

ECO 209Y
MACROECONOMIC THEORY
Solution to Problem Set 7
(Odd numbers only)

1. a) The money multiplier is:

$$mm = \frac{1 + cu}{cu + re} = \frac{1 + 0.15}{0.15 + 0.10} = \frac{1.15}{0.25} = 4.6$$

b) If the public already holds all the cash it wants, any change in money holdings represents an increase in demand deposits only. Therefore, the currency-deposit ratio for any increase in money holdings is zero. Thus the money multiplier for any increase in the stock of high-powered money becomes now

$$mm = \frac{1 + cu}{cu + re} = \frac{1}{0.10} = 10$$

c) When the Bank of Canada purchases \$1 billion in government bonds from the public the stock of high-powered money is increased by \$1 billion. Therefore, the stock of money is increased by this amount times the money multiplier, that is,

$$\begin{aligned} \Delta M &= mm \Delta H \\ &= 10 (\$1 \text{ Billion}) \\ &= \$10 \text{ Billion} \end{aligned}$$

The final changes in the balance sheets of the public, the commercial bank, and the Bank of Canada are as follows:

Public			Commercial Bank			Bank of Canada				
(1) GB	-1		(1) D _{CB}	+1	(1) D	+1	(1) GB	+1	(1) D _{CB}	+1
(1) D	+1									
(2) D	+9	(2) L	+9	(2) L	+9	(2) D	+9			

where GB = Government of Canada bonds; L = commercial bank's loans to the public; D = demand deposits of the public; D_{CB} = commercial bank's deposits at the Bank of Canada; (1) indicates the initial change; and (2) indicates the aggregate of all subsequent changes.

When the public deposits the cheques for \$1 billion written by the Bank of Canada on itself in their chequing accounts, the commercial bank holds a claim on the Bank of Canada for \$1 billion. Therefore, the deposit of the commercial bank at the Bank of Canada increases by \$1 billion. Since now the commercial bank holds excess reserves, it can increase loans to the public by the amount of these excess reserves (90% of the initial change of \$1 billion in demand deposits). Since there is no cash drain, the public will deposit the amount of this loan in their chequing accounts and the process will continue until all excess reserves are eliminated. Therefore, since

the commercial bank wants to hold 10% of demand deposits as cash reserves, an increase in reserves of \$1 billion can support an increase in demand deposits of \$10 billion. Accordingly, loans of the commercial bank to the public increase by \$9 billion (10% of the change in demand deposits is kept as cash reserves and the rest is loaned out).

- d) The changes in the balance sheets of the public, the commercial bank, and the Bank of Canada are as follows:

Public		Commercial Bank		Bank of Canada	
(1) GB	+1	(1) D _{CB}	-1	(1) D	-1
(1) D	-1	(2) L	-9	(1) GB	-1
(2) D	-9	(2) L	-9	(1) D _{CB}	-1
	(2) L				
	-9				

When the Bank of Canada sells \$1 billion Government of Canada bonds to the public demand deposits initially decrease by \$1 billion. The Bank of Canada now holds a claim on the commercial bank equal to the sum of the cheques written on it by the public (\$1 billion). Therefore, the deposits of the commercial bank in the Bank of Canada decrease by \$1 billion. A decrease in cash reserves of \$1 billion calls for a decrease in demand deposits of \$10 billion. However, demand deposits have decreased only by \$1 billion and thus the commercial bank is holding insufficient cash reserves. Therefore, demand deposits must be further reduced by \$9 billion. In order to do so, the commercial bank must recall loans by the amount of \$9 billion. Since there is no change in the quantity of currency in circulation (outside the banking system), the change in money supply is equal to the change in demand deposits alone, i.e., the change in money supply is equal to \$10 billion (the initial \$1 plus the further \$9 billion).

- e) When the government borrows from the public to finance a deficit it is involved in debt financing. The government borrows from the public by selling government bonds to them. In the first stage, then, the balance sheets of the public, the government, the commercial bank, and the Bank of Canada are changed as follows:

Public		Government	
(1) GB	+1	(1) D _G	+1
(1) D	-1	(1) GB	+1
Commercial Bank		Bank of Canada	
(1) D _{CB}	-1	(1) D	-1
		(1) D _G	+1
		(1) D _{CB}	-1

where D_G are government deposits at the Bank of Canada.

In order to pay for the bonds the public reduces their demand deposits by \$1 billion. The Government of Canada deposits this \$1 billion in its account with the Bank of Canada. The Bank of Canada holds now a claim on the commercial bank for \$1 billion and thus reduces the deposits of the commercial bank by this amount. In this first stage, therefore, both the stock of high-powered money and the money supply decrease by \$1 billion.

Now the Government of Canada has \$1 billion in its account with the Bank of Canada to spend. Once this amount is spent, the balance sheets change in the following way:

Public				Government			
(1) GB	+1			(1) D _G	+1	(1) GB	+1
(1) D	-1			(2) D _G	-1		
(2) D	+1						
Commercial Bank				Bank of Canada			
(1) D _{CB}	-1	(1) D	-1			(1) D _G	+1
(2) D _{CB}	+1	(2) D	+1			(1) D _{CB}	-1
						(2) D _G	-1
						(2) D _{CB}	+1

Once the government spends the \$1 billion, its deposits at the Bank of Canada decrease and the public's demand deposits increase by this amount. The commercial bank holds now a claim on the Bank of Canada for \$1 billion and thus its deposits at the Bank of Canada are increased by this amount. At the end of the second stage, then, both the stock of high-powered money and the money supply go back to the levels they were before the bond sale.

- f) When the government borrows from the Bank of Canada to finance a deficit it is involved in money financing. The government borrows from the Bank of Canada by selling government bonds to it. In the first stage, then, the balance sheets of the public, the government, the commercial bank, and the Bank of Canada are changed as follows:

Public				Government			
				(1) D _G	+1	(1) GB	+1
Commercial Bank				Bank of Canada			
				(1) GB	+1	(1) D _G	+1

In order to pay for the bonds the Bank of Canada increases the deposit of the government by \$1 billion. Now the Government of Canada has \$1 billion in its account with the Bank of Canada to spend. Once this amount is spent, the balance sheets change in the following way:

Public				Government			
(2) D	+1			(1) D _G	+1	(1) GB	+1
				(2) D _G	-1		
Commercial Bank				Bank of Canada			
(2) D _{CB}	+1	(2) D	+1	(1) GB	+1	(1) D _G	+1
						(2) D _G	-1
						(2) D _{CB}	+1

When the government spends its deposits at the Bank of Canada it writes cheques to the public by \$1 billion and thus the public's demand deposits increase by this amount. Now the commercial bank holds the government's cheques and thus has a claim on the Bank of Canada for \$1 billion. Therefore, the commercial bank's deposits at the Bank of Canada increase by \$1 billion. At the

end of the second stage, then, both the stock of high-powered money (ΔD_{CB}) and the money supply (ΔD) have increased by \$1 billion.

At the end of the second stage, however, the commercial bank is holding excess cash reserves (an increase in cash reserves of \$1 billion can support an increase in demand deposits of \$10 billion, but the latter have increased by only \$1 billion – excess reserves equal \$900 million). The commercial bank will loan out any excess reserve and through the multiplying mechanism loans and demand deposits will increase by \$9 billion. Therefore, at the end of the third stage the changes in the balance sheets look like this:

Public				Government			
(2) D	+1			(1) D_G	+1	(1) GB	+1
(3) D	+9	(3) L	+9	(2) D_G	-1		
Commercial Bank				Bank of Canada			
(2) D_{CB}	+1	(2) D	+1	(1) GB	+1	(1) D_G	+1
(3) L	+9	(3) D	+9			(2) D_G	-1
						(2) D_{CB}	+1

So at the end of the third stage, the stock of high-powered money has increased by \$1 billion and the money supply by \$10 billion.

- g) In this case, the public does not deposit the whole \$1 billion in their chequing accounts. The public keeps a fraction of any change in their money holdings in cash. This fraction is

$$\frac{CU}{CU + D} = \frac{cu}{cu + 1} = \frac{0.15}{0.15 + 1} = \frac{0.15}{1.15} = 0.13$$

Therefore, when the public receives \$1 billion for their bonds, it keeps \$130 million in cash and deposits \$870 million in its chequing accounts.

Public		Commercial Bank		Bank of Canada	
(1) GB	-1	(1) CU_B	-0.13	(1) D	+0.87
(1) CU_P	+0.13	(1) D_{CB}	+1	(1) GB	+1
(1) D	+0.87			(1) D_{CB}	+1

where CU_p = currency held by the public, CU_b = currency held by the commercial bank in its vaults (till money), and $CU = CU_p + CU_b$.

At the end of the first stage, therefore, both the stock of high-powered money ($\Delta CU + \Delta D_{CB}$) and the money supply ($\Delta CU_p + \Delta D$) have increased by \$1 billion. The commercial bank's cash reserves have increased by \$870 million ($\Delta RE = \Delta CU_b + \Delta D_{CB}$).

Now the commercial bank is holding excess cash reserves ($\$870 - \$87 = \$783$ million). Therefore, the commercial bank will loan out any excess reserves and through the multiplying mechanism the balance sheets will change as follows (note that the total (final) increase in money supply is equal to the change in the stock of high-powered money times the money multiplier ($\$1 \text{ billion} \times 4.6 = \4.6 billion)):

Public		Commercial Bank		Bank of Canada					
(1) GB	-1	(1) CU _B	-0.13	(1) D	+0.87	(1) GB	+1	(1) D _{CB}	+1
(1) CU _P	+0.13	(1) D _{CB}	+1						
(1) D	+0.87	(2) L	+3.6	(2) D	+3.13				
(2) CU _P	+0.47	(2) CU _B	-0.47						
(2) D	+3.13								

At the end of the second stage, therefore, the stock of high powered money has increased by \$1 billion (equal to the change in D_{CB}), the money supply has increased by \$4.6 billion (equal to the changes in CU_P and D), the commercial bank's cash reserves have increased by \$0.4 billion (equal to the changes in CU_B and D_{CB}). Note that the commercial bank holds no excess reserves since the change in cash reserves (\$0.4 billion) represents 10% of the change in demand deposits (\$4 billion).

3. a) $M_D = 4Y - 1000i \rightarrow 1000 = 4(1500) - 1000i \rightarrow 1000i = 5000 \rightarrow i = 5$.
The money multiplier is 1 over the target reserve ratio, i.e., $1/0.1 = 10$. Therefore, since the money supply is 1000, the level of cash reserves is 100.
- b) $M_D = 4Y - 1000i = 4(1750) - 1000(4) = 3000$.
- c) $M_D = 4Y - 1000i = 4(1500) - 1000(4) = 2000$. Since $M_S = 1000$, there is now an excess demand of 1000 in the money market. Therefore, the commercial banks are experiencing a situation of too little cash reserves. For the money supply to increase by 1000, the cash reserves should go up by 100 (given the target reserve ratio of 0.1).
- d) The commercial banks will sell government securities to the Bank of Canada in order to increase their cash reserves. Since at $Y = 1750$ the quantity demanded of money is 3000, the money supply must increase by 2000 which implies that the cash reserves of the banking system should increase by 200.