Inflation Targeting, Price-Level Targeting, the Zero Lower Bound, and Indeterminacy[*]

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Abstract

We compare inflation targeting and price-level targeting in a simple New Keynesian framework, with particular attention to multiple steady-states, regions of indeterminacy in parameter space, and global stability. First, we show that the well-known problem of multiple steady-state equilibria under a Taylor rule with inflation targeting is absent under a modified Taylor rule and price-level targeting. Second, we show that the model's dynamics in the neighbourhood of the steady state are determinate for a much wider range of parameter values under price-level targeting, in particular for different values of the central bank's reaction function. Third, we compare the size and shape of the convergent manifolds under the two different monetary policy regimes.

1   Introduction

There is an extensive literature that compares the performance of inflation

targeting (henceforth IT) and price-level path targeting (henceforth PT) in terms

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of their abilities to stabilize the economy and enhance economic welfare.[1] Under

rational expectations and perfect central-bank credibility, PT has the advantage

of conveying an expectational advantage in the face of an unexpected innovation

to inflation. Under PT, the central bank commits to offsetting the impact of an

unexpected inflation shock on the price level. In response to a positive inflation

shock, the central bank commits to achieving a rate of inflation that is

temporarily lower than the long-run target rate. This lowers expectations of

future inflation, and firms that set their prices for several periods will choose a

lower price than in response to the same shock under IT.[2]

Many of the studies comparing the performance of IT and PT have used New

Keynesian models solved by approximating agents' first order conditions in the

neighbourhood of a zero-inflation deterministic steady state. Some studies take

into account the zero lower bound on the central bank's policy rate while still

approximating first order conditions around the zero-inflation steady steady

state.[3]

A fundamental problem with this approach is that it ignores the possibility of

multiple steady states. Benhabib, Schmitt-Grohé and Uribe (2001, 2001b,

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[1] See Ambler (2009) for a survey on PT that discusses its potential advantages and disadvantages compared to PT.

[2] We know that the optimal (Ramsey) interest rate rule in the canonical Keynesian model implies a stationary price level. This result was shown by Clarida, Galí and Gertler (1999) and Woodford (1999) (with just equations (1) and (2) below and where the central bank chooses the inflation rate to minimize a quadratic loss function that depends on inflation and the output gap. This suggests that simple PT rules may do better than simple IT rules.

[3] Adam and Billi (2006) linearize the equations of the model and then use projection methods to account for the zero bound on the central bank's policy rate. Amano and Ambler (2009) use higher-order approximations and use a smooth approximation to the kink in the central bank's reaction function at the zero bound.

henceforth BSU) showed that the zero lower bound on the central bank's policy rate implies that under IT there must be two deterministic steady-state equilibria. The literature comparing IT and PT has focused for the most part on the equilibrium where the central bank achieves its target for the inflation rate, while ignoring a second "liquidity-trap" equilibrium at which the nominal interest rate is stuck at or near the zero bound and the inflation rate is negative. The inflation rate is close to satisfying the Friedman rule, which in the context of the New Keynesian model is highly suboptimal because it implies a large, negative output gap.

Mendes (2011) recently conjectured that history-dependent policy rules could eliminate the multiplicity of steady-state equilibria (he focused on stochastic steady states) and demonstrated that this is the case for a simple rule where the central bank's desired policy rate depends negatively on the time spent at the zero bound. The PT regime is an example of history dependence since past innovations to inflation are corrected. It is therefore possible that a PT regime conveys additional benefits compared to IT by eliminating low-inflation equilibria.

This paper compares IT and PT in a simple New Keynesian model with particular attention to the existence of multiple steady states, regions of indeterminacy in parameter space, and global stability. We show the following.

1. There is only one deterministic steady-state equilibrium under PT.

2. Under PT, there exists a second "quasi steady state" in the deterministic

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case in which the gap between the price level and its optimal path grows without bound over time. This quasi steady state corresponds to the low-inflation deterministic steady state under IT.

3. Under PT, there can only be one stochastic steady state equilibrium. If the policy rate is at the lower bound, insofar as agents expect that the interest rate will eventually leave the lower bound in response to a positive shock to inflation, the central bank's commitment to moving the price level back to its target path means that the only possible value for the unconditional expectation of inflation is the central bank's target inflation rate.

4. The model's dynamics are determinate for a much wider range of parameter values under PT than under IT. In particular, determinacy depends much less on the parameter values of the central bank's interest rate reaction function. [Section still to come.]

5. When we drop the assumption of rational expectations and introduce adaptive learning in order to apply global stability analysis from the engineering literature, the PT regime is globally stable, while (as shown by Evans, Guse and Honkapohja, 2008) under IT a negative shock can push the economy into a region of its state space where it veers off toward a low-inflation steady state. [Section still to come.]

6. Following Brunner and Strulik (2002) and using backward integration to solve the model, we show that the size of the convergent manifold under PT is much larger than under IT. [Section still to come.]

2 Theoretical Framework

We consider the basic New Keynesian macroeconomic model given by the following set of equations.[4] The equations can be derived in the standard way by linearizing firms' and households' first order conditions around a deterministic steady state.[5]

$$\pi_t = (1 - \beta)\pi^* + \beta E_t \pi_{t+1} + \varphi y_t, \tag{1}$$

where π_t is inflation, π^* is trend or target inflation, y_t is the output gap, and E_t is the mathematical expectations operator conditional on information available at time t. We assume here that $\pi^* > 0$, so that the central bank aims for a positive inflation rate in the long run.[6]

The New Keynesian IS equation given by

$$y_t = E_t y_{t+1} - \frac{1}{\gamma}\left(i_t - E_t \pi_{t+1} - r_t\right), \tag{2}$$

where r_t is the natural real interest rate and i_t is the short-term nominal interest rate, set directly by the central bank.

The natural rate of interest follows the stochastic process given by

$$r_t \sim N\left(r, \sigma_r^2\right). \tag{3}$$

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[4]We follow much of the literature and Adam and Billi (2006) and Mendes (2011) in particular in using linearized equations except for the central bank's interest rate reaction function.

[5]See Galí (2008) for a detailed derivation of the equations of the standard New Keynesian model. The equations of the model are more complicated when linearized around a non-zero steady-state rate of inflation. See Bakhshi et al. (2007) for details.

[6]Mendes (2011) considers negative values of π^*, and shows that the Friedman rule is impossible in the presence of stochastic shocks to the real rate of interest.

Under IT, the model is completed by the following Taylor rule.

$$i_t^d = r_t + \pi^* + \rho_\pi \left(\pi_t - \pi^* \right) + \rho_y y_t + v_t, \tag{4}$$

where i_t^d is the desired nominal rate of interest, and where $\rho_\pi > 1$ so that the Taylor principle is satisfied. The actual nominal rate of interest is given by

$$i_t = \max \left(0, i_t^d \right), \tag{5}$$

so that the nominal interest rate is subject to a zero lower bound.

Under PT, the monetary policy rule becomes

$$i_t^d = r_t + \pi^* + \rho_p \left(p_t - p_t^* \right) + \rho_y^* y_t + v_t, \tag{6}$$

where p_t is the price level (in logs) and where π_t^* is the projected path of the price-level target (also in logs). The price-level target path evolves according to

$$p_t^* = p_{t-1}^* + \pi^*,$$

where once again π^* is trend inflation. This allows for a price-level target that is not necessarily constant. The realized nominal interest rate is still given by (5). The main distinguishing feature between IT and PT is whether or not unexpected shocks that affect the inflation rate are corrected in the long run or not.

Under PT, it will be convenient to consider the following transformed version of

the model, which introduces the deviation between the price level and its target path as an extra state variable. The Phillips curve (1) can be transformed as follows:

$$(p_t - p_{t-1}) = (1 - \beta) \pi^* + \beta \mathrm{E}_t (p_{t+1} - p_t) + \varphi y_t$$

$$\Rightarrow (p_t - p_t^*) - (p_{t-1} - p_{t-1}^*) + (p_t^* - p_{t-1}^*)$$

$$= (1 - \beta) \pi^* + \beta \mathrm{E}_t (p_{t+1} - p_{t+1}^*) - \beta (p_t - p_t^*) + \beta (p_{t+1}^* - p_t^*) + \varphi y_t.$$

Since $(p_t^* - p_{t-1}^*) = (p_{t+1}^* - p_t^*) = \pi^*$, we get

$$(p_t - p_t^*) - (p_{t-1} - p_{t-1}^*) = \beta \mathrm{E}_t (p_{t+1} - p_{t+1}^*) - \beta (p_t - p_t^*) + \varphi y_t. \quad (7)$$

The New Keynesian IS curve (2) becomes

$$y_t = \mathrm{E}_t y_{t+1} - \frac{1}{\gamma} \left(i_t - \mathrm{E}_t (p_{t+1} - p_{t+1}^*) + (p_t - p_t^*) + \pi^* - r_t \right). \quad (8)$$

The other equations of the model require no transformations.

3 Deterministic Steady States

3.1 Deterministic Steady State under IT

Even before the crisis hit, some researchers questioned the stability properties of the IT framework. BSU (2001, 2001b) showed that IT regimes must theoretically have two steady states under perfect foresight. There is one equilibrium in which

inflation is equal to its target. The other equilibrium is a "liquidity-trap" equilibrium with the nominal interest rate stuck at or near its lower bound and characterized by deflation.

The logic of their argument is illustrated in Figure 1 (taken from Mendes, 2011). The Fisher equation gives a linear relation (with a slope equal to one) between steady-state inflation and the nominal interest rate. The Taylor rule together and the zero lower bound imply a kinked relation between inflation and the nominal interest rate. The positively-sloped segment of this curve has a slope greater than one if the Taylor principle is satisfied. This means that there must be two points of intersection between the two curves and two steady states. The steady state with a zero nominal interest rate has the property that $\pi = -r$. This satisfies the Friedman rule, but the equilibrium is "bad" in this context because it implies a negative output gap which is potentially quite large depending on the parameters of the model.

The result derived by BSU holds under perfect foresight. Evans, Guse and Honkapohja (2008) showed the possibility of large shocks like the ones that initiated the Great Recession leading to deflationary spirals in environments with expectations formed using an adaptive learning rule. Adaptive learning is particularly relevant when analyzing possible changes in the monetary policy framework: subsequent to changes in the framework itself, individuals will typically have to learn how the new regime functions. Their inflation expectations will adapt as they learn how the new regime functions.

We show in Appendix A that there are exactly two deterministic steady states for

our model under IT, in line with the results of BSU. The liquidity-trap steady state, in particular, has the properties that

$$\pi = -r,$$

and

$$y = -\frac{(1-\beta)}{\varphi}\left(r + \pi^*\right).$$

The first of these two equations is just the Friedman rule. This would be socially optimal in a model without nominal rigidities and with a well-defined money demand function. Here, it is not, since it also involves a negative output gap, which is potentially quite large if the φ parameter is small, i.e. if inflation is relatively insensitive to the output gap, which will be the case if with large nominal price rigidities (firms adjust their prices infrequently) or large real rigidities (firms' optimal reset prices are not very sensitive to the output gap).

4   Deterministic Steady State under PT

We show in Appendix B that there can only be one true deterministic steady state in which the deviation of the price level from its target path is constant, that is

$$\left(p_{t+1} - p^*_{t+1}\right) = (p_t - p^*_t) = \left(p_{t-1} - p^*_{t-1}\right) \equiv p^d$$

This equilibrium must have the property that the deviation of the price level p^d must be equal to zero, which also implies a zero output gap. This result would seem to imply that the economy cannot remain stuck indefinitely at the zero lower bound.

However, there is also a "quasi-steady-state equilibrium", equivalent to the liquidity-trap equilibrium in the IT case. We characterize this quasi steady state in Appendix B, starting from the assumption that the realized interest rate is at the lower bound. All of the model's variables are constant in this quasi steady state except for the price-level gap $(p_t - p_t^*)$, which must be decreasing at a rate equal to $-(r + \pi^*)$.

Since the price-level gap is not at rest, and the central bank's desired interest rate is also decreasing over time. However, there is no feedback from this gap to the rest of the model as long as the realized nominal interest rate is stuck at zero. There is no mechanism to pry the economy away from this low-inflation quasi steady state.

We argue in the next section that as long as agents expect that, sooner or later, a shock will push the economy away from the liquidity-trap quasi steady state, so that the unconditional expectation of the realized nominal interest rate is bounded above zero, then the only possible stochastic steady state is one in which the inflation rate is equal on average to its target rate.

5 Stochastic Steady States

5.1 Stochastic Steady State under IT

This case has been covered in detail by Mendes (2011). He shows that there can be either two, one, or zero stochastic steady states in a model like the one developed here. The two-steady-state case is similar to the deterministic case and holds when the volatility of stochastic shocks to the real rate of interest is sufficiently low. If the volatility of the real interest rate is sufficiently high, the expected nominal interest rate for any given rate of inflation increases.[7]

5.2 Stochastic Steady State under PT

In Appendix C, we show that if there is a stochastic steady state under the PT regime, it must have the characteristic that the gap between the price level and the desired price-level path is constant. This immediately implies that the inflation rate is on average equal to the target rate of inflation. We also show that the low-inflation deterministic quasi steady state of the previous subsection does not exist when we consider stochastic steady states.

The stochastic steady state under PT has several interesting properties. As noted in the previous paragraph, the unconditional expectation of the inflation rate is equal to target inflation. This means that there is no inflationary bias or deflationary bias under PT. The expected value of the output gap is zero. The

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[7]The nominal interest rate i_t has a distribution truncated at zero, and its unconditional expectation is an increasing function of the variance of the innovation to r_t given by σ_r^2.

expected value of the realized interest rate is just the unconditional mean of the real interest rate plus the target inflation rate. There is a wedge between the unconditional expectation of the desired interest rate and the realized interest rate. This follows from equation (5) which implies that the realized interest rate is a (left) truncated variable compared to the desired interest rate. Taking unconditional expectations of the modified Taylor rule leads immediately to the following expression for the relation between the wedge and the expected price-level gap.

$$\mathrm{E}p^d = -\frac{1}{\rho_p}\left(\mathrm{E}i - \mathrm{E}i^d\right) < 0.$$

The unconditional expectation of the price level gap is negative, and depends inversely on ρ_p, the parameter that determines how strongly the central bank reacts to the price-level gap. Under pure price-level targeting with $\rho_p \to \infty$, the expected price level gap and the wedge between the realized and desired interest rate disappear. This means that as the central bank reacts more and more strongly against deviations of the price level from its target path, the probability of hitting the lower bound goes goes to zero.

The economic intuition for these results is straightforward. Given the modified Taylor rule, the central bank has a commitment to restore the price level to its target path after any shock. Even if the economy is at in an equilibrium in which the zero lower bound on the policy rate is binding, agents expect that sooner or later a positive shock will occur that will move the economy away from the lower bound. Then, along the transition path back to the target price-level path,

inflation will be higher than the target rate π^*. Averaging over periods where the economy is at the zero bound and periods where it is not, inflation is equal to the target rate.

We also show that the desired nominal interest rate, while lower on average than the realized nominal interest rate because of the zero-bound problem, is arbitrarily close to the realized interest rate on average as the central bank reacts more and more strongly to price-level deviations, that is to say for large values of the ρ_p parameter.

6   Determinacy in Parameter Space

We use the techniques described in Ratto (2008) to analyze the stability of the model in parameter space. Dittmar and Gavin (2004) already explored, in the context of a standard New Keynesian model, regions of the parameter space under IT and PT and concluded that the model's dynamics were determinate for a wider range of parameter values under PT than under IT.

The advantage of the methodology proposed by Ratto (2008) is that it explores the parameter space in a systematic way, and uncovers the parameters that are most important for determining stability versus instability and indeterminacy. [Section incomplete.]

7 Global Stability

7.1 Global Stability in A Model with Learning

The mathematical techniques used to analyze global stability have mostly been developed for dynamic systems with a standard definition of stability. In dynamical systems used in engineering and physics, state variables are typically all predetermined. Without forward-looking or jump variables, saddlepoint stability is clearly not desirable.

In a model with learning, all state variables in the model become backward-looking, and techniques from the engineering literature for dynamical systems can be applied without modification.

[Section incomplete.]

7.2 Global Stability in the Model with Rational Expectations

The development of techniques for analyzing global stability are less developed for economic models with forward-looking or non-predetermined state variables. Here we use backward integration (see Brunner and Strulik, 2002) to analyze the nature of the stable manifold leading to the unique steady-state equilibrium under PT, and compare it to the stable manifold leading to the high-inflation steady state under IT.

[Section incomplete.]

8 Conclusions

Appendices

A Deterministic Steady State under IT

We show the existence of precisely two deterministic steady states in this case. Dropping time subscripts from the equations of the model gives

$$\pi = (1 - \beta)\pi^* + \beta\pi + \varphi y, \tag{9}$$

$$i = r + \pi, \tag{10}$$

$$i^d = r + \pi^* + \rho_\pi \pi - \rho_\pi \pi^* + \rho_y y, \tag{11}$$

$$i = \max\left(0, i^d\right). \tag{12}$$

There are two possible cases, $i = i^d > 0$ and $i = 0$. First consider the case with a positive nominal interest rate in the steady state. Equations (10) and (11) together imply that

$$\pi = \pi^* + \rho_\pi \pi - \rho_\pi \pi^* + \rho_y y$$

$$\Rightarrow (1 - \rho_\pi)\left(\pi - \pi^*\right) = \rho_y y,$$

while equation (9) implies

$$(1 - \beta)\left(\pi - \pi^*\right) = \varphi y.$$

We have two linear equations in two unknowns, the first of which has a positive slope and the second of which has a negative slope. The unique solution is $y = 0$ and $\pi = \pi^*$. This is the steady state where inflation is equal to target inflation and the output gap is zero.

Now consider the case where $i = 0$. Equation (11) now just gives the level of the desired interest rate in the deterministic steady state, which must be negative. The Fisher relation (10) gives

$$\pi = -r.$$

Substituting into (9) and solving gives the following unique solution for the output gap:

$$y = -\frac{(1 - \beta)}{\varphi} (r + \pi^*) .$$

This is the low-inflation steady state. It is clearly an undesirable steady state given the model. The inflation rate is equal to the negative of the real interest rate, which satisfies the Friedman rule, but the economy is stuck with a negative income gap which is potentially quite large. It would be theoretically possible to eliminate the negative output gap by setting $\pi^* = -r$. This is just the Friedman rule. As is well-known, it also has the advantage of equating the real rates of return on money and short-term bonds, leading to a socially-optimal level of real money balances (of course money demand is does not explicitly enter our model). While this works in a deterministic setting, Mendes (2011) shows that it leads to non-existence of the steady state when stochastic shocks to the real interest rate are added to the model.

B Deterministic Steady State under PT

B.1 True Steady State

First, consider a true steady state in which all of the model's state variables are constant, in particular

$$\left(p_{t+1} - p_{t+1}^*\right) = (p_t - p_t^*) = \left(p_{t-1} - p_{t-1}^*\right) \equiv p^d,$$

where p^d is the deviation of the price level from its target path. The value of p^d is possibly different from zero, but in fact it is easy to show that this cannot be the case. The transformed version of the New Keynesian Phillips curve (2) immediately gives

$$0 = \varphi y \Rightarrow y = 0.$$

Substituting into the New Keynesian IS curve (8), we immediately get

$$i = r + \pi^*.$$

The only true steady state has an output gap of zero and a positive nominal interest rate. The modified Taylor rule (6) then implies that

$$(p_t - p_t^*) = p^d = 0.$$

The price level follows its target path in the steady state.

B.2 Quasi Steady State

If we start by simply assuming $i = 0$, we can back out the following solutions for the other variables of the model in the long run. The untransformed version of the New Keynesian IS curve (2) then immediately implies that

$$\pi = -r.$$

Once again, we have the Friedman rule, but this will again imply a negative output gap in the steady state. Substituting in the transformed version of the New Keynesian IS curve (8) gives

$$\left(p_{t+1} - p_{t+1}^*\right) - \left(p_t - p_t^*\right) = -r - \pi^*,$$

which implies (using the transformed version of the New Keynesian Phillips curve) that

$$y = -\frac{(1-\beta)}{\varphi}\left(r + \pi^*\right).$$

We get the same solution for inflation, the output gap, and the nominal interest rate as in the liquidity-trap steady state under IT.

The solution is a "quasi" steady state because one of the model's state variables, the gap between the price level and its target path, is not at rest. With a negative rate of inflation, this gap decreases without bound, and the central bank's desired interest rate also decreases without bound. However, since the constraint of the zero bound is binding in this equilibrium, there is no feedback from the

price-level gap to the rest of the model.

C   Stochastic Steady State under PT

As noted in the text, Mendes (2011) gives and exhaustive treatment of the
stochastic steady state under IT.

As shown by Mendes, the liquidity-trap equilibrium under IT involves an
expected nominal interest rate that remains constant and that is superior to the
expected desired interest rate. The lower bound makes the realized interest rate a
left-truncated normal random variable, whose expectation depends positively on
the variance of the underlying shocks in the model.

Here, we consider the existence of either a steady state in which the
unconditional expectations of all of the model's state variables are constant, or a
quasi steady state in which all variables have constant unconditional means
except for possibly the gap between the price level and its desired path and the
desired interest rate. In the quasi steady state, the unconditional expectation of
the inflation rate is constant so that

$$\mathrm{E}\left(p_t^d - p_{t-1}^d\right) \equiv \mathrm{E}\Delta p_t^d \quad \forall t$$

$$\equiv \mathrm{E}\Delta p^d$$

is constant. This implies that $\mathrm{E}p_t^d$ is a deterministic function of time.
Dropping time subscripts, and taking unconditional expectations of variables, we

get

$$\mathrm{E}\Delta p^d = \beta\mathrm{E}\Delta p^d + \varphi\mathrm{E}y,$$

$$\mathrm{E}y = \mathrm{E}y - \frac{1}{\gamma}\left(Ei - \mathrm{E}\Delta p^d - \pi^* - r\right),$$

$$\mathrm{E}i_t^d = r + \pi^* + \rho_p\mathrm{E}p_t^d + \rho_y^*\mathrm{E}y,$$

$$\mathrm{E}i = \mathrm{E}\max\left(0, i_t^d\right).$$

We immediately have a contradiction. The expectation of the realized interest

rate depends on the expectation of a nonlinear function of a variable that is not

constant, so it cannot be constant. So there is no steady state that satisfies the

criterion that variables other than the gap between the price level and its target

path (and the desired interest rate) are constant.

So if a stochastic steady state with these properties exists, it must be the case that

the unconditional expectation of the deviation of the price level is constant. This

means that we must have

$$\mathrm{E}\Delta p^d = 0.$$

From the first equation we must have $\mathrm{E}y = 0$. If the stochastic equilibrium exists,

it must be the case that the unconditional expectation of the output gap is zero.

The intuition for this result is straightforward. With any expected inflation rate

that is different from π^*, the expected price-level gap must be changing over

time. The modified Taylor rule then implies that the unconditional expectation of

the desired interest rate must be changing over time, which implies that the

unconditional expectation of the realized nominal interest rate cannot be

constant.

We then get, from the New Keynesian IS curve, that

$$\mathrm{E}i = r + \pi^*.$$

Substituting into the modified Taylor rule gives

$$\mathrm{E}i^d = \mathrm{E}i + \rho_p \mathrm{E}p^d.$$

If the shocks of the model (the shock to the real interest rate and the shock to the modified Taylor rule itself) are normally distributed, the unconditional distributions of the variables in the model must be normal, and the realized real interest rate is a truncated normal distribution. It is left-truncated, so it must be the case that

$$\mathrm{E}i > \mathrm{E}i^d.$$

We have

$$\mathrm{E}p^d = -\frac{1}{\rho_p}\left(\mathrm{E}i - \mathrm{E}i^d\right) < 0.$$

On average, there will be a non-zero price-level gap. Its expected value is negative and depends on the strength with which the central bank varies its desired interest rate in response to the price-level gap. Under pure price-level gap targeting, as $\rho_p \to \infty$, the expected price-level gap tends to zero. The interpretation of this is clear. If the central bank reacts strongly against price-level deviations from the desired price-level path, the zero bound will

rarely be binding and the desired nominal interest rate will be close, on average, to the realized nominal interest rate.

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Figure 1:

Good vs. Evil: Two Long-Run Equilibria