Notes for Computing a Stationary Recursive Equilibrium in: "Intergenerational Persistence of Earnings: The Role of Early and College Education," *American Economic Review* (December 2004), 94(5), pp. 1354-78.

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The aggregate production function $Y = AH$ implies that the wage rate per unit of human capital is equal to A (and in our calibration A is normalized to one). Given a tax rate τ (calibrated to match public expenditures in education as a fraction of GDP), our computational task amounts to finding the equilibrium g (government expenditures in early education). For a given g, we solve for the optimal policy rules and the associated invariant distribution of people across states (using successive approximations) to calculate government revenues and expenditures in college education. Since the government budget must balance every period, we use the government budget equation to update g until convergence. Our computational strategy is to solve for the equilibrium g and parameter values from our calibration at the same time, but in what follows we describe in detail the steps required to solve for the recursive competitive equilibrium assuming that parameter values are given.

1. Guess a level of per capita government expenditures in early education g^i starting in iteration $i = 0$.

2. We make the state space discrete. We construct log-spaced grids for human capital with 60 points, innate ability with 15 points, and acquired ability with 60 points. The grid for innate ability π is chosen to approximate the AR(1) process for π using Tauchen's

method as described in the paper. We choose the grid for acquired ability $\hat{\pi}$ with a lower bound equal to $\hat{\pi(1)} = \pi(1)g^{\gamma}$ and upper bound $\hat{\pi(60)} = \pi(15)(5g)^{\gamma}$. The human capital grid has lower bound $h(1) = \hat{\pi(1)}$ and upper bound $h(60) = \bar{p}\hat{\pi(60)}$. These upper and lower bounds were chosen in general in order not to affect the equilibrium solution of the model.

3. The college completion function $q(\hat{\pi})$ and the college subsidy function $\kappa(h)$ are obtained for each point in the grid.

4. We iterate on the value function for old parents:

   a. Make initial guesses for the value function of old parents, $V_o^{i=0,j=0}$ starting in iteration $j = 0$.

   b. Given w, g, and $V_o^{i,j}$, obtain $V_y^{i,j+1}$ from young parent's problem and compute optimal policy functions. These function are easily computed by searching over the grids.

   c. Given w, and $V_y^{i,j+1}$, obtain $V_o^{i,j+1}$ from old parent's problem and compute optimal policy functions.

   d. Check for convergence of value functions by comparing $V_o^{i,j}$ and $V_o^{i,j+1}$. Our metric for convergence is the sup norm of

$$\frac{||V_o^{i,j+1} - V_o^{i,j}||}{2 * (V_o^{i,j+1} + V_o^{i,j})}.$$

   If convergence criteria is satisfied then we move to the next step, otherwise we iterate on the value function until convergence. Our tolerance criterion is to achieve accuracy up to 4 decimal places.

5. We iterate on the distribution of young parents starting from iteration $z = 0$:

   a. Guess a distribution of young parents across states $\mu_y^{i,z=0}$ and use this guess together with optimal policy functions to obtain the implied distribution of old parents μ_o.

   b. With μ_o, optimal policy functions, and laws of motion for the exogenous states obtain $\mu_y^{i,z+1}$.

   c. Continue until convergence of μ_y^i. We use the sup norm of the absolute vector difference as our metric.

6. Using the invariant distribution and optimal policy functions, compute g^{i+1} by imposing the budget constraint of the government. Compare with g^i. Iterate from step 1. until convergence of

$$\left| \frac{g^{i+1}}{g^i} - 1 \right|.$$