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Journal of Monetary Economics 52 (2005) 3–29

Journal of
MONETARY
ECONOMICS

www.elsevier.com/locate/econbase

Rational speculation and exchange rates[☆]

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Received 23 April 2004; received in revised form 4 August 2004; accepted 4 August 2004

Available online 13 December 2004

Abstract

Models of exchange rates have typically failed to produce results consistent with the key fact that real and nominal exchange rates move in ways *not* closely connected to current (or past) macroeconomic variables. Models that rely on the same shocks to drive fluctuations in macroeconomic variables and exchange rates typically imply counterfactually-strong comovements between them. We develop a model in which new information leads agents to change their rational beliefs about risk premia on foreign exchange markets. These changes in risk premia work through asset markets to cause real and nominal exchange rates to change *without* corresponding changes in GDP, productivity, money supplies, and other key macro variables.

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JEL classification: F31; F41; G15

Keywords: Exchange rates; Risk premium; Speculation; Exchange rate disconnect

[☆]We thank Olivier Jeanne, Maurice Obstfeld, Kenneth Rogoff and seminar participants at various universities and conferences for their comments and suggestions. All errors are our own. The views expressed in this paper are those of the authors and not necessarily represent those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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1. Introduction

The purpose of this paper is to propose a new approach to explaining exchange rates and to implement this approach within a specific model. A new approach is needed because old approaches have failed:

(1) Purchasing power parity performs miserably, so it cannot play any key role in a satisfactory explanation of exchange rates.

(2) Exchange rates fail to follow the strong cyclical patterns implied by most models in which the *same* shocks (whether monetary or real) drive both business cycles and exchange rates. In standard exchange-rate models based on sticky prices and monetary shocks, a monetary expansion simultaneously raises domestic real GDP (by more than it raises foreign GDP) and creates a (temporary) depreciation of home currency. Consequently, these models almost generically imply a strong positive correlation between depreciations and (relative) business-cycle booms.¹ However, the data show almost *no* relationship of any kind between exchange rates and ratios of business cycles.² Dynamic general equilibrium models based on technology shocks have the same kind of problem. A model that tries to generate both business cycles and exchange-rate changes with technology shocks almost invariably implies counterfactual correlations between exchange rates and business cycles.³

(3) The behavior of exchange rates seems not to matter much for the behavior of other macroeconomic variables. As [Baxter and Stockman \(1989\)](#) have documented, the exchange rate regime seems to have little systematic effects on the business cycle properties of macroeconomic aggregates, aside from the real exchange rate. In fact, as [Flood and Rose \(1995\)](#) have elegantly shown, the exchange rate appears to have “a life of its own,” disconnected from other macroeconomic variables. Like stock prices or other asset prices, exchange rates show little relation to current or past macroeconomic variables or international-trade variables. The new approach we propose in this paper is intended to address this puzzling fact.

(4) The pioneering work of Richard Meese and Ken Rogoff two decades ago ([Meese and Rogoff, 1983](#)) has held up remarkably well: with minor caveats, a simple random-walk model of exchange rates forecasts as well or better than alternative statistical models or statistical implementations of existing economic models.

¹[Chari et al. \(2002\)](#), for example, develop a quantitative, calibrated model of exchange rates based on monetary shocks operating through sticky prices. While their model succeeds in matching many features of the data, it implies a strongly counterfactual relation between exchange rates and international ratios of (detrended) GDP or consumption.

²See [Stockman \(1998\)](#). [Leonard and Stockman \(2002\)](#) document the joint statistical behavior of exchange rates and GDP-ratios in a nonparametric framework.

³These are far from the only serious criticisms of standard exchange-rate models. For example, identification of monetary shocks, like identification of technology shocks, poses a key problem for models of exchange rates as well as for models of business cycles. Sticky-price models based on monetary shocks must confront the persistence puzzle: the problem that half-lives of exchange rates (even when statistically detrended in ways that reduce their half-lives!) are far longer than half-lives of business cycles (while the models imply that they should be about the same).

(5) Practitioners in foreign-exchange markets typically believe that large exchange-rate swings are not justified by fundamentals. Most attribute those swings to speculation, with associated changes in interest rates and risk premia. Speculation in foreign exchange markets involves sales or purchases of interest-bearing assets denominated in some currency (with opposite transactions in assets denominated in another currency). When speculators sell yen-denominated assets to buy Euro-denominated assets, interest rates rise on yen-denominated assets and fall on Euro-denominated assets, and these interest-rate changes can be interpreted as representing changes in risk premia. (Although the nominal return on a short-term treasury bill may be riskless, its real return is not.)

We propose a new approach that focuses on the effects of speculation—perhaps rationally reflecting new information or perhaps “irrationally” exuberant or fearful—and the resulting changes in risk premia, by returning to the old idea that exchange rates are determined in asset markets. This approach combines the idea of international segmentation in product markets as in Dumas (1992), Sercu et al. (1995), Ohanian and Stockman (1997), Sercu and Uppal (2000), and Obstfeld and Rogoff (2000), with incomplete international risk sharing. Loosely, product market segmentation and incomplete risk sharing each eliminate “marginal rate of substitution equals relative price” conditions that would otherwise bind real exchange rates to contemporaneous *product market* conditions.⁴ Consequently, these features of the model allow asset markets to determine the (expected) *growth rate* of the exchange rate (through a forward-looking stochastic difference equation) and eliminate all equilibrium conditions that bind changes in exchange rates to changes in other contemporaneous variables.⁵

Breaking the strong link between product markets and exchange rates opens the possibility for speculative activities on asset markets to play a key role in exchange-rate determination. While traditional macroeconomic forcing variables, such as monetary shocks and productivity shocks, can (and must) play roles in the model, this framework naturally focuses attention on shocks that affect risk premia—and expectations—and that can potentially affect exchange rates without much effect on other macroeconomic aggregates. While models based solely on shocks to macroeconomic fundamentals must essentially *ignore* the Flood–Rose critique (or

⁴Models of exchange rates derived from individual optimization typically involve optimization conditions that equate the real exchange rate (defined as the relative price of foreign to domestic goods) with a marginal rate of substitution. In fact, that condition played the central role in the equilibrium approach to exchange rates proposed in Stockman (1980) and developed in Lucas (1982) and Svensson (1985). Also, complete international risk sharing implies the equalization of the real exchange rate with the ratio of marginal utilities of consumption across countries. Substantial evidence, however, has made it clear that these equations are grossly at variance with the data. Any good theoretical explanation of exchange rates must find a way to *avoid* these conditions.

⁵Asset markets alone, however, do not tie down the *level* of the exchange rate path. The level of the exchange rate depends on other features of the model, including the possibility of future product-market arbitrage (which puts endpoint restrictions on the forward-looking difference equation) and wealth effects of exchange rate changes (operating through household wealth constraints) that affect the marginal utilities that play the role of “parameters” in the difference equation.

what Obstfeld and Rogoff, 2000, term the “exchange-rate disconnect puzzle”), a focus on speculation raises the possibility of explaining that disconnect.

This paper implements these ideas in a stochastic two-country general equilibrium model built along the lines of Svensson and Van Wijnbergen (1989) and Obstfeld and Rogoff (1995) in which firms in each country set prices in advance in the buyer’s currency and consumers are restricted to trading discount bonds only. The model is driven by shocks to the money supplies and productivity levels, and we consider a process for these shocks characterized by two regimes that differ in their second moments. We address the question of whether rational speculation and the associated changes in risk premia brought about by changes in the regime of the economy, can generate changes in exchange rates without much response in other macroeconomic aggregates.

Our motivation for focusing on speculation derives from both casual observation evidence and statistical evidence. Within two years after the Euro was introduced in January 1999 with high expectations and great fanfare, it had lost 30 percent of its value against the U.S. dollar. The Euro fell from \$1.18 in January 1999 to less than \$0.83 in October 2000, before rising again to \$0.95 three months later, in January 2001. Other seemingly inexplicable changes in exchange rates are almost daily occurrences. From 1991 to 1995, the Japanese yen rose in value from 130 yen per U.S. dollar to less than 90 in 1995, then fell to more than 140 yen per dollar in 1998 before rising to 101 in December 1999 and then falling to almost 120 by March 2001. Is there a reasonable explanation of such episodes that does *not* rely upon speculation?

Risk premia on forward foreign-exchange markets are highly variable. Unlike exchange rates, evidence suggests that risk premia have predictable components and are strongly correlated with expected changes in exchange rates.⁶ Perhaps participants on foreign exchange markets have rational expectations and the compensation that they require for bearing risk corresponds to standard finance models (e.g. Hodrick, 1987). Or, perhaps, foreign exchange markets are dominated by noise traders or other irrational speculators (e.g. Krugman, 1989, or Krugman and Miller, 1993) whose actions implicitly reflect changes in risk premia that have little to do with those models. Regardless, the variation in implied risk premia is substantial.

The idea that speculation plays a key role in exchange-rate volatility is, of course, not new. Krugman (1989) and Krugman and Miller (1993), for example, have

⁶Backus et al. (1993) estimated that standard deviations of the predictable components of the excess return from currency speculation, interpretable as risk premia, average about 0.7–0.8% per month. Updating their results through 2000 produces similar (and slightly higher) standard deviations estimates of variability in risk premia. Similarly updated estimates show that the forward-premium puzzle is alive and well, implying (by Fama’s 1984, argument) a strong negative correlation between the risk premium and the expected change in the exchange rate (expected depreciation), implying that currencies perceived to be riskier than others are more likely to be expected to appreciate. This strong relation between the risk premium and expected depreciation contrasts markedly the absence of relationships between exchange rates and other macro variables that Flood and Rose document, and the predictability of the risk premium contrasts markedly with the lack of much predictability of exchange rates by the variables that standard models suggest.

stressed the role of speculation in exchange-rate behavior.⁷ Krugman and Miller, for example, argue that “the real-world case for exchange-rate stabilization has always rested on fears of excessive speculation, not on the microeconomic concerns of the optimal-currency-area approach.” Assuming an exchange-rate equation that depends, among other variables, on a term that reflects the foreign-exchange risk premium, they discuss a “decision by... investors to shift from domestic- to foreign-currency assets” in response to an exogenous (non-rational) change in the risk premium. Although Krugman and Miller characterize speculation as “irrational,” their argument and the equations they use to explain it apply equally to “rational” speculation that reflects new information relevant to the size of the risk premium.

The effects of (rational) time-varying risk premia on exchange rates have also been studied in general equilibrium models.⁸ Hodrick (1989) showed how the second moments of exogenous variables affect risk premia and exchange rates in a flexible price model. More recently, Obstfeld and Rogoff (2002) have studied the role of risk in an explicitly stochastic sticky-price model and show analytically that the model allows for large effects of risk premia on exchange rates. In their model, however, changes in exchange rates induced by time-varying risk premia are necessarily correlated with changes in other macro aggregates through the optimizing conditions for consumers. In a general equilibrium framework similar to ours, Devereux and Engel (2002) have explored the role of noise traders in foreign exchange markets in explaining the high volatility of exchange rates and their disconnect from other macroeconomic variables. In their model, exogenous shocks to the beliefs of foreign exchange traders generate what can be interpreted as a time-varying exogenous (non-rational) risk premium. In our model, changes in risk premia are endogenous and rationally calculated.

2. The model

The world economy consists of two countries, denominated home and foreign, each specialized in the production of a composite traded good. We assume that the two markets are segmented and that monopolistically competitive firms in each country set prices one period in advance in the currency of the buyer (local currency pricing). We also assume that asset markets are incomplete and restrict households to trade a riskless nominal bond denominated in home currency and a riskless nominal bond denominated in foreign currency.⁹ These features of the model allow

⁷Of course, the idea that exchange rates reflect expectations about future fundamentals is commonplace in exchange-rate models, and goes back at least to the more sophisticated monetary models of exchange rates in the 1970s, such as Hodrick (1978), and to Dornbusch’s famous overshooting model (Dornbusch, 1976).

⁸Abel (1988) and others have studied the effects of time-varying risk premia on asset prices more generally.

⁹This setup, which involves *complete* segmentation of goods’ markets, is an extreme version of the incomplete-markets model in Ohanian and Stockman (1997), which combines the iceberg-cost model of segmentation (Sercu and Uppal, 2000) with shifts in risk premia.

for expectations about period $t + 1$ variables to affect exchange rates in period t directly, *aside* from any interest-rate effects on money-demand. In this paper we explore whether new information revealed in period t that affects expectations about $t + 1$ variables can induce a change in period- t exchange rate *without* creating large changes in other period- t macroeconomic variables.

The model is driven by exogenous shocks to the growth rate of the money supply and productivity levels in each country. We consider two regimes for the exogenous processes, which differ in the pattern of covariances between money and productivity shocks in each country. In what follows, we describe the home country's economy. The foreign country's economy has an identical structure, with all foreign variables denoted by an $*$.

2.1. Households

2.1.1. Preferences

The lifetime expected utility of the home representative household is

$$U_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(c_t, l_t, \frac{M_{t+1}}{P_t} \right) \right],$$

where E_0 denotes the mathematical expectation conditional on information available in period $t = 0$ and $\beta \in (0, 1)$ is the discount rate. The momentary utility function u depends on c_t , an index of consumption to be defined below, on labor effort l_t , and on real money balances held from period t to period $t + 1$, M_{t+1}/P_t .

2.1.2. Consumption and price indices

There is a continuum of domestic goods indexed by $i \in [0, 1]$ and a continuum of foreign goods indexed by $j \in [0, 1]$, which are imperfect substitutes in consumption. The consumption index c_t is defined as

$$c_t = \left[\omega^{1/\gamma} c_{h,t}^{(\gamma-1)/\gamma} + (1 - \omega)^{1/\gamma} c_{f,t}^{(\gamma-1)/\gamma} \right]^{\gamma/(\gamma-1)}, \quad \gamma > 0 \quad \text{and} \quad \omega \in (0, 1),$$

where $c_{h,t}$ and $c_{f,t}$ represent date- t consumption of home and foreign composite goods, respectively. The parameter γ represents the elasticity of substitution between the two composite goods and the weight ω determines the household's bias for the domestic composite good. The home and foreign composite goods $c_{h,t}$ and $c_{f,t}$ are defined as

$$c_{h,t} = \left(\int_0^1 c_{h,t}(i)^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)} \quad \text{and} \quad c_{f,t} = \left(\int_0^1 c_{f,t}(j)^{(\theta-1)/\theta} dj \right)^{\theta/(\theta-1)}, \quad \theta > 1,$$

where $c_{h,t}(i)$ and $c_{f,t}(j)$ denote date- t domestic consumption of home and foreign goods of type i and j , respectively. The parameter θ denotes the elasticity of substitution between any two goods produced in the same location.

Let $P_{h,t}(i)$ and $P_{f,t}(j)$ denote the home-currency prices of home and foreign goods of type i and j , respectively. Given these prices, the consumption-based money price

index P_t associated with the consumption indices defined above is given by

$$P_t = [\omega P_{h,t}^{1-\gamma} + (1 - \omega) P_{f,t}^{1-\gamma}]^{1/(1-\gamma)},$$

where the price indexes $P_{h,t}$ and $P_{f,t}$ for each composite good are given by¹⁰

$$P_{h,t} = \left(\int_0^1 P_{h,t}(i)^{1-\theta} di \right)^{1/(1-\theta)} \quad \text{and} \quad P_{f,t} = \left(\int_0^1 P_{f,t}(i)^{1-\theta} di \right)^{1/(1-\theta)}.$$

Taking prices for all individual goods as given, each period the consumer allocates optimally a given level of total consumption among the differentiated goods. This static allocation problem yields the demand functions

$$c_{h,t}(i) = \omega \left(\frac{P_{h,t}}{P_{h,t}(i)} \right)^\theta \left(\frac{P_t}{P_{h,t}} \right)^\gamma c_t \tag{2.1}$$

and

$$c_{f,t}(j) = (1 - \omega) \left(\frac{P_{f,t}}{P_{f,t}(j)} \right)^\theta \left(\frac{P_t}{P_{f,t}} \right)^\gamma c_t. \tag{2.2}$$

2.1.3. The budget constraint

Home and foreign households can trade nominally riskless discount bonds denominated in home and foreign currencies. Let Q_t denote the price in period t (in home currency units) of one discount bond paying with certainty one unit of home currency at $t + 1$, and let D_{t+1} denote the number of these bonds held by the home household between time t and $t + 1$. Similarly, let Q_t^* denote the price at time t (in foreign currency units) of one discount bond paying with certainty one unit of foreign currency at $t + 1$ and let B_{t+1} denote the number of these bonds held by the home household between time t and $t + 1$. To rule out equilibria which admit unbounded borrowing, or Ponzi schemes, we impose exogenous upper bounds, a_t and a_t^* , on the number of one-period bonds that a household can issue.¹¹

The household's intertemporal budget constraint, in units of home currency, is

$$P_t c_t + M_t + Q_t D_{t+1} + e_t Q_t^* B_{t+1} \leq P_t w_t l_t + M_{t-1} + D_t + e_t B_t + \Pi_t + P_t T_t, \tag{2.3}$$

where T_t denotes real transfers from the government (which can be negative in the case of taxes), Π_t represents profits of domestic firms (which we assume to be owned by the domestic household) and $P_t w_t l_t$ represents nominal labor earnings.

¹⁰The price index P is defined as the minimum expenditure necessary to buy one unit of composite good c , taking as given the prices P_h and P_f . These price indices have analogous interpretations. See Obstfeld and Rogoff (1996, Chapter 10) for a derivation of these price indices, as well as for a derivation of the demand functions (2.1) and (2.2).

¹¹The borrowing constraint is time dependent, reflecting the fact that the model is non-stationary. We assume that the borrowing constraint is constant in the stationary version of the model.

The household's optimization problem is summarized by

$$\max_{c_t, l_t, D_{t+1}, B_{t+1}, M_{t+1}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(c_t, l_t, \frac{M_{t+1}}{P_t} \right) \right] \quad (2.4)$$

subject to the budget constraint (2.3), the borrowing constraints, $D_{t+1} \geq -a_t$ and $B_{t+1} \geq -a_t^*$, and taking D_0 , B_0 , and M_0 as given.

2.2. Firms and market structure

Firm i produces its good according to the production function $z_t l_t(i)$, where $l_t(i)$ represents labor input and z_t is an aggregate (country-specific) productivity shock. Because all goods are imperfect substitutes in consumption, each individual firm has some market power determined by the price elasticity of demand, θ .

We assume that, due to high costs of arbitrage to consumers, each individual monopolist can price-discriminate across countries. Furthermore, we assume that firms set prices in the currency of the buyer one period in advance.¹²

The price setting problem of monopolist i is then to maximize expected profits conditional on $t-1$ information, by choosing $P_{h,t}(i)$ and $P_{h,t}^*(i)$, i.e. firm i solves

$$\max_{P_{h,t}(i), P_{h,t}^*(i)} E_{t-1} [\rho_t \Pi_t(i)] \quad (2.5)$$

subject to $z_t l_t(i) = c_{h,t}(i) + c_{h,t}^*(i)$ and the downward sloping demand functions for $c_{h,t}(i)$ and $c_{h,t}^*(i)$. The term ρ_t denotes the pricing kernel used to value date- t profits, which are random as of $t-1$. Date- t profits of monopolist i (in home currency units), $\Pi_t(i)$, are given by

$$\Pi_t(i) = P_{h,t}(i) c_{h,t}(i) + e_t P_{h,t}^*(i) c_{h,t}^*(i) - P_t w_t l_t(i),$$

where w_t denotes real wages in units of consumption good c . Note that $P_{h,t}(i)$ and $P_{h,t}^*(i)$ are denominated in units of home and foreign currency, respectively. The country's nominal exchange rate in period t , e_t , converts the revenues from sales in the foreign country into home currency.

2.3. Government

The government issues the local currency, has no expenditures and runs a balanced budget every period. Therefore, nominal transfers are given by

$$P_t T_t = M_{t+1}^s - M_t^s,$$

¹²This assumption is usually referred to in the literature as pricing-to-market. This assumption implies that, on impact, unanticipated changes in the nominal exchange rate do not affect consumer prices. This implication of the model, albeit extreme, is broadly consistent with empirical evidence suggesting a low short-run pass-through of exchange rate changes to consumer prices. See Engel (2002), for example, for a discussion and references on this topic. Obstfeld and Rogoff (2000) argue that wholesale import prices exhibit higher exchange rate pass-through than consumer prices.

where M_t^s is the domestic money stock in period t . This stock is stochastic and evolves according to

$$M_{t+1}^s = (1 + g_t)M_t^s,$$

where g_t is a random variable to be described later.

3. Bond markets, risk premia, and the disconnect puzzle

Home and foreign agents trade both bonds and goods in international markets. The first-order conditions for bonds denominated in home currency are given by

$$Q_t \lambda_t \geq \beta E_t[\lambda_{t+1}] \tag{3.1}$$

and

$$Q_t \frac{\lambda_t^*}{e_t} \geq \beta E_t \left[\frac{\lambda_{t+1}^*}{e_{t+1}} \right], \tag{3.2}$$

where λ_t represents the nominal marginal utility of consumption of the home household, $u_{c,t}/P_t$, and λ_t^* is the analogous term for the foreign household.¹³ Similarly, the first-order conditions for foreign bond holdings are given by

$$e_t Q_t^* \lambda_t \geq \beta E_t[e_{t+1} \lambda_{t+1}] \tag{3.3}$$

and

$$Q_t^* \lambda_t^* \geq \beta E_t[\lambda_{t+1}^*]. \tag{3.4}$$

As can be seen from Eqs. (3.2) and (3.3), the current cost of holding an additional bond denominated in home (foreign) currency for the foreign (home) agent depends directly on the nominal exchange rate. Therefore, the optimization conditions for bond holdings imply that the nominal exchange rate in period t depends explicitly on expectations of next period’s variables (which determine next period’s benefit from holding an additional bond). This dependence of the nominal exchange rate on expectations is implied by the optimization conditions in bond markets and it is, therefore, distinct from the usual dependence of exchange rates on expectations operating through nominal interest rates on the demand for money (as in “monetary models” of exchange rates).

Unlike the optimization conditions for bond holdings, the optimization conditions for goods do not depend directly on the exchange rate since prices are set one period in advance in the buyer’s currency (and hence, do not respond to unanticipated changes in the nominal exchange rate). In fact, in our model, the only equilibrium

¹³Note that Eqs. (3.1) and (3.2) hold with equality when the corresponding household is not borrowing constrained. Because there is no upper bound on the quantity of home bonds that can be purchased, and this asset has zero net supply worldwide, it follows that at all times at least one of the two households is not borrowing constrained. Therefore, at least one of these two equations always holds with equality, allowing us to always determine the bond price, Q_t . A similar argument applies to Q_t^* and Eqs. (3.3) and (3.4).

conditions that depend on the nominal exchange rate at date t are the Euler equations for bond holdings and the household's budget constraint. Therefore, changes in expectations about future variables that translate into changes in the current nominal exchange rate do not necessarily imply changes in other macroeconomic variables (apart from wealth effects from exchange rate changes operating through the budget constraint), i.e. shocks that affect expectations have the potential to affect current exchange rates with little effect on other macroeconomic aggregates and, hence, to be an important aspect in solving what Obstfeld and Rogoff (2000) term the disconnect puzzle.

We can see now why asset market incompleteness and product market segmentation (together with local currency pricing) are critical features in addressing the exchange rate disconnect puzzle. First, consider asset markets. If the model were to include a complete set of state-contingent nominal assets, then complete risk-sharing across countries implies that $e_t = (P_t/u_{1,t})(u_{1,t}^*)(P_t^*)$.¹⁴ This condition equates the ratio of nominal marginal utilities of consumption across countries to the nominal exchange rate and it ties exchange rate movements with movements in other macroeconomic variables, namely, the consumption ratio across countries. Now, turn to product market segmentation and firm's pricing. If product markets were not segmented, or if product markets were segmented but prices were pre-set in the seller's currency, then agents would arbitrage away any price differentials and the law of one price would hold for every good. The law of one price implies, as Eqs. (2.1) and (2.2) show, that the ratio of imports to consumption of domestic goods is a function of the nominal exchange rate. In this case, exchange rate movements would be directly tied with movements in the ratio of imports to domestic goods. In short, when either asset markets are complete or goods markets are integrated, the nominal exchange rate is tied down by additional equilibrium conditions that do not appear in our model. These additional equilibrium conditions would imply that changes in exchange rates would be directly tied to changes in other macroeconomic variables, such as relative consumption across countries or the ratio of imports to consumption of domestic goods.

We now rewrite the optimization conditions for bond holdings. Combining Eqs. (3.3) and (3.4), it follows that (in an interior solution)

$$e_t = \frac{\lambda_t^*}{\lambda_t} \frac{E_t[e_{t+1}\lambda_{t+1}]}{E_t[\lambda_{t+1}^*]}. \quad (3.5)$$

The forward exchange rate f_t , must satisfy covered interest parity (which is a no-arbitrage condition) and is given by $f_t = e_t Q_t^*/Q_t$. Making use of Eqs. (3.5), (3.1) and (3.4) it follows that the forward price of foreign exchange is given by

$$f_t = \frac{E_t[e_{t+1}\lambda_{t+1}]}{E_t[\lambda_{t+1}]}.$$

¹⁴See Sercu and Uppal (2000).

Let us now define the risk premium on foreign assets (for the home household) as $rp_t = f_t - E[e_{t+1}]$. Eq. (3.5) can then be rewritten as

$$e_t = \frac{Q_t}{Q_t^*} (rp_t + E_t[e_{t+1}]). \quad (3.6)$$

Finally, taking into account that prices are set one period in advance, it follows that the risk premium is given by

$$rp_t = \frac{cov_t(e_{t+1}, u_{c,t+1})}{E_t[u_{c,t+1}]} \quad (3.7)$$

Eq. (3.6) is a first-order stochastic difference equation for the exchange rate, roughly showing that its expected growth rate depends on the home household's perception of the relative risk of holding the two nominal assets rp_t , normalized by the level of the exchange rate. Moreover, new information that leads households to revise their perceptions of risk can potentially affect the expected growth rate of the exchange rate *without*, in principle, affecting (much) current macroeconomic variables. That observation is the source for optimism that an approach to exchange rates based on speculation can avoid the exchange-rate-disconnect puzzle. Note that if the remaining portion of the model implied a terminal condition for Eq. (3.6), then such a revision in perceptions of risk would change the current level of the exchange rate so that it could subsequently rise or fall—at the rate required by Eq. (3.6)—toward that terminal condition. The currency might depreciate on impact so that it could be expected to appreciate over time at a rate that compensates investors for the change in relative risk.¹⁵ Brennan and Xia (2004) present evidence that risk premia in foreign exchange markets vary over time and move with the corresponding risk premia in bond markets. Moreover, they show that when pricing kernel volatilities are included in a regression of the (differenced, log) exchange rate on the lagged forward premium, those pricing kernel volatilities enter significantly and help explain changes in exchange rates.

Empirically, the measured size of variation in the risk premium, while large, is smaller than variation in exchange rates themselves. Estimates of standard deviations of U.S. dollar risk premia from Backus et al. (1993) range from 0.36 percent per month against the Canadian dollar to 0.93 percent per month against the British pound, with a mean standard deviation of 0.70 percent per month and a median of 0.78. These high standard deviations generate large expected returns to speculators. The simple foreign-exchange investment strategy discussed by Backus et al. based solely upon the sign of the forward premium, $(f_t - e_t)/e_t$, yields Sharpe ratios (ratios of mean returns to standard deviations of returns) ranging from 0.17 to 0.29, significantly higher than Sharpe ratios of around 0.14 for investments in the stock market. The first-order stochastic difference equation for the exchange rate generated by the model has the potential to amplify this variation in risk premia,

¹⁵The first-order stochastic difference Eq. (3.5) describes the expected growth rate of the exchange rate. Appendix A discusses how the remainder of the model interacts with this equation to determine the *levels* of current and expected future exchange rates.

depending on how the other terms in that equation covary with the risk premium. Apparently, some amplification of this variability will be essential for a successful explanation of exchange-rate variability based on changes in speculators' perceptions of risk.

3.1. Generating changes in risk premia

We now introduce exogenous shocks with time-varying second moments. Changes in second moments cause endogenous changes in the risk premium. We consider two possible regimes for the shocks to money growth and productivity. These regimes, denoted by Ω_t , are characterized by different second moments and imply different levels of the risk premium. We assume that the regime variable Ω_t follows a Markov process.

As Eq. (3.7) shows, the foreign exchange risk premium arises from the covariance between the nominal exchange rate and the marginal utility of consumption. When next period's covariance between the nominal exchange rate and the marginal utility of consumption is high, the foreign bond tends to pay a high (low) real return when the marginal utility of consumption is also high (low). Therefore, the foreign bond is relatively more risky to the home agent the lower the covariance between e_{t+1} and $u_{c,t+1}$ is.

Because prices are sticky in our model, monetary shocks alone affect both consumption and the nominal exchange rate and, therefore, can generate a foreign exchange risk premium.¹⁶ Furthermore, the size of the risk premium generated by shocks to home and foreign money supplies depends on the covariance of these shocks. In the extreme case in which the two countries are in a fixed exchange rate regime, the foreign exchange risk premium is zero. In a flexible exchange rate regime, however, the covariance between the nominal exchange rate and the marginal utility of consumption of the home agent is negative (and the risk premium is negative). Therefore, when the economy switches to a regime with a lower covariance between money supplies across countries, the variability of the nominal exchange rate increases and the bond denominated in each agent's own currency becomes less risky relative to the alternative asset, i.e. the bond denominated in home (foreign) currency becomes relatively less risky to the home (foreign) agent than the bond denominated in foreign (home) currency. This experiment captures the idea of a portfolio shift towards domestic assets for both agents.

We are interested in a regime shift that affects *all* households' perceptions of the relative risk in the same direction and that leads to attempted portfolio shifts for all households towards one currency (a "safe heaven" effect on one currency). We, therefore, model regime shifts as changes in the covariance between monetary shocks and productivity shocks within a country (holding variances and cross-country covariances fixed). While the model outlined above includes completely-predetermined nominal prices, shifts in the within-country covariance of monetary and

¹⁶Engel (1999) studies the behavior of the risk premium in a two-country general equilibrium model with sticky nominal prices and monetary shocks.

productivity shocks can change risk premia only if prices are *not completely* predetermined. Consequently, we modify the model so that prices are only partially preset. Our formulation approximates a model in which a subset of firms can re-adjust prices after the realization of current shocks. Details appear in Appendix B.

To implement the idea that exchange rates respond to changes in asset market conditions we assume that agents observe the regime variable Ω_t at the beginning of period t and that the regime variable is persistent. Therefore, a change in the regime variable in period t affects both the current regime and the probability distribution of next period's variables. The regime variable conveys information about the future because it is persistent; consequently, changes in regime affect the risk premia set in asset markets.¹⁷ Below, we measure the effects on current nominal and real exchange rates and other macroeconomic variables of a change in the regime variable. The particular relation between the regime variable and the covariance structure of productivity and money shocks that we study in this paper is described in Section 4. Finally, in the numerical implementation, we restrict agents to trade internationally only the bond denominated in foreign currency.¹⁸ Since all agents in the home country are identical, holdings of the bond denominated in home currency are zero in equilibrium (that is, $D_{t+1} = 0, \forall t$) but its equilibrium price is still given by Eq. (3.1).

We focus on the symmetric and stationary equilibrium. Consequently, all firms located in the same country make the same choices (and therefore we drop the firm- and goods-index henceforth) and endogenous variables are stationary functions of the current state of the world (to be defined below). To make the economy stationary, we deflate all nominal variables by the level of the relevant money supply¹⁹ and restrict attention to Markov stochastic processes for the exogenous shocks.

The aggregate state of the world when the pricing decisions are made (before the realization of current shocks) is fully characterized by the realization of the shocks in the previous period, $s_{-1} \equiv (z_{-1}, z_{-1}^*, g_{-1}, g_{-1}^*, \Omega_{-1})$ and by the distribution of wealth between the two countries, B . Let $s^m \equiv (s_{-1}, B)$ denote the aggregate state for the monopolists. Consumers make their choices after the realization of current period shocks. Consequently, the relevant aggregate state of the world for their decisions also includes these shocks; we denote this state by $s^c = (s^m, s)$.

¹⁷To the extent that a regime shift affects current allocations and other macroeconomic variables, this modeling strategy reduces the ability of the model to replicate the facts of the “exchange-rate disconnect puzzle.” Alternatively, we could model changes in the risk premium as reflecting information relevant to predicting a *future* regime shift. One could model this with an information variable that signals future shifts in regime.

¹⁸Note that this restriction implies we do not allow international trade of forward contracts in currencies.

¹⁹Nominal variables denominated in home (foreign) currency are deflated by the home (foreign) money stock at the beginning of the period, M_t (M_t^*). The nominal exchange rate is deflated by the ratio of foreign to home money stocks.

A stationary and symmetric equilibrium for this economy is defined as a collection of

- optimal decision rules for the home and foreign consumers, $l(s^c)$, $B'(s^c)$, $M(s^c)$, $c_h(s^c)$, $c_f(s^c)$ and similarly for the foreign consumer;
- optimal pricing rules for home and foreign firms, $P_h(s^m)$, $P_h^*(s^m)$ and similarly for the foreign firm;
- equilibrium wage rates $w(s^c)$, $w^*(s^c)$ and
- equilibrium bond price $Q(s^c)$ and nominal exchange rate $e(s^c)$

that satisfy the following conditions:

- (i) consumers' decision rules solve the consumers' problem,
- (ii) firms' pricing rules solve the firms' problem and
- (iii) market clearing conditions for bond and money markets hold.²⁰

4. Numerical exercises

4.1. Calibration

We study the properties of this economy by approximating numerically the stationary, symmetric equilibrium of the model. This section specifies the functional forms and the parameter values used in solving the model. Our calibration assumes that the world economy is symmetric, implying that both countries share the same specific functional forms and parameter values. Moreover, we assume that each time period corresponds to one quarter.

4.1.1. Preferences

The momentary utility function is given by

$$u\left(c, l, \frac{M}{P}\right) = \frac{1}{1-\sigma} \left[\left(ac^\eta + (1-a) \left(\frac{M}{P} \right)^\eta \right)^{\frac{\zeta}{\eta}} (1-l)^{1-\zeta} \right]^{1-\sigma},$$

where $\sigma > 0$, $\eta > 0$, $\zeta \in (0, 1)$, and $a \in (0, 1)$.

The preference parameter values used are described in Table 1. See Duarte (2003) for a discussion of these values.

4.1.2. Exogenous shocks

The regime variable Ω , is assumed to determine the covariance between shocks to productivity and money growth in each country. This variable can take two values, Ω^1 and Ω^2 , and evolves according to the symmetric Markov process with transition probabilities: $\pi_{ii} = \pi$ and $\pi_{ij} = 1 - \pi$, $i, j = 1, 2$. In our benchmark calibration, we set $\pi = 0.9$.

²⁰Equilibrium in labor and goods markets was already imposed in the firm's problem.

Table 1
Preference parameters

β	σ	η	ζ	a	γ	ω
0.99	2	-1.56	0.32	0.73	1.5	0.85

The vector of exogenous shocks to productivity and money growth rates, $\mathbf{s}_t = (z_t, z_t^*, g_t, g_t^*)$, follows the autoregressive process:

$$\mathbf{s}_t = A\mathbf{s}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{4.1}$$

where A is a (4×4) matrix of coefficients and $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \Sigma_t | \Omega_t)$. Note that the variance–covariance matrix $\Sigma_t | \Omega_t$ depends on the realization of the regime variable in period t , Ω_t . In particular,

$$\Sigma_t | \Omega_t = \begin{bmatrix} \sigma_z^2 & 0 & \sigma_{zg,t} & 0 \\ 0 & \sigma_{z^*}^2 & 0 & \sigma_{z^*g^*,t} \\ \sigma_{zg,t} & 0 & \sigma_g^2 & 0 \\ 0 & \sigma_{z^*g^*,t} & 0 & \sigma_{g^*}^2 \end{bmatrix},$$

where

$$\sigma_{zg,t} = \begin{cases} \sigma_1 & \text{if } \Omega_t = \Omega^1, \\ \sigma_2 & \text{if } \Omega_t = \Omega^2 \end{cases}$$

and

$$\sigma_{z^*g^*,t} = \begin{cases} \sigma_2 & \text{if } \Omega_t = \Omega^1, \\ \sigma_1 & \text{if } \Omega_t = \Omega^2 \end{cases}$$

with $\sigma_1 < \sigma_2$.

In all the exercises in this paper we set

$$A = \begin{bmatrix} 0.9825 & 0.0155 & 0 & 0 \\ 0.0155 & 0.9825 & 0 & 0 \\ 0 & 0 & 0.81 & 0 \\ 0 & 0 & 0 & 0.81 \end{bmatrix}$$

and $\sigma_z = \sigma_{z^*} = 0.00675$, and $\sigma_g = \sigma_{g^*} = 0.0114$. These values are obtained from estimating separately a bivariate autoregressive process for (z_t, z_t^*) and univariate autoregressive processes for g_t and g_t^* . The bivariate autoregressive process for (z_t, z_t^*) was estimated using estimated Solow residuals for the U.S. and Canada, while the univariate autoregressive processes for g_t and g_t^* were estimated using U.S. data for M1. See Duarte (2003) for a description of these regressions.

To specify completely the process in (4.1) we also need to assign values to σ_1 and σ_2 . We chose these two values so that the correlation between the innovations to productivity and money growth shocks is -0.9 and $+0.9$, respectively. With these choices for σ_1 and σ_2 we try to magnify the importance of switches in regime in our results.

4.2. Responses to money and productivity shocks

First consider the impulse response functions for shocks to the growth rate of money and productivity. The figures reported here set the price adjustment parameter α at 0.5.

Fig. 1 depicts the impulse response functions to a shock to the growth rate of money in the home country. In period $t = 2$ the rate of money growth rises by 1.975%, which corresponds roughly to 1.75 standard deviations of σ_g .

On impact, prices in the home country increase approximately 3% and the nominal and real exchange rates depreciate approximately 7% and 3.7%, respectively. Because in this period the home consumer needs to hold more real money balances, home consumption rises approximately 3.8%. Accordingly, also home output and labor rise, roughly 3.4% and 5.2%, respectively.²¹

This period, the shock to home money supply is transmitted to the foreign country only through the increased home demand for foreign goods. Therefore also foreign consumption, labor, and output increase slightly in period $t = 2$.

Finally, in period $t = 2$ the home household also increases its average bond holdings, in order to intertemporally substitute its temporary increase in wealth. The long-run change in bond holdings is very small, implying that this monetary shock does not generate relevant permanent wealth effects.²²

One period after the shock, prices denominated in the home currency adjust fully to their new long-run level, while the home money supply rises gradually to its new long-run level. Because all firms are allowed to reset prices in period $t = 3$ and the path for the home money supply is known, all real effects of the money shock die out after one period. Therefore, all real variables (including the real exchange rate) return to their original level in period $t = 3$.

We now turn to the productivity shock. Fig. 2 depicts the impulse response functions to a 1.17% increase in home productivity (which also corresponds to approximately 1.75 standard deviations of σ_z).

On impact, all prices adjust partially to the increased home productivity. Therefore, this period all prices decrease, and due to the higher productivity of

²¹If prices could not adjust on impact ($\alpha = 0$), then in the period of the shock the behavior of the real exchange rate would mimic the nominal exchange rate, by depreciating approximately 7%. The adjustment of all other real variables would be bigger as well. Home consumption, output, and labor would increase, on impact, 6.5%, 8.7%, and 5.7%, respectively.

²²See Chari et al. (2002) for a discussion of these small permanent wealth effects.

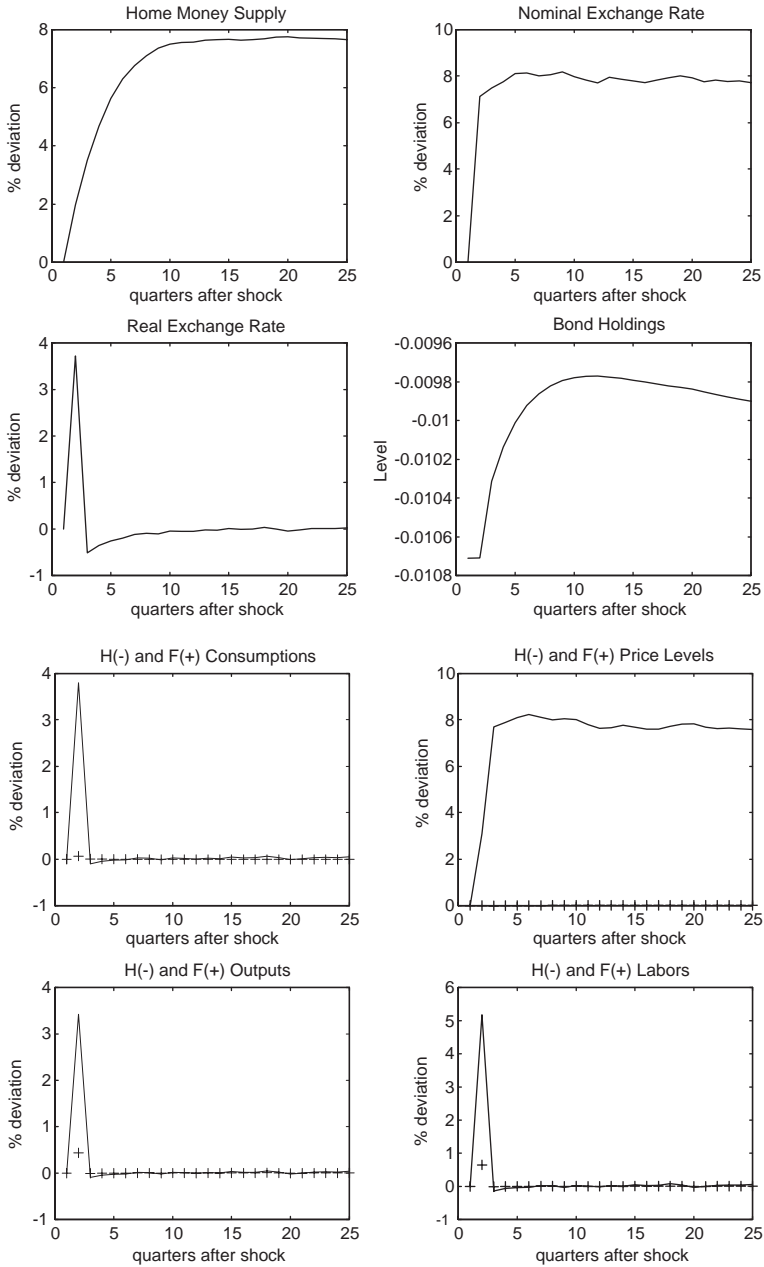


Fig. 1. Monetary shock.

home firms, the relative price of home goods falls in both countries. Because both consumers have a bias for the local good, the consumer price index decreases more in the home country than in the foreign country. In response to the higher labor

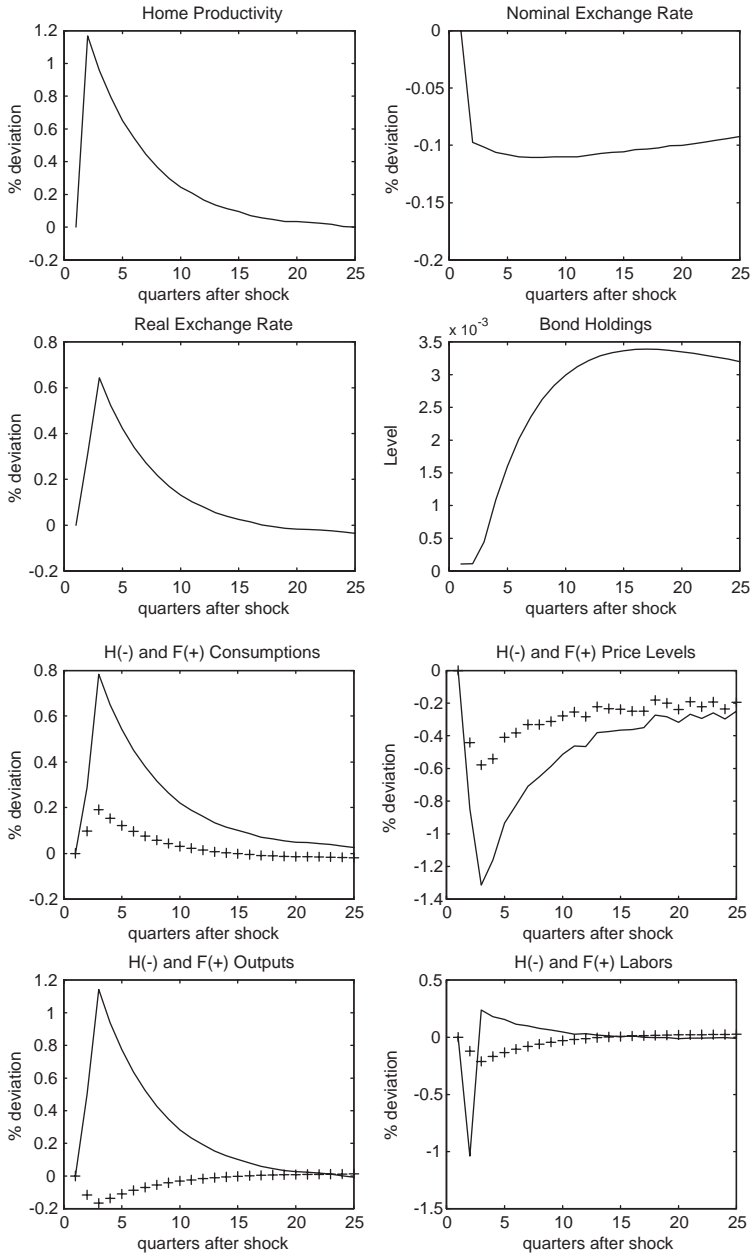


Fig. 2. Productivity shock.

productivity, home firms demand less labor and, in equilibrium, the home household works less but consumes more in period $t = 2$. The home productivity shock affects the foreign country only by lowering home demand for its goods, and we observe a

very small reduction in foreign labor and output. On impact, the real exchange rate depreciates reflecting the adjustment in the price levels, while the nominal exchange rate appreciates slightly.²³

The following period, firms adjust their prices to the new productivity level. All prices decrease further and the relative price of home goods falls further in both countries. Thus total consumption increases in both countries and both households substitute consumption of home goods for foreign goods. Consequently, home output and labor rise, while foreign output and labor fall.

In response to the higher labor productivity, the home household also accumulates bond holdings, although this effect is small. Therefore, as the productivity shock dies out, variables return gradually to approximately their original levels. The new long-run level of the nominal exchange rate is slightly lower than the initial one, consistent with the home household's higher bond holdings.

4.3. Responses to a change in regime

We now examine the effects of regime shocks, which alter the covariance between productivity and monetary shocks. These regime shocks are intended to represent episodes of "speculation" by generating changes in risk premia that speculators demand on foreign exchange markets. Those changes in risk premia are then intended to operate through asset markets to generate first-order effects on the exchange rate while creating much smaller effects on other macroeconomic variables such as output, employment, and consumption.

The model implies that changes in the risk premium require changes in the cross-country difference of covariances between the real returns on nominal bonds and the marginal utility of consumption. Regime changes can affect this covariance because monetary and productivity shocks generate different patterns of responses in consumption, the price level, and the exchange rate. However, the results discussed above show that the model generates extremely small responses of macroeconomic aggregates to productivity shocks (compared to the responses to monetary shocks). We conclude that a model with this structure probably requires some source of shocks other than productivity to generate the sizes of the risk premia observed in the data and sufficient to generate large fluctuations in exchange rates.

Fig. 3 depicts the impulse response functions to a change in the regime, from $\Omega_t = \Omega^1$ in period 1 to $\Omega_t = \Omega^2$ in period 2.²⁴ That is, at $t = 1$ the covariance between home monetary and productivity shocks is negative, and the covariance between foreign monetary and productivity shocks is positive, while at $t = 2$ the covariance between home monetary and productivity shocks becomes positive, and the covariance between foreign shocks becomes negative. Since the magnitude of the

²³If, on impact, all goods' prices are fixed ($\alpha = 0$), then in response to the higher home labor productivity, the home agent works and consumes less in period $t = 2$. The fall in consumption reflects the substitutability of leisure and consumption in the utility function.

²⁴These figures depict the average response of the system to a change in the information variable in 1000 simulations of 100 periods each.

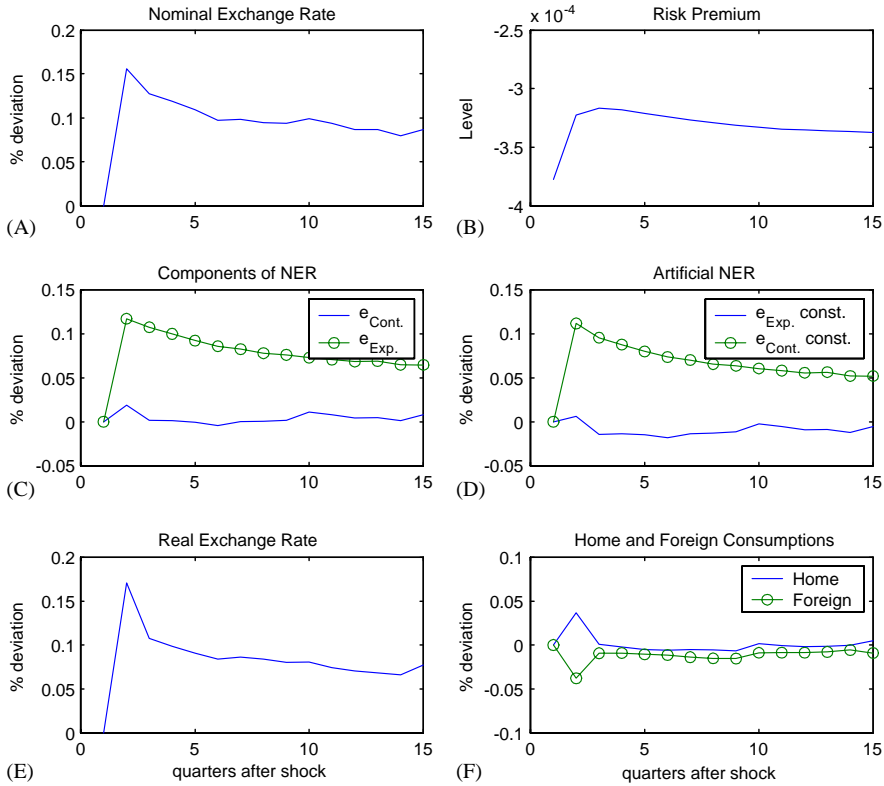


Fig. 3. Temporary change in regime.

responses to productivity shocks are so small relative to those of shocks to the money supply, we raise the standard deviation of productivity shocks by a factor of 4.4 (implying that the response of consumption to a one standard deviation increase of σ_z or σ_g is of the same order of magnitude).

In interpreting the figures, note that the *only* source of persistence in the model is the regime. The model has abstracted from *all* other sources of persistence, such as longer-lived, staggered price setting, or capital. Consequently, all responses in the figures after the impact response reflect regime persistence. In period 3 (one period after the shock is realized), the chance that Ω_t remains at Ω^2 is 0.9. Over time, that probability falls toward 0.5, and the economy moves toward an unconditional steady-state. This is the sole source of persistence in the figures.

The regime shock generates a larger percentage change in the nominal exchange rate than in other variables. Panel A shows that the exchange rate rises by 0.16% on impact, as the risk premium rises from -0.00038 to -0.00032 .²⁵

²⁵The steady-state level of the risk premium is negative due to the asymmetry in the model: with trade *only* in foreign-currency bonds, home households, but not foreign households, bear exchange-rate risk in holding those bonds.

Panels C and D show that the change in the exchange rate occurs mainly through changes in expectations of future variables — working through asset markets and represented by the term $e_{\text{Exp}} \equiv \frac{E[e^{\lambda}]}{E[\lambda^*]}$ — than through changes in allocations and prices, represented by the term $e_{\text{Cont}} \equiv \frac{\lambda^*}{\lambda}$ in (3.5). The expectations term (e_{Exp}) rises by 0.12% while the contemporaneous term (e_{Cont}) rises by 0.02% in response to the regime. Panel D plots the response of two artificial nominal exchange rates, constructed by simply maintaining either e_{Exp} or e_{Cont} at its value in the period before the shock occurs. The responses of these artificial variables correspond roughly to the responses of e_{Exp} and e_{Cont} , indicating that the asset-market term accounts for the larger fraction in the response of the nominal exchange rate.

Fig. 3 shows that a regime change which makes domestic currency “riskier” causes initial depreciation of domestic currency, followed by (expected) appreciation. The impact depreciation is necessary for subsequent expected appreciation (as in Dornbusch, 1976, though in a different context). Note that the magnitudes of all the changes in Fig. 3 are quite small; to explain the data the model would need either to create larger risk premia, which would create larger changes in response to regime shifts, or larger shocks (e.g. a larger regime shift). However, one magnitude that is *not* small is the size of the initial depreciation *relative* to the change in subsequently expected appreciation. Fig. 3 shows that a relatively small change in expected annual appreciation (to compensate for changed risk) corresponds to a much larger impact depreciation.

Appreciation after the impact depreciation occurs in Fig. 3 for a second reason: although regimes in our model are highly persistent, they do occasionally switch back. Part of the expected appreciation shown in the figure reflects this feature. To separate these two sources of expected appreciation, Fig. 4 shows impulse responses conditional on the event that the regime remains in its new state for 10 consecutive quarters. In that case, the risk premium rises further over time, after its increase on impact. Comparing the two figures, the expected appreciation occurs at about half the rate in Fig. 4 as in Fig. 3, indicating that about half of the expected appreciation is due to each factor. Consider a change in risk that requires compensation of one-percent per year through expected appreciation. Fig. 4 shows that such a change requires an impact depreciation exceeding 10 percent; clearly not *all* magnitudes predicted by our model are small!

Our results illustrate the possibility of large asset-market “speculation” effects on exchange rates. However, our model, like previous models, has not generated sufficiently large changes in risk premia to match the data (or, as a result, sufficiently large Sharpe ratios to match those found in the data by Backus et al., 1993). Because the changes in risk premia are too small to match the data, the resulting exchange-rate variability is also too small. Clearly, a full theory of risk premia and their effects on exchange rates requires that economists revise their models, or develop new ones. For example, Backus et al. (1993) develop a theoretical model of the risk premium, but even when choosing parameters to maximize its standard deviation, their model generates a standard deviation that is only about half that in the data.

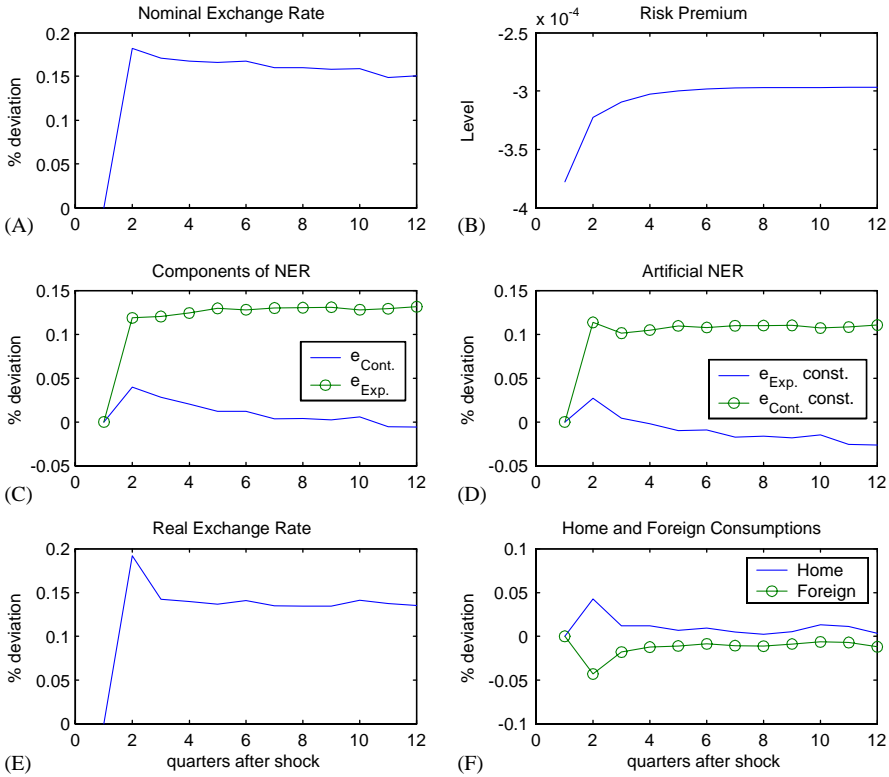


Fig. 4. Permanent change in regime.

5. Conclusion

This paper has argued that speculation, with implied changes in risk premia, is likely to play a key role in explaining the behavior of exchange rates. With sufficient international segmentation in product markets, we have argued that exchange rates follow a forward-looking, first-order stochastic difference equation that includes terms involving risk premia. Consequently, the current exchange rate can be affected by changes in that risk premium, generating persistent deviations from uncovered interest parity. We have argued that changes in exchange rates, generated in this way, need *not* be strongly correlated with changes in other macroeconomic variables (aside from the risk premium itself).

We have implemented this idea in a standard two-country monopolistic-competition model with sticky prices and pricing to market, with markets for final products that are completely segmented internationally, and a model of regime shifts—affecting the covariances of shocks—intended to create “rational”

speculation in the sense of altering equilibrium risk premia.²⁶ The model generates a strong connection between changes in exchange rates and changes in risk premia, but the magnitude of the risk premium is too small to match the data, and so the exchange-rate changes they produce are also too small. Nevertheless, the model predicts that even small changes in the risk premium can generate significantly larger impact effects on the exchange rate, suggesting that larger, more realistic changes in the risk premium may generate substantial exchange rate variability.

Future research might adopt model variations from the equity-premium literature to (try to) raise the implied size of and variation in risk premia. A second area for research would be to examine the degree of “irrational” speculation necessary to generate sufficient exchange-rate variability from the difference-equation for the exchange rate. One could ask whether models like the one in this paper can generate sufficient exchange-rate variability *conditional* on variation in the risk premium. Another area for future research involves exploiting information in the term structure of the risk premium to infer the expected persistence of risk-premium changes and the implied magnitude of exchange rate changes.

Appendix A. Exchange rate level

The first-order stochastic difference equation (3.5) describes the expected growth rate of the exchange rate; the remainder of the model interacts with this equation to determine the *levels* of current and expected future exchange rates. A key factor in the remainder of the model involves the relative wealth of the two countries at date t . (As noted before, with complete markets, the ratio of expectations would drop out of Eq. (3.5), preventing speculation in asset markets from influencing the exchange rate.)

The simplest way to understand this interaction between Eq. (3.5) and the remainder of the model is to consider in this appendix a simplified model with a very simple set of behavioral responses. In particular, consider momentarily a two-period nonstochastic model, with $t = 1, 2$, for given initial conditions $B_1 = -B_1^* \neq 0$, M_0 , and M_0^* , and terminal conditions $B_3 = B_3^* = 0$. In addition, set $D_t = D_t^* = 0$, for $\forall t$.

The home representative household’s intertemporal budget constraint could then be rewritten as

$$B_1 + \frac{\varphi_1}{e_1} + \frac{Q^* \varphi_2}{e_2} = 0,$$

²⁶Our modeling choice represented a compromise between two extremes. On the one hand, the regime must show persistence to generate speculation in our model. (If changes in regime were serially uncorrelated, then the asset-market term in our exchange rate equation, as a ratio of expectations, would be a constant, independent of the current state.) On the other hand, high persistence in regimes reduces the magnitude of changes in exchange rates. One interesting area for future research would be to introduce a regime variable, conveying information about future regime changes before they occur. That regime (information) variable would introduce rational speculation into the model without requiring the high degree of serial correlation in the regime that we assume. Consequently, the model may be able to generate larger changes in exchange rates for any given change in risk premia.

where

$$\varphi_t \equiv P_t w_t l_t + M_{t-1} + \Pi_t + P_t T_t - M_t - P_t c_t$$

and the intertemporal budget constraint for the foreign representative household could then be rewritten as

$$B_1^* + \varphi_1^* + Q^* \varphi_2^* = 0,$$

where

$$\varphi_t^* \equiv P_t^* w_t^* l_t^* + M_{t-1}^* + \Pi_t^* + P_t^* T_t^* - M_t^* - P_t^* c_t^*.$$

Suppose that φ_t , φ_t^* , and Q^* are fixed, and approximate the exchange-rate equation (3.5) by

$$e_1 = \Theta e_2 \tag{A.1}$$

for some parameter Θ .²⁷ Although this simplified set of equations is nonstochastic, we can loosely identify a change in the parameter Θ with a change in the real risk of holding nominal bonds. Because the real return on a nominal bond is proportional to the inverse of the inflation rate, an increase in Θ corresponds to a rise in the risk premium, i.e. it corresponds to an increase in the risk of holding home-currency-denominated bonds (which, in the complete model, is due to a rise in the covariance between the marginal utility of consumption and the inverse of the inflation rate). (An increase in the parameter Θ can also be identified with a *fall* a risk of holding *foreign* nominal bonds, since the model is symmetric in the two countries.) Such an increase in home-currency risk (or fall in foreign-currency risk) requires an offsetting increase in the rate of home-currency appreciation (or fall in the rate of depreciation), i.e. an increase in Θ .

Solving for the exchange rate from the foreign intertemporal budget constraint and Eq. (A.1), and using the domestic budget constraint, we obtain

$$e_2 = \frac{-(\varphi_1 + \Theta Q^* \varphi_2)}{\Theta(\varphi_1^* + Q^* \varphi_2^*)}$$

and

$$e_1 = \frac{-(\varphi_1 + \Theta Q^* \varphi_2)}{\varphi_1^* + Q^* \varphi_2^*}.$$

²⁷The exchange rate enters budget constraints multiplying both debt and export revenue in foreign currency. If initial debt is large relative to these revenues, then the main effect of exchange rate changes on the budget constraint operates through its effect on the value of nominal debt. Consequently, one can treat φ_t and φ_t^* as approximately constant. However, if B_1 is *not* large relative to the current revenue denominated in foreign currency, then one cannot ignore the effects of a change in the exchange rate on φ_t and φ_t^* . If, for example, $B_1 = 0$, then only these other terms appear in the equation, and help determine the exchange-rate level.

A rise in Θ affects the exchange rate in both periods and its effect depends on the wealth distribution across countries in both periods (B_2/B_1),

$$\frac{d \ln e_1}{d \ln \Theta} = Q^* \frac{B_2}{B_1},$$

and

$$\frac{d \ln e_2}{d \ln \Theta} = Q^* \frac{B_2}{B_1} - 1,$$

i.e. if the home country is a net international creditor at the beginning of the first period ($B_1 > 0$), the effect of an increase in Θ on the current and future exchange rate is proportional to the share of initial debt that the foreign country repays in the first period.

This simplified set of equations illustrates the interaction between the effects of the exchange-rate difference Eq. (3.5) and the rest of the model, for which the exchange rate matters because it affects the value of nominal debt in the households' budget constraints (in particular, it affects the value of domestic-currency debt in the foreign household's budget constraint, and foreign-currency debt in the home household's constraint). The example also shows that incomplete markets are necessary to generate a unique equilibrium exchange rate (as mentioned earlier), because allocations would not depend on individual households' budget constraints in a complete-market model; instead, complete contingent securities would provide, on a state-by-state basis, the resources to finance optimal allocations. Consequently, the exchange rate would appear in the model *only* in the difference equation (3.5). While the simplified set of equations can help illustrate these points, it relies on a very loose approximation and ignores households' optimal behavioral responses.

Appendix B. The model with partially adjustable prices

Using the algebra of expected values, the term e_{Exp} in Eq. (3.5) can be rewritten as

$$e_t^{Exp} = \frac{E[u_{c,t+1}]E\left[\frac{1}{P_{t+1}}\right] + cov\left(u_{c,t+1}, \frac{1}{P_{t+1}}\right)}{E[u_{c,t+1}^*]E\left[\frac{1}{e_{t+1}P_{t+1}^*}\right] + cov\left(u_{c,t+1}^*, \frac{1}{e_{t+1}P_{t+1}^*}\right)}. \tag{B.1}$$

A change in the regime variable, as long as this variable is persistent, will affect the covariance terms in the above expression by affecting the covariance between future shocks to money growth and productivity in both countries. In our framework, however, firms set prices one period in advance and, on impact, prices do not respond to shocks. Consequently, the covariance terms in Eq. (B.1) are zero in our model. However, if prices were to adjust to current shocks, future money and productivity shocks would affect both future price levels and marginal utilities of consumption. This would allow a change in regime to affect the term e^{Exp} by affecting the covariance terms in Eq. (B.1).

In this appendix we describe the price setting problem of firms, in order to allow them to adjust partially their prices to current shocks. As before, firms set prices for period t one period in advance at $t - 1$, before observing date- t shocks. However, we assume that after uncertainty is resolved, all firms can adjust partially their prices to the ones that would occur in the flexible price equilibrium, i.e. for home firm i , the price effectively charged to home consumers in period t , $P_{h,t}^e(i)$, is a linear combination of the price pre-set in advance, $P_{h,t}(i)$, and the price that would occur in the flexible price equilibrium, $P_{h,t}^{\text{flx}}(i)$, i.e.

$$P_{h,t}^e(i) = \alpha P_{h,t}(i) + (1 - \alpha) P_{h,t}^{\text{flx}}(i) \quad (\text{B.2})$$

and similarly for all other three prices.

The price level in the home country in period t is now given by

$$P_t = [\omega (P_{h,t}^e)^{1-\gamma} + (1 - \omega) (P_{f,t}^e)^{1-\gamma}]^{1/(1-\gamma)}.$$

When firms choose their prices at $t - 1$, they know that the effective price at t is a given linear combination of the price they set at $t - 1$ and the price that would occur in the flexible price equilibrium, which they take has given. So, the price setting problem of home firms in period $t - 1$ becomes

$$\begin{aligned} \max_{P_{h,t}(i), P_{h,t}^*(i)} & E_{t-1} [\rho_t \{ (\alpha P_{h,t}(i) + (1 - \alpha) P_{h,t}^{\text{flx}}(i)) c_{h,t}(i) \\ & + e_t (\alpha P_{h,t}^*(i) + (1 - \alpha) P_{h,t}^{\text{flx}*}(i)) c_{h,t}^*(i) - P_t w_t l_t(i) \}] \end{aligned}$$

subject to the resource constraint $z_t l_t(i) = c_{h,t}(i) + c_{h,t}^*(i)$ and the downward sloping demand functions for $c_{h,t}(i)$ and $c_{h,t}^*(i)$. The first-order conditions with respect to $P_{h,t}(i)$ and $P_{h,t}^*(i)$ are

$$E_{t-1} \left[\rho_t \left((1 - \theta) c_{h,t}(i) + \theta \frac{P_t w_t c_{h,t}(i)}{\alpha P_{h,t}(i) + (1 - \alpha) P_{h,t}^{\text{flx}}(i)} \right) \right] = 0$$

and

$$E_{t-1} \left[\rho_t \left((1 - \theta) e_t c_{h,t}^*(i) + \theta \frac{P_t w_t c_{h,t}^*(i)}{\alpha P_{h,t}^*(i) + (1 - \alpha) P_{h,t}^{\text{flx}*}(i)} \right) \right] = 0.$$

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