given that the debt in default incorporates missed interest rate payments, the debt portfolio at the beginning of the next period consists of $((1+r){b_S}', (1+r){b_L}')$. The government exits the current default at the beginning of next period with probability ξ. The government exits the default by extending α new bonds for each bond in default. The variable $d' = \hat{d}\left((1+r)\alpha {b_S}', (1+r)\alpha {b_L}', y', g^{*\prime}\right)$ denotes the next-period default decision on the bonds that are extended as part of the debt exchange, and $q_S^{DD\prime} = q_S^D\left(\alpha(1+r){b_S}', \alpha(1+r){b_L}', y', g^{*\prime}\right)$ denotes the value of a one-period bond in default once the government defaults on the bonds that are extended as part of the debt exchange.

The value of long-term bonds depends on the defaulting rule as well as on the borrowing rule. The reason for the latter is that the value of a long-term bond depends on default decisions taken in all future periods, which in turn depends on future debt levels. Formally:

$$q_L({b_S}', {b_L}', y, g^*) = E_{y',g^{*\prime}} \left[M\left(g^*, g^{*\prime}\right) \left[(1-d')\left(1 + (1-\delta){q_L}'\right) + d' q_L^{D\prime} \right] \mid y, g^* \right], \tag{8}$$

where ${q_L}' = q_L({b_S}'', {b_L}'', y', g^{*\prime})$ denotes the next-period value of long-term bonds when the government repays its debt obligations, ${b_S}'' = \hat{b}_S\left({b_S}', {b_L}', y', g^{*\prime}\right)$ denotes the next-period borrowing decision for short-term debt, ${b_L}'' = \hat{b}_L\left({b_S}', {b_L}', y', g^{*\prime}\right)$ denotes the next-period borrowing decision for long-term debt, and $q_L^{D'} = q_L^D\left({b_S}', {b_L}', y', g^{*\prime}\right)$ denotes the next-period value of a long-term bond in default.

As with the one-period bonds, the value of a long-term bond in default depends on the next-period value of bonds in default for the cases in which the government remains in default and in which the government makes a debt exchange and defaults again in the next-period. In addition to that, the value of a long-term bond in default depends on the market value of the long-term bonds that take part of the debt exchange for the cases in which the government exits the current default and does not default in the next period. Namely,

$$q_L^D({b_S}', {b_L}', y, g^*) = E_{y', g^{*\prime}}\left[M\left(g^*, g^{*\prime}\right)(1+r)\left[(1-\xi)q_L^{D\prime} + \xi\alpha\left((1-d')\left(1+(1-\delta){q_L}'\right) + d' q_L^{DD\prime}\right)\right] \mid y, g^*\right], \tag{9}$$

where $q_L^{D\prime} = q_L^D\left((1+r){b_S}', (1+r){b_L}', y', g^{*\prime}\right)$ denotes the next-period value of a long-term bond in default if the default does not end in the next-period, ${q_L}' = q_L(b_S^{D\prime\prime}, b_L^{D\prime\prime}, y', g^{*\prime})$ denotes the next-period value of a long-term bond if the government exits the default, $b_S^{D\prime\prime} = \hat{b}_S\left((1+r)\alpha {b_S}', (1+r)\alpha {b_L}', y', g^{*\prime}\right)$ denotes the next-period short-term borrowing decision after exiting the default, $b_L^{D\prime\prime} = \hat{b}_L\left((1+r)\alpha {b_S}', (1+r)\alpha {b_L}', y', g^{*\prime}\right)$ denotes the next-period long-term borrowing decision after exiting the default, and $q_L^{DD\prime} = q_L^D\left((1+r)\alpha {b_S}', \alpha(1+r){b_L}', y', g^{*\prime}\right)$ denotes the value of a long-term bond in default once the government defaults on the bonds that are extended in the debt exchange.

Equilibrium concept A Markov Perfect Equilibrium is then characterized by value functions $\hat{V}$, $\hat{V}^R$, $\hat{V}^D$, policy rules $\hat{d}$, $\hat{b}_S$, $\hat{b}_S$, and bond price functions q_S and q_L such that

(a) $\hat{d}(b_S, b_L, y, g^*) = \underset{d\in\{0,1\}}{Argmax}\left\{dV^D(b_S, b_L, y, g^*) + (1-d)V^R(b_S, b_L, y, g^*)\right\}$ for all b_S, b_L, y, g^*, where V^R satisfies (5) and $V^D = \hat{V}^D$.

(b) The functions $\hat{b}_S(b_S, b_L, y, g^*)$ and $\hat{b}_L(b_S, b_L, y, g^*)$ jointly solve (5) for all b_S, b_L, y, g^*, given $\hat{V}$, q_S and q_L

(c) $\hat{V}$, $\hat{V}^R$, and $\hat{V}^D$ satisfy functional equations (2)-(4),

(d) and the bond price functions q_S and q_L satisfy functional equations (6)-(9).

2.3 A framework without debt dilution

In this section, we propose a modification to the model presented in Section 2.1 that will allow us to study an economy without debt dilution and, in turn, to measure the effects of debt dilution. We eliminate debt dilution—caused by borrowing decisions—by introducing a borrowing-contingent debt covenant. The covenant specifies that if the sovereign borrows, it has to pay each holder of previously-issued long-term bonds the difference between the counterfactual bond price that would have been observed absent new borrowing in the current period and the observed bond price.[18] This covenant eliminates debt dilution by disentangling the current value of long-term bonds from future borrowing decisions.

We assume that a default on borrowing-contingent payments triggers acceleration and cross-default clauses that make all the government's debt obligations become current: if the government selectively defaults on borrowing-contingent payments, it has to cancel all current and future debt obligations, discounting future debt obligations at the risk-free rate. Consequently, selectively defaulting on borrowing-contingent payments cannot be better than buying back all government debt. Therefore, the next subsection presents the recursive formulation of the framework with borrowing-contingent payments without giving the government the option of selectively defaulting on borrowing-contingent payments (but giving the government the option to buy back its debt).

2.4 Recursive formulation of the framework without debt dilution

As in Section 2.1, when the government wants to buy back its long-term debt, it does so at the secondary-market price. The departure from the baseline model is that when the government issues debt, it has to make a compensation payment $\mathcal{C}$ to each holder of previously issued long-term debt. The compensation payment is such that the holders of previously issued long-term

[18]These payment obligations do not only depend on the borrowing decision (how many bonds are issued in the current period). They also depend on the current income realization and the debt level, given that the counterfactual bond price that would have been observed absent new borrowing requires knowledge of all shock realizations, the debt level, and the bond price function. While this price is easy to compute in our simulations, it would be difficult to determine in practice. In Section 4.4 we show that most benefits from eliminating dilution can be obtained with borrowing-contingent payments that do not depend on this counterfactual price.

debt are not adversely affected by current borrowing decisions. Formally, the compensation payment satisfies

$$\mathcal{C}(b_L, y, g^*, {b_S}', {b_L}') = Max\left\{q_L(0, b_L(1-\delta), y, g^*) - q_L({b_S}', {b_L}', y, g^*), 0\right\}. \tag{10}$$

This means that each holder of previously issued long-term debt is compensated exactly for the difference between the price at which it would have been able to trade its long-term bonds in the absence of short or long-term debt issuances (represented by $q_L(0, b_L(1-\delta), y, g^*)$) and the market price of its long-term bonds, which depends on the government's current borrowing decisions and is represented by $q_L({b_S}', {b_L}', y, g^*)$.

The government's budget constraint when it repays its debt reads as

$$c = y - b_L - b_S + q_L({b_S}', {b_L}', y, g^*)\left({b_L}' - (1-\delta)b_L\right) + q_S({b_S}', {b_L}', y, g^*){b_S}' - (1-\delta)b_L\mathcal{C}(b_L, y, g^*, {b_S}', {b_L}'). \tag{11}$$

Assuming that the bond price is decreasing in both short-term and long-term debt (we find this is the case in our numerical solution), the price of a long-term bond depends also on future compensation payments, that is

$$q_L({b_S}', {b_L}', y, g^*) = E_{y', g^{*\prime}}\left[M\left(g^*, g^{*\prime}\right)\left[(1-d')\left(1 + (1-\delta)\left({q_L}' + \mathcal{C}'\right)\right) + d' q_L^{D\prime}\right] \mid y, g^*\right], \tag{12}$$

where $\mathcal{C}' = \mathcal{C}({b_L}', y', g^{*\prime}, \hat{b}_S\left({b_S}', {b_L}', y', g^{*\prime}\right), \hat{b}_L\left({b_S}', {b_L}', y', g^{*\prime}\right))$ denotes the next-period compensation payment when the government does not default and follows the borrowing rules $\hat{b}_S$ and $\hat{b}_L$.

The price of a long-term bond in default satisfies:

$$q_L^D({b_S}', {b_L}', y, g^*) = E_{y', g^{*\prime}}\left[M\left(g^*, g^{*\prime}\right)(1+r)\left[(1-\xi)q_L^{D\prime} + \xi\alpha\left((1-d')\left(1 + (1-\delta)\left({q_L}' + \mathcal{C}^{D\prime}\right)\right) + d' q_L^{DD\prime}\right)\right] \mid y, g^*\right], \tag{13}$$

where $\mathcal{C}^{D\prime} = \mathcal{C}((1+r)\alpha {b_L}', y', g^{*\prime}, \hat{b}_S\left((1+r)\alpha {b_S}', (1+r)\alpha {b_L}', y', g^{*\prime}\right), \hat{b}_L\left((1+r)\alpha {b_S}', (1+r)\alpha {b_L}', y', g^{*\prime}\right)$ denotes the next-period compensation payment when the government exits the default and stays current on its debt.

The equilibrium definition differs in two ways with respect to the one for the baseline model: i) the budget constraint (11) replaces the budget constraints in (24) and (5), and ii) the functional equations (12)-(13) replace the functional equations (8)-(9).

3 Calibration

Table 1 presents the baseline parameterization. We use a peripheral European economy (Spain) to discipline the parameter values corresponding to the sovereign borrower. A period in the model refers to a quarter. The domestic endowment process follows an AR(1) process:

$$\log(y_t) = (1-\rho)\,\mu_y + \rho\log(y_{t-1}) + \varepsilon_t, \tag{14}$$

with $\varepsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

We estimated equation (14) using quarterly real GDP data from Spain from the first quarter of 1960 to the first quarter of 2013. The data counterpart of $log(y_t)$ is the deviation of the natural logarithm of GDP from its linear trend. García-Cicco et al. (2010) and Alvarez-Parra et al. (2013) estimate a standard business-cycle model for small open economies in which aggregate output is affected by two shocks to the state of technology: a standard stationary shock, and a nonstationary shock that affects the growth rate of productivity. They find that the role of the nonstationary technology shock significantly diminishes once the estimation procedure includes an ad-hoc state-dependent interest rate scheme at which the sovereign can borrow in foreign markets. For instance, García-Cicco et al. (2010) find that the nonstationary (stationary) technology shock accounts for 7.4 (84.2) percent of the variance of output growth using Mexican data. The contribution of the nonstationary technology shock to the variance of other aggregate variables is even lower. Given that our model features a state-dependent interest scheme (which is endogenous), the findings in those papers suggest that allowing for a nonstationary output shock would likely have a modest contribution to business cycle dynamics. In addition, it would require the use of an additional state variable. For those reasons, we chose to specify the domestic endowment process as a stationary AR(1) process.

We assume that the representative agent in the sovereign economy has a coefficient of relative risk aversion γ of 2 and a discount factor β of 0.98. Those values are within the range of accepted values in studies of business cycles in small open economies. For instance, those are the values used in García-Cicco et al. (2010) and in Alvarez-Parra et al. (2013).

With respect to the parameters governing the pricing kernel, we use NIPA data for the U.S. to estimate the process for bond holders' consumption growth. As in Piazzesi and Schneider (2007), bond holders' consumption consists of personal consumption expenditures in nondurable goods and in services. Equation (1) is then estimated using data from the second quarter of 1952 to the fourth quarter of 2005.[19] The parameter β^* is chosen so that the mean quarterly real interest rate equals 1 percent. The parameter γ^* coefficient of relative risk aversion for bond holders is taken from Piazzesi and Schneider (2007).

The domestic government remains in exclusion for an average of three years after a default. This is the estimate obtained by Dias and Richmond (2009) for the median duration of exclusion using their partial access definition of re-entry.[20] A three-year exclusion period is also in the range of the estimates reported by Gelos et al. (2011).

The recovery rate of debt in default (α) is assumed to take a value of 0.63. This is the average recovery rate reported by Cruces and Trebesch (2013) using a sample of 180 default episodes between 1970 and 2010.

As in Chatterjee and Eyigungor (2012), we assume a quadratic loss function for income during a default episode $\phi(y) = max\left\{d_0 y + d_1 y^2, 0\right\}$. In the context of a sovereign default model with long-term debt, they show that this function allows the equilibrium default model to match the behavior of the spread in the data by affecting the sensitivity of the cost of defaulting to the domestic income shock, and through that, the sensitivity of bond prices to debt levels. Aguiar and Gopinath (2006) provide a discussion of the role of the cost of defaulting on the equilibrium spread behavior.

We assume that the minimum issuance price for long-term debt ($\underline{q}$) equals 70% of the mean

[19] This is the same sample period that is used in Piazzesi and Schneider (2007).

[20] According to Dias and Richmond (2009) definition of partial market access, a sovereign in default is said to regain access to capital markets in the first year in which there are positive transfers (in the form of bond and commercial bank loans) to the domestic public or private sector

risk-free price for long-term debt. This implies a maximum annual yield to maturity of 9.7 percent.[21] This is higher than the maximum yield to maturity at which the Spanish government has issued debt since 2008 (6.97 percent for a 10 year bond issued on November 11th. 2011) and is higher than the yield to maturity at which any European government issued government debt since 2008 with one exception: the Italian government issued a 7 year bond at a yield to maturity of 10.96 percent on December 13th, 2012 (see Trebesch and Wright, 2013).[22] In the simulations, the minimum issuance price for long-term debt is binding in 0.007 percent of the periods.

The two parameters that define the output cost of defaulting and the rate of decay in long-term bonds $(1-\delta)$ are calibrated to match i) the average duration of government debt, ii) the level of government debt, and iii) the average long-term interest rate spread. We use data from 2008 to 2013 from Spain to calculate those moments.[23] The reason for choosing that sample period is that the interest rate spread of Spanish government debt was around zero between 1999 and 2007 (and even negative in some periods) and that prior to the beginning of the Euro, the Spanish government issued debt denominated in local currency.

4 Results

As discussed in Krusell and Smith (2003), there may be multiple Markov Perfect Equilibria in infinite-horizon economies. In order to avoid this problem, we solve for the limit of the equilibrium of the finite-horizon economy. That is, we solve for the equilibrium of finite-horizon economies

[21]The annualized yield to maturity of a long-term bond (i_L) is calculated as

$$i_L = \left(\frac{1}{q_L} + 1 - \delta\right)^4 - 1$$

[22]We thank Christoph Trebesch and Mark Wright for sharing their data with us.

[23]We use the Macaulay definition of duration, which with the coupon structure of long-term bonds assumed in this paper is calculated as

$$D = \frac{\frac{b_L+b_S}{1+i} + b_L \sum_{t=2}^{\infty} t \frac{(1-\delta)^{t-1}}{(1+i)^t}}{\frac{b_L+b_S}{1+i} + b_L \sum_{t=2}^{\infty} \frac{(1-\delta)^{t-1}}{(1+i)^t}},$$

where i denotes the constant yield to maturity of long-term bonds.

Borrower's risk aversion	γ	2	Prior literature
Borrower's discount factor	β	0.98	Prior literature
Output autocorrelation coefficient	ρ	0.97	Spanish GDP
Standard deviation of innovations	σ_ϵ	1.04%	Spanish GDP
Mean log output	μ_y	$(-1/2)\sigma_\epsilon^2$	Mean output level = 1
Bond holders' risk aversion	γ^*	59	Piazzesi and Schneider (2007)
Bond holders' discount factor	β^*	0.99614	Mean real rate = 1%
Bond holders' consumption autocorrelation coefficient	ρ^*	0.329	US private consumption
Bond holders' std. dev. of consumption innovations	σ_ϵ^*	0.4722%	US private consumption
Bond holders' mean consumption growth	μ_g^*	0.8%	US private consumption
Minimum issuance price for long-term debt	$\underline{q}$	0.693	Trebesch and Wright (2013)
Duration of defaults	ξ	0.083	Dias and Richmond (2009)
Recovery rate of debt in default	α	0.63	Cruces and Trebesch (2013)
Duration of long-term bond	δ	0.0225	Calibrated to fit targets
Output loss while in default	d_0	-0.698	Calibrated to fit targets
Output loss while in default	d_1	0.8	Calibrated to fit targets

Table 1: Parameter values. The source for Spanish GDP is Banco de España. The source for US consumption is the Bureau of Economic Analysis. The value for β is taken from García-Cicco et al. (2010) and Alvarez-Parra et al. (2013). They also use a relative risk aversion coefficient of domestic agents equal to 2.

until the number of periods is large enough that the value functions and bond prices for the first and second periods of this economy are sufficiently close. We then use the first-period equilibrium functions as the infinite-horizon-economy equilibrium functions. We provide more details of the computation in the appendix.

As in Hatchondo et al. (2010), we solve the model numerically using value function iteration and interpolation. First, we show that the baseline model can account for salient features of business cycle dynamics in Spain (and in other small open economies). Second, we show that debt dilution accounts for a significant fraction of the default premium in the benchmark economy. Third, we present the welfare gains from eliminating dilution. Fourth, we show that most gains from eliminating dilution can be obtained with simpler borrowing-contingent payment schemes that may be easier to implement. Fifth, we compare the allocation without dilution with the allocation that the government could attain if it could trade a full range of one-period Arrow-Debreu claims contingent on local income realizations. Finally, we compare the allocation without dilution with the allocation that the government could attain if it did not face a limited commitment problem.

4.1 Quantitative implications in the baseline economy

Table 2 reports moments in the data and in our simulations. Given that there has not been a sovereign default in Spain in recent years, we report results for sample paths without defaults. The exception is the default frequency, which we compute using all simulation periods. We generate 1,000 sample paths of 300 periods each and take the last 74 periods of each sample path if no default was observed in the last 100 periods of the sample path. We focus on samples of 74 periods because we compare the artificial data generated by the model with Spanish data from the first quarter of 1995 to the second quarter of 2013.

The moments reported in Table 2 are chosen so as to illustrate the ability of the model to replicate distinctive business cycle properties of small open economies. In Table 2, the standard deviation of a variable x is denoted by $\sigma(x)$. The coefficient of correlation between x and z is denoted by $\rho(x, z)$. Moments are computed using detrended series. Trends are computed

	Spain	With dilution
Moments targeted in the calibration		
Debt / mean annual income (in %)	61.8	61.8
Debt duration (years)	6.00	5.95
Spread of long-term debt (in %)	2.04	2.10
Non-targeted moments		
Debt obligations within 1 year/ Total debt (in %)	21.1	23.2
Spread of short-term debt (in %)	0.86	0.73
$\sigma(c)/\sigma(y)$	1.15	1.50
$\sigma\,(TB/y)$ (in %)	0.98	1.20
$\rho\,(TB/y, y)$	-0.72	-0.73
Defaults per 100 years		2.78

Table 2: Business cycle statistics. The second column is computed using data from Spain. We use data from 1995 to 2013 for aggregate private consumption (c), trade balance (TB), and GDP (y). The logarithm of private consumption, the logarithm of income and the trade balance to output ratio were detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. We report deviations from the trend. The source for consumption, income, and trade balance is Haver Analytics. We use data from 2008 to 2013 for the interest rate spread and debt statistics. The short-term spread was computed using the yield of 3 month government bonds in Spain and Germany. The long-term spread was computed using yield to maturity of 8 year government bonds in Spain and Germany. The source for the data on bond yields is Bloomberg. The source for government debt and the ratio of debt obligations maturing within the next year is the Government Statistics Database of the European Central Bank. The source for government debt duration is JP Morgan. Column 3 reports the mean of the value of each moment in 1,000 simulation samples. We take the last 74 periods (quarters) of samples in which no default occurs in the last 100 periods of each sample.

using the Hodrick-Prescott filter with a smoothing parameter of $1,600$. In terms of consumption volatility and the co-movement between the trade balance and output, business cycle dynamics in Spain resemble more the ones of emerging economies than the ones of advanced small open economies (see Aguiar and Gopinath, 2007). Given that the sample period with positive interest rate spread in Spain that we use is relatively short, we chose not to compare the co-movement between the spread and other macroeconomic variables.[24]

The sovereign spread consists of the extra yield to maturity delivered by a defaultable bond over the yield to maturity of a default-free bond with the same structure of coupon payments. We report the annualized spread

$$R_j^s = \left(\frac{1+i_j}{1+r_j}\right)^4 - 1, \text{ for } j = S, L. \tag{15}$$

The term R_S^s (R_L^s) denotes the spread of a short-term (long-term) bond. The yield to maturity of a long-term bond is computed as in footnote 21, whereas the yield to maturity of a default-free long-term bond (r_L) is computed using the price of a default-free long-term bond with the same structure of coupon payments (which depends on bond holders' current consumption growth rate). The yield to maturity of a defaultable short-term bond is computed as $i_S = 1/q_S - 1$ and the yield to maturity of a default-free short-term bond is computed as $r_S = 1/E_{g^{*\prime}}(M(g^*, g^{*\prime}) \mid g^*) - 1$.

The level of long-term debt is calculated as the present value of future payment obligations discounted at the average short-term risk-free rate, i.e., $b_L(\delta + r)^{-1}$.

Table 2 shows that the baseline model with dilution approximates well the moments used as targets and it is broadly aligned with the data with respect to non-targeted moments: consumption is more volatile than income, the trade balance is countercyclical and the spread on short-term debt is lower than the spread on long-term debt. The model not only mimics the average duration of debt but approximates well a standard measure of short-term debt obligations: the fraction of total debt obligations that mature within the next year. Estimating the default probability in the data is elusive. Using a sample of 68 countries between 1970 and 2010, Cruces and Trebesch (2013) find a frequency of 6.6 defaults every 100 years. Aguiar and Gopinath

[24]In our simulations the spread is countercyclical, as observed in most emerging economies (see Neumeyer and Perri (2005) and Uribe and Yue (2006)).

(2006) target a frequency of 3 defaults per 100 years based on the data reported by Reinhart et al. (2003). The default frequency in our benchmark simulations is closer to the second number.

The basic mechanism that determines the maturity structure is similar to the one studied in Arellano and Ramanarayanan (2012) and Niepelt (2008). In what follows we use first order conditions to illustrate trade-offs the government faces when it issues debt. We do not assume differentiability when we find numerical solutions. If it is assumed that the constraint on the long-term bond price is not binding in the current or next period, and that it is optimal to repay the debt in the current period, the first order condition for long-term debt accumulation reads as

$$
\begin{aligned}
&u'(c)\left[q_L\left(b'_S, b'_L, y, g^*\right) + \frac{\partial q_L\left(b'_S, b'_L, y, g^*\right)}{\partial b'_L}\left(b'_L - (1-\delta)b_L\right) + \frac{\partial q_S\left(b'_S, b'_L, y, g^*\right)}{\partial b'_L} b'_S\right] = \\
&\beta E_{y', g^{*\prime}}\left[(1-d')u'(c')(1+(1-\delta)q'_L) - d'\frac{\partial \hat{V}^D\left(b'_S, b'_L, y', g^{*\prime}\right)}{\partial b'_L} \mid y, g^*\right], \qquad (16)
\end{aligned}
$$

where d' and q'_L are defined as in equations (6) and (8), and c' denotes next-period consumption when the next-period government follows the defaulting rule $\hat{d}$, and borrowing rules $\hat{b}_S$ and $\hat{b}_L$.

The first order condition for short-term debt accumulation reads as

$$
\begin{aligned}
&u'(c)\left[q_S\left(b'_S, b'_L, y, g^*\right) + \frac{\partial q_L\left(b'_S, b'_L, y, g^*\right)}{\partial b'_S}\left(b'_L - (1-\delta)b_L\right) + \frac{\partial q_S\left(b'_S, b'_L, y, g^*\right)}{\partial b'_S} b'_S\right] = \\
&\beta E_{y', g^{*\prime}}\left[(1-d')u'(c') - d'\frac{\partial \hat{V}^D\left(b'_S, b'_L, y', g^{*\prime}\right)}{\partial b'_S} \mid y, g^*\right], \qquad (17)
\end{aligned}
$$

The left-hand side of equations (16)-(17) represents the net marginal benefit of borrowing. By issuing one extra short-term (long-term) bond, the government obtains less than $q_S\left(b'_S, b'_L, y, g^*\right)$ ($q_L\left(b'_S, b'_L, y, g^*\right)$) goods in the current period. The reason is that the last bond issued affects the price of all the short-term and long-term bonds issued in the current period. This is captured by the second and third terms of the left hand side of equations (16)-(17). The right-hand side of equations (16)-(17) represents the future marginal cost of borrowing: by borrowing more, the government decreases expected future consumption.

As in Arellano and Ramanarayanan (2012), the government wants to smooth out the cost of issuing different types of debt and it wants to mitigate the exposure to rollover risk by avoiding choosing debt portfolios that may lead to low consumption levels in next-period states in which it is costly to issue debt. Figures 1-2 illustrate the differential exposure to rollover risk derived from issuing short or long-term debt. Figure 1 shows that the yield of short-term debt increases with the choice of short-term debt. The reason is that if the government chooses to end the period with a higher debt level, bond holders demand a higher premium on short-term bonds in anticipation of the higher default probability expected for the next period. Given that the incentives to default decrease with income, a higher current domestic income realization anticipate higher future domestic income realization and, thus, lowers the next-period default probability. The left (right) panel of Figure 1 shows how the borrowing opportunities in short-term debt shrink in response to an adverse domestic (foreign) shock. Both sources of risk limit the fraction of short-term debt that the government wants to carry.

Figure 2 shows that the yield of long-term debt increases with the choice of long-term debt. In this case, if the government chooses to end the period with a higher debt stock, bond holders demand a higher premium in anticipation of the higher default probability for the next-period (as in the case for short-term debt) and for subsequent periods. The exposure to rollover risk is lower when the government issues long-term debt for two reasons: First, the government does not need to pay back its entire stock of long-term debt at the beginning of each period. It is only liable for a fraction of it. Second, the effect of the shock to the pricing kernel on the set of borrowing opportunities is smaller than in the case of short-term debt. This is because of the mean reversion in bond holders' consumption growth process. Besides, it is consistent with the lower volatility of long-term yields relative to the short-term yields of U.S. government debt.

The first order conditions (16)-(17) show that if the government enters the period with a positive stock of long-term debt, the government dilutes the value of those claims when it issues long-term or short-term debt (in the numerical solution we observe a negative relationship between q_L and b_L, and between q_L and b_S). The current government may find it optimal to avoid debt dilution in the next-period by issuing only short-term debt, which cannot be diluted away. The simulations of the baseline model show that this strategy is not optimal for the government,

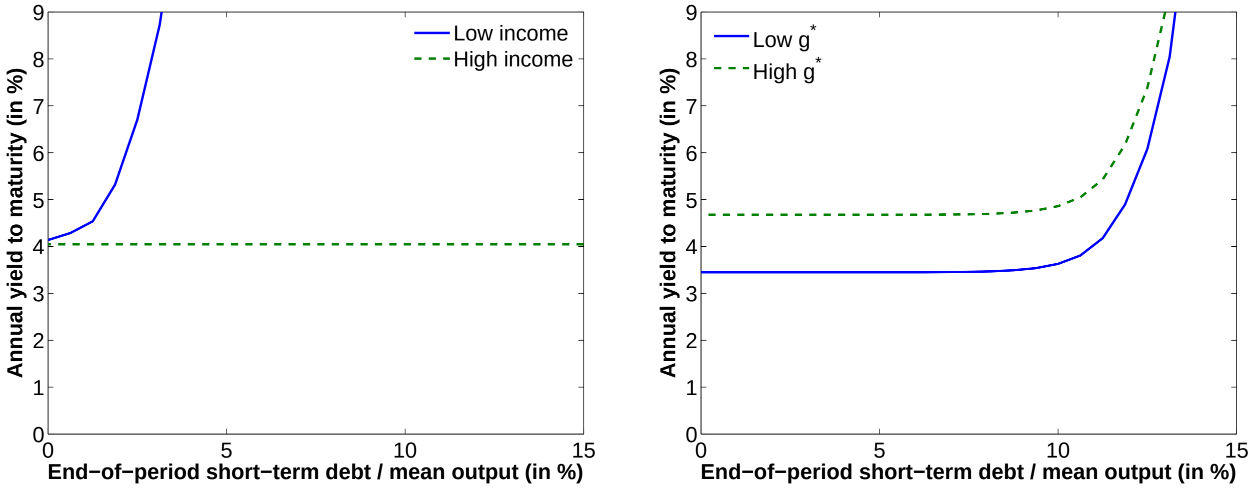

Figure 1: Menu of combinations of yield to maturity of short-term debt (i_S) and end-of-period short-term debt levels (b'_S) from which the government can choose in the baseline economy. Both graphs were computed assuming that the government does not issue long-term debt in the current period and enters the period with a long-term debt level equal to the mean long-term debt level observed in the simulations. The left panel assumes that $g^* = \mu_g$ and the right panel assumes that $log(y) = \mu_y$. The low (high) value of y in the left panel corresponds to a domestic income realization that is one standard deviation below (above) the unconditional mean. The low (high) value of g^* in the right panel corresponds to a bond holders' consumption growth realization that is one standard deviation below (above) the unconditional mean.

possibly because of the high exposure to rollover risk implied by a portfolio of only short-term debt. In the next two subsections we discuss the quantitative implications of eliminating debt dilution.

4.2 Dilution and default risk

This subsection measures the quantitative effects of debt dilution. Table 3 shows that debt dilution accounts for 78% of the default frequency in the simulations of the baseline model. The number of defaults per 100 years decreases from 2.78 in the baseline to 0.61 in the model without debt dilution. Debt dilution also accounts for 84% of the long-term spread paid by the sovereign.[25] The mean debt-to-income ratio decreases by 8.9%.

[25] To compute the spread on long-term debt in the no-dilution economy, we need to compute the price of a default-free long-term bond that pays the same compensation $\mathcal{C}$ that the defaultable long-term bond pays in the economy without dilution. We do so by finding the function

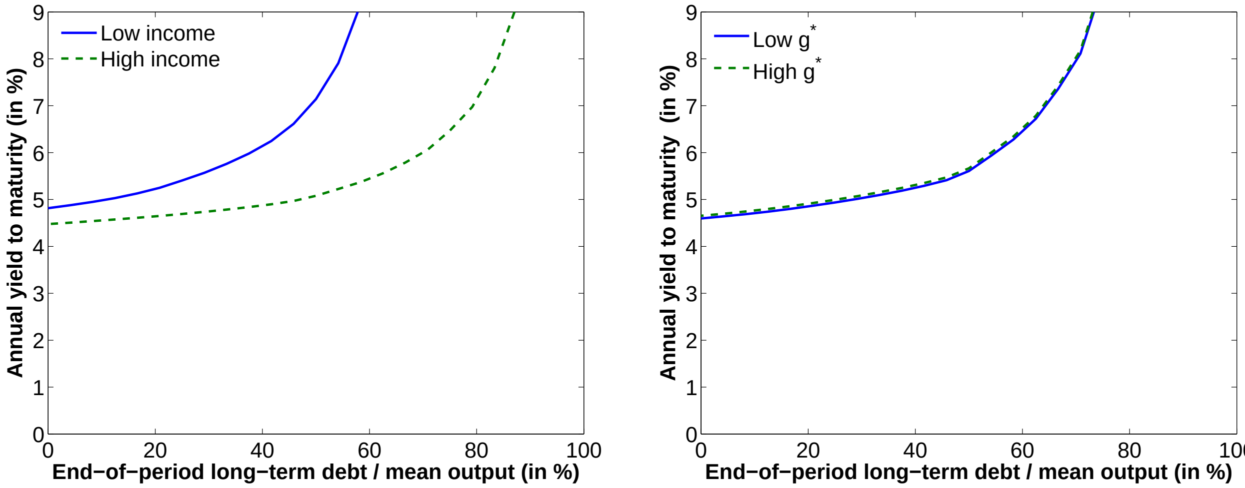

Figure 2: Menu of combinations of yield to maturity of long-term debt (i_L) and end-of-period long-term debt levels ($\frac{b'_L}{\delta+r}$) from which the government can choose in the baseline economy. Both graphs were computed assuming that the government does not issue short-term debt in the current period ($b'_S = 0$). The left panel assumes that $g^* = \mu_g$ and the right panel assumes that $log(y) = \mu_y$. The low (high) value of y in the left panel corresponds to a domestic income realization that is one standard deviation below (above) the unconditional mean. The low (high) value of g^* in the right panel corresponds to a bond holders' consumption growth realization that is one standard deviation below (above) the unconditional mean.

In order to shed light on how eliminating debt dilution affects the government's optimal decisions it is illustrative to consider how the elimination of debt-dilution affects the first order conditions. Assuming that bond prices are decreasing in debt (as we find it is the case for the parameterization we study), that the government chooses to borrow in the current period,

---

$$q_L^{DF}(b_S{}', b_L{}', y, g^*; i) = \frac{1}{1+i} E_{g^{*\prime}, y'} \left[1 + (1-\delta)\left(\mathcal{C}(b'_L, b_S{}'', b_L{}'', y', g^{*,\prime}) + q_L^{DF}(b_S{}'', b_L{}'', y', g^{*\prime}; i)\right) \mid y, g^*\right],$$

that denotes the price of a default-free long-term bond that pays the coupon and compensation $\mathcal{C}$ every period, where i denotes the constant rate at which future payments are discounted, $b_S{}'' = \hat{b}_S^{\text{No dil}}\left(b_S{}', b_L{}', y', g^{*\prime}\right)$, and $b_L{}'' = \hat{b}_L^{\text{No dil}}\left(b_S{}', b_L{}', y', g^{*\prime}\right)$. The superindex "No dil" is used to denote functions in the economy without dilution. The yield of a defaultable bond i thus depend on the state $(b_S{}', b_L{}', y, g^*)$ and is defined as the rate i^* that satisfies

$$q_L^{DF}\left(\hat{b}_S^{\text{No dil}}(b_S, b_L, y, g^*), \hat{b}_L^{\text{No dil}}(b_S, b_L, y, g^*), y, g^*; i^*\right) = q_L^{\text{No dil}}\left(\hat{b}_S^{\text{No dil}}(b_S, b_L, y, g^*), \hat{b}_L^{\text{No dil}}(b_S, b_L, y, g^*), y, g^*\right).$$

The annualized interest rate spread is calculated as in equation (15).

	With dilution	Without dilution
Debt / mean annual income (in %)	61.8	56.3
Debt duration (years)	5.95	7.70
Debt obligations within 1 year/ Total debt (in %)	23.0	13.5
Spread of long-term debt (in %)	2.10	0.34
Spread of short-term debt (in %)	0.73	0.39
$\sigma(c)/\sigma(y)$	1.50	2.07
$\sigma(TB/y)$ (in %)	1.20	2.53
$\rho(TB/y, y)$	-0.73	-0.69
Defaults per 100 years	2.78	0.61

Table 3: Business cycle statistics in the baseline economy and in the economy without dilution.

and abstracting from the constraint on long-term bond prices, the first-order conditions in the economy without dilution are given by

$$u'(c^{\text{No dil}})\left[q_L^{\text{No dil}}(b'_S, b'_L, y, g^*) + \frac{\partial q_L^{\text{No dil}}(b'_S, b'_L, y, g^*)}{\partial b'_L} b'_L + \frac{\partial q_S^{\text{No dil}}(b'_S, b'_L, y, g^*)}{\partial b'_L} b'_S\right] =$$
$$-\beta E_{y',g^{*\prime}}\left[\left(1 - d'^{,\text{ No dil}}\right)\frac{\partial \hat{V}^{R,\text{ No dil}}\left(b'_S, b'_L, y', g^{*\prime}\right)}{\partial b'_L} + d'^{,\text{ No dil}}\frac{\partial \hat{V}^{D,\text{ No dil}}\left(b'_S, b'_L, y', g^{*\prime}\right)}{\partial b'_L} \mid y, g^*\right], \quad (18)$$

for long-term debt borrowing and

$$u'(c^{\text{No dil}})\left[q_S^{\text{No dil}}(b'_S, b'_L, y, g^*) + \frac{\partial q_L^{\text{No dil}}(b'_S, b'_L, y, g^*)}{\partial b'_S} b'_L + \frac{\partial q_S^{\text{No dil}}(b'_S, b'_L, y, g^*)}{\partial b'_S} b'_S\right] =$$
$$-\beta E_{y',g^{*\prime}}\left[\left(1 - d'^{,\text{ No dil}}\right)\frac{\partial \hat{V}^{R,\text{ No dil}}\left(b'_S, b'_L, y', g^{*\prime}\right)}{\partial b'_S} + d'^{,\text{ No dil}}\frac{\partial \hat{V}^{D,\text{ No dil}}\left(b'_S, b'_L, y', g^{*\prime}\right)}{\partial b'_S} \mid y, g^*\right]. \quad (19)$$

for short-term borrowing. The super-index "No dil" is used to denote functions in the economy without dilution. The partial derivative

$$\frac{\partial \hat{V}^{R,\text{ No dil}}(b_S, b_L, y, g^*)}{\partial b_L} = -u'(c^{\text{No dil}})\left(1 + (1-\delta) q_L^{\text{No dil}}(0, (1-\delta) b_L, y, g^*)\right)$$

when the government dilutes debt in the current period, i.e., $q_L^{\text{No dil}}(0,(1-\delta)b_L,y,g^*) > q_L^{\text{No dil}}\left(b_S'^{\text{No dil}}, b'\right.$
where $b_L'^{\text{No dil}} = \hat{b}_L^{\text{No dil}}(b_S,b_L,y,g^*)$ and $b_S'^{\text{No dil}} = \hat{b}_S^{\text{No dil}}(b_S,b_L,y,g^*)$.[26]

The comparison of equations (16)-(17) and (18)-(19) shows how our modification to the baseline model affects the tradeoffs faced by the government when it issues debt. In equations (16)-(17), the government only internalizes as a cost the negative effect that short-term and long-term bond issuances have on the value of long-term bonds issued in the current period. It does not internalize as cost the negative effect that short-term and long-term bond issuances have on the value of the long-term debt issued in previous periods. In contrast, equations (18)-(19) show that with our modification to the baseline model, when the government issues debt, it internalizes the dilution in the value of debt issued in previous periods.

Equations (18)-(19) illustrate the tradeoffs faced by the government for given bond prices. But the change in the government's tradeoffs also affect the bond price schedules the government faces when issuing debt. We depict that in Figure 3. The figure presents the yield demanded by bond holders as a function of the end-of-period long-term debt stock.

Figure 3 shows that a shift in the government's choice set plays an important role in accounting for the reduction in spreads implied by the elimination of debt dilution: Even for the same debt levels, spread levels are higher in the benchmark than in the economy without dilution. In the model without dilution, borrowing-contingent payments weaken the government's incentives to issue debt and thus imply lower future issuance levels. For any debt level, the expectation of lower future issuance levels implies a lower default probability. This allows the government to pay a lower yield when it issues debt in the current period.

The fact that the government faces a lower yield in the economy without dilution does not

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[26]

$$\frac{\partial \hat{V}^{R,\text{ No dil}}(b_S,b_L,y,g^*)}{\partial b_L} = -u'(c^{\text{No dil}}) + (1-\delta)\beta E_{y',g^{*\prime}}\left[\frac{\partial \hat{V}^{\text{No dil}}\left(0,(1-\delta)b_L,y',g^{*\prime}\right)}{\partial b_L'} \mid y, g^*\right]$$

if the government does not issue debt in the period ($b_L'^{\text{No dil}} = (1-\delta)b_L$ and $b_L'^{\text{No dil}} = 0$) and

$$\frac{\partial \hat{V}^{R,\text{ No dil}}(b_S,b_L,y,g^*)}{\partial b_L} = -u'(c^{\text{No dil}})\left(1+(1-\delta)q_L^{\text{No dil}}\left(b_S'^{\text{No dil}}, b_L'^{\text{No dil}}, y, g^*\right)\right)$$

if the government buys back long-term debt ($b_L'^{\text{No dil}} < (1-\delta)b_L$) and does not dilute debt ($q_L^{\text{No dil}}(0,(1-\delta)b_L,y,g^*) \leq q_L^{\text{No dil}}\left(b_S'^{\text{No dil}}, b_L'^{\text{No dil}}, y, g^*\right)$).

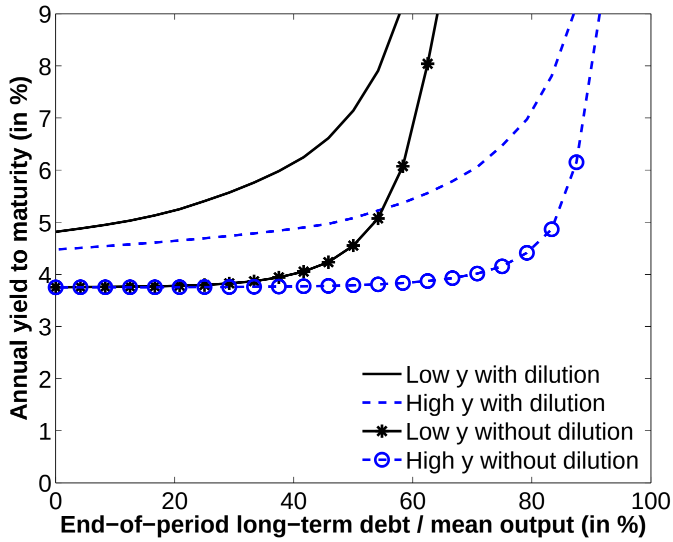

Figure 3: Menu of combinations of yield to maturity of long-term debt (i_L) and end-of-period long-term debt levels ($\frac{b'_L}{\delta+r}$) from which the government can choose in the baseline economy and in the economy without dilution. The graph assumes that the government does not issue short-term debt in the current period ($b'_S = 0$) and that $g^* = \mu_g{}^*$. The low (high) value of y corresponds to a domestic income realization that is one standard deviation below (above) the unconditional mean.

mean that it is less costly to issue debt. Figure 3 does not incorporate the cost of compensating bond holders for the dilution in the value of their debt claims. Figure 3 shows that this cost is more sensitive to debt issuances at lower domestic income realizations, given that future default probability becomes more sensitive to debt levels at lower domestic income realizations. This makes issuing debt more costly at lower domestic income realizations. Figure 4 illustrates this property of the model. For sufficiently low income levels, the government finds it optimal not to issue any debt in the current period and, thus, it does not need to compensate bond holders. For moderately low income levels, the government finds it optimal to issue debt (and compensate bond holders) but debt issuances are lower than what is observed in the economy with dilution. For sufficiently high income levels, the government exploits the lower yield by borrowing more than what it does in the economy with dilution. Figure 4 also helps us understand why consumption volatility is higher in the economy without dilution. At low domestic income realizations, the obligation to compensate bond holders limit the use of debt issuances as a tool to buffer adverse domestic income shocks. Thus, the government is less effective in mitigating

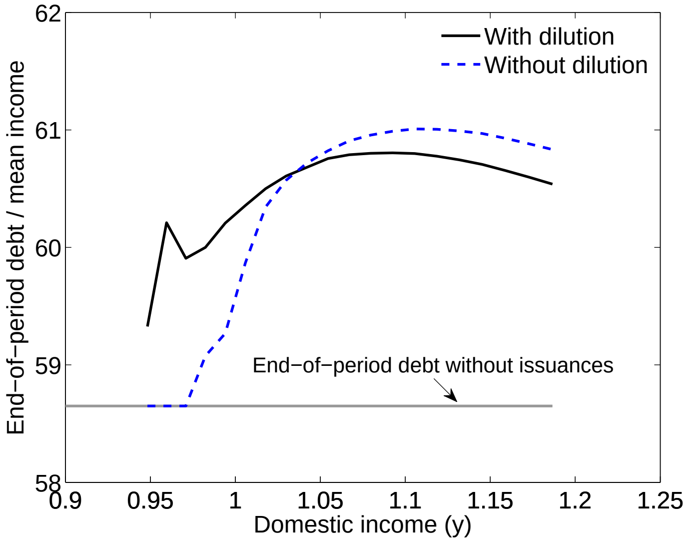

Figure 4: Normalized end-of-period debt $\left(\left(\hat{b}_S + \hat{b}_L/(r+\delta)\right)/(4e^{\mu_y})\right)$ as a function of the domestic income realization. The graph restricts to domestic income realizations at which the government repays its debt in the current period. It is assumed that $g^* = \mu_g$, the initial long-term debt equals the mean total debt level in the simulations and the government starts with no short-term debt

the effects of low income realizations on consumption in the economy without dilution.

The previous discussion shows that there is a trade-off between reducing the frequency of defaults (which are costly because of the output losses and impaired ability to borrow after the default) and increasing the consumption volatility. In the next subsection we show that welfare increases with the elimination of debt dilution, which suggests that the first effect dominates in terms of welfare.

4.3 Welfare gains from eliminating dilution

We measure welfare gains as the constant proportional change in consumption that would leave a consumer indifferent between continuing living in the benchmark economy (with dilution) and moving to an economy without dilution. The welfare gain of moving from the benchmark economy to the economy without dilution is given by

$$\left(\frac{\hat{V}^{\text{No dil}}(b_S, b_L, y, g^*)}{\hat{V}^{\text{Dil}}(b_S, b_L, y, g^*)}\right)^{\left(\frac{1}{1-\gamma}\right)} - 1.$$

Figure 5 presents welfare gains from implementing the borrowing-contingent payments that eliminate dilution. The figure considers two initial debt portfolios: in one the government enters the period with no debt, and in other the government enters the period with a debt portfolio equal to the average portfolio observed in the simulations of the economy with dilution. Figure 5 shows that for both cases there are positive welfare gains from eliminating dilution. Eliminating dilution reduces the frequency of defaults, and with that it reduces the deadweight losses caused by defaults.

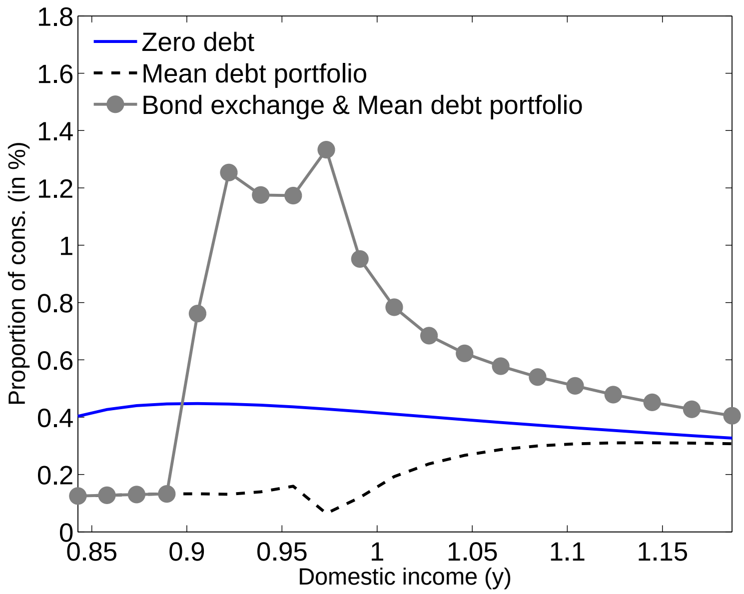

Figure 5: Consumption compensation (in percentage terms) that makes domestic agents indifferent between living in an economy with or without dilution. The graph was computed for two debt portfolios at the beginning of the reform period: in one there is no debt and in the other the debt portfolio equals the average debt portfolio in the simulations of the baseline economy (with dilution). We computed two welfare measures for the latter. The solid blue line assumes that the government starts compensating bond holders in the current period. The solid line with circles assumes that the government buys back outstanding long-term bonds at the price that would have been observed in the baseline economy and then issues debt with the borrowing-contingent payment covenant. In all exercises it is assumed that $g^* = \mu_g$. A positive number means that domestic agents prefer the economy without dilution.

The dashed line in Figure 5 assumes that the government eliminates dilution (in a period with a representative debt portfolio) by changing the terms of the long-term contracts that were issued in the past: the government amends the existing long-term debt contracts by unilaterally introducing the borrowing-contingent payments that eliminate dilution. This is advantageous for the affected bond holders because the value of their debt claims (after the introduction of borrowing-contingent payments) appreciate, but transiting to the new stationary equilibrium with lower debt is costly for the government. This explains why welfare gains from eliminating dilution at states with positive initial debt are lower than at states with no debt. Figure 5 also presents welfare gains for the case in which the government captures existing bondholders' capital gains from the introduction of the borrowing-contingent payments that eliminate debt dilution. We assume that the government captures these gains through a debt exchange: The government makes a take-it-or-leave-it debt buyback offer with the promise that these borrowing-contingent payments will be implemented only if the debt-exchange offer is accepted. Thus, the government offers bondholders to buy back previously issued bonds at the price that would have been observed if borrowing-contingent payments were never implemented.[27] That price is lower than the no-dilution price at which the government would be able to issue debt after introducing borrowing-contingent payments. By assuming that the government makes a take-it-or-leave-it offer, we focus on the extreme case in which the government reaps all capital gains.[28] The case in which borrowing-contingent payments are introduced without a debt exchange constitutes the other extreme case in which bondholders enjoy all these gains.

It should be mentioned that one may want to take our measure of the welfare gain with a grain of salt. In particular, one could argue that our measure is too low. Since there is no production in our setup, we cannot capture productivity gains from reducing the level of interest rates.[29] Several studies find evidence of significant spillover effects from interest rates to

[27]Formally, the budget constraint under repayment reads as

$$c = y - b_L\left(1 + (1-\delta)q_L^{\text{Dil}}\left(\hat{b}_S^{\text{Dil}}(b_L, b_L, y, g^*), \hat{b}_L^{\text{Dil}}(b_L, b_L, y, g^*), y, g^*\right)\right) - b_S + q_L^{\text{No dil}}(b_S', b_L', y, g^*)b_L' + q_S^{\text{No dil}}(b_S', b_L', y, g^*)b_S'$$

in the period of the exchange and as in equation (11) in all future periods. The superindex "Dil" refers to functions in the economy with dilution and the superindex "No dil" refers to functions in the economy without dilution.

[28]It is also assumed that there are no costs triggered by that debt exchange.

[29]The development of a sovereign default framework that accommodates effects of interest rates on factors

aggregate productivity (through the allocation of factors of production), and of a significant role of interest rate fluctuations in the amplification of shocks (see, for example, Mendoza and Yue (2012), Neumeyer and Perri (2005), and Uribe and Yue (2006)).

4.4 Alternative borrowing-contingent payments

Implementing the borrowing-contingent payments that eliminate dilution would require knowledge of the fundamentals that determine bond prices and the mapping from fundamentals onto bond prices. In this section, we study the effects of two simpler debt covenants.

First, we study the case in which borrowing-contingent payments are a predetermined fixed share of the resources obtained when the government issues debt. Formally, this amounts to solving the economy without dilution but with the following compensation payment schedule

$$\mathcal{C}\left(b_L, y, g^*, b_S', b_L'\right) = \begin{cases} \frac{\tau Max\{q_L(b_S', b_L', y, g^*)(b_L' - (1-\delta)b_L) + q_S(b_S', b_L', y, g^*)b_S', 0\}}{(1-\delta)b_L} & \text{if } b_L' + b_S' > (1-\delta)b_L \text{ and} \\ 0 & \text{otherwise,} \end{cases} \tag{20}$$

instead of the one specified in equation (10). Equation (20) implies that existing holders of long-term debt collectively receive a fraction τ of the revenues the government obtains from current debt issuances when the government increases it's debt stock beyond the debt level that would have been observed in the absence of current debt issuances. Notice that the compensation schedule in equation (20) does not involve the calculation of any counterfactual bond price. The compensation schedule depends only on the current market prices of debt, current issuance volumes, and on the existing stock of long-term debt. We search for the ex-ante optimal share of long-term borrowing revenues the government should promise to holders of its long-term bonds. We find that this share is such that on average the government pays to holders of long-term debt issued in previous periods 7 percent of its issuance revenues.[30]

Second, we study the effects of a simple borrowing-contingent payment schedule that is a

allocation is the subject of ongoing research (see, for example, Mendoza and Yue (2012) and Sosa Padilla (2010)).

[30]It should be mentioned that promising higher borrowing-contingent payments is not necessarily costly for the government because it also allows the government to sell bonds at a higher price.

decreasing function of the post-issuance bond price $q_L(b'_S, b'_L, y, g^*)$ but that does not depend on the bond price that would have been observed in the absence of borrowing.[31] In particular, we assume that the covenant specifies that if the government issues debt in the current period, it has to pay $\bar{q}_L - q_L(b'_S, b'_L, y, g^*)$ per long-term bond issued in previous periods. The parameter $\bar{q}_L$ represents a constant reference value of long-term bonds below which the government compensates the holders of its long-term debt.

$$\mathcal{C}\left(b_L, y, g^*, b'_S, b'_L\right) = \begin{cases} Max\left\{\bar{q}_L - q_L(b'_S, b'_L, y, g^*), 0\right\} & \text{if } b'_L + b'_S > (1-\delta)b_L \text{ and} \\ 0 & \text{otherwise,} \end{cases} \tag{21}$$

We search for the optimal value of $\bar{q}_L$ and find that this value is 2.5% lower than the average price of a risk-free long-term bond without borrowing-contingent payments.

Third, we study an economy without compensations but with debt-elastic coupon payments. We consider the case in which coupon payments increase with debt. Formally, the budget constraint under repayment reads as

The government's budget constraint when it repays its debt reads as

$$c = y - x(b_L, b_S)b_L - b_S + q_L(b_S{}', b_L{}', y, g^*)\left(b_L{}' - (1-\delta)b_L\right) + q_S(b_S{}', b_L{}', y, g^*)b_S{}',$$

where the coupon x satisfies

$$x(b_L, b_S) = 1 + \psi Max\left\{b_S + \frac{b_L}{\delta + r} - \bar{b}, 0\right\} \tag{22}$$

Equation (22) says that the coupon paid per-long term bond increases with the total outstanding debt only for debt levels that are above $\bar{b}$.[32] This scheme is likely to i) have better insurance properties than the compensations that mitigate debt dilution and ii) deter borrowing

[31] Notice that the compensation payment implied by equation (20) is an increasing function of the post-issuance long-term bond price $q_L(b'_S, b'_L, y, g^*)$ whereas the borrowing-contingent payment that eliminates dilution is a decreasing function of $q_L(b'_S, b'_L, y, g^*)$.

[32] We focus on cases in which $\psi > 0$. For simplicity, we approximate the present discounted value of future long-term debt obligations by $\frac{b_L}{\delta+r}$. In fact, the present value of future long-term debt obligations depend on future borrowing decisions which are state-contingent and we do not know its shape prior to solving for the model. The functional equation for long-term bond prices also change in this environment. The issuance price of a long-term bond satisfies

at sufficiently high debt levels. The better insurance property derives from the the procyclicality of borrowing in our benchmark economy, i.e., periods with low debt tend to be periods with low income. The scheme is likely to reduce borrowing past the threshold $\bar{b}$ given that current borrowing increases the coupon payments of all long-term debt issued in the past. That is costly for the government. We search for the optimal values of $(\bar{b}, \psi)$. We find that the optimal $\psi = 0.3$ and $\bar{b} = 2$, which is equivalent to a mean debt ratio of 50 percent of annual output.

Table 4 presents simulation results for these two simpler borrowing-contingent payments. The table shows that with borrowing-contingent payments that depend on a reference long-term bond price, the government achieves a 66% of the ex-ante welfare gain it achieves with the borrowing-contingent payments that eliminate dilution and it reduces the default frequency by even more than what it does with the borrowing-contingent payments that eliminate dilution. We find that the debt-covenant with the optimal revenue share is not that effective in bringing down the default frequency and is less beneficial in terms of welfare. The optimal debt-elastic coupon scheme is successful in reducing consumption volatility but it achieves a modest reduction in the default frequency compare to the compensation payments that eliminate dilution.

In summary, our findings indicate that simple borrowing-contingent payments could also reduce default risk significantly and be welfare enhancing. In what follows we present two alternative economies that help us gauge the role of debt dilution relative to the two sources of inefficiency in the environment we study: the lack of complete markets and the government's limited liability.

$$q_L({b_S}', {b_L}', y, g^*) = E_{y', g^{*\prime}} \left[M\left(g^*, g^{*\prime}\right) \left[(1-d') \left(x\left({b_S}', {b_L}'\right) + (1-\delta){q_L}' \right) + d' q_L^{D\prime} \right] \mid y, g^* \right],$$

and the price of a long-term bond in default satisfies

$$q_L^D({b_S}', {b_L}', y, g^*) = E_{y', g^{*\prime}} \left[M\left(g^*, g^{*\prime}\right)(1+r) \left[(1-\xi) q_L^{D\prime} + \xi\alpha \left((1-d') \left(x\left(\alpha {b_S}', \alpha {b_L}'\right) + (1-\delta){q_L}' \right) + d' q_L^{DD\prime} \right) \right] \mid y, g^* \right]$$

	With dilution	Without dilution	Fixed share	Reference price	Elastic coupon
Debt / mean annual income (in %)	61.8	56.3	53.4	54.7	49.3
Debt duration (years)	5.95	7.70	7.67	7.55	7.48
Debt obligations within 1 year/ Total debt (in %)	23.0	13.5	30.0	12.3	12.2
Spread of short-term debt (in %)	0.73	0.39	1.02	0.32	0.40
Spread of long-term debt (in %)	2.10	0.37	2.79	0.88	0.60
$\sigma(c)/\sigma(y)$	1.50	2.07	1.22	3.01	1.06
$\sigma(TB/y)$ (in %)	1.20	2.53	0.75	2.76	0.2
$\rho(TB/y, y)$	-0.73	-0.69	-0.61	-0.69	-0.51
Defaults per 100 years	2.78	0.61	2.10	0.49	1.68
Welfare gain (% of cons.)		0.41	0.15	0.27	0.2

Table 4: Simulation results for different borrowing-contingent payments. The welfare gain corresponds to the average gain for the case of zero debt.

4.5 Borrowing-contingent payments vs. one-period state-contingent claims

Equation (11) makes clear that the borrowing-contingent payments needed to eliminate dilution are state-contingent. In this subsection, we explore how those borrowing-contingent payments perform against claims contingent on the domestic income realization. It would be best for the government to transfer resources across periods using contracts with payoffs that are conditional on the past history of state realizations. For tractability reasons, in this subsection we consider a market structure in which the government can only issue one-period Arrow-Debreu securities that pay off conditional on the next-period domestic income realization.[33] The government is subject to the same limited liability constraint that is present in the benchmark economy.[34]

[33]The borrowing-contingent payments that eliminate debt-dilution depend on the domestic and foreign shock. Given the difference in risk aversion between foreign bond holders and domestic agents, we opted to abstract from contracts with payments contingent on the foreign growth rate.

[34]Issuing long-term debt allows the government to bring resources forward from future periods, not only from the subsequent period. That is not an option in the economy with one-period Arrow-Debreu securities. Clearly, this limitation is not an issue in the absence of the limited liability constraint. But it can dampen the welfare gain from issuing Arrow-Debreu securities in the presence of such constraint.

We assume that the government chooses how much it promises to pay next period for each realization of next-period domestic income y' (payments can be negative). The government is subject to the same cost of defaulting that is present in the benchmark economy. We simplify the model by assuming that the government can only promise payments for which it would choose not to default (we discuss this assumption below and argue that it is unlikely to have a significant effect on the results).

As in the rest of the paper, we focus on Markov Perfect Equilibria. Let $\hat{W}(b, y, g^*)$ denote the value function when the government follows the borrowing rule $\hat{b}$ and the defaulting rule $\hat{d}$ in the current and every future period. That is:

$$\hat{W}(b, y, g^*) = \hat{d}(b, y, g^*) \hat{W}^D(b, y, g^*) + (1 - \hat{d}(b, y, g^*)) \hat{W}^R(b, y, g^*), \tag{23}$$

where the continuation value under repayment $\hat{W}^R$ satisfies:

$$\hat{W}^R(b, y, g^*) = u(c) + \beta E_{y', g^{*\prime}} \left[\hat{W}(\hat{b}(b, y, g^*; y'), y', g^{*\prime}) \mid y, g^* \right], \tag{24}$$

subject to

$$c = y - b + q(g^*) \int \hat{b}(b, y, g^*; y') dy'.$$

The function $\hat{b}(b, y, g^*; y')$ denotes the number of Arrow-Debreu securities issued against a next-period domestic income realization of y' when the current state is (b, y, g^*). The function $q(g^*)$ denotes the price of those securities.

The function $\hat{W}^D$ denotes the continuation value at the beginning of next-period when the next-period government defaults on its debt and then follows the decision rules $(\hat{d}, \hat{b})$ in every future period (in which it is not in default). Formally,

$$\hat{W}^D(b, y, g^*) = u(c) + \beta E_{y', g^{*\prime}} \left[(1 - \xi) \hat{W}^D(b', y', g^{*\prime}) + \xi \hat{W}(\alpha b', y', g^{*\prime}) \mid y, g^* \right], \tag{25}$$

subject to

$c = y - \phi(y)$, and

$b' = b(1 + r)$.

Let W^R denote the maximum utility the current government can attain if it repays its debt and expects every future government to follow the borrowing and defaulting rules $(\hat{b}, \hat{d})$. Namely:

$$W^R(b, y, g^*) = \max_{b'(y')} \left\{ u(c) + \beta E_{y', g^{*\prime}} \left[\hat{W}(b'(y'), y', g^{*\prime}) \mid y, g^* \right] \right\}, \tag{26}$$

subject to

$$c = y - b + q(g^*) \int b'(y') dy', \text{ and}$$

$$b'(y') \leq \bar{b}(y'),$$

where the maximum borrowing against next-period states with domestic income y' satisfies

$$\bar{b}(y') = \sup \left\{ \tilde{b} : \hat{W}^R(\tilde{b}, y', g^*) \geq \hat{W}^D(\tilde{b}, y', g^*) \quad \forall g^* \right\}. \tag{27}$$

Equation (27) implies that the government never starts a period with a debt level at which it defaults. This assumption simplifies the problem and it is unlikely to have a significant effect on the results. For example, Figure 6 illustrate that the equilibrium value functions $\hat{W}^R$ and $\hat{W}^D$ are not much sensitive to the realization of bond holders consumption growth and, thus, the maximum debt levels with a positive repayment probability is very close to the maximum debt levels with certain repayment.

The function W^D denotes the optimal continuation value for a government that has decided to default in the current period and expects every future government to follow the borrowing and defaulting rules $(\hat{b}, \hat{d})$. Given that a defaulting government cannot borrow in the current period, $W^D = \hat{W}^D$.

A Markov Perfect Equilibrium is then characterized by value functions $\hat{W}$, $\hat{W}^R$, $\hat{W}^D$, policy rules $\hat{d}$, $\hat{b}$, and bond price function q such that

(a) $\hat{d}(b, y, g^*) = \underset{d \in \{0,1\}}{Argmax} \left\{ dW^D(b, y, g^*) + (1-d) W^R(b, y, g^*) \right\}$ for all b, y, g^*, where W^R satisfies (26) and $W^D = \hat{W}^D$.

The function $\hat{b}$ solve (26) for all b,y, g^*, given $\hat{W}$, and q

(b) $\hat{W}$, $\hat{W}^R$, and $\hat{W}^D$ satisfy functional equations (23)-(25),

(c) and the bond price function q satisfies

$$q(g^*) = E_{g^{*\prime}} \left[M(g^*, g^{*\prime}) \right]$$

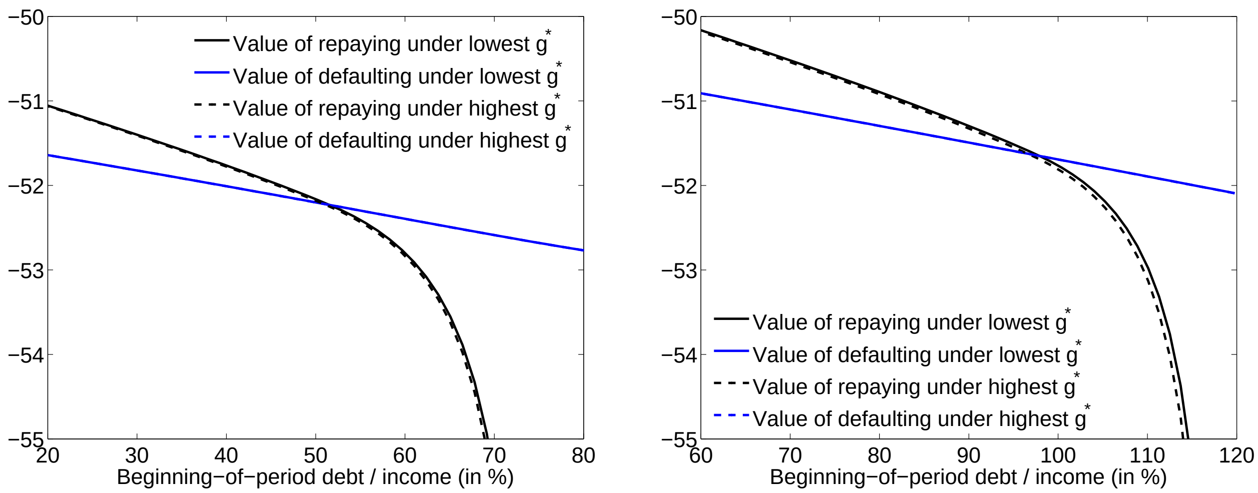

Figure 6: Value function of repayment ($\hat{W}^R$) and defaulting ($\hat{W}^D$) for two domestic income realization. The panel on the left (right) corresponds to a current domestic income realization that is one standard deviation below (above) the unconditional mean. The lowest (highest) value for g^* corresponds to a current bond holders consumption growth that is 2 standard deviations below (above) the unconditional mean. The beginning of period debt to income ratio was calculated as $b/(4y)$.

.

We solve for the equilibrium using the same parameter values presented in Table 1. Table 5 summarizes simulation results in the benchmark economy, in the economy with the borrowing-contingent payments that eliminate dilution, and in the economy with state-contingent claims. The table shows that the average ex-ante welfare gain from moving to an economy with claims contingent on the domestic income realization amounts to a permanent increase in consumption of 1.12%. There are two sources of welfare gains from using claims contingent on the domestic income realization. First, issuing claims contingent on the domestic income realization allows the government to avoid defaults without compromising the government's ability to bring resources forward. In fact, the government is able to obtain more resources when it issues claims contingent on the domestic income realization (as reflected in the higher debt level). Second, the consumption process is more disentangled from the income process in the economy with state-contingent claims (as reflected in the lower consumption volatility).

Welfare gains from moving to an economy with borrowing-contingent payments are 37 per-

	With dilution	Without dilution	State-contingent claims
Debt / mean annual income (in %)	61.8	56.3	75.0
$\sigma(c)/\sigma(y)$	1.50	2.07	0.67
$\sigma\,(TB/y)$ (in %)	1.20	1.39	1.39
$\rho\,(TB/y, y)$	-0.73	-0.69	0.81
Defaults per 100 years	2.78	0.61	0
Welfare gain (% of cons.)		0.41	1.12

Table 5: Business cycle statistics in the benchmark economy and in the economies with the borrowing-contingent payments that eliminate dilution and with one-period claims contingent on domestic income realizations. The moments in the economy with state-contingent claims correspond to averages over the last 72 periods of each sample path. The welfare gain corresponds to the average gain for the case of zero debt.

cent of those from introducing state-contingent claims. In contrast with state-contingent claims, borrowing-contingent payments do not eliminate defaults completely, reduce slightly the government's debt, and increase consumption volatility.

Our stylized framework is likely to overstate the advantages of introducing claims contingent on the domestic income realization over the introduction of borrowing-contingent payments. First, identifying the state is much more difficult in the real world than in our stylized model. In our model income shocks are the only source of uncertainty. However, Tomz and Wright (2007) argue that other determinants of the sovereigns' willingness to repay besides aggregate income play an important role in accounting for sovereign defaults. Identifying the shocks that affect default risk, measuring these shocks, and writing debt contracts with payments contingent on these shocks may be difficult. Second, writing debt claims contingent on income may suffer from verifiability and moral hazard issues that are not present in our stylized model. A government could manipulate the GDP calculation and final GDP data are available with a significant lag. It has also been argued that debt claims contingent on GDP may introduce moral hazard problems by weakening the government's incentives to implement growth-promoting policies, which improves the likelihood of repayment.[35]

[35]See, for instance, Krugman (1988). These issues could be addressed by indexing debt contracts to variables

4.6 Borrowing-contingent payments vs. no defaults

In this subsection we present an alternative benchmark to gauge the gains from eliminating dilution: We remove the limited commitment assumption. Formally, we solve for a recursive competitive equilibrium characterized by a value function V and borrowing rule $\hat{b}$ such that:

(a) The value function V solves the functional equation:

$$V(b, y, g^*) = \max_{b'} \left\{ u(c) + \beta E_{y', g^{*\prime}} \left[V(b', y', g^{*\prime}) \mid y, g^* \right] \right\}, \tag{28}$$

subject to

$$c = y - b + q(g^*)b',$$

$$b' \leq \bar{b}.$$

(b) The borrowing rule $\hat{b}$ attains the maximum of (28) for all (b, y, g^*),

(c) and the bond price function q satisfies

$$q(g^*) = E_{g^{*\prime}} \left[M(g^*, g^{*\prime}) \right]$$

.

We solve for the equilibrium using the same parameter values presented in Table 1. We assume that $\bar{b} = 60y$. This bound is close to the natural borrowing limit for the income grid used to solve for the model. The results are reported in Table 6. As expected, the government exploits the laxer borrowing constraint to bring resources forward and carries a significantly higher debt level. The consumption volatility is larger than in the benchmark economy and than in the economy without dilution. This is a consequence of the lower consumption levels caused by the cost of servicing a significantly larger debt stock. Fluctuations in domestic income have a proportional larger effect on consumption given that debt levels are close to the maximum debt.

that the government cannot control such as commodity prices or trading partners' growth rates (see for instance Caballero (2002)).

	With dilution	Without dilution	Without defaults
Debt / mean annual income (in %)	61.8	56.3	1,485
$\sigma(c)/\sigma(y)$	1.50	2.07	3.29
$\sigma\,(TB/y)$ (in %)	1.20	1.39	1.28
$\rho\,(TB/y, y)$	-0.73	-0.69	-0.46
Defaults per 100 years	2.78	0.61	0
Welfare gain (% of cons.)		0.41	11.41

Table 6: Business cycle statistics in the benchmark economy and in the economies with the borrowing-contingent payments that eliminate dilution and with one-period claims contingent on domestic income realizations. The moments in the economy with no defaults correspond to averages over the last 72 periods of each sample path. Each path has 1,000 periods. The welfare gain corresponds to the average gain for the case of zero debt.

5 Conclusions

We solved the canonical Eaton and Gersovitz (1981) model extended in three dimensions: i) the average duration of sovereign debt is endogenous, ii) bond holders are risk averse and are subject to shocks, and iii) the recovery rate of debt in default is positive. We introduced a debt covenant mandating that after each time the government borrows, it has to pay each holder of previously issued long-term bonds the difference between the long-term bond price that would have been observed absent current-period borrowing and the observed price. This covenant eliminates debt dilution—caused by borrowing decisions—by disentangling the value of long-term bonds from future borrowing decisions. We measured the effects of debt dilution by comparing the simulations of this model with the ones of the baseline model without borrowing-contingent payments. We found that even without commitment to future repayment policies and without optimally designed contingent claims, if the sovereign eliminates debt dilution, the default probability decreases 78%. We also showed that most gains from eliminating dilution can be obtained with borrowing-contingent payments that depend only on observed long-term bond prices.

Our findings indicate that governments could benefit from reducing future borrowing levels, which governments could also achieve through fiscal rules. Eliminating debt dilution should be an important motivation for the implementation of fiscal rules that could reduce significantly the risk of debt crises and the mean and volatility of interest rates (see Hatchondo et al. (2011)). Fiscal crises are occurring in countries with fiscal rules in part because of the weak enforcement of these rules. The borrowing-contingent covenants studied in this paper could enhance the enforcement of fiscal rules by providing market incentives to lower future borrowing levels.

References

Aguiar, M. and Gopinath, G. (2006). 'Defaultable debt, interest rates and the current account'. *Journal of International Economics*, volume 69, 64–83.

Aguiar, M. and Gopinath, G. (2007). 'Emerging markets business cycles: the cycle is the trend'. *Journal of Political Economy*, volume 115, no. 1, 69–102.

Alvarez-Parra, F., Brandao-Marques, L., and Toledo, M. (2013). 'Durable goods, financial frictions, and business cycles in emerging economies'. *Journal of Monetary Economics*, volume 60(6), 720–736.

Arellano, C. (2008). 'Default Risk and Income Fluctuations in Emerging Economies'. *American Economic Review*, volume 98(3), 690–712.

Arellano, C. and Ramanarayanan, A. (2012). 'Default and the Maturity Structure in Sovereign Bonds'. *Journal of Political Economy*, volume 120, no. 2, 187–232.

Asquith, P., Beatty, A., and Weber, J. (2005). 'Performance pricing in bank debt contracts'. *Journal of Accounting and Economics*, volume 40, 101128.

Athreya, K., Tam, X. S., and Young, E. R. (2007). 'Does Rising Income Risk Lead to Better Risk Sharing?' Manuscript, University of Virginia.

Benjamin, D. and Wright, M. L. J. (2008). 'Recovery Before Redemption? A Theory of Delays in Sovereign Debt Renegotiations'. Manuscript.

Bi, R. (2006). 'Debt Dilution and the Maturity Structure of Sovereign Bonds'. Manuscript, University of Maryland.

Bianchi, J. (2011). 'Overborrowing and Systemic Externalities in the Business Cycle'. *American Economic Review*, volume 101(7), 3400–3426.

Bizer, D. S. and DeMarzo, P. (1992). 'Sequential Banking'. *Journal of Political Economy*, volume 100, 41–61.

Bolton, P. and Jeanne, O. (2009). 'Structuring and Restructuring Sovereign Debt: The Role of Seniority'. *Review of Economic Studies*, volume 76, 879–902.

Bolton, P. and Skeel, D. A. (2005). 'Redesigning the International Lender of Last Resort'. *Chicago Journal of International Law*, volume 6, no. 1, 177–201.

Borensztein, E., Chamon, M., Jeanne, O., Mauro, P., and Zettelmeyer, J. (2004). 'Sovereign Debt Structure for Crisis Prevention'. IMF occasional paper 237.

Borensztein, E. and Panizza, U. (2009). 'The Costs of Sovereign Default'. *IMF Staff Papers*, volume 56(4), 683–741.

Boz, E. (2011). 'Sovereign Default, Private Sector Creditors, and the IFIs'. *Journal of International Economics*, volume 83, 70–82.

Caballero, R. (2002). 'Coping with Chile.s external vulnerability: a financial problem'. In N. Loayza and R. Soto (eds), Economic growth: sources, trends and cycles, Banco Central de Chile, Santiago, Chile, 377-415.

Calvo, G. (1988). 'Servicing the Public Debt: The Role of Expectations'. *American Economic Review*, volume 78 (4), 647–661.

Carey, M., Prowse, S., Rea, J., and Udell, G. (1993). 'he Economics of the Private Placement Market'. Board of Governors of the Federal Reserve System.

Chamon, M. and Mauro, P. (2006). 'Pricing growth-indexed bonds'. *Journal of Banking & Finance*, volume 30, 3349–3366.

Chatterjee, S., Corbae, D., Nakajima, M., and Ríos-Rull, J.-V. (2007). 'A Quantitative Theory of Unsecured Consumer Credit with Risk of Default'. *Econometrica*, volume 75, 1525–1589.

Chatterjee, S. and Eyigungor, B. (2012). 'Maturity, Indebtedness and Default Risk'. *American Economic Review*, volume 102(6), 2674–2699.

Chatterjee, S. and Eyigungor, B. (2013). 'Debt Dilution and Seniority in a Model of Defaultable Sovereign Debt'. Federal Reserve Bank of Philadelphia Working Paper No. 13-30.

Choi, S. J., Gulati, M., and Posner, E. A. (2011). 'Political Risk and Sovereign Debt Contracts'. John M. Olin Law & Economics Working Paper No. 583.

Claessens, S., Mody, A., and Vallée, S. (2012). 'Paths to Eurobonds'. IMF Working Paper.

Cruces, J. J. and Trebesch, C. (2013). 'Sovereign Defaults: The Price of Haircuts'. *American Economic Journal: Macroeconomics*, volume 5(3), 85–117.

Cuadra, G., Sanchez, J. M., and Sapriza, H. (2010). 'Fiscal policy and default risk in emerging markets'. *Review of Economic Dynamics*, volume 13(2), 452–469.

D'Erasmo, P. and Mendoza, E. (2013). 'Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default'. NBER Working Paper No. 19477.

Detragiache, E. (1994). 'Sensible buybacks of sovereign debt'. *Journal of Development Economics*, volume 43, 317–333.

Dias, D. A. and Richmond, C. (2009). 'Duration of Capital Market Exclusion: An Empirical Investigation'. Working Paper, UCLA.

Durdu, C. B., Nunes, R., and Sapriza, H. (2013). 'News and sovereign default risk in small open economies'. *Journal of International Economics*, volume 91(1), 1–17.

Eaton, J. and Fernandez, R. (1995). 'Sovereign debt'. NBER Working Paper No. 5131.

Eaton, J. and Gersovitz, M. (1981). 'Debt with potential repudiation: theoretical and empirical analysis'. *Review of Economic Studies*, volume 48, 289–309.

Eichengreen, B. and Lindert, P. H., editors (1989). *The International Debt Crisis in Historical Perspective*. The MIT Press.

Epstein, L. G. and Zin, S. E. (1989). 'Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework'. *Econometrica*, volume 57(4), 937–969.

European Council (2012). 'Towards a Genuine Economic and Monetary Union, Report by the President of the European Council, in collaboration with the Presidents of the Commission, the Eurogroup and the ECB'.

Fishlow, A. (1985). 'Lessons from the Past: Capital Markets During the 19th Century and the Interwar Period'. *International Organization*, volume 39(3), 383–439.

G-10 (2002). 'Report of the G-10 Working Group on Contractual Clauses'.

García-Cicco, J., Pancrazi, R., and Uribe, M. (2010). 'Real Business Cycles in Emerging Countries?' *American Economic Review*, volume 100, 2510–2531.

Gelos, G., Sahay, R., and Sandleris, G. (2011). 'Sovereign Borrowing by Developing Countries: What Determines Market Access?' *Journal of International Economics*, volume 83, 243–254.

Hatchondo, J. C. and Martinez, L. (2009). 'Long-duration bonds and sovereign defaults'. *Journal of International Economics*, volume 79, 117125.

Hatchondo, J. C. and Martinez, L. (2013). 'Sudden Stops, Time Inconsistency, and the Duration of Sovereign Debt'. *International Economic Journal*, volume 27 (1), 217–228.

Hatchondo, J. C., Martinez, L., and Roch, F. (2011). 'Fiscal rules and the sovereign default premium'. Mimeo, University of Chicago.

Hatchondo, J. C., Martinez, L., and Sapriza, H. (2010). 'Quantitative properties of sovereign default models: solution methods matter'. *Review of Economic Dynamics*, volume 13, no. 4, 919–933. Doi:10.1016/j.red.2010.03.001.

IMF (2002). 'The Design and Effectiveness of Collective Action Clauses'.

IMF (2003). 'Proposed Features of a Sovereign Debt Restructuring Mechanism'.

Kletzer, K. M. (1984). 'Asymmetries of Information and LDC Borrowing with Sovereign Risk'. *The Economic Journal*, volume 94, 287–307.

Krueger, A. O. and Hagan, S. (2005). 'Sovereign Workouts: an IMF Perspective'. *Chicago Journal of International Law*, volume 6, no. 1, 203–218.

Krugman, P. (1988). 'Financing vs. forgiving a debt overhang'. *Journal of Development Economics*, volume 29(3), 253–268.

Krusell, P. and Smith, A. (2003). 'Consumption-savings decisions with quasi-geometric discounting'. *Econometrica*, volume 71, no. 1, 365–375.

Levy Yeyati, E. and Panizza, U. (2011). 'The elusive costs of sovereign defaults'. *Journal of Development Economics*, volume 94, 95–105.

Li, W. and Sarte, P.-D. (2006). 'U.S. consumer bankruptcy choice: The importance of general equilibrium effects'. *Journal of Monetary Economics*, volume 53, 613–631.

Livshits, I., MacGee, J., and Tertilt, M. (2008). 'Consumer Bankruptcy: A Fresh Start,'. *American Economic Review*, volume 97, 402–418.

Lizarazo, S. (2012). 'Contagion of financial crises in sovereign debt markets'. Working Paper.

Lizarazo, S. (2013). 'Sovereign risk and risk averse international investors'. *Journal of International Economics*, volume 82(2), 317–330.

Mendoza, E. and Yue, V. (2012). 'A General Equilibrium Model of Sovereign Default and Business Cycles'. *The Quarterly Journal of Economics*, volume 127(2), 889–946.

Neumeyer, P. and Perri, F. (2005). 'Business cycles in emerging economies: the role of interest rates'. *Journal of Monetary Economics*, volume 52, 345–380.

Niepelt, D. (2008). 'Debt Maturity without Commitment'. Manuscript, Study Center Gerzensee.

Paulus, C. G. (2002). 'A Legal Order for Insolvencies of States'. Working Paper, Humboldt University.

Piazzesi, M. and Schneider, M. (2007). 'Equilibrium Yield Curves'. *NBER Macroeconomics Annual*, pages 389–472. Vol. 21, ed. Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, Cambridge, MA: MIT Press.

Reinhart, C., Rogoff, K. S., and Savastano, M. A. (2003). 'Debt Intolerance'. *Brookings Papers on Economic Activity*, pages 1–74.

Rodgers, C. (1965). 'The corporate trust indenture project'. *Business Lawyer*, volume 20, 551–571.

Rudebusch, G. D. and Swanson, E. T. (2012). 'The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks'. *American Economic Journal: Macroeconomics*, volume 4 (1), 105–143.

Sachs, J. and Cohen, D. (1982). 'LDC borrowing with default risk'. National Bureau of Economic Research Working Paper 925.

Sanchez, J. M. (2010). 'The IT Revolution and the Unsecured Credit Market'. Federal Reserve Bank of St. Louis Working Paper.

Saravia, D. (2010). 'On The Role and Effects of IMF Seniority'. *Journal of International Money and Finance*, volume 29(6), 1024–1044.

Smith, C. W. and Warner, J. B. (1979). 'On Financial Contracting: An Analysis of Bond Covenants'. *Journal of Financial Economics*, volume 7, 117–161.

Sosa Padilla, C. (2010). 'Sovereign Defaults and Banking Crises'. Manuscript, University of Maryland.

Sturzenegger, F. and Zettelmeyer, J. (2006). 'Defaults in the 90s'. Manuscript, Universidad Torcuato Di Tella.

Tauchen, G. (1986). 'Finite state markov-chain approximations to univariate and vector autoregressions'. *Economics Letters*, volume 20 (2), 177–181.

Tirole, J. (2002). 'Financial Crises, Liquidity, and the International Monetary System'. Princeton University Press.

Tomz, M. and Wright, M. L. J. (2007). 'Do Countries Default in "Bad Times"?' *Journal of the European Economic Association*, volume 5, 352–360.

Trebesch, C. and Wright, M. L. J. (2013). 'A Default Measure of Sovereign Default'. Mimeo.

UN (2004). 'Strategic issies in managing sovereign debt for sustained development: an issues paper for the multi-stakeholder dialogue on debt'. United Nations.

Uribe, M. and Yue, V. (2006). 'Country spreads and emerging countries: Who drives whom?' *Journal of International Economics*, volume 69, 6–36.

Weil, P. (1989). 'The Equity Premium Puzzle and the Risk-Free Rate Puzzle'. *Journal of Monetary Economics*, volume 24(3), 401–421.

Wright, M. L. J. (2005). 'Coordinating Creditors'. *The American Economic Review, Papers and Proceedings*, volume 95, no. 2, 388–392.

Yue, V. (2010). 'Sovereign default and debt renegotiation'. *Journal of International Economics*, volume 80, no. 2, 176–187.

A Numerical algorithm for the baseline economy

The algorithm iterates on two value functions, $\hat{V}^R$ and $\hat{V}^D$ and four price functions q_L, q_L^D, q_S, and q_S^D until convergence is attained.

We approximate $\hat{V}^R$ and $\hat{V}^D$ using linear interpolation for y and bi-dimensional tensor-product spline interpolation for b_S and b_L using the routines BS2IN and BS2VL from the IMSL library. We discretize the stochastic process for g^* using Tauchen (1986). We use grids of evenly distributed grid points. We use 20 grid points for y, 25 grid points for b_S, 25 grid points for b_L, and 5 grid points for g^*.

The expectations $E_{y,g^*}\left[f(b'_S, b'_L, y', g^{*\prime} \mid y, g^*)\right]$ for $f = \hat{V}(b'_S, b'_L, y', g^{*\prime})$,$\hat{V}^D(b'_S, b'_L, y', g^{*\prime})$, $q_L(\hat{b}_S(b'_S, b'_L$ $q^D_L(b'_S, b'_L, y', g^{*\prime}))$, and $q^D_S(b'_S, b'_L, y', g^{*\prime}))$ are calculated using 50 Gauss-Legendre quadrature points over y' and the 5 grid points over $g^{*\prime}$.[36]

The algorithm used to solve for the equilibrium with interpolation works as follows. First, we specify initial guesses for V^R, V^D, q_L, and q_S. We use as initial guesses the continuation values at the last period of the finite-horizon version of the model, i.e., for values of (b_S, b_L, y, g^*) on the grid for asset levels and endowment shocks, $q_L = q_S = 0$,

$$V^R(b_S, b_L, y, g^*) = u(y - b_L - b_S), \text{ and}$$

$$V^D(b_S, b_L, y, g^*) = u(y - \phi(y)).$$

Second, we solve the optimization problem defined in (5) for each point on the grid. In order to solve for the optimum, we first find a candidate value for the optimal borrowing level using a global search procedure: We first search over 20 points for b_S. For each of these values, we search over 15 points for b_L and find the b_L that attains the maximum value of the objective function defined in (5). That value is used as initial guess in a one-dimensional optimization routine UVMIF from the IMSL library. That conditional optimization routine is defined over b_L for a fixed b_S. We then use the value of b_S that attains the maximum value (and its corresponding optimal value for b_L) as the initial guess in a two-dimensional optimization routine that uses Powell algorithm. We update $\hat{V^D}$, q_S, and q_L using functional equations (4), and (6)-(9).

If the maximum distance between the updated values for $\hat{V^R}$, $\hat{V^D}$, q_S, and q_L and their previous ones is below 10^6, a solution has been found. If it is not, we repeat the optimization exercise using the new continuation values $\hat{V^R}$ and $\hat{V^D}$, and bond prices q_S, and q_L.

Using a two dimensional tensor-product spline to interpolate over b_S and b_L works best when the function $\hat{V}^R$ is differentiable over b_S and b_L. This need not be the case in the current setup

[36] In order to speed up the code we use a version of parameterized expectations to approximate the expectations of those functions. That is, at the beginning of each iteration, we compute those expectations for a grid of 50 values for b_S, 50 values for b_L and the same grid points for y and g^* at which the problem is solved. We use a bi-dimensional tensor product spline interpolation over b_S and b_L when solving the optimization problem and for updating $\hat{V}^D$.

because of the constraint that the long-term bond price cannot be lower than $\underline{q}_L$. However, we find that constraint is binding only for state realizations in which the government would have chosen to default. For instance, Figure 7 depicts the value function for a low domestic income realization. The non-differentiability introduced by the default decision is significantly milder than in models without positive recovery.

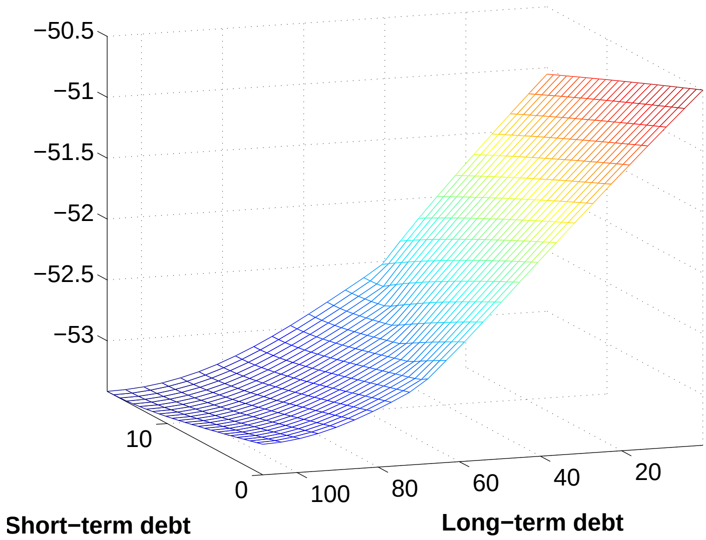

Figure 7: Value function $\hat{V}^R$ for a domestic income realization y that is 2 standard deviations below the mean and $g^* = \mu_g$.