Multi-Dimensional Screening: Buyer-Optimal Learning and Informational Robustness

RAHUL DEB and ANNE-KATRIN ROESLER, University of Toronto, Canada

ACM Reference Format:
Rahul Deb and Anne-Katrin Roesler. 2021. Multi-Dimensional Screening: Buyer-Optimal Learning and Informational Robustness. In *Proceedings of the 22nd ACM Conference on Economics and Computation (EC '21), July 18–23, 2021, Budapest, Hungary.* ACM, New York, NY, USA, 2 pages. https://doi.org/10.1145/3465456.3467611

What is the optimal mechanism that a monopolist should use to sell multiple goods to a single buyer? Despite being a classic economic problem, multi-dimensional screening is notoriously intractable. Even if the seller has just two goods and the buyer's values are additive, independent, and identically distributed, the optimal mechanism is hard to characterize generally. In this paper, we study a general version (with arbitrarily many goods and non-additive values) of this problem but with the novel feature of buyer learning. As it turns out, introducing this new feature makes the model tractable and in certain environments—including the one with independent and additive values—makes pure bundling an optimal mechanism.

The buyer in our model initially has an unknown type $(\theta_1, \ldots, \theta_n)$ that is drawn from a commonly known exchangeable distribution, where each $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right] \subset \mathbb{R}_+$. The buyer's type determines his value $\kappa_b \sum_{i \in b} \theta_i$ for any bundle $b \subseteq \{1 \ldots, n\}$ of goods where $\kappa_b \geq 0$ is a non-negative constant. We assume a weak free-disposal property, which requires that the value of the grand bundle (that is, the bundle of all n goods) is greater than any other bundle for every type. This class of value functions allows for goods to be complements or substitutes and, importantly, includes additive values ($\kappa_b = 1$ for all bundles b) as a special case. The buyer learns about his type via a signal. Upon privately observing the signal realization, the buyer forms a posterior estimate of his value for different bundles.

Our aim is to derive the seller's optimal mechanism under two different informational environments. We first characterize the *buyer-optimal outcome*: this is the signal and the corresponding optimal mechanism for the seller that generate the *maximal consumer surplus*. Specifically, an information designer first publicly picks the signal (the signal realization remains private to the buyer) to maximize consumer surplus anticipating that the seller will choose an optimal mechanism in response. We show that the buyer-optimal outcome is generated by a signal that makes "pure bundling" (selling the grand bundle at a given price) an optimal mechanism for the seller. Additionally, we show that the seller's profit is minimized: there is no other signal and corresponding optimal mechanism that yield a lower profit. Thus, we show that the seller's profit in the buyer-optimal outcome is the solution to a min-max problem where an adversarial nature picks a signal with the aim of minimizing the profits of a seller who best-responds with an optimal mechanism.

We then derive the *optimal informationally robust mechanism* for the seller: this is an optimal mechanism for a seller who does not know how the buyer learns and who evaluates profits according

EC '21, July 18–23, 2021, Budapest, Hungary

ACM ISBN 978-1-4503-8554-1/21/07.
https://doi.org/10.1145/3465456.3467611

to a worst-case criterion. Here, the timing is reversed: the seller first chooses the mechanism, following which nature picks the signal to minimize the seller's profit. Therefore, in this case, the seller's profit from the optimal informationally robust mechanism is the solution to a max-min problem. Once again, we show that pure bundling is optimal for the seller, but, in this case, she randomizes over the price for the grand bundle. Moreover, we derive this result by showing that the seller's profit, in this case, is exactly equal to her profit from the buyer-optimal outcome; that is, the optimal value of the objective function in the max-min and the min-max problems coincide.

In our view, the solutions to both problems are individually economically interesting and have distinct implications. At a high level though, both demonstrate different important properties of pure bundling. The buyer-optimal outcome is a natural theoretical benchmark. The seller of a single good always finds it optimal to screen by simply posting a price. By contrast, optimal multidimensional screening can, and frequently does, involve complex menus and randomization even when values are additive and each θ_i is independently and identically distributed. Such elaborate screening helps sellers maximize profits, but the effect on the consumer is unclear. For instance, complex screening might lead to Pareto improvements where both the seller and the buyer are better off because the efficiency of trade increases. The buyer-optimal outcome is a natural benchmark to study the tradeoff between mechanism complexity and the efficiency of trade because the seller best responds to the most advantageous information structure for the buyer. Here, the optimal mechanism takes the very simple form of pure bundling, and we show that trade is efficient.

The selling practices of multi-product retailers are scrutinized by regulators who specifically express concerns about and pursue litigation against practices like tying and bundling by large firms. Under the buyer-optimal signal, not only does pure bundling not cause consumer harm, it leads to the highest possible consumer surplus and efficient trade. This suggests that the information available to buyers is an important factor that should determine whether or not bundling needs to be scrutinized. Of course, this also raises the question of whether and, if so, which advertising practices should be regulated in the interest of consumers.

Conversely, the optimal informationally robust mechanism provides a positive explanation for why we should expect to observe pure bundling in practice. Despite having historical data from different markets, sellers are unlikely to have very precise estimates of a buyer's value distribution. In particular, it is impossible for a seller to predict what information the buyer has or will acquire in any particular period. Our results show that pure bundling (albeit with a random price) provides the highest revenue guarantee. This is perhaps one reason why, in practice, we do not observe very complex screening that depends on fine details of the type distribution (as is possible in multidimensional screening). Instead, pure bundling is the common way that digital goods such as streaming services are sold. This setting is a good fit for the model: sellers such as Netflix and Spotify have considerable market power, and disposal is free.

A full version of this paper is available at https://arxiv.org/pdf/2105.12304.pdf .