# Corrigendum to "Implementation with Contingent Contracts"

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There is a mistake in the proof of Lemma 2 in Deb and Mishra (2014) – the partition we construct is not an $f$-ordered partition as claimed. That said, *we stress that the statement of the Lemma and all the results in the paper are correct.* In this corrigendum, we provide a simple correction to the proof of the lemma. We are very grateful to Kiho Yoon who brought the error to our attention - he also provides an alternate proof of our main result in Yoon (2015). In what follows, we assume that the readers are familiar with the notation in Deb and Mishra (2014).

In the proof of Lemma 2 in Deb and Mishra (2014), we recursively constructed a partition and claimed that it was an $f$-ordered partition. To see the mistake in our construction, choose an scf $f$ and a set of possible types $V_i = \{v_i^1, v_i^2, v_i^3, v_i^4\}$ of agent $i$ that are related as follows: $v_i^1 \succ^f v_i^2 \succeq^f v_i^3 \succeq^f v_i^4$. The maximal set $\widetilde{V}_i$ in $V_i$ with respect to $\succ^f$ is $\{v_i^1, v_i^3, v_i^4\}$ and, hence, $M(V_i) = \{v_i^1, v_i^4\}$. [2] The partition resulting from our construction is then $\{M(V_i), M(V_i \setminus M(V_i))\}$ which is $\{\{v_i^1, v_i^4\}, \{v_i^2, v_i^3\}\}$. Clearly, this is not an $f$-ordered partition as $v_i^3 \succeq^f v_i^4$, violating property P2 of $f$-ordered partition.

In the above example, an $f$-ordered partition is $\{\{v_i^1\}, \{v_i^2, v_i^3, v_i^4\}\}$. One way to obtain this correct partition is to recursively remove maximal sets corresponding to the relation given by the 'transitive closure' of $\succ^f$. We do this formally in the corrected proof below.

Lemma 2: Suppose the type space is finite and $f$ is an acyclic scf. Then, the type space can be $f$-ordered partitioned.

*Proof:* We begin by defining a new relation $\rhd^f$ as follows: for every $v_i, v_i'$, we say $v_i \rhd^f v_i'$ if there is a finite sequence $\{v_i^1, \ldots, v_i^k\}$ such that $v_i \equiv v_i^1 \succeq^f v_i^2 \succeq^f \ldots \succeq^f v_i^k \equiv v_i'$, with at least one of the above relations strict ($\succ^f$). The proof now follows in several steps.

Step 1. First, we show that $\rhd^f$ is acyclic. Consider a sequence $v_i^1, \ldots, v_i^k$ such that $v_i^1 \rhd^f \ldots \rhd^f v_i^k$. Assume for contradiction $v_i^k \rhd^f v_i^1$. So, we get $v_i^1 \rhd^f \ldots \rhd^f v_i^k \rhd^f v_i^{k+1} \equiv v_i^1$. Between any consecutive types $v_i^j$ and $v_i^{j+1}$ in this sequence, we have $v_i^j \succeq^f \ldots \succeq^f v_i^{j+1}$, with strict relation holding for at least one. Hence, we get a finite sequence $v_i^1 \succeq^f \ldots \succeq^f v_i^{k+1} \equiv v_i^1$

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[2]Recall that a type $v_i$ is maximal in $V_i' \subseteq V_i$ with respect to $\succ^f$ if there exists no type $v_i' \in V_i'$ such that $v_i' \succ^f v_i$. Further, $M(V') := \{v \in \widetilde{V'} : \ v' \not\succeq^f v \ \forall \ v' \in V' \setminus \widetilde{V'}\}$.

with strict relation holding for at least one. Acyclicity of $f$ implies that $v_i^1 \not\succeq^f v_i^1$, contradicting the reflexivity of $\succeq^f$.

STEP 2. Next, choose any nonempty subset $V_i' \subseteq V_i$, and let $\widehat{M}(V_i') \subseteq V_i'$ be the set of maximal elements of $V_i'$ corresponding to the relation $\rhd^f$. Formally

$$\widehat{M}(V_i') := \{v_i \,|\, v_i \in V_i' \text{ and there is no } v_i' \in V_i' \text{ such that } v_i' \rhd^f v_i\}.$$

Since $\rhd^f$ is acyclic, $\widehat{M}(V_i')$ is non-empty.

STEP 2(A). $v_i' \not\succ^f v_i$ for every $v_i, v_i' \in \widehat{M}(V_i')$. To see this, if $v_i' \succ^f v_i$, then $v_i' \rhd^f v_i$, contradicting $v_i \in \widehat{M}(V_i')$.

STEP 2(B). $v_i' \not\succeq^f v_i$ for every $v_i \in \widehat{M}(V_i')$ and every $v_i' \in (V_i' \setminus \widehat{M}(V_i'))$. Assume for contradiction $v_i' \succeq^f v_i$. Since $v_i' \notin \widehat{M}(V_i')$, there exists a $v_i''$ such that $v_i'' \rhd^f v_i'$. Hence, there is a finite sequence, $v_i'' \succeq^f \ldots \succeq^f v_i' \succeq^f v_i$, with strict relation holding at least once. As a result, we have $v_i'' \rhd^f v_i$, contradicting the fact that $v_i \in \widehat{M}(V_i')$.

STEP 3. Now, we recursively define a partition. We set $V_i^1 = \widehat{M}(V_i)$. Having defined, $(V_i^1, \ldots, V_i^{k-1})$, we define $R^k := V_i \setminus (V_i^1 \cup \ldots \cup V_i^{k-1})$. If $R^k = \emptyset$, we stop, else, we set $V_i^k := \widehat{M}(R^k)$. Suppose $(V_i^1, \ldots, V_i^K)$ is the partition created. We show that $(V_i^1, \ldots, V_i^K)$ satisfies Property P1 and P2. To do that, pick $v_i, v_i' \in V_i^j$ for some $j$. Since $V_i^j = \widehat{M}(R^j)$, by Step 2(A), we have $v_i' \not\succ^f v_i$. So, Property P1 is satisfied.

Similarly, pick $v_i \in V_i^j$ and $v_i' \in (V_i^{j+1} \cup \ldots \cup V_i^K)$. Since $V_i^j \equiv \widehat{M}(R^j)$ and $(V_i^{j+1} \cup \ldots \cup V_i^K) \equiv R^j \setminus V_i^j$, by Step 2(B), we get $v_i' \not\succeq^f v_i$. So, Property P2 is satisfied. ■

We end this note by returning to the example. There, $v_i^1 \rhd^f v_i^2$, $v_i^1 \rhd^f v_i^3$ and $v_i^1 \rhd^f v_i^4$. So, the set $\{v_i^1\}$ is maximal in $V_i$ with respect to $\rhd^f$. Since, none of $v_i^2$, $v_i^3$ or $v_i^4$ are related, the maximal subset of $\{v_i^2, v_i^3, v_i^4\}$ is the set itself. Hence, recursively removing the maximal sets subject to $\rhd^f$ yields an $f$-ordered partition.

# References


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YOON, K. (2015): "Implementation with Contingent Contracts: Comment," *working paper*.