Scope and Quality in International Networks

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April 13, 2012

Abstract

This paper develops a theoretical framework about firms' quality, scope and boundary decisions. In our framework, a firm's choice of scope (or number of suppliers) affects its revenue share in equilibrium. With a larger scope, the firm has to hire more suppliers, which means the firm as well as each supplier get smaller shares of revenue in equilibrium, thus their incentive for investing in inputs qualities are lower, resulting in lower quality outputs, and thus lower revenue. Therefore, there is a trade off between larger scope (which implies an output with higher functionality), and higher revenue share (which improves the incentive of quality investments and implies an output with higher quality). In equilibrium, a firm's organizational choices will depend both on its productivity, and the kind of industry it is in. In industries where scope and firm productivity are complements, higher productivity firms choose to integrate and lower productivity firms choose to outsource. In industries where scope and firm productivity are substitute, higher productivity firms choose to outsource while lower productivity firms choose to integrate.

1 Introduction

In many markets, such as those for TVs and white goods, there are pronounced differences in the role played by quality. For TVs, quality is the key to success: the largest firms have the highest quality, they have the highest level of R&D and patenting, and they rely on an extensive, vertically integrated supply network in which suppliers also invest in developing quality. In refrigerators, the key to success is productive efficiency: the largest firms have low quality (they have a past record of R&D and patenting that established the brand name), and rely on an extensive network of arm's length suppliers.

Company	Market Share	Quality (Share sold to rich)	Share of Shipments Owned	Suppliers	Share of Suppliers Owned	Buyer Patents	Comment on Patents (ordered by shipments)
Samsung	19%	22%	80%	9	33%	1,633	Tatung 78; AU Optronics 378; Hannstar 212
Sony	14%	31%	89%	6	83%	431	AU Optronics 378
Vizio (Amtran)	13%	6%	0%	2	0%	0	Foxconn 64; Amtran 11
LG Electronics	11%	17%	47%	6	33%	1,239	AU Optronics 378; TPV 0;
Panasonic	8%	3%	77%	8	50%	459	Ranko 0; B M Nagano 0; Hitachi 1,114
Mitsubishi	Small	5%	25%	4	50%	222	Futaba 0; Samsung 1,663
Toshiba	Small	4%	1%	8	13%	535	AU Optronics 378; BenQ 21; Orion 3
Phillips	Small	2%	27%	11	9%	326	LG Philips 552; Quanta 14; Suzhou 0
Sharp	Small	2%	46%	11	36%	1,589	Orion 3; Funai 28; Goh 0; Hirata 0; Kuroda 0

Buyer Name	Volume	Share of Volume Owned	Number of Suppliers	Share of Supplier Owned	Buyer Patents	Supplier Patents
LG Corp	97,131	1.00	13	0.38	475	All are 0
General Electric Co (GE)	84,713	0.00	21	0.10	304	Samsung 364; Sampo 0; Ghuangsho Wanbao 0
Sanyo Electric Co Ltd	38,762	0.00	9	0.11	137	Samsung 364; Matsushita 95
Haier Group	14,088	0.74	7	0.29	7	All are 0
Whirlpool Corp	11,918	0.03	10	0.20	498	Xingxing 0; Daewoo 98; Meda 0
The Mackle Company	5,972	0.00	11	0.00	0	All are 0
BSH Bosch & Siemens	2,959	0.34	2	0.50	106	Daewoo 98
Emerson	2,541	0.00	1	0.00	18	Totomak 0
Costco Wholesales Corp	1,829	0.00	2	0.00	0	Xingxing 0; Whirlpool 498
Sub-Zero Freezer Co Inc	297	0.00	1	0.00	12	Scott Tech 0

Table 1: Supplier Networks and Relationship-Specific Investments in the TV and Fridge Markets

[Weak — empirics are still to be done] Table 1 provides some evidence of this. The top panel deals with TVs. The two top suppliers have the highest quality (as measured by the share of sales to customers with income in excess of $200,000 as well as by Consumer Reports), 80% of their purchases from suppliers are from vertically integrated suppliers, and both have large numbers of TV-related patents and buy from vertically integrated suppliers, and both have large numbers of TV-related patents and buy from suppliers with large numbers of TV-related patents (including AU Optronics, a high-end manufacturer of flat panels). In contrast, the lower-quality firms (Vizio) have small numbers of suppliers who are at arm's length and have modest numbers of patents. In the refrigerator market, the typical firm has an even larger number of suppliers, but these are typically not owned by the firm and often have more fridge-related patents than the firm.

This paper examines the nexus between quality, supplier networks, and the make-or-buy decision. It tries to understand both across industries and within industries why we see such variation. We begin with the observation that consumer's valuations of products depends two dimensions of quality. First, it depends on what we refer to as 'functionality'. The archetypical example is a smartphone such as Apple's iPhone 4, which embodies dozens of functions including phone, wifi, camera, etc. Less archetypically, but more common empirically, functionality deals with 'product lines': a firm produces many sizes of TVs and many types of refrigerators. The second dimension of consumer's valuation is the quality of each function/product, here understood in a more conventional sense of reliability or better design. We model the firm's choices of functions or, alternatively, the size of the product line. In conjunction with suppliers it then comes up with a blueprint for exactly what the function will look like. We assume that each function is developed in conjunction with a single supplier and that the development of a blueprint requires relationship-specific, non-contractible investments from both the firm and its supplier. The larger are the investments, the higher is the quality of the function.

This paper identifies two determinants of functionality and quality of individual functions. First, the greater the functionality–i.e., the larger the number of suppliers–the greater is the hold-up problem. This is modeled by assuming that the firm and all N suppliers engage in multilateral bargaining so that, roughly speaking, each receives a share $1/(N+1)$ of revenues. This creates a tension: products with many functions and hence many suppliers generate more revenue, but also create a larger hold-up problem. For reasons familiar from the property rights theory of the firm (specifically, Antràs (2003)) to overcome the hold-up problem larger firms will vertically integrate their suppliers.

The second determinant deals with a managerial tension: some managers are good at cutting costs, others are good at identifying functions that are valued by consumers and building the supplier networks needed to deliver these functions. Specifically, in "ideas-oriented' industries more productive firms are firms with a high marginal return to more functions (productivity and functions are complements in the profit function). These industries behave like TVs in that the most productive firms will have many integrated suppliers and will have high levels of relationship-specific investments in each function's quality. The starting point of this paper is two strands of the literature. The first is work by Antràs (2003) and Antràs and Helpman (2004) on the vertical integration decision when there is a single firm and supplier, each making relationship-specific, non-contractible investments. The second is work by Acemoglu et al. (2007) about the optimal size of production networks in which a single firm (who makes only contractible investments) deals with many suppliers who are making non-contractible investments. We depart from this literature in a large number of ways.

2 Setup

2.1 Preferences and Production

Representative consumer's preferences:

$$U = \left\{ \int_{\omega} \left[\varphi(\omega)^{\nu} y(\omega) \right]^{(\sigma-1)/\sigma} d\omega \right\}^{\sigma/(\sigma-1)}$$

where ω is a product index, $y(\omega)$ is a consumption level, $\varphi(\omega)^{\nu}$ is a demand shifter (ν is a parameter and φ is explained in detail below), and σ is the elasticity of substitution. We assume $\sigma > 1$ and $\nu(\sigma - 1) < 1$.

Production of a variety has three stages. The firm first decides on a level of functionality N, that is, on the number of functions the product will have or on the number of products in the product class. For example, an iPhone 4 has many functions (wifi, voice recognition, apps etc.) and a Mercedes has many products (sports car, sedan etc.). Second, the firm identifies N suppliers, each of which will help the firm develop one of the functions. This blueprint or 'ideas' stage involves non-contractable inputs from both the firm and the supplier. Third, in the 'production' stage the final good is produced in a complete-contracting environment. The ideas stage is the key stage and we discuss it in detail next.

In the ideas stage each function is developed using the shared inputs of the firm and the supplier. For simplicity, we assume that each function is developed by the firm with the help of a single supplier.[1] A function can be of variable quality. For example, voice recognition is better in some cell phones than in others and compressors are better in some refrigerators than in others. Let q_j be the quality of function $j = 1, \ldots, N$. It depends on the firm's input h_j and the seller's input m_j:

$$q_j = h_j^{\eta} m_j^{1-\eta} / \hat{\eta}$$

where $\hat{\eta} \equiv \eta^{\eta}(1-\eta)^{1-\eta}$. Quality q_j and inputs (h_j, m_j) are non-contractible.

Consumer valuation of functionality and function quality are captured by the demand shifter

$$\varphi = D(N, \theta) \min\{q_1, q_2, ..., q_N\} \tag{1}$$

where $\theta \in [0, 1]$ is a firm index that replaces ω; it plays no role yet, but we will later interpret it as the firm's productivity as in Melitz (2003).

The particular functional form in equation (1) is not all that important to our argument.

[1]It is possible to allow for multiple suppliers of a single function — this would allow the firm to reduce the hold-up problem.

We obtain similar results with either a utility function that is CES in functions (of which equation (1) is the special case of perfect complements) or with O-Ring utility.[2] What is very important is that the buyer and all suppliers are *essential* in a Shapley-value sense. That is, $\varphi = 0$ if any player is not part of the team. This will ensure that the buyer's Shapley value is decreasing in the size of the team (N). Restated, essentiality rather than the functional form of equation (1) is what provides our key modelling assumption, namely, that *more functionality comes at the cost of greater hold-up.*[3]

The marginal cost of input $j \in \{h, m\}$ is $C_j(N, \theta)$. For simplicity, we assume that $C_j(N, \theta) = w_j C(N, \theta)$ where the constant w_j captures the prices of inputs and other things that are log-separable from N and θ. Note that both D and C depend on θ. Not surprisingly, we will find (roughly) that only D/C matters. This is the usual point that demand shifters and productivity are isomorphic. We assume below that D/C is increasing in θ.

Demand for the final product y is

$$y = A\varphi^{\alpha} p^{-\sigma} \text{ where } \alpha \equiv \nu(\sigma - 1) \in (0, 1).$$

The firm is a monopolistic competitor and sets price equal to $[\sigma/(\sigma - 1)]c$. This generates revenues

$$R = \hat{A}\varphi^{\alpha} = \hat{A}\left[D(N, \theta) \min\{q_1, q_2, ..., q_N\}\right]^{\alpha} \tag{2}$$

where $\hat{A} \equiv \sigma^{-\sigma}[(\sigma - 1)/c]^{\sigma-1}A$.

2.2 Timing

1. The firm and all the suppliers observe θ.

[2] For CES, $\left\{\sum_{j=1}^{N} q_j^{\beta}\right\}^{1/\beta}$ which, under the symmetry that we impose below, becomes $N^{1/\beta}q$. For O-Ring, $B(N)\Pi_{j=1}^{N} q_j$, which when symmetry is imposed and logs taken becomes $\ln B(N) N \ln(q)$. These different specifications affect the functional form of the optimal inputs (h_j, m_j), but otherwise do not matter.

[3] In contrast, AAH do not assume that players are essential. In their setup, the Shapley value is independent of N.

2. The firm chooses organizational form $k = O, V$, adopts technology N, and offers contract $\{\tau_j\}_{j=1}^{N}$, where τ_j is an upfront payment to supplier j.

3. Potential suppliers decide whether to apply for the contracts and the firm chooses N suppliers from applicant pool.

4. The firm and the suppliers simultaneously choose their investment levels $\{(h_j, m_j)_{j=1,2,...,N}\}$.

5. The firm and the suppliers bargain over the division of future revenue. At this stage, the firm and the suppliers can decide to withdraw their investments.

6. Ideas are generated (φ is determined). Output is produced and sold. Revenue is divided according to the bargaining agreement.

2.3 Hold-up

We assume that in the negotiation stage, if supplier j decides to withdraw from the production process, the quality of his input drops from q_j to $\Delta^k q_j$, where $k \in \{O, V\}$ and $\Delta^O < \Delta^V$.

3 Equilibrium

3.1 SSPE

We define an SSPE as a tuple $\{N, \tau, h, m\}$, where N is the firm's choice of functionality. τ is the firm's up-front payment to every supplier, that is, $\tau_j = \tau$ for $j = 1, ..., N$. Similarly, h is the firm's investment for each function, and m is each supplier's investment. That is $(h_j, m_j) = (h, m)$, for $j = 1, ..., N$.

SSPE can be characterized by backward induction as in AAH. Since this is familiar (and notationally difficult) territory, we jump immediately to the revenue in any SSPE. This is given by $R = \hat{A}\left\{D(N, \theta)h^{\eta}m^{1-\eta}/\hat{\eta}\right\}^{\alpha}$.

Lemma 1. *In every SSPE*[4]*, the firm's Shapley value under organizational form $k \in \{O, V\}$ is $\gamma^k(N)R$ where*

$$\gamma^k(N) = \frac{\delta^k N + 1}{N + 1}.$$

where $\delta^k \equiv (\Delta^k)^\alpha$. Each seller's Shapley value is $(1 - \gamma^k(N))R/N$.

In AAH, the firm's share of revenue γ^k is independent of N. Here, organizations with more suppliers face larger hold-up problems. This is reflected in the fact that γ^k is decreasing in N. This has an important implication. If in our model γ^k were independent of N then the choice of number of suppliers and choice of organizational form would not interact. Specifically, the choice of organizational form would be determined as in Antràs (2003) or as in AH (2004) with $f_V = f_O$ i.e., if η is large all firms integrate and if η is small all firms outsource. Here, a productive firm may want to have a large N and this will lead to a smaller share of revenue (a small γ^k); the firm may find it optimal to offset this loss of revenue by moving from the O form to the V form, which has the effect of increasing the firm's revenue share from γ^O to γ^V. **In essence, productive firms will want to vertically integrate to offset the endogenously greater hold-up problem that comes with having more suppliers.**

3.2 Optimal Choice of Idea Inputs (h_j, m_j)

The firm's problem is familiar from Antras (2003) and Antras and Helpman (2004). It may be written as:

$$\max_{(h_1,h_2,\ldots,h_N)} \gamma^k(N)\frac{\hat{A}}{\hat{\eta}^\alpha}\left[D(N,\theta)\min_{1\le j\le N}\{h_j^\eta m_j^{1-\eta}\}\right]^\alpha - c_h C(N,\theta)\sum_{j=1}^{N} h_j \tag{FP1}$$

$$\text{s. t. } m_j = \arg\max_{m_j} \frac{1-\gamma^k(N)}{N}\frac{\hat{A}}{\hat{\eta}^\alpha}\left[D(N,\theta)\min_{1\le j\le N}\{h_j^\eta m_j^{1-\eta}\}\right]^\alpha - c_m C(N,\theta) m_j \tag{IC1}$$

$$\tau_j + \frac{1-\gamma(N)}{N}\frac{\hat{A}}{\hat{\eta}^\alpha}\left[D(N,\theta)\min_{1\le j\le N}\{h_j^\eta m_j^{1-\eta}\}\right]^\alpha - c_m C(N,\theta) m_j \ge 0 \tag{PC1}$$

Note that the firm does not choose m_j. The suppliers choose this subject to the incentive

[4] The existence of a SSPE is proved in Appendix ?.

compatibility constraint (IC1). The participation constraint (PC1) means that the supplier's surplus (which equals his up-front payment τ_j, plus his Shapley value, less his cost of investment) should be greater than or equal to his outside option. We set the outside option to 0 as this will allow us to exploit some powerful monotone comparative statics tools.

We assume that $\alpha < 1$ so that the supplier's problem (the maximand of IC1) and the firm's problem (FP1) are concave.

Assumption 1. $0 < \alpha < 1$

Lemma 2. *In any SSPE, the unique solution (h_j, m_j) to the firm's problem is:*

$$m^k(N,\theta,\eta) = \kappa A \frac{1-\eta}{c_m} \left\{ \left[\gamma^k(N)\right]^{\alpha\eta} \left[1-\gamma^k(N)\right]^{1-\alpha\eta} \right\}^{1/(1-\alpha)} \left[\frac{D(N,\theta)^\alpha}{NC(N,\theta)}\right]^{1/(1-\alpha)} \tag{3}$$

$$h^k(N,\theta,\eta) = \frac{\gamma^k(N)}{1-\gamma^k(N)} \, \frac{c_m/(1-\eta)}{c_h/\eta} \, m^k(N,\theta,\eta). \tag{4}$$

where $\kappa \equiv \{\alpha(\sigma-1)^{\sigma-1}\sigma^{-\sigma}c^{1-\sigma}\}^{1/(1-\alpha)}$.

These are messy expressions, but ones that are not fundamentally new. The only new insight comes from equation (4): h/m will vary *within an industry* not only because different firms choose different organizational forms k, but also because they choose different-sized organizations and this effects h^k/m^k via the effects of N on γ^k. Thus, our framework offers a natural explanation of the enormous within-industry heterogeneity in relationship-specific investments that we see in the data.[5]

[5]There are two main (old) insights from equations (4). First and obviously, the optimal input levels are both less that the first-better (contractible) input levels. Second, h^k/m^k equals the first-best input ratio if and only if $\gamma^k = 1/2$. This points to how the Grossman-Hart logic plays out in this model. When η is large so that the firm's investment is most important, the firm wants to choose a form that will raise h^k/m^k. This is the form with the larger γ^k and, since $\gamma^V(N) > \gamma^O(N)$, vertical integration is preferred.

3.3 Optimal Choice of Organization Size N and Form $\{O, V\}$

Plugging in the Lemma 2 optimal inputs into the firm's problem FP1, the firm's problem simplifies to

$$\max_{k\in\{O,V\},N\in(1,\infty)} \Pi^k(N,\theta,\eta) = \kappa A G(N,\theta)\Psi(\gamma^k(N),\eta), \qquad (P2)$$

where

$$G(N,\theta) \equiv \left[\frac{D(N,\theta)}{NC(N,\theta,\eta)}\right]^{\frac{\alpha}{1-\alpha}},$$

$$\Psi(\gamma,\eta) \equiv \frac{1-\alpha[\gamma\eta+(1-\gamma)(1-\eta)]}{[\gamma^\eta(1-\gamma)^{1-\eta}]^{-\frac{\alpha}{1-\alpha}}},$$

and κ is a constant that depends on $(\sigma, \eta, c, c_h, c_m, \nu)$.

It is now apparent that only $G = D/(NC)$ matters, not D or NC separately.[6] Note that up to this point we have not said anything about θ. It is now clear that the appropriate assumption is that G is increasing in θ.

Assumption 2. *$G(N,\theta)$ is strictly increasing in $\theta \in [0,1]$.*

This is a good spot to compare our model with that of Antras and Helpman (2004, equation 10), where $N = 1$. Their model has an almost identical profit function: In our notation it is basically $\Pi^k(1,\theta,\eta) = \theta^{\sigma-1}\Psi(\gamma,\eta)$ where, as is standard in Melitz-like models, $G(1,\theta) = \theta^{\sigma-1}$. However, there are four differences of note. (1) $N \neq 1$ is a choice variable. (2) There are no fixed costs of organizations (f_V and f_O in their notation). Recall that in their model, when there are no fixed costs as is the case here (or even when there are fixed costs and $f_V = f_O$) then their model reduces to Antras (2003). That is, when η is small all firms outsource and when η is large all firms vertically integrate. (3) The most important difference is that $\Psi(\gamma^k(N),\eta)$ depends on N. In Antras (or Antras and Helpman with $f_V = f_O$), the firm chooses the organizational form k that maximizes $\Psi(\gamma^k(1),\eta)$ where $\gamma^k(1)$ and η are

[6] Note that in the expressions for h^k and m^k, what matters is D^α/NC, so D and NC matter separately. However, they only matter for the *levels* of h^k, m^k and hence for quality q_j. They do not matter separately for anything else whatsoever.

parameters. In our setting, there is an interaction between the choice of organization and the choice of functionality. The larger is the organization (N), the smaller is γ^k. This creates a tension: the firm might want to grow bigger in order to have higher demand, but this exacerbates the hold-up problem. In short, we have heterogeneity of organizational forms without fixed costs because we have endogenized the extent of the hold-up problem.

This is also a good spot to compare our profit function to that in AAH. First, in their model only the supplier makes a relationship-specific investment ($\eta = 0$) so that the firm always outsources. Second, in their model the Shapley value is completely determined by exogenous parameters so that there is no trade-off between size and hold-up.

We now make assumptions that make it easier to solve for the optimal N. We will use first-order conditions and so ignore the integer constraint on N. The following assumption ensures that there is a unique optimal N^k and that it is bounded away from 1 and infinity.

Assumption 3. *(1)* $\frac{\partial^2 \ln G(N,\theta)}{\partial(\ln N)^2} < 0$, *(2)* $\lim_{N\to 1} \frac{\partial \ln G(N,\theta)}{\partial \ln N} > \frac{1}{2}$, *and (3)* $\lim_{N\to\infty} \frac{\partial \ln G(N,\theta)}{\partial \ln N} < 0$.

Note that some of our main results rely on monotone comparative static arguments and thus do not require convexity or uniqueness.[7]

4 Two Types of Industries

There are two types of industries, ideas-oriented and cost-oriented. In ideas-oriented industries, consumers highly value functionality N so that $D_N > 0$ is salient. Further, high-productivity firms are also the firms that develop the best functions in the sense that each function generates a high marginal revenue. Mathematically, D_N is increasing in θ or $D(N, \theta)$ is log supermodular in (N, θ). One can get at this same notion of ideas-oriented industries from the cost side by noting that in these industries, high-productivity firms are really good at managing the integration of complex designs. With complex designs, more functionality raises the marginal costs for each supplier because each firm-supplier pair must ensure

[7] This might be overstated: Be more careful.

its design is compatible with all the other suppliers' designs. That is $C_N > 0$. However, high-productivity firms are better able to manage these rising costs: C_N is decreasing in θ or $C(N, \theta)$ is log submodular in (N, θ). Whether tackled from the demand side or the supply side, both imply the following:

Assumption 4. *Ideas-oriented industries: $G(N, \theta)$ is log supermodular in (N, θ).*

In cost-oriented industries, the manager is good at keeping costs down ($C_\theta < 0$), but this focus on lower costs is to the exclusion of a focus on good functionality. Porter's (1996) "What is strategy" is precisely about the tension between being cost-efficient and being able to build a product with many intertwined functions that support each other and cannot be disentangled (cream-skimmed) by competitors. This means that in cost-oriented indudstries, high-productivity firms do not get a big bang for their functionality. Mathematically, D_N is decreasing in θ.

Assumption 5. *Cost-oriented industries: $G(N, \theta)$ is log submodular in (N, θ).*

5 Ideas-Oriented Industry

5.1 Heterogeneity of Organizational Forms

Theorem 1. *There exist two threshold values of η, $\underline{\eta}$ and $\overline{\eta}$, with $0 < \underline{\eta} < \overline{\eta} < 1$, such that:*

1. *For $\eta < \underline{\eta}$ industries, all firms choose outsourcing;*

2. *For $\eta > \overline{\eta}$ industries, all firms choose vertical integration;*

3. *For $\underline{\eta} < \eta < \overline{\eta}$ industries, there exists a $\theta^*(\eta)$, such that*

   (a) firms with $\theta < \theta^*(\eta)$ choose outsourcing;

   (b) firms with $\theta > \theta^*(\eta)$ choose vertical integration;

   (c) $\theta^*(\eta)$ is strictly decreasing in η.

Compared to Antràs (2003), AH (2004, 2008) and AAH (2007), we have heterogeneity of organizational form within industry that does not rely on assumption about fixed organizational cost. In Antràs (2003), all firms outsource in small η industries. In AH (2004, 2008), productive firms integrate because of higher fixed cost of integration ($f_V > f_O$). In AAH (2007), firms never integrate because there is not firm relationship-specific investment.

5.2 The Trade-Off Between Hold-up and Organizational Size

Theorem 2. *In industries with $\underline{\eta} < \eta < \overline{\eta}$, the following results are true:*

1. *$N^O(\theta, \eta)$, $N^V(\theta, \eta)$ and $N^*(\theta, \eta)$ are strict increasing in θ.*

2. *$\gamma^O(N^O(\theta, \eta))$ and $\gamma^V(N^V(\theta, \eta))$ are strictly decreasing in θ.*

3. *$N^O(\theta^*(\eta), \eta) < N^V(\theta^*(\eta), \eta)$ and $\gamma^O(N^O(\theta^*(\eta), \eta)) < \gamma^V(N^V(\theta^*(\eta), \eta))$.*

Parts 1 and 2 of theorem 2 capture the key tradeoff of the paper: a more productive firm has a larger supplier network (larger N), but also a larger hold-up problem (a smaller Shapley value or share of revenue γ). Part 3 deals with a firm that is just indifferent between the two organizational forms. As the firm moves from O to V, two offsetting things happen to its share of revenue. The direct effect is the improved outside option ($\delta^O < \delta^V$), which raises its share of revenue. The indirect effect is that the firm increases it supplier network ($N^O < N^V$) which lowers the firm's share of revenue. Part 3 states that the direct effect dominates.

5.3 Component Quality q, Overall Quality φ, and Revenues R

By Lemma 2

$$q^k(N, \theta, \eta) = \left\{ \frac{\alpha \hat{A}}{\hat{\eta}^{1-\alpha}} \frac{D(N, \theta)^\alpha \gamma^k(N)^\eta [1 - \gamma_k(N)]^{1-\eta}}{NC(N, \theta, \eta)} \right\}^{1/(1-\alpha)},$$

$$\varphi^k(N, \theta, \eta) = \left\{ \frac{\alpha \hat{A}}{\hat{\eta}^{1-\alpha}} \frac{D(N, \theta) \gamma^k(N)^\eta [1 - \gamma_k(N)]^{1-\eta}}{NC(N, \theta, \eta)} \right\}^{1/(1-\alpha)}.$$

and R is proportional to $\varphi^k(N, \theta, \eta)$.

Log supermodularity is enough for our previous results. However, it is not enough to ensure that higher-productivity firms have higher quality and revenues. The issue is that we have argued that either $D_N > 0$, $C_N > 0$ or both so that G_N is not signed. The natural assumption in ideas-oriented industries is that the demand-side benefits of higher functionality are not too offset by supply-side costs.

Assumption 6. *Ideas-oriented industries:* $\partial \ln G / \partial \ln N > -1/2$.

Theorem 3. $q(\theta, \eta)$, $\varphi(\theta, \eta)$ *and* $R(\theta, \eta)$ *are strictly increasing in* θ.

6 Ideas-Oriented Industries – Intuition (NEEDS MAJOR RE-WRITE)

In ideas-oriented industries, log supermodularity of $G(N, \theta, \eta)$ immediately implies that $N^k(\theta, \eta)$ is increasing in θ. More productive firms have more functionality and a larger number of suppliers.

Consider Antràs (2003) or Antràs and Helpman (2004), which here means that $N = 1$ is fixed. They show that if the firm could choose its revenue share (call it γ), it would choose γ to maximize $\Psi(\gamma)$. The optimal γ is given in equaiton (10) of Antràs and Helpman (2004).[8] Generalizing this to our setting, suppose the firm could choose N and γ, or in terms of primatives, N and δ. $\delta \in [0, 1)$ is a continuous choice variable that replaces the parameters δ^O and δ^V. As in 1, define $\gamma(N, \delta) \equiv (\delta N + 1)/(N + 1)$. The firm chooses (N, δ) to maximize profits or, equivalently log profits

$$\pi(N, \delta) \equiv g(N, \theta) + \psi(\gamma(N, \delta), \eta))$$

[8] In our notation their equation (10) is $\gamma(\eta) = \left\{[\eta(\alpha\eta + 1 - \alpha) - \sqrt{\eta(1-\eta)(1-\alpha\eta)(\alpha\eta + 1 - \alpha)}]/[2\eta - 1]\right\}$.

where $g = \ln G$ and $\psi \equiv \ln \Psi$. The first order conditions are $\psi_\gamma \gamma_\delta = 0$ or

$$\psi_\gamma(\gamma, \eta) = 0 \tag{5}$$

and $g_N + \psi_\gamma \gamma_N = 0$ or

$$g_N(N, \theta) = \frac{1-\delta}{(1+N)^2} \psi_\gamma(\gamma, \eta) \tag{6}$$

where $(1-\delta)/(1+N)^2 = -\partial\gamma/\partial N$.

Equation (5) establishes that the optimal choice delivers an optimal γ, denoted $\gamma(\eta)$, is independent of θ. The $\delta(N)$ curve is the set of pairs (N, δ) for which $\gamma(N, \delta) = \gamma(\eta)$ i.e., $[1 + \delta(N)]/[1 + N] = \gamma(\eta)$ or

$$\delta(N) = \gamma(\eta) + [\gamma(\eta) - 1]/N.$$

$\delta(N)$ is plotted in figure 1. Equation (6) establishes that the optimal N depends on θ. For example, if g is supermodular than N increases in θ. In this case, less-productive firms are to the southwest of $\delta(N)$ and more-productive firms are to the northeast.[9]

[9] By assumption 3, N is finite so that $\delta(N)$ has an upper bound $\gamma(\eta)$. Since $N \geq 1$, $\delta(N)$ has a lower bound $2\gamma(\eta) - 1$.

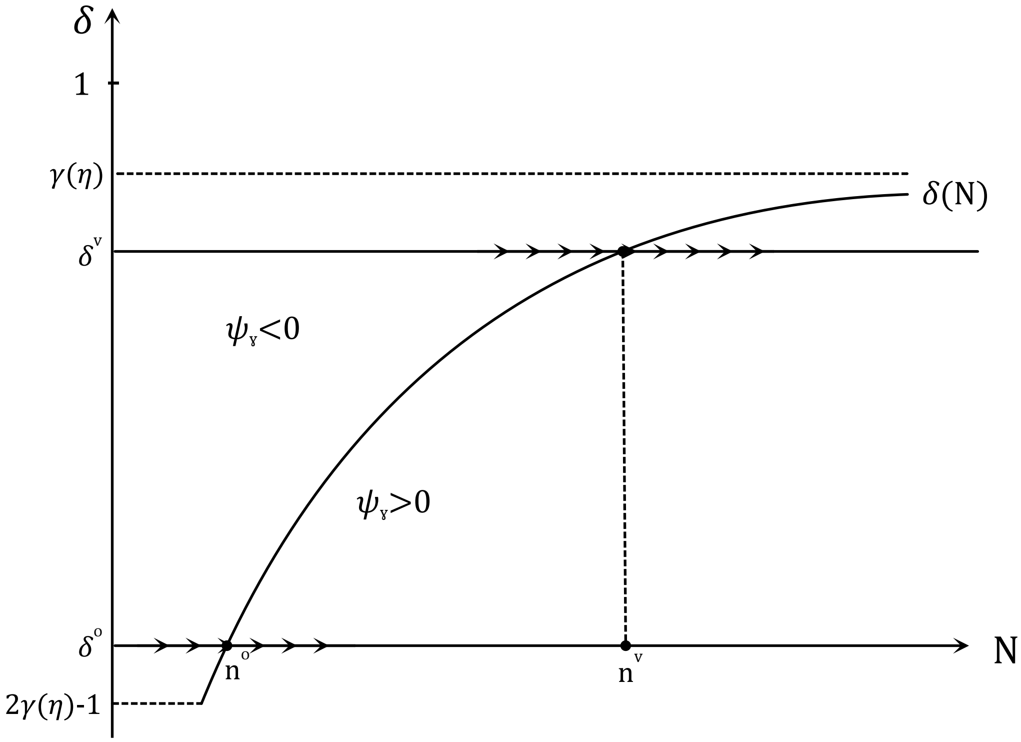


Figure 1: $\delta(N)$ in Ideas-Oriented Industries

When η is close to either 0 or 1, figure 1 shows that Antras and Helpman logic goes through exactly as in their work. They show that $\psi_\gamma < 0$ above $\delta(N)$ and $\psi_\gamma > 0$ below $\delta(N)$. When η is close to 0 then $\gamma(\eta)$ is close to 0 and the $\delta(N)$ curve is everywhere below δ^O. Hence $\delta^V > \delta^O > \delta(N)$ for all N. But then for all N, $\psi_\gamma < 0$ and the firm does best by lowering $\gamma(N,\delta)$. Since $\gamma(N,\delta)$ is increasing in δ, the firm does best by lowering δ. Restated, the firm prefers δ^O to δ^V or outsourcing to vertical integration.

When η is close to 1, $\gamma(\eta)$ is close to 1 and the $\delta(N)$ curve is everywhere above δ^V. Now $\psi_\gamma > 0$ and the argument is reversed: the firm prefers to raise δ. Hence, the firm prefers δ^O to δ^V or outsourcing to vertical integration.

For intermediate values of

so that $\delta^V > \delta^O > \gamma(\eta)$. That is, very small, $\delta^V > \delta^O > \delta(N)$, all firms choose outsourcing because δ^O is closer to their "ideal" choices of δ and N. If η is very large, $\delta^O < \delta^V < \delta(N)$, all firms choose integration because δ^V is closer to their "ideal" choices of δ and N. So there must be an η in between where some θ such that firms with productivity θ^* is indifferent between outsourcing and vertical integration.

of all the firms within an industry. $\delta(N)$ is an "iso-γ" line because the γ that maximizes ψ does not depend on firms' productivities and is thus the same for all firms within the same industry. As AH shows, $\psi_\gamma < 0$ above $\delta(N)$ and $\psi_\gamma > 0$ below $\delta(N)$. Since N is fininte by our assumption, $\delta(N)$ has an upperboud $\gamma(\eta)$ and a lower bound $2\gamma(\eta)-1$. If η is very small, $\delta^V > \delta^O > \delta(N)$, all firms choose outsourcing because δ^O is closer to their "ideal" choices of δ and N. If η is very large, $\delta^O < \delta^V < \delta(N)$, all firms choose integration because δ^V is closer to their "ideal" choices of δ and N. So there must be an η in between where some θ such that firms with productivity θ^* is indifferent between outsourcing and vertical integration. Call it $\theta^*(\eta)$. Firms with the low θ are to the SW of $\delta(N)$. They would want to outsource because δ^O is closer to their ideal choice. As θ increases, firms' choices of δ and N moves to the NE. When a firm's ideal choice is above δ^O, they would want a higher δ. They can only do so by choosing δ^V, which might be too large for some firms such that it contracts these firms' profits. So these firms choose δ^O even though it is not ideal. But as θ increases, the ideal choices of firms with productivity θ move NE. δ^O gets farther and farther away from these firms' ideal choice, while δ^V gets closer. Until θ reaches $\theta^*(\eta)$, where firms with productivity $\theta^*(\eta)$ are indifferent between choosing δ^O and δ^V. When $\theta > \theta^*(\eta)$, firms will find it optimal to choose δ^V because δ^O is way too low for them.

Turning to theorem 2, for a firm that is indifferent between outsourcing and vertical integration, denote by N^O and N^V his choice of funtionality under outsourcing and vertical integration, and γ^O and γ^V as his revenue shares under these two organizational forms. Since (δ^O, N^O) is below $\delta(N)$ and (δ^V, N^V) is above $\delta(N)$, it must be that $\psi_\gamma(\gamma^O, \eta) > 0$ and $\psi_\gamma(\gamma^V, \eta) < 0$. According to the first order condition, this means that $g_N(N^O, \theta^*, \eta) > 0$ and $g_N(N^V, \theta^*, \eta) < 0$. Since $g_{NN} < 0$, this implies that $N^O < N^V$. Thus if this $\theta^*(\eta)$ firm jumps from outsourcing to vertical integration, his choice of N increases.

We know that a firm's revenue share $\gamma = (\delta N+1)/(N+1)$ is increasing in δ and decreasing in N. So as the $\theta^*(\eta)$ jumps from outsourcing to vertical integration, his δ increases, which increases γ, but his N increases, which decreases γ. Which effect dominates? If we think of the firm's gain by choosing vertical integration, the answer is straightforward. As θ increases, firm would increase N within the same organizational form. Meanwhile, γ decreases because δ is constant within the same organizaitonal form. As θ increases even further, firms will find their revenue share γ has decreased to a point that they want to increase it by jumping to V. Since the firm's purpose for jumping into V is a larger revenue share, he will not chooce too large an N such that γ is actually lower than when he was outsourcing. This means that even though the firm will choose a larger N, he will not make N so large that γ^V is actually lower than γ^O. So it must be that $\gamma^V > \gamma^O$.

6.1 Cost-Oriented Industry

Recall that in cost-oriented industries, $G(N, \theta)$ is log supermodular. This immediately implies that the most productive, high profitability firms will have smaller supplier networks. This means in Figure 1, high θ is to the SW of $\delta(N)$. So high productivity firms will find it optimal to outsource because δ^O is closer to their ideal choice, vice versa.

Theorem 4. *There exist two threshold values of η, $\underline{\eta}$ and $\overline{\eta}$, with $0 < \underline{\eta} < \overline{\eta} < 1$, such that:*

1. *For $\eta < \underline{\eta}$ industries, all firms choose outsourcing;*
2. *For $\eta > \overline{\eta}$ industries, all firms choose vertical integration;*
3. *For $\underline{\eta} < \eta < \overline{\eta}$ industries, there exists a $\theta^*(\eta)$, such that*
   - *(a) firms with $\theta > \theta^*(\eta)$ choose outsourcing;*
   - *(b) firms with $\theta < \theta^*(\eta)$ choose vertical integration;*
   - *(c) $\theta(\eta)$ is strictly increasing in η.*

Theorem 5. *In industries with $\underline{\eta} < \eta < \overline{\eta}$, the following results are true:*

1. $N^O(\theta, \eta)$, $N^V(\theta, \eta)$ *and* $N^*(\theta, \eta)$ *are strict decreasing in* θ.

2. $\gamma^O(N^O(\theta), \eta)$ *and* $\gamma^V(N^V(\theta), \eta)$ *are strictly increasing in* θ.

3. $N^O(\theta^*(\eta), \eta) > N^V(\theta^*(\eta), \eta)$ *and* $\gamma^O(N^O(\theta^*(\eta), \eta)) < \gamma^V(N^V(\theta^*(\eta), \eta))$.

6.2 Quality and Revenues – Approach 1

As before, higher productivity translates into higher profits, but it need not translate into lower In addition, higher functionality does not raise D by very much (D_N is small) so that the small benefits of increased functionality are offset by the higher costs ($\partial(NC)/\partial N = NC_N + C > 0$):

Assumption 7. *Cost-oriented industries:* $\partial \ln G/\partial \ln N < -1/2$.

Theorem 6. $q(\theta, \eta)$, $\varphi(\theta, \eta)$ *and* $R(\theta, \eta)$ *are strictly increasing in* θ.

6.3 Quality and Revenues – Approach 2

Alternatively, we could drop assumption 7 and keep assumption 6. Then we would get that higher productivity firms have lower component quality q, lower overall quality φ and lower sales. Then to generate higher sales, we could load productivity onto the marginal cost of the production stage (c above). Specifically, if we assume that in cost-oriented, c is sharply decreasing in productivity θ, then I think we would get:

Theorem 7. $q(\theta, \eta)$ *and* $\varphi(\theta, \eta)$ *are strictly decreasing in* θ *and* $R(\theta, \eta)$ *is strictly increasing in* θ.

A Proof of Lemma 1

Each player's Shapley value is the average of her contributions to all coalitions that consist of players ordered below her in all permutations of the order. A coalition generates one of three possible values.

1. In a coalition without the firm, the value is 0.

2. In a coalition with the firm and all the suppliers, the value is revenue $R = \hat{A}D(N,\theta)^{\alpha}q^{\alpha}$, where $q = h^{\eta}m^{1-\eta}/\hat{\eta}$ as in the statement of the Lemma.

3. In a coalition with the firm, but not all the suppliers, the minimum quality is $\delta^k q$ so that the value is revenue $\delta^k R = \hat{A}D(N,\theta)^{\alpha}(\Delta^k q)^{\alpha}$, where $\delta^k \equiv (\Delta^k)^{\alpha}$.

Consider the firm's contribution. Pick a permutation (a ranking of each player from 0 to N) and let $g(B)$ be the firm's rank in this permutation. If $g(B) < N$ then there is at least one supplier not in the coalition and the firm's contribution is $\delta^k R$ i.e., case 3 less case 1. If $g(B) = N$ then all suppliers are in the coalition and the firm's contribution is R i.e., case 2 less case 1. The share of permutations with $g(B) = N$ is $1/(N+1)$. The share of permutations with $g(B) < N$ is $N/(N+1)$. Therefore, the firm's Shapley value is

$$R\frac{1}{N+1} + \delta^k R\frac{N}{N+1} = \frac{\delta^k N + 1}{N+1}R$$

The value generated by a coalition of all players is R (case 2). Since the Shapley value is efficient, suppliers must receive

$$R - \frac{\delta^k N + 1}{N+1}R = \frac{1-\delta^k}{N+1}NR$$

The Shapley value is symmetric so that all suppliers have the same Shapley value. Dividing the above expression by the N suppliers gives each supplier's Shapley value: $[(1-\delta^k)/(N+1)]R$.

B Shapley Value under CES and O-Ring Production Function

B.1 CES

Suppose the demand shifter is:

$$\varphi = D(N,\theta)N^{-1/\beta}Q$$

where $Q = \left(\sum_{j=1}^{N} q_j^{\beta}\right)^{1/\beta}$. In a symmetric equilibrium, $q_j = q$ for all j. Revenue is $R = \hat{A}\varphi^{\alpha} = \hat{A}D(N,\theta)^{\alpha}q^{\alpha}$. The scale effect from CES is killed by $N^{-1/\beta}$. Same as the previous section, a coalition generates one of three possible values:

1. In a coalition without the firm, the value is 0.
2. In a coalition with the firm and all the suppliers, the value is revenue $R = \hat{A}D(N,\theta)^{\alpha}q^{\alpha}$.
3. In a coalition with the firm, but not all the suppliers, the overall quality is $Q = \left[nq^{\beta} + (N-n)(\Delta^k q)^{\beta}\right]^{1/\beta} = [n + (N-n)(\Delta^k)^{\beta}]^{1/\beta}q$, where n is the number of suppliers who are in the coalition. And the corresponding value is $\hat{A}D(N,\theta)^{\alpha}N^{-\alpha/\beta}Q^{\alpha} = \hat{A}D(N,\theta)^{\alpha}N^{-\alpha/\beta}[n + (N-n)(\Delta^k)^{\beta}]^{\alpha/\beta}q^{\alpha} = [n/N + (1-n/N)(\Delta^k)^{\beta}]^{\alpha/\beta}R$.

Let $g(B) \in \{0, ..., N\}$ be the firm's rank within a permutation. If $g(B) = N$ then all the suppliers are in the coalition and the firm's contribution is $R = \hat{A}D(N,\theta)^{\alpha}q^{\alpha}$. If $g(B) = n < N$ then n suppliers are in the coalition and the firm's contribution is $\{[1-(\Delta^k)^{\beta}]n/N + (\Delta^k)^{\beta}\}^{\alpha/\beta}R$. Note that $g(B) = N$ is a special case of $g(B) = n$ with $n = N$. The share of permutations with $g(B) = n$ is $1/(N+1)$ for $n = 0, ..., N$. So the firm's Shapley value is

$$\frac{1}{N+1}\sum_{n=0}^{N}\{[1-(\Delta^k)^{\beta}]n/N + (\Delta^k)^{\beta}\}^{\alpha/\beta}R = \gamma(N,\Delta^k)R$$

where $\gamma(N, \Delta^k) \equiv \frac{\sum_{n=0}^{N}\{[1-(\Delta^k)^\beta]n/N+(\Delta^k)^\beta\}^{\alpha/\beta}}{N+1}$. It is obvious that $\gamma(N, \Delta^k)$ is strictly decreasing in N, meaning that the firm's share of revenue decreases as the number of suppliers becomes larger. And $\gamma(N, \Delta^k)$ is increasing in Δ^k, meaning that under vertical integration, the firm gets a higher share of revenue, conditional on the same functionality N.

As an additional illustration, we show that when $\beta \to -\infty$, the Shapley value here converges to the that in the main text (since min function is a special case of CES function when $\beta \to -\infty$).

Start from $\gamma(N, \Delta^k) = \frac{1}{N+1}\sum_{n=0}^{N}[n/N+(1-n/N)(\Delta^k)^\beta]^{\alpha/\beta}$. Write it as $\frac{1}{N+1}\{1+(\Delta^k)^\alpha+\sum_{n=1}^{N-1}[(n/N)1^\beta+(1-n/N)(\Delta^k)^\beta]^{\alpha/\beta}\}$. When $\beta \to -\infty$, the term behind the summation mark becomes $\left(\min\{1, \Delta^k\}\right)^\alpha = (\Delta^k)^\alpha \equiv \delta^k$, so $\gamma(N, \Delta^k)$ can be written as $\frac{1}{N+1}\{1+\delta^k N\} = \frac{\delta^k N+1}{N+1}$, which is what we have for the min function.

B.2 O-Ring

Suppose the overall quality is an o-ring function of the individual functions:

$$Q = \prod_{j=1}^{N} q_j$$

Then demand shifter is $\varphi = D(N, \theta)Q$ and revenue is $\hat{A}D(N, \theta)^\alpha Q^\alpha$. In a symmetric equilibrium, $q_j = q$ for all j. Revenue is $R = \hat{A}D(N, \theta)^\alpha q^{\alpha N}$. Again, there are three values that can be generated by a coalition:

1. In a coalition without the firm, the value is 0.

2. In a coalition with the firm and all the suppliers, the value is revenue $R = \hat{A}D(N, \theta)^\alpha q^{\alpha N}$.

3. In a coalition with the firm, but not all the suppliers, the overall quality is $Q = q^n(\Delta^k q)^{N-n} = (\Delta^k)^{N-n}q^N$, where n is the number of suppliers who are in the coalition. And the corresponding value is $\hat{A}D(N, \theta)^\alpha Q^\alpha = (\Delta^k)^{\alpha(N-n)}R$. Note that gain, case 2 is a special case of case 3 when $n = N$.

Using the same logic, the firm's Shapley value is

$$\frac{1}{N+1}\sum_{n=0}^{N}(\delta^k)^{N-n}R = \gamma(N,\delta^k)R$$

where $\delta^k \equiv (\Delta^k)^\alpha$, and $\gamma^k(N,\delta^k) \equiv \frac{1-(\delta^k)^{N+1}}{(1-\delta^k)(N+1)}$. It can also be shown that $\gamma(N,\delta^k)$ is increasing in Δ^k and decreasing in N.

C Proof of the Existence of an SSPE

Lemma 3. *There exists a symmetric equilibrium, such that* $(h_j, m_j) = (h, m)$ *for* $j = 1, ..., N$, *where* (h, m) *is uniquely solved by:*

$$h^k(N,\theta,\eta) = \arg\max_{h'} \gamma_k(N)\frac{\hat{A}}{\hat{\eta}^\alpha}D(N,\theta)^\alpha h'^{\alpha\eta}m^{\alpha(1-\eta)} - Nw_hC(N,\theta)h'$$

and

$$m^k(N,\theta,\eta) = \arg\max_{m'} \frac{1-\gamma_k(N)}{N}\frac{\hat{A}}{\hat{\eta}^\alpha}D(N,\theta)^\alpha h^{\alpha\eta}m'^{\alpha(1-\eta)} - w_mC(N,\theta)m'$$

Proof. First, consider firm's problem:

$$\max_{(h_1,h_2,...,h_N)} \gamma^k(N)\frac{\hat{A}}{\hat{\eta}^\alpha}D(N,\theta)^\alpha min\{h_1^{\alpha\eta}m_1^{\alpha(1-\eta)}, ..., h_N^{\alpha\eta}m_N^{\alpha(1-\eta)}\} - w_hC(N,\theta)\sum_{j=1}^{N}h_j$$

Suppose all suppliers stick to their equilibrium strategies. Firm's problem can be simplified to

$$\max_{(h_1,h_2,...,h_N)} \gamma^k(N)\frac{\hat{A}}{\hat{\eta}^\alpha}D(N,\theta)^\alpha min\{h_1^{\alpha\eta}, h_2^{\alpha\eta}..., h_N^{\alpha\eta}\}m^{\alpha(1-\eta)} - w_hC(N,\theta)\sum_{j=1}^{N}h_j$$

If the firm deviates by choosing $(h_1, h_2, ..., h_N) \neq (h, h, ..., h)$:

1. Firm always chooses $(h_1, h_2, ..., h_N)$ such that $h_1 = h_2 = ... = h_N$. If not, then the firm can do strictly better by lowering the levels of all $h_i > \min_{j=1,2,...,N}\{h_j\}$ to $h_i =$

$\min_{j=1,2,...,N}\{h_j\}$. So firm's problem can be further simplified to:

$$\max_{h'} \gamma^k(N)\frac{\hat{A}}{\hat{\eta}^\alpha}D(N,\theta)^\alpha h'^{\alpha\eta}m^{\alpha(1-\eta)} - w_h C(N,\theta)Nh'$$

2. It is never optimal for the firm to choose $h' \neq h$ because the objective function is strictly concave in h', so $h' = h$ is, by definition, the unique maximizer of the objective function.

Now consider supplier j's problem:

$$\max_{m_j} \frac{1-\gamma^k(N)}{N}\frac{\hat{A}}{\hat{\eta}^\alpha}D(N,\theta)^\alpha min\{h_1^{\alpha\eta}m_1^{\alpha(1-\eta)}, ..., h_N^{\alpha\eta}m_N^{\alpha(1-\eta)}\} - w_m C(N,\theta)m_j$$

Suppose the firm and all the other players stick to the equilibrium strategy. Supplier j's problem can be written as:

$$\max_{m_j} \frac{1-\gamma^k(N)}{N}\frac{\hat{A}}{\hat{\eta}^\alpha}D(N,\theta)^\alpha h^{\alpha\eta} min\{m^{\alpha(1-\eta)}, m_j^{\alpha(1-\eta)}\} - w_m C(N,\theta)m_j$$

If supplier j deviates by choosing $m_j \neq m$, supplier j will be strictly worse off because supplier's objective function is strictly concave in m_j, that means $m_j = m$ is the unique maximizer of the supplier's objective function.

□

D Proof of Lemma 2

Substituting in $h_j = h$ and $m_j = m$ to the firm and the supplier's problems in the above lemma and solve for h and m gives the following expressions:

$$h^k(N,\theta,\eta) = \left\{\frac{\alpha\hat{A}}{\hat{\eta}}\frac{D(N,\theta)^\alpha}{NC(N,\theta)}\left[\frac{\eta\gamma^k(N)}{w_h}\right]^{1-\alpha+\alpha\eta}\left[\frac{(1-\eta)(1-\gamma^k(N))}{w_m}\right]^{\alpha-\alpha\eta}\right\}^{1/(1-\alpha)}$$

$$m^k(N,\theta,\eta) = \left\{ \frac{\alpha \hat{A}}{\hat{\eta}} \frac{D(N,\theta)^{\alpha}}{NC(N,\theta)} \left[\frac{\eta\gamma^k(N)}{w_h} \right]^{\alpha\eta} \left[\frac{(1-\eta)(1-\gamma^k(N))}{w_m} \right]^{1-\alpha\eta} \right\}^{1/(1-\alpha)}$$

Substituting them into the definition of q, φ and R, we can get the following expressions:

$$q^k(N,\theta,\eta) = \left\{ \alpha\hat{A} \frac{D(N,\theta)^{\alpha}}{NC(N,\theta)} \left(\frac{\gamma^k(N)}{w_h} \right)^{\eta} \left(\frac{1-\gamma^k(N)}{w_m} \right)^{1-\eta} \right\}^{1/(1-\alpha)}$$

$$\varphi^k(N,\theta,\eta) = \left\{ \alpha\hat{A} \frac{D(N,\theta)}{NC(N,\theta)} \left(\frac{\gamma^k(N)}{w_h} \right)^{\eta} \left(\frac{1-\gamma^k(N)}{w_m} \right)^{1-\eta} \right\}^{1/(1-\alpha)}$$

$$R^k(N,\theta,\eta) = \left\{ \alpha\hat{A}^{1/\alpha} \frac{D(N,\theta)}{NC(N,\theta)} \left(\frac{\gamma^k(N)}{w_h} \right)^{\eta} \left(\frac{1-\gamma^k(N)}{w_m} \right)^{1-\eta} \right\}^{\alpha/(1-\alpha)}$$

E Proof of Theorem 1 and 2

Recall that the firm's problem is

$$\max_{k\in\{O,V\}, N\in[1,\infty)} \Pi^k(N,\theta,\eta) = \hat{A}^{1/(1-\alpha)} G(N,\theta)\Psi(\gamma^k(N),\eta)$$

The log-transformation of this problem can be written as:

$$\max_{k\in\{O,V\}, N\in[1,\infty)} \pi^k(N,\theta,\eta) = \tilde{a} + g(N,\theta) + \psi(\gamma^k(N),\eta)$$

where $\pi^k(N,\theta,\eta) \equiv \ln \Pi^k(N,\delta,\eta)$, $\tilde{a} \equiv 1/(1-\alpha)\ln\hat{A}$, $g(N,\theta) \equiv \ln G(N,\theta)$, and $\psi(\gamma,\eta) \equiv \ln\Psi(\gamma,\eta)$.

As explained in the remarks, we will adopt the methodology used in AH (2004), where we allow the firm to choose δ as a continuous variable. So the above problem can be written as:

$$\max_{N,\delta} \pi(N,\delta,\theta,\eta) = \tilde{a} + g(N,\theta) + \psi(\gamma(N,\delta),\eta)$$

where $\gamma(N,\delta) \equiv \frac{\delta^\alpha N+1}{N+1}$, and $g(N,\theta)$, $\psi(\gamma)$ are the same as defined before. Denote the solution to the first problem (where we choose the optimal N taking k as given) as $N^k(\theta)$, and the solution to the second problem (where we choose the optimal N and δ) as $N(\theta)$ and $\delta(\theta)$. Denote the 'ultimate' optimal solution as $N^*(\theta)$.

We solve for these two theorems in two steps:

1. If there exists a θ^* such that $\pi^V(N^V(\theta^*,\eta),\theta^*,\eta) = \pi^O(N^O(\theta^*,\eta),\theta^*,\eta)$, then
   (a) $N^V(\theta^*,\eta) > N^O(\theta^*,\eta)$, $\gamma^V(N^V(\theta^*,\eta)) > \gamma^O(N^O(\theta^*,\eta))$;
   (b) θ^* is unique and firms with $\theta > \theta^*$ integrate and firms with $\theta < \theta^*$ outsource;
   (c) $\theta^*(\eta)$ is strictly decreasing in η.
2. $\theta(\eta)$, $\underline{\eta}$ and $\overline{\eta}$ exist.

The following proofs solve these two steps progressively.

E.1 $\pi(N,\delta,\theta,\eta)$ is strictly concave in (N,δ)

$\pi(N,\delta,\theta,\eta)$ is strictly concave in (N,δ) iff its Hessian matrix is negative definite. Its Hessian matrix, using first order conditions to simplify, can be written as

$$\begin{pmatrix} \pi_{NN} & \pi_{N\delta} \\ \pi_{\delta N} & \pi_{\delta\delta} \end{pmatrix} = \begin{pmatrix} g_{NN} + \psi_{\gamma\gamma}\gamma_N^2, & \psi_{\gamma\gamma}\gamma_N\gamma_\delta \\ \psi_{\gamma\gamma}\gamma_N\gamma_\delta, & \psi_{\gamma\gamma}\gamma_\delta^2 \end{pmatrix}$$

It is negative definite iff $g_{NN}, \psi_{\gamma\gamma} < 0$.

$$\psi_{\gamma\gamma} = -\left\{\frac{\alpha(2\eta-1)}{1-\alpha[\gamma\eta+(1-\gamma)(1-\eta)]}\right\}^2 - \frac{\alpha}{1-\alpha}\left[\frac{\eta}{\gamma^2} + \frac{1-\eta}{(1-\gamma)^2}\right] < 0.$$

So $\pi(N,\delta,\theta,\eta)$ is strictly concave iff $g_{NN} = \frac{\alpha}{1-\alpha}\left\{\frac{\partial^2 \ln D(N,\theta)}{(\partial \ln N)^2} - \frac{\partial^2 \ln C(N,\theta,\eta)}{(\partial \ln N)^2} + 1\right\} < 0$, or $\frac{\partial^2 \ln C(N,\theta,\eta)}{(\partial \ln N)^2} > \frac{\partial^2 \ln D(N,\theta)}{(\partial \ln N)^2} + 1$. This is equivalently to saying that $G(N,\theta,\eta)$ is log-concave in N.

E.2 $N(\theta,\eta)$ and $\delta(\theta,\eta)$ strictly increasing in θ

Since $\pi(N,\delta,\theta,\eta)$ is strictly concave in (N,δ), the optimal (N,δ) is determined by the two first order conditions, $\pi_N = 0$ and $\pi_\delta = 0$. Differentiating these two equations with respect to θ and rearranging,

$$\begin{pmatrix} dN/d\theta \\ d\delta/d\theta \end{pmatrix} = \begin{pmatrix} \frac{1}{det}\pi_{\delta\delta} & -\pi_{N\delta} \\ -\pi_{N\delta} & \pi_{NN}\pi_{N\theta} \\ \pi_{\delta\theta} & \end{pmatrix}$$

where det is the determinant of the Hessian matrix.Using first order conditions to simplify,

$$\begin{pmatrix} dN/d\theta \\ d\delta/d\theta \end{pmatrix} = \frac{g_{N\theta}\psi_{\gamma\gamma}}{det}\begin{pmatrix} \gamma_\delta^2 \\ -\gamma_\delta\gamma_N \end{pmatrix}$$

We know that $\psi_{\gamma\gamma} < 0$, $det > 0$. $\gamma_\delta = \frac{N}{N+1} > 0$ and $\gamma_N = \frac{\delta-1}{(N+1)^2} < 0$. So $\frac{dN}{d\theta}, \frac{d\delta}{d\theta} > 0$ iff $g_{N\theta} > 0$ or $\frac{\partial \ln D(N,\theta)}{\partial\theta} > \frac{\partial \ln C(N,\theta,\eta)}{\partial\theta}$.

E.3 $N^k(\theta,\eta)$ strictly increasing in θ

We know that $\pi^k(N,\theta,\eta) = \tilde{a} + g(N,\theta,\eta) + \psi(\gamma^k(N),\eta)$, and $\pi^k_{N\theta} = g_{N\theta}$. So $g_{N\theta} > 0$ implies $\pi^k_{N\theta} > 0$, and that $N^k(\theta,\eta)$ is strictly increasing in θ.

E.4 Properties of θ^* (If it exists)

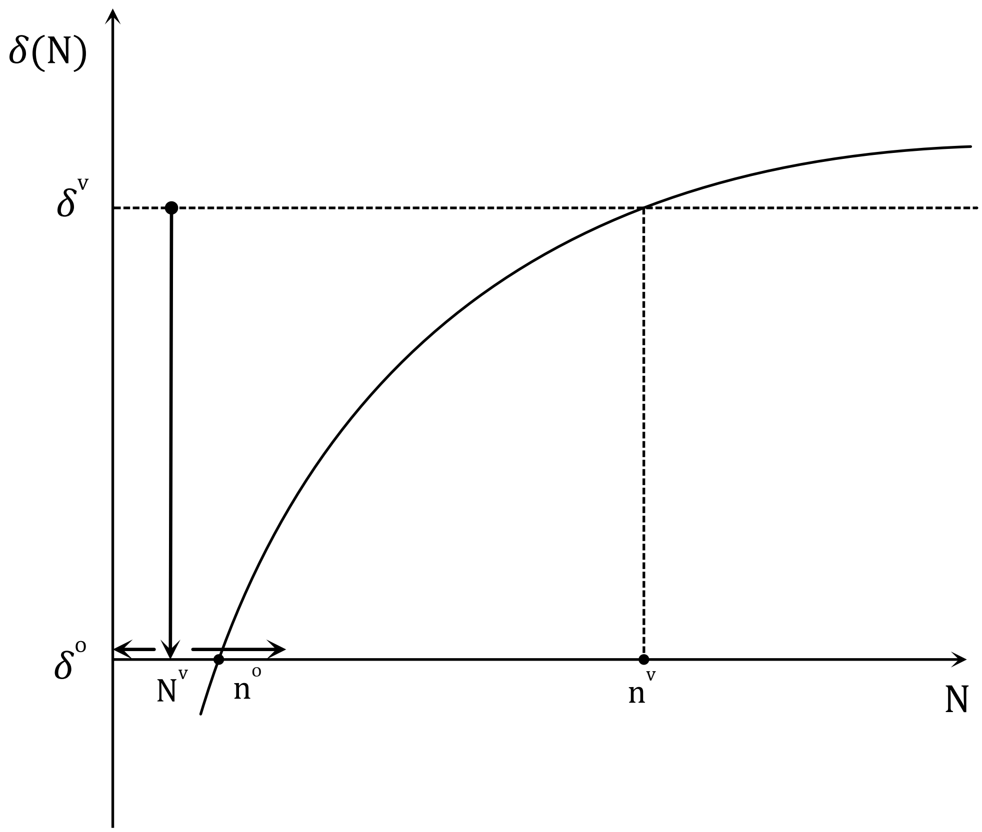
E.4.1 $N^V(\theta^*,\eta) > N^O(\theta^*,\eta)$, $\gamma^V(N^V(\theta^*,\eta)) > \gamma^O(N^O(\theta^*,\eta))$.

Suppose $\delta(N)$ crosses $\delta = \delta^O$ at (n^O,δ^O), and crosses $\delta = \delta^V$ at (N^V,δ^V). Since $\delta(N)$ is increasing in N and $\delta^O < \delta^V$, $n^O < n^V$. Depending on the values of N^O and N^V, there are 9 cases as shown in the table below. In short, we use elimination to show that if there exists a θ^* such that $\pi^V(N^V(\theta^*,\eta),\theta^*,\eta) = \pi^O(N^O(\theta^*,\eta),\theta^*,\eta)$, then we must be in the middle cell of the table (the one where $n^O < N^O, N^V < n^V$). Then we prove that in this cell, $N^V(\theta^*,\eta) > N^O(\theta^*,\eta)$, and $\gamma^V(N^V(\theta^*,\eta)) > \gamma^O(N^O(\theta^*,\eta),\eta)$. For simplicity, we write

$N^O(\theta^*, \eta)$, $N^V(\theta^*, \eta)$, $\gamma^O(N^O(\theta^*, \eta))$ and $\gamma^V(N^V(\theta^*, \eta))$ as N^O, N^V, γ^O and γ^V.

	$N^V < n^O$	$n^O < N^V < n^V$	$n^V < N^V$
$N^O < n^O$	N/A	N/A	N/A
$n^O < N^O < n^V$	N/A	$N^V > N^O$,$\gamma^V > \gamma^O$	N/A
$n^V < N^O$	N/A	N/A	N/A

1. $N^V < n^O$

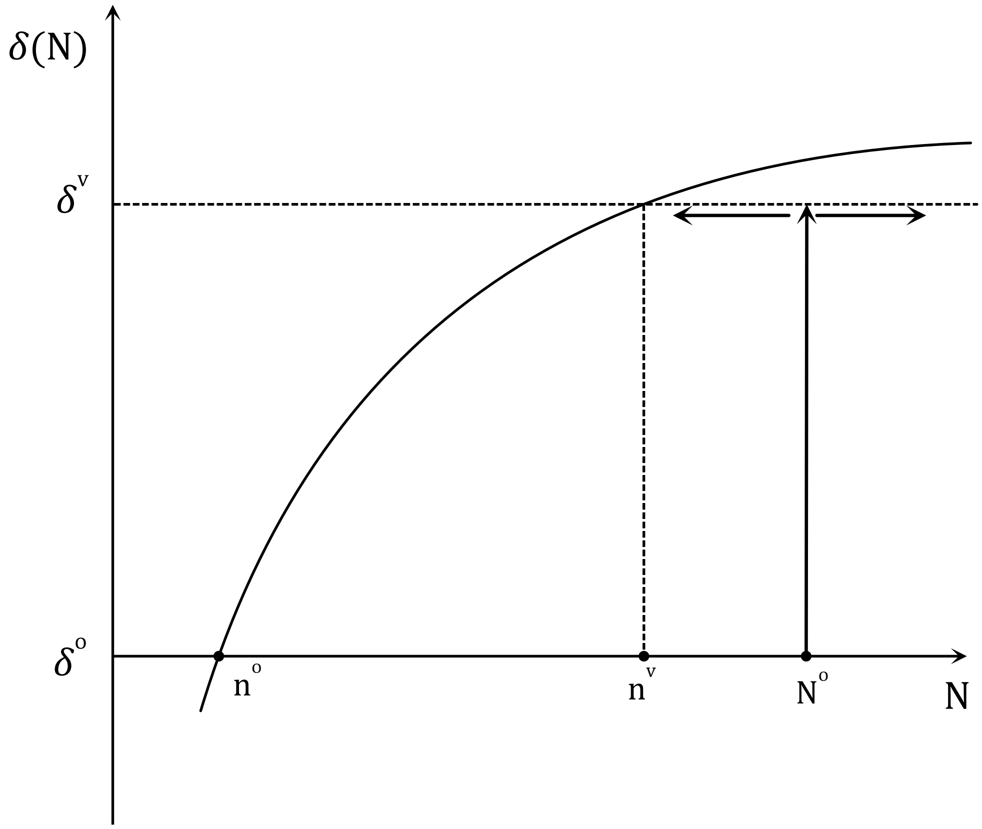


This case cannot happen. Suppose $N^V < n^O$. Start from (N^V, δ^V) and move to (N^V, δ^O). The value of $\pi(N, \delta, \theta, \eta)$ increases because all the points along this route are above $\delta(N)$, meaning that $\pi_\delta(N, \delta, \theta, \eta) < 0$ along this route as we decrease the value of δ while keeping N constant. Thus

$$\pi(N^V, \delta^V, \theta^*, \eta) < \pi(N^V, \delta^O, \theta^*, \eta) \leq \max_{N \in (1,\infty)} \pi(N, \delta^O, \theta^*, \eta)$$

In the above inequality, the left term is $\pi^V(N^V(\theta^*), \theta^*, \eta)$ and the right term is $\pi^O(N^O(\theta^*, \eta), \theta^*, \eta)$. So $\pi^V(N^V(\theta^*), \theta^*, \eta) < \pi^O(N^O(\theta^*, \eta), \theta^*, \eta)$. This contradicts with the definition of θ^*.

2. $N^O > n^V$



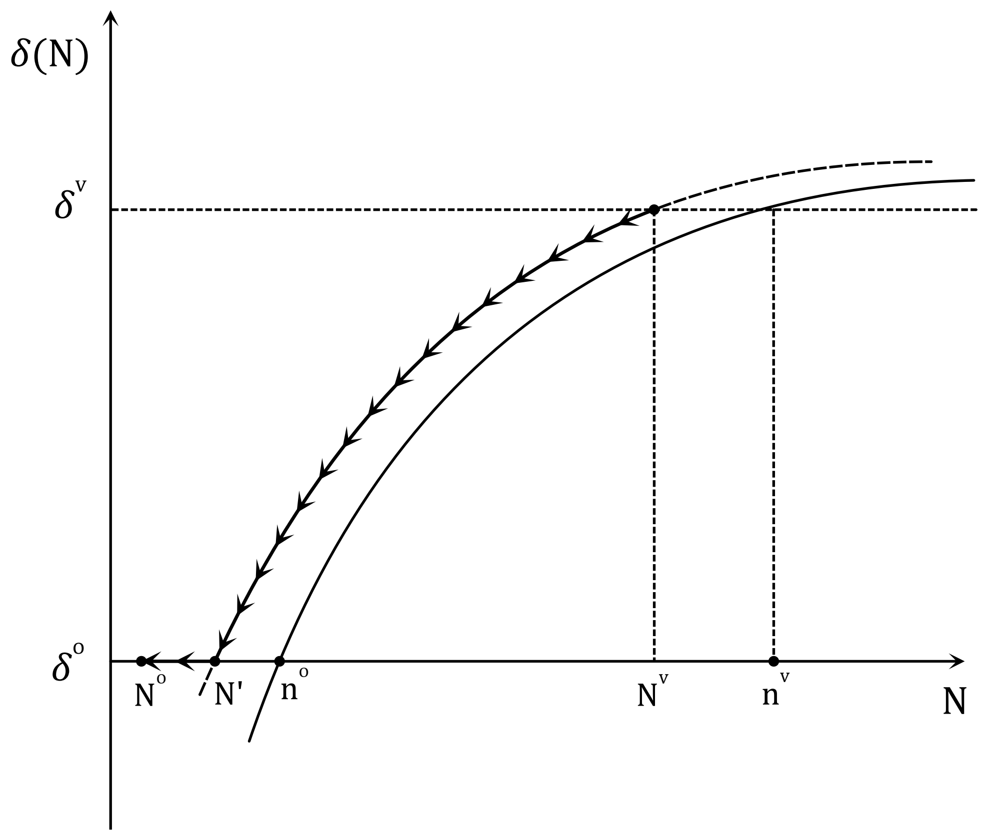
This case cannot happen. Suppose $N^O > n^V$. Start from (N^O, δ^O) and move to (N^O, δ^V). The value of $\pi(N, \delta, \theta, \eta)$ increases because all the points along this route are below $\delta(N)$, meaning that $\pi_\delta(N, \delta, \theta, \eta) < 0$ as we increase the value of δ while keeping N constant. Thus

$$\pi(N^O, \delta^O, \theta^*, \eta) < \pi(N^O, \delta^V, \theta^*, \eta) \leq \max_{N \in [1,\infty)} \pi(N, \delta^V, \theta^*, \eta)$$

In the above inequality, the left term is $\pi^O(N^O(\theta^*, \eta), \theta^*, \eta)$ and the right term is $\pi^V(N^V(\theta^*, \eta), \theta^*, \eta)$. So $\pi^O(N^O(\theta^*, \eta), \theta^*, \eta) < \pi^V(N^V(\theta^*, \eta), \theta^*, \eta)$. This contradicts with the definition of θ^*.

3. $N^O < n^O, n^O < N^V < n^V$ This case cannot happen either. To see this, draw an iso-γ line through (N^V, δ^V). Suppose it crosses $\delta = \delta^O$ at (N', δ^O). There are two cases: $N' \geq N^O$ and $N' < N^O$. Since in this case, both N^O and N^V are above $\delta(N)$, it must be that $\psi_\gamma(\gamma^O, \eta), \psi_\gamma(\gamma^V, \eta) < 0$. By Lemma 6, N^k satisfies $\pi^k_N(N^k) = g_N(N^k, \theta^*) + \psi_\gamma(\gamma^k)\gamma^k_N = 0$. So $\psi_\gamma(\gamma^k, \eta) < 0$ implies $g_N(N^k, \theta^*) < 0$ because $\gamma^k_N = \frac{\delta^k - 1}{(N+1)^2} < 0$. Thus in this case where $N^O < n^O$ and $n^O < N^V < n^V$, $g_N(N^k, \theta^*) < 0$ for $k \in \{O, V\}$.

   (a) $N^O \leq N' < N^V$



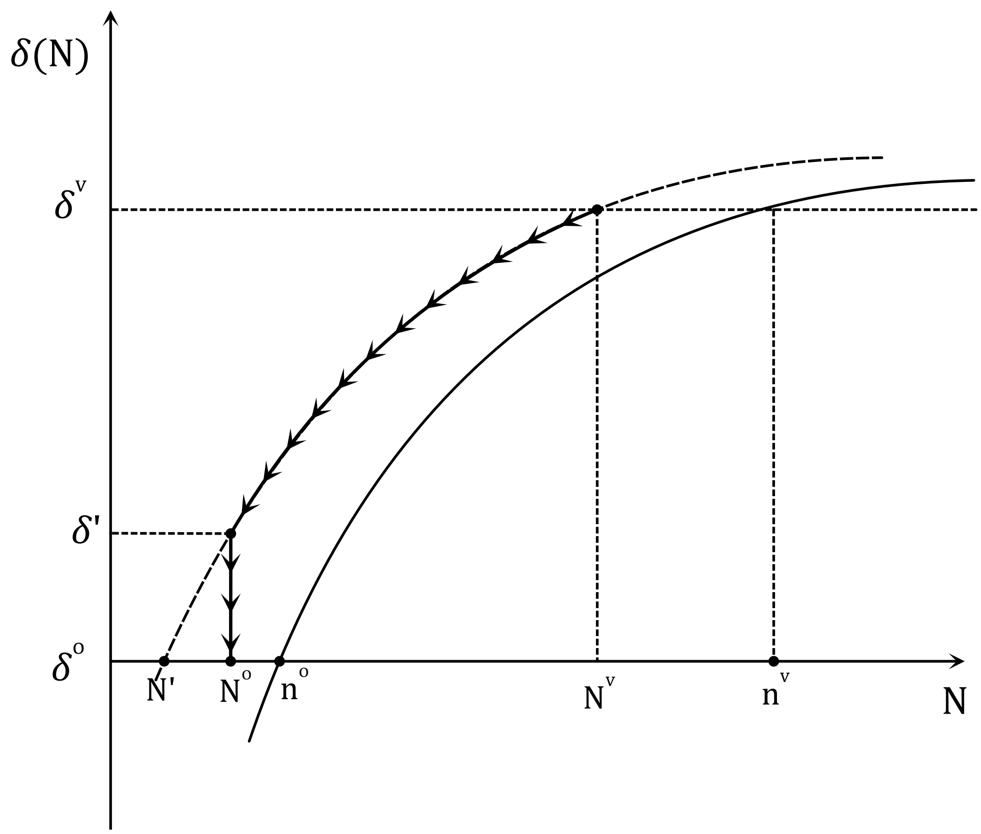
By Assumption 4, $g_{NN}(N, \theta) < 0$. So $g_N(N^V, \theta^*) < g_N(N', \theta^*) \leq g_N(N^O, \theta^*) < 0$. If we move from (δ^V, N^V) to (δ^O, N') along the iso-γ line, $\psi(\gamma, \eta)$ remains constant. But $g(N, \theta)$ increases because $g_N(N, \theta)$ remains negative as we decrease the value of N. Then move from (N', δ^O) to (N^O, δ^O). Both $g(N, \theta^*, \eta)$ and $\psi(\gamma)$ increase because $g_N(N, \theta^*), \psi_\gamma(\gamma, \eta)$ remain negative as we decrease N and γ by decreasing

N while keeping δ constant. Thus

$$\pi(N^V, \delta^V, \theta^*, \eta) < \pi(N', \delta^O, \theta^*, \eta) \leq \pi(N^O, \delta^O, \theta^*, \eta)$$

The left term in the above inequality is $\pi^V(N^V, \theta^*, \eta)$ and the right term is $\pi^O(N^O, \theta^*, \eta)$. So $\pi^V(N^V, \theta^*, \eta) < \pi^O(N^O, \theta^*, \eta)$. This contradicts with the definition of θ^*.

(b) $N' < N^O < N^V$

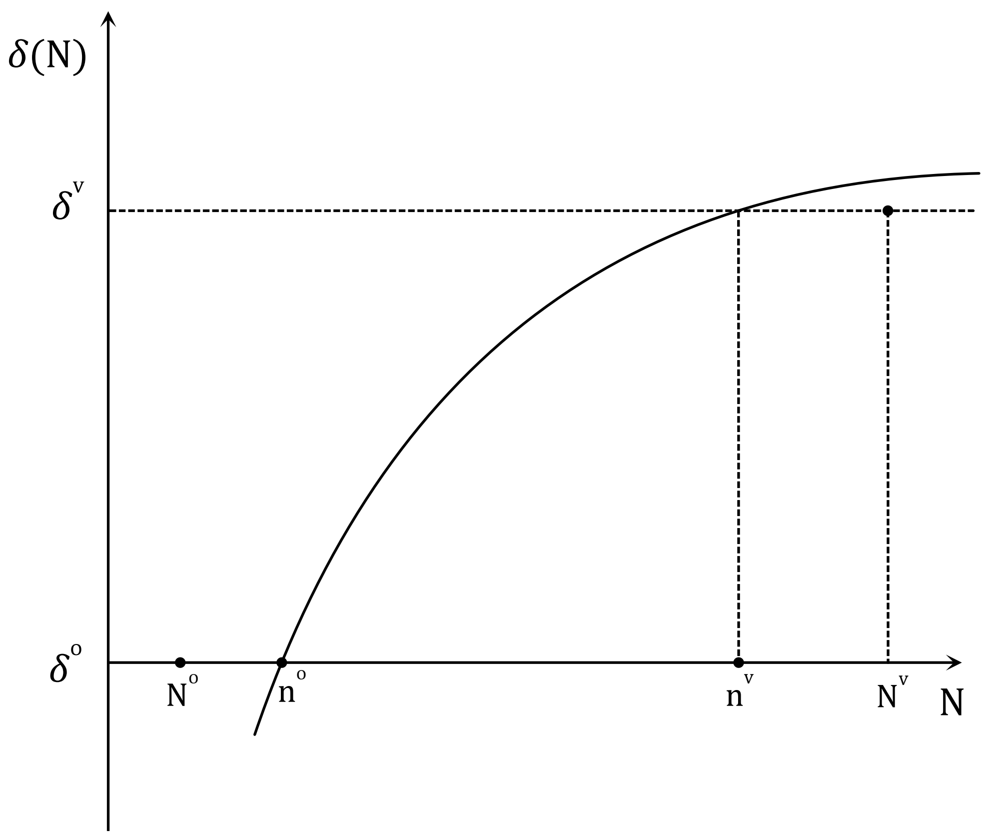


In this case, $g_N(N^V, \theta^*) < g_N(N^O, \theta^*) < 0$ still holds. But $g_N(N', \theta^*, \eta)$ may be positive. Move from (δ^V, N^V) to $(\delta^{'}, N^O)$ along the iso-γ line. $\psi(\gamma)$ remains constant. $g(N, \theta^*, \eta)$ increases because $g_N(N, \theta^*, \eta)$ remains negative as N decreases from N^V to N^O. Then move from $(\delta^{'}, N^O)$ to (δ^O, N^O). $g(N, \theta^*, \eta)$ remains constant. $\psi(\gamma, \eta)$ increases because all points along this route are above $\delta(N)$ meaning that $\psi_\gamma(\gamma, \eta) < 0$ as we decrease the value of γ by decreasing δ while keeping N constant. Thus the value of $\pi(N, \delta, \theta^*, \eta)$ increases along this route. Thus

$$\pi(N^V, \delta^V, \theta^*, \eta) < \pi(N^O, \delta', \theta^*, \eta) < \pi(N^O, \delta^O, \theta^*, \eta)$$

The left term in the above inequality is $\pi^V(N^V, \theta^*, \eta)$, and the right term is $\pi^O(N^O, \theta^*, \eta)$. So $\pi^V(N^V, \theta^*, \eta) < \pi^O(N^O, \theta^*, \eta)$. This contradicts with the definition of θ^*.

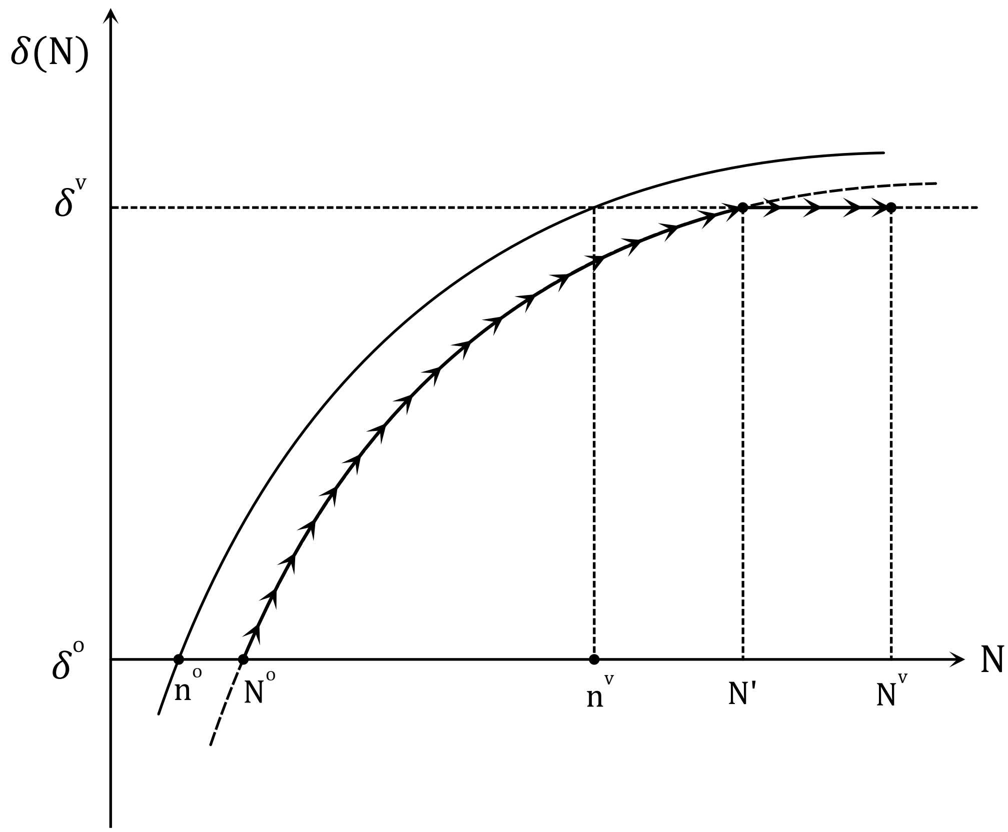
4. $N^O < n^O, N^V > n^V$



This case cannot happen either. In this case, N^O is above $\delta(N)$ and N^V is below $\delta(N)$. So $\psi_\gamma(\gamma^O, \eta) < 0 < \psi_\gamma(\gamma^V, \eta)$. Using the first order conditions for N^O and N^V, $g_N(N^O, \theta^*) < 0 < g_N(N^V, \theta^*)$. This implies $N^O > N^V$ because $g_{NN}(N, \theta) < 0$. This contradicts with $N^O < N^V$. So this case cannot happen.

5. $n^O < N^O < n^V, N^V > n^V$ In this case, both N^O and N^V are below $\delta(N)$. Draw an iso-γ line through (N^O, δ^O). Suppose it crosses $\delta = \delta^V$ at (N', δ^V). There are two possible cases: $N' \le N^V$ and $N' > N^V$.

   (a) $N^O < N' \le N^V$

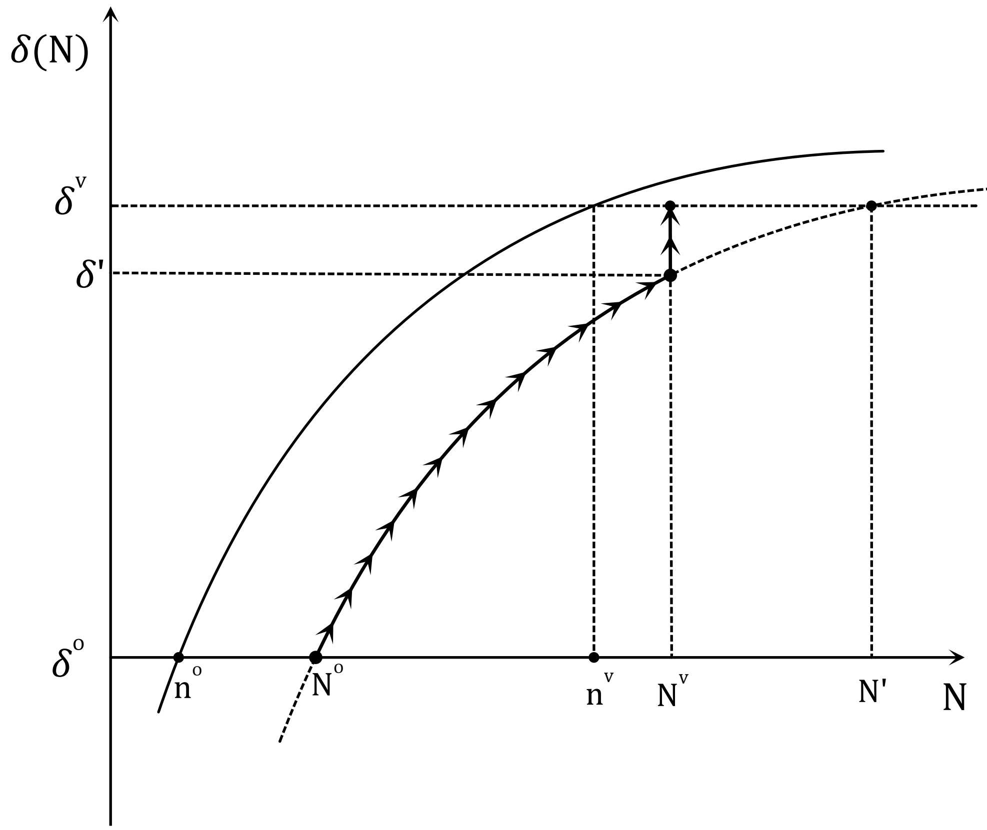


$N^O < N' < N^V$ implies $g_N(N^O, \theta^*) > g_N(N', \theta^*) > g_N(N^V, \theta^*) > 0$. So as we move from (N^O, δ^O) to (N', δ^V) along the iso-γ line, $\psi(\gamma, \eta)$ remains constant. But $g(N, \theta)$ increases because $g_N(N, \theta^*, \eta)$ remains positive as N increases from N^O to N'. Then move from (N', δ^V) to (N^V, δ^V). Both $g(N, \theta, \eta)$ and $\psi(\gamma, \eta)$ increase because g_N and $\psi_\gamma(\gamma, \eta)$ remain positive as we increase N and γ by increasing N and keeping δ constant. Thus

$$\pi(N^O, \delta^O, \theta^*, \eta) < \pi(N', \delta^V, \theta^*, \eta) \le \pi(N^V, \delta^V, \theta^*, \eta)$$

The left term is just $\pi^O(N^O, \theta^*, \eta)$ while the right term is just $\pi^V(N^V, \theta^*, \eta)$. So we reach at $\pi^O(N^O, \theta^*, \eta) < \pi^V(N^V, \theta^*, \eta)$. This contradicts the definition of θ^*.

(b) $N^O < N^V < N'$



In this case, we still have $g_N(N^O, \theta^*) > g_N(N^V, \theta^*) > 0$ since both N^O and N^V are below $\delta(N)$. But $g_N(N', \theta^*)$ may be negative. Start at (N^O, δ^O). Move to (N^V, δ') along the iso-γ line. $\psi(\gamma)$ remains constant. But $g(N, \theta^*)$ increases because $g_N(N, \theta^*)$ remains positive as N increases from N^O to N^V. So $\pi(N^O, \delta^O \theta^*, \eta) < \pi(N^V, \delta', \theta^*, \eta)$. Then move from (N^V, δ') to (N^V, δ^V). $g(N, \theta^*)$ remains constant because N does not change. But $\psi(\gamma, \eta)$ increases because this whole route is below $\delta(N)$, meaning that $\psi_\gamma(\gamma, \eta)$ remains positive as γ increases (we increase δ while keeping N constant). Thus

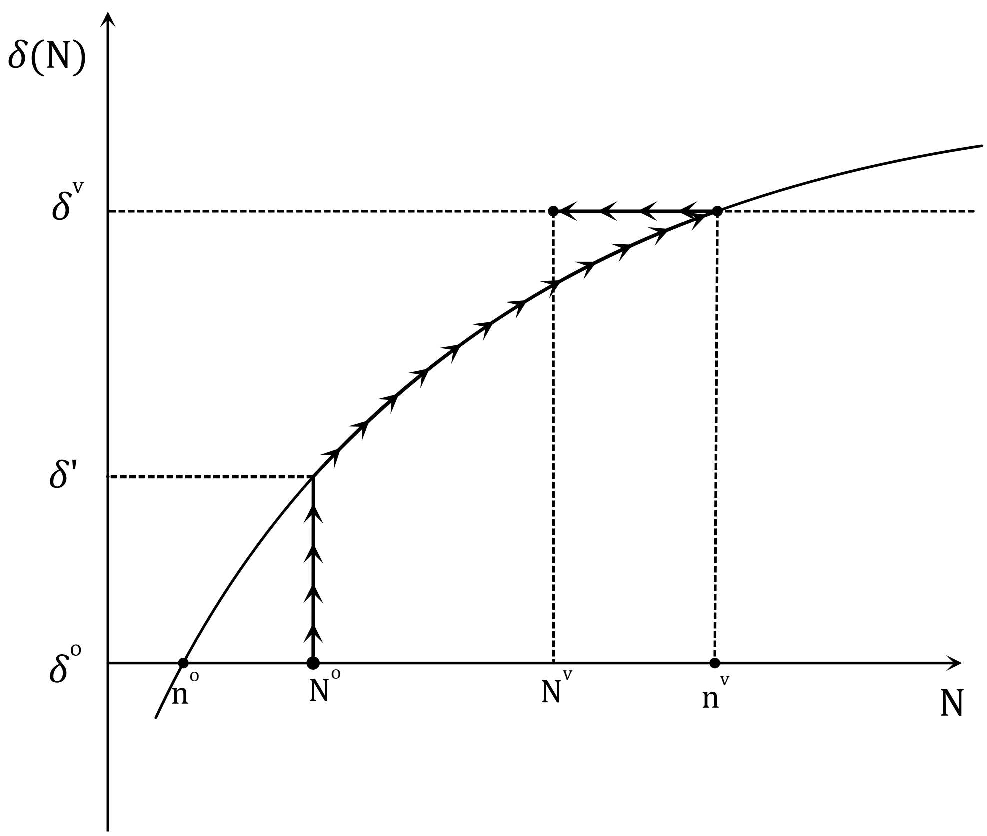
$$\pi(N^O, \delta^O, \theta^*, \eta) < \pi(N^V, \delta', \theta^*, \eta) < \pi(N^V, \delta^V, \theta^*, \eta)$$

The left term is $\pi^O(N^O, \theta^*, \eta)$ and the right term is $\pi^V(N^V, \theta^*, \eta)$. So $\pi^O(N^O, \theta^*, \eta) < \pi^V(N^V, \theta^*, \eta)$. This contradicts the definition of θ^*.

6. $n^O < N^O, N^V < n^V$ Now we have ruled out all other cases except for this one last case. This means if there exists a θ^* such that $\pi^O(N^O, \theta^*, \eta) = \pi^V(N^V, \theta^*, \eta)$, then it must be that $n^O < N^O, N^V < n^V$. Now we prove that $N^O < N^V$, and $\gamma^O < \gamma^V$.

   (a) $N^O < N^V$ In this case, (N^O, δ^O) is below $\delta(N)$ and (N^V, δ^V) is above $\delta(N)$. So $g_N(N^O, \theta^*) > 0 > g_N(N^V, \theta^*)$. $g_{NN}(N, \theta) < 0$ implies that $N^O < N^V$.

   (b) $\gamma^O < \gamma^V$ Refer to the figure below:



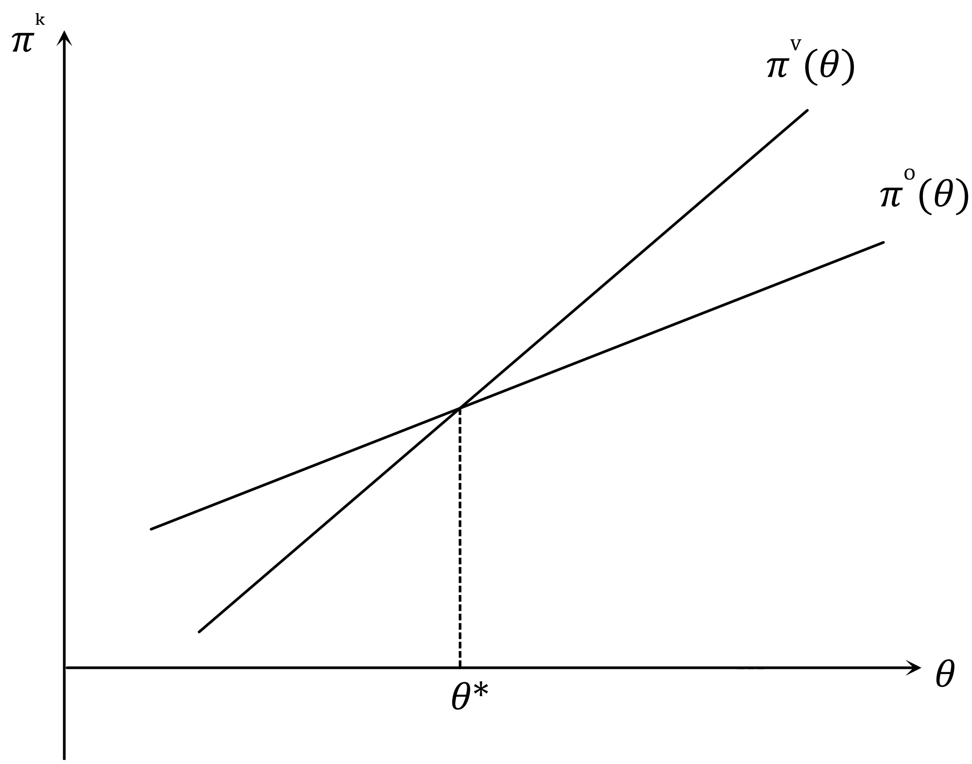
Start at (N^O, δ^O) and move to (N^O, δ'), the γ increases as we increase δ and keep N constant. Then move from (N^O, δ') to (n^V, δ^V) along $\delta(N)$, γ remains constant because $\delta(N)$ is an iso-γ line itself (with γ constant at $\gamma(\eta)$). Then move from

(n^V, δ^V) to (N^V, δ^V), γ increases because N decreases and δ remains constant. Thus The left term is γ^O and the right term is γ^V. So $\gamma^O < \gamma^V$.

E.4.2 θ^* unique, with $\theta > \theta^*$ firms integrate, $\theta < \theta^*$ firms outsource.

By Envelope Theorem, $\frac{d}{d\theta}\pi^k(\theta,\eta) = \pi^k_\theta(\theta,\eta) = g_\theta(N^k(\theta,\eta),\theta) > 0$ iff $g_\theta(N^k(\theta,\eta),\theta) = \frac{\alpha}{1-\alpha}\left\{\frac{\partial \ln D(N,\theta)}{\partial\theta} - \frac{\partial \ln C(N,\theta,\eta)}{\partial\theta}\right\} > 0$, or $\frac{\partial \ln D(N,\theta)}{\partial\theta} > \frac{\partial \ln C(N,\theta,\eta)}{\partial\theta}$.

We have shown in the previous section that $N^V(\theta^*,\eta) > N^O(\theta^*,\eta)$. So $g_\theta(N^V(\theta^*,\eta),\theta^*) > g_\theta(N^O(\theta^*,\eta),\theta^*)$ because $g_{N\theta} > 0$ by Assumption ?. Thus $\pi^V_\theta(\theta^*,\eta) > \pi^O_\theta(\theta^*,\eta)$.



$\pi^O(\theta,\eta)$ and $\pi^V(\theta,\eta)$ have single crossing because if they cross more than once, $\pi^V_\theta(\theta^*,\eta) > \pi^O_\theta(\theta^*,\eta)$ would be violated at least once. Therefore θ^* is unique. Moreover, for $\theta > \theta^*$, it must be that $\pi^V(\theta,\eta) > \pi^O(\theta,\eta)$. So firms with $\theta > \theta^*$ choose integration, and vice versa.

E.4.3 $\theta^*(\eta)$ is decreasing in η

We have shown that if θ^* exists, it is unique for a given η, thus $\theta(\eta)$ is a function. By the definition of θ^*,

$$\pi^V(N^V, \theta^*(\eta), \eta) = \pi^O(N^O, \theta^*(\eta), \eta)$$

By Implicit Function Theorem,

$$\frac{d\theta(\eta)}{d\eta} = -\frac{\pi^V_\eta(N^V, \theta(\eta), \eta) - \pi^O_\eta(N^O, \theta(\eta), \eta)}{\pi^V_\theta(N^V, \theta(\eta), \eta) - \pi^O_\theta(N^O, \theta(\eta), \eta)}$$

where $\pi^V_\theta(N^V, \theta(\eta), \eta) - \pi^O_\theta(N^O, \theta(\eta), \eta) = g_\theta(N^V, \theta(\eta), \eta) - g_\theta(N^O, \theta(\eta), \eta)$. Since $N^V > N^O$ and $g_{N\theta} > 0$, $g_\theta(N^V, \theta(\eta), \eta) > g_\theta(N^O, \theta(\eta), \eta)$. So the denominator is positive.

$\pi^V_\eta(N^V, \theta(\eta), \eta) - \pi^O_\eta(N^O, \theta(\eta), \eta) = \psi_\eta(\gamma^V, \eta) - \psi_\eta(\gamma^O, \eta)$. $\gamma^V > \gamma^O$ and $\psi_{\gamma\eta} > 0$ implies $\psi_\eta(\gamma^V, \eta) - \psi_\eta(\gamma^O, \eta) > 0$. Thus the numerator is also positive. So $d\theta(\eta)/d\eta < 0$, and $\theta(\eta)$ is strictly decreasing in η, if it exists.

E.5 $1 < N(\theta, \eta) < \infty$

The strict concavity of $\pi(N, \delta, \theta, \eta)$ implies that $\pi_{NN} < 0$ for all values of δ, θ and η. So $1 \leq N(\theta, \eta) < \infty$ if $\lim_{N \to 1} \pi(N, \delta, \theta, \eta) \geq 0$ and $\lim_{N \to \infty} \pi(N, \delta, \theta, \eta) < 0$. $N^k(\theta, \eta)$ is a special case of $N(\theta, \eta)$ when δ takes the value of δ^k. Thus $1 < N(\theta, \eta) < \infty$ implies $1 < N^k(\theta, \eta) < \infty$. We know that

$$\pi_N(N, \delta, \theta, \eta) = \frac{\alpha}{1-\alpha}\left\{\partial\left[\ln\frac{D(N,\theta)}{NC(N,\theta)}\right]/\partial N\right\} - \frac{1-\delta}{(N+1)^2}\cdot\psi_\gamma(\gamma(N,\delta), \eta),$$

$$
\begin{aligned}
& \lim_{N\to 1} \pi_N(N,\delta,\theta,\eta) \\
= \quad & \frac{\alpha}{1-\alpha} \cdot \lim_{N\to 1} \partial \left[\ln \frac{D(N,\theta)}{NC(N,\theta)} \right] / \partial N - \frac{1-\delta^\alpha}{4} \cdot \psi_\gamma\left(\frac{\delta^\alpha+1}{2}, \eta\right) \\
> \quad & \frac{\alpha}{1-\alpha} \cdot \lim_{N\to 1} \partial \left[\ln \frac{D(N,\theta)}{NC(N,\theta)} \right] / \partial N - \frac{1-\delta^\alpha}{4} \cdot \psi_\gamma\left(\frac{\delta^\alpha+1}{2}, 1\right) \\
= \quad & \frac{\alpha}{1-\alpha} \left\{ \lim_{N\to 1} \partial \left[\ln \frac{D(N,\theta)}{NC(N,\theta)} \right] / \partial N - \frac{(1-\delta^\alpha)^2}{2(1+\delta^\alpha)(2-\alpha-\alpha\delta^\alpha)} \right\} \\
\geq \quad & \frac{\alpha}{1-\alpha} \left\{ \lim_{N\to 1} \partial \left[\ln \frac{D(N,\theta)}{NC(N,\theta)} \right] / \partial N - \lim_{\delta\to 0} \frac{(1-\delta^\alpha)^2}{2(1+\delta^\alpha)(2-\alpha-\alpha\delta^\alpha)} \right\} \\
= \quad & \frac{\alpha}{1-\alpha} \left\{ \lim_{N\to 1} \partial \left[\ln \frac{D(N,\theta)}{NC(N,\theta)} \right] / \partial N - \frac{1}{4-2\alpha} \right\} \\
> \quad & \frac{\alpha}{1-\alpha} \left\{ \lim_{N\to 1} \partial \left[\ln \frac{D(N,\theta)}{NC(N,\theta,\eta)} \right] / \partial N - \frac{1}{2} \right\}
\end{aligned}
$$

$\lim_{N\to 1} \pi_N(N,\delta,\theta,\eta) > 0$ if $\lim_{N\to 1} \partial \ln \left[\frac{D(N,\theta)}{NC(N,\theta,\eta)} \right] / \partial N > 1/2$.

$$
\begin{aligned}
\lim_{N\to\infty} \pi_N(N,\delta,\theta,\eta) = \quad & \frac{\alpha}{1-\alpha} \cdot \lim_{N\to\infty} \partial \left[\ln \frac{D(N,\theta)}{NC(N,\theta,\eta)} \right] / \partial N - 0 \cdot \psi_\gamma(\delta^\alpha) \\
= \quad & \frac{\alpha}{1-\alpha} \cdot \lim_{N\to\infty} \partial \left[\ln \frac{D(N,\theta)}{NC(N,\theta,\eta)} \right] / \partial N
\end{aligned}
$$

$\lim_{N\to\infty} \pi_N(N,\delta,\theta,\eta) < 0$ iff $\lim_{N\to\infty} \partial \left[\ln \frac{D(N,\theta)}{NC(N,\theta,\eta)} \right] / \partial N < 0$.

Thus we have proved that $\lim_{N\to 1} \pi(N,\delta,\theta,\eta) > 0$ and $\lim_{N\to\infty} \pi(N,\delta,\theta,\eta) < 0$. So $N = 1$ and $N = \infty$ can never be optimal, thus $1 < N(\theta,\eta) < \infty$ for any value of δ. This implies that $1 < N^k(\theta,\eta) < \infty$ for $k = O, V$.

E.6 Existence of $\theta(\eta)$, $\underline{\eta}$ and $\overline{\eta}$.

We have shown in the previous section that $1 \leq N^k(\theta,\eta) < \infty$. So there must be an upper bound and a lower bound for the continuous function $N(\theta,\eta)$. Define $\underline{\mathrm{N}}$ and $\overline{N}$ as [10]:

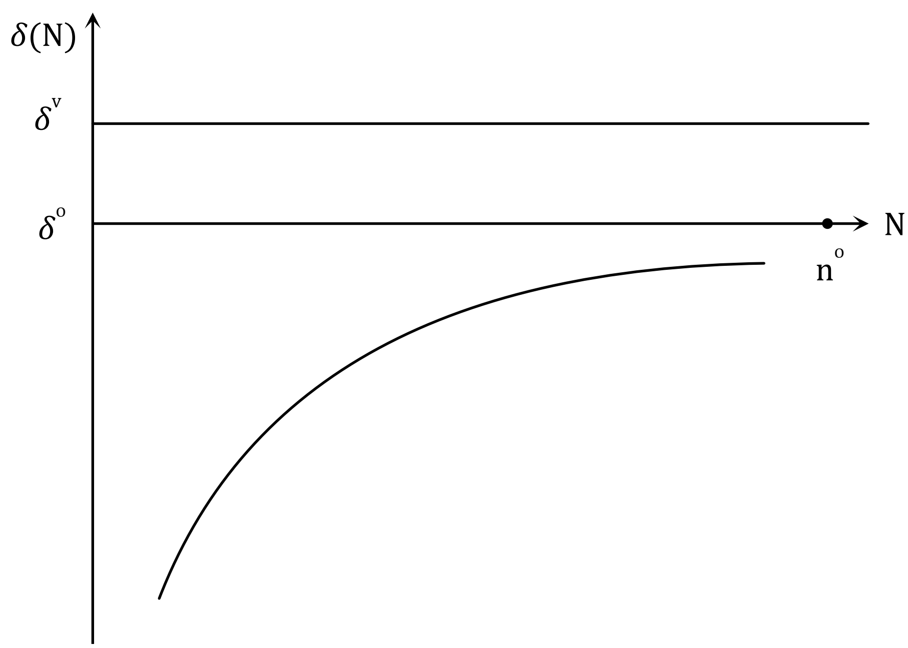
$$\underline{\mathrm{N}} = \inf_{\theta\in[0,1],\eta\in(0,1)} N(\theta,\eta)$$

and

$$\overline{N} = \sup_{\theta\in[0,1],\eta\in(0,1)} N(\theta,\eta)$$

Recall that if $\delta(N)$ crosses $\delta = \delta^O$ and $\delta = \delta^V$, the crossing points are (δ^O, n^O) and (δ^V, n^V), where $n^O = \frac{1-\gamma(\eta)}{\gamma(\eta)-\delta^O}$, and $n^V = \frac{1-\gamma(\eta)}{\gamma(\eta)-\delta^V}$.

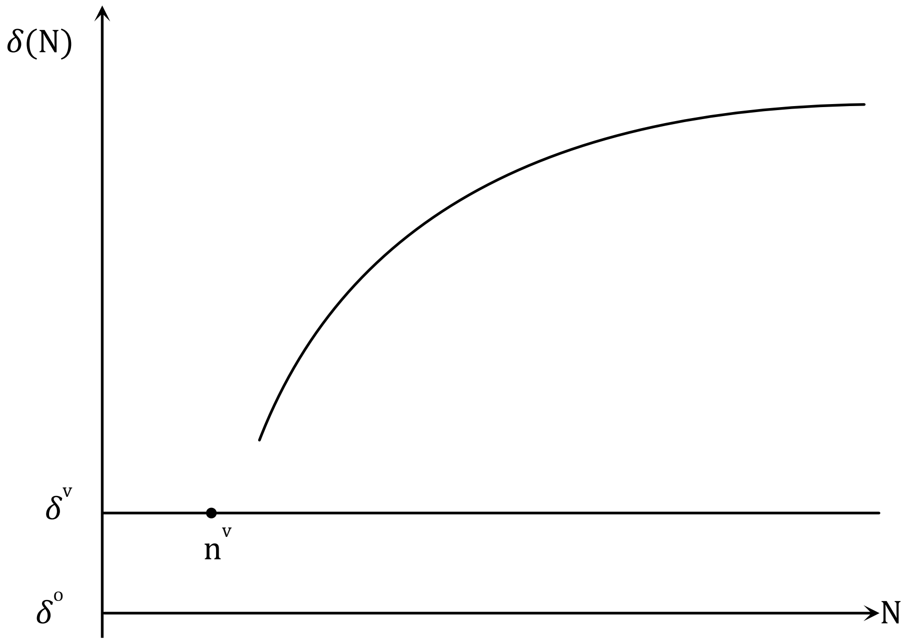
When $\eta < \gamma^{-1}(\frac{\delta^O\overline{N}+1}{\overline{N}+1})$, $n^O = \frac{1-\gamma(\eta)}{\gamma(\eta)-\delta^O} > \overline{N}$. This means $\delta(N)$ is always below $\delta = \delta^O$:



In this case, all firms will choose outsourcing (under the same logic in Appendix ?) i.e., $\pi^O(\theta,\eta) > \pi^V(\theta,\eta)$, for all θ.

When $\eta > \gamma^{-1}(\frac{\delta^V\underline{\mathrm{N}}+1}{\underline{\mathrm{N}}+1})$, $n^V = \frac{1-\gamma(\eta)}{\gamma(\eta)-\delta^V} < \underline{\mathrm{N}}$. This means $\delta(N)$ is always above $\delta = \delta^V$:

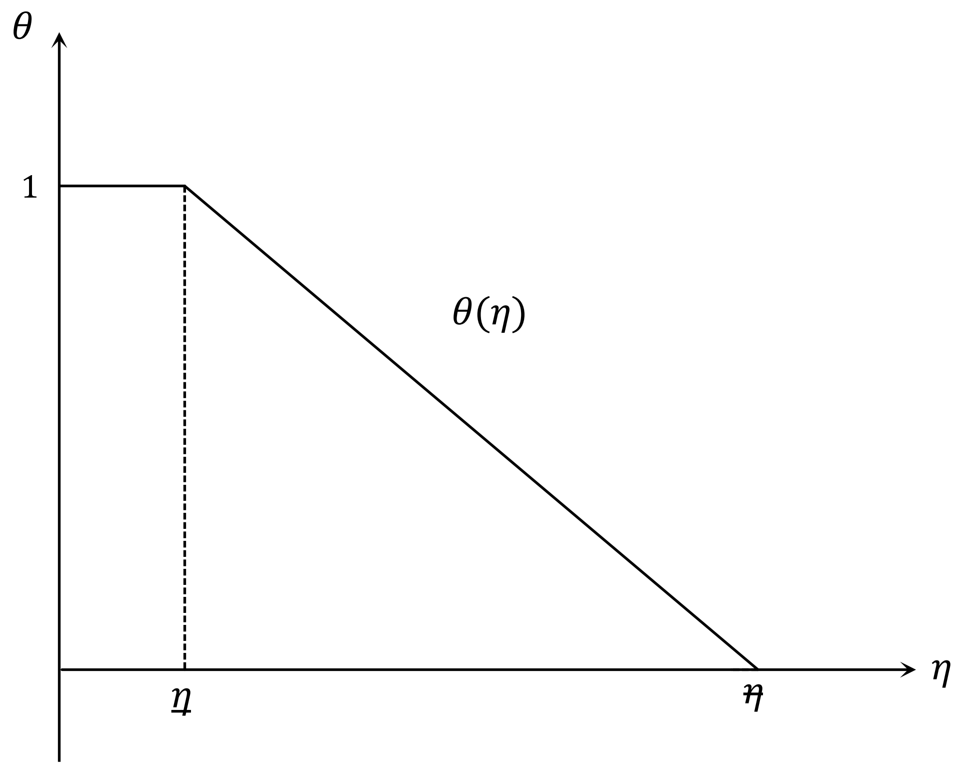
[10] $\underline{\mathrm{N}}$ and $\overline{N}$ exist because of the Completeness Axiom.



In this case, all firms choose vertical integration, i.e., $\pi^V(\theta, \eta) > \pi^O(\theta, \eta)$, for all θ.

Combining the above two figures, $\pi^V(\theta, \eta) - \pi^O(\theta, \eta) < 0$ when $\eta < \gamma^{-1}(\frac{\delta^O+1}{\overline{N}+1})$ and > 0 when $\eta > \gamma^{-1}(\frac{\delta^V \underline{\mathrm{N}}+1}{\underline{\mathrm{N}}+1})$. By continuity, for each value of θ, there must be at lease one $\eta(\theta)$ such that η is such that $\pi^V(\theta, \eta) = \pi^O(\theta, \eta)$. And there cannot be more than one η that satisfies this condition because this would violate the condition that $\theta(\eta)$ is strictly decreasing in η (see Appendix ?). Thus there is a one-to-one mapping from θ to η. Since $\theta(\eta)$ is strictly decreasing, $\eta(\theta)$ is also strictly decreasing over the interval $\theta \in [0, 1]$. So $\eta(1) \leq \eta(\eta) \leq \eta(0)$.

Define $\underline{\eta} \equiv \eta(1)$ and $\overline{\eta} \equiv \eta(0)$. For $\eta < \underline{\eta}$, $\pi^V(\theta, \eta) - \pi^O(\theta, \eta) < 0$ for all θ, all firms choose outsourcing; For $\eta > \overline{\eta}$, $\pi^V(\theta, \eta) - \pi^O(\theta, \eta) > 0$, all firms choose vertical integration. (This is because $\pi^V(\theta, \eta) - \pi^O(\theta, \eta)$ is strictly increasing in η when $\theta(\eta)$ exists, as shown in Appendix ?) If we were to draw a graph of $\theta(\eta)$, it should look like this:



F Proof of Theorem 3

By Lemma 2, the equilibrium value of q can be expressed as a function of N, θ and η:

$$q^k(N,\theta,\eta) = \left\{ \alpha \hat{A} \frac{D(N,\theta)^\alpha}{NC(N,\theta)} \left(\frac{\gamma^k(N)}{w_h} \right)^\eta \left(\frac{1-\gamma^k(N)}{w_m} \right)^{1-\eta} \right\}^{1/(1-\alpha)}$$

$$\varphi^k(N,\theta,\eta) = \left\{ \alpha \hat{A} \frac{D(N,\theta)}{NC(N,\theta)} \left(\frac{\gamma^k(N)}{w_h} \right)^\eta \left(\frac{1-\gamma^k(N)}{w_m} \right)^{1-\eta} \right\}^{1/(1-\alpha)}$$

$$\frac{dq^k}{d\theta} = \frac{\partial q^k}{\partial N} \frac{dN^k}{d\theta} + \frac{\partial q^k}{\partial \theta}$$

We have proved that $dN^k/\theta > 0$. So $dq^k/d\theta > 0$ if $\partial q^k/\partial\theta > 0$ and $\partial q^k/\partial N > 0$, or equivalently, if $\partial \ln q^k/\partial\theta > 0$ and $\partial \ln q^k/\partial N > 0$.

$$\frac{\partial \ln q^k}{\partial \theta} = \frac{1}{1-\alpha}\left\{\alpha\frac{\partial \ln D(N,\theta)}{\partial \theta} - \frac{\partial \ln C(N,\theta)}{\partial \theta}\right\}$$

$$\frac{\partial \ln q^k}{\partial N} = \frac{1}{1-\alpha}\left\{\alpha\frac{\partial \ln D(N,\theta)}{\partial N} - \frac{\partial \ln C(N,\theta)}{\partial N} - \frac{1}{N} + \eta\frac{\partial \ln \gamma^k(N)}{\partial N} + (1-\eta)\frac{\partial \ln[1-\gamma^k(N)]}{\partial N}\right\}$$

Similarly, the equilibrium value of φ can also be expressed as a function of N, θ and η,

$$\begin{aligned}\varphi^k(N,\theta,\eta) = & \quad D(N,\theta)q^k(N,\theta,\eta) \\ = & \left\{\frac{\alpha\hat{A}}{\hat{\eta}^{1-\alpha}}\frac{D(N,\theta)\gamma^k(N)^\eta[1-\gamma_k(N)]^{1-\eta}}{NC(N,\theta,\eta)}\right\}^{1/(1-\alpha)}\end{aligned}$$

$d\varphi^k/d\theta > 0$ if $\partial \ln \varphi^k/\partial\theta > 0$ and $\partial \ln \varphi^k/\partial N > 0$.

$$\frac{\partial \ln \varphi^k}{\partial \theta} = \frac{1}{1-\alpha}\left\{\frac{\partial \ln D(N,\theta)}{\partial \theta} - \frac{\partial \ln C(N,\theta)}{\partial \theta}\right\}$$

$$\frac{\partial \ln \varphi^k}{\partial N} = \frac{1}{1-\alpha}\left\{\frac{\partial \ln D(N,\theta)}{\partial N} - \frac{\partial \ln C(N,\theta)}{\partial N} - \frac{1}{N} + \eta\frac{\partial \ln \gamma^k(N)}{\partial N} + (1-\eta)\frac{\partial \ln[1-\gamma^k(N)]}{\partial N}\right\}$$

$\partial \ln q^k/\partial\theta > 0$ implies $\partial \ln \varphi^k/\partial\theta > 0$ because $\partial \ln D(N,\theta)/\partial\theta > 0$. $\partial \ln q^k/\partial N > 0$ implies $\partial \ln \varphi^k/\partial N > 0$ because and $\partial \ln D(N,\theta)/\partial N > 0$.

$\partial \ln q^k/\partial\theta > 0$ iff

$$\alpha\frac{\partial \ln D(N,\theta)}{\partial \theta} - \frac{\partial \ln C(N,\theta)}{\partial \theta} > 0,$$

or

$$\partial\left[\ln\frac{D(N,\theta)^\alpha}{NC(N,\theta,\eta)}\right]/\partial\theta > 0.$$

$\partial \ln q^k/\partial N > 0$ iff

$$\alpha\frac{\partial \ln D(N,\theta)}{\partial N} - \frac{\partial \ln C(N,\theta)}{\partial N} - \frac{1}{N} + \eta\frac{\partial \ln \gamma^k(N)}{\partial N} + (1-\eta)\frac{\partial \ln[1-\gamma^k(N)]}{\partial N} > 0.$$

This can be re-written as

$$\partial\left[\ln\frac{D(N,\theta)^\alpha}{NC(N,\theta)}\right]/\partial N > -\eta\frac{\partial \ln \gamma^k(N)}{\partial N} - (1-\eta)\frac{\partial \ln[1-\gamma^k(N)]}{\partial N}$$

where $-\eta\frac{\partial \ln \gamma^k(N)}{\partial N} - (1-\eta)\frac{\partial \ln[1-\gamma^k(N)]}{\partial N}$ has an upper-bound of 1/2.[11]. So a sufficient condition for $\frac{\partial \ln q^k}{\partial N} > 0$ is $\partial\left[\ln\frac{D(N,\theta)^\alpha}{NC(N,\theta,\eta)}\right]/\partial N > 1/2$.

We conclude that if $\partial\left[\ln\frac{D(N,\theta)^\alpha}{NC(N,\theta,\eta)}\right]/\partial\theta > 0$ and $\partial\left[\ln\frac{D(N,\theta)^\alpha}{NC(N,\theta,\eta)}\right]/\partial N > 1/2$, $q^k(\theta,\eta)$ and $\varphi^k(\theta,\eta)$ are strictly increasing in θ. It is immediate from $R = \hat{A}\varphi^\alpha$ that R is also strictly increasing in θ.

G Proof of Theorem 4 and 5

The proof of Theorem 4 and 5 follows the same logic as the proof of Theorem 1 and 2.

H Proof of Theorem 6

$$\frac{dq^k}{d\theta} = \frac{\partial q^k}{\partial N}\frac{dN^k}{d\theta} + \frac{\partial q^k}{\partial \theta}$$

Using the same logic as before, we prove in the previous section that $dN^k/\theta > 0$. So $dq^k/d\theta > 0$ if $\partial q^k/\partial\theta > 0$ and $\partial q^k/\partial N < 0$, or equivalently, if $\partial \ln q^k/\partial\theta > 0$ and $\partial \ln q^k/\partial N < 0$.

$$\frac{\partial \ln q^k}{\partial \theta} = \frac{1}{1-\alpha}\left\{\alpha\frac{\partial \ln D(N,\theta)}{\partial \theta} - \frac{\partial \ln C(N,\theta,\eta)}{\partial \theta}\right\}$$

[11] $-\frac{\partial \ln\{\gamma^k(N)^\eta[1-\gamma_k(N)]^{1-\eta}\}}{\partial N} = \frac{1-(\delta^k)^\alpha}{(N+1)^2}\left\{\frac{\eta}{\gamma^k} - \frac{1-\eta}{1-\gamma^k}\right\} = \frac{1-(\delta^k)^\alpha}{(N+1)^2}\left\{\left[\frac{1}{\gamma^k} + \frac{1}{1-\gamma^k}\right]\eta - \frac{1}{1-\gamma^k}\right\} < \frac{1-(\delta^k)^\alpha}{(N+1)^2}\frac{1}{\gamma^k} = \frac{1-(\delta^k)^\alpha}{(N+1)((\delta^k)^\alpha N+1)} < \frac{1}{N+1} < 1/2$

$$\frac{\partial \ln q^k(N,\theta,\eta)}{\partial N} = \frac{1}{1-\alpha}\left\{\alpha\frac{\partial \ln D(N,\theta)}{\partial N} - \frac{\partial \ln C(N,\theta,\eta)}{\partial N} - \frac{1}{N} + \frac{\partial \ln\{\gamma^k(N)^\eta[1-\gamma_k(N)]^{1-\eta}\}}{\partial N}\right\}$$

Similarly, $d\varphi^k/d\theta > 0$ if $\partial \ln \varphi^k/\partial\theta > 0$ and $\partial \ln \varphi^k/\partial N > 0$.

$$\frac{\partial \ln \varphi^k}{\partial \theta} = \frac{1}{1-\alpha}\left\{\frac{\partial \ln D(N,\theta)}{\partial \theta} - \frac{\partial \ln C(N,\theta,\eta)}{\partial \theta}\right\} \frac{\partial \ln \varphi^k(N,\theta,\eta)}{\partial N} = \frac{1}{1-\alpha}\left\{\frac{\partial \ln D(N,\theta)}{\partial N} - \frac{\partial \ln C(N,\theta,\eta)}{\partial N}\right.$$

$\partial \ln \varphi^k/\partial\theta > 0$ implies $\partial \ln q^k/\partial\theta > 0$ because $\partial \ln D(N,\theta)/\partial\theta < 0$. $\partial \ln \varphi^k/\partial N < 0$ implies $\partial \ln q^k/\partial N < 0$ because and $\partial \ln D(N,\theta)/\partial N > 0$.

$\partial \ln \varphi^k/\partial\theta > 0$ iff

$$\frac{\partial \ln D(N,\theta)}{\partial \theta} - \frac{\partial \ln C(N,\theta,\eta)}{\partial \theta} > 0$$

or

$$\partial\left[\ln\frac{D(N,\theta)}{NC(N,\theta,\eta)}\right]/\partial\theta > 0$$

$\partial \ln q^k/\partial N < 0$ iff

$$\alpha\frac{\partial \ln D(N,\theta)}{\partial N} - \frac{\partial \ln C(N,\theta,\eta)}{\partial N} - \frac{1}{N} + \frac{\partial \ln\{\gamma^k(N)^\eta[1-\gamma_k(N)]^{1-\eta}\}}{\partial N} < 0, or \partial\left[\ln\frac{D(N,\theta)}{NC(N,\theta,\eta)}\right]/\partial N < -\frac{\partial \ln\{\gamma$$

where $-\frac{\partial \ln\{\gamma^k(N)^\eta[1-\gamma_k(N)]^{1-\eta}\}}{\partial N}$ has a lower-bound of $-1/2$.[12] So a sufficient condition for $\frac{\partial \ln q^k}{\partial N} > 0$ is $\partial\left[\ln\frac{D(N,\theta)^\alpha}{NC(N,\theta,\eta)}\right]/\partial N < -1/2$.

So we conclude that if $\partial\left[\ln\frac{D(N,\theta)}{NC(N,\theta,\eta)}\right]/\partial\theta > 0$ and $\partial\left[\ln\frac{D(N,\theta)^\alpha}{NC(N,\theta,\eta)}\right]/\partial N < -1/2$, $q^k(\theta,\eta)$ and $\varphi^k(\theta,\eta)$ are strictly increasing in θ. It is immediate from $R = \hat{A}\varphi^\alpha$ that R is also strictly

[12] $-\frac{\partial \ln\{\gamma^k(N)^\eta[1-\gamma_k(N)]^{1-\eta}\}}{\partial N} = \frac{1-(\delta^k)^\alpha}{(N+1)^2}\left\{\frac{\eta}{\gamma^k} - \frac{1-\eta}{1-\gamma^k}\right\} = \frac{1-(\delta^k)^\alpha}{(N+1)^2}\left\{\left[\frac{1}{\gamma^k} + \frac{1}{1-\gamma^k}\right]\eta - \frac{1}{1-\gamma^k}\right\} > -\frac{1}{1-\gamma^k}\frac{1-(\delta^k)^\alpha}{(N+1)^2} = -\frac{1}{N(N+1)} > -1/2$

increasing in θ.

I Proof of Theorem 7

[TBA.]