

# Learning to Play Nash from the Best

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## Abstract

I experimentally investigate the effects of the ability to imitate successful others on convergence to the one-shot Nash equilibrium. I study a two-player game (Potters and Suetens Forthcoming) in which a single parameter determines the existence of strategic complementarities. I generally confirm previous findings when learning from others is not possible: games with strategic complementarities converge more robustly to the Nash equilibrium than those without. However, I find the reverse with the ability to learn from successful others as this information significantly improves convergence in games without strategic complementarities but has no effect and possibly a negative effect on games with complementarities.

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## 1 INTRODUCTION

It seems trite, but identification of a Nash equilibrium, even if unique, has predictive power insofar as agents actually play equilibrium strategies. In many environments, however, agents plausibly will not immediately identify and play equilibrium strategies, but will rather need to learn good strategies. This may be difficult, as play against other learners hinders both mapping actions into payoffs and match-play prediction. The link between learning and equilibrium convergence has been examined both theoretically (see Fudenberg and Levine (1998) for a survey) and experimentally (see Camerer (2003) for a survey).

In terms of Nash equilibrium convergence, the existence of strategic complementarities has been shown to be important both theoretically and experimentally. Games with strategic complementarities (Milgrom and Roberts 1991, Milgrom and Shannon 1994) have robust dynamic stability properties: under learning dynamics consistent with adaptive learning, they converge to the set of Nash equilibria that bound the serially-undominated set. These learning dynamics include Bayesian learning, fictitious play, adaptive learning, Cournot best reply and many others. These games include the supermodular games of Topkis (1979), Vives (1985, 1990), Cooper and John (1988), and Milgrom and Roberts (1990). Experimentally, Chen and Gazzale (2004) provide evidence that in games with a unique, interior Nash equilibrium, those with strategic complementarities generally converge more robustly to the Nash equilibrium than those without.

Additionally, the amount and type of information available likely affects the ability to find good strategies. In addition to feedback from own actions, information can be divided into two dimensions: the amount of information a player has about the underlying structure of the game and the amount of information a player has about the play and outcomes of similar-type agents.<sup>1</sup> Most experiments on learning in games (and much of the theoretical literature as well) consider a very particular information condition. First, they consider environments of full information. That is, players usually know the mapping of actions into payoffs, the stability of that mapping, the number of players, the rules for matching players into games, and the number of periods.<sup>2</sup> On the other hand, subjects generally receive limited or no information about the actions chosen or payoffs received by agents facing the same environment, and thus precludes the possibility of learning from similar players. How well this particular information condition matches a particular environment is an empirical question. The goal of this study is to assess the effects of learning from the play of similar

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<sup>1</sup>I use similar type to mean those players perceived to have the same payoff function, and opponents or matched players to mean those players whose actions directly affect a particular player's payoffs. In some symmetric games, such as a some repeated oligopolies, the two groups will overlap.

<sup>2</sup>Of course, the experimenter does not know how agents process this information, nor their beliefs about the rationality of others. Both of these factors will introduce uncertainty.

others on equilibrium convergence.

Learning from others may effectively reduce computation costs in complex and unfamiliar environments (Conlisk 1980, Pingle 1995).<sup>3</sup> Observation of others may influence choices on two broad levels. First, information about either the actions chosen or payoffs received by similar players can change the probability that a player sticks with her current action. Second, conditional on deciding to change actions, a player may either imitate or innovate.<sup>4</sup> Clearly, imitation is only feasible if a player receives information about the strategies chosen by similar players, and might be an even more attractive option if the player learns of the payoffs associated with these actions. Information about play of others may also affect innovative behavioral rules by improving an agent's ability to best respond, both by improving the map between actions and payoffs and, in competitive environments, opponent play prediction. More subtly, information about the play of others might provide information on actions to previously unconsidered actions, and a player may weight her own experience differently than the experiences of others.

Experimentally, various studies have shown that people do learn from others when appropriate information is available. In a non-game context, Offerman and Sonnemans (1998) show that individuals learn both from personal experience and by imitating successful others. In strategic environments, the robust findings are that players do use information about the actions and payoffs of others when available, and this affects outcomes. Erev and Rapoport (1998) find that giving subjects information about other players' payoffs moves the game away from the Nash equilibrium in a market entry game. However, in a common value auction experiment, Armantier (2004) finds that the revelation of others' actions and payoffs speeds Nash-equilibrium convergence. Information about the actions of others affects ultimatum game outcomes (Harrison and McCabe 1996, Duffy and Feltovich 1999, Bohnet and Zeckhauser 2004), with differences in what subjects learn about the play of others resulting in inconsistent findings across studies.

The literature on learning in Cournot oligopolies provides a clear lesson on the importance of both a player's relationship to these others and the content of information received about their play. Vega-Redondo (1997) shows that if firms tend to imitate the most successful firm, the long-run outcome of this industry will tend to the Walrasian competitive outcome.<sup>5</sup> Subsequent experiments generally supported this finding (Huck,

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<sup>3</sup>Payoff-function instability increases environment complexity and might also increase the attractiveness of imitation. Squintani and Välimäki (2002) consider games in which the dominant action changes over time, and find that as long as changes in the environment are relatively infrequent, a behavioral rule which mixes imitation and experimentation is rather successful in coordinating on the dominant action.

<sup>4</sup>I follow the distinction proposed by Schlag (1998): "A behavioral rule is imitative, as compared to *innovative*, if the agent adopts the strategy of someone he observed whenever he changes his strategy."

<sup>5</sup>A similar result is found by Selten and Ostmann (2001) using their concept of an "imitation equilibrium."

Normann and Oechssler 1999, Huck, Normann and Oechssler 2000, Offerman, Potters and Sonnemans 2002), although not universally (Bosch-Domenech and Vriend 2003).<sup>6</sup> However, a model of imitation presented by Schlag (1998, 1999) predicts the Cournot outcome. The difference, pointed out by Apesteguia, Huck and Oeschssler (2007), is not how agents imitate (almost surely in Vega-Redondo versus probabilistically in Schlag), but rather whom one imitates. Whereas Vega-Redondo envisions imitating direct competitors, Schlag models an environment where a player receives information about, and might imitate the actions of, a player in a different market. Apesteguia et al. (2007) find support for both predictions.

Laboratory studies of Cournot oligopolies also differ in what information is provided about the actions and payoffs of same-type players. For example, Huck et al. (2000) find that the level of aggregation matters. In markets where players receive only the aggregate output of rivals, play tends toward Cournot-Nash prediction, whereas detailed information pushes markets toward the competitive equilibrium. Offerman et al. (2002) note that collusive, Cournot-Nash, and competitive outcomes can be the result of different behavior rules. By varying the type of information about the play of others, and thus the behavior rules, available to players, they are able to qualitatively induce each of the outcomes. Dixon, Sbriglia and Somma (2006) find evidence that the provision of average payoffs increases the likelihood of collusive outcomes, and find support for the hypothesis that players follow aspirational rules (Palomino and Vega-Redondo 1999, Dixon 2000, Oechssler 2002). Altavilla, Luini and Sbriglia (2006) carefully examine the distinction between “imitate average” and “imitate best,” and find that the former induces an increase in cooperation, while the latter induces more competition.

This study evaluates, in a systematic way, the effect of learning from successful others on convergence to a unique Nash equilibrium across multiple related environments. In order to focus purely on learning and minimize its strategic consequences, I look at two-player games with random rematching and fixed types. I focus on the following questions. Limiting attention to high-information environments, does the ability to learn from successful others improve convergence to the Nash equilibrium? To what extent does imitation’s impact depend on the existence of strategic complementarities? If, in fact, the ability to imitate others does improve convergence to the Nash equilibrium, is the information a complement to or substitute for strategic complementarities?

To address these questions, I use a  $3 \times 2$  experiment design. In the spirit of Potters

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Selten and Apesteguia (2005) finds experimental support for this solution concept in a game based on Salop’s (1979) model of spatial competition.

<sup>6</sup>Bosch-Domenech and Vriend (2003) employ a clever design in which the difficulty of the decision problem varies (i.e., possibly the relative benefit of imitation), but the ability to imitate remains constant. While they cannot reject the hypothesis of Cournot-Nash play in any treatment, the treatment with triopolies and the hardest decision problem comes closest to the Walrasian outcome.

and Suetens (Forthcoming), I use a two-person game easily modified from one with strategic complementarities to one without. I look at three games: two with strategic complementarities varying in the number of Pareto-superior action combinations, and a game without strategic complementarities. For each game, I run one set of treatments in which players receive information about the preceding-round play of the highest-earning same-type player, and one in which a player has access to only her own actions, payoffs and history. I can thus assess the effect of information in learning to play Nash strategies across games.

In many ways, this study is closest to Duffy and Feltovich (1999) in that players receive in each round information about a same-type player, and this player is not a possible opponent. Our studies differ in a couple of important dimensions. First, the mapping from actions into payoffs is relatively straightforward in the games they study (the ultimatum game and the best-shot game studied by Prasnikar and Roth (1992)). In this experiment, I chose payoff functions with non-obvious mappings from actions into payoffs. Second, in the games they study, learning is primarily about backward induction and convergence to the subgame-perfect equilibrium. I study simultaneous move games. Finally, fairness considerations likely play a significant role in their games, particularly the ultimatum game. My use of complex payoff functions limits this motivation to not best respond.

In the base case when learning from others is not possible, I largely confirm the previous finding that games with strategic complementarities converge more robustly to a unique Nash equilibrium than games without (Chen and Gazzale 2004). I further find that in games without strategic complementarities, the ability to learn from same-type players significantly improves convergence to the Nash equilibrium. However, in games with strategic complementarities, the availability of such information decidedly does not improve convergence, with regression evidence suggesting it actually worsens convergence. The differences in the effect of public information is so profound that that it reverses the convergence ordering: with learning from others, the game without strategic complementarities converged more strongly than those with.

## 2 STRATEGIC COMPLEMENTARITIES AND THE GENERAL ECONOMIC ENVIRONMENT

In this section, I first introduce games with strategic complementarities and their theoretical properties. I then introduce a family of games, and derive conditions under which an instance of this game has strategic complementarities.

Games with strategic complementarities (Milgrom and Shannon 1994) are those in which, given an ordering of strategies, higher actions by one player provide an incentive for the others to increase their actions. These games include supermodular games. Supermodular games are games in which: a) the incremental return to any player from increasing her strat-

egy is nondecreasing in the strategy choices of other players (*increasing differences*); and b) components of a player’s multi-dimensional strategy space are complements (*supermodularity*). Particularly for smooth functions in  $\mathbb{R}^n$ , class membership is easy to check. With  $X_i$  the strategy space and  $\pi_i$  player  $i$ ’s payoff function, the following theorem characterizes increasing differences and supermodularity.

**Theorem 1** (Topkis (1978)). *Let  $\pi_i$  be twice continuously differentiable on  $X_i$ . Then  $\pi_i$  has increasing differences in  $(x_i, x_j)$  if and only if  $\partial^2 \pi_i / \partial x_{ih} \partial x_{jl} \geq 0$  for all  $i \neq j$  and all  $1 \leq h \leq k_i$  and all  $1 \leq l \leq k_j$ ; and  $\pi_i$  is supermodular in  $x_i$  if and only if  $\partial^2 \pi_i / \partial x_{ih} \partial x_{il} \geq 0$  for all  $i$  and all  $1 \leq h < l \leq k_i$ .*

The supermodularity requirement is automatically satisfied in a one-dimensional strategy space. Note that a supermodular game is a game with strategic complementarities, but the converse is not necessarily true.

Supermodular games have interesting theoretical properties. In particular, Milgrom and Roberts (1990) prove that learning algorithms consistent with adaptive learning converge to the set bounded by the largest and the smallest Nash equilibrium strategy profiles in these games. A sequence is consistent with adaptive learning if players “eventually abandon strategies that perform consistently badly in the sense that there exists some other strategy that performs strictly and uniformly better against every combination of what the competitors have played in the not too distant past” (Milgrom and Shannon 1994, pp. 176–77). This includes numerous learning dynamics, such as Bayesian learning, fictitious play, adaptive learning, and Cournot best reply. While strategic complementarity is sufficient for convergence, it is not a necessary condition, and thus, games without strategic complementarities may also converge under specific learning algorithms.

While the theory on games with strategic complementarities predicts convergence to the unique Nash equilibrium, the theory does not address the importance of strategic complementarities relative to other factors which may induce convergence. In particular, the theory assumes that new information consists only of feedback from own choices, whereas in many environments, an agent may have access to the actions and payoffs of other agents in similar environments. I choose games allowing an assessment of the impact strategic complementarities relative to the ability to learn from same-type others in terms of convergence to Nash play.

Specifically, I start with profit functions used by Potters and Suetens (Forthcoming) in their study on the effect of strategic complementarities on cooperation in repeated interactions. I consider a two-player game with the generic profit function

$$\pi_i(x_i, x_j) = a_i x_i + b_i x_j - c_i x_i^2 - d_i x_j^2 + e_i x_i x_j, \quad (1)$$

with  $x_i$  player  $i$ 's choice and  $x_j$  her match's. I choose relatively complex profit functions for a couple of reasons. First, I want to induce an environment of complete information where learning is important. Second, I would like to generate games whose main salient differences are the satisfaction of the strategic-complementarities condition and the proportion of action combinations resulting in Pareto-superior outcomes. As I show below, this profit function satisfies these requirements.

With profits given by equation (1), player  $i$ 's best-response function is

$$BR_i(x_j) = \frac{a_i + e_i x_j}{2c_i}. \quad (2)$$

Solving for mutual best responses, the Nash Equilibrium occurs at

$$\{x_1; x_2\} = \left\{ \frac{4c_1c_2}{4c_1c_2 - e_1e_2} * \frac{a_1 + \frac{e_1a_2}{2c_2}}{2c_1}; \frac{4c_1c_2}{4c_1c_2 - e_1e_2} * \frac{a_2 + \frac{e_2a_1}{2c_1}}{2c_2} \right\} \quad (3)$$

The following proposition characterizes the necessary and sufficient conditions for supermodularity.

**Proposition 1.** *The two-player game characterized by 1 is supermodular if and only if  $e_i \geq 0$  for  $i = \{1, 2\}$ .*

The proof is simple. The supermodularity condition is automatically satisfied as the strategy space is one dimensional. Second, I use Theorem 1 to check for increasing differences. As  $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} = e_i$ ,  $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} \geq 0$  if and only if  $e_i \geq 0$ .

### 3 EXPERIMENT DESIGN

I employ a  $3 \times 2$  experiment design. The first dimension in which treatments vary is the profit functions. The profit function of type-1 players does not vary across treatments, and actions are strategic complements for type-1 players as  $e_1 > 0$ . In one game, *Strategic Complements A (SCA)*, I set  $e_2 > 0$ , thus by Proposition 1 the game is supermodular. In a second game, *Mixed (MIX)*, I set  $e_2 < 0$  and thus the game is not supermodular.<sup>7</sup> In my final game, *Strategic Complements B (SCB)*, I make the game supermodular ( $e_2 > 0$ ). However, I adjust the parameters so that the number of action combinations resulting in outcomes Pareto superior to the Nash equilibrium is equal to that in the *Mixed* game. I control for Pareto-superior actions as attraction to this region (as found in Chen and Gazzale (2004) and Gazzale and Geissler (2006)) might confound with the intended treatment effect.

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<sup>7</sup>The two-player game with strategic substitutes is in fact a game with strategic complements. Consider equation (1) with  $e_1 < 0$ ,  $e_2 < 0$ . With  $\bar{x}_2$  player 2's maximal feasible strategy, redefining player 2's strategy space to  $y_2 = \bar{x}_2 - x_2$  satisfies the condition in Theorem 1.

[Table 1 about here.]

The other dimension in which treatments vary is the information provided to players. In the *No Public (None)* treatments, at the end of each round a player's monitor displays her action, her profit, and her match's action. In the other set of treatments, *Best Public (Best)*, the player of a given type receiving the highest payoff in a round is the public player of that type for that round. In addition to the information presented in the *No Public* treatments, in the *Best Public* treatments, all type  $i$  players are informed of the action and payoff of the type- $i$  public player, as well as the action chosen by the type- $j$  player matched with type- $i$  public player. Table 1 outlines the four treatments and the number of sessions of each.

### 3.1 The Economic Environment

Letting  $x_1$  be the choice of a type-1 player and  $x_2$  be the choice of her type-2 match, the payoff function for type-1 players in all treatments is

$$\pi_1 = 92x_1 - 6x_2 - 10x_1^2 + x_2^2 + 6x_1x_2. \quad (4)$$

The payoff function for type-2 players depends on the game:

$$\text{Strategic Complements A: } \pi_2 = 104x_2 - 26x_1 - 10x_2^2 + x_1^2 + 8x_1x_2 \quad (5)$$

$$\text{Strategic Complements B: } \pi_2 = 104x_2 + 2x_1 - 10x_2^2 - 3x_1^2 + 8x_1x_2. \quad (6)$$

$$\text{Mixed: } \pi_2 = 216x_2 - 40x_1 - 10x_2^2 + 3x_1^2 - 8x_1x_2; \quad (7)$$

I chose these parameters for several reasons. The first set of reasons consider the resulting Nash equilibria. These parameters induce the same Nash equilibrium in all games. When substituted back into equation 3, the Nash equilibrium is  $(x_1^*, x_2^*) = (7, 8)$  in all games, which has a couple of desirable properties. First, as subjects choose  $x_i \in \{0, 1, \dots, 19, 20\}$  this Nash equilibrium does not lie in the center of the strategy space, nor are either of the equilibrium choices a multiple of 5. Second, the Nash equilibrium is not symmetric. These features mitigate the risk of equilibrium values chosen because they are focal.

[Figure 1 about here.]

The next set of reasons for parameter selection are based on considerations about the resulting best-response functions. I find the best response functions by substituting the parameters into equation 2. In all games, the best-response function for type-1 players is

$$BR(x_2) = \frac{92 + 6x_2}{20}.$$



In both Strategic Complements games, the best-response function for type-2 players is

$$BR(x_1) = \frac{104 + 8x_1}{20},$$

whereas in the Mixed game, it is

$$BR(x_1) = \frac{216 - 8x_1}{20}.$$

In all games the absolute value of the slope of player two's best-response function is  $\frac{2}{5}$ . As a result, number of strategies deleted in each round of iterated deletion of dominated strategies is the same across games. In Figure 1, I depict the best-response functions for the Strategic Complements games (Figure 1(a)) and for the Mixed game (Figure 1(b)).

I also considered payoff implications in selecting parameters. Payoffs, at least near the equilibrium, are relatively even in order to mitigate fairness concerns.<sup>8</sup> At the Nash equilibrium, type-1 players earn 506, whereas type-2 players earn 507 in all games. Also, with the generic profit functions, each game offers the possibility of players of either type earning significant losses in a particular round. The chosen parameters moderate the size of possible negative profits.<sup>9</sup> Finally, I needed to make sure that selected parameters provide equivalent incentives to best respond. In all treatments, the payoffs from a choice 1 away from the best response when the match plays the equilibrium choice are 98% of Nash equilibrium profits, while a choice 2 away from the best response is 92% of equilibrium profits.<sup>10</sup>

[Figure 2 about here.]

Finally, parameters control the range of actions resulting in Pareto superior outcomes. In the Strategic Complements A game, 10.7% of all choice combinations result in payoffs Pareto superior to equilibrium payoffs, while only 2.7% of choice combinations are Pareto superior in the Mixed game. I selected parameters so that the proportion of Pareto Superior outcomes in Strategic Complements B is exactly equal to that in *MIX*. In Figure 2, I show the action combinations resulting in payoffs Pareto superior to the Nash equilibrium.

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<sup>8</sup>Numerous studies have found inequality aversion. See Fehr and Schmidt (1999) for an overview. Inequality aversion may play a role even in environments where players may learn from same-type others, as Duffy and Feltovich (1999) find that providing subjects with more information made it likely that they would play a more equitable strategy even if it gave them a lower monetary payoff.

<sup>9</sup>The worst payoff for a type-1 player is -2160, which is equivalent to the loss of 5.3 rounds of equilibrium payoffs. For type-2 players, worst payoffs are equivalent to the loss of 4.9 rounds of equilibrium payoffs in the MIX game and 3.8 rounds in the games with strategic complementarities.

<sup>10</sup>Due to nature of the payoff functions, decreasing the relative payoff of a near best response comes at the cost of significantly decreasing the worst payoffs.

### 3.2 Information

Treatments vary in the information a player receives. In the *No Public* treatments, a player does not receive any information about the choices or outcomes of other same-type players. At the end of each round, the computer reports to each player her choice, her payoff, and the choice of her match. In the other set of treatments, *Best Public*, the player of each type earning the most points in a round is designated the public player. At the end of the round, in addition to the information received in the *No Public* treatments, each player of a given type sees the choice of the public player of her type, the public player's profit, and the choice of the public player's match.

### 3.3 Experiment Procedures

In each session, there are 12 players randomly split into two groups — six type-1 players (Red players in the instructions) and six type-2 (Blue) players. Subjects maintain the same type throughout the experiment. At the beginning of each session, players sit at a PC terminal where they are given printed instructions. These instructions include the payoff functions of both types. The instructions are then read aloud and subjects are encouraged to ask questions. The instruction period typically lasts around 10 minutes. After the instructions are read aloud, but before the first round of the experiment, subjects complete a brief quiz to ensure instruction comprehension. I paid subjects \$0.10 for each question correctly answered on the first attempt, with subjects needing to (eventually) provide a correct response to all questions. In Appendix A, I include the instructions for the Strategic Complements A, Best Public treatment.

At the beginning of each round, each type-1 player is randomly and anonymously matched with a type-2 player. In each round, subjects choose an integer between 0 and 20. Once all subjects have chosen a value, a subject's computer monitor displays round outcome, which varies by information condition. Throughout the session, players have access to outcomes from all previous rounds including that of the public player in each round in the Best Public treatments. This process is repeated until all 60 rounds have been completed. There are no practice rounds.

In short, subjects know both their own and their match's profit functions as well as having a history of their choices and payoffs from all previous rounds. While I did not provide subjects with profit tables, I did place paper and a pen at each player's terminal. Therefore, I implement a game of complete information where subjects can solve for both best response functions and hence the Nash equilibrium. However, I do not know how subjects will use this information, nor their beliefs about the rationality of others.

I conducted 30 independent, computerized sessions at George Mason University's ICES

Laboratory. I programmed and conducted the experiments with z-Tree (Fischbacher 2007). All subjects were George Mason students and no one participated in multiple sessions meaning there were 360 subjects. Each session lasted approximately one hour. The exchange rate was \$1 for each 2,000 points. Average earnings across these 30 sessions was approximately \$25, including a \$10 participation fee.<sup>11</sup>

#### 4 HYPOTHESES

Given the experiment design, I now formally state my hypotheses. In order to do so, I first define a measure of convergence. In theory, convergence is achieved when the equilibrium strategy is chosen without subsequent deviation. I use a measure, introduced by Chen and Gazzale (2004), better suited for a laboratory setting. This measure,  $L_b(t_1, t_2)$ , is the proportion of Nash-equilibrium play over the block of rounds from  $t_1$  to  $t_2$ . This block convergence measure helps smooth out inter-round convergence variation.

Payoff functions are somewhat flat near equilibrium, and therefore small deviations from equilibrium are not very costly. The small cost of local deviations combined with a relatively large strategy space leads me to consider  $\varepsilon$ -Nash play, defined as a choice within  $\pm 1$  of the equilibrium choice. Therefore, the  $\varepsilon$ -equilibrium prediction is  $(\varepsilon-x_1^*, \varepsilon-x_2^*) = (\{6, 7, 8\}, \{7, 8, 9\})$ . I define levels of  $\varepsilon$ -convergence appropriately.

Based on theories presented in Section 2, I formally state the hypotheses about the level of convergence across treatments.

**Hypothesis 1.** *In the No Public information condition, the level of convergence will be greater in the strategic complements games (SCA and SCB) than in the Mixed game.*

Hypothesis 1 is based on the theoretical prediction that games with strategic complementarities converge to the unique Nash equilibrium, as well Chen and Gazzale’s (2004) finding that supermodular games converge more robustly to the Nash equilibrium than similar non-supermodular games.

**Hypothesis 2.** *In the Best Public information condition, the level of convergence is greater in games with strategic complementarities (SCA and SCB) than in the Mixed game.*

Hypothesis 2 is consistent with the theoretical prediction that games with strategic complementarities converge to the unique Nash equilibrium under when learning from same-type others is not possible.

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<sup>11</sup>In order to mitigate the effects of possible early-round negative earnings, subjects entered round 1 with 14,000 points representing \$7 of the \$10 participation fee. Thus, only \$3 of the participation fee was guaranteed. As a result, at no time did any subject have negative total points.

**Hypothesis 3.** *In the strategic complements games (SCA and SCB), changing from the No Public to the Best Public information condition significantly increases the level of convergence.*

**Hypothesis 4.** *In the Mixed game, changing from the No Public to the Random Public information condition significantly increases the level of convergence.*

Hypotheses 3 and 4 are consistent with the idea that the additional information provided in the *Best Public* treatments will improve the ability of subjects to best respond both by allowing them to develop a better intuition for their payoff function and by providing additional information about match play. Further, this environment closely, but not perfectly, matches the environment for which Cartwright (2007) shows a learning dynamic mixing imitation and better reply converges to the Nash equilibrium.<sup>12</sup> Hypotheses 3 is also consistent with the theoretical results of Kandori and Rob (1995), who show that imitative learning converges to equilibrium play in supermodular games.

## 5 EXPERIMENT RESULTS

[Figure 3 about here.]

[Figure 4 about here.]

In this section, I compare convergence to equilibrium play across treatments. Figure 3 summarizes type-1 player choices across periods for all treatments, while Figure 4 presents the same for type-2 players. The box represents the range of the twenty-fifth and seventy-fifth percentiles of choices, while the whiskers extend to the minimum and maximum choices in each round. The horizontal line within each box represents the median choice. The figures show that for both player types, play in later rounds generally remained above the Nash equilibrium in the strategic complements games. For each treatment of Mixed, however, median play approached the Nash equilibrium. Finally, notwithstanding one type-2 subject who conducted an interesting experiment over the final 10 rounds, information about the play of the highest-earning same-type player appears to greatly improve convergence in the Mixed game.

[Figure 5 about here.]

Figure 5 displays the per-round evolution of Nash and  $\varepsilon$ -Nash play across treatments. It is clear that in the strategic complements games, while play evolves toward the neighborhood of the Nash equilibrium, there is little evidence of convergence to the strict Nash

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<sup>12</sup>The two-player games with random rematching do not satisfy the semi-anonymity assumption.

equilibrium. Furthermore, the Best Public information condition does not appear to have improved convergence in these games. (If anything, it appears to even worsen convergence to the neighborhood of the Nash equilibrium.) However, this information does appear to have a rather large positive effect in the Mixed game.<sup>13</sup>

To analyze Nash-equilibrium convergence across treatments, I compare the level of  $\varepsilon$ -convergence in rounds 41–58 of each treatment.<sup>14</sup> Table 2 reports the level of convergence ( $L_b(41, 58)$ ) to the  $\varepsilon$ -Nash equilibrium for each session in each of the six treatments. It also reports the alternative hypotheses and the corresponding  $p$ -values of one-tailed permutation tests.<sup>15</sup> Panel A looks at the proportion of joint  $\varepsilon$ -Nash equilibrium play, whereas Panels B and C look at proportion of type-1 and type-2  $\varepsilon$ -Nash equilibrium play, respectively.<sup>16</sup> I now formally test the hypotheses regarding convergence levels and present my main results.

[Table 2 about here.]

**Result 1.** (*Level of  $\varepsilon$ -Nash Convergence in Rounds 41–58*):

(i) *In the No Public information condition, the level of Nash-equilibrium convergence is significantly higher<sup>17</sup> in the strategic complements game than in the Mixed game when the Pareto-superior space is held constant (SCB vs. MIX). Convergence in the play of type-2 players is weakly higher in SCA than in MIX, but there is no significant difference in the other measures ( $\varepsilon$ - $x_1^*$ ,  $(\varepsilon$ - $x_1^*$ ,  $\varepsilon$ - $x_2^*$ )).*

(ii) *In the Best Public information condition, the level of Nash equilibrium convergence in the Mixed game is significantly greater than in the games with strategic complementarities (SCA and SCB).*

(iii) *In the strategic complements games (SCA and SCB), changing from the No Public to the Best Public information condition does not significantly change the level of Nash-equilibrium convergence.*

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<sup>13</sup>The Figures 8–9 in Appendix B depict the evolution of Nash and  $\varepsilon$ -Nash across treatments by player type.

<sup>14</sup>I look at rounds 41–58 as I am interested in the results of learning, but would like to avoid confounding end-game effects.

<sup>15</sup>The permutation test, also known as the Fisher randomization test, is a nonparametric version of a difference of two means  $t$ -test. It pools all independent observations and finds a  $p$ -value which represents the exact probability of observing a difference between random divisions of the pooled sessions at least as large as the inter-treatment difference. The permutation test uses all the information in the sample and therefore has 100 percent power efficiency.

<sup>16</sup>Table 7 in Appendix B presents the same information for strict convergence to Nash play in rounds 41–58.

<sup>17</sup>When analyzing the data, a significance level of five percent or less is referred to as *significant*, while a significance level between five and ten percent is referred to as *weakly significant*.

(iv) *In the Mixed game, changing from the No Public to the Best Public information condition significantly increases the level of Nash-equilibrium convergence.*

**SUPPORT:** Table 2 reports the permutation test results for Result 1.

By part (1) of Result 1, I find partial support for Hypothesis 1. The strategic-complements game with a relatively small Pareto-superior space converged better than the mixed game under the standard information condition, and the strategic complements game with a relatively large Pareto superior space did so in one dimension. This general result is not unexpected given the theoretical properties of games with strategic complementarities and previous experiment findings.

By part (2) of Result 1, I strongly reject Hypothesis 2. Furthermore, I accept the hypothesis that in the Best Public information condition, convergence is significantly greater in the Mixed game than in either of the games with strategic complements.

By part (4) of Result 1, I accept Hypothesis 4. This is the first experimental result showing that learning from same-type players can improve convergence to equilibrium in games without strategic complementarities.

By contrast, I reject Hypothesis 3 by part (3) of Result 1. This surprising result suggests that in games with strategic complementarities, imitation does not improve convergence. I now turn to a regression analysis suggesting that learning from high-earning same-type players actually leads play away from the Nash equilibrium in these games.

I estimate three probit models where the dependent variables are indicator variables for joint  $\varepsilon$ -Nash play ( $\varepsilon$ - $x_1^*$ ,  $\varepsilon$ - $x_2^*$ ), and  $\varepsilon$ -Nash play by type-1 ( $\varepsilon$ - $x_1^*$ ) and type-2 ( $\varepsilon$ - $x_2^*$ ) players. To control for learning effects, I include variables for round and round squared. I include indicator variables for the strategic complements games ( $\mathbf{D}_{\text{SCA}}$  and  $\mathbf{D}_{\text{SCB}}$ ), for the Best Public information condition ( $\mathbf{D}_{\text{Best}}$ ), as well as for their interactions. The Mixed game under the No Public information condition is the omitted category. By interacting the treatment indicators with the round variable, I allow learning rates to differ across treatments.

[Table 3 about here.]

I present estimated coefficients in Table 3.<sup>18</sup> I report probability derivatives, and robust standard errors with clustering at the session level. I focus on Model (1) looking at joint  $\varepsilon$ -Nash play. The positive coefficient on round combined with a negative coefficient on round squared (both significant) suggest that equilibrium-play improvement decreases in later rounds. In the No Public information condition, the coefficients are consistent with all games having the same general level of early-round equilibrium play, but with the strategic

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<sup>18</sup>Ai and Norton (2003) advise care in the interpretation of interaction terms in nonlinear models. I include coefficient estimates from OLS models of all regressions in Appendix B. Qualitative results are the same.

complements games (*SCB* in particular) converging more rapidly to the neighborhood of the Nash equilibrium. In addition, whereas the permutation-test results showed only a weakly significant difference in one convergence measure comparing Strategic Complements A and Mixed in the No Public condition, I estimate a positive and significant coefficient on the interaction of round and the *SCA* indicator in terms of type-2 player  $\epsilon$ -Nash play, and a weakly positive coefficient in terms of joint Nash play.

The estimated coefficients suggest that the change from No Public to Best Public in the Mixed game has a weakly significant early-round effect and a significantly positive effect over time. These effects in no way carry over to the strategic complements games. The coefficients on the interactions between round and these games' Best Public indicators are both significantly negative (although only weakly in the case of *SCB*), and the interaction between Best Public and the *SCA* game is negative, relatively large, and precisely estimated. Interestingly, the strategic complementarities learning effects in the No Public information condition are largely given back in the Best Public information condition. The results for equilibrium play by type-1 and type-2 players is largely consistent with the results for joint equilibrium play.

While the conservative permutation tests suggest that the Best Public information condition does not significantly affect equilibrium convergence in the strategic complements games, the estimated coefficients in the probit models suggests that, if anything, Best Public has a negative effect in these games. This effect is especially pronounced in the game where relatively many actions lead to outcomes Pareto superior to the Nash equilibrium (*SCA*). It is natural to ask what strategies were being played in lieu of equilibrium strategies.

In Figures 3 and 4, it appears that both player-types converge to choices above the Nash actions in strategic complements games. This trend seems particularly strong in the Best Public treatment. The payoff structure in the strategic-complements games, and particularly that of the Strategic Complements A game, presents the possibility that players will reach a “collusive” outcome. This term is misleading as it suggests that players of each type intentionally choosing to not myopically best respond. Given that subjects were randomly rematched after each round and did not know with whom they were matched, such intentional collusion would be difficult, if not impossible, to sustain. It is possible, however, that some subjects of both types were satisfied with the high profits they were earning in these “collusive” situations, and therefore continued satisficing instead of trying to find the best response to expected match choice.

[Figure 6 about here.]

In Figure 6, I depict a round-by-round measure of outcomes Pareto superior to the Nash equilibrium. To control for the proportion of action space resulting in outcomes Pareto

superior to Nash (*psRatio*), I divide the proportion of Pareto superior-play by *psRatio*. Thus a score of 4 means that Pareto-superior outcomes were 4 times more likely than if subjects randomly chose actions. In Figure 6(a), I present the evolution of Pareto superior outcomes for both information conditions for Strategic Complements A. In the Best Public treatment, there is an almost immediate attraction to this region. By round 10, the likelihood of Pareto superior play was already three times more likely than chance. In the No Public sessions, play gravitates more much more slowly to this region. However, Pareto-superior action combinations were never three times more likely than chance.

As Strategic Complements B and Mixed have the exact same number of Pareto-superior action combinations, it is informative to compare these games directly. I compare these two games in both the No Public (Figure 6(b)) and Best Public (Figure 6(b)) information conditions. In the No Public treatments, Pareto-superior outcomes were not rare in either game, although they seem more likely in Strategic Complements B. In the Best Public treatments, there is a clear separation of games. The Best Public information condition in no way decreases the likelihood of Pareto-superior outcomes in Strategic Complements B. These outcomes are consistently at least seven times more likely than chance. However, the Best Public information condition nearly extinguished Pareto-superior play in later-round Mixed-game play.

[Table 4 about here.]

Formal comparisons of Pareto-superior outcomes confirm the graphical intuitions. In Table 4, I present the proportion of Pareto-superior outcomes for blocks of twenty rounds by session for each treatment, as well the overall proportion across all sessions. I also report the alternative hypotheses and  $p$ -values of one-tailed permutation tests. The level of Pareto-superior play in the Strategic Complements A game is significantly higher in the Best Public information condition than in the No Public information condition over the first 40 rounds, and particularly in the first 20 rounds. While the level of Pareto-superior play does not significantly vary across information conditions in the Strategic Complements B game, the Best Public information condition has a negative effect in the final 20 rounds in the Mixed game. Finally, while in No Public the level of Pareto-superior play is significantly higher in *SCB* than in *MIX* over the first 20 rounds and weakly higher in the second 20 rounds, in Best Public the level of Pareto-superior play is significantly higher in Strategic Complements B across all 60 rounds.

### 5.1 Best Responding across Treatments

Given our surprising results on the effects of learning from others on convergence to the Nash prediction and on convergence to Pareto-superior outcomes in the strategic complements



games, I look at treatment effects on best responding. I do so for a couple of reasons. First, best replying is a necessary condition for Nash equilibrium play. Second, best responding to expected match play is at the heart of a variety of learning models such as fictitious play and Cournot best reply. Understanding why subjects do not best respond provides evidence on the question of non-convergence.

[Figure 7 about here.]

In Figure 7, I graphically depict best responding across treatments. In No Public, there is little difference in Player-1 best-response likelihood across treatments (Figure 7(a)). This is relatively unsurprising as I kept the Player-1 profit function constant across games. Differences in Player-2 best-response likelihoods across No Public treatments (Figure 7(c)) are small, with perhaps the Mixed game lagging the strategic complements games in later rounds. Switching to the Best Public information condition gives some inter-game separation (Figures 7(b) and 7(d)), with the Mixed game perhaps having the highest likelihood of best-response for both player types.

A more formal analysis of best responding needs to consider that a failure to best respond may be due to difficulties in calculating the best response. Given a desire to best respond, an accurate prediction of match play will improve ability to best respond. To control for ability to predict match strategy in round  $t$ , I construct the variable  $-\mathbf{SD3}_t$ , the standard deviation of the current and last two match choices. More stable match play (i.e., smaller standard deviation of current and previous match play) makes it easier for a player to best respond if she so desired. I multiply the standard deviation by negative one to make it a positive measure of play stability and thus measure ease of match-play prediction.<sup>19</sup>

[Table 5 about here.]

To determine the effects of the interaction between strategic complementarities, the ability to learn from others, and other factors on best responding, I estimate probit models of round- $t$  best responding across treatments with robust standard errors adjusted for clustering at the individual-level (Table 5). In Models (1) and (2), the dependent variables are best-response indicators for type-1 and 2 players. Additionally, I define an  $\varepsilon$ -Best Response as playing within  $\pm 1$  of the best response to the match's choice, and use the associated indicator variables as the dependent variables in Models (3) and (4).

To control for general learning effects, I include round. I include indicator variables for the strategic complements games, for the Best Public information condition, as well as for their interactions. As before, the Mixed game under the No Public information condition

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<sup>19</sup>The estimated coefficients in Table 5 are largely insensitive to use of longer histories of match play.

is the omitted category. I include the indicator variable  $(\varepsilon)\mathbf{BR}_{t-1}$  set equal to one if the subject  $(\varepsilon)$ best responded in the preceding round. The main coefficients of interest are those on the measure of match-choice stability  $(-\mathbf{SD3}_t)$  and this variable's interaction with the treatment indicators.

I focus on  $\varepsilon$ -best responding, Models (3) and (4). Coefficients on treatment indicators relatively small and imprecisely estimated, consistent with little inter-game difference in early-round best responding. Coefficients on round and its interactions with treatment indicators are generally positive and significant, suggesting that learning to best respond extended past controlled-for match-choice prediction. In particular, interactions with the indicators for the strategic-complements games are generally positive and significant, consistent with the generally better equilibrium convergence of these games in the absence of learning from others. The positive and significant coefficient on round's interaction with the indicator for the Best Public information condition suggests that not all of the improvement from this information in the Mixed game was due to match-play stability. Consistent with the failure of information about the play of others in improving convergence in the strategic-complements games, the coefficients on interaction of these treatments with round are uniformly negative and significant.

I estimate positive and significant coefficients on the measure of match-play stability, consistent with best responding increasing in match-play predictability. While the coefficients on most of the interactions between this measure and the treatment indicators are small and imprecisely estimated, the interaction between match-play stability and the SCA-Best Public treatment is negative, large in magnitude, and precisely estimated. This suggests that the improvement in best response resulting from ease in predicting match choice largely disappears in this treatment.<sup>20</sup> Estimated coefficients in Models (1) and (2) are largely in agreement, although due to relatively low overall levels of exact best response, the coefficients capturing the effects of match-play stability are less precisely estimated.

## 6 EFFECT OF PUBLIC INFORMATION ON INDIVIDUAL DECISIONS

In this section, I take a more in-depth look at how players use the information they receive in Best Public treatments. This analysis suggests that there are differences across games in the use of public information, which may help explain why more information in the Mixed game pushes play towards the Nash equilibrium, but does not in the strategic complements games. More concretely, I find evidence that while type-2 players in the Mixed game use

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<sup>20</sup>Wald tests on the hypothesis that coefficients  $-\mathbf{SD3}_t + (-\mathbf{SD3}_t \times \mathbf{D}_{\text{SCA}} \times \mathbf{D}_{\text{Best}}) = 0$  yield  $p$ -values of 0.2174 and 0.4624 for  $\varepsilon - BR_1$  and  $\varepsilon - BR_2$  respectively. For  $-\mathbf{SD3}_t + (-\mathbf{SD3}_t \times \mathbf{D}_{\text{SCB}} \times \mathbf{D}_{\text{Best}}) = 0$ , the resulting  $p$ -values of 0.2174 and 0.4624 for  $\varepsilon - BR_1$  and  $\varepsilon - BR_2$  respectively.

high-quality information to best respond, for subjects in strategic complements games, the information most useful in best responding does not induce this behavior to the same extent.

I estimate probit models with robust standard errors adjusted for clustering at the individual level. In Model (1), the dependent variable is an indicator variable equal to one if the type-1 player best responds round  $t$ , while in Model (2) the dependent variable is the indicator for the type-2 player best responding. Models (3) and (4) look at  $\varepsilon$ -best responding for type-1 and 2 players. I limit observations to those from Best Public treatments.

I use many of the same regressors that I used in the analysis presented in Table 5. I control for the ability to predict match choice with the variable  $-\mathbf{SD3}_t$ , the standard deviation of the current and last 2 match choices. Second, I use round and an indicator for ( $\varepsilon$ -)best responding in the preceding round to control for learning effects independent of our other variables. I use indicator variables to control for treatment effects, and interact them with round.

The goal is to see how players in different games use public information in making choices for the next round. I focus on the use of public information which should help a player best respond. There are two components to the usefulness of public information. First, the better the public player has played, the more useful is the information. Using  $p$  to subscript the public player of type  $i$  and  $-p$  her match, I define  $\gamma_i = \frac{\pi_p}{\max \pi_i(x-p)}$ .  $\gamma$  thus measures how well the public player did given her match's choice. Second, the closer the public player's situation is to my situation, the more useful the information. Using  $-i$  to subscript the match of player  $i$ , I define public-information relevance  $\rho_i = \frac{1}{1+(x-p-x_i)^2}$ .  $\rho$  measures the proximity of the choice of a player's match to that of the public player's match. Combining these two measures, I measure the usefulness of public information as  $\mathbf{InfoQual}_i = \gamma_i \rho_i$ . I include  $\mathbf{InfoQual}_{i,t-1}$ , and its interaction with the treatment indicators, in the best-response regression models.

[Table 6 about here.]

I report estimated coefficients in Table 6. I first note that the coefficients support the previous finding that while play stability improved player-2  $\varepsilon$ -best responding in *MixBest*, this improvement was absent in *SCABest*. Furthermore, in the Mixed game, high-quality public information facilitated the best response of the type-2 player, although this effect was only weakly significant in terms of player-2  $\varepsilon$ -best responding. It did not improve best-responding in the *SCB* game, as the coefficient on the interaction between information quality and the SCB game is negative and at least as large as the positive coefficient on the information-quality proxy.<sup>21</sup>

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<sup>21</sup>For type-2 players, Wald test on the hypothesis that coefficients  $\mathbf{InfoQual}_{t-1} + (\mathbf{InfoQual}_{t-1} \times \mathbf{D}_{\mathbf{SCB}})$

## 7 INTERPRETATION AND DISCUSSION

Economists have long appreciated that real agents might not identify and play Nash equilibrium strategies. There has thus been a long literature that looks at the conditions under which learning agents will converge to equilibrium play.

In most laboratory studies of learning in games, a player receives feedback only about payoffs she receives from her choices. Under this particular information assumption, experimenters have looked at many environments to determine whether or not individual learning may lead to Nash-equilibrium play. One factor which has been shown to be important is the existence of strategic complementarities. For example, in reviewing the experimental literature on incentive-compatible mechanisms for pure public goods, Chen (2008) finds that mechanisms with strategic complementarities, such as the Groves-Ledyard mechanism under a high punishment parameter, converge robustly to the efficient equilibrium (Chen and Plott 1996, Chen and Tang 1998). Conversely, those far away from the threshold of strategic complementarities do not seem to converge (Smith 1979, Harstad and Marrese 1982).

In this study, play generally converged to the neighborhood of Nash play in the strategic complements games under the standard information condition, at least relative to the Mixed game. Introducing the information needed to learn from successful same-type players did not have the intuitive effect in all games. This information dramatically improved the Nash convergence of the Mixed game. This is important as the Mixed game belongs to a class of games which have generally not shown good convergence properties. While this finding points in the direction of a reassessment of previous laboratory results—convergence in games without strategic complementarities might depend on the information structure—the generality of this finding awaits further investigation.

The ability to learn from high-earning same-type players did not improve convergence in games with strategic complementarities. Furthermore, regression evidence suggests it impeded equilibrium play in the SCA game. In addition to strategic complementarities, the SCA game is notable for a fairly large proportion of actions resulting in outcome Pareto-superior to the Nash equilibrium. Whereas 2.7% of action combinations in the Strategic Complements B and Mixed games are Pareto superior to Nash, this figure is 10.7% in the Strategic Complements A game. While play somewhat gravitated to this region in No Public, play immediately went to this region in the Best Public information condition. Likewise, comparing the Strategic Complements B and Mixed games (i.e., holding constant the number of Pareto-superior action combinations), I find significant differences in Pareto-superior outcomes, particularly in the Best Public information condition. I do note that the attrac-

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= 0 yield  $p$ -values of 0.9918 and 0.2316 for  $BR_2$  and  $\varepsilon - BR_2$  respectively. For  $\mathbf{InfoQual}_{t-1} + (\mathbf{InfoQual}_{t-1} \times \mathbf{D}_{SCA}) = 0$ , the  $p$ -values are 0.5358 and 0.4334

tion to Pareto-superior action combinations in games with strategic complementarities is not unique to this study. Despite the overall convergence to equilibrium play in the strategic complements games studied by Chen and Gazzale, there was also a good deal of attraction to Pareto-superior outcomes. Averaging across games and excluding their “near” supermodular treatment, Pareto-superior play over the final twenty rounds was 18 times more likely than chance in games with strategic complementarities, and only 3.7 times more likely in games without.<sup>22</sup>

It is also interesting that Potters and Suetens (Forthcoming) find that strategic complementarities facilitate Pareto-superior outcomes with repeated play (as opposed to random rematching). My results are consistent with learning from others being someone of a substitute for repeated play in this regard.

Continued play in the Pareto-superior region requires a failure to best respond. I investigate underlying causes. I first find that the ease of predicting match play has differential effects across treatments. While match-play stability generally increases the likelihood of best response, I cannot reject the hypotheses that it has no effect in the Best Public treatment of the Strategic Complements A game. I next find that differences best responding in the Best Public information condition are not primarily driven by differences in the information content of public information. Information most helpful in helping a player best respond (i.e., the public player did well against a match who chose an action similar to my match’s choice) weakly improved best responding in the Mixed game. However, I cannot reject the hypothesis that it had no effect on best responding in the Strategic Complements B game. Interestingly, Cartwright (2007) assumes a “success property” whereby the identification of a better reply serves as a “success example” for others of the same type. This assumption did not appear to always hold in this study.

The geometry of best-responses plausibly explains the difference in the persistence of Pareto-superior play between games with and without strategic complementarities. Combining Figures 1 and 2, best responses tightly bracket the Pareto-superior region in the strategic complements games and very loosely bracket this area in the Mixed game. Therefore, starting from a Pareto-superior outcome, one player’s move toward a best response will still leave the other player with a “good” payoff in the strategic complements games, with the subjective assessment of good plausibly influenced by closeness to publicly known profits. In the Mixed game, as best responses are often far from Pareto superior, one player’s move towards a best response will often leave the other type with a “poor” payoff and provides an added incentive for this second player to explore new strategies. Information about the actions of high-earning same-type players facilitates a better response.

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<sup>22</sup>Pareto-superior play was over 28 times more likely than chance in the game near the strategic complements threshold.

This discussion highlights two avenues by which public information as implemented in this study might affect choices. First, information about the *choices* of others enables conformity and assists in choice selection conditional on search for an action. Second, information about the *payoffs* of others may affect player aspirations and thus whether one chooses to explore. Disentangling these effects is left for future work.

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## APPENDIX A EXPERIMENT INSTRUCTIONS

This version uses the Blue Player payoff function from Strategic Complements A. I put in brackets text that appears only in Best Public treatments.

### Introduction

- You are about to participate in a session in which one of numerous alternatives is selected in each of 60 rounds. This is part of a study intended to provide insight into certain features of decision processes. If you follow the instructions carefully and make good decisions you may earn a considerable amount of money. You will be paid in cash at the end of the experiment.
- *During the experiment, I ask that you please do not talk to each other.* If you have a question, please raise your hand and an experimenter will assist you.

### Procedures

- Prior to the start of the actual experiment, there will be a series of review questions to test your understanding of these Experiment Instructions. You may use these Experiment Instructions in answering the review questions. You will be paid \$0.10 for each correctly answered question.
- At the beginning of the experiment, you will be randomly assigned a type: either Red or Blue. There will be 6 Red players and 6 Blue players. You will remain the same type for the entire experiment.
- In each round, you will make a Choice and enter it at the computer terminal. You may choose any whole number from 0 through 20.
  - Prior to clicking the **OK** button on the computer screen, you may change your Choice.
  - *Once you have clicked **OK**, you cannot change your Choice for the round.*
- In each round, you will be randomly matched with one of the other type players. Each Red player is randomly matched with a Blue player, and each Blue player is randomly matched with a Red player. You will not know the identity of your Match. The Points you earn in each round depend only on your Choice and the Choice of your Match for that round.
- Letting  $X_r$  represent the Choice of the Red player in a pair and  $X_b$  the Choice of a Blue player in a pair, Points earned in a round are as follows:  
**Red Player:**  $\text{Points} = 92X_r - 6X_b - 10X_r^2 + X_b^2 + 6X_rX_b$ .

**Blue Player:**  $\text{Points} = 104X_b - 26X_r - 10X_b^2 + X_r^2 + 8X_rX_b$ .

- [At the end of each round, the computer will select one Public Red player and one Public Blue player. The Public Red player will be the Red player earning the most points in the round. The Public Blue player will be the Blue player earning the most points in the round. If there is a tie for most points earned by a type, the computer will randomly select one of the tied players.]
- At the end of each round, your screen will display:
  - Your Choice
  - The Choice of your Match for that round
  - Points you earned
  - [The Choice of the Public Player for your type. (Red players will see the Public Red player’s Choice. Blue players will see the Public Blue player’s Choice.)]
  - [The Choice your Public player’s Match.]
  - [Points earned by the Public player for your type.]
- At all points in the experiment, you may access the information from previous rounds (Choice, Match’s Choice, and Points earned [for you and the Public player]) in the History Box displayed on your screen.
- The experiment will continue for 60 rounds.

### Experiment Earnings

- There will be no practice rounds. The Points you earn in every round will be added to your Total Points. Should you earn negative points in a round, they will be subtracted from your Total Points.
- Your participation fee for the experiment is \$10. \$3 is guaranteed. At the beginning of the experiment, the remaining \$7 is converted into Points and added to your Total Points.
- You will earn \$1 for each 2,000 Points you accumulate in the experiment.
- Your Earnings =  $(\text{Total Points})/2000 + \$3 + \text{Review Question Earnings}$

I encourage you to earn as much cash as you can. Are there any questions?

APPENDIX B ADDITIONAL FIGURES AND TABLES

[Figure 8 about here.]

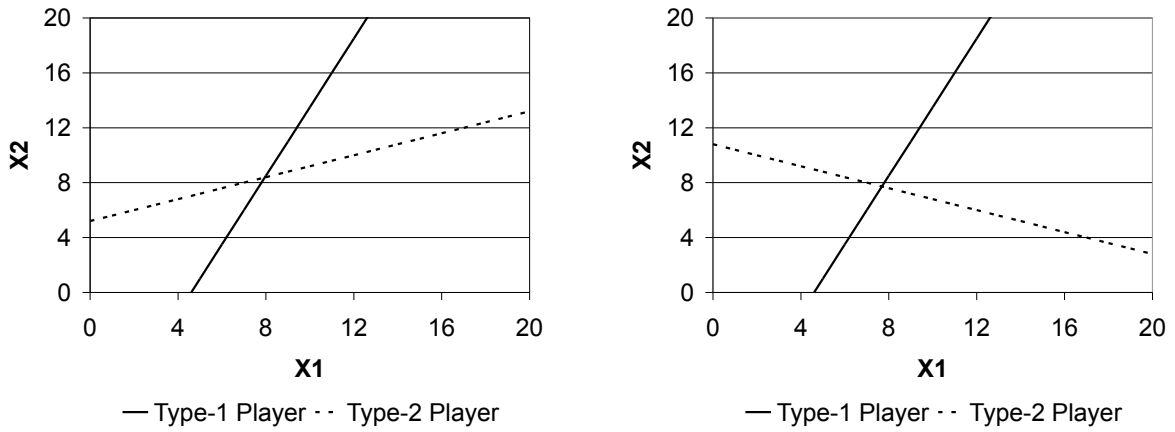
[Figure 9 about here.]

[Table 7 about here.]

[Table 8 about here.]

[Table 9 about here.]

[Table 10 about here.]



(a) Best-response functions in Strategic Complements treatments (b) Best-response functions in Mixed treatments

Figure 1: Best-response functions.

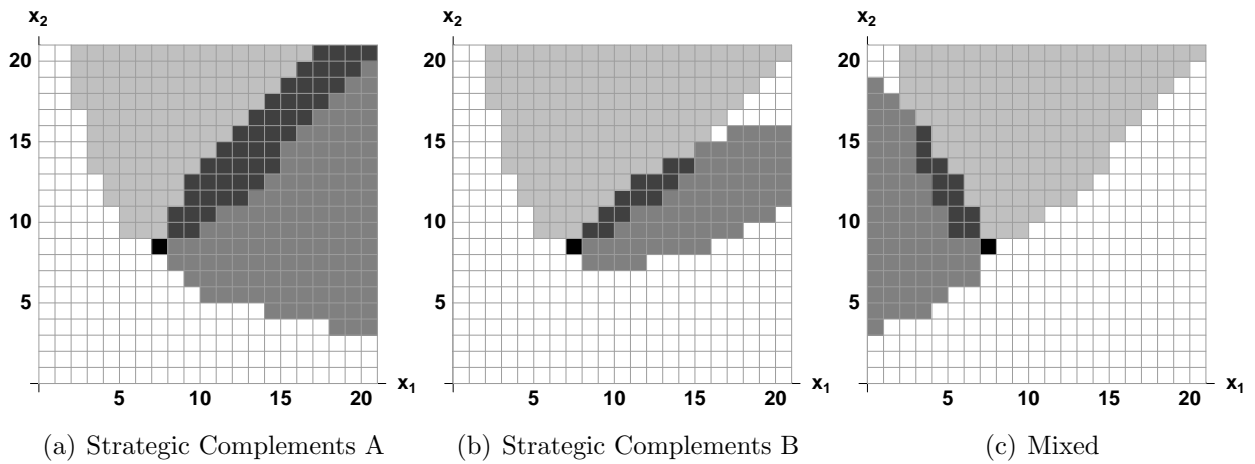
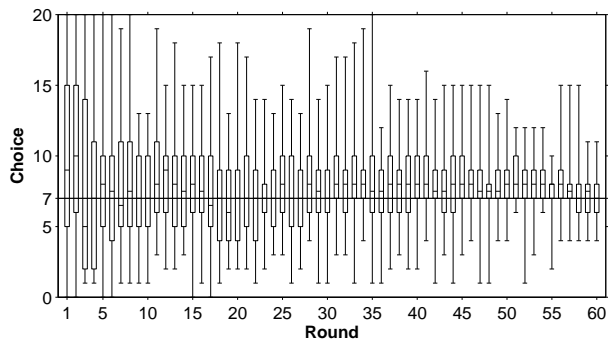
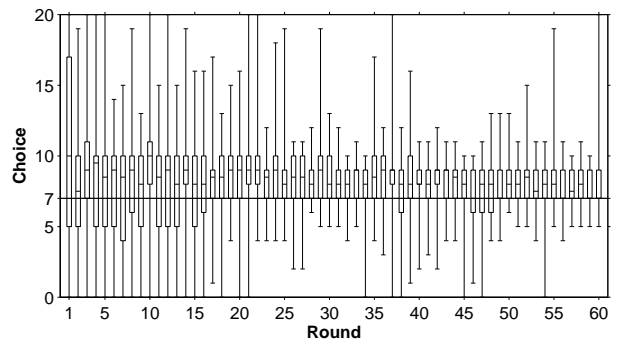


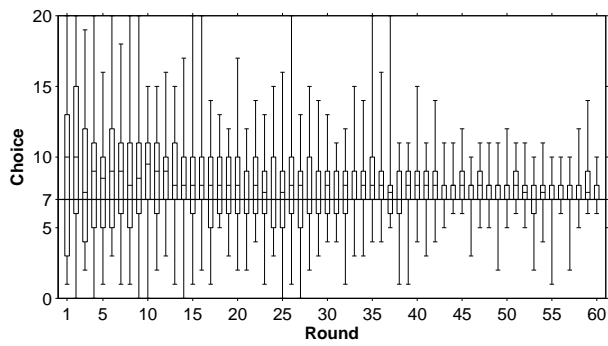
Figure 2: Dark Gray: action combinations Pareto superior to Nash equilibrium ( $\pi_1(x_1, x_2) \geq 506, \pi_2(x_1, x_2) \geq 507$ ); Light Gray:  $\pi_1(x_1, x_2) \geq 506, \pi_2(x_1, x_2) < 507$ ; Medium Gray:  $\pi_1(x_1, x_2) < 506, \pi_2(x_1, x_2) \geq 507$ ; Black: Nash equilibrium  $\{x_1^*, x_2^*\}$ .



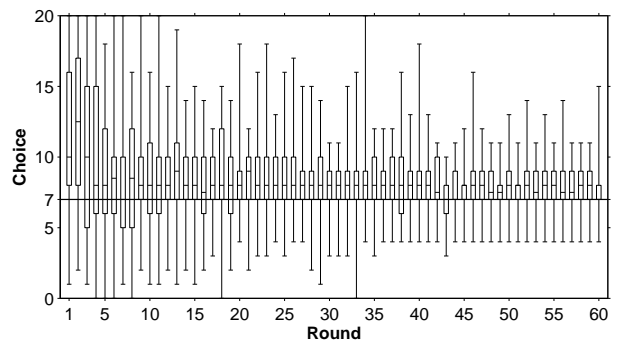
(a) Player 1: SCA, No Public



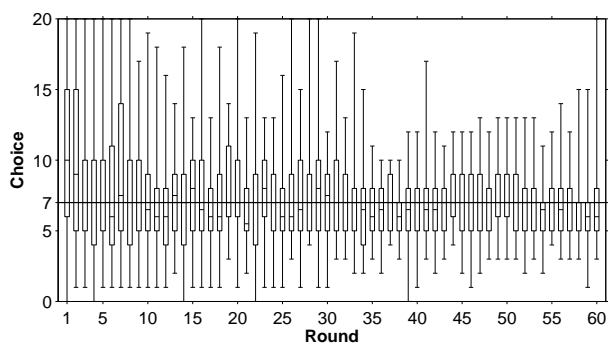
(b) Player 1: SCA, Best Public



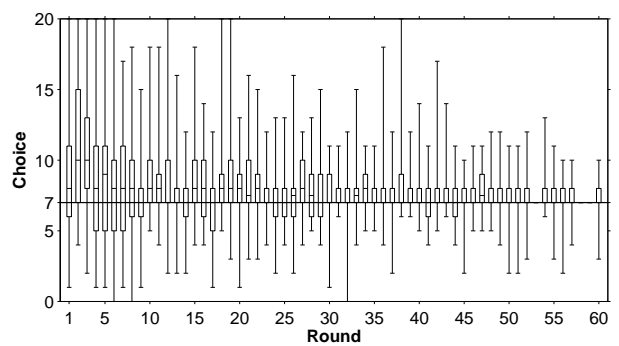
(c) Player 1: SCB, No Public



(d) Player 1: SCB, Best Public

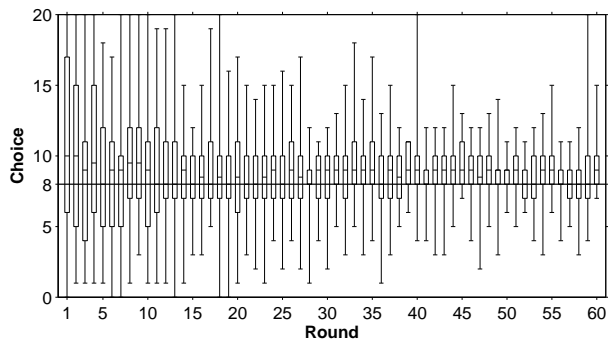


(e) Player 1: MIX, No Public

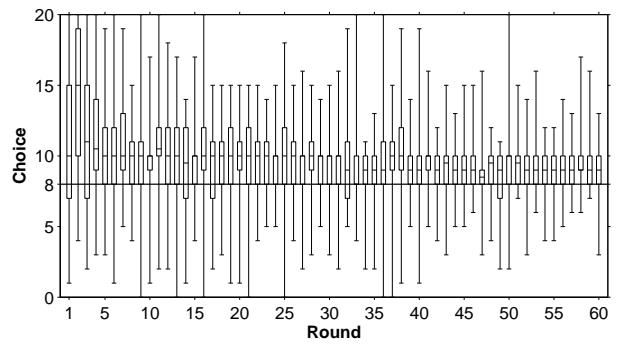


(f) Player 1: MIX, Best Public

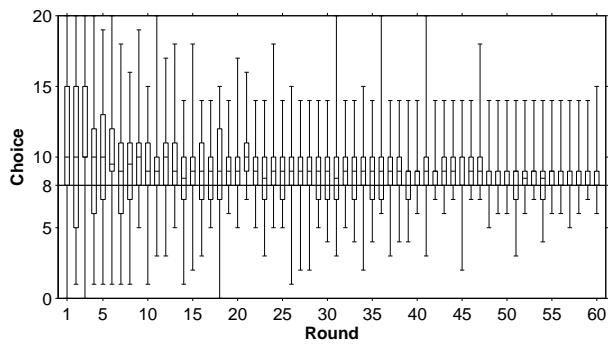
Figure 3: Distribution of Choices: Player 1.



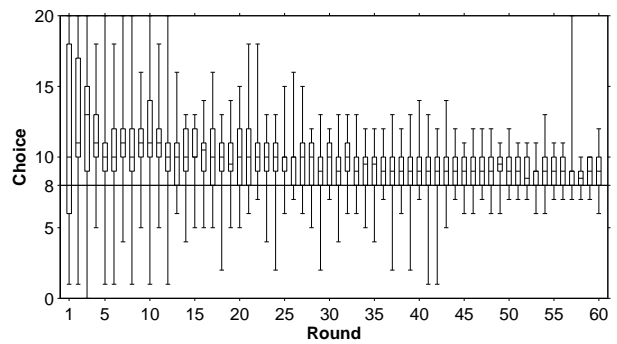
(a) Player 2: SCA, No Public



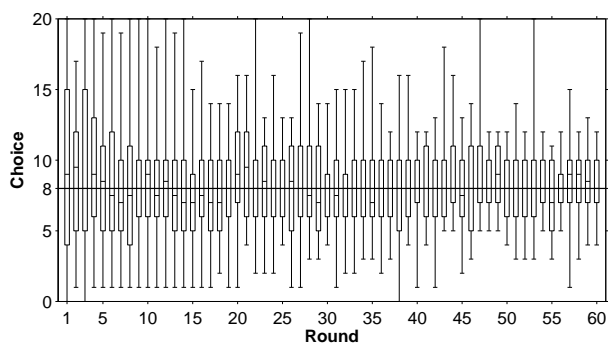
(b) Player 2: SCA, Best Public



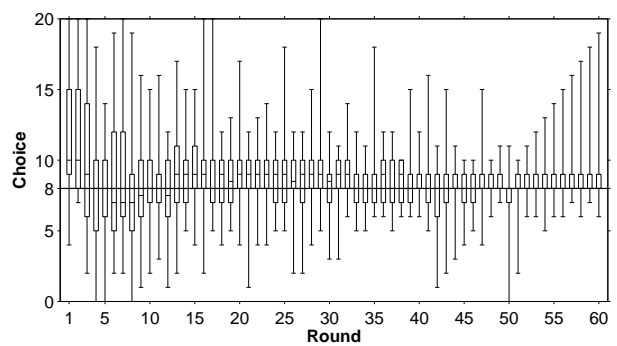
(c) Player 2: SCB, No Public



(d) Player 2: SCB, Best Public

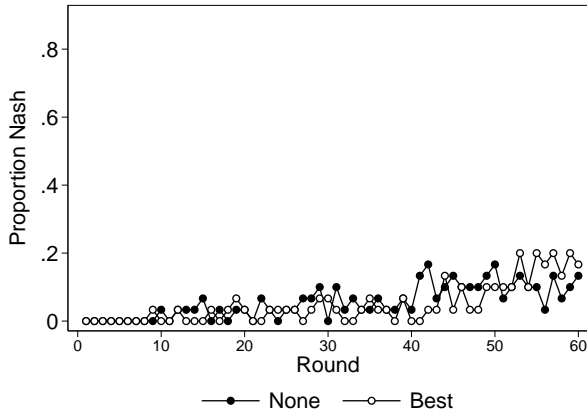


(e) Player 2: MIX, No Public

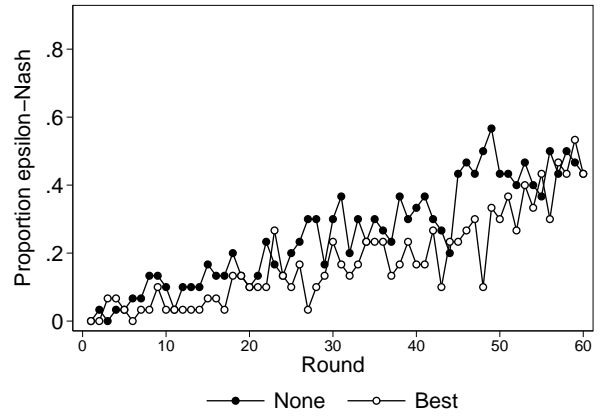


(f) Player 2: MIX, Best Public

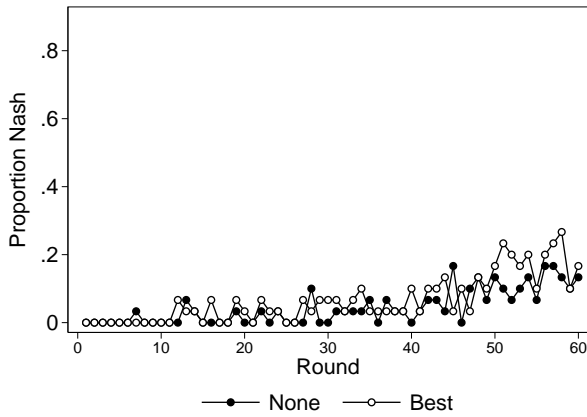
Figure 4: Distribution of Choices: Player 2.



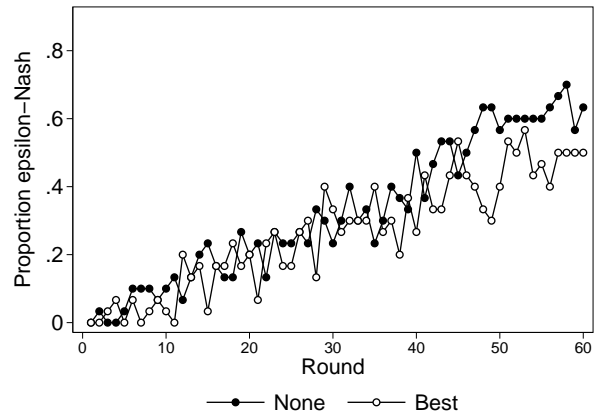
(a) Proportion Nash: SCA



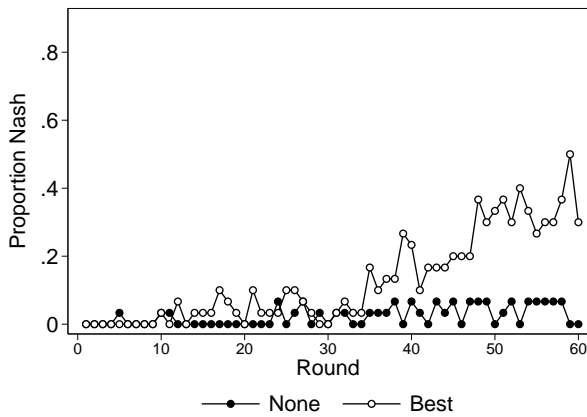
(b) Proportion  $\epsilon$ -Nash: SCA



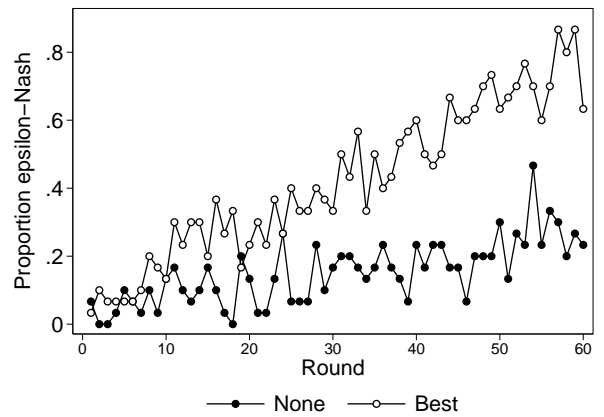
(c) Proportion Nash: SCB



(d) Proportion  $\epsilon$ -Nash: SCB



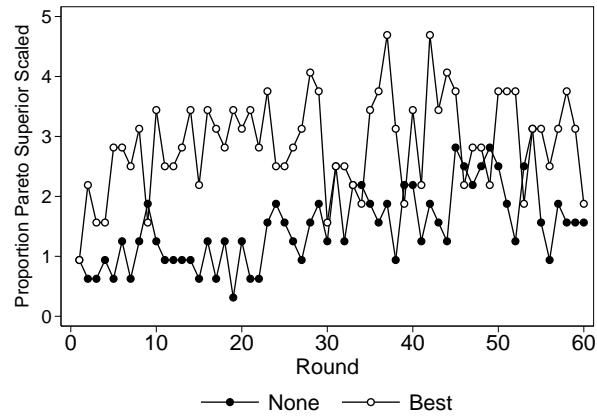
(e) Proportion Nash: MIX



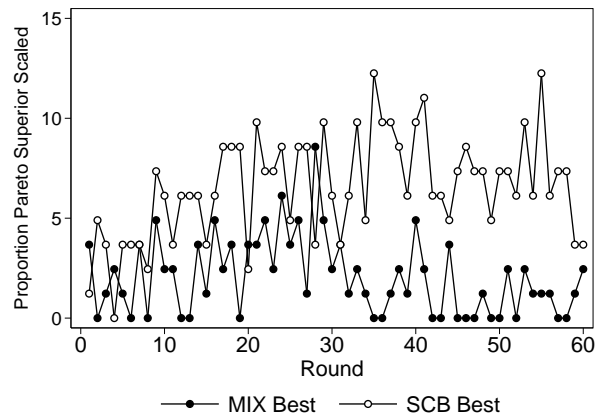
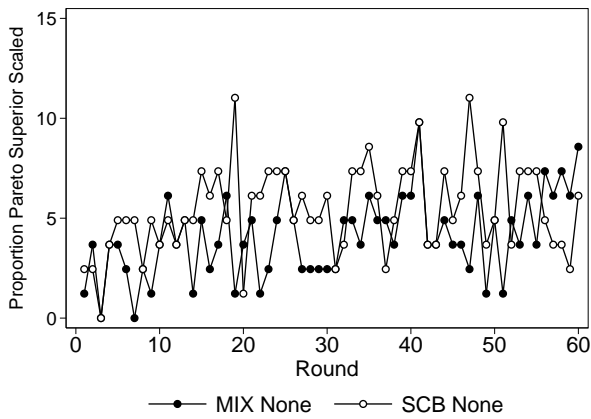
(f) Proportion  $\epsilon$ -Nash: MIX

Figure 5: Per-Round Nash and  $\epsilon$ -Nash Play.



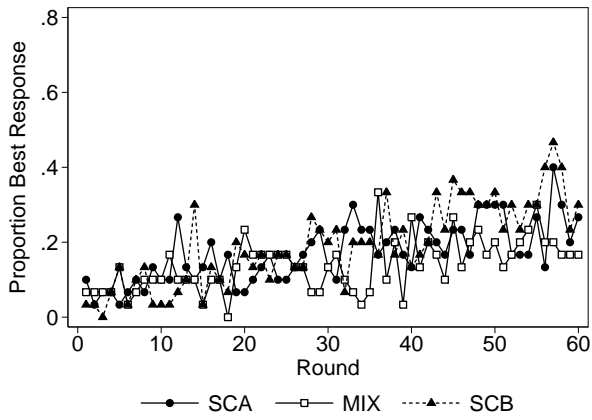


(a) Strategic Complements A

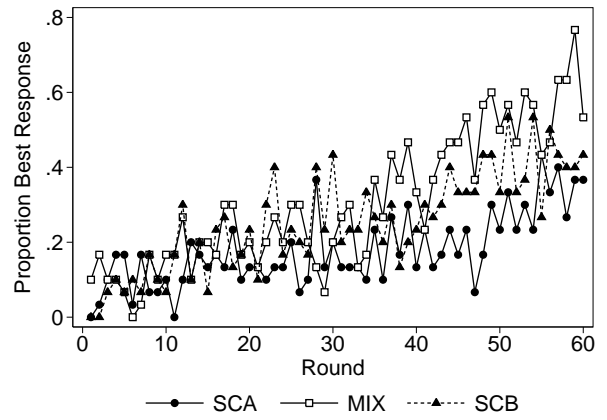


(b) No Public: Strategic Complements B vs. Mixed (c) Best Public: Strategic Complements B vs. Mixed

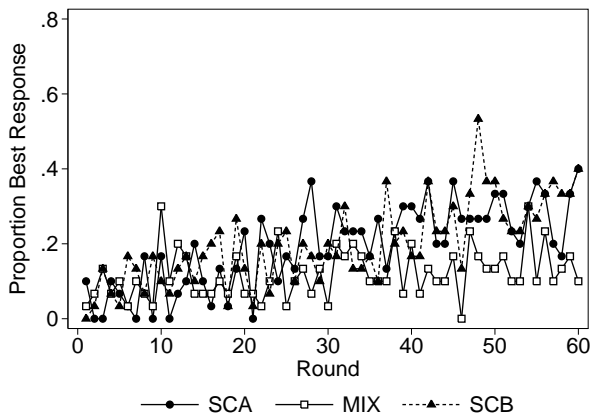
Figure 6: Outcomes Pareto superior to Nash equilibrium divided by the proportion of action combinations resulting in Pareto superior outcomes.



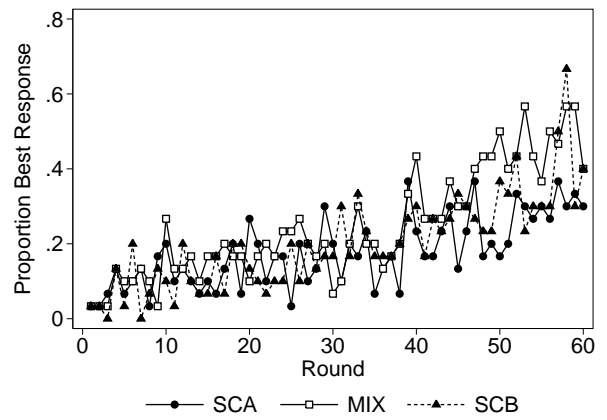
(a) Type-1 Player: None



(b) Type-1 Player: Best

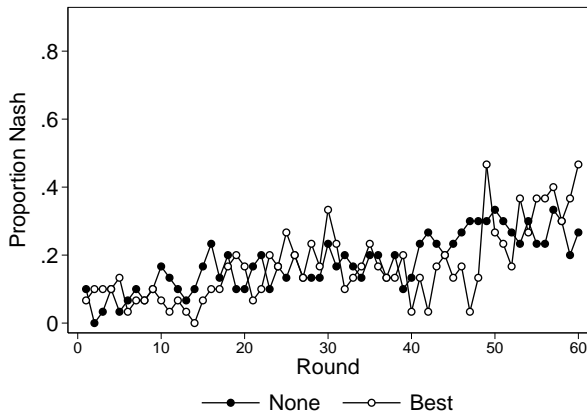


(c) Type-2 Player: None

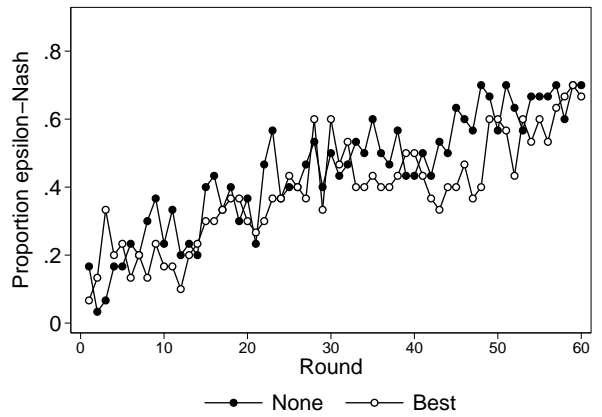


(d) Type-2 Player: Best

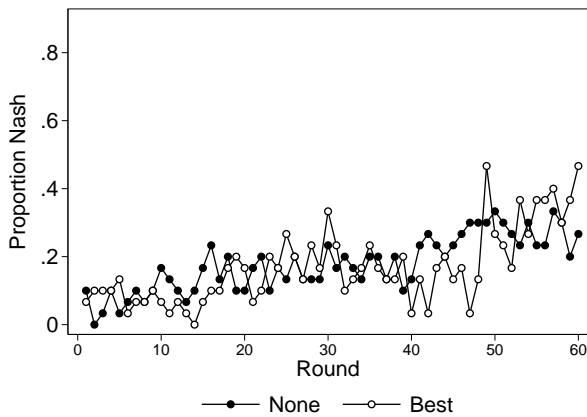
Figure 7: Best Responding by Player Type.



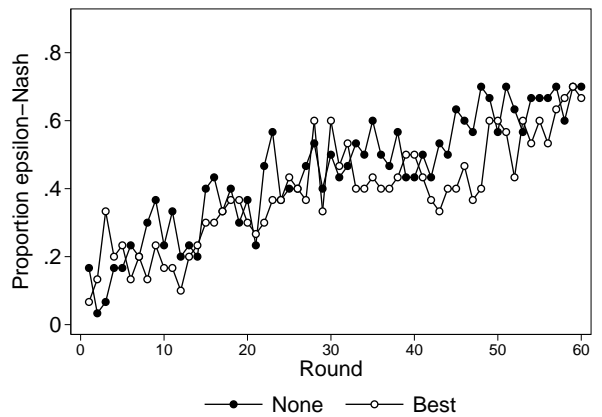
(a) SCA: Player 1 Nash Play



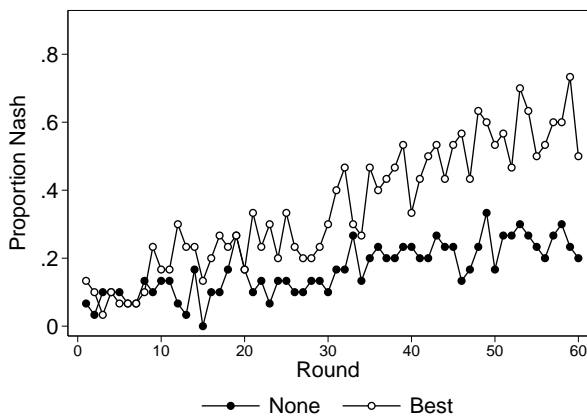
(b) SCA: Player 1  $\epsilon$ -Nash Play



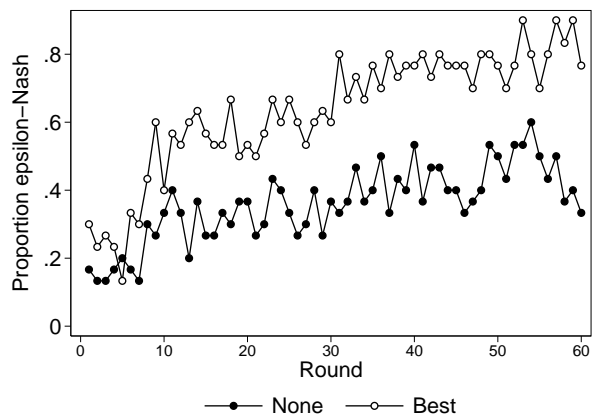
(c) SCB: Player 1 Nash Play



(d) SCB: Player 1  $\epsilon$ -Nash Play

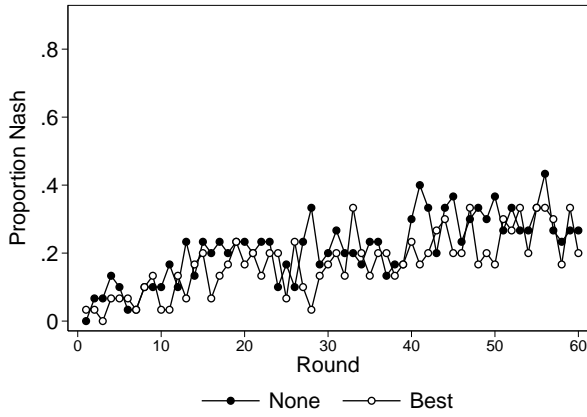


(e) MIX: Player 1 Nash Play

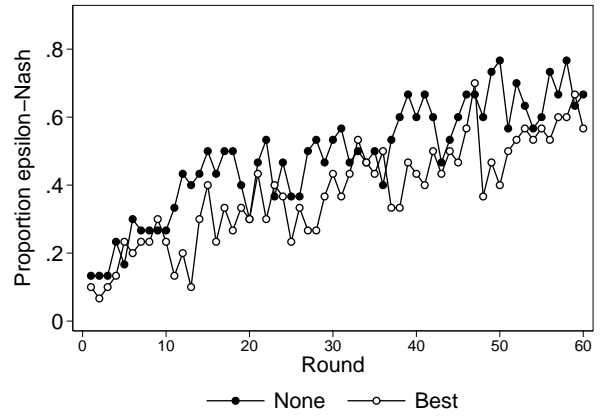


(f) MIX: Player 1  $\epsilon$ -Nash Play

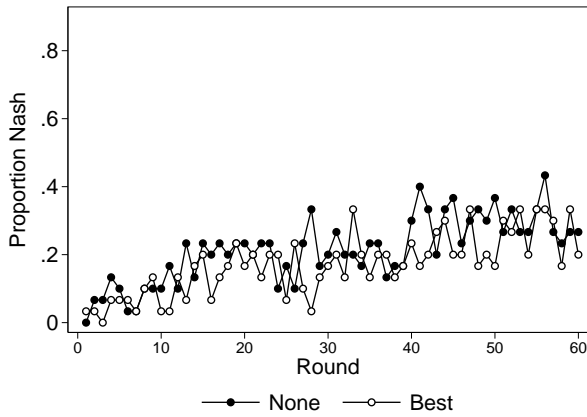
Figure 8: Per-Round Nash and  $\epsilon$ -Nash Play by Type-1 Players.



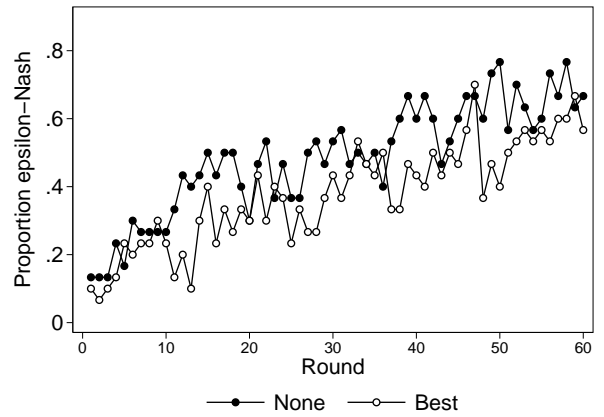
(a) SCA: Player 2 Nash Play



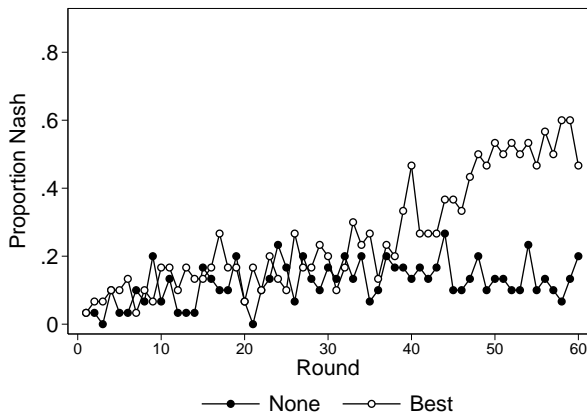
(b) SCA: Player 2  $\epsilon$ -Nash Play



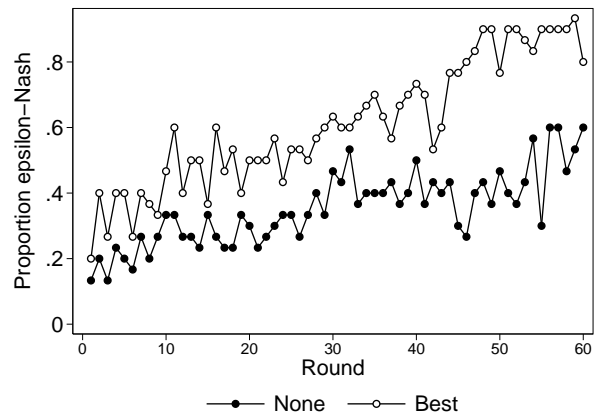
(c) SCB: Player 2 Nash Play



(d) SCB: Player 2  $\epsilon$ -Nash Play



(e) MIX: Player 2 Nash Play



(f) MIX: Player 2  $\epsilon$ -Nash Play

Figure 9: Per-Round Nash and  $\epsilon$ -Nash Play by Type-2 Players.

	No Public	Best Public
Strategic Complements A ( <i>SCA</i> )	5 Sessions	5 Sessions
Mixed ( <i>MIX</i> )	5 Sessions	5 Sessions
Strategic Complements B ( <i>SCB</i> )	5 Sessions	5 Sessions

Table 1: Experiment Sessions

Panel A:	Proportion of $\varepsilon$ -Nash Equilibrium choices ( $\varepsilon-x_1^*$ , $\varepsilon-x_2^*$ )					Permutation Tests		
	Session 1	Session 2	Session 3	Session 4	Session 5	Overall	$H_1$	p-value
SCA:None	0.231	0.222	0.426	0.343	0.852	0.415	SCA:None<SCA:Best	0.778
SCA:Best	0.676	0.120	0.139	0.352	0.185	0.294	SCA:Best<MIX:Best	0.012**
MIX:None	0.324	0.130	0.324	0.343	0.019	0.228	MIX:None<SCA:None	0.119
MIX:Best	0.824	0.519	0.463	0.796	0.685	0.657	MIX:None<MIX:Best	0.000***
SCB:None	0.278	0.769	0.380	0.667	0.750	0.569	SCB:Best<MIX:Best	0.083*
SCB:Best	0.806	0.148	0.685	0.287	0.250	0.435	MIX:None<SCB:None	0.012**
							SCB:None<SCB:Best	0.805
							SCA:Best<SCB:Best	0.171
							SCA:None<SCB:None	0.183
Panel B:	Proportion of Player 1 $\varepsilon$ -Nash equilibrium choices ( $\varepsilon-x_1^*$ )					Permutation Tests		
SCA:None	0.583	0.343	0.676	0.519	0.907	0.606	SCA:None<SCA:Best	0.794
SCA:Best	0.861	0.370	0.352	0.556	0.343	0.496	SCA:Best<MIX:Best	0.020**
MIX:None	0.583	0.519	0.491	0.537	0.130	0.452	MIX:None<SCA:None	0.150
MIX:Best	0.917	0.667	0.759	0.861	0.713	0.783	MIX:None<MIX:Best	0.000***
SCB:None	0.509	0.861	0.509	0.806	0.900	0.717	SCB:Best<MIX:Best	0.111
SCB:Best	0.852	0.593	0.880	0.417	0.491	0.646	MIX:None<SCB:None	0.044**
							SCB:None<SCB:Best	0.694
							SCA:Best<SCB:Best	0.123
							SCA:None<SCB:None	0.202
Panel C:	Proportion of Player 2 $\varepsilon$ -Nash equilibrium choices ( $\varepsilon-x_2^*$ )					Permutation Tests		
SCA:None	0.370	0.593	0.667	0.639	0.935	0.641	SCA:None<SCA:Best	0.883
SCA:Best	0.796	0.333	0.306	0.676	0.454	0.513	SCA:Best<MIX:Best	0.016**
MIX:None	0.556	0.259	0.602	0.630	0.065	0.422	MIX:None<SCA:None	0.075*
MIX:Best	0.880	0.731	0.630	0.889	0.944	0.815	MIX:None<MIX:Best	0.000***
SCB:None	0.463	0.898	0.704	0.824	0.852	0.748	SCB:Best<MIX:Best	0.123
SCB:Best	0.954	0.250	0.796	0.639	0.574	0.643	MIX:None<SCB:None	0.012**
							SCB:None<SCB:Best	0.750
							SCA:Best<SCB:Best	0.191
							SCA:None<SCB:None	0.179

Note: Significant at: \* 10-percent level; \*\* 5-percent level; \*\*\* 1-percent level.

Table 2: Level of  $\varepsilon$ -Nash Convergence in Rounds 41–58

VARIABLES	(1) eNash	(2) eNashR	(3) eNashB
<b>Round</b>	0.010*** (0.002)	0.011*** (0.002)	0.010*** (0.003)
<b>Round<sup>2</sup></b>	-0.000** (0.000)	-0.000*** (0.000)	-0.000* (0.000)
<b>D<sub>SCA</sub></b>	0.000 (0.060)	-0.047 (0.054)	0.051 (0.089)
<b>D<sub>SCB</sub></b>	-0.015 (0.069)	-0.040 (0.074)	0.044 (0.071)
<b>Round × D<sub>SCA</sub></b>	0.003* (0.002)	0.004*** (0.002)	0.003 (0.003)
<b>Round × D<sub>SCB</sub></b>	0.006*** (0.002)	0.006** (0.003)	0.006** (0.002)
<b>D<sub>Best</sub></b>	0.094* (0.050)	0.156** (0.065)	0.108 (0.089)
<b>Round × D<sub>Best</sub></b>	0.005*** (0.002)	0.005* (0.002)	0.006 (0.004)
<b>D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.168*** (0.055)	-0.172* (0.089)	-0.210* (0.109)
<b>D<sub>SCB</sub> × D<sub>Best</sub></b>	-0.083 (0.090)	-0.056 (0.119)	-0.252*** (0.091)
<b>Round × D<sub>SCB</sub> × D<sub>Best</sub></b>	-0.005* (0.003)	-0.006** (0.003)	-0.006 (0.004)
<b>Round × D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.007** (0.003)	-0.008** (0.004)	-0.005 (0.004)
Observations	10800	10800	10800
Log pseudo-likelihood	-5329	-6792	-6706

*Notes:* Coefficients are probability derivatives. Robust standard errors in parentheses are adjusted for clustering at the session level. Significant at: \* 10-percent level; \*\* 5-percent level; \*\*\*1-percent level.

Table 3: Probit models of  $\varepsilon$ -Nash play. Model 1: Joint  $(\varepsilon - x_1^*, \varepsilon - x_2^*)$ ; Model 2: Type-1 player  $(\varepsilon - x_1^*)$ ; Model 3: Type-2 Player  $(\varepsilon - x_2^*)$

Panel A: Proportion Pareto Superior: Rounds 1–20										Permutation Tests	
Treatment	Session 1	Session 2	Session 3	Session 4	Session 5	Overall	$H_1$			p-value	
SCA:None	0.108	0.092	0.125	0.125	0.058	0.102	SCA:None<SCA:Best			0.020**	
SCA:Best	0.325	0.417	0.058	0.175	0.408	0.277					
MIX:None	0.050	0.117	0.067	0.117	0.058	0.082	MIX:Best<MIX:None			0.119	
MIX:Best	0.050	0.033	0.033	0.092	0.075	0.057	MIX:None<SCB:None			0.024**	
SCB:None	0.117	0.142	0.133	0.150	0.075	0.123	SCB:None<SCB:Best			0.417	
SCB:Best	0.058	0.175	0.083	0.108	0.233	0.132	MIX:Best<SCB:Best			0.024**	
Panel B: Proportion Pareto Superior: Rounds 21–40										Permutation Tests	
SCA:None	0.225	0.242	0.217	0.100	0.067	0.170	SCA:None<SCA:Best			0.048**	
SCA:Best	0.225	0.450	0.183	0.258	0.475	0.318					
MIX:None	0.075	0.150	0.033	0.092	0.217	0.113	MIX:Best<MIX:None			0.266	
MIX:Best	0.192	0.025	0.100	0.033	0.067	0.083	MIX:None<SCB:None			0.099*	
SCB:None	0.217	0.158	0.175	0.117	0.142	0.162	SCB:None<SCB:Best			0.183	
SCB:Best	0.025	0.317	0.258	0.192	0.267	0.212	MIX:Best<SCB:Best			0.040**	
Panel C: Proportion Pareto Superior: Rounds 41–58										Permutation Tests	
SCA:None	0.296	0.407	0.130	0.093	0.139	0.213	SCA:None<SCA:Best			0.115	
SCA:Best	0.065	0.463	0.370	0.278	0.509	0.337					
MIX:None	0.028	0.111	0.065	0.130	0.306	0.128	MIX:Best<MIX:None			0.016**	
MIX:Best	0.009	0.028	0.056	0.000	0.028	0.024	MIX:None<SCB:None			0.730	
SCB:None	0.296	0.065	0.222	0.120	0.130	0.167	SCB:None<SCB:Best			0.674	
SCB:Best	0.028	0.278	0.037	0.324	0.343	0.202	MIX:Best<SCB:Best			0.016**	

Note: Significant at: \* 10-percent level; \*\* 5-percent level; \*\*\* 1-percent level.

Table 4: Proportion of Outcomes Pareto Superior to Nash Equilibrium



VARIABLES	(1) BR1	(2) BR2	(3) eBR1	(4) eBR2
<b>Round</b>	0.002** (0.001)	0.001 (0.001)	0.002** (0.001)	0.002** (0.001)
<b>D<sub>SCA</sub></b>	0.072 (0.047)	-0.016 (0.052)	-0.021 (0.053)	0.049 (0.067)
<b>D<sub>SCB</sub></b>	-0.019 (0.056)	-0.034 (0.051)	-0.037 (0.059)	0.018 (0.069)
<b>Round × D<sub>SCA</sub></b>	-0.000 (0.001)	0.003*** (0.001)	0.003** (0.001)	0.003** (0.001)
<b>Round × D<sub>SCB</sub></b>	0.002 (0.001)	0.003** (0.001)	0.006*** (0.001)	0.006*** (0.002)
<b>D<sub>Best</sub></b>	0.008 (0.045)	-0.019 (0.057)	0.065 (0.064)	0.041 (0.072)
<b>Round × D<sub>Best</sub></b>	0.004*** (0.001)	0.004*** (0.001)	0.005*** (0.002)	0.006*** (0.002)
<b>D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.072 (0.047)	-0.004 (0.068)	-0.103 (0.082)	-0.114 (0.093)
<b>D<sub>SCB</sub> × D<sub>Best</sub></b>	0.035 (0.078)	-0.029 (0.066)	-0.019 (0.091)	-0.095 (0.101)
<b>Round × D<sub>SCB</sub> × D<sub>Best</sub></b>	-0.003* (0.001)	-0.004*** (0.002)	-0.005** (0.002)	-0.006** (0.002)
<b>Round × D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.004** (0.002)	-0.003* (0.002)	-0.007*** (0.002)	-0.006** (0.003)
<b>−SD3<sub>t</sub></b>	0.003 (0.006)	0.013** (0.007)	0.014** (0.007)	0.014** (0.006)
<b>−SD3<sub>t</sub> × D<sub>SCA</sub></b>	0.016* (0.009)	0.002 (0.009)	0.007 (0.010)	0.018* (0.011)
<b>−SD3<sub>t</sub> × D<sub>SCB</sub></b>	0.002 (0.012)	-0.007 (0.009)	0.011 (0.013)	0.006 (0.010)
<b>−SD3<sub>t</sub> × D<sub>Best</sub></b>	0.007 (0.009)	0.003 (0.010)	0.015 (0.012)	0.018 (0.013)
<b>−SD3<sub>t</sub> × D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.023* (0.013)	-0.013 (0.013)	-0.040** (0.017)	-0.046*** (0.017)
<b>−SD3<sub>t</sub> × D<sub>SCB</sub> × D<sub>Best</sub></b>	-0.012 (0.016)	-0.006 (0.013)	-0.030 (0.019)	-0.018 (0.018)
<b>BR<sub>t−1</sub></b>	0.181*** (0.016)	0.136*** (0.018)		
<b>ε−BR<sub>t−1</sub></b>			0.252*** (0.018)	0.182*** (0.017)
Observations	10260	10260	10260	10260
Log pseudo-likelihood	-4847	-4738	-6311	-6383

Notes: Coefficients are probability derivatives. Robust standard errors in parentheses are adjusted for clustering at the individual level. Significant at: \* 10-percent level; \*\* 5-percent level; \*\*\*1-percent level.

Table 5: Probit models of (ε−)best responding across all treatments.

VARIABLES	(1) BR1	(2) BR2	(3) eBR1	(4) eBR2
<b>Round</b>	0.007*** (0.001)	0.004*** (0.001)	0.007*** (0.001)	0.008*** (0.002)
<b>D<sub>SCA</sub></b>	-0.015 (0.040)	0.005 (0.045)	-0.146** (0.066)	-0.052 (0.066)
<b>D<sub>SCB</sub></b>	0.004 (0.053)	-0.035 (0.048)	-0.091 (0.069)	-0.036 (0.076)
<b>Round × D<sub>SCA</sub></b>	-0.003*** (0.001)	-0.001 (0.001)	-0.002 (0.002)	-0.002 (0.002)
<b>Round × D<sub>SCB</sub></b>	-0.002* (0.001)	0.001 (0.001)	-0.001 (0.002)	0.000 (0.002)
<b>−SD3<sub>t</sub></b>	0.011 (0.008)	0.015** (0.007)	0.029*** (0.010)	0.030*** (0.011)
<b>−SD3<sub>t</sub> × D<sub>SCA</sub></b>	-0.008 (0.011)	-0.010 (0.010)	-0.033** (0.014)	-0.026* (0.014)
<b>−SD3<sub>t</sub> × D<sub>SCB</sub></b>	-0.012 (0.011)	-0.012 (0.010)	-0.021 (0.014)	-0.009 (0.015)
<b>InfoQual<sub>t−1</sub></b>	0.030 (0.032)	0.093*** (0.028)	0.015 (0.042)	0.070* (0.037)
<b>InfoQual<sub>t−1</sub> × D<sub>SCA</sub></b>	0.015 (0.047)	-0.073* (0.043)	0.061 (0.056)	-0.037 (0.056)
<b>InfoQual<sub>t−1</sub> × D<sub>SCB</sub></b>	0.027 (0.040)	-0.093** (0.039)	0.082 (0.051)	-0.113** (0.051)
<b>BR<sub>t−1</sub></b>	0.187*** (0.023)	0.150*** (0.027)		
<b>ε − BR<sub>t−1</sub></b>			0.241*** (0.026)	0.192*** (0.023)
Observations	5130	5130	5130	5130
Log pseudo-likelihood	-2624	-2474	-3153	-3175

*Notes:* Coefficients are probability derivatives. Robust standard errors in parentheses are adjusted for clustering at the individual level. Significant at: \* 10-percent level; \*\* 5-percent level; \*\*\*1-percent level.

Table 6: Probit models of ( $\varepsilon$ -)best responding in Best Public treatments by player type.

Panel A:	Proportion of Nash Equilibrium choices ( $x_1^*, x_2^*$ )					Permutation Tests		
	Session 1	Session 2	Session 3	Session 4	Session 5	Overall	$H_1$	p-value
SCA:None	0.028	0.009	0.148	0.111	0.241	0.107	SCA:None<SCA:Best	0.560
SCA:Best	0.333	0.000	0.037	0.083	0.046	0.100	SCA:Best<MIX:Best	0.039**
MIX:None	0.037	0.037	0.130	0.028	0.000	0.046	MIX:None<SCA:None	0.151
MIX:Best	0.481	0.167	0.111	0.333	0.250	0.269	MIX:None<MIX:Best	0.004***
SCB:None	0.019	0.176	0.130	0.065	0.092	0.096	SCB:Best<MIX:Best	0.127
SCB:Best	0.435	0.000	0.231	0.019	0.019	0.141	MIX:None<SCB:None	0.087*
							SCB:None<SCB:Best	0.337
							SCA:Best<SCB:Best	0.353
							SCA:None<SCB:None	0.571
Panel B:	Proportion of Player 1 Nash equilibrium choices ( $x_1^*$ )					Permutation Tests		
SCA:None	0.130	0.046	0.426	0.361	0.389	0.270	SCA:None<SCA:Best	0.615
SCA:Best	0.519	0.102	0.139	0.315	0.093	0.162	SCA:Best<MIX:Best	0.012**
MIX:None	0.426	0.213	0.306	0.204	0.037	0.237	MIX:None<SCA:None	0.377
MIX:Best	0.787	0.444	0.426	0.500	0.565	0.544	MIX:None<MIX:Best	0.000***
SCB:None	0.185	0.546	0.296	0.222	0.370	0.324	SCB:Best<MIX:Best	0.071*
SCB:Best	0.694	0.046	0.546	0.111	0.185	0.317	MIX:None<SCB:None	0.182
							SCB:None<SCB:Best	0.508
							SCA:Best<SCB:Best	0.306
							SCA:None<SCB:None	0.298
Panel C:	Proportion of Player 2 Nash equilibrium choices ( $x_2^*$ )					Permutation Tests		
SCA:None	0.139	0.213	0.380	0.259	0.556	0.309	SCA:None<SCA:Best	0.714
SCA:Best	0.565	0.065	0.167	0.287	0.148	0.246	SCA:Best<MIX:Best	0.056*
MIX:None	0.102	0.065	0.296	0.204	0.019	0.137	MIX:None<SCA:None	0.040**
MIX:Best	0.602	0.250	0.306	0.630	0.435	0.444	MIX:None<MIX:Best	0.004***
SCB:None	0.074	0.315	0.324	0.361	0.352	0.285	SCB:Best<MIX:Best	0.111
SCB:Best	0.620	0.167	0.389	0.157	0.083	0.283	MIX:None<SCB:None	0.040**
							SCB:None<SCB:Best	0.500
							SCA:Best<SCB:Best	0.600
							SCA:None<SCB:None	0.386

Note: Significant at: \* 10-percent level; \*\* 5-percent level; \*\*\* 1-percent level.

Table 7: Level of Nash Convergence in Rounds 41–58

VARIABLES	(1) eNash	(2) eNashR	(3) eNashB
<b>Round</b>	0.002 (0.002)	0.010*** (0.002)	0.008*** (0.003)
<b>Round<sup>2</sup></b>	0.000 (0.000)	-0.000*** (0.000)	-0.000 (0.000)
<b>D<sub>SCA</sub></b>	-0.027 (0.035)	-0.043 (0.042)	0.039 (0.073)
<b>D<sub>SCB</sub></b>	-0.056 (0.042)	-0.037 (0.058)	0.039 (0.058)
<b>Round × D<sub>SCA</sub></b>	0.004 (0.003)	0.004** (0.002)	0.003 (0.003)
<b>Round × D<sub>SCB</sub></b>	0.008*** (0.002)	0.006** (0.002)	0.005** (0.002)
<b>D<sub>Best</sub></b>	0.014 (0.034)	0.148** (0.056)	0.104 (0.071)
<b>Round × D<sub>Best</sub></b>	0.008*** (0.002)	0.004** (0.002)	0.005* (0.003)
<b>D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.049 (0.047)	-0.162** (0.076)	-0.193* (0.099)
<b>D<sub>SCB</sub> × D<sub>Best</sub></b>	0.007 (0.059)	-0.063 (0.104)	-0.237** (0.088)
<b>Round × D<sub>SCB</sub> × D<sub>Best</sub></b>	-0.010** (0.004)	-0.006* (0.003)	-0.006 (0.004)
<b>Round × D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.011*** (0.004)	-0.007** (0.003)	-0.005 (0.003)
Constant	0.049** (0.021)	0.161*** (0.029)	0.162*** (0.048)
Observations	10800	10800	10800
Log pseudo-likelihood	-5516	-7136	-7046

*Notes:* Coefficients are probability derivatives. Robust standard errors in parentheses are adjusted for clustering at the session level. Significant at: \* 10-percent level; \*\* 5-percent level; \*\*\*1-percent level.

Table 8: OLS models of  $\epsilon$ -Nash play. Model 1: Joint  $(\epsilon - x_1^*, \epsilon - x_2^*)$ ; Model 2: Type-1 player  $(\epsilon - x_1^*)$ ; Model 3: Type-2 Player  $(\epsilon - x_2^*)$

VARIABLES	(1) BR1	(2) BR2	(3) eBR1	(4) eBR2
<b>Round</b>	0.002** (0.001)	0.001 (0.001)	0.002** (0.001)	0.002** (0.001)
<b>D<sub>SCA</sub></b>	0.045 (0.032)	-0.020 (0.041)	-0.022 (0.047)	0.045 (0.061)
<b>D<sub>SCB</sub></b>	-0.031 (0.045)	-0.046 (0.041)	-0.032 (0.054)	0.022 (0.063)
<b>Round × D<sub>SCA</sub></b>	0.000 (0.001)	0.003*** (0.001)	0.003** (0.001)	0.003** (0.001)
<b>Round × D<sub>SCB</sub></b>	0.002* (0.001)	0.003*** (0.001)	0.005*** (0.001)	0.005*** (0.001)
<b>D<sub>Best</sub></b>	-0.038 (0.039)	-0.043 (0.049)	0.070 (0.058)	0.047 (0.064)
<b>Round × D<sub>Best</sub></b>	0.006*** (0.001)	0.005*** (0.001)	0.004*** (0.001)	0.005*** (0.002)
<b>D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.021 (0.047)	0.021 (0.060)	-0.104 (0.075)	-0.117 (0.086)
<b>D<sub>SCB</sub> × D<sub>Best</sub></b>	0.059 (0.064)	0.004 (0.063)	-0.025 (0.083)	-0.100 (0.093)
<b>Round × D<sub>SCB</sub> × D<sub>Best</sub></b>	-0.005*** (0.002)	-0.006*** (0.002)	-0.004* (0.002)	-0.005** (0.002)
<b>Round × D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.005** (0.002)	-0.004** (0.002)	-0.005*** (0.002)	-0.005** (0.002)
<b>-SD3<sub>t</sub></b>	0.003 (0.004)	0.009** (0.005)	0.013** (0.006)	0.011** (0.005)
<b>-SD3<sub>t</sub> × D<sub>SCA</sub></b>	0.011* (0.006)	0.003 (0.007)	0.005 (0.008)	0.016* (0.009)
<b>-SD3<sub>t</sub> × D<sub>SCB</sub></b>	0.000 (0.009)	-0.005 (0.007)	0.009 (0.011)	0.006 (0.009)
<b>-SD3<sub>t</sub> × D<sub>Best</sub></b>	0.005 (0.008)	0.004 (0.008)	0.013 (0.011)	0.017 (0.011)
<b>-SD3<sub>t</sub> × D<sub>SCA</sub> × D<sub>Best</sub></b>	-0.016 (0.010)	-0.012 (0.011)	-0.034** (0.015)	-0.041*** (0.015)
<b>-SD3<sub>t</sub> × D<sub>SCB</sub> × D<sub>Best</sub></b>	-0.009 (0.013)	-0.007 (0.011)	-0.024 (0.016)	-0.016 (0.016)
<b>BR<sub>t-1</sub></b>	0.189*** (0.016)	0.146*** (0.019)		
<b>ε - BR<sub>t-1</sub></b>			0.242*** (0.018)	0.172*** (0.016)
Constant	0.071*** (0.024)	0.112*** (0.032)	0.256*** (0.036)	0.255*** (0.043)
Observations	10260	10260	10260	10260
Log pseudo-likelihood	-4870	-4712	-6626	-6704

*Notes:* Coefficients are probability derivatives. Robust standard errors in parentheses are adjusted for clustering at the individual level. Significant at: \* 10-percent level; \*\* 5-percent level; \*\*\*1-percent level.

VARIABLES	(1) BR1	(2) BR2	(3) eBR1	(4) eBR2
<b>Round</b>	0.007*** (0.001)	0.005*** (0.001)	0.006*** (0.001)	0.007*** (0.001)
<b>D<sub>SCA</sub></b>	0.021 (0.035)	0.030 (0.041)	-0.149** (0.060)	-0.062 (0.059)
<b>D<sub>SCB</sub></b>	0.014 (0.046)	-0.006 (0.045)	-0.089 (0.064)	-0.044 (0.068)
<b>Round × D<sub>SCA</sub></b>	-0.004*** (0.001)	-0.002 (0.001)	-0.001 (0.001)	-0.002 (0.002)
<b>Round × D<sub>SCB</sub></b>	-0.003* (0.001)	-0.000 (0.001)	-0.001 (0.001)	0.001 (0.002)
<b>−SD3<sub>t</sub></b>	0.008 (0.007)	0.010 (0.006)	0.026*** (0.009)	0.026*** (0.009)
<b>−SD3<sub>t</sub> × D<sub>SCA</sub></b>	-0.006 (0.009)	-0.007 (0.008)	-0.030** (0.012)	-0.023* (0.012)
<b>−SD3<sub>t</sub> × D<sub>SCB</sub></b>	-0.010 (0.009)	-0.009 (0.009)	-0.018 (0.012)	-0.008 (0.013)
<b>InfoQual<sub>t−1</sub></b>	0.032 (0.031)	0.105*** (0.030)	0.010 (0.035)	0.056* (0.033)
<b>InfoQual<sub>t−1</sub> × D<sub>SCA</sub></b>	0.004 (0.043)	-0.087** (0.043)	0.062 (0.048)	-0.025 (0.051)
<b>InfoQual<sub>t−1</sub> × D<sub>SCB</sub></b>	0.025 (0.040)	-0.107*** (0.040)	0.077* (0.043)	-0.093** (0.046)
<b>BR<sub>t−1</sub></b>	0.191*** (0.022)	0.160*** (0.027)		
<b>ε − BR<sub>t−1</sub></b>			0.232*** (0.025)	0.182*** (0.022)
<b>Constant</b>	0.025 (0.030)	0.032 (0.034)	0.328*** (0.046)	0.279*** (0.046)
<b>Observations</b>	5130	5130	5130	5130
<b>Log pseudo-likelihood</b>	-2704	-2503	-3312	-3336

*Notes:* Coefficients are probability derivatives. Robust standard errors in parentheses are adjusted for clustering at the individual level. Significant at: \* 10-percent level; \*\* 5-percent level; \*\*\*1-percent level.

Table 10: OLS models of (ε−)best responding in Best Public treatments by player type.