Introduction: Basic Facts and Neoclassical Growth Model

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Overview

- Facts and the neoclassical growth model
  - Basic facts
  - Framework: Neoclassical growth model with distortions
  - Discussion, extensions
Some Facts

- Large income differences across countries at any point in time (recent history)
- Growth not systematically related to the level of development
- Strong positive relationship between real investment rates (or real capital to output ratios) and real GDP per capita across countries
## GDP per capita across Countries and Time

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GDP per worker, ratio of 5% richest and poorest countries

Source: Duarte and Restuccia (2006)
### GDP per Capita across Countries and Time

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GDP per capita 1960 and 2011

Source: Jones (2016)
Growth in GDP per worker, 1960-1996

Source: Duarte and Restuccia (2006)
Real Investment to Output Ratio

Source: Restuccia and Urrutia (2001)
**Neoclassical Growth Model**

- Understand factors behind choice of investment
- By spelling out the economic forces determining investment we would be in better position to understand investment rate differences across countries
- And, therefore, part of income differences across countries
- Start with optimal growth and then consider a competitive market economy with distortions
A Model of Optimal Growth

Preferences and Endowments

- Large number of infinitely-lived homogeneous households
- Preferences over consumption goods at each date, $c_t$, described by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

(1)

where $\beta \in (0, 1)$

- Households are endowed with one unit of productive time per period and $k_0 > 0$ units of the capital stock
- Assume population is constant over time ($L$ normalized to one)
A Model of Optimal Growth Technology

- At each date only one good produced with the following production function,
  \[ Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \]  
  (2)
  where \( A_t \) is total factor productivity (TFP), assumed to be constant over time.
- Output can be allocated to consumption and investment
  \[ C_t + X_t \leq Y_t \]  
  (3)
- The capital accumulation is given by,
  \[ K_{t+1} = (1 - \delta)K_t + X_t \]  
  (4)
  where one unit of investment \( X_t \) transforms into one unit of capital next period.
A benevolent social planner cares about the utility of the representative household

Chooses sequences of consumption and investment to maximize (1) subject to (2), (3), and (4)

Formally,

\[
\max \{C_t, X_t, K_{t+1}\}_{t=0}^\infty \sum_{t=0}^{\infty} \beta^t u(C_t)
\]

subject to

\[
C_t + X_t \leq AK_t^\alpha L_t^{1-\alpha} \quad t = 0, 1, 2, ...
\]

\[
K_{t+1} = (1 - \delta)K_t + X_t \quad t = 0, 1, 2, ...
\]

\[C_t, K_{t+1} \geq 0 \text{ and } K_0 > 0 \text{ given}\]
CHARACTERIZATION OF PLANNER’S PROBLEM

- Under mild conditions on $u(\cdot)$ the resource constraint is satisfied with equality (no output will be wasted) and the constraints that $C_t$ and $K_{t+1}$ be non-negative are not binding.

- Can substitute the two constraints to eliminate $X_t$ and $C_t$ from the problem and express the maximization problem as a choice of $K_{t+1}$.

- The first order condition with respect to the capital stock next period gives,

$$u'(C_t) = \beta u'(C_{t+1}) \left[ \alpha AK_{t+1}^{\alpha-1} + (1 - \delta) \right] \quad t = 0, 1, 2, \ldots \quad (5)$$

- Planner is equating the marginal cost of postponing consumption to the marginal benefit.
**Definition: Steady State**

- A steady state solution to the planner’s problem is a $K_s$ such that $K_0 = K_s$ implies $K_{t+1} = K_t = K_s$ for all $t$.

- In a steady state the capital stock is constant over time.

- Notice that if the capital stock is constant then investment, consumption, and output are also constant.

- Applying the steady state definition to the equation (5)

\[ 1 = \beta[\alpha AK_s^{\alpha-1} + (1 - \delta)] \]

which implies

\[
K_s = \left( \frac{\alpha A}{\frac{1}{\beta} - (1 - \delta)} \right)^{\frac{1}{1-\alpha}}
\]
In steady state $X_s = \delta K_s$, then $\frac{X_s}{Y_s} = \delta \frac{K_s}{Y_s}$

Therefore the investment to output ratio,

$$\frac{X_s}{Y_s} = \frac{\alpha \delta}{\left[ \frac{1}{\beta} - (1 - \delta) \right]}$$

The investment rate is determined by technology and preference parameters that are assumed to be constant across countries

Explore competitive equilibrium version of the neoclassical model with distortions
Competitive Equilibrium

- Decentralization of the planner’s solution as the solution to optimization problems of consumers and firms in competitive markets
- Households own the capital stock and supply capital and labor services to firms at competitive rental rates
- Households are endowed with equal ownership shares of all firms in the economy
- Large number of firms operating constant returns to scale output technology
- Firms hire capital and labor services at competitive rates to maximize profits
- Let $w_t$ and $r_t$ be the rental rates of labor and capital at each date in terms of the consumption good at date $t$
Given prices, profits from firms $\pi_t$, and $k_0$, the problem of the household is to choose sequences of $c_t$, $x_t$, and $k_{t+1}$ to maximize the present discounted value of utility.

Formally,

$$\max_{\{c_t, x_t, k_{t+1}\}_t=0} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + x_t = w_t + r_t k_t + \pi_t \quad t = 0, 1, 2, \ldots$$

$$k_{t+1} = (1 - \delta) k_t + x_t \quad t = 0, 1, 2, \ldots \quad k_0 > 0$$

Notice that since households do not value leisure they allocate all their time to work in the market.
Given prices, the firm’s problem is to choose capital and labor so as to maximize profits.

Because the problem of the firm is static (the decisions of firms today do not affect their decisions tomorrow), we can write their problem as a sequence of one-period maximization problems,

\[
\max_{K_t, L_t > 0} \pi_t = [AK_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t K_t]
\]
**Definition: Competitive Equilibrium**

A *competitive equilibrium* is a sequence of prices \( \{w_t, r_t\}_{t=0}^{\infty} \), allocations for the household \( \{c_t, x_t, k_{t+1}\}_{t=0}^{\infty} \), allocations for firms \( \{K_t, L_t\}_{t=0}^{\infty} \), and profits \( \{\pi_t\}_{t=0}^{\infty} \) such that:

(i) Given prices, profits, and \( k_0 > 0 \), the allocations for the household solve the household’s problem

(ii) Given prices, the allocations for firms solve the firm’s problem

(iii) Markets clear:
- Output \( c_t + x_t = Y_t \)
- Capital \( k_t = K_t \)
- Labor \( 1 = L_t \)
The industrial organization of this economy is irrelevant because of the constant returns to scale assumption on the output technology.

First order conditions from the firm’s problem

\[ K_t : \quad \alpha AK_t^{\alpha-1} L_t^{1-\alpha} = r_t, \]

\[ L_t : \quad (1 - \alpha) AK_t^\alpha L_t^{-\alpha} = w_t. \]

Market clearing conditions imply

\[ r_t = \alpha Ak_t^{\alpha-1}, \quad w_t = (1 - \alpha) Ak_t^\alpha, \]

and therefore, the competitive equilibrium wage and rental rate of capital are equated to the marginal product of labor and capital.
It is straightforward to show that at these prices, the demand of capital and labor from firms imply zero profits in equilibrium \( (\pi_t = 0 \text{ for all } t) \).

Also note that the competitive assumption together with the Cobb-Douglas specification of the output technology imply that the share of capital in income is equal to \( \alpha \).

The household’s first order condition with respect to the capital stock next period is given by,

\[
u'(c_t) = \beta u'(c_{t+1}) \left[ r_{t+1} + (1 - \delta) \right] \quad t = 0, 1, 2, \ldots \tag{6}\]
Definition: Steady State

- A *steady state competitive equilibrium* is a competitive equilibrium with $k_s$ such that $k_0 = k_s$ imply $k_{t+1} = k_t = k_s$ for all $t$

- Notice that at the equilibrium prices, equation (6) determines an allocation of capital that exactly corresponds to the solution of the planner’s problem

- Not surprising since in this environment the fundamental welfare theorems hold, allocations of the planner’s problem and the competitive equilibrium coincide
Competitive Equilibrium with Distortions

- The determination of the investment to output ratio in the neoclassical growth model does not leave much room for thinking about investment rate differences across countries.
- Inspection of the Euler equation for capital accumulation of the consumer –equation (6)– does give some hints about the factors that may be relevant in understanding investment rate differences across countries.
- Households would equate the marginal cost and benefit of trading consumption inter-temporally.
- Government policies such as taxes and other distortionary practices can change the returns to investing – so do inefficiencies in producing investment goods.
- Many possibilities, here consider a tax to investment.
Consider a government that taxes household’s investment at proportional rate \( \tau \) and rebates the proceeds back to households as a lump-sum subsidy transfer.

The budget constraint of the household becomes,

\[
c_t + (1 + \theta)x_t = w_t + r_t k_t + T_t \quad t = 0, 1, 2, \ldots
\]

where \( T_t \) is a lump-sum transfer.
**Definition: Competitive Equilibrium**

A *competitive equilibrium* is a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, allocations for the household $\{c_t, x_t, k_{t+1}\}_{t=0}^{\infty}$, allocations for firms $\{K_t, L_t\}_{t=0}^{\infty}$, and government transfers $\{T_t\}_{t=0}^{\infty}$ such that:

(i) Given prices, transfers, and $k_0 > 0$, the allocations for the household solve the household’s problem

(ii) Given prices, the allocations for firms solve the firm’s problem

(iii) Markets clear: Output $c_t + x_t = Y_t$, capital $k_t = K_t$, and labor $1 = L_t$

(iv) The government’s budget is balanced every period

$$T_t = \theta X_t$$
The Euler equation for capital accumulation from households now satisfies,

\[ u'(c_t) = \beta u'(c_{t+1}) \left[ \frac{r_{t+1}}{1 + \theta} + (1 - \delta) \right] \quad t = 0, 1, 2, ... \]

Notice that with \( \theta > 0 \) the allocation of capital in this version of the model will differ from the optimal planner’s solution.

The tax will discourage investment as it lowers the return to investing relative to the cost of foregone consumption.
It is straightforward to show that the steady state capital stock is inversely related to the tax rate.

Substituting the rental rate for capital and imposing the steady state condition in the Euler equation,

\[ 1 = \beta \left[ \frac{\alpha A k_s^{\alpha - 1}}{(1 + \theta)} + (1 - \delta) \right] \]

which implies

\[ k_s = \left( \frac{\alpha A}{(1 + \theta) \left[ \frac{1}{\beta} - (1 - \delta) \right]} \right)^{\frac{1}{1-\alpha}} \]
Steady-State Implications

- In steady state,

\[
\frac{K_s}{Y_s} = \frac{K_s}{AK_s^\alpha} = \frac{K_s^{1-\alpha}}{A} = \left(\frac{\alpha}{(1 + \theta)\left[\frac{1}{\beta} - (1 - \delta)\right]}\right)
\]

and hence does not depend on \(A\).

- Researchers exploit this property in growth and development accounting to separate the contribution of TFP and capital intensity to output differences by writing the production function in intensive form

\[
Y = A^{\frac{1}{1-\alpha}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}}
\]
Steady-State Implications

- In steady state $x_s = \delta k_s$, therefore,

$$\frac{X_s}{Y_s} = \frac{\alpha \delta}{(1 + \theta) \left[ \frac{1}{\beta} - (1 - \delta) \right]} \equiv s$$

- Differences in tax rates may help in accounting for investment rate differences across countries.

- Higher taxes imply lower capital accumulation, lower investment rates, lower capital to output ratios, and lower output per worker.

- A tax to capital income has the same qualitative implications than a tax on investment (see Restuccia and Urrutia, 2001 for an application to investment taxes and differential productivity on investment goods).

- What matters is effective wedges that affect the return to capital investment.
Relative Price of Investment

![Graph showing relative price of investment data points and regression lines.]

Source: Restuccia and Urrutia (2001)
Let $y = Y/L$ be output per worker

Then for any arbitrary countries $i$ and $j$

$$\frac{y_i}{y_j} = \left( \frac{A_i}{A_j} \right)^{\frac{1}{1-\alpha}} \left( \frac{s_i}{s_j} \right)^{\frac{\alpha}{1-\alpha}}$$

Solow assumed $A_i = A_j = 1$ (i.e., no differences in technology across countries), then

$$\frac{y_i}{y_j} = \left( \frac{s_i}{s_j} \right)^{\frac{\alpha}{1-\alpha}}$$
Can differences in investment to output ratios explain labor productivity differences across countries?

\[
\frac{y_i}{y_j} = \alpha \left( \frac{s_i}{s_j} \right)^{1/3} \left( \frac{s_i}{s_j} \right)^{2/3} = \left( \frac{s_i}{s_j} \right)^{3/3}
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- Large differences in investment rates imply small differences in output per worker if reproducible capital is physical capital.
- A broader notion of capital, e.g. human capital, may provide amplification (see Erosa et al. 2010; Manuelli and Seshadri 2014).
What determines human capital differences across countries? Standard theory implies TFP an important factor. Development accounting or even modern approach to human capital differences leaves a large role for TFP differences. What accounts for TFP differences?