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## ECO 209Y <br> MACROECONOMIC THEORY

## Solution to Problem Set 3-4

(Odd numbers only)

1. a) First, we must obtain the aggregate expenditure function:

$$
\begin{aligned}
\mathrm{AE} & =\mathrm{C}+\mathrm{I}+\mathrm{G} \\
& =\bar{C}+\mathrm{c} \mathrm{YD}+\bar{I}+\bar{G} \\
& =\bar{C}+\mathrm{c}(\mathrm{Y}-\mathrm{t} Y+\overline{T R})+\bar{I}+\bar{G} \\
& =\bar{C}+\bar{I}+\bar{G}+\mathrm{c} \overline{T R}+\mathrm{c}(1-\mathrm{t}) \mathrm{Y} \cdot \overline{T R}
\end{aligned}
$$

b) In equilibrium $Y=A E$, therefore

$$
\begin{aligned}
& \mathrm{Y}=\bar{C}+\bar{I}+\bar{G}+\mathrm{c} \overline{T R}+\mathrm{c}(1-\mathrm{t}) \mathrm{Y} \\
& {[1-\mathrm{c}(1-\mathrm{t})] \mathrm{Y}=\bar{C}+\bar{I}+\bar{G}+\mathrm{c} \overline{T R}}
\end{aligned}
$$

$$
Y^{*}=\frac{1}{1-c(1-t)} \overline{A E}
$$

$$
\text { where } \overline{A E}=\bar{C}+\bar{I}+\bar{G}+\mathrm{c} \overline{T R} .
$$

c) $\alpha_{A E}=\frac{\Delta Y^{*}}{\Delta \overline{A E}}=\frac{1}{1-c(1-t)}$
d) i) Given $\mathrm{C}=\overline{\mathrm{C}}+\mathrm{cYD}$,

$$
M P C_{Y D}=\frac{\Delta C}{\Delta Y D}=c .
$$

ii) Since $M P C_{Y D}+M P S Y D=1$, then $M P S_{Y D}=1-M P C Y D=1-c$.
iii) Given $\mathrm{YD}=\mathrm{Y}-\mathrm{t} Y+\overline{T R}=(1-\mathrm{t}) \mathrm{Y}+\overline{T R}$,

$$
\begin{aligned}
\mathrm{C} & =\bar{C}+\mathrm{cYD} \\
& =\bar{C}+\mathrm{c}[(1-\mathrm{t}) \mathrm{Y}+\overline{T R}] \\
& =\bar{C}+\mathrm{c} \overline{T R}+\mathrm{c}(1-\mathrm{t}) \mathrm{Y},
\end{aligned}
$$

$$
M P C_{Y}=\frac{\Delta C}{\Delta Y}=c(1-t)
$$

3. a) $\mathrm{AE}=\mathrm{C}+\mathrm{I}+\mathrm{G}+\mathrm{NX}$

$$
\begin{aligned}
& =400+0.6 \mathrm{Y}+\mathrm{NX} \\
& =400+0.6 \mathrm{Y}+(150-10-0.14 \mathrm{Y}) \\
& =540+0.46 \mathrm{Y}
\end{aligned}
$$

In equilibrium $Y=A E$, therefore
$Y=540+0.46 Y \rightarrow(1-0.46) Y=540 \rightarrow 0.54 Y=540 \rightarrow Y^{*}=540 / 0.54=1000$
Given the above, the multiplier is:
$\alpha_{A E}=1 / 0.54=1.85$ or $\alpha_{A E}=1 /[1-$ slope of $A E$ curve $=1 /[1-c(1-t)+m]$
b) The marginal propensity to import involves an additional leakage that reduces the multiplier.
c) $\mathrm{NX}=\mathrm{X}-\mathrm{Q}$

$$
\begin{aligned}
& =150-(10+0.14 \mathrm{Y}) \\
& =140-0.14 \mathrm{Y}
\end{aligned}
$$

d) $\Delta Y^{*}=\alpha_{A E} \Delta X=1.85(50)=92.5$
$\Delta N X=\Delta X-0.14 \Delta Y$
$=\Delta X-0.14 \alpha_{A E} \Delta X$
$=\Delta X-0.14$ (1.85) $\Delta X$
$=(1-0.26) \Delta X$
$=0.74 \Delta \mathrm{X}$
$=0.74$ (50)

$$
=37
$$

5. a) $A E=C+I+G+N X$

$$
\begin{aligned}
= & 420+(4 / 5)[Y-(1 / 6) Y+100]+160+180-40 \\
& =800+(2 / 3) Y \\
Y= & A E \rightarrow Y=800+(2 / 3) Y \rightarrow(1 / 3) Y=800 \rightarrow Y^{*}=2,400
\end{aligned}
$$

To achieve full employment Y must increase by: $\Delta \mathrm{Y}=2,700-2,400=300$
A change in $G$ will increase $Y$ as follows: $\Delta Y=\alpha_{A E} \Delta \overline{A E}=\alpha_{A E} \Delta \bar{G}$ and the value of the multiplier is: $\alpha_{A E}=1 /(1-$ slope of $A E)=1 /(1-1 / 3)=3$. Therefore, $\Delta \bar{G}=\Delta Y / \alpha_{A E}=300 / 3=100$
b) We've seen that $\triangle A E=100$ is required to increase $Y$ to the level of full employment. A change in TR will cause $A E$ to change as follows: $\Delta \overline{A E}=c \Delta \overline{T R}$.
Therefore, the required change in TR is: $\Delta \overline{T R}=\Delta \overline{A E} / \mathrm{C}=100 /(4 / 5)=125$.
c) We've seen that $\Delta \overline{\mathrm{AE}}=100$ is required to increase $Y$ to the level of full employment. An equal change in $G$ and TA will cause $A E$ to change as follows:

$$
\Delta \overline{A E}=\Delta \bar{G}-\mathrm{c} \Delta \bar{T}=(1-\mathrm{c}) \Delta \bar{G}=(1-4 / 5) 300=60 .
$$

Therefore, this change in fiscal policy will not be sufficient since overall autonomous expenditure will not increase by the required amount.
d) If the marginal propensity to save (s) decreases, then the marginal propensity to consume (c) increases since $c+s=1$. Therefore, since $\alpha_{A E}=1 /(1-$ slope of $A E)$ and $c$ is the slope of the $A E$ curve, then as s decreases the expenditure multiplier increases. The economic explanation is as follows. As s decreases, a larger fraction of each additional dollar of YD will be spent and thus the multiplying impact on Y will be larger.
7. a) $\mathrm{AE}=\mathrm{C}+\mathrm{I}+\mathrm{G}+\mathrm{X}-\mathrm{Q}$

$$
=500+0.8 Y D+500+0.13 Y+1500+2000-0.1 Y
$$

$$
=4500+0.8 Y D+0.03 Y
$$

$$
Y D=Y-1000-0.1 Y+1000=0.9 Y
$$

$$
=4500+0.8(0.9 Y)+0.03 Y
$$

$$
=4500+0.75 Y
$$

$\alpha_{\mathrm{AE}}=1 /(1-$ slope of AE curve $)=1 /(1-0.75)=1 / 0.25=4$
b) We must equate Y and AE to find equilibrium Y :
$Y=4500+0.75 Y \rightarrow 0.25 Y=4500 \rightarrow Y^{*}=4500 / 0.25=\$ 18,000$
Since $\mathrm{Y}_{\mathrm{fe}}=\$ 20,000$, there is a recessionary gap of $\$ 2,000$.
c) $\Delta Y=\alpha_{A E} \Delta A E \rightarrow \Delta A E=\Delta Y / \alpha_{A E}$

And since income must increase by $\$ 2,000$ (i.e., $\Delta Y=\$ 2,000$ ) to achieve full employment and the expenditure multiplier is 4 (i.e., $\alpha_{A E}=4$ ), then $\Delta A E=\$ 2,000 / 4=\$ 500$.
d) $B S=T A-G-T R$

$$
\begin{aligned}
& =1000+0.1 Y^{*}-1500-1000 \\
& =0.1(18000)-1500 \\
& =1800-1500 \\
& =300 \rightarrow \text { That is, there is a government surplus of } \$ 300 .
\end{aligned}
$$

e) Since the government is running a budget surplus of $\$ 300$, $G$ must increase to eliminate this surplus. But an increase of $\$ 300$ in $G$ will not achieve a balance budget. Indeed, an increase in $G$ will cause $Y$ to rise and thus government revenues (TA) will also increase. Therefore, it is the increase in expenditures $(\Delta \mathrm{G})$ minus the increase in revenues $(\Delta T A)$ that must be equal to $\$ 300$, i.e., $\Delta \mathrm{G}-\Delta \mathrm{TA}$ $=300$.
Since $\Delta T A=0.1 \Delta Y$ and $\Delta Y=\alpha_{A E} \Delta G=4 \Delta G$, then $\Delta T A=0.1(4 \Delta G)=0.4 \Delta G$.
Therefore, $\Delta \mathrm{G}-\Delta \mathrm{TA}=\$ 300 \rightarrow \Delta \mathrm{G}-0.4 \Delta \mathrm{G}=\$ 300 \rightarrow 0.6 \Delta \mathrm{G}=\$ 300 \rightarrow \Delta \mathrm{G}=\$ 300 / 0.6=\$ 500$.
That is, an increase of $\$ 500$ in $G$ will cause equilibrium income to increase by $\$ 2,000$ and thus government revenues will increase by $\$ 200$. Therefore, the budget surplus will decrease by $\$ 300$ and a balanced budget will be achieved.
f) Since the economy was in a recession with a recessionary gap of \$2,000 and the government was, at the same time, running a budget surplus of $\$ 300$, it's obvious that the insufficient aggregate demand might have been partly the result of insufficient government expenditure on goods and service and/or of taxes being too high. Indeed, in a recession we would expect the government to be running a deficit and not a surplus, and thus it becomes evident that $G$ might have been too low or taxes too high. Therefore, it makes economic sense for the government to increase its expenditures on goods and service since this action will contribute to move the economy towards full-employment income while reducing the unnecessary budget surplus without necessarily moving the government budget into the red.
g) We have seen in part c) above that autonomous AE has to increase by $\$ 500$ in order to achieve fullemployment equilibrium income of $\$ 20,000$. Therefore, $G$ should increase by $\$ 500$.
In the short run, i.e., before any impact on income of the increase in $G$ is perceived, the government would be running a deficit of $\$ 200$. Indeed,

$$
\text { BS }==\$ 1000+0.1(\$ 18,000)-\$ 2,000-\$ 1,000=-\$ 200
$$

In the long run, i.e., when a new equilibrium income of $\$ 20,000$ is achieved as a result of the increase in $G$, the government would be running a balanced budget. Indeed,

$$
\text { BS }==\$ 1000+0.1(\$ 20,000)-\$ 2,000-\$ 1,000=\$ 0 .
$$

