Tax evasion: Is this a government fight, or can anyone join?

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Abstract

We study optimal tax and auditing policies in an economy where consumers play a key role in enforcing firm's sales tax compliance. If consumers request a purchase receipt, they act as tax enforcers (individual auditing). To reward agents for their auditing effort, the government returns a share of the tax revenue in the form of tax rebate. The government also audits firms directly (government auditing). We characterize the optimal policies, i.e., the retail sales tax, auditing probability and the rebate tax rate and show that the individual auditing and the firm's concealment technologies and the unitary audit cost play key roles in the determination of optimal policies. The trade-off faced by the government when choosing between alternative auditing policies is also analyzed. We show that compliance is higher when both auditing policies are in place but welfare is higher if individual auditing is the only tax enforcement policy.

Keywords: Optimal Taxation, Indirect Tax Evasion, Tax Enforcement and Auditing.

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1 Introduction

Tax evasion has a significant impact on public finances.[1] A great deal of effort has been invested by tax authorities around the world to design and implement policy measures to tackle this problem. In most countries, the main approach towards tax evasion remains focused on deterrence by improving detection probabilities or increasing evasion penalties. However, in recent decades, a more enabling approach to curb tax avoidance and evasion has received increasing attention in the literature (Renooy et al., 2004; SBC, 2004; Slemrod, 1992; Williams, 2006).

The enabling approach views individuals as social actors who are usually inclined to comply with the law (Kagan and Scholz, 1984; Murphy, 2008). Cooperation rather than coercion is pursued. The role of the tax authorities is not so much to act as a strict law enforcer that punishes bad behavior, but more as a service provider that secures future compliance (Murphy, 2005). Understanding the interplay between the deterrence and enabling approaches and how alternative policy instruments can be used to reduce tax evasion is an important avenue of research we pursued.

In this paper, we study optimal tax and auditing policies in an economy where consumers play a key role in enforcing firm's compliance with the tax code. Labor is the only factor of production and firms are competitive. A single consumption good is produced and sold to households. To finance its expenditures the government relies on (proportional) sales taxes only. We exclude the possibility of under-reporting prices and collusion between consumers and firms[2] and we focus on retail sales tax (RST) evasion, a form of evasion that has received rather less consideration in the literature.

The RST is, in general, remitted only by retail businesses, triggered by sales to final consumers. We follow Gordon and Li (2009) in the sense that receipts are not automatically issued as firms attempt to evade sales taxes. If consumers request a purchase receipt, which is costly in terms of (leisure) time, the transaction leaves a paper trail and the firm is obliged to remit taxes to the tax authority.[3] In this case, consumers act as tax enforcers and we denote it *individual auditing*. To

[1] For a comprehensive discussion of different aspects of this phenomenon see Schneider and Enste (2000).

[2] See Huang, Ueng and Yang (2010) and Chang and Lai (2004), respectively.

[3] Our modeling of the individual auditing technology is closer in spirit to shopping-time models of money, where there are transaction costs expressed in units of time and money is utilized by the household to economize the time spent on carrying out transactions. In our approach without money, a greater amount of purchases with receipt will require more auditing (non-leisure) time. Time spent on auditing is increasing in the amount of purchases with

reward buyers for their auditing effort, the government returns a portion of the tax collected to them. Through this policy, consumers can change their own attitudes towards tax evasion, promote a sense of tax morality and elicit a change in the behavior of firms (i.e., force them to collect and remit taxes). On the sales without receipt, firms take into account concealment costs and the risk of being audited by the government (*government auditing*) to decide whether to collect taxes voluntarily.

The first analysis of optimal sales taxes was undertaken by Ramsey (1927), which focused on the problem of designing sales taxes to raise a given amount of revenue at the least possible distortionary cost in a single-person economy. We follow the Ramsey primal approach (Lucas and Stokey, 1983; Chari, Christiano and Kehoe, 1991) to characterize the optimal policies, i.e., the retail sales tax, auditing probability and the rebate tax rate. There is an extensive literature in the Ramsey (primal approach) tradition that studies optimal monetary and fiscal policy (Chari, Christiano and Kehoe, 1994; Correia and Teles, 1996; Jones, Manuelli and Rossi, 1997) when structural imperfections such as tax evasion exist (Cavalcanti and Villamil, 2003; Arbex and Turdaliev, 2011). However, with few exceptions, firms are absent from the modern theory of optimal taxation.[4] To the best of our knowledge, our paper is the first to study optimal sales tax in the Ramsey framework and to propose tax rebates to consumers as a policy to enforce the tax code and curb tax evasion.

We show that the individual auditing and the firm's concealment technologies and the unitary audit cost play key roles in the determination of optimal policies. If the unit cost of audit is zero, the planner will set the optimal retail sales tax and the optimal expected penalty to the same level such that firms are indifferent between collect taxes voluntarily or evade them and being caught by the government. This result is also verified when the firm's concealment cost is zero for any audit (positive) cost. An interesting result emerges regarding the individual auditing technology. If consumers either spend a fixed amount of their time on auditing or do not spend time requesting receipts, the government does not have to reward the agent's behavior and the optimal rebate rate is zero.

Tax and enforcement policies have distinct effects on the consumers' allocations, i.e., consump-

receipts and the individual auditing technology exhibits decreasing returns to scale.

[4]See for instance, Kopczuk and Slemrod (2006), Dharmapala, Slemrod, and Wilson (2011) and Slemrod (2008).

tion and leisure, via the economy's price system. When choosing these policies optimally the planner has to take into account the effect of these auditing policies on the good's price and consequently on the economy's welfare. In line with Usher (1986), Kaplow (1990) and Cremer and Gahvari (1993) studies on tax evasion and optimal commodity tax, we analyze the policy trade-off using the Ramsey dual approach to optimal taxation.

Consider, for instance, the trade-off the planner faces when deciding to change either the audit probability *(government auditing)* or the tax rebate rate (*individual auditing*) to fight tax evasion. These policies affect consumers' welfare through the price of the good produced and the net income of the consumers. Moreover, both auditing procedures are costly for the government. To reward agents for their role of tax enforcers, the government returns a share of its tax revenue in the form of tax rebate, while to engage in auditing procedures directly, the government incurs a cost proportional to the fraction of the firms it audits. We show that the marginal rate of substitution between these policies is such that consumers' welfare and net tax revenue are constant must be the same.

Lastly, our numerical simulations compare three tax enforcement policy regimes. First we consider an economy without *individual auditing* where the only instrument available to the government to enforce the tax code is the standard (*government auditing*) detection probability. Then we allow for both *government and individual auditing* policies together. And finally, we study the case when the government transfer completely to consumer the role of auditing the economy, i.e., the detection probability by the government is constraint to be zero. The comparison of these enforcement schemes suggests that compliance is higher when both auditing policies are in place but welfare is higher if individual auditing is the only tax enforcement policy.

In practice, there has also been a noticeable increase in the use of measures that encourage compliance by preventing people from engaging in tax evasion, enable the legalization of previously informal work and change attitudes (Williams and Renooy, 2008). For instance, European Union Member States have targeted consumers with direct tax measures and wage cost subsidies. In the household repair, maintenance and improvement sector, a sector notorious for the prevalence of undeclared work, tax rebates on home maintenance expenses have been available in France under the

Universal Service Employment Cheque scheme.[5] Tax reductions for labor costs on home repairs and household services have been introduced in Italy, Luxembourg and Sweden. Businesses registered in the Danish Home Service Scheme provide services to households for which the government reimbursed a proportion of the cost (Williams and Renooy, 2008). The State of São Paulo (Brazil) has a program that entitles consumers who request sales receipts a rebate of up to thirty percent of the taxes collected in their retail purchases. Puerto Rico has turned sales receipts into lottery tickets, printing contest numbers on each receipt and holding weekly drawings for cash prizes. The main goal of these programs is to discourage businesses from dodging sales tax by making unrecorded cash sales and increase compliance of hard-to-tax establishments such as bars, restaurants and small bakeries.

Finally, a related literature that studies tax evasion focus on the interactions between consumers and firms. To improve tax collection, Tanzi and Shome (1993) suggest a cross tax-control between buyer and seller where the transaction receipt is the main proof of purchase. Das-Gupta and Gang (1996) propose a matching mechanism, essentially a comparison of buyer's and seller's record of transactions, to improve auditing of value added tax. Boadway, Marceau and Mongrain (2002) and Chang and Lai (2004) study an economy where tax evasion requires a collusion behavior for every pair of agents (firm and consumer).

The paper proceeds as follows. Section 2 presents the model economy. In Sections 3 and 4 we study optimal tax and auditing policies as well as the policy trade-off faced by the government in an economy where consumers play a role of firm's compliance with the tax code. Section 5 presents a numerical exercise and Section 6 offers concluding comments. To improve readability, proofs are collected in the Appendix.

2 The Economic Environment

Consider an economy populated by a continuum (of measure one) of identical agents, a continuum of identical firms and a government. Time evolves in discrete periods indexed by t. Firms are

[5] Individuals can purchase cheques from their local bank or post office to pay for domestic service work in the home. For example, for a job valued at €20, the employer can contribute €10 and of the remaining €10 due to be paid by the individual household, 50% can be recovered in the form of a tax credit.

competitive. They hire labor to produce a single consumption good, which is sold to households. The government must finance a stream of exogenous consumption expenditures. It can do so by taxing firms's sales at a proportional rate. The retail sales tax (RST) is remitted to the government only by firms, triggered by sales to final consumers (Slemrod, 2008). This remittance system creates opportunities for avoidance and evasion, as firms may attempt to evade sales taxes by not reporting sales to the tax authority.

The government audits the economy direct and indirectly. It can either engage in standard auditing or encourage consumers to request purchase receipts and act as tax enforcers (tax police). Both auditing methods are costly and we explore them in more details below. We discuss the problem of a representative firm, the consumer's problem and define the competitive equilibrium for this economy.

2.1 Firms

Each firm produces output Y_t using labor L_t as its only input under a constant returns technology: $Y_t = L_t$. Output is sold at a consumer price p_t and sales are subject to a tax rate τ_t. The enforcement of the tax rules is stochastic and occurs either due to consumer's actions or due to government's auditing measures and procedures. We sketch the scenario of tax evasion by means of the following two-stage game.

First, firms sell the good to consumers at a price p_t and the seller must remit any sales tax triggered by this sale. However, the remittance of taxes to the government will depend on whether consumers request or not a receipt of the transaction. Let a_t be the proportion of a consumer's purchases for which he asks for receipts in period t. We denote this mechanism *individual auditing*, where the consumer acts to enforce the tax code. In this case, firms are obliged to collect taxes in the amount of $a_t \tau_t p_t L_t$.

In the second stage, out of the proportion of sales that is not subject to individual auditing, i.e., $(1 - a_t)p_t L_t$, the firm will decide whether to evade taxes or not. The firm may attempt to evade taxes by reporting only a fraction γ_t of its sales. In order to avoid being detected by the tax authorities evading taxes the firm spends resources to conceal it. Each unit of sales concealed from the authorities entails a resource cost of $H(\xi_t)$ to the firm, where $\xi_t = (1 - a_t)(1 - \gamma_t)$. The concealment cost is an increasing and convex function of the proportion ξ_t of sales not reported, either because consumers did not asked for a receipt, $(1 - a_t)$, or firms decided to hide a fraction of its sales, $(1 - \gamma_t)$.[6] Finally, firms may be discovered evading retail sales taxes with probability $\theta_t \in [0, 1]$, which we denote *government auditing*. If caught evading taxes, a firm must pay a tax rate $\rho_t \geq \tau_t$, i.e., the standard RST τ plus a surcharge factor.

Hence, firms choose prices and the proportion of sales it wants to collect taxes to maximize expected profits in each period $t \geq 0$,

$$\Pi^e = \{1 - a_t\tau_t - (1 - a_t)[\gamma_t\tau_t + (1 - \gamma_t)\theta_t\rho_t + (1 - \gamma_t)H(\xi_t)]\}\, p_tL_t - w_tL_t. \qquad (1)$$

where w_t is the nominal wage. Firms' behavior is characterized by the following first order necessary and sufficient conditions:

$$\tau_t = \theta_t\rho_t + H(\xi_t) - (1 - \gamma_t)H_\gamma(\xi_t) \qquad (2)$$

$$w_t = [1 - T_t - \xi_t H(\xi_t)]p_t \qquad (3)$$

where $T_t = a_t\tau_t + (1 - a_t)[\gamma_t\tau_t + (1 - \gamma_t)\theta_t\rho]$ is tax a firm expects to pay to the tax administrator on each unit of output sold.

Equation (2) reveals the optimum choice of firms regarding the percentage of their sales they want to report to the tax authorities, after consumers decide the fraction of purchases with receipt a_t. A firm chooses γ_t such that the cost of honestly report this amount and remit the appropriate sales taxes is equal to the hiding cost plus the penalty rate in the event of being detected by the government. The condition (3) rules out corner solutions and allows us to focus on the interesting problem of tax evasion when a positive (and not infinite) quantity of output is produced and sold.

[6] The firm will incur concealment costs regardless whether it issues sales receipts (and collect the appropriate amount of taxes) or is detected by the government evading taxes (and has to pay a penalty). Without such (concealment) strategy firms would be automatically detected without the necessity of auditing by a simple process of verification.

Combining equations (2) and (3) we obtain an equilibrium condition for the firm as follows:

$$\frac{w_t}{p_t} = P(a_t, \gamma_t), \tag{4}$$

where $P(a_t, \gamma_t) = [1 - H(\xi_t) + (a_t + (1 - a_t)\gamma_t)(1 - \gamma_t)H_\gamma(\xi_t) - \theta_t\rho_t]$ and equation (4) reflects how the real wage, via sales price, is affected by the amount of sales with receipts, the amount of sales voluntarily reported and the concealment cost.

2.2 Households

The representative household supplies L_t units of labor to the market at the wage w_t. He devotes the remainder of his time to either leisure h_t or auditing d_t. Preferences are represented by the following utility function

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - L_t - d_t) \tag{5}$$

where the instantaneous utility function u is increasing in the first and decreasing in the second argument and strictly concave, $\beta \in (0, 1)$ is the discount rate, c_t is consumption and the time endowment is normalized to 1 so that leisure is $h_t = 1 - L_t - d_t$.

Upon the purchase of goods for consumption, a consumer has also to decide whether or not to request a receipt of his purchase. The very act of asking for a receipt is what we call *individual auditing* and this activity is time consuming (leisure cost). Every sale triggers remittances of sales taxes to the government as long as receipts of purchases are generated. When a consumer does not request the receipt, firms have the opportunity to evade taxes. If receipt is issued, it leaves a paper trail of the transaction and sales taxes must be remitted.

Auditing takes time. Such request of a fiscal receipt can be associated with a longer time standing in the line waiting for assistance or even to fill out paper work. The amount of time employed in auditing is described by the technology

$$d_t = d(a_t) = \bar{d}a^\delta \tag{6}$$

where a_t is the proportion of goods purchased and a receipt is requested, $\bar{d} \in (0, 1)$ is an upper bound on the leisure cost of auditing activities and δ is the auditing technology parameter.[7] When the auditing technology exhibits decreasing returns to scale, $\delta \in (0,1)$, it is less costly, in terms of leisure time, for a consumer to request a receipt for an additional unit of purchased good. Auditing time is increasing in the share of consumer's purchases for which he asks receipts for, i.e., $d'(a_t) > 0$. The auditing technology satisfies $d(0) = 0$, meaning that if individuals do not request receipts in any of their purchases no time is spent in auditing and $d(1) = \bar{d}$. This last assumption is to avoid a degenerate behavior of time allocation and guarantee an interior solution for labor and auditing time.[8]

Since individual auditing is costly, without further incentives, consumers would not ask for receipts. To reward agents for their role of tax enforcers, the government offers a tax rebate based on the amount of taxes collected due to consumers' auditing actions. The amount of tax rebate an agent receives in time t depends on the amount of purchases with receipts that triggered tax remittances to the government in the previous period, i.e., $p_{t-1}a_{t-1}\tau_{t-1}L_{t-1}$.

The representative consumer faces the following budget constraint:

$$p_t c_t = w_t L_t + \phi_{t-1}\left(p_{t-1}a_{t-1}\tau_{t-1}L_{t-1}\right) \tag{7}$$

where p_t is the consumer price (equal to the producer price since we abstract from consumption tax), $\phi \in [0,1]$ is the tax rebate rate. Notice that agent's auditing effort in a previous period affects his current choice of consumption and leisure as the tax rebate received from the government represents additional income available for consumption.

2.3 The government and aggregate resources

The economy as a whole faces the following aggregate resource constraint

$$c_t + G_t + \theta_t N_t + \xi_t H(\xi_t) L_t = L_t \tag{8}$$

[7] The idea that spending time on consumption activities might be substitute to labor is also addressed on Boadway and Gahvari (2006), Gahvari (2007), Gahvari and Yang (1993) and Kleven (2004). Our interpretation of the time spent on calling for receipts has the same spirit of their idea about time spent on consumption activities.

[8] This function is usually related with the information transmission technology within the firm. Modern firms (better internet connections, computerized fiscal receipts emission) with better technologies make the transmission less (time) costly for consumers.

where G_t is the government consumption, c_t is the aggregate private consumption (since agents are identical the aggregate behaves like any one of them) and N is a fixed unit cost of audit. The government finances its expenditures by levying proportional taxes on firms' sales. By Walras law, the government's period-by-period budget constraint is implied by the household's budget constraint and the resource constraint. The government budget constraint is given by

$$p_t \left(G_t + \theta_t N_t \right) + \phi_{t-1} \left(p_{t-1} a_{t-1} \tau_{t-1} L_{t-1} \right) = T_t p_t L_t. \tag{9}$$

The terms on the left side are, respectively, government expenditures, auditing spending and tax rebate payments. The right-hand side of this equation contains government revenues generated by individual auditing, firm's voluntary compliance and penalties imposed on firms detected evading taxes.

2.4 Competitive equilibrium

The individual maximizes (5) subject to (7) by choosing the allocations c_t, h_t and a_t, taking as given prices and government policy instruments. The individual's optimization problem yields the following first-order conditions

$$u_1(t) = \mu_t p_t \tag{10}$$

$$u_2(t) = \mu_t w_t + \beta \mu_{t+1} \phi_t p_t a_t \tau_t \tag{11}$$

$$u_2(t) d'(a_t) = \beta \mu_{t+1} \phi_t p_t \tau_t L_t \tag{12}$$

where μ is the Lagrange multiplier on the consumer budget constraint, equation (7). From equations $(10) - (12)$, we obtain the following equilibrium conditions:

$$u_2(t) = u_1(t) \frac{w_t}{p_t} + \beta u_1(t+1) \phi_t \frac{p_t}{p_{t+1}} a_t \tau_t \tag{13}$$

$$u_2(t) d'(a_t) = \beta u_1(t+1) \phi_t \frac{p_t}{p_{t+1}} \tau_t L_t \tag{14}$$

Equation (13) shows the relative price of leisure with respect to consumption when the agent engages in non-zero individual auditing. Equation (14) illustrates the trade-off agents face when deciding to request sales receipts. Consumers will choose the proportion of their purchase with receipts a_t to the extent that the disutility of auditing $u_2(t)d'(a_t)$ is equal to the utility of more consumption next period due to the tax rebate received from the government $(\beta u_1(t+1)\phi_t \frac{p_t}{p_{t+1}}\tau_t L_t)$.

Combining equation (13) and (14) we obtain the following equilibrium condition:

$$\frac{w_t}{p_t} = \frac{u_2(t)}{u_1(t)}\left(\frac{L_t - a_t d_a(t)}{L_t}\right), \tag{15}$$

Equation (15) represents the real wage from the consumer's problem and it shows the relative price of leisure with respect to consumption when the agent engages in non-zero individual auditing. Notice that if $a_t = 0$ the relationship between w_t and p_t is the standard marginal rate of substitution between leisure and consumption, i.e., $u_2(t)/u_1(t)$.

Definition 1 *A competitive equilibrium is a policy $\Upsilon = \{\tau_t, \theta_t, \rho_t, \phi_t\}_{t=0}^{\infty}$, government spending $\bar{G} = \{g_t, N_t\}_{t=0}^{\infty}$, private agents' allocations $x = \{c_t, a_t, L_t, \gamma_t\}_{t=0}^{\infty}$, a price system $\bar{P} = \{p_t, w_t\}_{t=0}^{\infty}$ such that given the policy, government spending and the price system, the resulting household's and firm's allocations choices maximize the consumer's utility and firms' profits and satisfies the government's budget constraint, the economy's resource constraint and market clearing conditions.*

3 Optimal Tax and Audit Policy

We study the Ramsey problem using the primal approach to optimal taxation (Lucas and Stokey, 1983; Chari, Christiano and Kehoe, 1991). At the beginning of each period, the government announces its program of tax rates and individuals behave competitively.[9] The objective of the social planner is to choose values of its tax instruments such that the agent's utility and firm's profit are maximized (i.e., taking into account the equilibrium reaction of private agents to the policy instruments) and the government is constrained to raise a given amount of revenue.

[9] We follow the majority of the literature in assuming that the government can commit to follow a long-term program for taxing labor income. We assume that there are institutions that effectively solve the time inconsistency problem so that the government can commit to the tax plan it announces in the initial period.

The Ramsey problem is a programming problem of finding the optimum within a set of allocations that can be implemented as a competitive equilibrium with distortionary taxes. In other words, the Ramsey problem is to choose a process for tax and enforcement policies $\{\tau_t, \theta_t, \rho_t, \phi_t\}$, which maximizes social welfare and satisfies (7) and an implementability constraint, a second-best optimal tax problem (Chari and Kehoe, 1999). In this approach, taxes and prices are eliminated, so that the government can be thought as directly using the allocations as controls. We assume that evasion penalties (ρ) are at their statutory level and constant over time and the Ramsey planner chooses optimally the retail sales tax τ, the probability of detection θ and the rebate rate ϕ.

The implementability constraint is obtained from the consumers' intertemporal budget constraint by expressing prices and auditing instruments in terms of allocations through the marginal efficiency conditions of consumers - equations (10) and (11). That is,

$$\sum_{t=0}^{\infty} \beta^t \left[u_1(t) c_t - u_2(t) L_t \right] = W_0, \tag{16}$$

where $W_0 = u_1(0) p_{-1} \phi_{-1} \tau_{-1} a_{-1} L_{-1}$.

Definition 2 *A Ramsey equilibrium in this economy is a policy* Υ*, an allocation rule* x *and a price rule* $\bar{P}$ *that satisfy the following two conditions:* (i) *the policy* Υ *maximizes* (5) *subject to the government budget constraint* (9) *with allocations and prices given by* x *and* $\bar{P}$ *and* (ii) *for every* Υ'*, the allocation* $x(\Upsilon')$*, the price rule* $\bar{P}(\Upsilon')$ *and the policy* Υ' *constitute a competitive equilibrium.*

The social planner is also constraint to choose allocations that are feasible in this economy. This implies that the resource constraint must be imposed on the planner's problem. Following the primal approach to the Ramsey problem, we must express this constraint in terms of allocations only, but notice that the audit probability θ appears in the equation (8). This implies that we need to express θ in terms of allocations, which is related to an interesting aspect of our problem.

Recall that the social planner's goal is to maximize a representative agent's utility subject to raising an exogenously determined amount of revenue for the government, taking into account the equilibrium reactions by consumers and firms to the distortionary tax system. Notice that in the derivation of the implementability constraint, equation (16), we used equations (10) and (11) only, which leaves (2), (3) and (12) as conditions that must also be imposed. Imposing these conditions on the Ramsey planner problem means that, given allocations, the real wage paid by the firm and the real wage received by the consumer, equations (4) and (15), coincide. This strategy is similar to the one employed by Jones, Manuelli and Rossi (1997), Domeij (2005) and Reinhorn (2009) when addressing different problems.

Combining equations (4) and (15), and solve for θ_t we obtain:

$$\begin{aligned}\theta_t &= \frac{1}{\rho_t}\left[1 - H(\xi_t) + (1-\gamma_t)(a_t + (1-a_t)\gamma_t)H_\gamma(\xi_t) - \frac{u_2(t)}{u_1(t)}\left(\frac{L_t - a_t d_a(t)}{L_t}\right)\right], \\ &= \Phi(c_t, L_t, a_t, \gamma_t; \rho_t). \end{aligned} \tag{17}$$

We then substitute $\theta_t = \Phi(c_t, L_t, a_t, \gamma_t; \rho_t)$ into equation (8) and rewrite the economy's resource constraint as follows:

$$c_t + G_t + \Phi(c_t, L_t, a_t, \gamma_t; \rho_t)N_t + \xi_t H(\xi_t)L_t = L_t. \tag{18}$$

Proposition 1 *The household's allocations and the date* 0 *policy* Υ_0*, in a competitive equilibrium satisfy the economy's resource constraint* (18) *and the implementability constraint* (16)*. Furthermore, given household's choices and* Υ_0*, prices and policies can be constructed for all dates, which together with the choices and date* 0 *policies constitute a competitive equilibrium.*

Proof. See Appendix $A.1$. ■

The planner's maximization problem can thus be written as follows:

$$\max_{\{c_t, L_t, a_t, \gamma_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - L_t - d(a_t)) \tag{19}$$

subject to (16), (18), and $\bar{G}$, N, τ_0, ϕ_0, θ, ρ given. Our goal is to study this economy in the steady state and we will focus our attention on the first-order conditions for period 1 and onwards. The first-order conditions for period zero are not presented.

The implementability constraint is similar to the objective function in the sense that both are discounted infinite sum of terms. Thus, given the Lagrange multiplier λ_1 (which is endogenous), it is possible to write the pseudo-welfare function of the social planner as

$$Z(c_t, L_t, a_t; \lambda_1) \equiv u(c_t, 1 - L_t - d(a_t)) + \lambda_1 \left[u_1(t) c_t - u_2(t) L_t \right]$$

where λ_1 is the Lagrange multiplier on (16). Evaluating the first order conditions of problem (19) at the steady state, and after some manipulation, we obtain

$$Z_c^* = [1 + \Phi_c^* N] \lambda_2^* \tag{20}$$

$$Z_L^* = [\Phi_L^* N + \xi^* H(\xi^*) - 1] \lambda_2^* \tag{21}$$

$$Z_a^* = [\Phi_a^* N - ((1 - \gamma^*) H(\xi^*) - \xi^* H_{a^*}) L^*] \lambda_2^* \tag{22}$$

$$\Phi_\gamma^* N = [(1 - a^*) H(\xi^*) - \xi^* H_{\gamma^*}] L^* \tag{23}$$

where λ_2 is the Lagrange multiplier on the resource constraint, equation (18) and Z_c^*, Z_L^*, Z_a^*, Φ_c^*, Φ_L^*, Φ_a^*, Φ_γ^* are defined as follows:

$$Z_c^* = u_1^* + \lambda_1^* \left[u_{11}^* c^* + u_1^* \right]$$

$$Z_L^* = -u_2^* - \lambda_1^* \left[u_{22}^* L^* + u_2^* \right]$$

$$Z_a^* = -u_2^* d_{a^*}$$

$$\Phi_c^* = \frac{1}{\rho} \left(\frac{u_{11}^*}{(u_1^*)^2} \frac{u_2^*}{L^*} \right) (L^* - a^* d_a^*)$$

$$\Phi_L^* = -\frac{1}{\rho} \left(\frac{1}{u_1^* L^*} \right) \left[u_{22}^* (L^* - a^* d_a^*) + u_2^* \frac{a^* d_a^*}{L^*} \right]$$

$$\Phi_a^* = \frac{1}{\rho} \left\{ -H_a^* + (1 - \gamma^*) \left[(1 - \gamma^*) H_\gamma^* + (a^* + (1 - a^*) \gamma^*) H_{\gamma a}^* \right] - \left(\frac{u_2^*}{u_1^* L^*} \right) (d_a^* + a^* d_{aa}^*) \right\}$$

$$\Phi_\gamma^* = \frac{1}{\rho} \left\{ -H_\gamma^* - (a^* + (1 - a^*) \gamma^*) H_\gamma^* + (1 - \gamma^*) \left[(1 - a^*) H_\gamma^* + (a^* + (1 - a^*) \gamma^*) H_{\gamma\gamma}^* \right] \right\}$$

Following the primal approach to the Ramsey planner's maximization problem (19) we find the optimal allocations c^*, L^*, a^* and γ^* and with them we can recover not only the optimal policies but also good's price. The following proposition show that policy instruments and prices are affected by the unit audit cost, the individual auditing technology and the concealment cost function.

Proposition 2 *Suppose that the evasion penalty ρ is fixed. If the solution to the Ramsey problem converges to a steady state, and the government has access to a retail sales tax τ, an auditing probability θ and a tax rebate ϕ, then in steady state, these instruments and good's price are as follows:*

$$\tau^* = \left[\Phi_L^* - \Phi_c^* \frac{Z_L^*}{Z_c^*} + \Phi_\gamma^* \left(\frac{1-a^*-(1-a^*)\gamma^*}{(1-a^*)L^*}\right)\right] N - M^*, \quad (24)$$

$$\theta^* = \frac{1}{\rho}\left\{\left[\Phi_L^* - \Phi_c^* \frac{Z_L^*}{Z_c^*} - \Phi_\gamma^* \left(\frac{a^*+(1-a^*)\gamma^*}{(1-a^*)L^*}\right)\right] N - M^*\right\}, \quad (25)$$

$$\phi^* = \frac{Z_c^*\left\{\left[(1-\gamma^*)H(\xi^*) - \xi^* H_{a^*}\right]L^* - \Phi_a^* N\right\}}{\beta u_1^* L^*(1+\Phi_c^* N)\left\{\left[\Phi_L^* - \Phi_c^* \frac{Z_L^*}{Z_c^*} + \Phi_\gamma^* \left(\frac{1-a^*-(1-a^*)\gamma^*}{(1-a^*)L^*}\right)\right] N - M^*\right\}}, \quad (26)$$

$$p^* = \left[1 - H(\xi^*) + (a^* + (1-a^*)\gamma^*)(1-\gamma^*)H_{\gamma^*} - \theta^*\rho\right]^{-1}. \quad (27)$$

where c^*, L^*, a^* and γ^* are the optimal steady state allocations and

$$M^* = \frac{Z_L^*}{Z_c^*} + \frac{u_2^*}{u_1^*}\left(\frac{L^* - a^* d_a^*}{L^*}\right)$$

Proof. See Appendix $A.2$. ■

Notice that if the unit cost of audit N is zero, from expressions (24) and (25), the government will set the retail sales tax and the audit probability such that the following condition is satisfied: $\tau^* = \theta^*\rho$. This means that the expected penalty for tax evasion is equal to the retail sales tax. Moreover, the optimal tax rebate rate is negatively related to the auditing probability as these two auditing policies are substitutes tools that the government has to fight tax evasion. This result is also verified for any audit (positive) cost N when the firm's concealment cost is zero, i.e., $H(\xi^*) = 0$. Intuitively, the planner will set the optimal retail sales tax and the optimal expected penalty to the same level such that firm's are indifferent between collect taxes voluntarily or evade them and being caught by the government.

An interesting result emerges regarding the individual auditing technology described by expression (6). If we assume $\delta = 0$ this means that consumers spend a fixed amount of their time on auditing, i.e., $d(a) = \bar{d}$ and $d_a^* = 0$. Given that this is a sunk cost for consumers, the government does not have to reward the agent's behavior and the optimal rebate rate is zero ($\phi^* = 0$). A zero tax rebate rate is also optimal if, on the other hand, consumers do not spend time requesting receipts (for instance, due to technological progress on the tax remittance process). This can be illustrated using our auditing technology function, equation (6), and assuming $\bar{d} = 0$, which also implies $d_a^* = 0$. In this case, $Z_a^* = 0$ and in order to satisfy (12), following the proof of Proposition 2, it is optimal to set $\phi^* = 0$.

Tax and enforcement policies have distinct effects on the consumers' allocations, i.e., consumption and leisure, via the economy's price system. When choosing these policies optimally the planner has to take into account the effect of these auditing policies on the good's price and consequently on the economy's welfare. Both policy instruments θ and ϕ affect the price of the consumption good, and hence the household consumption, but the tax rebate rate has an additional effect on the consumer's time allocation.

In the Ramsey primal approach to optimal taxation used so far we used the consumer and firm first-order conditions to eliminate prices and tax rates. To be able to explore the trade-off faced by the government when choosing these policies and their effect on good's price, we employ an alternative approach, namely the Ramsey dual approach. In the dual formulation the government uses the tax rates or prices as controls and preferences and technology are represented by expenditure and cost functions, whereas in the primal model they are represented by utility and production functions and quantities are the controls variables. We study the policy trade-off using the dual approach in the next section.

4 Policy Trade-off

We now formulate a Ramsey problem in which the government chooses tax rates (dual) rather than allocations (primal). To simplify the analysis and to make our results comparable to the existing literature, in particular, Cremer and Gahvari (1993), we impose three assumptions. First, we normalize the wage rate $w = 1$. This implies that $p = 1/[1 - T - \xi_t H(\xi_t)]$ from the firm's first-order condition equation (3). The good's price is affected by tax enforcement policies τ, θ, ρ and allocations a, γ. Since the buyer's decision to request receipts is affected but the tax rebate rate and the firm's decision to voluntarily pay taxes is affected by the retail tax itself and auditing policy instruments we write the good's price as follows $p = F(\tau, \phi, \theta, \rho; \beta)$.

Second, we assume that $\beta_t = \mu_t/\mu_{t+1}$ for every t. This allows us to write the indirect utility function as $v(p, \phi, \tau; \beta)$ and its correspondent Roy's identity as $\partial v/\partial p = -\mu L/p$, where μ is the representative household's marginal utility of income. And third, we consider the consumer's problem in a steady state. This implies that the consumer's budget constraint, equation (7), can be rewritten as $pc = (w + \phi p \tau a)L$.

Using the first-order conditions $(10) - (12)$ of the consumer's problem, we derive the ordinary demand functions $c(w/p, \phi, \tau; \beta)$, $L(w/p, \phi, \tau; \beta)$ and $a(w/p, \phi, \tau; \beta)$ and the effect of policies on the agent's indirect utility as follows (details presented in the Appendix $A.4$): $\frac{\partial v}{\partial \tau} = \frac{\partial v}{\partial p}\frac{\partial p}{\partial \tau} - \mu p \phi a L$, $\frac{\partial v}{\partial \phi} = \frac{\partial v}{\partial p}\frac{\partial p}{\partial \phi} - \mu p \tau a L$ and $\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial p}\frac{\partial p}{\partial \theta}$.

Following the Ramsey dual approach, the planner chooses a retail sales tax τ, an auditing probability θ and a tax rebate rate ϕ in order to maximize the consumer's indirect utility $v(p, \phi, \tau; \beta)$ subject to its budget constraints, equation (9).[10]

Hence, the government's problem in the Lagrangian form is

$$\pounds = v(p, \phi, \tau; \beta) + \lambda \left[-pG - p\theta N + a\tau pL(1-\phi) + (1-a)pL(\gamma\tau + (1-\gamma)\theta\rho) \right], \tag{28}$$

where λ is the Lagrangian multiplier on the government budget constraint. The structure of this problem is now analogous to the standard optimal commodity tax problem (Sandmo, 1974, Atkinson and Stiglitz, 1980) with one relevant difference: the good's price is dependent on $\phi\tau$ and θ and, therefore, p is also an argument of the indirect utility of the consumer.

Different from traditional optimal taxation literature, it is important to point out that, in our problem, the optimal tax, auditing probability and tax rebate rate depend on the response of labor supply to theses policies rather than the changes in consumption. This occurs due to the relationship between labor supply decision and the size of production. In other words, although level of production is independent of the firm decision regarding evasion (γ) it is related, via labor supply L, to the individual auditing time allocation a.

[10] Government expenditures are assumed to be exogenously given and, for the purpose of what follows, can be considered as a lump-sum transfer to consumer. This assumption would affect the consumer budget constraint but not consumer allocation choices. This makes the problem simpler and the policy trade-off clearer since in this case supply and demand of goods are equal.

The first-order conditions of problem (28) with respect to τ, θ and ϕ are presented in the Appendix $A.3$. They essentially guarantee that the optimum choice of these policies must ensure that the net effect of policies on individual's welfare is equal to their net effect on tax collection. After some manipulation, we obtain the expressions below.

Proposition 3 *Optimal auditing and tax-rebate policies must satisfy:*

$$\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \theta} = \lambda\left[L\left(\frac{\partial T^e}{\partial p}\frac{\partial p}{\partial \theta}+\frac{\partial T^e}{\partial \theta}\right)+T^e\left(\frac{\partial L}{\partial p}\frac{\partial p}{\partial \theta}-N\right)\right], \tag{29}$$

$$\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \phi} - \mu p\tau a L = \lambda\left\{L\left(\frac{\partial T^e}{\partial p}\frac{\partial p}{\partial \phi}+\frac{\partial T^e}{\partial \phi}\right)+T^e\left[\left(\frac{\partial L}{\partial p}\right)\left(\frac{\partial p}{\partial \phi}+\frac{\partial L}{\partial \phi}\right)\right]\right\}, \tag{30}$$

$$\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \tau} - \mu p\phi a L = \lambda\left\{L\left(\frac{\partial T^e}{\partial p}\frac{\partial p}{\partial \tau}+\frac{\partial T^e}{\partial \tau}\right)+T^e\left[\frac{\partial L}{\partial \tau}+\left(\frac{\partial L}{\partial p}\frac{\partial p}{\partial \tau}\right)\right]\right\}, \tag{31}$$

where $T^e = a\tau pL(1-\phi) - (1-a)pL(\gamma\tau + (1-\gamma)\theta\rho) - p\theta N - pG.$

We can further characterize a fundamental relationship between optimal policies. Dividing equation (29) by (30) and equation (31) by (30) we obtain the trade-offs between the optimal audit probability θ and the optimal tax rebate rate ϕ and the trade-offs between the optimal retail sales tax τ and the optimal tax rebate ϕ. The following proposition summarizes these results.

Proposition 4 *Trade-offs between (i) the auditing probability and the tax rebate rate, (ii) the retail sales tax and the tax rebate rate and (iii) the auditing probability and the retail sales tax are represented by the following relationships:*

$$\frac{\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \theta}}{\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \phi} - \mu p a\tau L} = \frac{L\left(\frac{\partial T^e}{\partial \theta}+\frac{\partial T^e}{\partial p}\frac{\partial p}{\partial \theta}\right)+T^e\left(\frac{\partial L}{\partial p}\frac{\partial p}{\partial \theta}\right)-N}{L\left(\frac{\partial T^e}{\partial \phi}+\frac{\partial T^e}{\partial p}\frac{\partial p}{\partial \phi}\right)+T^e\left(\frac{\partial L}{\partial p}\frac{\partial p}{\partial \phi}+\frac{\partial L}{\partial \phi}\right)} \tag{32}$$

$$\frac{\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \tau} - \mu p\phi a L}{\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \phi} - \mu p a\tau L} = \frac{L\left(\frac{\partial T^e}{\partial \tau}+\frac{\partial T^e}{\partial p}\frac{\partial p}{\partial \tau}\right)+T^e\left(\frac{\partial L}{\partial \tau}+\frac{\partial L}{\partial p}\frac{\partial p}{\partial \tau}\right)}{L\left(\frac{\partial T^e}{\partial \phi}+\frac{\partial T^e}{\partial p}\frac{\partial p}{\partial \phi}\right)+T^e\left(\frac{\partial L}{\partial p}\frac{\partial p}{\partial \phi}+\frac{\partial L}{\partial \phi}\right)} \tag{33}$$

$$\frac{\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \theta}}{\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \tau} - \mu p\phi a L} = \frac{L\left(\frac{\partial T^e}{\partial \theta}+\frac{\partial T^e}{\partial p}\frac{\partial p}{\partial \theta}\right)+T^e\left(\frac{\partial L}{\partial p}\frac{\partial p}{\partial \theta}\right)-N}{L\left(\frac{\partial T^e}{\partial \tau}+\frac{\partial T^e}{\partial p}\frac{\partial p}{\partial \tau}\right)+T^e\left(\frac{\partial L}{\partial \tau}+\frac{\partial L}{\partial p}\frac{\partial p}{\partial \tau}\right)} \tag{34}$$

Consider, for instance, the trade-off the planner faces when deciding to change either the audit probability (government auditing) or the tax rebate rate (individual auditing) to curb tax evasion.

The left-hand side of equation (32) denotes the rate of substitution between the two policies (θ and ϕ) such that consumers' welfare remain constant. The standard (government) auditing policy only affects welfare through prices and, hence, it is the only component in the numerator of equation (32). On the other hand, the tax rebate distorts consumers' welfare via two channels, namely prices and income, which are represented in the denominator of equation (32). In the same fashion, the right-hand side of equation (32) represents the rate of substitution between θ and ϕ keeping net taxes constant.

Alternatively, this trade-off can be viewed as follows. Any vector of consumer prices and net income can be achieved by various combinations of vectors of taxes, rebates and auditing probabilities. At the optimum, however, τ, θ and ϕ must be such that the government's net tax revenues are maximal, conditional on prices and income. Otherwise, it would be possible to increase tax revenue while having utility unchanged which contradicts the optimality condition. The equality between the rates of substitution stated in (32) is obviously a necessary condition for the net revenue to be maximized.[11]

In summary, equation (32) states that at the optimum levels the rate of substitution between θ and ϕ is such that consumers' welfare and net tax revenue are constant must be the same. A similar reasoning applies to equations (33) and (34) regarding the trade-offs between the retail sales tax and the tax rebate rate and the auditing probability and the retail sales tax, respectively.

5 Numerical Exercise

5.1 Functional Forms and Parameterization

In this section we present the results of a numerical analysis of our model to obtain further insights into the relationship between alternative auditing instruments and retail sales tax and a quantitative sense of our theoretical results. We assume that the economy is in a steady state. For the purpose

[11] Cremer and Gahvari (1993) derive a similar expression when characterizing the relationship between an optimal commodity tax and the audit probability for firms (equation 14, pp. 269)

of this exercise, we use the following functional forms:

$$
\begin{aligned}
\textit{Preferences:} \quad & u(c_t, h_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \kappa \frac{(1-L_t-d(a_t))^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}, \\
\textit{Technology:} \quad & F(L_t) = L_t, \\
\textit{Auditing Technology:} \quad & d(a_t) = \bar{d} a_t^{\delta}, \\
\textit{Concealment Cost:} \quad & H(\xi_t) = \xi_t^{\varkappa},
\end{aligned}
$$

where $\xi_t = (1 - a_t)(1 - \gamma_t)$, κ is the weight on leisure, $\bar{d}$ is the upper bound on the leisure cost of auditing activities and δ and $\varkappa$ are the curvature parameter of the auditing technology and concealment cost functions, respectively. The parameters for our numerical exercise are set to values standard in this literature for the United States as follows. We set the discount factor $\beta = 0.96$ which implies a rate of time preference of 4 percent on an annual basis. Government purchases, G_t, are such that the steady-state ratio of government purchases to GDP generated by the model with initial policy is 20 percent of GDP.

The baseline value for σ is 1, which implies a relative risk aversion of 1. The baseline value for η is 0.4 (Blundell and MaCurdy, 1999). There are no available estimates in the literature regarding the parameters governing the individual auditing and concealment technologies. The weight on leisure, κ, is set to match initial steady-state non-leisure activities of $L + d(a) = 0.28$. In other words, we assume that, in equilibrium, people spend about one-quarter of their available time working and three percent of their time auditing. We investigate numerically the optimal tax policy and the optimal audit policy when the cost of auditing N varies as well as under different assumptions regarding the auditing technology and concealment cost functions. We choose the following baseline values: $N = 0.00$, $\bar{d} = 0.03$, $\delta = 0.30$ and $\varkappa = 5.00$ and later conduct sensitivity analysis.

This numerical exercise is divided in three steps. First we consider an economy without *individual auditing*, where the only instrument available to the government to enforce the tax code is the standard (*government*) auditing probability. Then we allow the government to provide a tax rebate to consumer for their role of tax enforcers to the extent that they request purchase receipts. In this second step, we consider the implications of both (*government* and *individual*) auditing policies together. Finally, we study the case when the government transfer completely to consumer the role of auditing the economy, i.e., the detection probability by the government is constraint to be zero. In all these cases, we characterize numerically the optimal retail sales tax, the optimal auditing probability, the optimal penalty rate and the optimal tax rebate and report compliance measures (a and γ) and allocations consumption, labor supply and auditing time (c, L and d). For reference, the Ramsey problem for the *government auditing* only and the *individual auditing* only schemes are presented in the Appendices $A.4$ and $A.5$, respectively.

5.2 Optimal Tax Policy and Individual Auditing

The results for our benchmark parameterization for an economy without individual auditing are presented in Table 1. When there are no costs to audit the economy, i.e., $N = 0$, the planner sets the retail sales tax τ^* equal to the expected penalty $\theta^*\rho^*$, according to Proposition (**??**) and guarantees (almost) full compliance of the firms with the tax code. As expected, when the cost of auditing increases the detection probability decreases and firms react to it by reducing the amount of sales voluntarily reported. The government increases the evasion penalty to its maximum value, i.e., $\rho^* = 1$ or 100 percent, at no cost. The fall in compliance shrinks the tax base and in order to finance an exogenously given government spending the retail sales tax must increase. Welfare losses are greater the higher the unitary cost of auditing as well as the higher taxes are, which increases deadweight losses.

Table 1 - Optimal Policies and Allocations

Government Auditing Only

		$N = 0.00$	$N = 0.001$	$N = 0.005$
Policies:	Retail Sales Tax τ^*	0.3116	0.3216	0.4013
	Evasion Penalty ρ^*	0.3200	1.0000	1.0000
	Detection Probability θ^*	0.9736	0.3213	0.3047
Compliance:	Sales Reported γ^*	0.9923	0.8652	0.5622
Allocations:	Consumption c^*	0.2210	0.2178	0.2035
	Leisure h^*	0.6790	0.6790	0.6790
	Welfare U^*	-0.9053	-0.9199	-0.9878

Next we consider the case when the government has two auditing policies at its disposal, namely a direct (via θ) and an indirect (via ϕ) auditing procedure. The benchmark values for the individual auditing technology and concealment cost are as follows: $\bar{d} = 0.03$, $\delta = 0.30$ and $\varkappa = 5.00$. In this case the government has to balance the impact of both θ and ϕ on good's price and consequently on consumers' welfare. Notice that when there are no cost for the government to engage in direct auditing, i.e., $N = 0$, the results are not necessarily the same as if the government had its direct auditing policy only. It is optimal for the government to audit the economy itself and have consumers do so too. By offering a tax rebate the government encourage individuals to ask for receipts which alleviates the need to audit the economy (via θ) at a higher rate. If the planner were to set the rebate rate equals to zero and increase the detection probability to increase compliance, it would increase good's price and that would make consumers worse off. This policy also allows for a decrease in sales tax.

Facing a higher cost of auditing the economy directly, for instance $N = 0.001$, the government balances the cost of auditing the economy itself and the cost of having consumers perform this task. Interestingly, as the unitary cost of auditing increases detection probability increases and tax rebate falls. This result suggests that, in our setup, when both auditing policies are available it is optimal to distort the labor-leisure decision the least and bear the cost of waste resources with direct auditing. As N increases, total compliance falls (staying roughly at 85 percent of sales) and retail sales tax increases accordingly. More importantly, this policy allows to keep retail sales taxes lower than in the *government auditing* only case (Table 1).

Table 2 - Optimal Policies and Allocations

Government and Individual Auditing

		$N = 0.00$	$N = 0.001$	$N = 0.003$	$N = 0.005$
Policies:	Retail Sales Tax τ^*	0.2801	0.2867	0.3007	0.3160
	Evasion Penalty ρ^*	0.9800	1.0000	1.0000	1.0000
	Detection Probability θ^*	0.2859	0.2864	0.2999	0.3146
	Tax Rebate ϕ^*	0.0775	0.0752	0.0707	0.0661
Compliance:	Purchases with receipt a^*	0.6444	0.6422	0.6374	0.6321
	Sales Reported γ^*	0.6005	0.6003	0.5997	0.5990
Allocations:	Consumption c^*	0.3306	0.3279	0.3221	0.3158
	Leisure h^*	0.5431	0.5430	0.5427	0.5424
	Auditing time $d(a^*)$	0.0263	0.0263	0.0262	0.0261
	Welfare U^*	-0.7628	-0.7713	-0.7897	-0.8103

Tables 3 and 4 show how the optimal policies and compliance measures are affected by a variety of parameter changes. We restrict our attention to the parameters pertinent to the auditing technology and the concealment cost function and fix the unitary cost of auditing at $N = 0.005$. For other values of N, results are similar, and are available upon request. Table 3 shows that the retail sales tax, evasion penalty, detection probability and sales voluntarily reported by firms are quite insensitive to changes in the auditing technology concavity parameter δ. On the other hand, as expected, the fraction of purchases with receipt a^* is negatively related to δ, while the tax rebate decreases as δ decreases. The intuition for this is as follows. For instance, when δ decreases from 0.30 to 0.20, it becomes less costly (at the margin) for consumers to request receipts so a^* increases. Because consumers can request more receipts at a lower (leisure) cost, the government can offer a lower tax rebate in return and hence ϕ^* is smaller.

It is interesting to consider two extreme cases, namely when there is a sunk cost of auditing and when it is costless to ask for receipts. If the cost of auditing is positive and fixed, i.e. when $\delta = 0.00$, agents will necessarily spend three percent of their time auditing ($\bar{d} = 0.03$), agents ask for receipts on all purchases and the tax rebate is zero reflecting the fact the government doesn't need to reward consumers for a task they will perform any way. Full compliance through purchases with receipt can also be obtained if no time is spent on auditing ($\bar{d} = 0.00$). Similarly no tax rebate is offered but the retails sales tax is lower. This is possible because the consumer can, keeping its optimal level of leisure constant, work more. It, hence, increases production, sales and consumption and welfare is higher in this case.

Table 3 - Optimal Policies and Compliance

Government and Individual Auditing ($N = 0.005^\star$, $\varkappa = 5.00^\star$)

	$\bar{d} = 0.03^\star$	$\bar{d} = 0.03^\star$		$\delta = 0.30^\star$	
	$\delta = 0.30^\star$	$\delta = 0.00$	$\delta = 0.20$	$\bar{d} = 0.00$	$\bar{d} = 0.02$
Retail Sales Tax τ^*	0.3160	0.2659	0.3160	0.2659	0.3021
Evasion Penalty ρ^*	1.0000	*n.r.*	1.0000	*n.r.*	1.0000
Detection Probability θ^*	0.3146	*n.r.*	0.3153	*n.r.*	0.3013
Tax Rebate ϕ^*	0.0661	0.0000	0.0428	0.0000	0.0425
Purchases w/ receipt a^*	0.6321	1.0000	0.6933	1.0000	0.6843
Sales Reported γ^*	0.5990	*n.r.*	0.5991	*n.r.*	0.5991

* : baseline values.

Table 4 shows that optimal policies and compliance are responsive to changes in the concealment technology parameter $\varkappa$. For a convex concealment cost function, as $\varkappa$ decreases from 5.00 to 1.50 it becomes more costly to hide and not report sales. Firms react by increasing the amount of sales they voluntarily report (from 59% to 62%) and the detection probability falls accordingly. In this case, total compliance $a^* + (1 - a^*)\gamma^*$ increases (from 0.8524 to 0.8697). Equilibrium good's price is lower when $\varkappa = 1.50$ and consumption and purchase with receipts increase. Even though the tax rebate rate ϕ^* falls the amount a consumer receives from the government ($\phi^* p^* a^* \tau^* L^*$) stays the same as the retail sales tax increases. Results presented in Tables 3 and 4 illustrate the fact that optimal policies, compliance and allocations are quite sensitive to different combinations of individual auditing technology and concealment costs

Table 4 - Optimal Policies and Compliance

Government and Individual Auditing ($N = 0.005^{\star}$)

	$\bar{d} = 0.03^{\star}$, $\delta = 0.30^{\star}$		
	$\varkappa = 1.50$	$\varkappa = 5.00^{\star}$	$\varkappa = 8.50$
Retail Sales Tax τ^*	0.3375	0.3160	0.3150
Evasion Penalty ρ^*	1.0000	1.0000	1.0000
Detection Probability θ^*	0.2011	0.3146	0.3150
Tax Rebate ϕ^*	0.0608	0.0661	0.0662
Purchases with receipt a^*	0.6562	0.6321	0.6332
Sales Reported γ^*	0.6211	0.5990	0.6000

* : baseline values.

Finally, we consider the possibility of eliminating government auditing, i.e., set $\theta = 0$, and allow individual auditing to be the only mechanism through which firms can be audited. We investigate how the optimal retail sales tax, the optimal tax rebate rate and the optimal compliance are affected by different auditing technology parameters. First notice that when $\theta = 0$, meaning that firms know that they will not be audit by the government, firms' compliance, on the amount of sales for which consumers do not ask receipts for, is zero ($\gamma^* = 0$). Moreover, firms would not bear costs to hide sales from the government if the chance of being caught is zero. In other words, if auditing probability on the part of government is constrained to be zero, firms have now incentive to pay taxes voluntarily and concealment costs are zero. In this case, compliance is solely determined by consumers' behavior.

Tables 5 presents the results. As the individual auditing technology parameter δ decreases from 0.30 to 0.20, total compliance a^* increases from 0.6439 to 0.7023, allowing the government to reduce the retail sales tax τ^* and the optimal tax rebate rate ϕ^*. If either $\delta = 0.00$ or $\bar{d} = 0.00$ full compliance is reached and no rebate is offered. Retail sales taxes are lower when $\bar{d} = 0.00$ because time not used for auditing is shifted to labor, which increases production, sales and, consequently, the tax base. It then allows the government to set a lower sales tax to finance a fixed exogenously given government spending.

Table 5 - Optimal Policies and Compliance

Individual Auditing Only

	$\bar{d} = 0.03^{\star}$	$\bar{d} = 0.03^{\star}$		$\delta = 0.30^{\star}$	
	$\delta = 0.30^{\star}$	$\delta = 0.00$	$\delta = 0.20$	$\bar{d} = 0.00$	$\bar{d} = 0.02$
Retail Sales Tax τ^*	0.4355	0.2301	0.3992	0.2046	0.3882
Tax Rebate ϕ^*	0.1718	0.0000	0.1688	0.0000	0.1718
Purchases w/ receipt a^*	0.6439	1.0000	0.7023	1.0000	0.6939
Consumption c^*	0.3306	0.3346	0.3291	0.3886	0.3483
Leisure h^*	0.5431	0.5354	0.5429	0.5114	0.5338
Welfare U^*	-0.7628	-0.7676	-0.7677	-0.6665	-0.7309

* : baseline values.

A comparison of (baseline) welfare levels across three tax enforcement policy regimes, namely (i) *government auditing* only (Table 1), (ii) *government and individual auditing* (Table 2) and (iii) *individual auditing* only (Table 5), suggests that welfare is higher when the government reward consumer for their role as tax enforcement. Consumers are better off in this case mainly due to the fact that labor income and tax rebate allow them to consume more, without giving up too much leisure.

It is also interesting to compare optimum compliance in a no-corner solution across these three tax enforcement regimes. When the government audits the economy directly (Table 1), compliance is only due to firms voluntarily payment of retail sales taxes on 56 percent of the their sales. If the government introduces the tax rebate policy to encourage consumers to act as tax enforcers (Table 2), optimum compliance increases to 85 percent, which breaks down as follows. Consumers request receipts on 63 percent of their purchases and firms pay taxes, voluntarily, on 60 percent of sales without receipt, i.e., $a^* + (1 - a^*)\gamma^* = 0.6321 + (1 - 0.6321).0.5990 = 0.8524$. If the government opts for the *individual auditing* policy only (Table 5), optimum compliance falls to 64 percent but it is still higher than under the government auditing regime only. The comparison of these three economies suggest that compliance is higher when both - *government and individual* - auditing polices are in place but welfare is higher if *individual auditing* is the only tax enforcement policy.

6 Conclusion

The paper investigates optimal tax and audit policy where both consumers and tax administrators can engage in auditing. In particular, our model allows consumers to play a role in auditing business to enforce compliance with the tax code and tax remittances to the government. To reward buyers for their auditing effort, the government returns a portion of the tax collected to them. We characterize the optimal retail sales tax, the auditing probability and the optimal rebate tax rate and show that the individual auditing and the firm's concealment technologies and the unitary audit cost play key roles in the determination of optimal policies. We also analyze the policy trade-off faced by the government. Numerically, our results suggest that compliance is higher when both auditing policies are in place but welfare is higher if individual auditing is the only tax enforcement policy. A natural extension of our model is to allow for heterogeneity across firms and consumers, good-specific sales tax rebates and additional tax instruments, for instance, income and consumption taxes. We pursue this in future research.

Appendix

A.1. Proposition 1

Proof. To show that any allocation that satisfy equations (16) and (18) can be decentralized as a competitive equilibrium we use these allocations together with the household's and firm's first-order conditions to construct the corresponding prices and taxes. The audit probability θ is determined using the firm's and household's equilibrium conditions (4) and (15). The retail sales tax τ is obtained by substituting equation (17) into the firm's first-order condition with respect to γ, equation (2). Once the retail sales tax is determined we recover the tax rebate rate ϕ using the consumer's first order condition with respect to a, equation (12). And, finally, we determine the real wage w/p using equation (4) from the firm's problem.

To show that any competitive equilibrium allocations satisfy equations (16) and (18), we proceed as follows: (a) Premultiply the household's budget constraint in period t with the associated Lagrangian multiplier $\beta^t \mu_t$ and sum over all periods $t \geq 0$. We proceed by solving for taxes and prices as a function of allocations using the household's and firm's first order conditions. This results in the implementability constraint, equation (16); (b) The resource constraint, equation (18), is implied by the household's and government's budget constraints, together with (17) derived from consumer's and firm's equilibrium conditions (4) and (15). Thus feasibility is satisfied. ■

A.2. Proposition 2

Proof. Combining the Ramsey's and firm's first-order conditions with respect to γ, equations (23) and (2), we obtain

$$\tau^* = \theta^* \rho + \frac{\Phi_\gamma^* N}{(1-a^*)L^*} \tag{35}$$

Combine equations (20) and (22) and equations (35) and (3) to obtain

$$\frac{w^*}{p^*} = \Phi_L^* N - [1 + \Phi_c^* N]\frac{Z_L^*}{Z_c^*} - \left(\frac{a^* + (1-a^*)\gamma^*}{(1-a^*)L^*}\right)\Phi_\gamma^* N - \theta^* \rho. \tag{36}$$

The optimal (government) auditing probability θ^*, equation (25), is implicitly determined by (36) and (15). Plugging (25) into (35) we obtain (24). Finally, combine equations (20) and (21) and consumer's first-order condition with respect to a, equation (12), and (35) to obtain (26). Normalizing the nominal wage $w = 1$, the optimal good's price, equation (27), is determined by equation (4). ■

A.3. (Dual) Ramsey Problem - First-Order Conditions

The first-order conditions for the Ramsey problem (28) with respect to $\{\tau, \theta, \phi\}$ are, respectively:

$$\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \tau} - \mu p \phi a L = \lambda \left\{ \begin{array}{l} L \left[\begin{array}{l} a(1-\phi) + \left(\frac{\partial a}{\partial p}\frac{\partial p}{\partial \tau} + \frac{\partial a}{\partial \tau}\right)(\tau(1-\phi) - \gamma\tau + (1-\gamma)\theta\rho) \\ +(1-a)(\gamma + (\frac{\partial \gamma}{\partial \tau} + \frac{\partial \gamma}{\partial a}(\frac{\partial a}{\partial p}\frac{\partial p}{\partial \tau} + \frac{\partial a}{\partial \tau}))(\tau - \theta\rho)) \end{array} \right] \\ + \left(\frac{\partial L}{\partial p}\frac{\partial p}{\partial \tau} + \frac{\partial L}{\partial \tau}\right)[a\tau(1-\phi) + (1-a)(\gamma\tau + (1-\gamma)\theta\rho)] \end{array} \right\}$$

$$\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \theta} = \lambda \left\{ \begin{array}{l} L \left[\begin{array}{l} \left(\frac{\partial a}{\partial p}\frac{\partial p}{\partial \theta}\right)(\tau(1-\phi) - (\gamma\tau + (1-\gamma)\theta\rho)) \\ +(1-a)\left[\left(\frac{\partial \gamma}{\partial \theta} + \frac{\partial \gamma}{\partial a}\frac{\partial a}{\partial p}\frac{\partial p}{\partial \theta}\right)(\tau - \theta\rho) + \rho(1-\gamma)\right] \end{array} \right] \\ + \left(\frac{\partial L}{\partial p}\frac{\partial p}{\partial \theta}\right)[a\tau(1-\phi) + (1-a)(\gamma\tau + (1-\gamma)\theta\rho)] - N \end{array} \right\}$$

$$\left(\frac{\mu}{p}\right) L\frac{\partial p}{\partial \phi} - \mu p \tau a L = \lambda \left\{ \begin{array}{l} L \left[\begin{array}{l} \left(\frac{\partial a}{\partial p}\frac{\partial p}{\partial \phi} + \frac{\partial a}{\partial \phi}\right)(\tau(1-\phi) - (\gamma\tau + (1-\gamma)\theta\rho)) \\ +(1-a)\left(\left[\frac{\partial \gamma}{\partial a}\left(\frac{\partial a}{\partial p}\frac{\partial p}{\partial \phi} + \frac{\partial a}{\partial \phi}\right)\right](\tau - \theta\rho) - a\tau\right) \end{array} \right] \\ + \left(\frac{\partial L}{\partial p}\frac{\partial p}{\partial \phi} + \frac{\partial L}{\partial \phi}\right)[a\tau(1-\phi) + (1-a)(\gamma\tau + (1-\gamma)\theta\rho)] \end{array} \right\}$$

A.4. Ramsey Problem - *Government Auditing* Only

The planner's maximization problem is written as follows:

$$\max_{\{c_t, L_t, \gamma_t\}} \sum_{t=0}^{\infty} \beta^t Z_1(c_t, L_t; \lambda_1)$$

subject to

$$c_t + G_t + X_1(c_t, L_t, \gamma_t; \rho_t)N_t + \xi_t H(\xi_t)L_t = L_t.$$

where $X(c_t, L_t, \gamma_t; \rho_t) = \frac{1}{\rho_t}\left[1 - H(\xi_t) + \gamma_t(1-\gamma_t)H_\gamma(\xi_t) - \frac{u_2(t)}{u_1(t)}\right]$, $Z_1(c_t, L_t; \lambda_1) \equiv u(c_t, 1 - L_t) + \lambda_1 [u_1(t)c_t - u_2(t)L_t]$ and λ_1 is the Lagrange multiplier on the implementability constraint.

Suppose that the evasion penalty ρ is fixed. If the solution to the Ramsey problem converges to a steady state, and the government has access to a retail sales tax τ and an auditing probability θ these instruments are as follows:

$$\begin{aligned}\tau^* &= \left[X_{1L}^* - X_{1c}^* \frac{Z_{1L}^*}{Z_{1c}^*} + X_{1\gamma}^* \left(\frac{1-\gamma^*}{L^*}\right)\right] N - M_1^*, \\ \theta^* &= \frac{1}{\rho}\left\{\left[X_{1L}^* - X_{1c}^* \frac{Z_{1L}^*}{Z_{1c}^*} - X_{1\gamma}^* \left(\frac{\gamma^*}{L^*}\right)\right] N - M_1^*\right\},\end{aligned}$$

where c^*, L^* and γ^* are the optimal steady state allocations, λ_2 is the Lagrange multiplier on the resource constraint, and Z_{1c}^*, Z_{1L}^*, X_{1c}^*, X_{1L}^*, $X_{1\gamma}^*$ are defined as follows:

$$\begin{aligned}M_1^* &= \frac{Z_{1L}^*}{Z_{1c}^*} + \frac{u_2^*}{u_1^*}, \\ Z_{1c}^* &= u_1^* + \lambda_1^* \left[u_{11}^* c^* + u_1^*\right], \\ Z_{1L}^* &= -u_2^* - \lambda_1^* \left[u_{22}^* L^* + u_2^*\right], \\ X_{1c}^* &= \frac{1}{\rho}\left(\frac{u_2^*}{(u_1^*)^2}\right), \quad X_{1L}^* = -\frac{1}{\rho}\left(\frac{u_{22}^*}{u_1^*}\right), \\ X_{1\gamma}^* &= \frac{1}{\rho}\left\{-2\gamma^* H_\gamma^* + (1-\gamma^*)\gamma^* H_{\gamma\gamma}^*\right\}\end{aligned}$$

A.5. Ramsey Problem - *Individual Auditing* Only

The planner's maximization problem is written as follows:

$$\max_{\{c_t, L_t, \gamma_t\}} \sum_{t=0}^{\infty} \beta^t Z_2(c_t, L_t; \lambda_1)$$

subject to

$$c_t + G_t = L_t.$$

where $Z_2(c_t, L_t; \lambda_1) \equiv u(c_t, 1-L_t) + \lambda_1 \left[u_1(t)c_t - u_2(t)L_t\right]$ and λ_1 is the Lagrange multiplier on the implementability constraint.

Suppose that the evasion penalty ρ is fixed. If the solution to the Ramsey problem converges to a steady state, and the government has access to a retail sales tax τ and a tax rebate ϕ, these instruments are as follows:

$$\begin{aligned}\tau^* &= \left(\frac{1}{a^*}\right)\left[1 - \frac{u_2^*}{u_1^*}\left(\frac{L^* - a^* d_{a^*}}{L^*}\right)\right], \\ \phi^* &= \frac{u_2^* a^* d_{a^*}}{\beta u_1^* L^*}\left[1 - \frac{u_2^*}{u_1^*}\left(\frac{L^* - a^* d_{a^*}}{L^*}\right)\right]^{-1}\end{aligned}$$

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