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Note

A testable model of consumption with externalities ☆

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Abstract

This paper presents a nonparametric model of interdependent preferences, where an individual's consumption may act as an externality on the preferences of other consumers. We assume that individual price consumption data is observed for all consumers. It is known that the general consumption model with externalities imposes few restrictions on the observed data, where the consistency requirement is Nash rationalizability. We motivate potential games as an important sub class of games where the family of concave potential games is refutable and imposes stronger restrictions on observed data. We use this framework to extend the analysis of Brown and Matzkin [D. Brown, R. Matzkin, Testable restrictions on the equilibrium manifold, Econometrica 64 (1996) 1249–1262] on refutable pure exchange economies to pure exchange economies with externalities. Finally, we discuss an application of this model to inter-household consumption data.

JEL classification: C14; C72; D11; D59

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1. Introduction

In general, economic theory assumes individual demand is the result of consumers maximizing independent utility functions subject to budget constraints. Some economists however, have questioned this assumption of convenience. As early as 1899, Veblen observed that social status was an important consideration for the nouveau riche of 19th century capitalist societies and used

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the term 'conspicuous consumption' to describe the practice of lavish spending to display wealth. Duesenberry in his classic work [17] concerning the consumption function problem, attempted to explain the statistical discrepancy between Kuznets' data on aggregate savings and income in the period 1869–1929, and budget study data for 1935–1936 and 1941–1942, by challenging the conventional assumption of independent preferences. Work on interdependent preferences can also be found in Hopkins and Kornienko [21], Pollak [26], Postlewaite [27], Schall [30] and Sobel [32].

The empirical research on interdependent preferences assume parametric specifications. Our analysis is nonparametric and extends Afriat's seminal nonparametric analysis of independent preferences [1] to interdependent preferences. Afriat provides nonparametric necessary and sufficient conditions for a finite set of observations on prices and individual demands to be consistent with independent utility maximization. The consistency requirement for interdependent preferences is pure-strategy Nash equilibrium. It has been shown that a general model of consumption with interdependent preferences imposes very weak restrictions on observed data (see Deb [15] and Carvajal [10]).

Bracha [4] introduced potential games as a model for interdependent preferences in the context of individual decision making under uncertainty and risk. We show that the class of utility functions which generate concave potentials impose refutable restrictions on observed data while still encompassing a large set of preferences. That is, we present necessary and sufficient conditions for constructing a concave potential function which Nash rationalizes the data.

Our model of interdependent preferences can be extended to study aggregate data. Carvajal [10] examines interdependent preferences using aggregate data, fixing a single good as the externality good. His results are largely negative (i.e., his model imposes no restrictions on aggregate data), although he does obtain positive results under the relatively strong assumption of weak separability in the externality good. By contrast, the concave potential model presented in this paper allows all goods to act as externalities. In spite of this general framework, we find that the concave potential model imposes nontrivial refutable restrictions on aggregate data. We show that this is a strengthening of the results of Brown and Matzkin [7] on refutable pure exchange economies to pure exchange economies with interdependent preferences.

The model of this paper can be used to study household consumption data. There has been a substantial amount of research on household consumption. The introduction of the collective household model of Chiappori [12,13] was the first notable divergence from the unitary approach, which modeled the household as a single unit, where members of the household were assumed to have common goals. This divergence was necessary as there was mounting empirical evidence that the unitary model was rejected on household data (see for example, Blundell, Pashardes and Weber [2], Browning and Meghir [9]). The collective model allows the members of the household to have different preferences and allows a household member's private unobserved consumption to act as an externality on another household member. In contrast, our framework assumes household member's have common goals (as in the unitary model) but allows a household's consumption to act as an externality on another household. An example of this, is the well known phenomenon of "keeping up with the Joneses." We discuss the application of our model to household data in Section 4.

We would like to mention a few other related works. Snyder [31] shows that the hypothesis of Pareto efficiency in the provision of public goods is falsifiable when finite data is available on market prices, production levels and individual incomes. Cherchye et al. [11] analyze a nonparametric version of the collective household model mentioned above when household level consumption is observed. Chiappori and Ekeland [14] show that the general version of the collective model is not identified, that is, individual preferences and the decision process cannot be uniquely inferred from household consumption data. In other words, they show that a continuum of different structural settings can generate identical observable household behavior.

The paper is organized as follows. In Section 2, we describe a general model of consumption with interdependent preferences. In Section 3, we describe the general class of potential games and in Section 3.1, we characterize observed data for consistency with concave potential games, as well as provide an example of aggregate data which refutes the model. Section 3.2 discusses the role played by concavity in the model. In Section 4, we discuss the application of this model to household consumption data and provide some concluding remarks.

2. The model

The economy consists of N individuals and L goods. Person i's consumption bundle is denoted by $x^i \in \mathbb{R}_+^L$. Person i has a utility function $u^i : \mathbb{R}_+^{NL} \to \mathbb{R}$. Person i's utility level is $u^i(x^1, \ldots, x^i, \ldots, x^N)$ (henceforth represented as $u^i(x^i, x^{-i})$) which depends not only on her consumption x^i but also on the consumption of the other individuals x^{-i} in the economy. Initially, we merely assume that $u^i(x^i, x^{-i})$ is strictly monotonic in individual consumption x^i and make no assumptions about the nature of the externalities x^{-i}. Prices are denoted by p. When we consider aggregate data we will denote person i's income by I^i and aggregate consumption by w.

We first consider the case where we observe market prices and individual level consumption data. The data consists of repeated observations of prices and consumption bundles in the economy. Hence, the observed data is of the form $D = \{(p_t, x_t^1, \ldots, x_t^i, \ldots, x_t^N)\}_{t=1}^T$, where the subscripts denote the time period of the observation. We pose the following question—when is the observed data consistent with utility maximization, where utility functions are interdependent? The relevant notion of individual optimization is Nash equilibrium. We say this data is *Nash Rationalizable* or simply *rationalizable*, if there exist utility functions u^i such that for all i and t, we have

$$x_t^i = \operatorname*{argmax}_{p_t x \leqslant p_t x_t^i} u^i\left(x, x_t^{-i}\right).$$

That is, each player chooses a consumption bundle in her budget set which is a best response to every other player's actions in each observation. For player i, it can be thought that for the subset of the data where every other players' actions stay the same, there are no externalities. Hence, it follows from Afriat's theorem that player i's actions must satisfy GARP (Generalized Axiom of Revealed Preference, see Varian [36]) on this subset of data. This is an intuitive necessary condition for rationalization. When no assumptions are made about the nature of the externalities, Deb [15] characterizes the general model by showing that this condition is also sufficient. He shows that while the general model is refutable, it imposes very weak restrictions on the observed data. Moreover, these restrictions disappear when aggregate data is observed. Carvajal [10] finds the same result for an economy where only one good acts as an externality.

3. Potential games and consumption

Potential games were introduced in Monderer and Shapley [24]. A classical example of a potential game is a Cournot oligopoly game. Monderer and Shapley [24] show that every congestion game (Rosenthal [29]) is a potential game. Other studies which use potential games are

Mäler et al. [23] in their study of the economics of shallow lakes, Konishi et al. [22] study a poll tax scheme for the provision for a public good, Garcia and Arbeláez [18] evaluate impacts of mergers in Colombian wholesale market for electricity. Bracha and Brown [5] propose a behavioral theory of choice, where the individual is a composite agent consisting of a rational and emotional process which is represented as a potential game.

Let $\Gamma(u^1, \dots, u^N)$ be an arbitrary game of N players where player i has strategy set $Y^i \subseteq \mathbb{R}^{n_i}$ (n_i being a positive integer). Player i's payoff function is $u^i : Y \to \mathbb{R}$ where $Y = Y^1 \times \cdots \times Y^N$. A function $P : Y \to \mathbb{R}$ is an *ordinal potential* if for every $i \in N$ and for every $y^{-i} \in Y^{-i}$

$$u^i(x, y^{-i}) - u^i(z, y^{-i}) > 0 \quad \text{iff} \quad P(x, y^{-i}) - P(z, y^{-i}) > 0 \quad \text{for all } x, z \in Y^i.$$

An *exact potential* or simply a *potential* is a function $P : Y \to \mathbb{R}$ such that for every $i \in N$ and for every $y^{-i} \in Y^{-i}$

$$u^i(x, y^{-i}) - u^i(z, y^{-i}) = P(x, y^{-i}) - P(z, y^{-i}) \quad \text{for all } x, z \in Y^i.$$

Γ is called an ordinal potential game if it admits an ordinal potential and a potential game if it admits an exact potential. For differentiable utility functions Deb [15] provides the following characterization.

Theorem 3.1. *(See Deb [15].) Given a game Γ. Assume the utility of each player is defined on an open convex set*[1] *Z such that $Y_1 \times \cdots \times Y_N \subseteq Z$, or in other words $u^i : Z \to \mathbb{R}$. Moreover assume the utility functions are twice continuously differentiable on Z. Then Γ is a potential game if and only if*

$$\frac{\partial^2 u^i}{\partial y^i_{m_i} \partial y^j_{m_j}} = \frac{\partial^2 u^j}{\partial y^j_{m_j} \partial y^i_{m_i}} \quad \text{for all points in } Z \text{ as well as for every } i, j \in N$$
$$\text{and for all } 1 \leqslant m_i \leqslant n_i,\ 1 \leqslant m_j \leqslant n_j.$$

The above result shows that the restriction to utility functions which generate a potential is simply a restriction on the mixed partial derivatives, much like concavity is a restriction on the Hessian. This class of games is large and it allows goods to be either positive or negative externalities, and this could differ across players. In the setting of consumption, the potential function must be strictly monotone in all arguments. This reflects the fact that each player's utility function is strictly monotone in individual consumption.

An example to motivate potential games as a model of consumption with externalities is the following. Assume that the Smith's and the Jones' utility functions are composed of two separate additive components—utility from consumption and utility from 'status.' Then a potential game implies that the Smiths and the Joneses care equally about status (that is, they have the same status term), however they may get different utilities from consumption. For example, although both the Smiths and the Joneses want to have a better car than their neighbor's, the Smiths settle for the minivan as they get more utility from going out to expensive dinners, even though they both have the same status terms in their utility functions. This can be modeled by making the consumption term of the Smiths, large with respect to the status term in their utility function. Moreover, by making the consumption term of the Smiths arbitrarily large, we can approximately model the case where the Joneses try to compete with the Smiths but not vice versa. Thus, although potential

[1] In most games, strategy sets are usually closed and convex, but it is straightforward to smoothly extend the utility function to an open convex set containing the strategy set.

games do restrict the possible set of preferences, they impose a reasonable restriction which still encompasses a large set of interdependent preferences.

Deb [15] shows that for two data points, potential games have the same explanatory power as the general model. This implies that the class of potential games also imposes the same weak restrictions on observed data as the general model. While this supports the intuition that preferences which generate potentials are fairly general, further restrictions are needed on the consumers' preferences in order to get a model which imposes nontrivial restrictions on observed data.

3.1. Concave potential games

Concavity is a desirable property of potential functions as it is for utility functions. In a smooth potential game with a concave potential function, a strategy profile is a pure-strategy Nash equilibrium, if and only if, it is a potential maximizer. In addition, if the potential function is strictly concave then there is a unique equilibrium. Neyman [25] shows that smooth strictly concave potential games have a unique Nash equilibrium and a unique correlated equilibrium. Since our characterization of concave potential games is constructive, this uniqueness allows us to predict future consumer behavior, by utilizing the tools developed in Bracha and Brown [5], who use the potential function to predict affective demand in insurance markets.

We now restrict our attention to utility functions which generate a potential which is concave in all arguments (we discuss the role played by concavity in more detail in Section 3.2). This additional restriction makes concave potential games refutable as can be seen by the following characterization of concave potential games.

Theorem 3.2. *For an observed data set* $D = \{(p_t, x_t^1, \dots, x_t^i, \dots, x_t^N)\}_{t=1}^T$ *the following are equivalent*:

(1) *The data set is Nash rationalized by utility functions which admit a strictly monotonic, concave, ordinal potential function.*
(2) *The following system of inequalities*

$$V_{t'} \leqslant V_t + \sum_{i=1}^{N} \lambda_t^i p_t \left(x_{t'}^i - x_t^i\right)$$

have positive solutions for potential values V *and strictly positive solutions for marginal utilities* λ *for all* i, t, t'.
(3) *The data set is Nash rationalized by utility functions which admit a continuous, strictly monotonic, concave potential function.*

Proof. (1) $\Rightarrow$ (2) Since the potential function V is concave, it is continuous and subdifferentiable and hence a Nash equilibrium must satisfy

$$\frac{\partial V(x_t^i, x_t^{-i})}{\partial x^i} \leqslant \lambda_t^i p_t \quad \text{for all } i \text{ and } t$$

where λ_t^i are the strictly positive Lagrangian multipliers. Also since V is concave we must have

$$V\left(x_{t'}^1,\ldots,x_{t'}^N\right) \leqslant V\left(x_t^1,\ldots,x_t^N\right) + \sum_{i=1}^N \frac{\partial V(x_t^1,\ldots,x_t^N)}{\partial x^i}\left(x_{t'}^i - x_t^i\right)$$

$$\Rightarrow \quad V\left(x_{t'}^1,\ldots,x_{t'}^N\right) \leqslant V\left(x_t^1,\ldots,x_t^N\right) + \sum_{i=1}^N \lambda_t^i p_t\left(x_{t'}^i - x_t^i\right).$$

Setting $V_t = V(x_t^1,\ldots,x_t^N)$ for all t, we have the required solutions for the inequalities.

(2) $\Rightarrow$ (3) For arbitrary consumption bundles $x^1,\ldots,x^N$ we define the potential as follows

$$V\left(x^1,\ldots,x^N\right) = \min_{1\leqslant t\leqslant T}\left[V_t + \sum_{i=1}^N \lambda_t^i p_t\left(x^i - x_t^i\right)\right].$$

Using an identical argument to Theorem 2.1 we can show $V(x_t^1,\ldots,x_t^N) = V_t$ for $1 \leqslant t \leqslant T$. Finally, for arbitrary person i, if $p_t x^i \leqslant p_t x_t^i$ then

$$\begin{aligned} V\left(x^i, x_t^{-i}\right) &\leqslant V_t + \lambda_t^i p_t\left(x^i - x_t^i\right) + \sum_{j\neq i} \lambda_t^i p_t\left(x_t^j - x_t^j\right) \\ &= V_t + \lambda_t^i p_t\left(x^i - x_t^i\right) \\ &\leqslant V_t. \end{aligned}$$

Hence, each person is best responding. Since the minimum of concave functions is concave, V is a concave function. Clearly it is also continuous and strictly monotone. Setting everyone's utility function equal to the potential function we have (3).

(3) $\Rightarrow$ (1) This is obvious and it completes the proof. □

We can also consider the implications of this model on aggregate data in an exchange economy, where consumers have interdependent preferences. The relevant notion of equilibrium is Nash–Walras equilibrium (see Ghosal and Polemarchakis [19]). Since our aim is to rationalize the data, we can follow Brown and Matzkin [7] and ignore the fact that we are no longer simply dealing with a consumption game but instead are in the setting of an abstract economy (see chapter 19 of Border [3]). This is clearly illustrated by the following.

An observed aggregate data set $D = \{(p_t, \{I_t^i\}_{i=1}^N, w_t)\}_{t=1}^T$ is consistent with Nash–Walras equilibrium, if we can find feasible consumptions x_t^i for each individual i at each observation t, such that $p_t x_t^i \leqslant I_t^i$ and $\sum_{i=1}^N x_t^i = w_t$, and each consumption bundle x_t^i is a best response to x_t^{-i} for all t, i. Since rationalization merely involves finding feasible consumption bundles such that p_t is an equilibrium price vector, we can avoid the complications of redefining our setting as an abstract economy.

The analogue of Theorem 3.2 for aggregate consumption data can be written as follows and the proof is omitted as it is a straightforward extension of Theorem 3.2.

Theorem 3.3. *For an observed data set $D = \{(p_t, \{I_t^i\}_{i=1}^N, w_t)\}_{t=1}^T$ the following are equivalent:*

(1) *There exist utility functions which admit a strictly monotonic, concave ordinal potential function such that at each observation t, p_t is a Nash–Walras equilibrium price vector for the exchange economy.*

(2) *The following system of inequalities*

$$V_{t'} \leqslant V_t + \sum_{i=1}^{N} \lambda_t^i p_t \left(x_{t'}^i - x_t^i\right),$$
$$p_t x_t^i = I_t^i,$$
$$\sum_{i=1}^{N} x_t^i = w_t$$

have positive solutions for potential values V, feasible consumptions x and strictly positive solutions for marginal utilities λ for all i, t, t'.

(3) *There exist utility functions which admit a continuous, strictly monotonic, concave potential function such that at each observation t, p_t is a Nash–Walras equilibrium price vector for the exchange economy.*

We will concentrate on the implications of the latter result. Any refutable restrictions on aggregate data will carry over to individual data. To show that the model of consumption using concave potential games is refutable, we need to show that the multivariate polynomial inequalities of Theorem 3.3 are refutable. To show that the system of inequalities is consistent we must construct an example of data which satisfies them (Example 3.1). To show that the inequalities are not always satisfied, it suffices to construct an example of data which violates the inequalities of Theorem 3.3 (Example 3.2). Hence it follows from the Tarski–Seidenberg theorem (Tarski [33]) that the system of inequalities are refutable as the inequalities are solvable for some but not all consumption data sets (see Brown and Kubler [6] for discussion).

The inequalities of Theorem 3.3 are simply the sum of the equilibrium inequalities of Theorem 2 in Brown and Matzkin [7]. This means that any aggregate data set that satisfies their inequalities will also satisfy ours. Hence, their model, which is the standard exchange economy model with independent preferences, is a special case of our concave potential model. A more intuitive explanation is the following.

Brown and Matzkin find necessary and sufficient conditions for the existence of independent utility functions and feasible consumption bundles, so that the observed aggregate data corresponds to an equilibrium of the exchange economy. These conditions are also necessary and sufficient for there to exist *concave* independent utility functions and feasible consumption bundles, so that the observed aggregate data corresponds to an equilibrium of the exchange economy. Concave independent utility functions constitute a concave potential game where the potential function is simply the sum of the utility functions. Hence, the exchange economy with independent preferences is a special case of a concave potential game. We now construct an example of aggregate data which satisfies the inequalities of Theorem 3.3 but does not satisfy the equilibrium inequalities of Brown and Matzkin. Hence, Theorem 3.3 strengthens their testable results to interdependent preferences.

Example 3.1. Consider the following aggregate consumption data

$$p_1 = (1, 2), \qquad I_1^1 = 14, \qquad I_1^2 = 1, \qquad w_1 = (3, 6),$$
$$p_2 = (2, 1), \qquad I_2^1 = 14, \qquad I_2^2 = 1, \qquad w_2 = (6, 3).$$

For player 1, every feasible consumption bundle in observation 1 is affordable under observation 2 and vice versa. However the same is not true for player 2. In particular the bundle $(1, 0)$ is

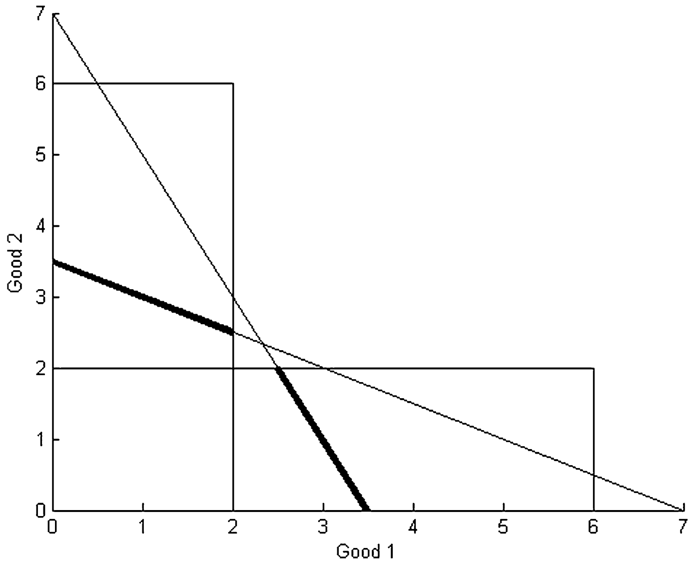

Fig. 1.

affordable for player 2 under period 1 prices and income but not under those of period 2. Thus although these observations would not satisfy the inequalities of Brown and Matzkin [7], we can find feasible consumptions x, potential levels V and Lagrangian multipliers λ such that they satisfy the inequalities of Theorem 3.3. In particular consider the following values

$$x_1^1 = (2,6), \qquad x_1^2 = (1,0), \qquad \lambda_1^1 = 1/4, \qquad \lambda_1^2 = 1, \qquad V_1 = 2,$$
$$x_2^1 = (6,2), \qquad x_2^2 = (0,1), \qquad \lambda_2^1 = 1/4, \qquad \lambda_2^2 = 3, \qquad V_2 = 1.$$

It is surprising to find that a general equilibrium model, where we allow interdependent preferences, has refutable restrictions on aggregate data. But this is in fact the case for the concave potential model as is shown by the following example.

Example 3.2. Consider the following aggregate consumption data:

$$p_1 = (1,2), \qquad I_1^1 = 7, \qquad I_1^2 = 7, \qquad w_1 = (2,6),$$
$$p_2 = (2,1), \qquad I_2^1 = 7, \qquad I_2^2 = 7, \qquad w_2 = (6,2).$$

The observed aggregate consumption only allows feasible individual consumptions that lie in the Edgeworth boxes shown in Figs. 1 and 2. Fig. 1 is similar to Fig. 1 in Brown and Matzkin [7] and it shows that for player 1, every feasible consumption bundle in observation 1 is affordable under observation 2 income and prices and vice versa. The feasible bundles for player 1 are shown by the dark shaded line in each box. Fig. 2 provides an equivalent analysis for player 2. This is the key difference between this example and that of Brown and Matzkin. The situation described by Fig. 1 was sufficient to violate their equilibrium inequalities. We now show that these data points cannot satisfy the inequalities of Theorem 3.3.

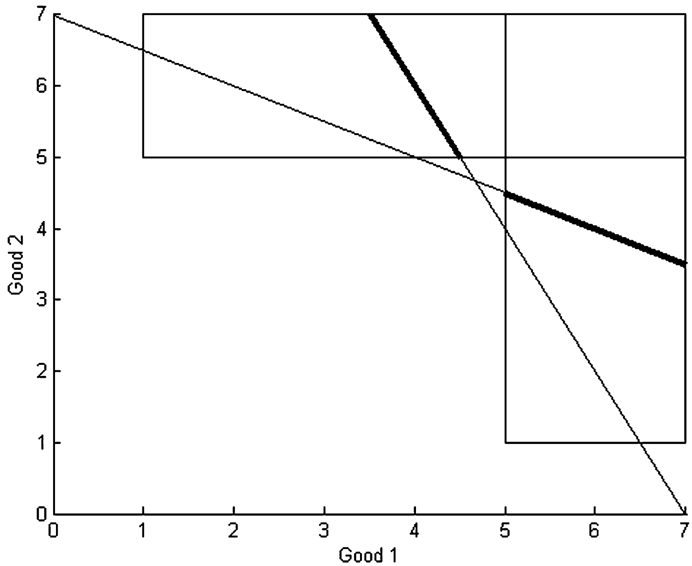

Fig. 2.

Let us individually consider the two inequalities which need to be satisfied:

$$V_1 \leqslant V_2 + \lambda_2^1 p_2\big(x_1^1 - x_2^1\big) + \lambda_2^2 p_2\big(x_1^2 - x_2^2\big), \tag{1}$$

$$V_2 \leqslant V_1 + \lambda_1^1 p_1\big(x_2^1 - x_1^1\big) + \lambda_1^2 p_1\big(x_2^2 - x_1^2\big). \tag{2}$$

For any feasible consumption bundles for player 1, we know from Fig. 1 that $p_2(x_1^1 - x_2^1) < 0$ and $p_1(x_2^1 - x_1^1) < 0$. Similarly from Fig. 2 we know that the same holds for player 2, that is, $p_2(x_1^2 - x_2^2) < 0$ and $p_1(x_2^2 - x_1^2) < 0$. Hence, inequality (1) will imply $V_2 > V_1$ as the λ's must be strictly positive. Similarly, (2) will imply $V_1 > V_2$ which is the contradiction we seek.

3.2. The role of concavity

The key insight in Afriat [1] is that at the observed data points, the utility levels and the gradients need to be related in a specific way (via the Afriat inequalities) in order for the data to be rationalizable. Many of the results in the revealed preference literature follow from this observation. In a model with externalities, consumer i best responding implies only that $\frac{\partial u^i(x^i, x^{-i})}{\partial x^i} = \lambda^i p$ but places no restriction on $\frac{\partial u^i}{\partial x^j}$ for $j \neq i$. As a result, we are free to choose $\frac{\partial u^i}{\partial x^j}$ when rationalizing the data and this is the reason that the general model imposes extremely weak restrictions on observed data. The main insight of this paper is using potential games to solve this problem. Restricting attention to potential games allows us to aggregate preferences of different individuals into a single potential function (equivalently, we can replace each person's utility function u^i by V). Since each person is best responding this implies that $\frac{\partial V(x^i, x^{-i})}{\partial x^i} = \lambda^i p$ for all i. Moreover, restricting attention to concave potential games implies that for any two observations $(p_t, x_t^1, \ldots, x_t^N)$ and $(p_{t'}, x_{t'}^1, \ldots, x_{t'}^N)$, it must be the case that $V(x_{t'}^1, \ldots, x_{t'}^N) \leqslant V(x_t^1, \ldots, x_t^N) + \sum_{i=1}^N \frac{\partial V(x_t^1, \ldots, x_t^N)}{\partial x^i}(x_{t'}^i - x_t^i)$. Thus, analogous to Afriat's theorem, potential levels $V(x_{t'}^1, \ldots, x_{t'}^N)$ and $V(x_t^1, \ldots, x_t^N)$ cannot be chosen arbitrarily and must be related through the above inequality. This results in the refutability of the concave potential model.

When we do not impose concavity, the above inequality need not hold. In fact, the only restriction on the potential function is the strict monotonicity of V in the natural partial order on $\mathbb{R}_+^{NL}$ (a consequence of strict monotonicity in individual consumption). In the classical model without externalities, if a bundle x_t is revealed preferred to another bundle $x_{t'}$, then $U_t \geqslant U_{t'}$. This is what ties the revealed preference axiom GARP to the Afriat inequalities and can be used to show the equivalence between rationalization by a utility function and rationalization by a concave utility function. However, in our model, the externalities can be both positive or negative and hence there is no natural revealed preference relation on the bundles of goods. Thus, it is not possible to use a modification of the standard argument to show the equivalence between rationalization by a potential and rationalization by a concave potential. In fact, in our setting, this equivalence does not hold and hence concavity is an important property for refutability of the model.

To make this intuition a little more concrete, we present an informal argument which shows that data which violates the inequalities of Theorem 3.2 can still be rationalized by a potential function. Consider an economy with 2 individuals where our data set consists of 2 data points or $D = \{(p_t, x_t^1, x_t^2), (p_{t'}, x_{t'}^1, x_{t'}^2)\}$. We restrict our attention to the case where $p_t x_t^1 > p_t x_{t'}^1$, $p_{t'} x_{t'}^1 > p_{t'} x_t^1$ and $p_t x_t^2 > p_t x_{t'}^2$, $p_{t'} x_{t'}^2 > p_{t'} x_t^2$. This was true of the data in Example 3.2 and using an identical argument, it is easy to show that this data cannot be rationalized by a concave potential. We now outline a construction[2] which can be used to find a potential function which rationalizes this data.

We define $C = (x_t^1, \circ) \cup (x_{t'}^1, \circ) \cup (\circ, x_t^2) \cup (\circ, x_{t'}^2)$ where

$$\big(\circ, x_s^2\big) = \big\{\big(y, x_s^2\big)\colon\ y \in \mathbb{R}_+^L\big\} \subset \mathbb{R}_+^{2L}, \quad \text{where } s \in \{t, t'\},$$
$$\big(x_s^1, \circ\big) = \big\{\big(x_s^1, y\big)\colon\ y \in \mathbb{R}_+^L\big\} \subset \mathbb{R}_+^{2L}, \quad \text{where } s \in \{t, t'\}.$$

Since neither $(x_t^1, x_t^2) \geqslant (x_{t'}^1, x_{t'}^2)$ nor $(x_t^1, x_t^2) \leqslant (x_{t'}^1, x_{t'}^2)$ in the natural partial order on $\mathbb{R}_+^{2L}$, we are free to choose either $V(x_t^1, x_t^2) \geqslant V(x_{t'}^1, x_{t'}^2)$ or $V(x_t^1, x_t^2) \leqslant V(x_{t'}^1, x_{t'}^2)$ which is critical for the following construction.

Consider the set $(x_t^1, \circ)$. It is possible to construct a function $V_t^1 : (x_t^1, \circ) \to \mathbb{R}$ such that at prices p_t, player 2 is maximizing V_t^1 by choosing x_t^2. In other words, V_t^1 is player 2's utility function when player 1 chooses x_t^1 and is consistent with the choice of x_t^2 at p_t. Similarly, we can construct functions $V_{t'}^1$, V_t^2, $V_{t'}^2$ for the sets $(x_{t'}^1, \circ)$, $(\circ, x_t^2)$, $(\circ, x_{t'}^2)$ respectively. Moreover, it is possible to construct these functions in such a way so that they agree at the observed consumption bundles or $V_t^1(x_t^1, x_t^2) = V_t^2(x_t^1, x_t^2)$, $V_t^1(x_t^1, x_{t'}^2) = V_{t'}^2(x_t^1, x_{t'}^2)$, $V_{t'}^1(x_{t'}^1, x_t^2) = V_t^2(x_{t'}^1, x_t^2)$ and $V_{t'}^1(x_{t'}^1, x_{t'}^2) = V_{t'}^2(x_{t'}^1, x_{t'}^2)$. Equivalently, it is possible to define a potential function $V : C \to \mathbb{R}$ on the subset C of $\mathbb{R}_+^{2L}$, such that

$$x_s^i = \operatorname*{argmax}_{p_s x \leqslant p_s x_s^i} V\big(x, x_s^{-i}\big)$$

for each $i \in \{1, 2\}$ and each $s \in \{t, t'\}$. Then using the extension theorem of Herden [20], this function can be lifted to the entire space $\mathbb{R}_+^{2L}$ preserving the only required property of V—strict monotonicity in the natural partial order on $\mathbb{R}_+^{2L}$. Hence, it is possible to rationalize data

[2] The complete construction can be found in the appendix of Deb [15].

which violates the inequalities of Theorem 3.2 using a potential function. Thus, concavity plays a crucial role in the refutability of the model.

As mentioned earlier, strict concavity in potential games yields unique equilibria which allows us to use our model to predict consumer behavior. That said, it is important to discuss what the assumption of concavity on the potential function implies for the underlying preferences. Utility functions which generate a concave potential need not be concave themselves. Ui [35] shows that requiring the potential to be concave imposes an additional constraint on the gradients of the underlying utility functions. He shows that a smooth potential game has a concave potential, if and only if, the payoff gradient of the game is monotone.[3] Formally, the payoff gradient is monotone if

$$\sum_{i=1}^{N}\left(\frac{\partial u^i(x_t^i, x_t^{-i})}{\partial x^i} - \frac{\partial u^i(x_{t'}^i, x_{t'}^{-i})}{\partial x^i}\right) \cdot \left(x_t^i - x_{t'}^i\right) \leqslant 0 \quad \text{for all } x_t, x_{t'} \in \mathbb{R}_+^{NL}.$$

This condition requires player i's utility function u^i to be concave in individual consumption x^i, however, u^i can be either concave or convex in the externality x^{-i} (see Deb [15] for an example of when u^i is convex in the externality). While the above condition is clearly not innocuous, it is possible to construct a wide variety of different preferences which satisfy it and as a result the concave potential model is a substantial generalization of the classical model without externalities.

4. Potential games and the household

A natural application of concave potential games is to analyze household consumption data. By assuming members of the household have common goals, the preferences of the household can be modeled by a single utility function and we can use Theorem 3.2 on household consumption data. In practice, it is easier to get consumption data at the level of the household, than it is to get consumption data for individuals within the household. Unobserved private consumption of household members is the reason that makes nonparametric tests of the collective model difficult to implement.

Browning and Chiappori [8] provide a rigorous theoretical framework for testing consistency of data with the collective model. They assume members of the household may have different, possibly divergent preferences, where each a household member's private consumption might act as an externality for another household member. The only assumption is that regardless of how decisions are made, the outcome must be efficient. Thus, if the household consists of a husband and a wife whose utilities are given by u^h and u^w, efficiency implies that the household's utility is given by $u^h + \lambda u^w$, where λ is a measure of the "distribution of power." The model's explanatory power lies in the fact that although the utility functions stay the same, λ can be different across different observations. Lastly, they assume individual consumption is not observed. They provide a parametric characterization of this model.

The assumption of changing "distributions of power" across observations seems somewhat ad hoc. Also, empirical studies such as Udry [34] show that even the assumption of efficiency is not as innocuous as it may seem. Finally, nonparametric tests of this model (Cherchye et al. [11])

[3] This condition is similar to the well known diagonal strict concavity condition defined in Rosen [28] which is widely utilized in the game theory literature.

are only testable for a special case of the collective model, moreover, Deb [16] shows them to be computationally inefficient even for this special case.

By contrast, the nonparametric test for the concave potential model is easily and efficiently implemented, since, if we observe household consumption, then the inequalities (of Theorem 3.2) are linear in the unknowns. Moreover, the assumption that households violate the unitary model because their preferences are influenced by the consumption of others is intuitive and the simplicity of the nonparametric test makes testing such a model an interesting empirical exercise. The intrahousehold collective model derives its explanatory power by allowing private consumptions of household members which in practice are unobserved. In contrast, our inter-household model utilizes the observed consumptions of the other households in the economy in order to rationalize the consumption of a particular household. Lastly, our approach is also constructive. We construct a concave potential function which Nash rationalizes the data, and this allows us to use the tools in Bracha and Brown [5] to predict future household behavior.

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