ECO 426 (Market Design) - Lecture 6

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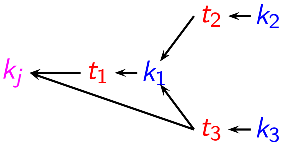
minimal chains and strategy proofness

- minimal chain and remove tail kidney is strategy proof
  - A patient t_i misreporting preferences in a round matters if it changes the allocation. After misreporting t_i can
    - be part of a cycle and receives k_j. Telling the truth leaves k_j and other kidneys available for later rounds. Telling the truth would give t_i a kidney at least as good as k_j;
    - be the lead patient of a w-chain. The wait list option is available to the patient in later rounds. Telling the truth would give an outcome at least as good as w to t_i.
- minimal chain and keep tail kidney is NOT strategy proof



TB priorities: $t_2 \succ t_3 \succ t_1$

t_3's preferences: $k_1 \succ k_j \succ k_2$

  - telling the truth the outcome for t_3 is k_2
  - by misreporting preferences t_2 can get k_j

School Choice

- Allocating students to schools, three type of problems
  - College Admission:
    - Students have preferences over schools **and** schools have preferences over students, both sides are strategic
    - "Welfare" of both schools and students matter
    - Many to one matching (Gale and Shapley 1962 paper)
  - Student Placement:
    - Only students have preferences
    - Schools are not "strategic players" they simply "rank" students according to objective test scores (e.g. standardized tests)
    - Students are the only "economic agents" (i.e. "welfare" of schools does not matter)
  - School Choice:
    - Only students have preferences
    - Schools are non strategic assign priorities (exogenous) to students, rather than ranks (endogenous)
    - Students are the only "economic agents"

Student Placement

- Model:
  - A set of **students**, I, with (strict) preferences over a set of **schools** S
  - For each school, s, a **"quota"** q_s of students (maximum number of students to take)
  - A set of **categories**, C, and for each school $s \in S$, a category $c_s \in C$ (e.g. medicine, engineering, law, management etc.)
  - An **exam score profile** $\{e^i_c\}_{i \in I, c \in C}$(i.e. e^i_c is the exam score of student i in category c) such that in each category all students are strictly ranked (i.e. no ties in any given category)
- Describing "preferences" of a school s by the students' exam scores in the school's category, c_s, one obtains an **associated college admissions**

Student Placement

- Objective: Find an assignment of students to schools such that no school exceeds its quota

  $\mu : I \to S \cup \emptyset$ s.t. no school has more students than its capacity

  the notation $\emptyset$ stands for the "no school option"
- Desirable properties:
  - **Individual rationality:** no student prefers the no school option to the school she is assigned to.
  - **No justified envy:** whenever a student i prefers another student j's assignment, $\mu(j)$, to her own (i.e. i envies j), i ranks worse than j in school $\mu(j)$'s category (i.e. the envy is not justified, j deserves more than i being in school $\mu(j)$.)
  - **No waste:** Whenever a student i prefers another school s to the one she is assigned to, school s has no empty slot.
  - **Pareto efficiency:** There is no assignment that makes no **student** worse off and some **student** better off.

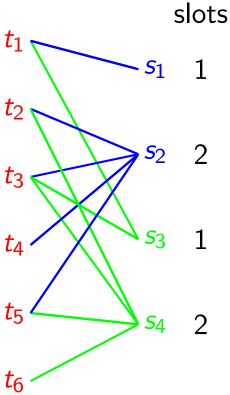
- Individual rationality + no justified envy + no waste coincide with a stability requirement
- Proposition. A school placement matching is individually rational and eliminates waste and justified envy, if and only if it is stable in the associated college admission problem.

Mechanisms for student placement: one category

- **One category serial dictatorship**
  - Assign to each student a priority equal to her test score rank
  - Let students choose their school sequentially, following the priority ordering
- When there is just one category, the serial dictatorship mechanism induced by the category's ranking is the only mechanism that
  - is Pareto eficient; **and**
  - eliminates justified envy.
- no-waste and individual rationality follow from Pareto efficiency

- Multi-category serial dictatorship.
  Example: Six students $\{i_1, i_2, \ldots, i_6\}$ two categories M and E four schools (two per category) s_1, s_2 and s_3, s_4. School 1 and 3 have quota 1, school 2 and 4 have quota 2. Student ranking in each category: $\{1, 2, 3, 4, 5, 6\}$ and $\{1, 3, 5, 2, 4, 6\}$.

- run the serial dictatorship separately in each category
- if a student is assigned to more than one school, change her preferences so that all school worse than best assigned school are not acceptable
- run the SD algorithm again
- repeat until no student is assigned to more than one school



Mechanisms for student placement: multi-category

- The multi-category serial dictatorship mechanism is used in Turkey in the centralized college student placement.
- Proposition: The multi-category serial dictatorship is equivalent to the DA **school optimal** mechanism.
- Limits: Pareto-efficiency is not guaranteed
  - Schools have no "preferences" only student welfare matter for pareto optimality
  - Some unstable matching might improve the welfare of some students without damaging any

    Example: Three students $\{i_1, i_2, i_3\}$ and three schools $\{s_1, s_3\}$ and $\{s_2\}$. Student ranking is $\{1, 3, 2\}$ and $\{2, 1, 3\}$ respectively. Preferences of students

i_1	s_2	s_1	s_3
i_2	s_1	s_2	s_3
i_3	s_1	s_2	s_3

    Unique stable matching is $(i_1, s_1)(i_2, s_2)(i_3, s_3)$
    Pareto dominated by $(i_1, s_2)(i_2, s_1)(i_3, s_3)$

Mechanisms for student placement: multi-category

- If a mechanism eliminates justified envy it cannot be Pareto efficient (i.e. the mechanism cannot guarantee that the outcome will be Pareto efficient).
- DA student optimal mechanism guarantees
  - no justified envy (multi-category SD: yes)
  - the outcome is Pareto efficient among those that satisfy no-justified envy (multi-category SD: NO)
    - no-justified envy is equivalent to stability
    - student optimal matching is favorite by all students among stable matchings
  - Strategy proof (multi-category SD: NO)

School Choice

- Historically children go to neigborhood schools
- **School choice programs** were introduced to give family more flexibility and also introduce competition between schools (i.e. eliminating the "monopoly" of schools over students in their neighborhood)
- In school choice programs factors other than the students' place of residence are considered to determine school attendance eligibility

School Choice problem

- Model:
  - A set of **students**, I, with (strict) preferences over a set of **schools** S
  - For each school, s, a **"quota"** q_s of students (maximum number of students to take)
  - Each school ranks students according to **priorities**. Multiple students might be assigned the same priority (i.e. ranking of students is not necessarily strict.)
    - priorities (exogenous) vs. test scores (endogenous)
    - no-justified envy is less critical

Boston mechanism

- (old) Boston Public School mechanism
  - In place in Boston until 2005
  - Still widely used elsewhere

1. Schools exogenously determine priority ordering over students
   - depend on: distance from school, whether sibling already attends school, lottery draw etc..
2. Students submit preference ranking over schools
3. Assignment is determined by
   - **Only** students' top choices are considered. Students are assigned to schools according to their priority until no space is left or all students are assigned
   - **Only** remaining students' second choices are considered.....

     ⋮

   - **Only** remaining students' k^{th} choices are considered....
   - Process ends when all students have been assigned a school

Boston mechanism

- Problem: Obviously fails strategy proofness
  - Would you list as top choice a very popular school where you have low priority?
  - Preference reporting as a "coordination game"

NYC schools

- NYC school choice: Semi decentralized
  - Student (about 90,000 high school students) submit up to five applications
  - Schools receive applications and either accept or wait-list applicants
  - Students accept and reject offers
  - More offers from waitlist are made (up to three rounds)
  - Students unassigned at the end of mechanism (about 30,000) are administratively assigned to a school.

Student proposing DA mechanism

- Treating school priorities as preferences, a college admission problem obtains
- Could use the student proposing DA mechanism as an alternative to Boston or NYC mechanism
  - Outcome is stable
  - Optimal for students among stable matchings
  - Strategy proof
- Problem: Can be inefficient (same example as in school placement problem)

TTC

- How doe we address efficiency? We can treat priorities as "owned" by students (i.e. priorities can be traded)
- Allow students to "trade priorities" using a TTC mechanism
- School Choice TTC mechanism
  - Break priority ties through a lottery (obtain strict priorities)
  - Each student points to favorite school
  - Each school points to student with highest priority
    - Remove all students in a cycle
    - Remove all schools in a cycle **if they have reached capacity**
  - Repeat until all students are assigned to a school
- School choice TTC mechanism is
  - Strategy proof (DA student optimal: yes)
  - Pareto efficient (DA student optimal: no)
- Efficiency improvements come from **"trading priorities"**

Boston and NYC School Choice programs

- NYC adopted the DA student optimal mechanism in Fall 2003
- Boston Public School program adopted the DA student optimal mechanism starting in 2006
  - DA algorithm easier to understand
    - Experimental study shows less preference manipulation in DA than TTC
  - School boards did not like the idea of "trading priorities"
- Big improvement in outcomes over previous mechanisms
  - Number of students administratively matched in NYC dropped to 3,000 from 30,000 after change in mechanism

Inefficiency from tie-breaking

- Coarse priorities are broken by a lottery draw to obtain strict priority ranking (needed for the algorithm)
- Breaking of priority creates an "artificial" ranking of students and might create inefficiencies
- Example: Three students $\{i_1, i_2, i_3\}$, three schools $\{s_1, s_2, s_3\}$. Priorities and preferences given by

$$\begin{array}{c|ccc} i_1 & s_2 & s_1 & s_3 \\ i_2 & s_3 & s_2 & s_1 \\ i_3 & s_2 & s_3 & s_1 \end{array} \qquad \begin{array}{c|cc} s_1 & i_1 & \{i_2, i_3\} \\ s_2 & i_2 & \{i_1, i_3\} \\ s_3 & i_3 & \{i_1, i_2\} \end{array}$$

  Break ties everywhere with ordering $1 \succ 2 \succ 3$
  - DA student optimal matching is $(i_1, s_1)(i_2, s_2)(i_3, s_3)$
  - Pareto dominated by $(i_1, s_1)(i_2, s_3)(i_3, s_2)$

- How to break ties?
  - single lottery (i.e one for all schools) vs multiple lotteries (i.e. on at each school)
  - single lottery seems to do better than multiple lotteries in simulations
  - NYC adopted a single lottery protocol

improvement cycles

- Look for (ex-post) improvements to the DA student proposing outcome
  - Search for a cycle of students $i_0, i_1, \ldots, i_N = i_0$ such that
    - Each student i_n prefers the school of student i_{n+1} to own
    - Student i_n has the highest priority among those who would like to switch to the school of student i_{n+1}
  - Students $i_0, i_1, \ldots, i_N = i_0$ form a **stable improvement cycle**
    - Starting from a stable matching and implementing a SIC yields a **Pareto improvement** and a new **stable** matching
    - Starting at a stable matching that is **not** student optimal a SIC exists
  - A student optimal matching can be reached applying SIC repeatedly until no SIC can be found
  - Limits: SIC procedure is not strategy proof.