ECO 209Y
Macroeconomic Theory and Policy

Lecture 5:
The IS-LM Model

INTRODUCTION OF THE INTEREST RATE

- We introduce the ***rate of interest*** (**i**) in three stages
- ***First***, we take **i** as an ***exogenous*** variable and see how it affects ***aggregate expenditure*** as it changes
  - Here, we examine the determination of **Y** (the ***endogenous*** variable) in the ***goods market*** for each level of **i**
- ***Second***, we take **Y** as an ***exogenous*** variable and see how it affects the ***demand for money*** as it changes
  - Here, we examine the determination of **i** (the ***endogenous*** variable) in the ***money market*** for each level of **Y**
- ***Finally***, we combine both frameworks to examine the ***simultaneous*** determination of **Y** and **i** in the economy

The Consumption Function

- When we assumed the rate of interest was fixed, we derived the following equation for the consumption function:

$$\boxed{C = (\bar{C} + c\overline{TR} - c\bar{T}) + c(1 - t)Y}$$

- Assuming now that the rate of interest is not fixed, we can write the consumption functions as follows:

$$\boxed{C = (\bar{C} + c\overline{TR} - c\bar{T}) + c(1 - t)Y - di}$$

where **d** describes the rate of change of planned consumption as the rate of interest changes

- For simplicity, however, we will ***assume*** that consumption expenditure does not depend on the rate of interest

THE INVESTMENT FUNCTION

- When we assumed that the rate of interest was fixed, investment was considered an ***exogenous*** variable → $\mathbf{I = \bar{I}}$

- Assuming now that the rate of interest is not fixed, we can write the consumption functions as follows:

$$\boxed{\mathbf{I = \bar{I} - bi}}$$

  where $\mathbf{\bar{I}}$ is autonomous investment (from both income and the rate of interest), **i** is the nominal rate of interest, and **b** measures the interest sensitivity of investment

- Note that investment depends on the ***real*** rate of interest (**r**), but since **P** is assumed fixed then **i = r**

THE INVESTMENT FUNCTION (CONT'D)

- We can express the equation $I = \bar{I} - bi$ in the following way:

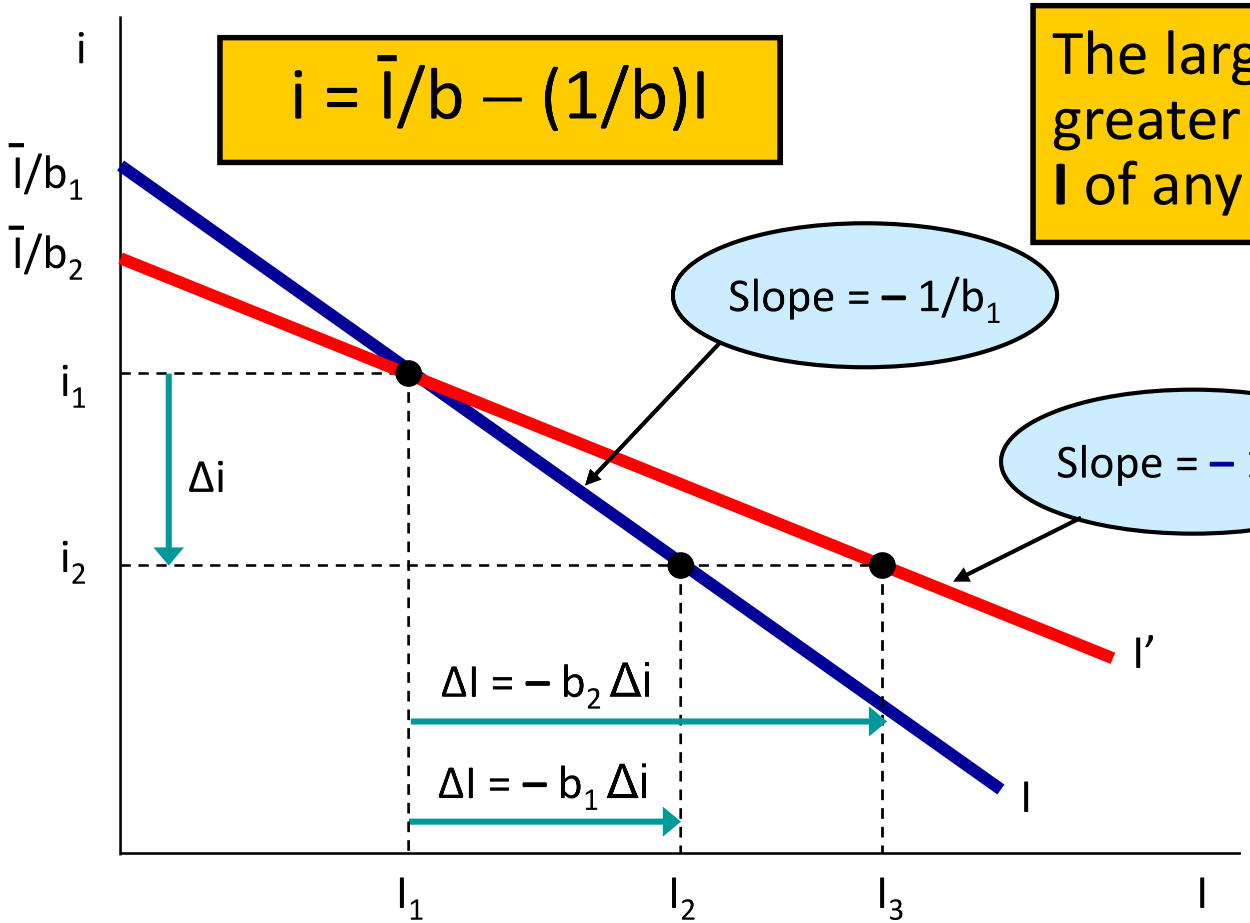
$$\boxed{i = \bar{I}/b - (1/b)I}$$

- The position of the **I** curve is determined by the level of autonomous investment spending ($\bar{I}$), and by the interest sensitivity of investment (**b**)

- The constant $\bar{I}/b$ is the vertical intercept of the curve, and the constant **1/b** is the absolute value of its slope

- Note that $\bar{I}$ captures particularly the impact of ***expected*** demand

THE INVESTMENT CURVE



The larger "**b**", the greater the impact on **I** of any change in **i**.

The Interest Rate and the Aggregate Expenditure Function

- Since the **investment** function (**I**) is now $\mathbf{I = \bar{I} - bi}$, the **aggregate expenditure** function (**AE**) becomes:

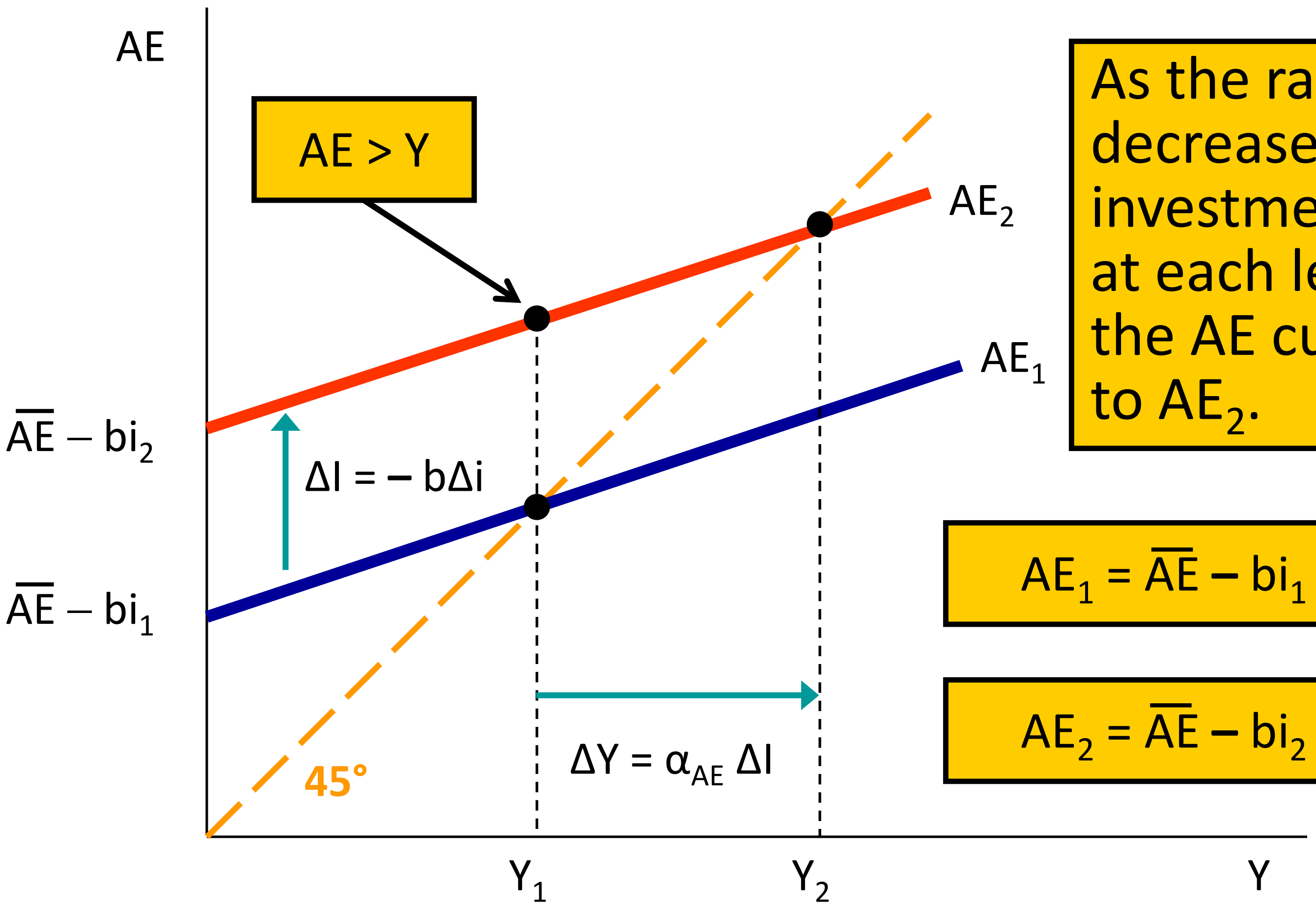
  $$AE = C + I + G$$
  $$= [\bar{C} - c\bar{T} + c\overline{TR} + c(1 - t)Y] + (\bar{I} - bi) + \bar{G}$$
  $$= \overline{AE} - bi + c(1 - t)Y$$

  where $\overline{AE} = \bar{C} - c\bar{T} + c\overline{TR} + \bar{I} + \bar{G}$

- The slope of the **AE** curve is, as before, $\mathbf{c(1 - t)}$; but the intercept has changed: before it was equal to $\overline{AE}$ and now it is equal to $\overline{AE} - bi$

- Therefore, there is one **AE** curve for each level of the interest rate

The Aggregate Expenditure Curve



As the rate of interest decreases to i_2, desired investment increases at each level of Y and the AE curve shifts up to AE_2.

$$AE_1 = \overline{AE} - bi_1 + c(1-t)Y$$

$$AE_2 = \overline{AE} - bi_2 + c(1-t)Y$$

The Algebraic Determination of Equilibrium Income

- Since there is one **AE** curve for each level of interest rate, there will be also one ***equilibrium*** income for each level of interest rate

- Since in equilibrium **Y = AE**, then

$$Y = \overline{AE} - bi + c(1 - t)Y$$

$$[1 - c(1 - t)]\, Y = \overline{AE} - bi$$

and

$$\boxed{Y = \frac{1}{1 - c(1 - t)}(\overline{AE} - bi)}$$

This relationship between **Y** and **i** is called the **IS** curve.

THE DERIVATION OF THE IS CURVE

- The relationship between the ***rate of interest*** and ***equilibrium income*** in the ***goods market*** is called the **IS** curve

- The **IS** curve shows combinations of the interest rate (**i**) and the level of income (**Y**) that ensure equilibrium in the goods market, i.e., combinations that make planned spending (**AE**) to be equal to output/income (**Y**)

- We can write the equation for the **IS** curve differently, placing the ***rate of interest*** by itself on the left-hand side of the equation

THE DERIVATION OF THE IS CURVE (CONT'D)

$$AE = \overline{AE} - bi + c(1 - t)Y$$

- We have seen that in equilibrium **Y = AE**, and then

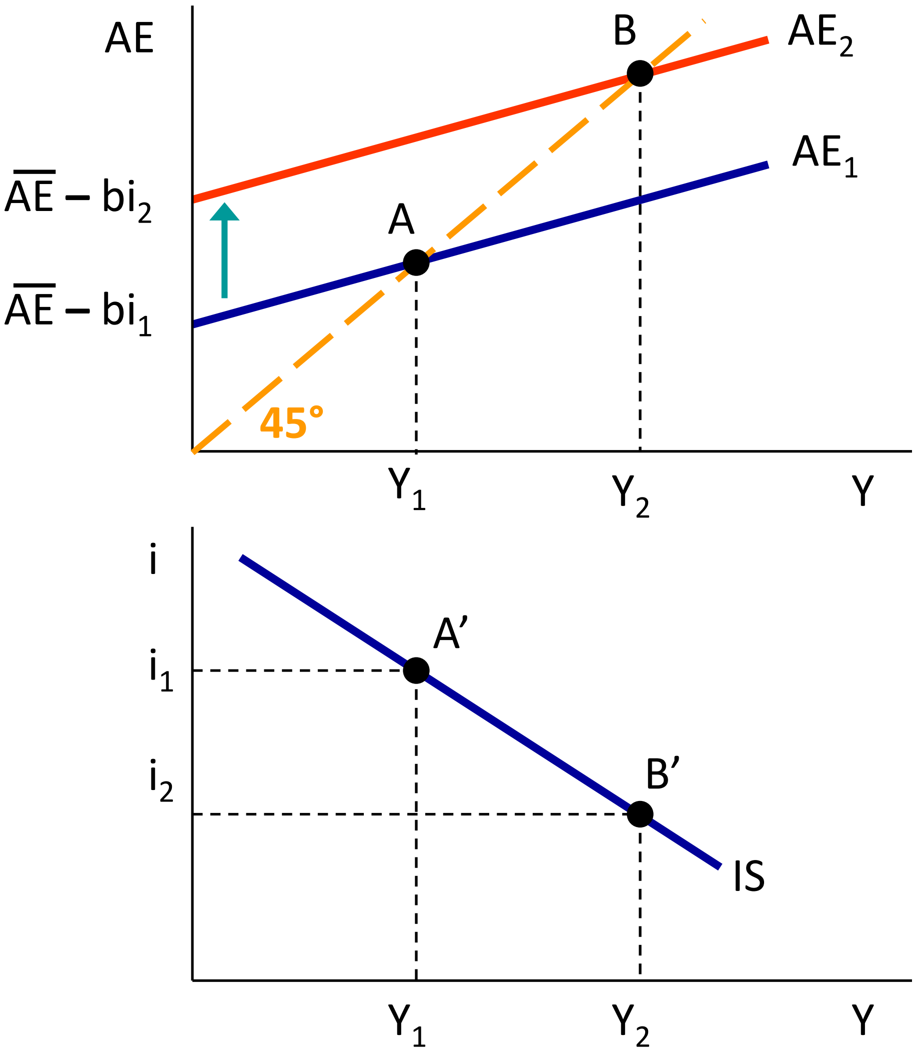
$$Y = \overline{AE} - bi + c(1 - t)Y$$

$$[1 - c(1 - t)]\, Y = \overline{AE} - bi$$

$$bi = \overline{AE} - [1 - c(1 - t)]\, Y$$

$$\boxed{i = \frac{\overline{AE}}{b} - \frac{1 - c(1 - t)}{b} Y}$$

The Derivation of the IS Curve



$$AE_1 = \overline{AE} - bi_1 + c(1-t)Y$$

The point A’ = (Y_1, i_1) is one point on the IS curve.

A decrease in the rate of interest to i_2 causes the AE curve to shift up to AE_2.

$$AE_2 = \overline{AE} - bi_2 + c(1-t)Y$$

The point B’ = (Y_2, i_2) is another point on the IS curve.

The Slope of the IS Curve

- The slope of the **IS** curve is negative and equal to:

$$-\frac{1-c(1-t)}{b} = -\frac{1}{b\,\alpha_{AE}}$$

where $\alpha_{AE} = 1/[1 - c(1 - t)]$ is the autonomous expenditure multiplier

- Therefore, the slope of the **IS** curve depends on the interest sensitivity of investment (**b**) and on the autonomous expenditure multiplier (α_{AE})

- Since $AE = \overline{AE} - bi + c(1 - t)Y$, the steeper the **AE** curve the flatter the **IS** curve (and vice versa)

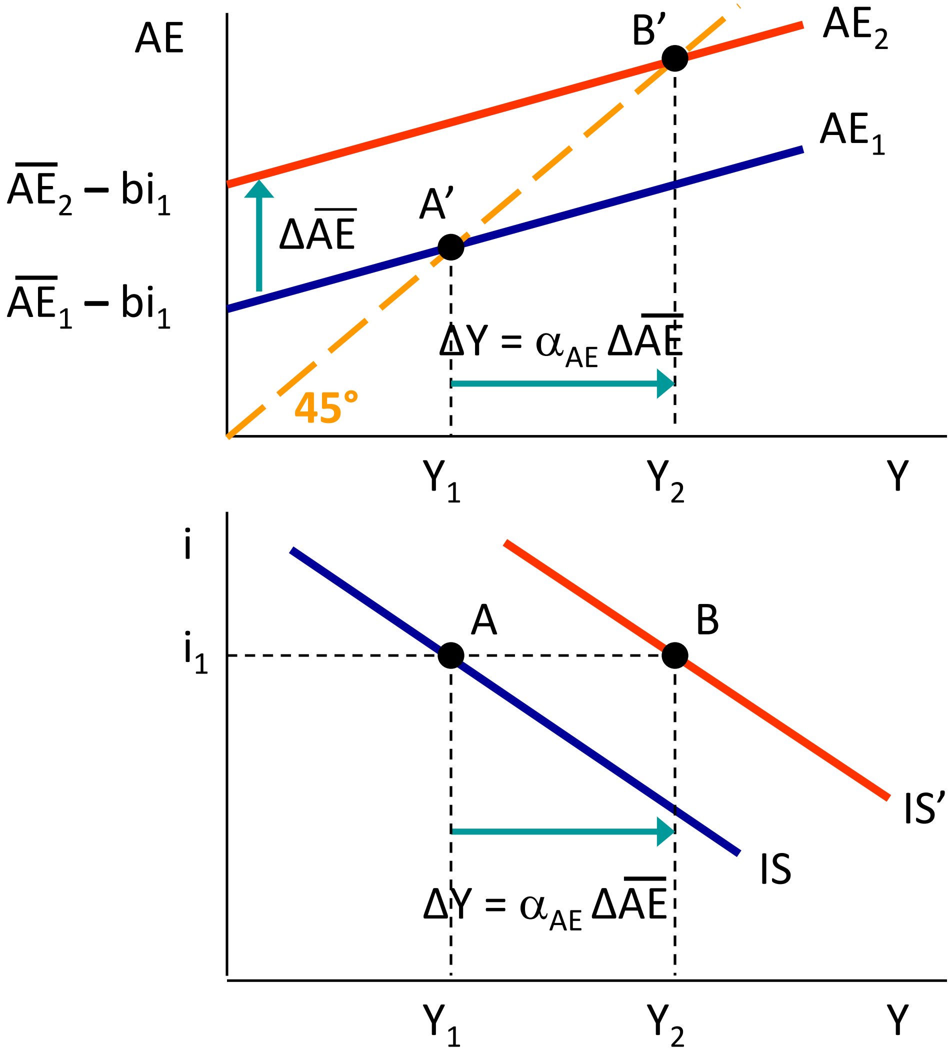
The Vertical Intercept of the IS Curve

- The intercept of the **IS** curve is $\overline{AE}/b$
  - Therefore, both changes in $\overline{AE}$ and **b** affect the intercept

- Let's consider only how changes in $\overline{AE}$ affect the position of the **IS** curve (thus, **b** will be assumed constant)

- For instance, as $\overline{AE}$ increases (without any change in the rate of interest), the **AE** curve shifts up by exactly $\Delta\overline{AE}$ and thus equilibrium income increases by $\Delta Y = \alpha_{AE}\,\Delta\overline{AE}$
  - Therefore, the **IS** curve shifts horizontally by exactly $\alpha_{AE}\,\Delta\overline{AE}$
  - Note that the vertical shift of the **IS** curve is equal to $\Delta\overline{AE}/b$

THE EFFECT OF A CHANGE IN $\overline{AE}$



Consider point A = (Y_1, i_1) on the IS curve.

The AE curve corresponding to point A on the IS curve is:

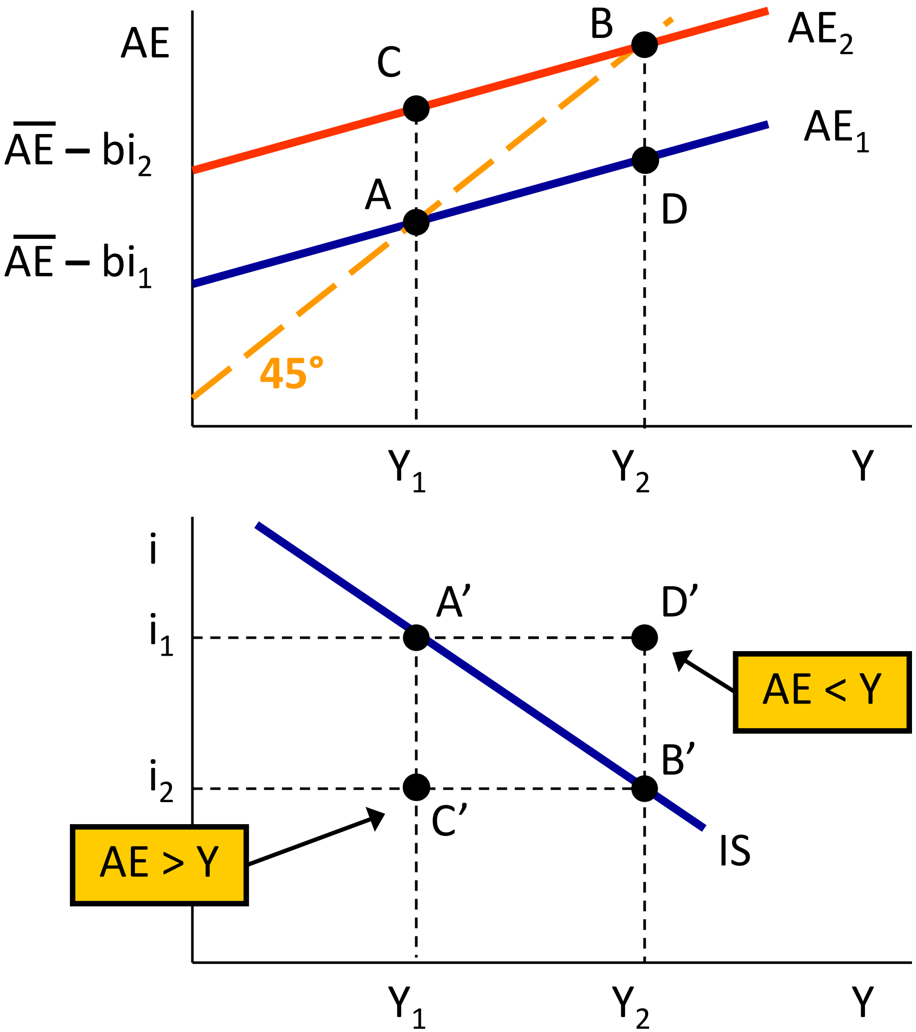
$$AE_1 = \overline{AE}_1 - bi_1 + c(1-t)Y$$

An increase in $\overline{AE}$ (with no change in i) causes the AE_1 curve to shift up to:

$$AE_2 = \overline{AE}_2 - bi_1 + c(1-t)Y$$

The AE_2 curve corresponds to point B = (Y_2, i_1) on a new IS curve.

Points Off the IS Curve



$$AE_1 = \overline{AE} - bi_1 + c(1 - t)Y$$

$$AE_2 = \overline{AE} - bi_2 + c(1 - t)Y$$

Points A' = (Y_1, i_1) and B' = (Y_2, i_2) are two points on the IS curve corresponding to points A and B in the top diagram.

Point C' = (Y_1, i_2) is off the IS curve and corresponds to point C on the AE_2 curve. At point C, AE > Y and thus any point below the IS curve represents a situation of excess demand.

Point D' = (Y_2, i_1) is off the IS curve and corresponds to point D on the AE_1 curve. At point D, AE < Y and thus any point above the IS curve represents a situation of excess supply.

THE ASSETS MARKET

- There are different types of assets in the economy:
  - ***Financial*** assets:
    - Money (i.e., currency and demand deposits)
    - Interest-bearing assets (saving accounts, bonds, etc.)
    - Stocks
  - ***Real*** assets (machinery, houses, art, etc.)

- For simplicity, we will ***assume*** that there are only two types of ***financial*** assets:
  - Money
  - Interest-bearing assets (which we are going to call ***bonds***)

NOMINAL WEALTH BUDGET CONSTRAINT

- At any time, an individual has a ***given*** financial wealth which she has to allocate between ***money*** and ***bonds***
- As already indicated, we will assume that ***money*** does not pay any return (***interest***), while ***bonds*** do
- Therefore, this is her ***nominal wealth budget constraint***:

  $$\boxed{WN = NDM + NDB}$$

  where **WN** is ***nominal*** financial wealth, **NDM** is the ***nominal*** demand for money, and **NDB** is the ***nominal*** demand for bonds
- Therefore, an individual has to ***choose*** under what type of assets she will hold her total financial wealth

Money and Bonds Markets

WN = NDM + NDB

- Since wealth ***not*** held in the form of money is held in the form of bonds, and vice-versa, the analysis of one market also gives us information for the other market
- When the demand for money increases, then the demand for bonds decreases; and when the demand for money decreases, the demand for bonds increases
- Therefore, we will focus our attention on the ***money market***

Cost-Benefit of Holding Money

- If an individual holds more of her financial wealth in the form of ***bonds***, then she will receive more interest on her financial wealth
  - This represents the ***opportunity cost*** of holding money
- If she holds more of her financial wealth in the form of ***money***, then she will be less likely not to have money available when she needs to make a payment
  - This represents the ***benefit*** of holding money
- Therefore, there is a ***trade-off***
  - An ***opportunity cost*** for holding money (the interest forgone)
  - A ***benefit*** for doing so (the less likely to be caught illiquid)

REAL AND NOMINAL DEMAND FOR MONEY

- The ***nominal*** demand for money is the demand for money expressed in a quantity of ***current*** dollars
- The ***real*** demand for money is the demand for money expressed in a quantity of dollars of the ***base period***
  - That is, the real demand for money is the nominal demand for money divided by the price level
- The ***real*** demand for money is called the demand for ***real balances***
- We will use the symbol **L** to denote the demand for real balances

Real Wealth Budget Constraint

- The ***real wealth budget constraint*** indicates that the ***demand for real balances*** (**L**) plus the ***demand for real bond holdings*** (**DB**) must add up to the ***real financial wealth*** (**W**):

$$\boxed{W = L + DB}$$

where **W = WN/P**, **L = NDM/P**, and **DB = NDB/P**

Assets Market Equilibrium

- In turn, real financial wealth (**W**) has to be equal to the total real ***supply*** of financial assets:

  $$W = M/P + SB$$

  where **M** is the nominal money stock, **M/P** is the real money stock, and **SB** is the real stock of bonds

- In equilibrium, then, **L + DB = M/P + SB**

  $$(L - M/P) + (DB - SB) = 0$$

- Therefore, if the money market is in equilibrium (**L = M/P**), then the bond market is also in equilibrium (**DB = SB**)
  - If **L > M/P**, then **DB < SB** (excess supply of bonds)
  - If **L < M/P**, then **DB > SB** (excess demand for bonds)

What is the Rate of Interest?

- Consider a ***perpetual*** bond, which is a promise to pay a fixed amount (***coupon***, C_B) to the holder of the bond every year and forever
  - For example, a newly issued bond that costs \$100 may have a coupon of \$5

- We must first make a distinction between the ***face value*** of the bond and its ***market price***
  - The ***face value*** of the bond is the amount of money that an individual must pay for the bond when it is issued (\$100 in our example)
  - The ***market price*** of the bond is the amount of money the individual will obtain when she sells her bond

Determination of the Rate of Interest (cont'd)

- The ***face value*** of the bond is ***fixed***, it does not depend on market forces (demand and supply)

- The ***market price*** of the bond, however, does depend on demand and supply

- The ***return*** or ***yield*** on the bond (**i**) is not equal to the coupon (**C_B**) divided by its face value, but to the coupon divided by its market price (**P_B**):

$$\boxed{i = C_B/P_B}$$

- In ***equilibrium***, the ***interest rate*** is equal to the ***yield*** on bonds (which represents the ***opportunity cost*** of holding money)

Determination of the Rate of Interest (cont'd)

- Suppose that there is an ***excess supply*** in the bonds market and the price of bonds falls
  - For instance, the bond with a face value of **\$100** and a coupon of **\$5** now has a lower ***market price***, say **\$80**
  - Hence, at the present time the ***yield*** on this bond is:

$$\boxed{i = \$5/\$80 = 6.25\%}$$

- Therefore, when the ***bond*** market is in disequilibrium (and thus the ***money*** market is also in disequilibrium), adjustments in the price of bonds restore equilibrium in both markets
  - For instance, if **DB < SB** (excess supply of ***bonds***) and thus **L > M/P** (excess demand for ***money***), the price of bonds falls and the interest rate rises to restore equilibrium

The Demand for Money

- The demand for money is the demand for ***real*** money balances (or real balances)

- The demand for real balances is ***assumed*** to depend on the ***nominal*** interest rate and the level of ***real*** income

- The demand for real balances depends on the ***opportunity cost*** of holding money, that is, on the interest forgone

  - In equilibrium, this forgone interest is equal to the ***nominal*** yield on bonds

- The higher the ***interest rate***, the higher the opportunity cost of holding real money balances, and therefore the lower the ***demand*** for real balances → ***negative*** relationship

The Demand for Money (cont'd)

- The demand for real balances also depends on the level of ***real*** income (**Y**)
  - ***Money*** balances are used to pay for transactions, and transactions increase with **Y** → ***positive*** relationship
- We can write the equation for the demand for real balances (**L**) as follows:

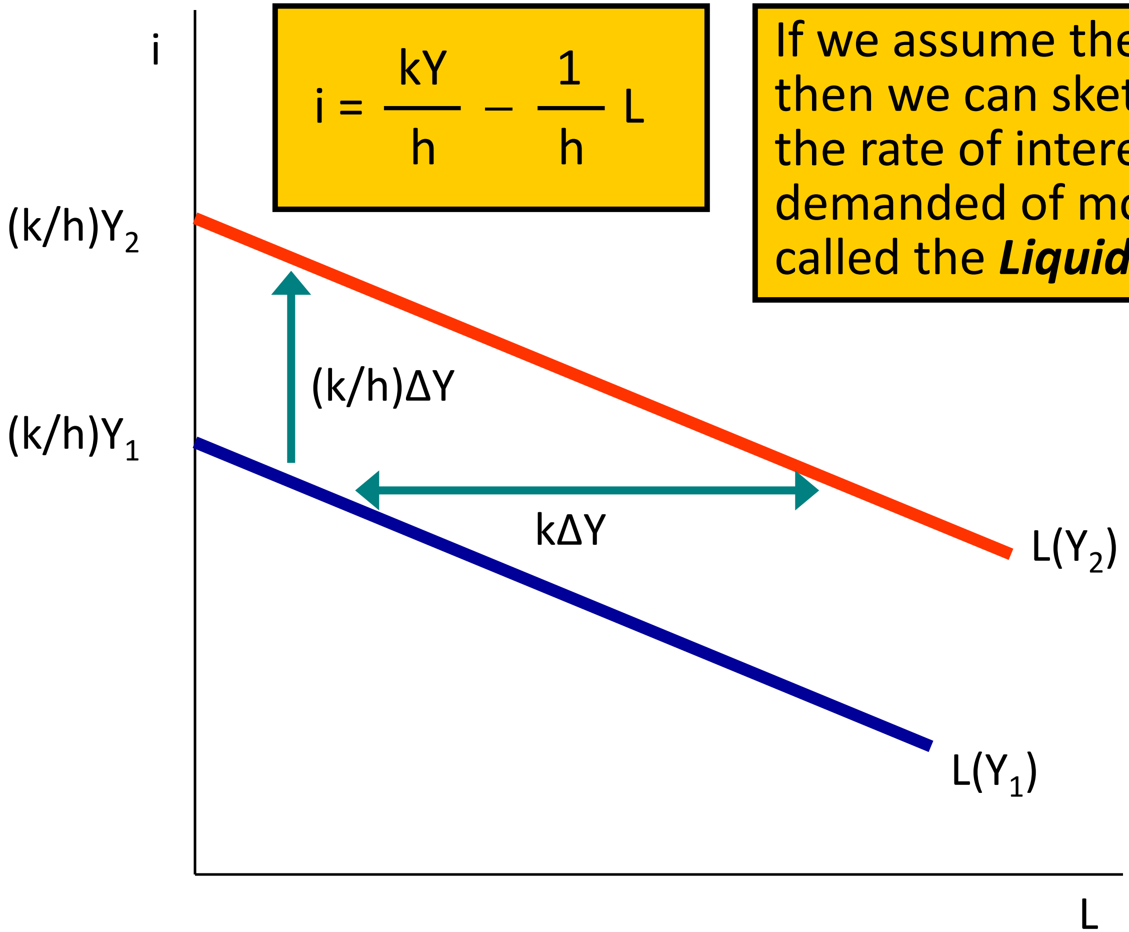
$$\boxed{L = kY - hi}$$

where **k > 0** represents the income-sensitivity and **h > 0** the interest-sensitivity of the demand for real balances

- We can rewrite this function in the following way:

$$\boxed{i = \frac{kY}{h} - \frac{1}{h} L}$$

The Liquidity Preference Curve



$$i = \frac{kY}{h} - \frac{1}{h} L$$

If we assume the level of income constant, then we can sketch the relationship between the rate of interest and the real quantity demanded of money. This relationship is called the ***Liquidity Preference***.

If $\mathbf{Y = Y_1}$, then the expression for the liquidity preference curve is:

$$\mathbf{i = (k/h)Y_1 - (1/h)\,L}$$

As **Y** increases to $\mathbf{Y_2}$, the liquidity preference curve shifts up to $\mathbf{L(Y_2)}$.

$$L = kY - hi$$

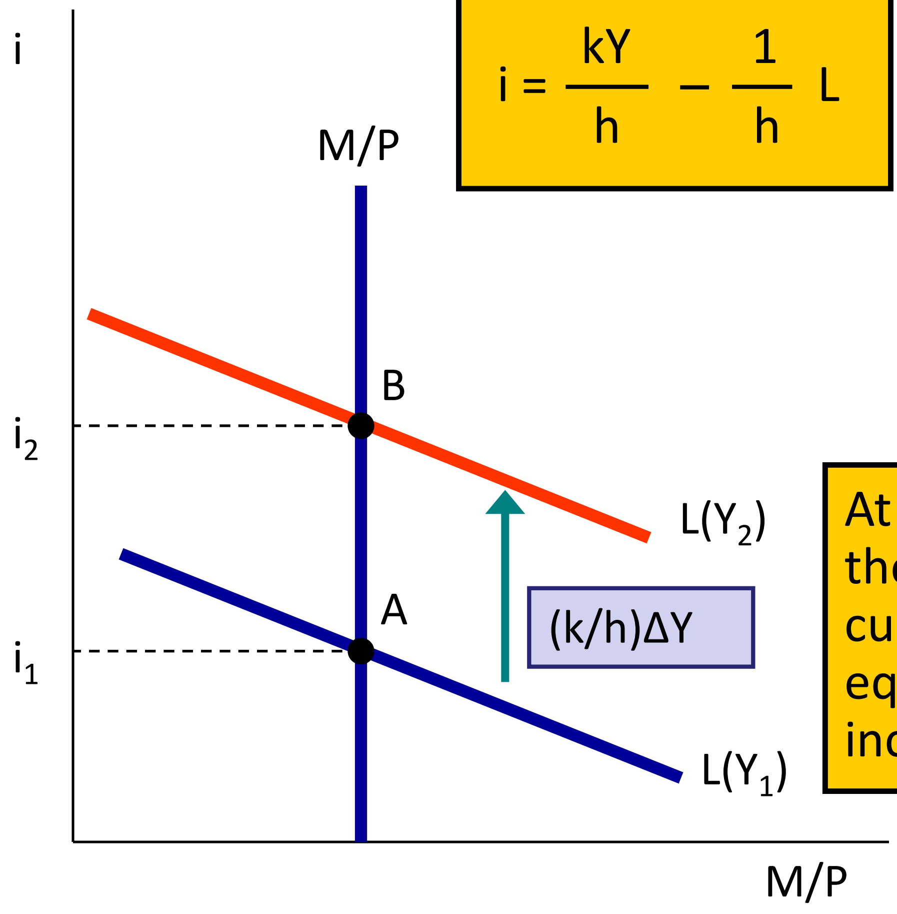
The Real Supply of Money

- The nominal money supply (**M**) is ***assumed*** to be ***exogenously*** determined by the Bank of Canada and thus we will take it as given ($\bar{M}$)

- Since the price level (**P**) is also assumed fixed, then the real money supply (**M/P**) is ***assumed*** to be fixed at $\bar{M}/\bar{P}$

- Therefore, the real money supply is ***assumed*** to be independent of both the rate of interest (**i**) and the level of real income (**Y**)
  - The real supply of money is ***assumed*** to be an ***exogenous*** variable

Equilibrium in the Money Market



$$i = \frac{kY}{h} - \frac{1}{h}L$$

At the level of income Y_1, the corresponding liquidity preference curve is $L(Y_1)$ and equilibrium interest rate is i_1.

At income increases to Y_2, the liquidity preference curve shifts to $L(Y_2)$ and equilibrium interest rate increases to i_2.

MONEY MARKET EQUILIBRIUM

- The money market is in equilibrium when the real demand for money (**L**) is equal to the real supply of money (**M/P**)
- And since $\mathbf{L = kY - hi}$, and $\mathbf{M/P = \overline{M}/\overline{P}}$, equilibrium is determined when

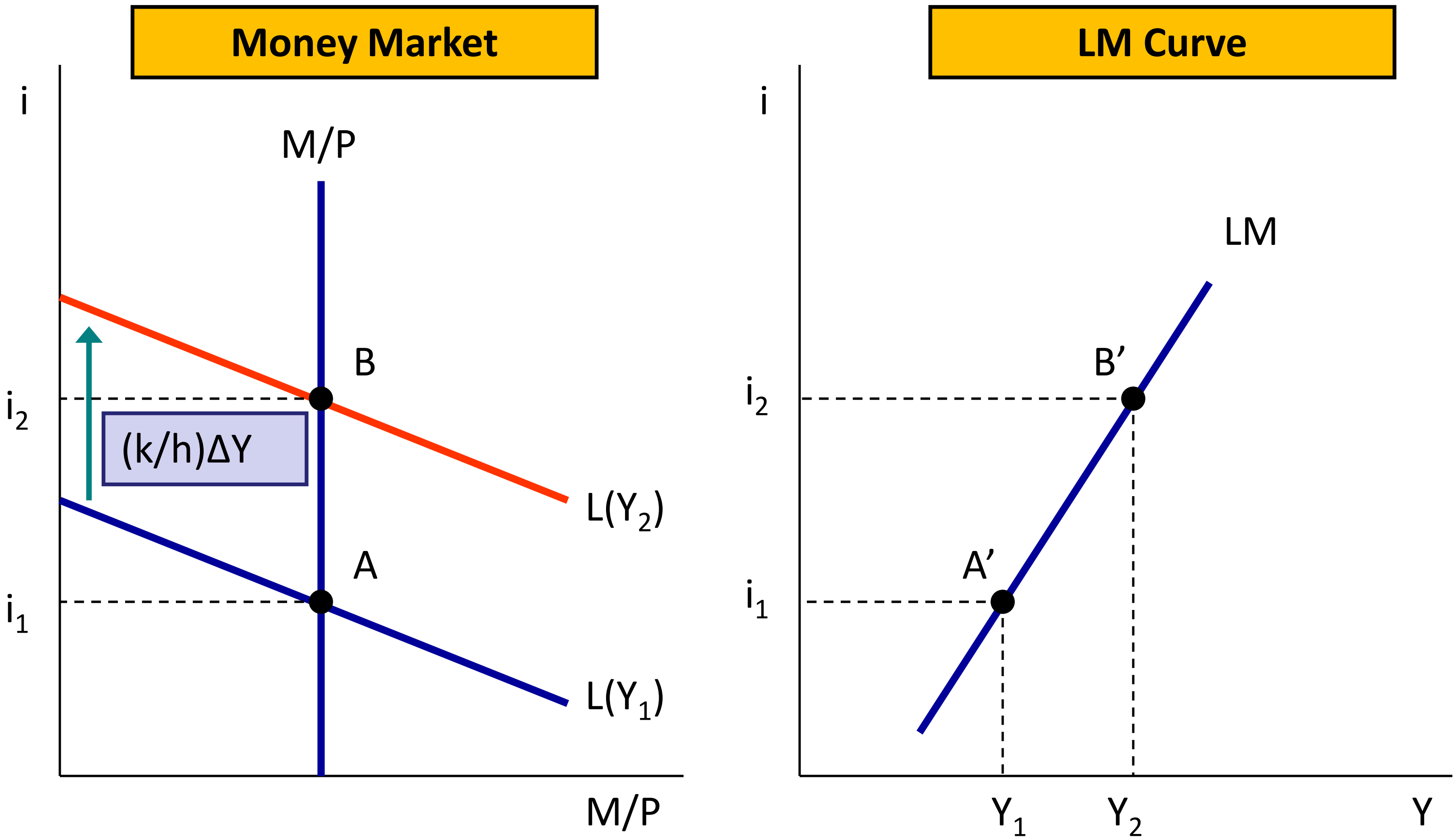
$$\boxed{\overline{M}/\overline{P} = kY - hi}$$

- Therefore, the money market is in equilibrium when:

$$\boxed{i = -\frac{\overline{M}/\overline{P}}{h} + \frac{k}{h}Y}$$

- This function indicates the relationship between the **i** and the level of **Y** when the money market is in equilibrium
  - This is the expression for the **LM** curve

Equilibrium in the Money Market and the LM Curve



The LM Curve

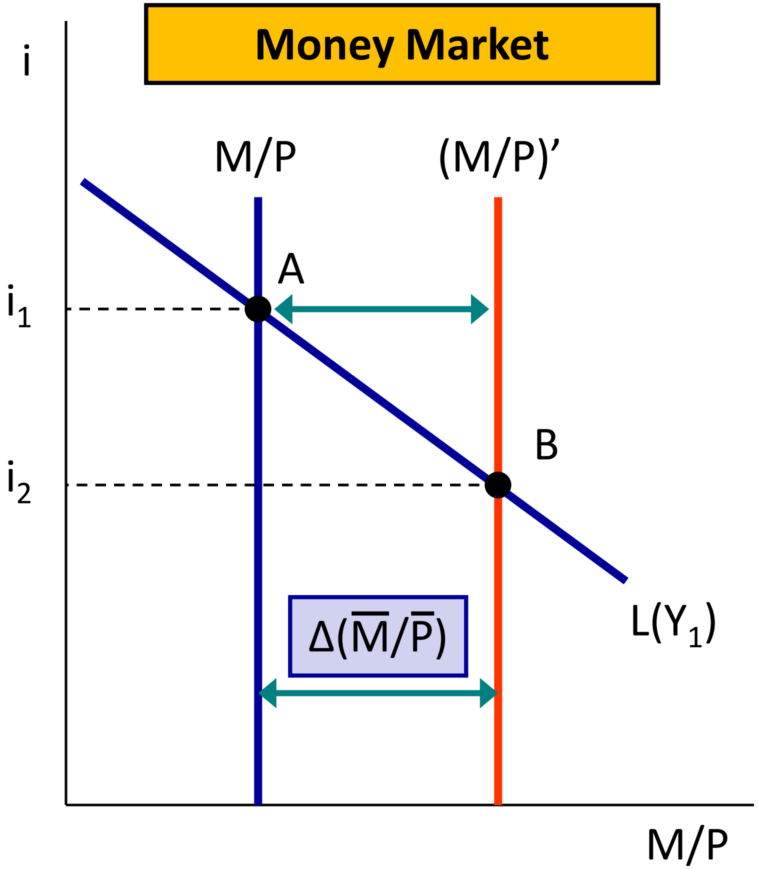
$$i = -(\overline{M}/\overline{P})/h + (k/h)Y$$

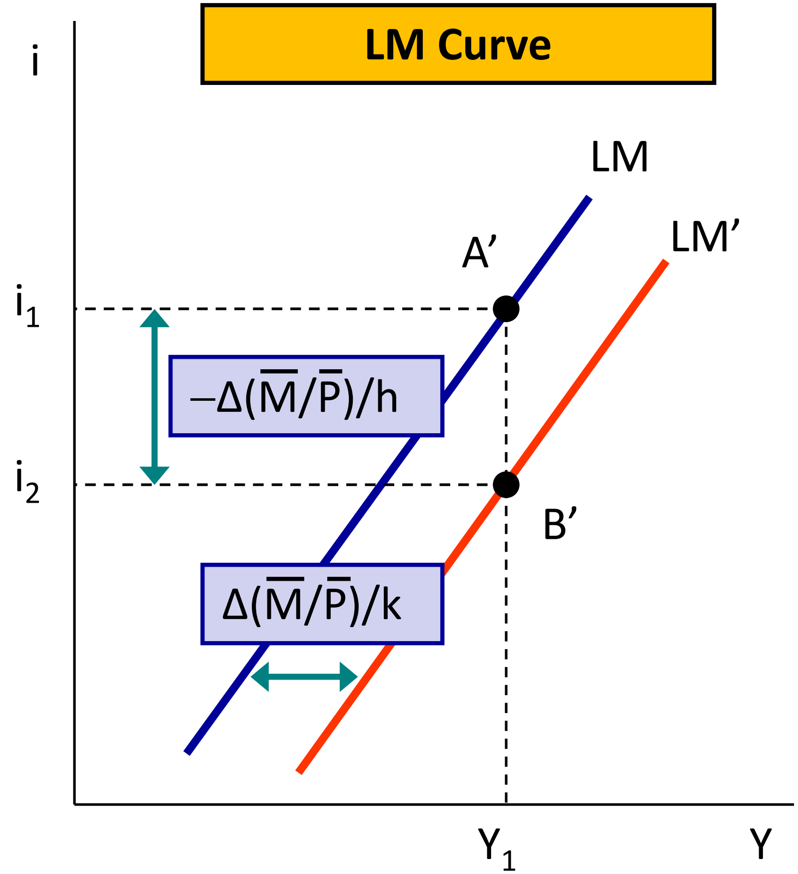
Liquidity Preference:
$i = (k/h)Y_1 - (1/h)\ L$

- The slope of the **LM** curve is ***positive*** and equal to **k/h**
  - Recall that the slope of the liquidity preference curve is **–1/h**
  - Therefore, the larger the interest sensitivity of demand for real balances, the flatter both the **L** and the **LM** curves
- The vertical intercept of the **LM** curve is $-(\overline{M}/\overline{P})/h$
  - Therefore, the position of the **LM** curve depends on the values of both **h** and $\overline{M}/\overline{P}$
  - That is, a change in $\overline{M}/\overline{P}$ will cause the **LM** curve to shift

EXOGENOUS INCREASE IN MONEY SUPPLY

$$i = -(\overline{M}/\overline{P})/h + (k/h)Y$$



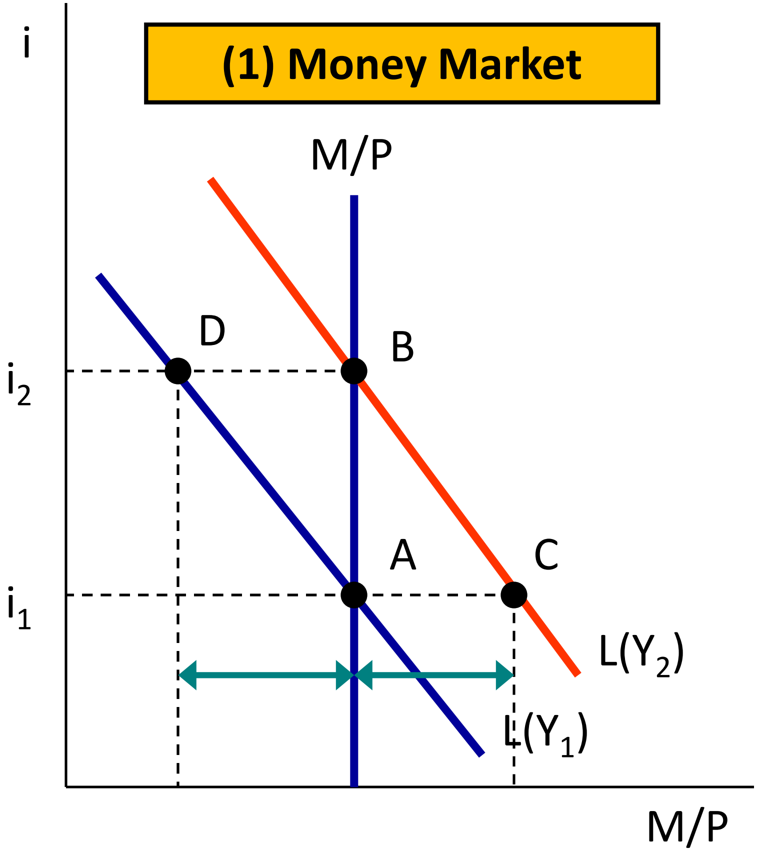


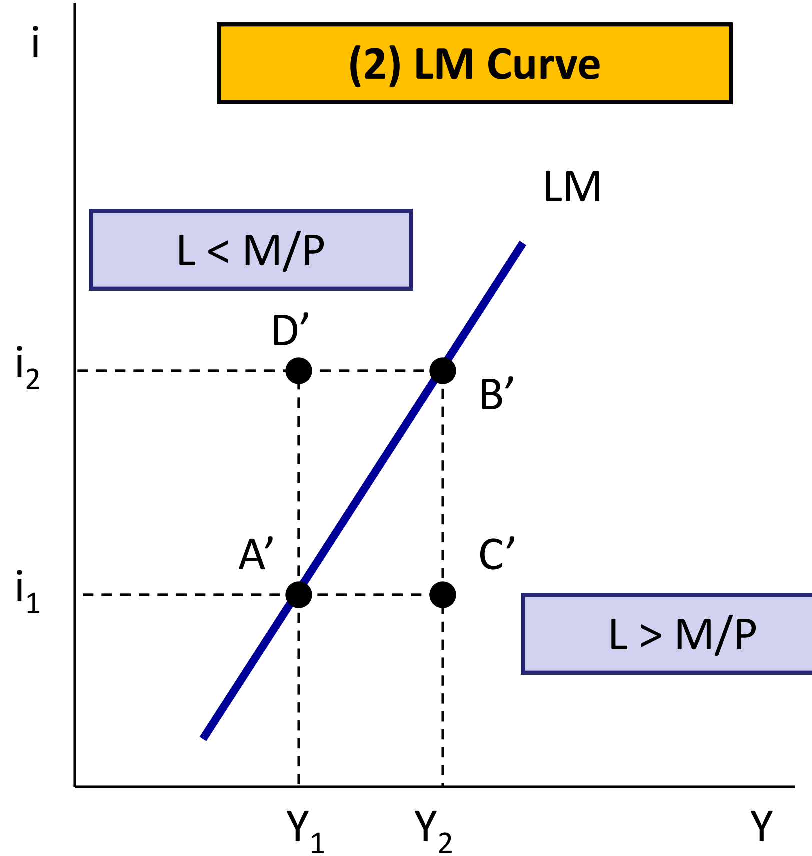
$$Y = (\overline{M}/\overline{P})/k + (h/k)\,i$$

POINTS OFF THE LM CURVE

Point C in diagram (1) corresponds to point C’ in diagram (2). At point C there is an excess demand in the money market.

Point D in diagram (1) corresponds to point D’ in diagram (2). At point D there is an excess supply in the money market.





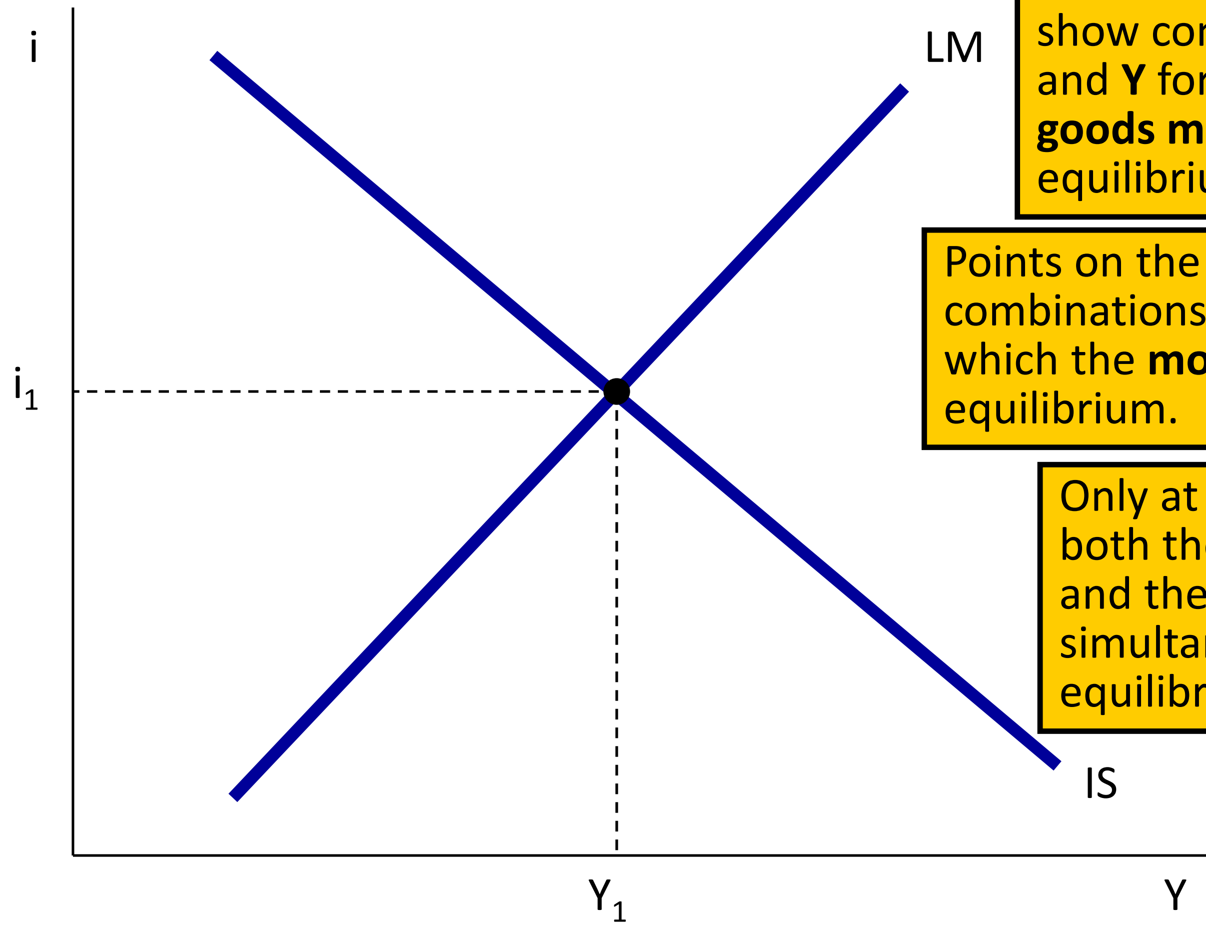
EQUILIBRIUM IN THE GOODS AND ASSETS (MONEY) MARKETS

- We have determined equilibrium in the ***goods market*** and the ***money market*** independently of each other
  - That is, we have derived a whole range of combinations of interest rate and income for which each market was in equilibrium

- Now we will derive a ***unique*** combination of the rate of interest and the level of income such that the goods market and the money market are ***both*** simultaneously in equilibrium

- In order to find this ***unique*** equilibrium, we must equate the **IS** curve and the **LM** curve
  - That is, equilibrium is achieved where the **IS** curve and the **LM** curve intersect

Goods and Money Markets Equilibrium



Points on the **IS** curve show combinations of **i** and **Y** for which the **goods market** is in equilibrium.

Points on the **LM** curve show combinations of **i** and **Y** for which the **money market** is in equilibrium.

Only at point ($\mathbf{Y_1}$, $\mathbf{i_1}$) are both the **goods market** and the **money market** simultaneously in equilibrium.

Determination of Income and Interest Rate Equilibrium

$$\text{IS: } i = \frac{\overline{AE}}{b} - \frac{1 - c(1 - t)}{b} Y$$

$$\text{LM: } i = \frac{-\overline{M}/\overline{P}}{h} + \frac{k}{h} Y$$

$$\frac{\overline{AE}}{b} - \frac{1 - c(1 - t)}{b} Y = \frac{-\overline{M}/\overline{P}}{h} + \frac{k}{h} Y$$

$$h\overline{AE} - h[1 - c(1 - t)]Y = -b(\overline{M}/\overline{P}) + bkY$$

$$h\overline{AE} + b(\overline{M}/\overline{P}) = \{h[1 - c(1 - t)] + bk\}Y$$

$$Y^* = \frac{h}{h[1 - c(1 - t)] + bk} \overline{AE} + \frac{b}{h[1 - c(1 - t)] + bk} \overline{M}/\overline{P}$$

$$= \frac{1}{1 - c(1 - t) + bk/h} \overline{AE} + \frac{1}{(h/b)[1 - c(1 - t)] + k} \overline{M}/\overline{P}$$

Determination of Income and Interest Rate Equilibrium (cont'd)

- To obtain now the equilibrium rate of interest we must plug the value for **Y*** in the expression for either the **IS** or the **LM** curve

$$\boxed{\text{IS:}\quad i^* = \frac{\overline{AE}}{b} - \frac{1-c(1-t)}{b}\,Y^*}$$

$$\boxed{\text{LM:}\quad i^* = \frac{-\overline{M}/\overline{P}}{h} + \frac{k}{h}\,Y^*}$$

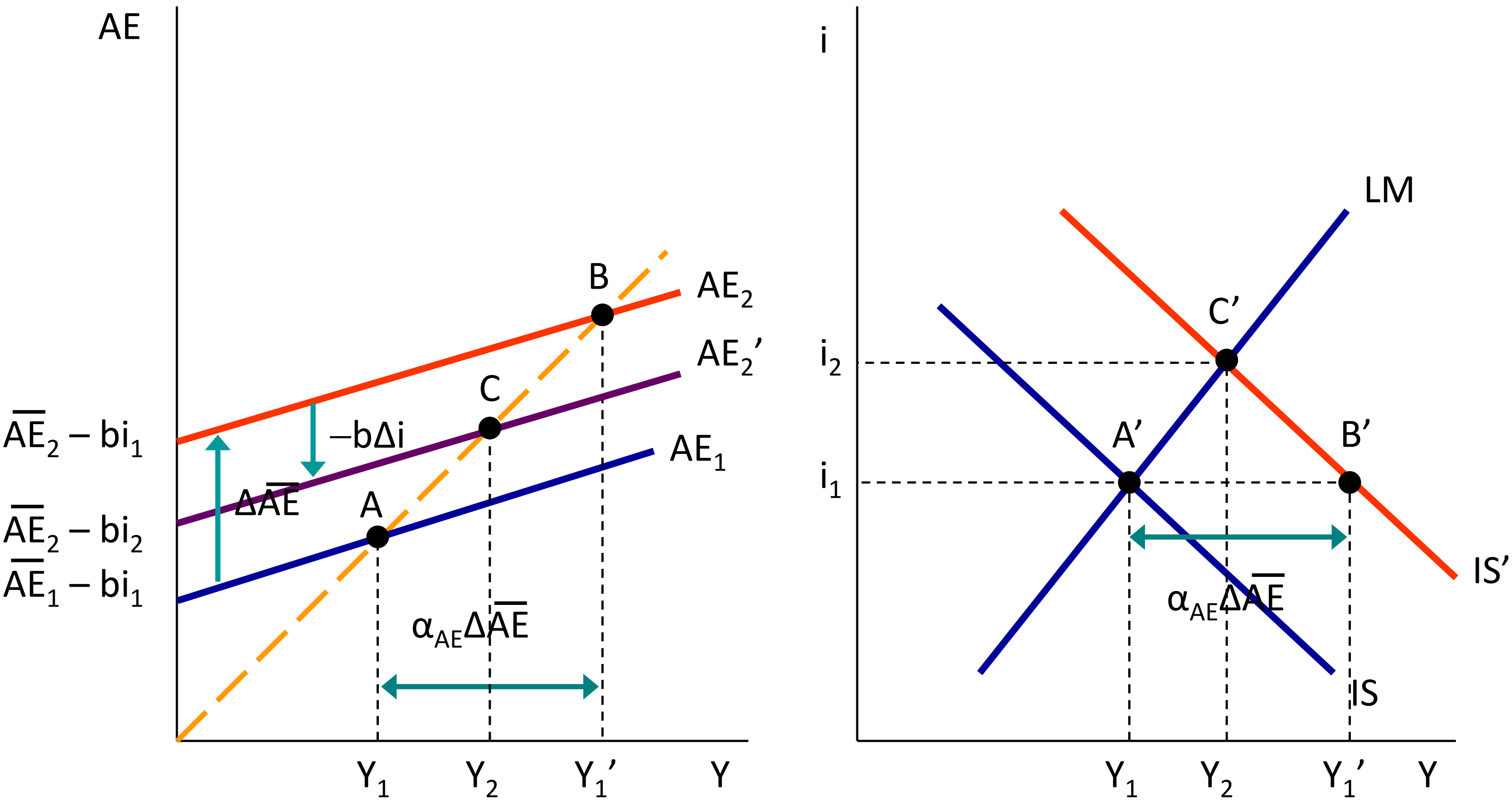
Changes in Equilibrium Income and Rate of Interest

$$\text{IS: } i = \frac{\overline{AE}}{b} - \frac{1-c(1-t)}{b}Y$$

$$\text{LM: } i = \frac{-\overline{M}/\overline{P}}{h} + \frac{k}{h}Y$$

- The equilibrium levels of income and interest rate change whenever the **IS** curve and the **LM** curve shift
- Therefore, any change in the position and/or slope of the **IS** curve (**$\Delta\overline{AE}$**, **Δb**, **$\Delta\alpha_{AE}$**) or any change in the position and/or slope of the **LM** curve (**$\Delta(\overline{M}/\overline{P})$**, **$\Delta k$**, **$\Delta h$**) will also change the income and interest rate equilibrium

The Impact of an Increase in Autonomous Expenditure



The Adjustment Mechanism in the Goods Market

- We have seen that points off the **IS** curve indicate situations of ***disequilibrium*** in the goods market
  - Points above the **IS** curve indicate situations of ***excess supply*** in the goods market (**ESG**)
  - Points below the **IS** curve indicate situation of ***excess demand*** in the goods market (**EDG**)

- Recall that we are assuming that demand (or aggregate expenditure) determines output
  - Whenever there is an ***excess supply*** in the goods market, **Y** decreases to restore equilibrium
  - Whenever there is an ***excess demand*** in the goods market, **Y** increases to restore equilibrium

The Adjustment Mechanism in the Money Market

- We have also seen that points off the **LM** curve indicate situations of ***disequilibrium*** in the money market
  - Point above the **LM** curve indicates situation of ***excess supply*** (**ESM**) in the money market
  - Points below the **LM** curve indicate situations of ***excess demand*** (**EDM**) in the money market

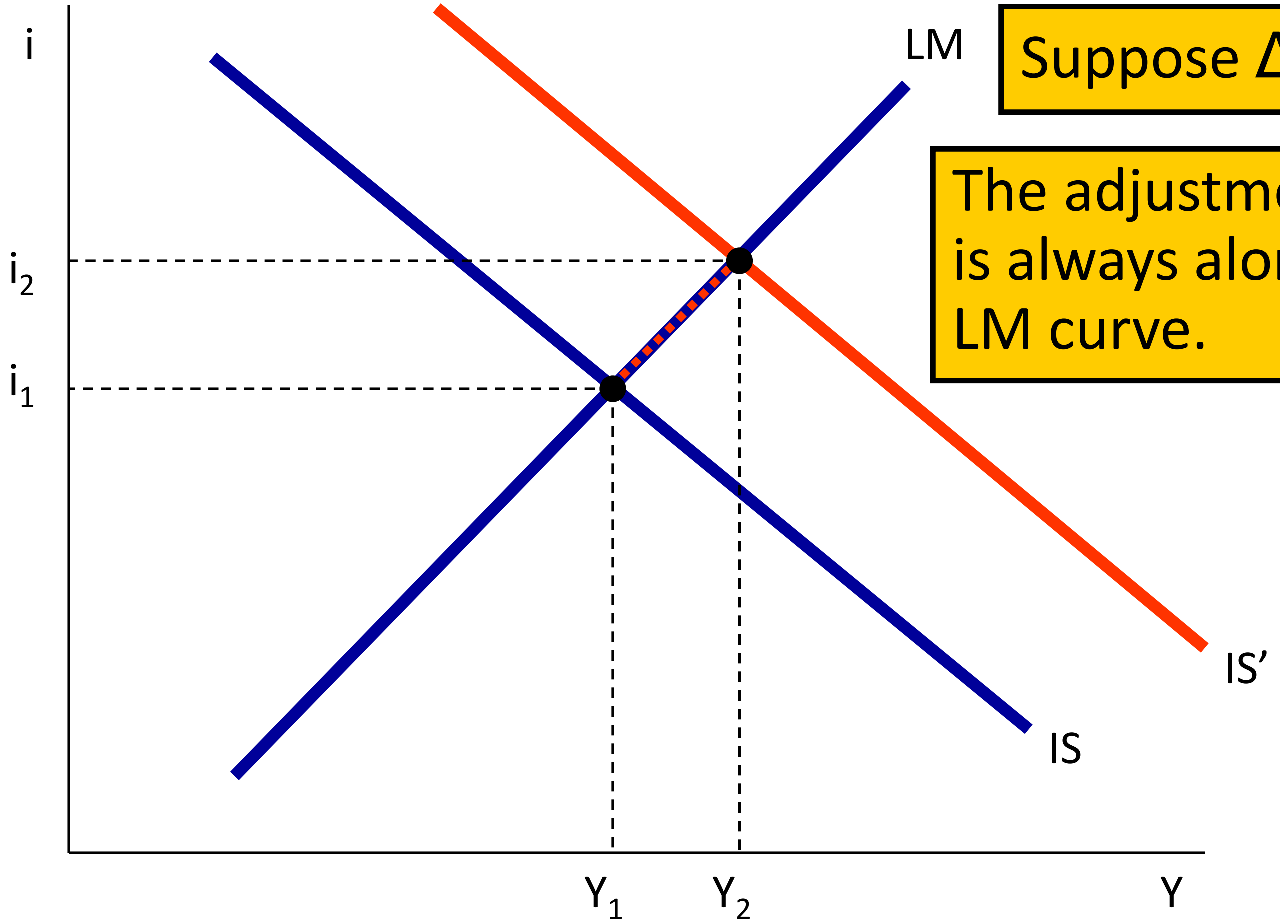
- Recall that changes in the rate of interest restore equilibrium in the money market
  - Whenever there is an ***excess supply*** in the money market, **i** decreases to restore equilibrium
  - Whenever there is an ***excess demand*** in the money market, **i** increases to restore equilibrium

The Adjustment Mechanism

- Note that the ***money market*** adjusts very quickly since the interest rate changes rapidly as bonds are bought and sold
  - Therefore, we are going to ***assume*** that the money market is ***always*** in equilibrium
  - This implies that the economy is always at a point on the **LM** curve

- On the other hand, the ***goods market*** adjusts relatively slowly because firms have to change their levels of production
  - This implies that the adjustment path is always along the **LM** curve

Changes in Equilibrium



Suppose $\Delta\overline{AE} > 0$.

The adjustment path is always along the LM curve.