Prof. Gustavo Indart
Department of Economics
University of Toronto

ECO 209Y
MACROECONOMIC THEORY

Solution to Problem Set 14
(Odd numbers only)

1. a) The IS curve represents all the combinations of interest rate and income (output) for which the goods market is in equilibrium. To derive the IS curve we must first find the AE curve:

$$AE = C + I + G + X - Q$$
$$= (10 + 0.8\ YD) + (80 - 10\ i + 0.1Y) + 20 + 20 - (10 + 0.2\ Y)$$
where $YD = Y - TA = Y - 0.25\ Y = 0.75\ Y$
$$= [(10 + 0.8\ (0.75\ Y)] + (80 - 10\ i + 0.1\ Y) + 20 + 20 - (10 + 0.2\ Y)$$
$$= (10 + 0.6\ Y) + (80 - 10\ i + 0.1\ Y) + 20 + 20 - (10 + 0.2\ Y)$$
$$= 120 + 0.5\ Y - 10\ i$$

and in equilibrium AE = Y,

$$Y = 120 + 0.5\ Y - 10\ i$$
$$(1 - 0.5)\ Y = 120 - 10\ i$$
$$Y = (1 / 0.5)\ (120 - 10\ i)$$
$$= 240 - 20\ i$$

so the equation for the IS curve is

$$i = 12 - 0.05\ Y$$

b) The LM curve represents all the combinations of interest rate and level of income (output) for which the money market is in equilibrium. To derive the LM curve we equate the demand for and supply of real balances:

$$L = (M / P)$$
$$120 + 0.5\ Y - 10\ i = 150/P$$
$$10\ i = 120 - 150/P + 0.5$$

so the equation for the LM curve is

$$i = 12 - 15/P + 0.05\ Y$$

c) The AD curve shows, for each given price level, the level of output (income) at which the goods market and the money market are simultaneously in equilibrium. To derive the AD curve we equate the equations for the IS and the LM curves and solve for Y or P:

$$12 - 0.05\ Y = 12 - 15/P + 0.05\ Y$$
$$Y = 150 / P \text{ or } P = 150/Y$$

d) The underlying assumption is that the firm maximizes profits. Another implicit assumption is that the production function exhibits <u>diminishing returns</u> to labour, and thus the MP_N curve is a decreasing function of the level of N. In addition, P and N must both be positive.

In order to find the labour demand curve we must equate either the value of the marginal product of labour to the nominal wage, or the marginal product of labour to the real wage. For this model, the function of the marginal product of labour curve is $10/N^{½}$.

$$W / P = 10 / N^{½}$$
$$N^{½} = (10\ P) / W$$
$$N = 100\ P^2 / W^2$$

e) With a fixed nominal wage of 3, the equilibrium level of employment is

$N = 100\ P^2 / 9$

Given this level of employment, we use the production function to compute the AS curve. (Recall that the AS curve relates the price level and the amount of output that firms are willing to produce.)

$Y = 20\ N^{½}$

$= 20\ (10\ P / 3)$

$= 200\ P / 3$ or $P = 3\ Y / 200$

f) To begin, we must find the equilibrium price level (P_0) in the economy by finding the intersection of the AD and AS curves.

$150 / P = 200\ P / 3 \rightarrow P^2 = 150\ (3) / 200 = 450 / 200 = 2.25 \rightarrow P_0 = 1.5$

Then we find the equilibrium level of income or output (Y_0),

$Y_0 = 200 P_0 / 3 = 200\ (1.5) / 3 = 100$

With $W = 3$ and $P_0 = 1.5$, the real wage is $3 / 1.5 = 2$, the equilibrium level of employment is:

$N_0 = 100\ P^2 / W^2 = 100 / 4 = 25$.

Given $Y_0 = 100$, we obtain the equilibrium interest rate from the IS curve:

$i_0 = 12 - 0.05\ (100) = 7$.

g) If the size of the labour force is LF = 30 and the equilibrium level of employment is

$N = 25$, then the unemployment rate is

$u = [(LF - N) / LF]\ 100$

$= [(30 - 25) / 30]\ 100$

$= 16.7$ percent

If the level of full employment is $N^* = 28$, then the natural rate of unemployment is

$u^* = [(LF - N^*) / LF]\ 100$

$= [(30 - 28) / 30]\ 100$

$= 6.7$ percent

The level of cyclical unemployment is $u - u^* = 10$ percent.

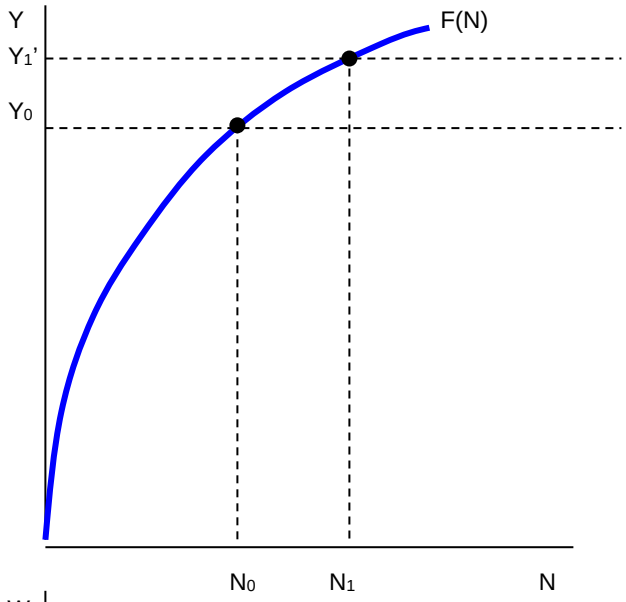
h) The graphs for this question are on the last page. It is wise in these questions to begin by identifying the initial equilibrium. In this case as Y_0, i_0, P_0, N_0, and W_0. As we know, expansionary fiscal policy shifts the IS curve to the right (to IS'). Now, before prices have changed, equilibrium in the IS-LM framework occurs at (i_1, Y_1). This shifts the AD curve to the right (to AD') passing through the point (P_0, Y_1). Now there is an excess demand in the economy which causes the price level to rise. This has two effects on the economy – on the demand and the supply sides:

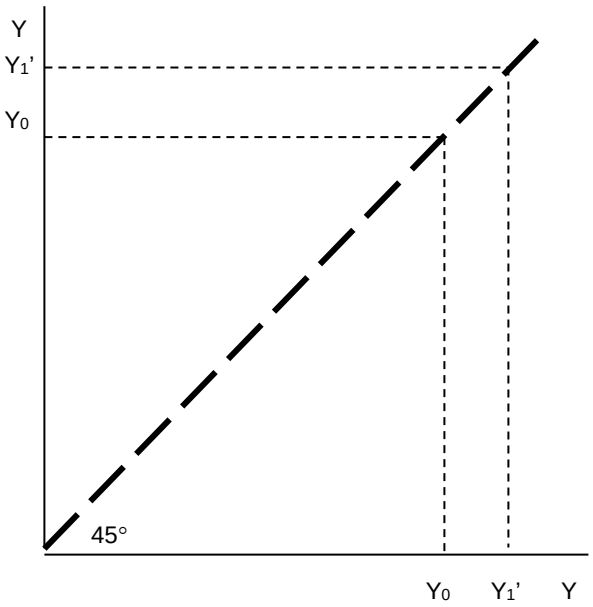
i) <u>on the supply side</u>, the increased price level shifts the labour demand schedule to the right (to $N^{D'}$ given the price level P_1). Given the fixed nominal wage, this increases the equilibrium level of employment (to N_1). Going through the production function, this results in a movement up the AS curve.

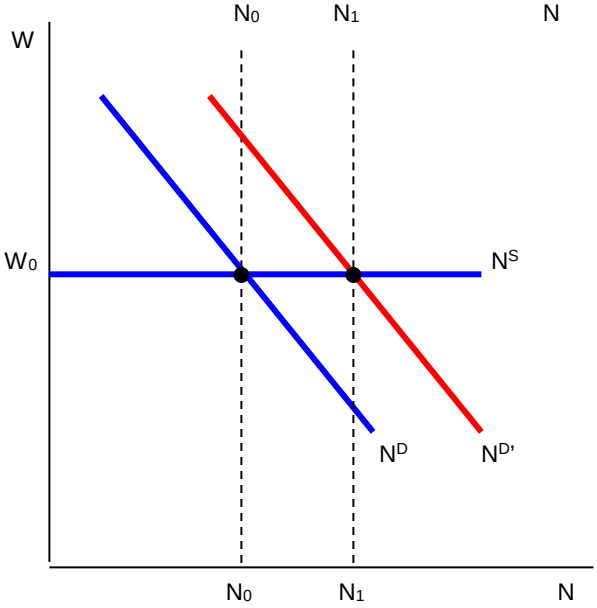
ii) <u>on the demand side</u>, the increased price level reduces the real money supply, which shifts the LM curve to the left (to LM given the price level P_1). Now, as in the normal case, a reduced real money supply causes an excess demand for money (or an excess supply of bonds) and this results in a fall in the price of bonds or an increase in the interest rate. In turn, this cuts off some investment spending. At any rate, in the IS-LM framework, the income level falls and the interest rate rises. In terms of the aggregate demand curve, this is represented by a movement up the AD' curve.

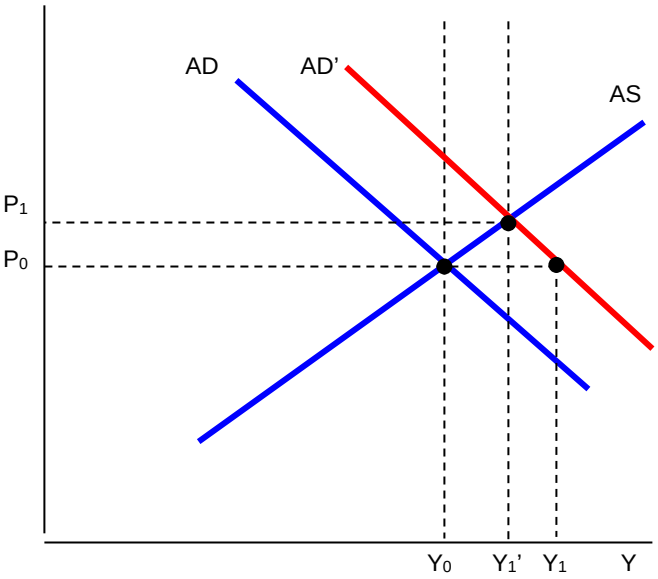
Finally, we compare this with the case of a classical aggregate supply curve. In the classical case the AS curve is vertical and as the price level rises both the labour demand and labour supply curves shift up by the same vertical distance and this leaves the equilibrium level of employment unchanged. This, in turn, means that the equilibrium level of output is unchanged. The price level must, therefore, rise enough to increase the interest rate enough to cut off enough investment to exactly match the increase in government spending. This shows the dichotomy between the real and nominal sides of the economy

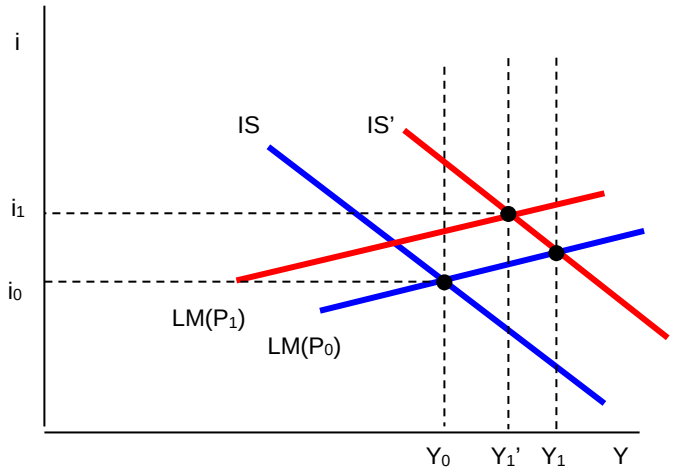
of the classical case. The real side (including employment, output and the real wage) is completely determined by the supply side. In this case, the demand side only determined the price level and the nominal interest rate.











3. a) $u^* = (LF - N^*) / LF \rightarrow N^* = LF - u^* LF = LF (1 - u^*) = 1\,000 (1 - 0.05) = 950$

$Y^* = 5 N^* = 5 (950) = 4\,750$

b) We need to define 3 relations in order to derive the short-run AS curve: 1) the wage-employment relation; 2) the employment-output relation' and 3) the price-cost relation.

We already have the last two:

- employment-output relation:
  $Y = 5 N \rightarrow N = Y / 5$
- price-cost relation:
  $P = (1 + 0.2) W / 5 = 0.24 W$

It only rests to define the wage-employment relation.

Given $g_w = - 5 (u - u^*)$ and that

(1) $g_w = (W - W_{-1}) / W_{-1} = (W - 100) / 100$

(2) $u = (LF - N) / LF = (1\,000 - N) / 1\,000 = 1 - 0.001 N$

(3) $u^* = 0.05$

Then,

$(W - 100) / 100 = - 5 (1 - 0.001 N - 0.05)$

$W - 100 = - 500 (0.95 - 0.001 N)$

$W = 100 - 475 + 0.5 N$

$W = - 375 + 0.5 N$

And since $N = Y / 5$,

$W = - 375 + 0.5 Y / 5 = - 375 + 0.1 Y$

And given that

$P = 0.24 W$

$P = 0.24 (- 375 + 0.1 Y)$

$P = - 90 + 0.024 Y$

c) AD = AS

$100 - 0.016 Y = - 90 + 0.024 Y$

$0.04 Y = 190$

$Y = 190 / 0.04 =$ **4 750**

$P = 100 - 0.016 Y = 100 - 0.016 (4\,750) = 100 - 76 =$ **24**

d) Since $Y = 4\,750$ and $Y^* = 4\,750$, then the economy is in long-run equilibrium and **$u = u^* = 0.05$**.

Alternatively, u = (LF – N) / LF and N = Y /5 = 4 750 / 5 = 950, and therefore

$u = (1\,000 - 950) / 1\,000 = 50 / 1\,000 = 0.05$